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Under Graduate Degree Programme NETAJI SUBHAS OPEN UNIVERSITY PHYSICS HPH CC-PH-01 SELF LEARNING MATERIAL

1 PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, discipline specific, generic elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade ''A''. UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for UG programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor

2 Printed in accordance with the regulations of the Distance Education Bureau of the Univer- sity Grants Commission. First Print : December, 2021 Netaji Subhas Open University Under Graduate Degree Programme Choice Based Credit System ((CBCS) Subject : Honours in Physics (HPH) Course : Physics Laboratory - I Code : CC - PH - 01

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5 Netaji Subhas Open University Unit - 1 q Extension of spring and to find out spring constant from vertical oscillations. 7-14 Unit - 2 q To find out modulus of rigidity from torsional oscillation of a wire. 15-26 Unit - 3 q Determination of Moment of Inertia of a Flywheel. 27-35 Unit - 4 q Determination of refractive index of a liquid by Travelling Microscope. 36-42 Unit - 5 q To Find the Fourier co-efficients of different periodic vibrations by graphical method. 43-50 Unit - 6 q To determine the co-efficient of viscosity of water by capillary flow method. 51-63 Unit - 7A q Determination of the acceleration due to gravity (g) using a Bar Pendulum. 64-73 Unit - 7B q Determination of the acceleration due to gravity (g) using a Kater's pendulum. 74-84 Unit - 8 q Determination of thermal conductivity of a bad conductor by Less' and Chorlton's method. 85-99 Unit - 9 q To determine the surface tension of a liquid by Jaeger's method. 100-112 Unit - 10A q Determination of the focal length of a concave lens by combination method. 113-121 Unit - 10B q Determination of the focal length of a convex lens by displacement method. 122-127 UG : Physics (HPH) Physics Laboratory - I Code : CC - PH - 01

6 Unit - 11 q To adjust a spectrometer for parallel rays by Schuster's method and to find out the angle of a prism. 128-141 Unit - 12 q To determine an unknown low resistance using Potentiometer. 142-150 Unit - 13A q Write a program in C to find sum and average of given number set. 151-153 Unit - 13B q Write a programme in C++ to find sum and average of given number set. 154-155 Unit - 14A q Write a programe in C to find out largest number and its position in a given number set. 156-157 Unit - 14B q Write a programe in C++ to find out largest number and its position in a given number set. 158-159 Unit - 15A q Write a program to arrange a number in ascending order for given number set by using C. 160-161 Unit - 15B q Write a program to arrange a number in ascending order for given number set using C++. 162-163 NSOU l CC - PH - 01 7 Unit - 1 q Extension of spring and to find out spring constant from vertical oscillations Contents : Sprint constant of a spring is measured considering its vertical oscillations. Introduction : The idea of an important parameter related to a spring, spring constant (K) came from Hooke's law. The spring constant is actually a measure of the stiffness of a spring. The common methods employed for measuring spring constant of a spring are statical method and dynamical method. (A) Extension of a spring, spring constant : It is known from Hooke's law, when an external force produces small extension or compression of a spring, the force applied or the restoring force developed in the process is directly proportional to the small extension or compression produced. The proportionality constant (K) which is the restoring force (F) per unit change in length (l) of the spring is known as spring constant (K) i.e., k = F/l. Unit is N/m. (B) Dynamical methods : The method for measuring spring constant of a spring is known as dynamical method where the vertical oscillations of a loaded spring is taken into account. (Fig. 1.1).In this method, the load hanging at the end of a spring is displaced through a small distance from its equilibrium position and released. The loaded spring will execute simple harmonic motion. By measuring the time periods for different loads, we can determine the spring constant (K) of the suspended spring. Time period of oscillation is given by 2 + = π M m T K

8 NSOU l CC - PH - 01 where m = sum of masses of spring and pan M = Mass placed on the pan. (c) Apparatus : Fig. 1.1 Here, R = A rigid support. M = Mass placed on hanger. P = A horizontal pointer attached to pan used to measure time period of vertical oscillation. A = A millimeter scale. S = A spring.

NSOU l CC - PH - 01 9 Objective : To determine the spring constant of a spring by considering its vertical oscillations. Theory : Definition : The restoring force developed due to unit change in length of a spring is known as its spring constant (K). SI unit of spring constnat is N/m. Working formula : (i) The spring constant (K) of a spring is given by ( ) ( ) 2 1 2 2 2 1 2 4π - ≡ - M M K T T ... (1.1) where, M 1 , M 2 = Masses applied successively at the lower end of the spring. T 1 , T 2 = Time period for vertical oscillations of the spring corresponding to masses M 1 and M 2 respectively. (ii) We find, 2 2 2 4 4 ? ? ? ? π π = + ? ? ? ? ? ? ? ? ? T M m K K , where m = mass of the spring The M — T 2 graph is a straight line where slope (tan φ) is 2 4π K . ∴ 2 4 tan π = φ K ... (1.2)

10 NSOU l CC - PH - 01 Procedure : 1. After placing some mass M 1 in the pan, it is displaced vertically downwards through a small distance and released. 2. The loaded spring is allowed to execute simple harmonic motion. 3. Total time required for a definite number of oscillations (say 20, 25, 30) are recorded by a stop-watch. Then time period T 1 (time for one oscillaiton) is calculated. 4. Now the load is increased to M 2 and following the step 3, the time period T 2 is calculated. 5. The experiment should be repeated with different loads. 6. A graph is now drawn with M along X-axis and corresponding T 2 along Y-axis. The graph would be a straight line (Fig.1). The slope of this straight line, 2 4 tan π φ = K and its intercept on the negative x-axis gives the mass of the spring (m). A O T 2 E D ΔM φ ΔT 2 M X Y m Fig.1

NSOU l CC - PH - 01 11 Experimental results : Determination of time period for different loads Table - 1 Least count of stop watch = ... sec Calculations : First Method : From equation (1.1), we know ( ) ( ) 2 1 2 2 2 1 2 4 K .... / π - = = - M M N m T T Substituting the different values of M 1 , M 2 and the corresponding values of T 1 and T 2 in the above expression, different values of K are calculated. Their mean value (K) is to be calculated. Second Method : The slope of the M – T 2 straight line graph is calculated from Fig.1 Slope of the graph = tan φ = 2 2 1 .... - - Δ = Δ T S kg M Again, 2 4 tan π φ = k No. of Load in the Number Total time Time period Mean T 2 obs. pan (M) in of taken (T) time period gm oscillations in see in sec (T) in sec in sec 2 1. 2. 3. 4. 5.

12 NSOU l CC - PH - 01 ∴ 2 4 .... / tan π = = φ K N m Result : Spring constant of the spring (K) = ...N/m Discussions : 1. The spring should be vertical and should oscillate vertically. 2. The amplitude of oscillation should be small, otherwise, the motion will not be simple harmonic. 3. To get the correct value of time period of oscillation, the pointer should move freely over the scale. 4. To get more occurate result (i) time period should be calculated taking higher number of oscillations. (ii) precise electronic stop-watch may be used. Maximum Proportional error : From equation (1.1) we get, ( ) ( ) 2 1 2 2 2 1 2 4π - = - M M K T T Therefore, max 2 | δ δ = K t K t since M is supplied and Total time (t) Number of oscillations =T δt = least count of stop watch, t is taken from experimental data. Conclusion : Measured value of the spring constant (K) is accurate within the errors involved in our experimental arrangement.

NSOU l CC - PH - 01 13 Key words : (i) Loaded spring (ii) spring constant (iii) simple harmonic motion. Summary : (i) Spring constant of a spring is defined and measured by considering the vertical oscillation of a loaded spring. (ii) The amplitude of vertical oscillation is made small to make the motion of loaded spring simple harmonic. (iii) For different loads placed at the end of the spring, time periods of oscillation are recorded precisely with a stop-watch having small value of least count. (iv) The value of spring constant (K) is calculated using equation (1.1) and also from M – T 2 graph. (v) Accuracy of measurement is checked. Model Questions and Answers : 1. On what factors the spring constant of a spring depend ? Ans. The spring constant of a spring depends an (i) the stiffness of spring material. (ii) the thickness of the wire from which spring is made. (iii) diameter of turns of coil. (iv) number of turns per unit length. (v) overall length of the spring.

14 NSOU l CC - PH - 01 2. How spring constant of a spring changes with the length of the spring? Ans. Spring constant (K) of a spring is inversely proportional to its length. 3. Why the amplitude of vertical oscillation of spring must be small? Ans. Consult discussion. 4. To draw graph in this experiment we use square of time period (T 2) instead of T. Why? Ans. We know, Time period of oscillation (T) = 2 + π m M K where, K = spring constant M = Mass in the pan m = mass of spring So, 2 2 · 4 = + π K T m M This shows that M – T graph will be a parabola whereas M – T 2 graph is a straight line which can be drawn more easily. 5. Out of two methods – statical and dymical, which one is preferable to find K and why? Ans. The statical method is more convenient than an dynamical method. This is because in statical method, we can take readings more accurately since the system is at rest, whereas, in dynamical method the accurate measurement of time period becomes difficult when motion of spring is rapid.

NSOU l CC - PH - 01 15 Unit - 2 q To find out modulus of rigidity from torsional oscillation of a wire Contents : Modules of rigidity of the material of a wire is measured using torsional oscillation of the wire. Introduction : It is a very simple and accurate method for the measurement of the rigidity modulus of the material of a wire in the laboratory using the principle of a torsional pendulum. Torsional Pendulum and Torsional Oscillation : Description : The torsional pendulum is shown in Fig. 2.1. It consists of a solid cylinder (A) suspended by a long, thin experimental wire (W) of uniform cross-section. One end of the wire a rigidly fixed to a torsion head (T) and the other end is connected to the centre of the cylinder by means of detachable pin (B). The cylinder can oscillate about the suspension wire as axis. In this case, the cylinder serves as the bob of the pendulum. Fig. 2.1 T W B A N P S L 1 L 2

16 NSOU l CC - PH - 01 When the suspension wire is twisted at the lower end using the cylinder, a restoring couple proportional to the angle of twist is produced. When the cylinder is released, it undergoes torsional oscillation due to the restoring couple. During torsional oscillation, a pointer (P) attached to the bottom of the cylinder moves over a circular scale (S) graduated in degrees. Objective : To determine the modulus of rigidity of the material of a wire from its torisional oscillation. Theory : Definition :

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Rigidity modulus is defined as the ratio of the shearing stress to shearing strain

within elastic limit. SI unit of rigidity modules is N/m 2 . Working formula : The modulus of rigidity of the material of a wire is given by 2 4 8 .....(2.1) π = Il n T r where, r = Radius of the wire l = Length of the wire I = Moment of inertia of the solid cylinder attached at the end of the suspension wire. T = Time period of torsional oscillation of the solid cylinder. If the axis of the cylinder coincides with the axis of rotation, NSOU l CC - PH - 01 17 2 1 2 =I MR ... (2.2) where, M = Mass of the cylinder R = Radius of the cylinder [using eq. (2.1) and (2.2)], the rigidity modulus of the material of a wire is ( ) 2 2 4 8 1 2 π = × l n MR T r .... (2.3) Procedure : 1. The diameter (D) of the cylinder (A) [Fig. 2.1] is measured by a slide callipers at least in five different places,. The mean diameter (D) and radius (R) of cylinder is then calculated. 2. The length (l) of the suspension wire between the torsion head (T) and the point where it is connected to the cylinder is measured by a metre scale thrice and its mean (l) is found out. 3. The diameter (d) of the suspension wire is measured by a screw gauge at five different places and their mean (d) is found out. The radius (r) of the wire is then determined. 4. Twisting the suspension wire and then an releasing it, the cylinder is allowed to execute torsional oscillation. Now with the help of a precision stop-watch, time taken by the pendulum for 15, 20, 25, 30, 35 etc. complete oscillations is noted. Time period (T) in each case is found out and their means (T) is them calculated.

18 NSOU l CC - PH - 01 Experimental results : (A) Mass (M) of the cylinder M = ... gm = ... kg (supplied) (B) Determination of the length (l) of the suspension wire. Mean length (l) = ... 3 + + = = cm =..........m (C) Determination of the radius (R) of the cylinder Vernier constant (v.c) of the slide callipers = value of 1smallest division of a main scale ( ) Total number of vernier divisions ( ) m n = ... cm. Instrument error(e) = ± y × v.c

NSOU l CC - PH - 01 19 Table-1 N. B. : (a) and (b) denote mutually perpendicular readings at a particular place ∴ D = ... cm = ...m So, mean radius of the cylinder ( ) ....( ) 2 = = D R m (D) Determination of the radius (r) of the wise Screw pitch (p) = ... mm Total number of circular scale divisions (N) = Reading in cm of No. of obs. Instrumental error (e) in cm Corrected diameter D = D′ – e in cm (a) 1. (b) (a) 2. (b) (a) 3. (b) (a) 4. (b) (a) 5. (b) Main Scale (s) Vernier Scale (v) = v.r × v. c Total R = S + V Mean Diameter (D′)

20 NSOU l CC - PH - 01 ∴ least count (l. c) = p N .... mm Instrumental error (e) = ± y × l.c = ± ..... mm Table-2 ∴ Mean radius of the wire ( ) 2 = = d r .... cm = ...m (a) 1. (b) (a) 2. (b) (a) 3. (b) (a) 4. (b) (a) 5. (b) Reading in mm of No. of obs. Instrumental error (e) in mm Corrected diameter d = d′ – e in mm linear scale (L) Circular Scale C = c . r× l.c Total = L + C Mean diameter (d 1) Mean diameter (d) in cm ....

NSOU l CC - PH - 01 21 (E) Determination of time period (T) Table-3 Calculation : From equation (2.3), we find the rigidity modulus of the material of the wire is ( ) 2 2 4 8 1 · 2 π = l n MR T r Substituting the values of l, M, T, r and R in the above expression, the value of n is obtained. Result : Rigidity modulus of the material of the wire n = ... N/m 2. Discussions : 1. The suspension wire must coincide with the axis of the cylinder. No. of obs. 1. 15 .... 2. 20 .... 3. 25 .... 4. 30 .... 5. 35 .... Number of oscillations (N) Time for oscillations (t) in sec Time period ( ) in sec = t T N Mean time period (T) in sec

22 NSOU l CC - PH - 01 2. As the radius (r) of the wire occurs in its fourth power and time period (T) and radius of cylinder (R) in their second power in the expression for rigidity modulus, their values should be determined very accurately. 3. The motion of the torsional pendulum should be purely rotational in horizontal plane. 4. As torsional couple is taken to be proportional to the angle of twist, the wire should not be twisted beyond elastic limit. 5. Torsional rigidity of the material of the wire can be measured with the help of this experiment. 6. Rigidity modulus (n) of the material of wire can also be determined by statical method. Maximum proportional error : We know from equation (2.1) and (2.2) ( ) 2 2 4 2 4 8 8 1 2 2 π π = = × Il l D n M T r T r where, 2 = D R ∴ Maximum proportional error = max 2 2 | 4. δ δ δ δ δ δ = + + + + n l t r M D n l t r M D (. . . Time period (T) = No.of oscilations t)

NSOU l CC - PH - 01 23 As M is large and given, δM M may be neglected. Here, δl = 0.2 cm (2 div of metre scale), δt = 0.2 × ..... s (2 divisions of stop watch) δr = ... cm (l. c. of screw gauge) δD = ... cm (v. c. of slide callipers) Putting a typical set of observed data for l. r, t and D we can calcuate max | δn n ∴ Maximum percentage error max 100% ....% | δ = × = n n Conclusion : Measured value of the modulus of rigidity (n) of the material of the wire is accurate within the erros involved in the experimental arrangement. Key words : (i) Modulus of rigidity (ii) Torsional pendulum and Torsional oscillation (iii) Moment of inertia (iv) Torsional rigidity (τ) Summary : (i) The measurement of the rigidity modules (n) of the mateiral of a wire by

24 NSOU l CC - PH - 01 dynamical method is discussed. Rigidity modulus is defined. (ii) The motion of the torsional pendulum is made purely rotational. (iii) To get accurate value of rigidity modulus, the radius (r) of the wire, period (T) of oscillation and radius (R) of the cylinder are measured very carefully. (iv) To measure time period (T) very accuretely, a stop watch having small value of least count is used. (v)

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The rigidity modulus (n) of the material of the given wire is calculated by using			

equation. (2.1) and (2.2) with the observed value of the quantities involved. (vi) Accuracy of measurment is checked. Model Questions and answers : 1. Define torisonal rigidity. Ans. Torsional rigidity (τ) is defined as the torque required to produce unit twist of suspension wire. 4 2 π τ = n r l where n, l, r bears usual meaning. 2. Does the value of rigidity modulus of the material of a wire depend on its length and diameter. Ans. No, the value of rigidity modulus depends on the material of wire.

NSOU l CC - PH - 01 25 3. If the temperature increases, how is the rigidity modulus of a wire affected? Ans. The rigidity modulus of a wire decreases with the increase of temperature. 4. On what factors the time period of torsional oscillation depend and how? Ans. The time period of torsional oscillation is 2= π τ I T or, 4 2 2= π π Il T n r 4 2 ? ? π ∴τ = ? ? ? ? n r l where, l, r are the length and radius of the wire respectively, n is the rigidity modulus and I is the moment of inertia of the suspended body. Again 2 1 2 I MR = (for cylinder) Thus T l ∝ , 2 1 T r ∝ , 1 T n ∝ , , T M T R ∝ ∝ So, time period increases with the increase of length of wire, mass, and radius of suspended body and decreases if diameter of wire and rigidity modulus increases. 5. On what factors the amount of twist depend? Ans. We know, torsional rigidity (τ) = Torque per unit twist

26 NSOU l CC - PH - 01 or, 4 n r = 2 π τ l ∴ The value of τ increases for higher value of rigidity modulus (n) or thicker wire (r high) and smaller length (l) of suspension wire. So greater the value of τ, smaller will be the amount of twist (θ). Therefore, θ depends on length, radius and material of the wire. 6. Is it necessary that oscillations should have small amplitude? Ans. No, the angle of oscillation may have any value within the elastic limit of suspension wire. 7. What is the harm if the suspension wire dose not coincide with the axis of the cylinder? Ans. In this case moment of inertia of cylinder 2 1 2 =I MR will not be valid. By the theorem of parallel axis, the measured moment of inertia will be higher than I by amount Md 2 where d is the distance between two axes and M is the mass of cylinder.

NSOU l CC - PH - 01 27 Unit - 3 q Determination of Moment of Inertia of a Flywheel Contents:Moment of mertia of a Fly-wheel about its axis of roation is measured. Introduction :As moment of inertia is intimately connected to the rotation of a body, its determination plays an important role to manufacture a machine producing rotational motion. A flywheel having large value of moment of inertia about its axis when connected to an engine, increases its power and ensures smooth running of the machine. So the measurement of the moment of inertia of a flwheel about its axis is very important. Description of a Flywheel Fig. 3.1

28 NSOU l CC - PH - 01 A flywheel is a large, heavy wheel or disc S with a long cylindrical axis passing through its centre which serves as the axis of rotation. The centre of gravity (C.G) of the flywheel lies on the axis of rotation, so when properly mounted on ball bearings in order to minimise friction, it may remain of rest in any desired position. The horizontal axle of the flywheel is kept at a convenient height from the ground. A tiny peg P is fixed at suitable position on the axle and a loop at the end of a piece of thin string/ thread is introduced loosely in the peg-other end of the string (thread) carries a suitable mass. Almost whole length of the string/thread is wound evenly round the axle. The length of the string/thread should be less than the height of the axle from the ground. Objective : To determine the moment of inertia of a flywheel about its axis of rotation. Theory : Definition : Moment of inertia of a body about an axis is the sum of the products of the mass of each particle in the body and the square of its distance from the axis of rotation. SI unit of moment of inertia is Kg m 2 . Working formula : Moment of inertia of a flywheel about its axis of rotation is given by 2 2 2 1 2 2 = 8 1 mgh t I n n n ? ? π + ? ? ? ? ... (3.1)

NSOU l CC - PH - 01 29 where, m = Mass suspended from the free end of string /thread wound evenly on the axle of the flywheel. h = Vertical height fallen through by the mass before the string leaves the axle of flywheel. t = Time for which the flywheel continues to rotate before coming to rest after the mass gets detached from the axle of flywheel. n 1 = Number of rotation made by the flywheel till the mass detached from the axle of flywheel. n 2 = Number of rotation made by the flywheel in time t. g = Acceleration due to gravity. Procedure : 1. One end of a string or thread is tied to a known mass m and a loop is made at the other end which is fastened to a peg P fixed on the axis of flywheel (See Fig. 3.1, chapter-1). 2. The string is then wrapped completely and evenly round the axle of the flywheel until the mass m is very near to the axle. Number of turns n 1 may be 4, 6, 8 etc. 3. The vertical height h fallen through by the mass before string gets detached from the axle of flywheel is actually the length of the string wrapped since h = 2πrn 1 , where r is the radius of the axle. The length of the string between the loop and the mark at the other end where the string left axle is measured by a scale which gives of the value of h. 4. Now the mass m is allowed to decend slowly under the action of gravity and the number of revolutions of flywheel n 1 during decend is noted.

30 NSOU l CC - PH - 01 5. At the very instant, the string has unwound itself and detached from the axle after n 1 turns, a stop watch is started. From this instant, the number of revolutions n 2 made by the flywheel before it comes to rest is recorded. The stopwatch is also stopped as soon as the flywheel stops. The stopwatch reading provides the value of t. Thus we get the value of n 2 and t. 6. The experiment is repeated for atleast three different masses (value of m may be 100 gm, 150 gm, 200 gm etc.) Experimental results : (A) Determination of n 1 , n 2 and t : Least count of the stop watch = ... sec Table-1 No. of obs. 1. 2. 3. Total load applied (m) kg Number of revolutions of the flywheel before the mass is detached from axle (n 1) Number of revolutions made by flywheel before it comes to rest after the mass gets detached (n 2) Time for n 2 revolutions (t) sec.

NSOU l CC - PH - 01 31 (B) Height of fall of mass (h) = length of string wrapped (see Procedure 3) = ... m Calculations : The moment of inertia of the flywheel about its axis of rotation is (eq. 3.1) 2 2 2 1 2 2 = 8 1 mgh t I n n n ? ? π + ? ? ? ? Substituting the values of g, m, h, t, n 1 and n 2 in the above expression, we get three values of I for three different masses. The mean value of I is then calculated. Result : The moment of inertia of the flywheel I = ... kgm 2 Discussions : 1. The string should be very thin and should be evenly wound on the axle i.e, there should be no overlapping of the various coils of the string. 2. The length of string must always be less then the height of the axle of flywheel from the ground so that it may be detached from the axle before it strikes the ground. 3. The loop which is made to slip over the peg should be quite loose so that when the string has unwound itself, it must leave the axle, there should not be any tendency to rewind in the opposite direction. 4. The length of the string between the loop and the mark at the other end where string left the axle before starting of experiment gives the value of h. This length is measured by a scale.

32 NSOU l CC - PH - 01 5. To make winding to whole number of turms of string on the axle, the winding should be stopped when the projecting peg is horizontal and winding is almost complete. 6. The stop watch should be started just after the string leaves the axle. 7. There should not be any kink in the string. 8. There should be least friction in the flywheel. 9. In this experiment, the expression for I (equation 3.1) has been derived neglecting the kinetic energy ( ) 2 1 2 mv of the falling mass m, considering large value of moment of inertia (I) of the flywheel. It we do not follow the above simplification the expression for I becomes, 2 2 2 2 2 1 2 8 1 ? ? - ? ? ? ? π ? ? = ? ? + ? ? ? ? mght mr n I n n where r is the radius of the flywheel. 10. The expression for the moment of inertia of the flywheel has been derived by considering the principle of conservation of energy. Maximum proportional error : We find from equation (3.1) 2 2 2 1 2 2 m gt I= 8 1 ? ? π + ? ? ? ? h n n n

NSOU l CC - PH - 01 33 Therefore, max I | 2· I h t h t δ δ δ = + Here, δh = 2 × smallest division of a metre scale δt = least count of stop watch, ∴ Maximum percentage error in I max I | 100% I δ = × = ... Conclusion : Measure value of the moment of Inertia (I) of the flywheel about its axis of rotation is accurate within the errors involved in our experimental arrangement. Key words : (i) Fly-wheel, (ii) Moment of inertia, (iii) Principle of conservation of energy Summary : (i) The moment of inertia of a body about an axis is defined and the moment of inertia of a flywheel about its axis of rotation is measured, by setting it in motion with a known amount of energy. (ii) In measuring the moment of inertia we have used a formula which is applicable for a flywheel having high value of moment of inertia. (iii) Number of revolution (n 1) of the flywheel before the mass gets detached from the axle is measured carefully. (iv) The number of revolutions (n 2) of the flywheel before it comes to rest after the mass is detached is also carefully measured.

34 NSOU l CC - PH - 01 (v) Moment of inertia of the flywheel is calculated using equation (3.1) (vi) Possible sources of error and precautions to be taken are discussed. (vii) Accuracy of measurement is checked. Model Questions answers 1. Does the moment of inertia of a body depend on its axis of rotation? Ans. Yes. Moment of inertia of a body depends on the axis of rotation since moment of inertia of a body about an axis is the sum of the products of the mass of each particle in the body and the square of its distance from the axis of rotation i.e. I = i i i m r 2 ∑ . 2. Can you perform your experiment with a load of any mass? Ans. No. The mass of the load must be sufficient to overcome friction and cause the wheel to rotate without external help. 3. Would you require long or short height of fall? Ans. To make n 2 and t large, a bit longer height of fall is required because it will cause w of high value. 4. Which quantity should be measured accurately in this experiment? Ans. As the time (t) for which the flywheel rotates after the mass gets detached from the axle occurs in the second power in the expression for I, it should be measured with higher accuracy. 5. Why the string used should be very thin? Ans. The string should be very thin so that its radius is very small in comparision to the radius is very small in camparison to the radius of the axle otherwise,

NSOU l CC - PH - 01 35 its radius should be added to that of the axle to get r. The radius (r) of the axle should be measured carefully if we employ the formula m I n n 1 2 ? ? = ? ? + ? ? ght r n n 2 2 2 2 2 8Π ? ? × - ? ? ? ? to measure moment of inertia.

36 NSOU l CC - PH - 01 Unit - 4 q Determination of refractive index of a liquid by Travelling Microscope Contents : The refractive index of a liquid is measured following the idea of Snell's law of the refraction of light. Introduction : There are different methods for the determination of refractive index of a liquid. The simplest method of measurement of refractive index of a liquid is the mechanical method such as using a travelling microscope. In this case, the refractive index of a liquid is measured by considering the relation between real depth and apparent depth of the liquid. Objective : To determine the refractive index of a liquid using a travelling microscope. Theory : Definition : The refractive index (μ) of a medium may be defined as the ratio of the velocity of light (c) of a given wavelength in vacuum to the velocity of light (v) in that medium i.e. c v μ = . Refractive index has no unit. Working formula : The refractive index of a liquid is given by

NSOU l CC - PH - 01 37 Realdepthof theliquid Apparentdepth of the liquid μ = u v μ = ... (4.1) 3 1 3 2 - μ = - R R R R where, R 1 = Microscope reading for the cross mark (made on the inner bottom of a beaker) when the beaker is empty, R 2 = Microscope reading for the image of cross mark when beaker contains some amount of experimental liquid. R 3 = Microscope reading for the liquid surface. Procedure : 1. The microscope tube is fixed vertical and the eye piece is distinctly focussed on the cross-wires. 2. The vernier constant of the vertical scale of the travelling microscope is determined. 3. Making a cross-mark on the inner bottom of an empty glass beaker, the cross- mark is sharply focussed by the microscope avoiding parallax. Main scale and vernier scale readings are takan. Three sets of readings are taken and their mean value (R 1) is calculated. 4. Now, taking some amount of experimental liquid in the beaker, the image of the cross-mark is sharply focussed by the microscope avoiding parallax. Main scale and vernier scale readings are taken. Repeating the process thrice, three sets of readings are taken and their mean value (R 2) is found out,

38 NSOU l CC - PH - 01 5. Then a small quantity of the lycopodium powder or cork dust is spread on the liquid surface and the microscope is sharply focussed on the powder without any parallax. After repeating the process thrice, three sets of readings are recorded. The mean of these three readings (R 3) is calculated. 6. Thus, the real depth of the liquid = u = R 3 – R 1 and the apparent depth of the liquid = v = R 3 – R 2 . From this data, the refractive index (μ) of the given liquid is determined by using eq.(4.1) 7. The whole process is repeated for two other depths of the liquid and refractive index (μ) for each depth is determined. The mean of these three values will give the correct value of μ. Experimental results : (A) Readings for the cross-mark when the beaker is empty Verrier constant (v.c) of the vertical scale of the travelling microsope valueof1samllestdivisionof mainscale = Total numberof vernierdivisions S = ... n cm = Table-1 No. of obs. 1. 2. 3. Main scale reading (s) cm Vernier scale reading (V) = (v.r × v.c) cm Total reading R 1 = (S + V) cm Mean reading (R 1) cm

NSOU l CC - PH - 01 39 Liquid depth Small No. of obs. Reading for the image of cross mark Main Scale (S) cm Vernier Scale (v) = (v. r×v.c) cm Total = R 2 = (S+V) cm Reading for the liquid surface Depth of liquid Mean (R 2) cm Main Scale (S) cm Vernier Scale (V) = (v. r × v.c) cm Total = R 3 = (S+V) cm Mean R 3 cm Real depth (u) = R 3 –R 1 cm Apparent depth (v) = R 3 – R 2 cm 1 2 3 Medium 1 2 3 Large 1 2 3 (B) Readings for the image of the cross-mark and for the liquid surface. Table-2 Calculations : The refractive index of the liquid ( ) u v μ = For three different liquid depths, the refrective index of liquid μ 1 , μ 2 , μ 3 are determined by using the above relation. Result : Experimental value of the refractive index of the given liquid 1 2 3 .... 3 μ +μ +μ μ = =

40 NSOU l CC - PH - 01 Discussions : 1. The axis of the microscope should be vertical. 2. The lycopodium powder or cock dusk shuld be sprinkled over the liquid surface very thinly. 3. Parallax between the image and the cross-wire should be avoided. 4. Liquid depth should not exceed the focal length of the objective of the microscope. 5. To help the focussing process, a piece of white paper should be kept below the beaker. Maximum proportional error : Refractive index μ = u v Therefore, max | u v u v δμ δ δ = + μ Here δu = δv = 2 × v. c of the microsope. u. v are taken from the data obtained. So, maximum percentage error max | 100% ....% δμ = × = μ Conclusion : The calculated value of refractive index is accurate within the error involved in our experimental arrangement. Key Words : (i) Refractive index (ii) Travelling microscope (ii) Parallax.

NSOU l CC - PH - 01 41 Summary : (i) Refracive index of a medium is defined. (ii) The determination of refractive index of a liquid by measuraing the real and apparent depth using a travelling microscope is discussed. (iii) The real depth and apparent depth of the liquid are measured avoiding parallax between the image and cross-wire. (iv) Accuracy of the measurement is checked. Model questions and answers : 1. Does the refractive index of a substance depend on the colour of light? Ans. Yes. Refractive index of a substance is high for light having smaller wavelengths and low for light having higher wavelengths. So, Refractive index of a substance for violet light (μ v) < Refractive index of the substance for red light (μ r) since λ violet > λ red 2. Is this method suitable for a volatile liquid? Ans. No. If the liquid is volatile, it will evaporate quickly and depth of liquid will change during experiment. 3. Can any depth of liquid be taken? Ans. No. The depth of liquid taken should not exceed the focal length of the objective of travelling microscope.

42 NSOU l HPH-CC-01 4. What happens if the cross-mark is given on the outer surface of the bottom of the beaker? Ans. When cross-mark is made on the outer surface of the botom of beaker, the thickness of the beaker comes into play and thus real deapth of liquid is not obtained. 5. Does the accuracy of result depend on the depth of liquid taken? Ans. Yes, greater depth gives more accurate result with less percentage error but it should not exceed the focal length of the objective of telescope. 6. Can you apply this method for a transparent solid? Ans. Yes. If the soid is taken in the form of a plate or block, its refractive index can be found out by a travelling microscope following same process.

NSOU l HPH-CC-01 43 Unit–5 q To Find the Fourier co-efficients of different periodic vibrations by graphical method. In this exercise, we shall find the Fourier co-efficients of a periodic function depicted in a graph. Definition : Any periodic function can be represented as the sum of an infinite series of cosine and sine functions with proper co-efficients. These co-efficients are called Fourier co-efficients. Theory : Any periodic function can be expressed as a series of cosine and sine functions. ( ) ( ) 0 2 2 ( ) cos sin 2 ∝ =-∝ π π ? ? = + + ? ? ? ? ∑ n n n a nx nx f x a b L L L is the periodicity of the function a 0 , a 1 , a 2 ,..., b 1 , b 2 ,....a −1 , a −2 , ..., b −1 , b −2 ,... are the Fourier co-efficients. The co-efficients can be expressed as /2 0 /2 2 ( ) L L a f x dx L - = ∫ ( ) /2 /2 2 2 ( )cos - π = ∫ L n L nx a f x dx L L ( ) /2 /2 2 2 ( )sin - π = ∫ L n L nx b f x dx L L Example : Figure 1 below shows a piece-wise continuous function. Fig. 1 : A periodic function

44 NSOU l HPH-CC-01 ABCDE is a periodic function ; AE = L is the period. Figure 1 shows two periods. ABCDE consist of four points ; AB, BC, CD, DE. Each part can be represented by a function like f(x) = p + qx. Each part has different values of p, q. Let x = 0 at A ; at B, x = x 1 , at C, x = x 2 ; at D, x = x 3 ; at E, x = L. ( ) 1 0 1 1 0 2 ? ? = + ? ? ∫ x a p q x dx L + ( ) 2 1 2 2 + ∫ x x p q x dx + ( ) 3 2 3 3 x x p q x dx + ∫ + ( ) 3 4 4 L x p q x dx ? ? + ? ? ∫ ( ) ( ) 1 1 1 1 0 2 cos 2 / x a p q x x L dx L ? ? = + π ? ? ∫ + ( ) ( ) 2 1 2 2 cos 2 / x x p q x x L dx + π ∫ + ( ) ( ) 3 2 3 3 cos 2 / x x p q x x L dx + π ∫ + ( ) ( ) 3 4 4 cos 2 / L x p q x x L dx ? ? + π ? ? ∫ ( ) ( ) 1 2 1 1 0 2 cos 4 / x a p q x x L dx L ? ? = + π ? ? ∫ + ( ) ( ) 2 1 2 2 cos 4 / x x p q x x L dx + π ∫ + ( ) ( ) 3 2 3 3 cos 4 / + π ∫ x x p q x x L dx+ ( ) ( ) 3 4 4 cos 4 / L x p q x x L dx + π ∫ ( ) ( ) 1 1 1 1 0 2 sin 2 / ? ? = + π ? ? ∫ x b p q x x L dx L + ( ) ( ) 2 1 2 2 sin 2 / + π ∫ x x p q x x L dx + ( ) ( ) 3 2 3 3 sin 2 / x x p q x x L dx + π ∫ + ( ) ( ) 3 4 4 sin 2 / L x p q x x L dx + π ∫ and so on.

NSOU l HPH-CC-01 45 EXPERIMENT : In figure 2, a graph represents a periodic function. Graph of a symmetric periodic function Figure 2 : A periodic function drawn on a graph The function is symmetric around x = 0 ; period length is 2 units. A symmetric function will have only cosine components. So, we have to find a 1 , a 2 ... Value of x ranges from − 1 to + 1, So the period L = 2. This range is divided into N = 40 divisions. Value of each segment is ∆x = L/N = 0.05. At each point, value of x can be given by x i = i ∆x ; i = 0 at the centre. i goes from − 1 to −N/2 towards left edge and it goes from 1 to N/2 towards right from centre. Since the function is symmetric, only one half is sufficient for calculation. We make a table with x i values and f(x i) values at each x i found from the graph taking only the right half.

46 NSOU l HPH-CC-01 Table 1 Values of f(x i) and its product with cos (2πnx i /2) for n = 1, 2, 3 i x i f(x i) f(x i) cos (πx i) f(x i) cos (2πx i) f(x i) cos (3πx i) 0 0 0.750 0.750 0.694 0.7500 1 0.05 0.730 0.721 0.546 0.6504 2 0.10 0.675 0.642 0.353 0.3968 3 0.15 0.600 0.535 0.148 0.0939 4 0.20 0.480 0.388 0.000 – 0.1483 5 0.25 0.375 0.265 – 0.068 – 0.2652 6 0.30 0.220 0.129 – 0.059 – 0.2092 7 0.35 0.100 0.045 0.040 – 0.0988 8 0.40 – 0.05 – 0.015 0.143 0.0405 9 0.45 – 0.15 – 0.023 0.250 0.0681 10 0.50 – 0.25 – 0.000 0.304 0.0000 11 0.55 – 0.32 0.050 0.291 – 0.1453 12 0.60 – 0.36 0.111 0.220 – 0.2912 13 0.65 – 0.375 0.170 0.113 – 0. 3704 14 0.70 – 0.365 0.215 0.000 – 0.3471 15 0.75 – 0.35 0.247 – 0.100 – 0.2475 16 0.80 –0.325 0.263 – 0. 176 – 0.1004 17 0.85 – 0.30 0.267 – 0.222 0.0469 18 0.9 – 0.275 0.262 – 0.250 0.1616 19 0.95 – 0.26 0.257 0.247 0.2317 20 1.00 – 0.25 0.250 0.750 0.2500 Total 0.3 5.529 2.73 0.4664

NSOU l HPH-CC-01 47 Calculation : ( ) /2 /2 0 /2 0 2 2 ( ) 2 L L L a f x dx f x dx L L - - = = ∫ ∫ The factor 2/L = 1 because L = 2. The integration will be replaced by summation for discrete x and f (x) values. 21 0 1 2 ( ) = = Δ ∫ i i a f x x Δx = 0.05 ; from the table ( ) 20 0 0.3 = = ∑ i i f x . So, a 0 = 0.03 Similarly, ( ) ( ) 20 1 0 2 cos 2 /2 i i i a f x x x = = π Δ ∑ = ( ) ( ) 20 0 0.1 cos 2 /2 0.1 5.5290 0.5529 = ∗ π = ∗ = ∑ i i i f x x ( ) ( ) 20 2 0 2 cos 4 /2 = = π Δ ∑ i i i a f x x x = ( ) ( ) 20 0 0.1 cos 4 /2 0.1 2.73 0.273 = ∗ π = ∗ = ∑ i i i f x x ( ) ( ) 20 3 0 2 cos 6 /2 = = π Δ ∑ i i i a f x x x = ( ) ( ) 20 0 0.1 cos 6 /2 0.1 0.4664 0.0466 i i i f x x = ∗ π = ∗ = ∑ Actual values are a 0 = 0 ; a 1 = 0.5 ; a 2 = 0.25 ; a 3 = 0.

48 NSOU l HPH-CC-01 SOURCES OF ERROR : There are basically two sources of error. Firstly, the values of each f(x i) measured from graph differs from true value due to eye-estimation. Secondly, the integration in the Fourier co-efficient expressions represents the area within the curve and x-axis. When the integration is replaced by a summation of discrete values, the area is replaced as a sum of rectangular areas. If the length of each segment Δx decreases, it converges to the actual area under the curve. DISCUSSION : We discuss briefly the importance of Fourier co-efficients. In many situations, physical quantities are periodic but discontinuous or piecewise discontinuos or rapidly varying. An example is the potential/electric permittivity distribution in a solid state/ photonic crystal. To extract physical properties, one has to solve second order differential equation like Schro JJ dinger equation or Helmholtz equation. Functions of above mentioned type are hard to incorporate in such equations. Fourier theorem help transform these functions into a series of continuous functions. Although the series is infinite, practically a finite number of terms are sufficient. So, one has to know the Fourier co-efficients to transform approximately a discontinuous function into a continuous function. Δx Δx f(x i – 1) f(x i + 1) f(x i) f (x)

NSOU l HPH-CC-01 49 EXERCISES : Find the Fourier coefficients of the curves shown in the following graphs. (i) Graph of a anti-symmetric periodic function Answer : b 1 = 0.4 ; b 2 = 0.5 (ii) Graph of a mixed periodic function Answer : a 0 = 0.1 ; a 1 = 0.25 ; b 1 = 0.5

50 NSOU l HPH-CC-01 Find the Fourier co-efficients (a –2 , a –1 , b –2 , b –1 , a 2 , a 1 , a 2 , b 1 , b 2) functions analytically and graphically of following : (iii) Graph of a square periodic function (iv) Graph of a triangular periodic function

NSOU l HPH-CC-01 51 Unit–6 q To determine the co-efficient of viscosity of water by capillary flow method Contents : The co-efficient of viscosity of water is measured by Poiseullie's method. Introduction : Viscosity is the general property of every fluid which acts only when fluids are in motion. Due to this property of fluid, resistance is developed against gradual deformation of a fluid by shear stress or tensite stress. At the molecular level, viscosity may be interpreted as the result of interaction between different molecules of a fluid. The measurement of the co-efficient of viscosity of water by considering its streamline flow through a capillary tube is one of the accurate methods used in the laboratory. Fig. 6.1 Fig. 6.1

52 NSOU l HPH-CC-01 Description : T → A horizontal capillary tube which is inserted into two small brass chambers A and B. V → Water reservoir connected to chamber A through the pinch-cock G 1 . E and F → Two limbs of a manometer H connected to chambers A and B respectively. The manometer is provided with a scale to measure difference (h) of water level in the two limbs of manometer. G 2 → Pinch-cocks provided with the entrance and exit tube of chamber B. CD → A glass tube open at both ends inserted in the water reservoir V. M → A thermometer used to measure temperature of water stored in the beaker P. Objective : We intend to measure the co-efficient of viscosity of water using its streamline flow through a capillary tube. Theory : Definition : The co-efficient of viscosity of a fluid (water) is defined as the tangential stress acting on any one of the two adjacent fluid (water) surfaces when there is unit vilocity gradient between them. SI unit of co-efficient of viscosity is N. S/m 2 = Poiseulle (Pl)

NSOU l HPH-CC-01 53 Working formula : The co-efficient of viscosity of water is given by 4 Pr 8 π η = lV ... (6.1) where, l = length of a uniform capillary tube. r = Internal radius of the capillary tube. P= Pressure difference under which water flows in streamlines through the horizontal capillary tube. V= volume of water flowing out per second. The pressure difference (P) is given by P = hρg .... (6.2) where, h = Height of water column producing pressure difference P ρ = Density of water. g = Acceleration due to gravity. Therefore, ( ) 4 r 8 π ρ η = h g lV ... (6.3) Procedure : 1. After cleaning the capillary tube T (Fig. 6.1) its length (l) is measured by a metre scale thrice and its mean (l) is determined.

54 NSOU l HPH-CC-01 2. To determine the internal radius (r) of the capilllary tube and r 2 , r 4 the steps given below are to be followed. (a) Introducing a long pellet of mercury in the capillary tube, the length (L) of the pellet is measured at different positions of the tube by a travelling microscope. (b) The mercury column is then taken in a crucible of known mass and the mass (m) of the mercury column is measured. The the square of internal radius (r 2) of the tube, 2 = ′ π ρ m r L is calculated where ρ′ is the density of mercury at room temperature. 3. By adjusting the pinch-cock G 1 (sometimes G 2) (Fig 4.1 of chap-1, unit-4), steady and small difference in the height (h) of water level in the two limbs of the manometer H is maintained such that water flows to the beaker in very slow stream. Then pressure difference (P) under which water flows is P = hρg where ρ is the density of water at room temperature, h is measured by the scale attached to the manometer. Noting the temperature of water in the beaker by a thermometer, ρ is found out from a table. 4. Now, the volume of water (V′) is collected is a measuring cylinder (preferably having lowest graduation 0.1 ml or 0.5 ml) for a given time (t), time is measured by a stop-watch. Hence, the volume of water flowing out through the tube per second, ( ) V V t ′ = is determined. 5. The experiment is repeated for atleast five different values of h and the corresponding values of rate of flow of water (V) is measured. 6. Then, a graph is plotted with h along x-axis and rate of flow of water (V) along

NSOU l HPH-CC-01 55 y-axis. The graph will be a straight line passing through the origin (Fig.1). By choosing a point (A) on the graph, its coordinate (h, v) is found out. Fig. 1 Experimental results : (A) Determination of length (l) of the capillary tube by a metre scale. Table-1 (B) Determination of the length (L) of the mercury column in the capillary tube by using a travelling microscope. V in m 3/S h in m (0,0) No. of obs Measured length (l) cm Mean length (l) (cm) length (l) (m) 1. 2. 3.

56 NSOU l HPH-CC-01 Vernier constant of the microscope (v.c) = ... cm Table-2 (C) Determination of radius (r) and r 4 of the capillary tube : Table-3 Room temperature (T) = ...°C Density of mercury at room temp (ρí) = ... kg/m 3 (from table) Mean length (L) of the mercury column (From table-2) (m) Mass of the crucible When empty (m 1) (gm) When with mercury (m 2) (gm) Mass of mercury column = m = m 2 − m 1 (gm) 2 = ′ π ρ m r L (cm 2) r 2 (m 2) r 4 (m 4) Position of mercury column Reading for the left end of mercury column Main scale (S) (cm) Vermier scale (V) (= v.r×vc) (cm) Total R 1 (= S + V) = (cm) Reading for the\ right end of mercury column Main scale (S) cm Total R 2 (= S + V) (cm) Vernier scale V (= v.r × v.c) (cm) Length of mercury column L = R 1 ~ R 2 (cm) Mean L (cm) L (m) 1. 2. 3.

NSOU l HPH-CC-01 57 (D) Determination of pressure difference (P) in terms of h. Table-4 (E) Determination of the rate of flow (V) of water by measuring cylinder Table-5 No. of Obs. Reading of the water level in the left arm of the manometer = (R 1) (cm) 1. 2. 3. 4. 5. Reading of the water level in the right arm of the monometer = R 2 (cm) Height of water level h = R 1 − R 2 (cm) Height h (m) No. of Obs. Height of water level (h) (cm) [From table-4] 1. 2. 3. 4. 5. Volume of Water collected (Ví) (cm 3) Mean Value of (Ví) (cm 3) Volume of water collected per sec 6 3 10 (m /s) V V t - ′ = × Time of collection of water (t) (sec) ... ...

58 NSOU l HPH-CC-01 Calculations : From eq. (6.3), we find the coefficient of viscosity of water ( ) 4 8 π ρ η = r h g lV (i) Measuring room temperature, we can find density of water (P) at that temperature from the table. (ii) Plotting h − V graph (Fig.1), we can find out h and V choosing a point on this straight line graph. (iii) Substituting the value of r, h, l, V, ρ, g in the above expression for η, we obtain the value of η. Result : The co-efficient of viscosity of water at room temperature ( ) .... C° is η = ... N.S/m 2 = ...Pl Discussions : 1. The capillary tube should be uniform. The radius (r) should be measured very accurately as it occurs in the fourth power in the expression of η. 2. The pressure difference (P) should be small, otherwise the flow of water through capillary tube will not be streamlined but turbulent, h should be less than 2 c h , where h c is the critical height (given).

NSOU l HPH-CC-01 59 3. To improve accuracy of the result, sufficient quantity of water must be collected in the measuring cylinder. 4. As η changes with temperature, the temperature of water should be noted carefully. 5. As capillary tube is horizontal, water coming out from open and may run back. To eliminate this, little grease or vaseline should be kept at the open end. Maximum proportional error : The co-efficient of viscosity of water ( ) 4 4 Pr 8 8 π ρ π η = = ′ r h g t lV lV ′ ? ? = ? ? ? ? ∵ V V t Therefore, the maximum proportional error max 4 | δη ′ δ δ δ δ δ = + + + + ′ η r h t l V r h t l V (. . . ρ and g are supplied) As, 2 = π ρ m r L , max 2 | r m L r m L δ δ δ = + Hence, max 2 | 2 δη ′ δ δ δ δ δ δ = + + + + + ′ η m L h t l V m L h t l V Knowing the maximum values of possible errors in the measurement of h, m, L, t, l, V′. we can determine max δη η

60 NSOU l HPH-CC-01 ∴ Maximum percentage error = max | 100% δη × η = ... % Conclusion : Measured value of the co-efficient of viscosity of water is accurate within the errors involved in the experimental arrangement. Key words : (i) Co-efficient of viscosity (ii) streamline and turbulent motion. (iii) critical height (iv) critical velocity. Summary : (i) The co-efficient of viscosity is defined. The measurement of the co-efficient of viscosity of water by using Poiseullie's equation is discussed. (ii) The streamline flow of water through the capillary tube is maintained by keeping small pressure difference across its ends. (iii) To calculate the rate of flow of water (V) through the tube accurately, a measuring cylinder with very small graduations is used. (iv) The straight line portion of (h – V) graph has been used in order to avoid error arising due to the kinetic energy of water and turbulent motion. (v) The co-efficient of viscosity (η) of water is calculated by using eq. (6.3) with the observed value of the quantities involved. (vi) Proper precautions have been discussed and accuracy of measurement is checked.

NSOU l HPH-CC-01 61 Model questions and answers : 1. What do you mean by 'streamline' and 'turbulent' motion? Ans. If the pressure difference under which a liquid flows in a horizontal capillary tube is small, the liquid particles move in straight paths parallel to the axis of the tube. This type of motion is called 'streamline' motion. On the otherhand, when pressure difference across the ends of the capillary tube is large, the liquid particles move in 'zig-zag' paths. This type of motion of liquid is called 'turbulent' motion. 2. What do you mean by 'critical height' and 'critical velocity'? Ans. Critical height (h C) : When the value of h and hence pressure difference across the ends of a capillary tube exceeds certain value (h c), then the motion of liquid flowing through the tube becomes turbulent. This height h c is called critical height. Critical velocity (V c) : There is a particular velocity of flow of liquid below which the motion is streamline and above which the motion is turbulent. This particuar velocity is called critical velocity. Critical velocity for a liquid depends on the density (ρ) of liquid, radius (r) of the capillary tube, the co- efficent of viscosity (η) of the liquid. The expression for critical velocity is r c K v η = ρ . K is called Reynold's number. 3. What is Reynold's number (K)? Ans. According to Reynold, the critical velocity (v c) marking the transition from

62 NSOU l HPH-CC-01 the streamline to the turbulent motion of a liquid is given by r c K v η = ρ where K is a number called Reynold's number (for narrow tubes K = 1000). ρ is

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the density of the liquid and r is the radius of capillary tube through which liquid is flowing. 4. Can

you perform this experiment with a tube of wider bore? Ans. No, In that case, a small pressure difference across the tube will cause the liquid flowing through it turbulent. 5. How does the co-efficient of viscosity change with temperature? Ans. In case of liquids, the co-efficient of viscosity decreases with the increase of temperature while in case of gases co-efficient of viscosity increases with the increase of temperature. 6. Why does the co-efficient of viscosity of water falls off with increasing temperature? Ans. When the temperature of water increases, its molecules gain energy. As a result, the intermolecular separation increases and the cohesive force responsible for viscosity decreases. So the co-efficient of viscosity of water will decrease with the rise of temperature. 7. Why should the capillary remain horizontal? Ans. The capillary tube through which the liquid flows must remain horizontal, otherwise the effect of gravity will come into play. 8. Is the velocity of water same everywhere inside the tube?

NSOU l HPH-CC-01 63 Ans. No, velocity of water is maximum along the axis of the tube and decrease towards the wall of tube from axis and minimum at the surface of contact with the tube. 9. Are there any assumptions in this method? Ans. Yes. First assumption : There is no acceleration of water along the axis of the tube, which is not true. Second assumption : The energy available due to pressure difference across the two ends of the capillary tube is spent entirely to overcome the viscous drag of the liquid flowing through the tube. This is also not true. 10. Is this method suitable for all types of liquids ? Ans. No. This method is only suitable for liquids having low viscosity. For highly viscous liquids stoke's method may be employed. 11. Should the pressure difference across the ends of the capillary tube remain constant during a particular set? Ans. Yes. If the pressure difference does not remain constant the rate of flow of water will change during a particular set. 12. Have you considered the effects of Kinetic energy of the liquid and its acceleration near the entrance end of the capillary tube? Ans. No, their effects produces error in the determination of η. To eliminate these effects, the formula for η should be ( ) 4 2 2 4 · 8 1 1.64 4 ? ? π η = ρ - ? ? ? ? + π ? ? r kV g h V r g where, k is a constant whose value is nearly one.

64 NSOU l HPH-CC-01 Unit–7A q Determination of the acceleration due to gravity (g) using a Bar Pendulum Contents : The acceleration due to gravity (g) is measured with the help of a bar pendulum. Introduction : A compound pendulum is a weighted rigid body of any shape capable of oscillating under gravity in a vertical plane about any horizontal axis passing through it. As a compound pendulum execute simple harmonic motion for small angular displacement, it helps us to find out the value of acceleration due to gravity (g). A bar pendulum (a long metallic bar) is actually a compound pendulum and is used to determine the value of g. Bar Pendulum : Fig. 7A.1 Bar Pendulum

NSOU l HPH-CC-01 65 Description : AB → A bar pendulum. It consists of a metal bar (Fe or Brass) about 1 m long. H → Series of circular holes on the bar of equal distances. K → Knife-edge which passes through any one of the holes. P → Platform with which knife-edge is fixed. L → Levelling screw used to make platform horizontal. The bar pendulum is suspended from a knife-edge and made to oscillate in a vertical plane. Objective : To determine the acceleration due to gravity (g) by means of a Bar Pendulum. Theory : Definition : The acceleration acquired by a freely falling body under gravity is known as acceleration due to gravity (g). SI unit of g is m/s 2 Working formula : The acceleration due to gravity (g) is given by 2 2 4 · L g T = π ... (7A.1) Where, T = Time period of oscillation of the bar pendulum.

66 NSOU l HPH-CC-01 L = Length of the simple equivalent pendulum. and L = l 1 + l 2 , l 1 and l 2 = Distances between the points of suspension from the centre of gravity (C.G) of pendulum, situated asymetrically an either side of the C. G, about which the time periods are same. Procedure : 1. The knife-edge is first made horizontal by means of levelling screws and tested with the help of a sprit-level placing on the platform. Two vertical lines are marked at the two ends (A and B) [Fig. 7A.1] of the bar pendulum. 2. The bar pendulum is placed horizontally on a sharp point and kept it in equilibrium position. The position of C. G of the bar is marked. 3. Now the bar is suspended on the knife-edge which is introduced in a hole nearest to one end (say, A) of the bar. 4. The lower vertical mark at the end B of bar is then focussed by a telescope avoinding parallax, keeping the bar at rest. 5. The pendulum is made to oscillate with very small amplitude (less than 4°) in a vertical plane. 6. Time for 30 complete oscillations is noted with the help of a precision stop- watch three times and mean time period (T) is determined. 7. The distance (d) of the hole i.e., axis of suspension from the fixed end A is measured by a metre scale. 8. The operations (3) to (7) are repeated for other holes situated on one side of C. G. The values of T and d are determined in each case. 9. Now, the bar is inverted and similar operations (3) to (7) are repeated for each hole (knife-edge) situated on the other side of C. G.

NSOU l HPH-CC-01 67 Time period (T) corresponding to each hole and the distance (d) of the holes,i.e, knife-edges from the same end A are determined. 10.

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A graph is then drawn with the distance of the holes (knife-edges) from the fixed end A along X-axis and the corresponding time period (T) along Y-axis.

The nature of the graph is shown in Fig.7A.1. Fig. 7A.2

68 NSOU l HPH-CC-01 11. A horizontal line PQRS is drawn which intersect the curves at P, Q, R, S. Measuring the distances PR and QS, the length of equivalent simple pendulum L is determined where 2 PR QS L + = . 12. We can calculate 'g' by using value of L and T from graph in equation. (7A.1). Experimental results : (A) Determination of time period (T) and distance (d) of holes (knife-edges) from one fixed end A.

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Table-1 Serial no of holes from one are fixed end A Time for 30 oscillations (S) Mean time (t) (S) Time period (T) (S) Distance of the hole (knife-edge) from the fixed end A (cm) On one side of C. G

on other side of C. G 1. 2. 3. etc. 1. 2. 3. etc. …

NSOU l HPH-CC-01 69 (B) Determination of L and T from d – T graph : Table-2 Calculations : We can calculate 'g' using (d – T) graph (Fig. 7A.1) : Drawing different horizontal lines, we can find different values of L and corresponding value of T (consult procedure-11) Then subestituing these values of L and T in equation (7A.1) 2 2 4 · = π L g T , we get three values of 'g'. Their mean is then found out. Result : The acceleration due to gravity (at the place of experiment) g = ...cm/s 2 = m/s 2 Discussions : 1. The amplitude of oscillation must be very small (within 4° of arc) during the No. of obs. Length PR from graph (7A.1) (cm) 1. 2. 3. Length QS from graph (7A.1) (cm) Length of equivalent simple pendulum 2 PR QS L + = (cm) Corresponding value of time period (T) (s) (from graph 7A.1) … … … … … … … … … … … … …

70 NSOU l HPH-CC-01 measurement of time period. 2. The knife-edges must be kept horizontal and the pendulum must oscillate in a vertical plane. There should not be any rotational motion of the pendulum during oscillation. 3. The time period should be noted very accurately as far as possible by using a precision stopwatch and a telescope. 4. There should not be any air current in the vicinity of the pendulum. 5. We can also calculate 'g' by drawing lT 2 along X-axis and l long Y-axis, where l is the distance of the holes from C. G. of the bar and T is the corresponding time period. The graph is a straight line. The slope of this curve, 2 4 g m = π . 6.

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The radius of gyration (k) of the pendulum about an axis

passing through its C. G. can be calculated with the help of (d – T) graph. Maximum proportional error : From equation (7A.1), we have, ( ) 2 2 2 2 4 · 2 4 · 30 ? ? π ? ? ? ? = π = x L L g T t where, L L 2 x = and 30 t T =

NSOU l HPH-CC-01 71 Therefore, max | 2· δ δ δ = + x x L g t g L t ∴ Maximum percentage error in g = max 100% ......% g g δ × = Knowing the smallest value of metre scale (δx = 0.2 cm) and that of stop watch (δт), the maximum percentage error can be calculated. Conclusion : The measured value of the acceleration due to gravity (g) is accurate within the errors involved in the experimental arrangement. Key Words : (i) Acceleration due to gravity ; (ii) Bar pendulum (ii) Radius of gyration (iv) Simple equivalent pendulum. Summary : (i) Acceleration due to gravity (g) is defined and of the method of determination of g by a Bar pendulum is discussed. (ii) The amplitude of oscillation is kept very small and the pendulum is allowed to oscillate in a vertical plane avoiding any rotational motion. (iii) The period of oscillation (T) is measured accurately by a precision stop-watch and a telescope for different values of distance (d) of the knife-edge from a fixed end. (iv) The length (L) of equivalent simple pendulum and the corresponding time period (T) is determined by drawing d – T graph. (v) The value of 'g' is determined by substituting the values of L and T in equation

72 NSOU l HPH-CC-01 (7A.1). (vi) The precautions to be taken for measurement of 'g' have been discussed. Evaluation of proportional error is mentioned. (vii)Accuracy of the measurement is checked. Model questions and answers : 1. What do you mean by 'centre of suspension' and ' centre of oscillation' of a compound pendulum? Ans. Centre of Suspension : It is the point where horizontal axis of oscillation meets the vertical sections of the pendulum taken through its C. G. Centre of oscillation : It is a point on the otherside of C. G. of the pendulum at a different distance than that of the centre of suspension such that the period of oscillation is the same as that about the centre of suspension. Centre of oscillation and centre of suspension are interchangeable. 2. What is simple equivalent pendulum? Ans. It is a simple pendulum whose length is such that its period is the same as that of a compound pendulum. 3. What would happen if the centre of suspension coincides with the C.G? Ans. When centre of suspension of the bar pendulum coincides with its C. G, the time period of oscillation of the pendulum will be infinite. 4. When is the period of Compound pendulum minimum? Ans. The time period of a compound pendulum is minimum when the distance

NSOU l HPH-CC-01 73 of the centre of suspension from its C. G is equal to

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the radius of gyration of the pendulum about an axis parallel to the axis of rotation and passing through

the C. G. 5. How many points are there about which the time periods of a compound pendulum are the same? Ans. There are four points, two on each side and collinear with the C. G. about which time periods are same. 6. Why is it necessary to make the knife-edges horizontal? Ans. The knife-edges are made horizontal to avoid the slipping of the pendulum and to make it oscillate in a vertical plane. 7. What are the different sources of error in this experiment? Ans. The different sources of error are (i) finite amplitude of oscillation (ii) the curvature of the knife edge. (iii) the buoyancy and viscosity of air (iv) yielding of the support (v) the flow of air alongwith the pendulum.
74 NSOU l HPH-CC-01 Unit–7B q Determination of the acceleration due to gravity (g) using a Kater's pendulum Contents : The acceleration due to gravity (g) is measured with the help of a Kater's pendulum. Introduction : Kater's pendulum which is also called 'reversible pendulum' is actually a compound pendulum— a rigid body of any shape capable of oscillating in a vertical plane under gravity about any axis passing through it. In the bar pendulum one knife-edge is used to oscillate it in a vertical plane whereas in Kater's pendulum two knife-edges are used. To determine 'g' more precisely, Bessel used two adjustable masses positioned on the pendulum rod between two knife-edges to make periods of oscillation about two knife-edges very nearly equal. The measurement of 'g' by Kater's pendulum with the help of Bessel's formula gives us very accurate result.
NSOU l HPH-CC-01 75 Kater's pendulum : Description : PQ → A Kater's pendulum. It is a metal (Brass) rod about 1m long. K 1 and K 2 → Two movable knife-edges positioned on the rod, turned inwards to face each other. A and B → Two equal cylinders placed beyond K 1 , K 2 A is made of box wood and 'B' is made of brass. C and D → Two equal cylinders, smaller in size placed between knife-edges K 1 and K 2 . C is made of box wood whereas D is made of brass. The pendulum is allowed to oscillate about any of the knife-edges in a vertical plane by placing the corresponding knife-edge on a metallic plate which is rigidly fixed on a permanent support. Objective : To determine the acceleration due to gravity by means of a Kater's pendulum. D A P C B Q K 2 K 1 Fig. 7B.1
76 NSOU l HPH-CC-01 Theory : Defintion : The acceleration produced in a freely falling body on account of the force of gravity is known as acceleraion due to gravity (g). SI unit of g is m/s 2. Working formula : The acceleration due to gravity (g) can be found out by using Bessel's formula given by 2 2 2 2 2 1 2 1 2 1 2 1 2 8 + - π = + + - T T T T g l l l l ..... (7B.1) Where, T 1 and T 2 = Time periods of the Kater's pendulum about its two knife- edges which are very nearly equal. l 1 and l 2 = Distances of the two knife-edges from the centre of gravity (C. G) of the pendulum. Procedure : 1. Keeping the cylinder B in the downward position, the pendulum is suspended from one of its knife-edges (say, K 1) [Fig. 7B.1]. A vertical sharp mark is drawn along the length of the pendulum and the mark is focussed through a telescope avoiding parallax keeping the pendulum at rest. 2. The pendulum is now allowed to oscillate freely about the knife-edge K 1 with very small amplitude in a vertical plane. The time for small number of oscillations (say. 5) is determined by means of a precision stop-watch reading upto 0.1s. 3. Now the pendulum is placed on the second knife-edge K 2 and again time for the same number of oscillation i.e.5 is measured. Usually these two times differ much.
NSOU l HPH-CC-01 77 4. Then the time for the same number of oscillations i.e, 5 about the knife- edges K 1 and K 2 are measured after shiffing the smaller cylinder D slightly in one direction. If the difference between these two times is increased, the cylinder D should be shifted in the opposite direction. Otherwise, the cylinder should be shifted in the same direction in subsequent adjustments. 5. The shifting of cylinder D should be continued in the same direction as before and times for more number of oscillations (say, 10, 15, 20 etc) about K 1 and K 2 are noted until the two times are nearly equal. 6. When times for more than 20 oscillations are noted, the difference between the times, about K 1 and K 2 for a given number of oscillations should be decreased by adjusting the position of smaller cylinder C slightly. Finally, these two times (T 1 , T 2) for 50 oscillations about K 1 and K 2 are made very nearly equal, their difference should be less than 2s. 7. Now the time for 50 oscillations about each knife-edge is measured thrice and the mean time for 50 oscillations about each knife-edge is calculated. From this, time periods T 1 and T 2 about the knife-edgs K 1 and K 2 are calculated. 8. To locate the centre of gravity (C. G) of the pendulum, the pendulum is removed from its support and balanced on a sharp wedge placed on the table. Then the balancing point is marked which is the position of C. G of the pendulum. The distances (l 1 , l 2) of the knife edges K 1 and K 2 from the C.G are measured by a metre scale. 9. Substituting the values of T 1 , T 2 , l 1 and l 2 in equation (7B.1), we can calculate the value of g.

78 NSOU l HPH-CC-01 Experimental results : (A) Determination of the times of oscillations at the preliminary observations Least count of the stop-watch = ... sec. Table-1 No. of obs Adjustments by shifting 1. 2. 3. 4. 5. 1. 2. 3. 4. 5. etc. No. of Oscillations observed Total time for oscillation about the knife- edge k 1 [t 1] (sec.) k 2 [t 2] (sec) Cylinder D (heavy mass) Cylinder C (lighter mass) 5 5 10 10 15 15 20 20 25 30 30 etc. etc etc

NSOU l HPH-CC-01 79 (B) Determination of final time periods T 1 and T 2 Table-2 (C) Determination of the distances of the Knife-edgs (K 1 , K 2) from the C. G of the pendulum Table-3 No. of obs Distance of K 1 from C. G l 1 (cm) 1. 2. 3. Mean l 1 (cm) Distance of K 2 from C. G. l 2 (cm) Mean l 2 (cm) ... ... No. of obs Oscillations about the Knife-edge 1. 2. 3. Time for 50 oscillations (sec) Mean time for 50 oscillations (sec) Time period (sec) (T 1 and T 2) 1. 2. 3. K 1 K 2 T 1 = ... T 2 = ...

80 NSOU l HPH-CC-01 Calculations : From equation (7B.1) we get, 2 2 2 2 2 1 2 1 2 1 2 1 2 8 + - π = + + - T T T T g l l l l Substituting the values of T 1 , T 2 , l 1 and l 2 in the above expression, the value of g is obtained. Result : The acceleration due to gravity g = ... m/s 2 T Discussion : 1. The amplitude of oscillaiton must be kept very small such that formula employed holds good. 2. The knife-edges must be adjusted horizontal and parallel to each other. 3. During preliminary observations, times of oscillations should be noted for smaller number of oscillations. Final observations for time periods should be noted for a large number of oscillations. 4. To increase accuracy, time periods should be measured by coincidence method. 5. Using the formula, 1 2 0 1 16 θ θ ? ? = - ? ? ? ? T T where T 0 and T are observed and corrected time period, θ 1 and θ 2 are the half angles of swing in radians at the start and end respectively, we can make correction for finite arc of swing. Maximum proporional error : We find from equation (7B.1)

NSOU l HPH-CC-01 81 2 2 2 2 2 2 1 2 1 2 1 2 1 2 8 + - π = + + - T T T T g l l l l The second term on the right hand side of the above expression is small compared to the first term since 2 2 1 2 T T≈ . Thus second term does not require much exact evaluation. Therefore, we may write 2 1 2 2 2 1 2 8 · l l g T T + = π + ∴ ( ) ( ) 2 2 1 1 1 2 max 2 2 1 2 1 2 | T T l l g g l l T T δ + δ + δ = + + + or, max 1 2 1 2 | 2· δ δ δ = + + g l T g l l T [since T 1 ≈ T 2] Here, δl = 0.2 cm (2 division of metre scale) δt = one smallest discussion of stop watch = ... sec Using l 1 + l 2 and T 1 from the observed data, we can calculate max | g g δ ∴ Maximum percentage error in g max | 100% ....% δ = × = g g Conclusion : Measured value of the acceleration due to gravity (g) is accurate within the errors involved in the experimental arrangement.

82 NSOU l HPH-CC-01 Key words : (i) Acceleration due to gravity ; (ii) Kater's pendulum ; (iii) Reversible pendulum. Summary : (i) Acceleration due to gravity (g) is defined and the method of determination of g by a Kater's pendulum is discussed. (ii) The amplitude of oscillation is kept very small. The pendulum is allowed to oscillate freely in a vertical plane. (iii) Keeping the knife-edges horizontal and parallel to each other, the periods of oscillation T 1 and T 2 about two knife-edges K 1 , K 2 are noted initially for small number of oscillations. The value of T 1 and T 2 are made nearly equal by adjusting the position of heavy cylinder D and increasing the number of oscillations upto 20 oscillations. (iv) When time for more than 20 oscillations are recorded, the difference between T 1 and T 2 is decreased by slightly adjusting the position of smaller cylinder C. (v) Finally, the difference between T 1 and T 2 for 50 oscillations are made very nearly equal by adjusting the position of smaller cylinder C. T 1 and T 2 are then calculated. (vi) The C.G of the pendulum is determined and value of distance of K 1 and K 2 from C.G (l 1 and l 2) are measured. (vii) Substituting the values of T 1 , T 2 , l 1 and l 2 in equation (7B.1), we get the value of g. (viii) The precautions to be taken for measurement of g have been discussed. Evaluation of Proportional error is mentioned. (ix) Accuracy of the measurement is checked.

NSOU l HPH-CC-01 83 Model questions and answers : 1. What are 'centre of suspension'and 'centre of oscillation'? Ans. Consult answer of Q. No. 1 of model question, unit-7A. 2. What is simple equivalent pendulum? Ans. See Answer of Q. No. 2 of model Q. unit 7A. 3. When the period of oscillation of a compound pendulum minimum? Ans. See answer of Q. No. 4 of model question, unit 7A. 4. What would happen if the centre of suspension coincides with the C. G? Ans. Consult answer of Q. No. 3 of model Q. Unit 7A. 5. Why should the knife-edges be horizontal? Ans. Otherwise, the bar may slip off and the pendulum would not oscillate in the vertical plane. 6. What are the uses of two adjustable cylinders in a Kater's pendulum? Ans. The heavier cylinder is used to make rough adjustment whereas the lighter cylinders are used to make fine adjustment. 7. What are the main sources of error in this experiment? Ans. Consult answer of Q. No. 7 of model Q. unit 7A. 8. How does this experiment give us correct result?

84 NSOU l HPH-CC-01 Ans. The working formula of this experiment containes two terms (i) first term contains l 1 + l 2 in the denominator. The distance between two knife-edges l 1 + l 2 can be measured accurately (ii) the denominator of the second term l 1 – l 2 involves certain inaccuracy since position of C. G can not be measured very accurately. But 2 2 1 2 T T- is very small since 1 2 ≈ T T , So error in l 1 – l 2 of second term effectively does not affect the result. Thus we can obtain most accurate result by this experiment.

NSOU l HPH-CC-01 85 Unit–8 q Determination of thermal conductivity of a bad conductor by Less' and Chorlton's method Contents : Value of

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thermal conductivity of a bad conductor in the form of a disc			

is measured using Leess' and Chorlton's method. Introduction : Thermal conductivity (k) is an important thermal property of a material which refers to its instrinsic ability to conduct heat. Depending on the value thermal conductivity, different substances are used for diffent purposes—as a conductor or insulator. So the determination of thermal conductivity of a material is very important. There are different methods for the determination of thermal conductivity of differnt substances such as, Searle's method is used to measure thermal conductivity of a good conductor (metal), thermal conductivity of glass (bad conductor) is measured in the laboratory taken in the form of tube. The determination of

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thermal conductivity of a bad conductor taken in the form of a thin disc by			

Lees' and Chorlton's method with Bedford's correction in the laboratory provides us most accurate result. Apparatus : Fig. 8.1

86 NSOU l HPH-CC-01 Description : C → Circular brass disc suspended by means of three strings from a ring on a retort stand. S → Circular bad conducting sheet of uniform thickness which is placed on C. A → A stream chamber placed on the sheet S. B → Bottom of the steam chamber, a thick circular metal plate (disc). T 1 and T 2 →Two thermometers used to record the temperatures of B and C. T 1 should read upto 0.2°C while T 2 should read upto 0.1°C. Diameters of B, C and S are all the same. Objective : To determine the thermal conductivity of a bad conductor using Lees' and Chorlton's method with Bedford's correction. Theory : Definition : Thermal conductivity of a material is defined as the rate at which heat is transferred by conduction normally per unit cross-sectional area of a slab made of that material from one face to other in the steady state when the temperature gradient is unity. S. I. unit of thermal conductivity is Js −1 m −1 K −1 = Wm −1 K −1 Working formula : Thermal conductivity of the material of a bad conducting sheet is given by 1 1 ( ) Qd K A = θ - θ .... (8.1) where, A = cross-sectional area of the bad conducting sheet.

NSOU l HPH-CC-01 87 d = thickness of the sheet. θ 1 , θ 2 = temperatures of the two opposite faces of the sheet in the steady state and θ 1 < θ 2 . Q = Quantity of heat conducted per second normally through the sheet in the steady state. Q is given by Q = ( ) 2 d ms dt θ θ .... (8.2) where, m = mass of the lower metal disc (C) of the apparatus [Fig. 6.1, chap-1 unit-6]

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s = Specific heat of the material of the lower disc. ( ) 2 d dt θ θ = rate of cooling			

of the lower disc at its steady temperature θ 2 under experimental condition. Bedford's correction factor 2 2 2 r h f r h + = + ... (8.3) where, r = radius of the lower circular metal disc (C) h = thickness of the lower disc. Therefore, considering Bedford's correction, thermal conductivity of the material of the sheet is ( ) 2 1 2 2 2 2 ? ? θ ? ? + ? ? = × + θ - θ dQ msd dt r h K r h A .... (8.4)

88 NSOU l HPH-CC-01 where, ( ) 2 θ θd dt = rate of cooling of the lower disc at its steady temperature θ 2 without the experimental sheet (S) on it. Procedure : 1. The mass (m) of the lower disc C (Fig. 8.1). 2. (a) To find the area (A) of the disc C (or sheet S), [Fig. 8.1, chap-1) a thread is wound round C, n times (say 5 or 6) avoiding overlapping. Total length (L 1) of the thread required is measured by a metre scale and number of turns (n) is noted. Then the circumference of the disc is 1 = L L n and radius of the disc C is 2 = π L r . Therefore, the cross-sectional area (A) of the disc C is 2 2 4 L A r = π = π . (b) The specific heat (s) of the material of the disc C is found out from the table of constants. 3. After determining the vernier constant of a slide-callipers, the thickness (h) of the lower disc C is measured at its different places and mean value of h is determined. 4. The thickness (d) of the bad conducting sheet (S) is measured by a travelling microscope. At first the vernier constant of the vertical scale of microscope is determined. Then a piece of paper with some cross marks (1,2,3 etc) on it at different places is attached to the upper disc B. Now a cross mark (say 1) is focussed by the microscope keeping the experimental sheet S between the discs B and C. The reading (R 1) of the microscope is noted. Then reading NSOU l HPH-CC-01 89 (R 2) of the same cross mark (1) is taken without the sheet S between B and C. Hence, the thickness of the sheet is, d = R 1 ~ R 2 . Following the same process the value of d are measured focussing other cross-marks. The mean value of thickness (d) is calculated. 5. After noting initial errors of the thermometers T 1 and T 2 , if any, steam is passed into the chamber A. Then temperatures of B and C are recorded using T 1 and T 2 at intervals of 5 minutes untril the thermometers show steady temperature for atleast 10 to 15 minutes. The correct value of steady temperatures θ 1 and θ 2 of discs B and C are recorded incorporating the initial errors of the thermometers. 6. Now the steam chamber A and the experimental sheet Sare removed. The lower disc C is then heated slowly by a burner until its temperature is raised about 10°C higher than its steady temperature θ 2 .Then burner is removed and the disc C is allowed to cool down. Recording of temperature of disc C by a stop-watch is started when its temperature is 5° or 6°C above its steady temperature θ 2 . The temperature is noted at an interval of 15 sec or 30 sec. until its temperature falls below its steady temperature θ 2 by about 5° or 6ºC. 7. To find ( ) 2 θ θd dt , we may draw a graph plotting time (t) along X-axis and the corresponding temperature (θ) along Y-axis during cooling. This curve is called cooling curve (Fig.1). A tangent is drawn at the point on the curve corresponding to steady temperature θ 2 of disc C. Measuring the slope of the tangent the value of ( ) 2 θ θd dt can be found out.

90 NSOU l HPH-CC-01 Let P be a point of the curve corresponding to temperature θ 2 . Then the slope of the tangent AB drawn at P = tan α = AC BC ( ) 2 θ θ = d dt . 8. Thus knowing the values of m, s, A, d, (θ 1 – θ 2), r, h and ( ) 2 d dt θ θ , the value of thermal conductivity K of the material of bad conducting sheet can be calculated using the equation (8.4). Experimental results : (A) Mass (m) and specific heat (s) of the material of lower disc C Mass of the lower disc C =

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m = ... kg (supplied) Specific heat of the material of the disc C = s = ... J kg –1 K –1 (

supplied) A P C B α Fig.1 Time (t) in s T emp (θ) in 0 C θ 2

NSOU l HPH-CC-01 91 No. of obs. Mean (L′) cm Circumference of S or C ( ) 100 ′ = L L m n Radius of sheet S or disc C ( ) 2 L r m = π Area of sheet S (A = πr 2)m 2 1. 2. 3. 4. 5. Length of thread for n turns (L′) cm (B) Determination of radius (r) and area (A) of the sheet S or disc C. Table-1 (C) Determination of thickness (h) of the lower disc C by slide callipers. Vernier constant of slide callipers = ... cm Instrumental error of slide callipers (e) = + ... cm Table-2 No. of obs. 1. 2. 3. 4. 5. Reading of Main scale (S) cm Vernier (V = v.r × v.c) cm Total (h′ = S+V) cm Mean (h′) cm correct (h = h 1 – e) cm correct thickness (h) m. ... ...

92 NSOU l HPH-CC-01 (D) Determination of the thickness (d) of the sheet S by a travelling microscope. vermier constant of the vertical scale of travelling microscope (v. c) = ... cm Table-3 (E) Time-temperature records of B and C for steady state : Room temperature = ... °C Initial Corrections, if any, between the thermometers = ...°C Table-4 No. of obs. 1. 2. 3. 4. 5. Thickness of the sheets (d) = (R 1 ~ R 2) cm Mean value of (d) cm Reading of cross-mark on B with sheet S between B and C Reading of the same cross- mark on B without the sheet S between B and C. Mean (d) m Main scale reading (S) cm Vernier reading (V = v. r × v. c) cm Total R 1 = (S + V) cm Main scale reading (S) cm Total R 2 = (S + v) cm Time in minutes → Temperature of disc B (°C)→ Temperature of disc C (°C)→ 0 5 10 15 20 25 30 35 40 ... ... = θ 1 = θ 1 = θ 1 ... ... = θ 2 = θ 2 = θ 2

NSOU l HPH-CC-01 93 (F) Time-temperature records of the disc C during its cooling. Room temperature = ...°C Table-5 Calculations : From equation (8.4), we get, ( ) ( ) 2 1 2 2 2 2 2 θ θ + = × + θ - θ d mRd dt r h K r h A The value of ( ) 2 d dt θ θ is obtained from the cooling curve (Procedure-7). Substituing the known values of m, s, d, A, r, h, (θ 1 – θ 2) and ( ) 2 d dt θ θ in the above expression, the value of thermal conductivity (K) is obtained. Result : Thermal conductivity of the material of the bad conducting sheet K = ... Wm –1 K –1 Discussions : 1. The diameter of the sheet S should be large in comparison with its thickness to minimise loss of heat due to radiation. 2. Steady temperatures θ 1 and θ 2 of disc B and C should be recorded for atleast Time in sec (t) → Temperature (θ) of disc C (°C)→ 0 15 30 45 50 65 ... ...

94 NSOU l HPH-CC-01 15 minutes when they remain steady. 3. During cooling, the temperature of the lower disc C should be recorded at an interval of 15 second or 1 2 minute if cooling rate is very slow. 4. The diameter of the sheet S should be made equal to those of discs B and C. 5. The apparatus should be screen off from direct heating from boiler by a wooden partition. 6. The rate of cooling of disc C is recorded without the experimental sheet on it. Bedford's correction must be used to get the rate of cooling under experimental condition. 7. We can also find rate of cooling ( ) d dt θ of disc C at its steady temperature θ 2 i.e, ( ) 2 d dt θ θ by drawing a graph with average temperature (θ) of the disc C along X-axis and the corresponding rate of cooling along Y-axis. The graph would be a straight line. Maximum proportional error : From equation (8.4), we get, ( ) ( ) 2 1 2 2 2 2 2 d msd dt r h K r h A θ θ + = × + θ - θ The above expression may be written as ( ) ( ) 2 1 2 2 2 2 4 ′ ″ θ - θ + = × + ? ? θ - θ ? ? π ? ? msd t r h K r h L

NSOU l HPH-CC-01 95 where, ( ) ′ ″ θ θ - θ = d dt t , 2 2 4 L A r = π = π Hence, maximum proportional error in K is ( ) max | 2 ′ ″ δ θ - θ δ δ δ δ δ = + + + + ′ ″ θ - θ K m d t L K m d t L ( ) 1 2 1 2 2 2 2 2 2 2 δ θ - θ δ + δ δ + δ + + + θ - θ + + r h r h r h r h (∴ s is known) As m is large, its contribution is negligible. Other errors are : δd = 2 × v. c of travelling microscope. ( ) ( ) 1 2 ′ ″ δ θ - θ = δ θ - θ = 2 × 1 division of thermometer δt ≈ 2 sec δh = v. c of slide callipers δL = 2 division of metre scale = 0.2 cm. Substituting the above errors and experimentally observed values of different quantities , we can calculate max | K K δ Therefore, maximum percentage error = max | 100% K K δ × = ... % Conclusion : Measured value of the thermal conductivity of a bad conductor is accurate within the errors involved in the experimental arrangement. Key words : (i) Thermal conductivity (ii) steady state (iii) Bedford's correction. Summary : (1) The measurement of thermal conductivity (K) of a bad conductor in the form of a disc using Lees' and Chorlton's method with Bedford's correction is discussed. (ii) In the steady or static part of the experiment the steady temperature θ 1 and θ 2 of the discs B and C are measured very accurately by the thermometers T 1 and T 2 .

96 NSOU l HPH-CC-01 (iii) In the cooling i.e, dynamic part of the experiment, the temperatures (θ) of the lower disc C and the corresponding time (t) during cooling are measurd very accurately wih the help of a precision stop watch and a sensitive thermometer T 2A without the sheet S on it. (iv) Drawing the (θ – t) curve, called cooling curve, the value of ( ) 2 d dt θ θ is calculated by drawing a tangent at the point on the curve corresponding to θ 2 . ( ) 2 d dt θ θ = slope of tangent drawn. (v) As greatest source of error lies in the measurement of rate of cooling ( ) 2 d dt θ θ , it must be calculated accurately. (vi) In the present method, the rate of cooling of disc C is determined without the experimental sheet on it. So to obtain correct value of ( ) 2 d dt θ θ under experimental condition, Bedford's correction term has been incorporated. (vii) Thermal conductivity (K) is calculated by using equation (8.4) with the observed value of the quantities involved. (viii) Precautions to be taken are discussed. (ix) Accuracy of measurement is checked. Model questions and answers : 1. What is thermometric conductivity? Ans. Thermometric conductivity or diffusivity of a substance is the ratio of thermal conductivity (K) and thermal capacity per unit volume. ∴ Thermometric conductivity (h) = K ps . Where, p and a are the density and specific heat of the substance.

NSOU l HPH-CC-01 97 2. Does the value of K depend on the dimension of the substance? Ans. No, the value of K depends only on the material of the substance. 3. Can this method be used to measure thermal conductivity of a good conducting disc? Ans. No. In case of a good conducting disc, the temperatures θ 1 and θ 2 will be nearly equal and measurment of their difference will be very difficult. 4. Can you apply this method to measure thermal conductivity of a liquid? Ans. Yes, In this case the experimental liquid is to be taken in a thin walled copper receptacle with edges made of bad conductors and then set it in place of the slab. 5. Why is the specimen taken in the form of a disc? Ans. When the specimen has small thickness and large area of cross-section, the amount of heat conducted increases. In this case, the radiation loss through the curved surface of the specimen is very small. This idea is considered to develop the theory of this experiment. 6. What are the factors on which the rate of cooling depend? Ans. The rate of cooling depends on the surface area, nature of surface and also on the temperature difference between the body and the surroundings. Rate of cooling increases with the increase of surface area and temperature difference. 7. Why do you measure the thickness of the slab (S) in situ? Ans. When the slab is placed between the two discs, its affective thickness decreases slightly and becomes less than its actual thickness. Therefore, the thickness (d) of the slab is to be measured under the experimental condition of heat flow. 8. What is he greatest source of error in this measurement? Ans. In this experiment, greatest source of error comes from the measurement of

98 NSOU l HPH-CC-01 ( ) 2 d dt θ θ , rate of cooling at the steady temperature θ 2 . 9. (a) What is Bedford's correction ? (b) If the correction is not made, what kind of error-random or systematic is introduced? (c) Can you perform the experiment without introducing Bedford's correction? Ans. (a) In actual experiment, the rate of cooling of the lower disc C at its steady temperature ( ) 2 . θ θd i e dt is measured with the experimental slab placed on it. In this case heat is radiated from the bottom and side surfaces of the disc C. However, in the present method the rate of cooling of disc C is measured without the slab on it. Heat is radiated here from top, bottom and side surfaces of the disc making ( ) 2 d dt θ θ higher. So, Bedford introduced a multiplying factor to correlate these two experiments which is known as Bedford's corretion. This correction factor is the ratio of the two radiating suface areas. (b) This error is called systematic error. (c) Yes, Bedford's correction will not be required if the rate of cooling of lower disc C is measured with the slab S on it. But the rate of cooling will be smaller in this case, hence the error in the measurement of d dt θ will be greater. 10. What is Newton's law of cooling? Ans. This law states that rate of radiation from a hot body is directly proportional to the temperature difference between the body and the surroundings provided this difference of temperature is small.

NSOU l HPH-CC-01 99 11. The experimental arrangement should be placed in a room where there is no irregular air current. Why? Ans. This experiment takes reasonable time to record the data of steady state and for drawing cooling curve. During this time the surrounding condition (air current and temperature) should remain same, otherwise there will be change in the rate of loss of heat due to radiation and convection.

100 NSOU l HPH-CC-01 Unit–9 q To determine the surface tension of a liquid by Jaeger's method Contents : The surface tension of a liquid (water) is measured by Jaeger's method. Introduction : Surface tension is a fundamental property of every liquid due to which the free surface of a liquid always try to contract spontaneously and occupy minimum surface area due to inter molecular attraction. There are several methods for the determination of surface tension of different liquids e.g., capillary rise method. Jaeger's method, Sessible drop method, drop-weight method, jet method. The expression of excess pressure inside a spherical air bubble formed inside a liquid is used to determine the surface tension of the liquid by Jaeger's method. This method is particularly suitable for determining the temperature variation of surface tension of a liquid but not for the determination of its absolute value. Apparatus : Fig. 9.1

NSOU l HPH-CC-01 101 Descriptions : C → A thin-walled glass tube with a fine capillary end O having diameter about 0.2 mm to 0.5 mm. B → A beaker containing experimental liquid (water) in which the capillary end O of tube C is vertically immersed. T → A glass tube connected to C by rubber tube. M → Manometer connected to glass tube T with the help of another rubber tube. Kerosine (generally) is taken as monometric liquid. W → Woulf's bottle where regulated supply of water from reservoir R is collected. P 1 , P 2 → Two screw caps, P 1 regulates supply of water from reservoir R to bottle W. P 2 regulates the pressure of air in the tube C. V → A vessel containing heating liquid in which beaker B is immersed. Th → A thermometer to record the temperature of experimental liquid. Sí → A stirrer. Objective ; We are concerned here to determine the surface tension of a liquid (water) with temperature by Jaeger's method. Theory : Defintion : The surface tension of a liquid

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is defined as the force acting per unit length on an imaginary line drawn on the liquid surface			

at rest. S. I. unit of surface tension is N/m. Working formula : The surface tension of a liquid (water) at a given temperature θ is given by 1 ( ) ( ) 2 = ρ - σ T f r g h d ... (9.1) where, r = radius of the spherical air bubble formed inside the experimental liquid (water) σ = density of the liquid (water) at temperature θ
102 NSOU l HPH-CC-01 h = maximum difference in heights of the manometric liquid at the moment the bubble breaks away. ρ = density of the manometric liquid at temperature θ g = acceleration due to gravity. f(r) = an unknown quantity and a definete funcion of the radius (r) having the same dimension as that of r. and ( ) 1 1 1 2 ( ) = ρ - σ T f r g h d ... (9.2) where, T 1 =Surface tension of the experimental liquid (water) at room temperture θ 1 σ 1 = density of the given liquid (water) at room temperature. h 1 = maximum difference in heights of the manometic liquid when bubble breaks away. Procedure : 1. At first, the densities of the experimental liquid (water) and of the manometric liquid (kerosine) are measured with the help of a specific gravity bottle if not supplied. 2. Making two scratch marks (say M 1 and M 2) on the tube C (Fig. 9.1, chap- 1, unit-7), their distances (say d 1 and d 2) from the orifice O of tube C are measured by means of a travelling microscope by successively focussing the orifice and scratch marks. 3. Now the experimental liquid (water) is taken in the beaker B in which the tube C is immersed vertically. Then liquid level is adjusted such that first scratch mark M 1 just coincides with the liquid level. So the depth of the orifice 0 below liquid level is d 1 . 4. The temperature of the heating liquid (starting from room temperature) kept in the vessel V is noted by thermometer Th after stirring by stirrer S′. This
NSOU l HPH-CC-01 103 temperature is actually the temperature of experimental liquid (water) in B. 5. Adjusting the rate of flow of liquid (water) by screw cap P 1 and regulating flow of air through the tube T by screw cap P 2 [Fig. 7.1, chap-1, unit-7], air bubbles are formed at the orifice O at the rate of approximately one per 10 second. 6. Determining the vernier constant of the vertical scale of the travelling microscope, the liquid level in the open arm of manometer is focussed sharply. The readings of the microscope corresponding to the highest liquid levels (at which bubble breaks away) for a number of bubbles are noted. The highest of the readings (R 1) is to be accepted. Similary, the lowest reading (R 2) of the microscope corresponding to the lowest liquid level in the closed arm of manometer is recorded. Thus the maximum difference of the liquid levels in the two arms of manometer at which bubble breaks is h 1 = (R 1 ~ R 2). 7. Then the value of f(r) is calculated by using known value of surface tension T 1 of the experiment liquid (water) at room temperature, its density σ 1 and the experimental values of h 1 , d and p is equation (9.2) 8. Again adjusting the liquid (water) level in the beaker, the depth of orifice below liquid level is made d 2 . The operations (5) to (7) are repeated and f (r) is calculated. The mean value of f(r) is then calculated. 9. To find out the value of surface tension of a liquid (water) at higher temperature, the vessel V is slowly heated to increase the temperature of the given liquid (water) in the beaker B. The temperature of liquid (water) is increased in steps of 5°C or 10°C and at each temperature the operations (3) to (6) are repeated for two depths d 1 and d 2 . Substituting the known values of f(r), h, d, σ, p and g in equation (9.1), we get the values of surface tension of the experimental liquid at different temperatures.
104 NSOU l HPH-CC-01 10. Thereafter, a graph is drawn with temperature (θ o C) along X-axis and the corresponding surface tension (T) along Y-axis. The graph is a straight line for water between 30°C – 80°C (Fig.1). From this graph, we can find out surface tension of the liquid at any arbitrary temperature. Fig. 1 Experimental results : (A) Determination of the density (σσσσσ) of the experimental liquid (water) and density (ρρρρρ) of the manometr liquid (kerosine) by specific gravity bottle. Room temperature (θ 1) = ... ºC density of water at room temperature (σ 1) = ... Kg/m 3 (supplied) (....... o C) Table-1 * omit the table if ρ and σ are supplied. Surface tension (N/m) Temperature (0 C) liquid Mass of the empty bottle (w 1) gm mass of bottle + liquid (w 2) gm mass of bottle + water at room temperature (w 3) gm Density of liquid 3 2 1 1 3 1 × kg/m w w w w - = σ - Experimental ... ... σ = ... kg/m 3 Manometric ... ... ρ = ... kg/m 3

NSOU l HPH-CC-01 105 (B) Measurement of the depth of the orfice below the liquid vernier constant (v.c) of the vertical scale of travelling microscope = ... cm Table-2 [N. B. Students may perform experiment for one depth (d) only, if not instructed for two depths] (C) Determination of f(r) Room temperature (θ 1) = ... o C Density of experimental liquid (water) at room temperature (σ 1) = ... kg/m 3 (supplied) Density of the manometric liquid (ρ) = ... kg/m 3 (from table-1) Microscope focussed on Lower scratch mark M 1 Upper scratch mark M 2 Orifice 0 Microscope reading Depth (cm) Depth (m) Main scale (S) cm vernier scale (V) = v. r × v c (cm) Total = (S + V) (cm) Mean (cm) ... ... R 1 = ... d 1 = (R 1 – R 0) d 1 = ... = ... R 2 = ... d 2 = (R 2 – R 0) d 2 = ... = ... R 0 = ...

106 NSOU l HPH-CC-01 Surface tension of the experimental liquid (water) at room temperature = ... N/m (supplied) Varnier constan of the microscope = ... cm, g = 9.8 m/s 2 Table-3 Mean of highest liquid level (R 1) (cm) d 1 = ... Depth of orifice O (m) Readings for constant lowest liquid level (R 2) in closed arm of manometer (cm) Readings for constant highest liquid level (R 1) in the open arm of manometer (cm) 1. 2. 3. 4. 5. 1. 2. 3. 4. 5. d 2 = ... Bubble number Main scale (S) Vernier scale (V) = v.r × v. c Total Rs. R 1 =S+V Main scale (S) Vernier scale (V) = v.r × u. c Total R 2 = S + V Mean of lowest liquid level (R 2) (cm) Maxi- mum differ- ence in height h R R 100 1 2 = - ? ? ? ? ? ? ? ? (m) f(r) (m) Mean f(r) (m)

NSOU l HPH-CC-01 107 (D) Record of temperature and manometric liquid level differences : vernier constant of the vertical scale of microscope (v.c) = ... cm Table-4 Mean of R 1 (cm) 1. No. of obs. Reading for constant lowest liquid level (R 2) is closed arm of manometer (cm) Readings for constant highest liquid level (R 1) in the open arm of manometer (cm) temperature of liquid (°C) Main scale (S) Vernier scale (V) = v.r × v. c Total R 2 = S + V Mean of R 2 (cm) etc etc etc etc etc etc etc etc etc etc etc etc etc Room temperature (θ 1) + 5°C = θ 2 ºC 3. Depth of orifice 0 (m) d 1 = ... 1. 2. 3. 4. 5. Bubble Number Main scale (S) Vernier scale (V) = v.r × v.c Total R 1 = S+ V (m) 1. 2. 3. 4. 5. d 2 = ... 2. Room temperature (θ 1) + 10°C θ = 3 ºC d 1 = ... d 2 = ... h 1 = ... h 2 = ... ( ) 1 2 100 R R h - =

108 NSOU l HPH-CC-01 (E) Temperature—Surface tension records g = 9.8 m/s 2 , ρ = ... kg/m 3 (from table-1) f(r) = ... m (from table-3) Table-5 Calculations : From equation (9.1), we get, 1 ( ) ( ) 2 = ρ - σ T f r g h d Substituting the known values of f(r), h, ρ, d, σ, g in the above expression, we can calculate value of T at a particular temperature (shown in table-5), but it is not the absolute value. But to show the variation of surface tension of the liquid with its temperature, θ – T curve is to be plotted (consult procedure-10). The curve shows that surface tension of a liquid decreses with the rise of temperature. Using this graph we can also find out surface tension of the liquid at any arbitrary temperature provided the temperature lies within the range of observed value. No. of obs. Depth of orifice (m) Corresponding h (m) (from table-4) Surface tension T = 1 2 f(r) × g(hρ – dσ) (N/m) Temperature of liquid (θ) (o C) 1. Density of experimental liquid (σ) (kg/m 3) [from table-1] Room temperature (θ 1) + 5°C = θ 2 = ...°C 2. Room temperature (θ 1) + 10°C = (θ 3 = ...°C d 1 = ... Mean T (N/m) h 1 = ... T 1 = ... T = ... d 2 = ... h 2 = ... T 2 = ... d 1 = ... h 1 = ... T 1 = ... T = ... d 2 = ... h 2 = ... T 2 = ... 3. 4. 5.

NSOU l HPH-CC-01 109 Result : The experimental value of the surface tension of the experimental liquid (water) at .....°C = ... N/m. Discussions : 1. The apparatus should be perfectly air tight. 2. The narrow experimental tube should be free from any cantamination like grease etc. 3. The manometric liquid should be of low density and non-volatile in nature. This makes the difference of liquid levels in the two limbs of manometer large. 4. Temperature of the liquid bath should be kept steady for a considerable time during the record of liquid level of manometer for each set. 5. Rate of bubbling should be made steady and slow for convenience of observation. Maximum Proportional error : From equation (9.1), we get, Surface tension ( ) ( ) 1 .....(9.1) 2 = ρ - σ T f r g h d During measurement of f(r), value of T at a particular temperature is supplied So, we may write ( ) ( ) max ( ) | δ ρ - σ δ = ρ - σ h d f r h d f r ... (9.3) Assuming small variation of T and other quantities ( ) max ( ) | ( ) δ ρ - σ δ δ = + ρ - σ h d f r T T f r h d or, max 2 ( ) | ( ) f r T T f r δ δ =

110 NSOU l HPH-CC-01 or, ( ) max | 2 [using 9.3] δρ + ρδ + σδ δ = × ρ - σ h h d T T h d As contribution of term hδρ is very small (∴ ρ is supplied) Therefore, ( ) max 2 | ...(9.4) ρδ + σδ δ = ρ - σ h d T T h d where, δh = δd = 2 × v.c of the microscope. Substituting measured values of h, σ we can calculate max | T T δ ∴ Maximum percentage error max | 100% ......% T T δ = × = Conclusion : Measured value of the surface tension of liquid is accurate within the errors involved in our experimental arrangement. Key Words : (i) Surface tension (ii) Specific gravity bottle. (iii) Excess pressure Summary : (i) Surface tension of a liquid (water) has been defined and its measurement by Jaeger's method is discussed. The expression for excess pressure inside a spherical air bubble formed inside a liquid is used to determine surface tension. (ii) To find surface tension of a liquid at temperature other than room temperature, the value of f(r) is calculated by using known value of surface tension of the liquid (water) at room temperature and the measured values of h, d, ρ and σ from equation (9.2).

NSOU l HPH-CC-01 111 (iii) Using this value of f(r) and the measured values of h at different temperatures for fixed depth, we have calculate surface tension of experimental liquid (water) at different temperatures, higher than room temperatures using equation (9.1). (iv) Drawing θ – T graph, we can calculate the surface tension of liquid at any temperature provided the temperature lies within the range of temperature observed. (v) Precautions have been discussed and the accuracy of measurement is checked. Model Questions and answers : 1. How does surface tension vary with temperature? Ans. With the rise of temperature surface tension of a liquid decreases and vanishes at the critical temperature of the liquid. For small range of temperaure, surface tension at θ°C is T θ = T 0 (1 – αθ) where T O = surface tension at 0°C α = temperature co-efficient of surface tension. 2. What will happen to surface tension if oil or grease is added to a liquid? Ans. Surface tension will reduce considerably. 3. What will happen if you increase the depth of orifice? Ans. With the increase of depth of orifice, the manometric level difference (h) will increase. 4. Can water or mercury be used in the manometer? Ans. No, Due to high density of mercury and water, the difference of liquid levels (h) in the manometer will be very small. So accurate measurement of h will not be possible. Therefore, manometric liquid should be of very low density e. g., Kerosine, Xylol.

112 NSOU l HPH-CC-01 5. How is bubble formed? Ans. When water enters the bottle W coming from reservoir R, the air above water level in bottle W is compressed. When this compressed air enters the tube C, it escapes through the orifice in the form of bubbles. 6. On what factors the accuracy of the experiment depend? Ans. The accuracy of this experiment depends on (i) the narrowness of capillary tube C (ii) the precise measurement of manometer reading just at the time of breaking away of the bubble.

NSOU l HPH-CC-01 113 Unit - 10A q Determination of the focal length of a concave lens by combination method Introduction : We can measure the focal length of a convex lens by displacement method or u – v method. But these methods can not be used to determine the focal length of a concave lens since a concave lens produces only virtual image which can not be cast on a screen. The focal length of a concave lens can be determined by employing any one of the following methods (i) combination method (ii) auxiliary lens method (iii) concave mirror method. In this unit, the determination of focal length of a concave lens by combination method has been discussed. If a convex lens of shorter focal length is kept in contact coaxially with a concave lens of longer focal length the combination will behave as a converging lens forming real image. We can measure the equivalent focal length of the combination by displacement method. By finding the focal length of convex lens of the combination and equivalent focal length, the focal length of the concave lens can be found out. Objective : To measure the focal length of a concave lens by combination method. Theory : Definition : The focal length of a lens is defined as the image distance when the object distance is infinity i.e., incident rays are parallel. S.I. unit of focal length is m. Working formula : The focal length of a concave lens is given by 1 2 1 Ff f F f = - ... (10.1)

114 NSOU l HPH-CC-01 where, f 1 = focal length of the convex lens of the combination. F = focal length of the combined lens. Focal length of a convex lens is given by the relation 2 2 4 D x f D - = ... (10.2) where, D = distance between the object and the screen which is greater than 4f. x = distance between two positions of the lens for which it forms sharp images on the screen. We can find f 1 and F using equation (10.2) Procedure : 1. Placing the object and the screen on their stands at two extreme ends of the optical bench, reading (a) of the object stand is noted from its index mark and bench scale. The position of the object stand should be kept fixed throughout the experiment. 2. Then index error (λ) between the object and the screen is to be determined. Keeping the index rod between the object and screen stand horizontally, the object and screen are made to touch the two ends of the index rod. The readings (x) and (y) of the two stands are taken, their difference (d) = X ~ Y gives the apparent length of the index rod. The length (l) of the index rod is measured by a metre scale. Then index error is λ = l – d. 3. Now the screen is placed at sufficient distance from the object and a convex lens is placed between the object and the screen. Adjusting the heights of the three stands the centres of object, screen and lens are placed on the same horizontal line. 4. Then the position of the screen is so adjusted that for two different positions of the convex lens which are small distance apart, we get two images—one NSOU l HPH-CC-01 115 magnified and other diminished. Keeping the position of the screen fixed, the reading (b) of the screen stand is noted from its index mark and bench scale. So, the apparent distance between the object and the screen is D 1 = a ~ b and correct distance, D = D 1 + λ. 5. The convex lens is then placed nearer to the object and its position is adjusted until a sharp magnified image is formed on the screen. The reading of the lens stand is taken. Repeating the process thrice, the mean of these readings (R 1) is calculated. This is known as first position of the lens. 6. Next, the convex lens is kept nearer to the screen and its position is adjusted carefully until a sharp disminished image is formed on the same screen. The position of lens stand called second position is noted. Repeating the operation three times, the mean of the readings (R 2) is found out. 7. So, the displacement of the convex lens is x = R 1 ~ R 2 . Using the values of x and D, the focal length (f 1) of the convex lens is determined using eq. (10.2) 8. Shifting the position of the screen, the operations (4) to (7) are repeated for atleast three different values of D and x. Then finding three values of f 1 , the mean value of f is determined. This gives the actual value of

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focal length (f 1) of the convex lens. 9. Now, the concave lens is kept in contact coaxially with the convex lens and the combination is placed between the object and the screen. The focal length (F) of the

combination is determined by displacement method in the same manner as adopted for the convex lens. 10. Substituting the values of f 1 , F in equation (10.1), the focal length (f 2) of concave lens is obtained.
116 NSOU l HPH-CC-01 Experimental results : (A) Determination of index error (λλλλλ) for object and screen positions : Table-1 (B) Determination of object-screen distance (D), displacement of lens (x) and focal length (f 1) of convex lens. Table-2 Length of the index rod (l) cm When two ends of the index rod touches the object and screen Apparent length of the index rod (d = X ~ Y) Index error λ = (l – d) cm Position of object (X) cm Position of screen (Y) cm ... ... 1. ... ... No. of obs. Position of Object (a) cm Screen (b) cm Apparent distance between Object and screen (D 1 = a ~ b) cm Corrected value of D = D 1 + λ Reading for lens in Displce- ment of the lens x = (R 1 ~ R 2) cm Focal length 2 2 1 4 - = D x f D cm Mean focal length f 1 cm First position (R 1) cm Mean R 1 cm Second Position (R 2) cm Mean R 2 cm ... ... 2. ... ... 3. ... ...
NSOU l HPH-CC-01 117 (C) Determination of the focal length (F) of the combined lens. Table-3 [Make a table same as Table-2] Calculations : From equation (10.1), we get 1 2 1 F f f F f = - Substituting the values of f 1 and F (from table-2 and table-3) in the above expression, the value of f 2 is obtained. Result : The focal length of the concave lens f 2 = ... cm Discussions : 1. To make proportional error minimum, the value of D (D < 4f) should be adjusted such that x is small. 2. The distance (D) between the object and the screen must be greater than 4f to get two real images for two different positions of the lens. 3. No index correction is necessary for x since it is the difference in the readings for two lens positions. 4. The focal length (f 1) of the convex lens must be less than the focal length (f 2) of the concave lens i.e., f 1 > f 2 so that lens combination behaves as a converging system producing real image. This is essential to find F. Maximum proportional error : The focal length of the concave lens is given by (Equation 10.1) 1 2 1 Ff f F f = -
118 NSOU l HPH-CC-01 Therefore, the maximum proportional error ( ) 2 1 1 max 2 1 1 | 10.3

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δ δ δ + δ δ = + + - The right hand side of equation (10.3) can be calculated by using the following expression. max 2 2 2 2 | f D D x x D f D D x δ δ + δ δ = + - 2 2 4 ? ? - = ? ? ? ? ∵ D x f D δD = 2 smallest divisions of bench scale δx = Distance moved by the lens without affecting focussing of the image. Using, the value of observed quantities, 2 max 2 | f f δ can be calculated from equation (10.3). F should have higher value than f 1 to reduce the error due to last term of equation (10.3). ∴ Maximum percentage error = 2 max 2 | 100% ....% δ × = f f Conclusion : The measured value of the focal length of the concave lens is accurate within the errors involved in the experimental arrangement. Key words : (i) Focal length (ii) displacement method (iii) Index error (iv) combination method. Summary : (i) Focal length of a lens is defined and the method of determination of the focal length (f 2) of a cancave lens by combination method is discussed. (ii) Index error (λ) between the object and the screen is determined in order to find the actual distance (D) between the object and the screen.

NSOU l HPH-CC-01 119 (iii) The focal length (f 1) of a convex lens and the focal length (F) of the combined lens is determined by adopting displacement method using equation (10.2). (iv) Substituting the values of f 1 , F in equation (10.1), the focal length (f 2) of concave lens is obtained. (v) Precautions for measurement of f 2 is discussed and evaluation of proportional error is mentioned. (vi) Accuracy of the mesurement is checked. Model questions and answers : 1. How many focal length a lens has and which one is measured here ? Are these focal lengths equal ? Ans. A lens has two focal lengths—first and second. First focal length is the object distance when image distance is infinity whereas second focal length is the image distance when object distance is infinity. In the experiment we are measuring second focal length of the lens. Two focal lengths of the lens will be equal when the lens is surrounded by the same media on both sides. 2. Does the focal length of a lens change with colour? Ans. Yes. We know that the lens maker's formula for a lens is ( ) 1 2 1 1 1 1 ? ? = μ - - ? ? ? ? f r r where, μ = Refractive index of the material of lens with respect to the surrounding medium. r 1 , r 2 = Radii of curvature of the two lens surfaces. f = focal length of the lens. The relation shows f that decreases with the increase of μ. It is known that refractive index of a medium is greater for violet light than for red light i.e, μ < μ v r . So focal length of a lens is greater for red light that for violet light. Focal length (f) of a lens also changes with curvature r, as r increases f decreases.

120 NSOU l HPH-CC-01 3. If it is found that the convex lens is not producing real image for any position of the lens, what is the reason? Ans. This will happen only when the distance (D) between the object and screen is less than four times focal length (f) of the convex lens i.e., D > 4f. 4. Can a concave lens produce real image? Ans. Yes. A concave lens can produce real image when the refractive index of the surrounding medium is greater than the refractive index of the lens material. 5. When you immerse the lens in water, will its focal lenths be same as before? Ans. No. The focal length of the lens in water will be almost four times greater than in air. 6. What will happen if the distance (D) between the object and screen is large ? Ans. If D is large, than the displacement of lens (x) will also be of large value. As a result, we will get very diminished image for one position of lens and highly magnified image for other position of lens. This will cause difficulty in focussing the images sharply. 7. Can you measure the size of the object from this experiment? Ans. Yes. If I 1 and I 2 be the sizes of the images for two positions of the lens, the size of the object 1 2 =O I I . 8. Is displacement method better than u – v method? Ans. Yes. In the displacement method, x and D can be measured mere accurately, the index corrction is necessary only for D and the thickness of the lens is not required. 9. In the combination method, can you use a convex lens of any focal length to measure the focal length of a concave lens? Ans. No. As a concave lens cannot produce real image, a convex lens of high power i.e., smaller focal length should be combined with the concave lens such that the combination behaves as a converging lens system.

NSOU l HPH-CC-01 121 10. Why index correction for x is not considered here? Ans. We do not measure index error (λ) because the displacement of lens is equal to the displcement of the index mark of the stand carrying the lens. 11. How can you measure the focal length of a convex lens where value is about 75 cm by using an optical bench of length 1.5 m? Ans. We can measrue the focal length (f) of this lens by displacement method since f 1 has high value. At first, you have to measure the focal length of a convex lens by displacement method with value 20-30 cm. Then combining this lens with the unknown lens of focal length (f 2) you measure combined focal lengths. (F) by displacement method. By using formula, 1 2 1 1 1 F f f = + we can find f 2 .

122 NSOU l HPH-CC-01 Unit–10B q Determination of the focal length of a convex lens by displacement method Introduction : There are different methods for the determination of focal length of a convex lens e.g., plane mirror method, u – v method, displacement method. The displacement method is more reliable than u – v method regarding the determination of focal length of a convex lens. Fig. 10 B.1

NSOU l HPH-CC-01 123 Theory : Working formula : The focal length of a convex lens is given by 2 2 4 D x f D - = ... (10.2) where, D = distance between the object and the screen which is greater than 4f. x = distance between two positions of the lens for which it forms sharp images on the screen. Procedure : [Same as 1-8 of unit 10A] Experimental results : [Make table-1 and table-2 of unit 10A] Calculation : From equation (10.2), we have, 2 2 4 D x f D - = Substituing the values of D, x in the above relation, we get the focal length of of convex lens. Result : The focal length of the given convex lens f = ... cm Discussions : [1 – 3], (same as of unit 10A) 4. We can also find f by drawing a graph with D along X-axis and 2 x D along y- axis. The graph is a straight lne. The point where the straight line intersects X-axis has coordinate (4f, 0). Thus D – 4f = 0 or, 4 D f = . Maximum proportional error : Focal length of convex lens

124 NSOU l HPH-CC-01 2 2 4 - = D x f D ∴ Maximum proportional error : max 2 2 2 2 | δ δ + δ δ = + - f D D x x D f D D x where, δD = 2 smallest division of bench scale. δx = Distance through which lens can be moved without affecting focusing. Using set of observed vales of D and x, we can calculate the maximum percentage error. = max | 100% ....% f f δ × = Conclusion : The measured value of the focal length of the convex lens is accurate within the errors involved in the experimental arrangement. Key words : (i) Focal length, (ii) displacement method, (iii) Index error. Summary : (i) The method of determination of focal length of a convex lens by diplacement method is discussed. (ii) Index error (λ) between the object and screen is determined to find the actual distance (D) between object and screen. (iii) The focal length of convex lens is determined by adopting displacement method finding D, x and using equation (10.2) (iv) Precaustions for measurement of f is discussed and evalutation of proportional error is mentioned. (v) Accuracy of the measurement is checked.

NSOU l HPH-CC-01 125 Model questions and answers : 1. How many focal length a lens has and which one is measured here ? Are these focal lengths equal ? Ans. A lens has two focal lengths—first and second. First focal length is the object distance when image distance is infinity whereas second focal length is the image distance when object distance is infinity. In the experiment we are measuring second focal length of the lens. Two focal lengths of the lens will be equal when the lens is surrounded by the same media on both sides. 2. Does the focal length of a lens change with colour? Ans. Yes. We know that the lens maker's formula for a lens is ( ) 1 2 1 1 1 1 ? ? = μ - - ? ? ? ? f r r where, μ = Refractive index of the material of lens with respect to the surrounding medium. r 1 , r 2 = Radii of curvature of the two lens surfaces. f = focal length of the lens. The relation shows f that decreases with the increase of μ. It is known that refractive index of a medium is greater for violet light than for red light i.e, μ < μ v r . So focal length of a lens is greater for red light that for violet light. Focal length (f) of a lens also changes with curvature r, as r increases f decreases. 3. If it is found that the convex lens is not producing real image for any position of the lens, what is the reason? Ans. This will happen only when the distance (D) between the object and screen is less than four times focal length (f) of the convex lens i.e., D > 4f. 4. Can a concave lens produce real image? Ans. Yes. A concave lens can produce real image when the refractive index of the surrounding medium is greater than the refractive index of the lens material.

126 NSOU l HPH-CC-01 5. When you immerse the lens in water, will its focal lenths be same as before? Ans. No. The focal length of the lens in water will be almost four times greater than in air. 6. What will happen if the distance (D) between the object and screen is large ? Ans. If D is large, than the displacement of lens (x) will also be of large value. As a result, we will get very diminished image for one position of lens and highly magnified image for other position of lens. This will cause difficulty in focussing the images sharply. 7. Can you measure the size of the object from this experiment? Ans. Yes. If I 1 and I 2 be the sizes of the images for two positions of the lens, the size of the object 1 2 =O I I . 8. Is displacement method better than u – v method? Ans. Yes. In the displacement method, x and D can be measured mere accurately, the index corrction is necessary only for D and the thickness of the lens is not required. 9. In the combination method, can you use a convex lens of any focal length to measure the focal length of a concave lens? Ans. No. As a concave lens cannot produce real image, a convex lens of high power i.e., smaller focal length should be combined with the concave lens such that the combination behaves as a converging lens system. 10. Why index correction for x is not considered here? Ans. We do not measure index error (λ) because the displacement of lens is equal to the displcement of the index mark of the stand carrying the lens. 11. How can you measure the focal length of a convex lens where value is about 75 cm by using an optical bench of length 1.5 m? Ans. We can measrue the focal length (f) of this lens by displacement method since f 1 has high value. At first, you have to measure the focal length of a convex

NSOU l HPH-CC-01 127 lens by displacement method with value 20-30 cm. Then combining this lens with the unknown lens of focal length (f 2) you measure combined focal lengths. (F) by displacement method. By using formula, 1 2 1 1 1 F f f = + we can find f 2 .

128 NSOU l HPH-CC-01 Unit–11 q Adjust a spectrometer for parallel rays by Schuster's Method and to find out the angle of a prism Contents : A spectrometer is adjusted for parallel rays by Schuster's method and the angle of a prism is measured. Introduction : Spectrometer is an indispensible scientific optical instrument used for wide varieties of purposes. In the laboratory, for the mesurement of different optical parameters by a spectrometer, the first thing we require to move the object to infinity and that is achieved by Schuster's method of adjustment of the spectrometer. Spectrometer is normally used for recording and analysing optical spectra as a method of analysis, for measuring angle of a prism and refractive indices. (A) Spectrometer : its construction Fig. 11.1 A spectrometer consists of the following parts : (i) prism table (P) (ii) collimator (C) (iii) Telescope (T) (iv) Circular scale (C.S)

NSOU l HPH-CC-01 129 (1) Prism table (P) : It is a circular, horizontal plate capable of rotation about the vertical axis of the instrument. It can be fixed at any desired height by screw S 1 . The table can be made horizontal by its three levelling screws E, F, G (fig. 11.1). On the surface of the table, a set of equidistant straight lines are ruled parallel and perpendicular to the line joining screws E and F. A series of concentric circles with the centre of the table as centre are also ruled on the prism table. The table can be rotated about a vertical axis coinciding with the axis of instrument. The angle of rotation can be measured by verniers V 1 and V 2 . The table can be fixed by fixing screw F 2 , a tangent screw T 2 can impart small rotation. (ii) Collimator (C) : It is a hollow horizontal tube with achromatic converging lens O 2 at one end and adjustable slit S at the other end. By rack and pinion arrangement R 2 (Fig. 11.1) the distance between the slit and lens can be changed. The axis of the collimator should be horizontal and perpendicular to the vertical axis about which prism table rotates. By means of screws c and d collimator tube is made horizontal. (iii) Telescope (T) It is a small astronomical telescope having objective lens O 1 which is an achromatic doublet of concave and convex lens and a Ramsden type eye piece E carring cross- wires. By adjusting rack and pinion arrangement R 1 , the distance between objective and cross-wires can be changed. The telescope axis can be made horizotal by means of its two screws and a and b . The telescope axis should be also horizontal and perpendicular to the vertical axis of rotation of the prism table. The telescope can be rotated about vertical axis and the amount of rotation can be measured by verniers V 1 and V 2 . Telescope is provided with a fixing screw F 1 and tangent screw T 1 . The whole apparatus is supported by levelling screws S 1 , S 2 and S 3 (Fig. 11.1)

130 NSOU l HPH-CC-01 (iv) The circular scale (C. S) : A circular scale graduated in degrees is provided with the spectrometer. The scale is rigidly attached to the telescope and turned with its rotation. There are two verniers V 1 and V 2 , 180° apart which rotate over the fixed circular scale with the rotation of the prime table. (B) Theory of Schuster's method of focussing : When parallel rays of light coming from an object passes through a prism, the emerging rays appear to come from a point which is the virtual image. If u = object distance from prism. v = image distance from prism. i, r = angle of incidence and angle or refraction at the first refracting surface. r 1 , i 1 = angle of incidence and angle of emergence at second refracting surface. The relation between u and v is 2 1 2 1 2 2 cos cos cos cos ? ? ? ? ? ? ? ? = ? ? ? ? ? ? i u r v i r ... (i) when prism is at the minimum deviation position (A 1 B 1 C 1), i = i 1 and r = r 1 , Therefore, at this position u = v.

NSOU l HPH-CC-01 131 Slant position or slanting position : Fig. 11.2 : Slant position of the prism After rotating the prism when the prism is placed at the position A 2 B 2 C 2 (Fig. 11.2), the angle of incidence (i) is greater than that at minimum deviation. This position of prism is called slant position. In this case, angle of incidence (i) is greater than angle of emergence (i 1) and 2 2 1 2 2 1 cos cos cos cos < i i r r . Thus v < u i.e, image (I 2) is formed at longer distance, appears thin and blurred. When image is sharply focussed by rack and pinion arrangement of telescope, image appears very thin. In this case, the telescope is focusssed for longer distance as image is formed at a longer distance from object. i 1 A 1 A 2 C 1 C 2 B 1 B 2 O I 2 i i < i 1

132 NSOU l HPH-CC-01 Normal position : Fig. 11.3 Normal position of the prism By rotating the prism when prism is taken to the position A 3 B 3 C 3 (Fig. 11.3), the angle of incidence is less than that at minimum deviation. This position of the prism is known as normal position. Under this condition, angle of incidence (i) is less than angle of emergence (i 1) i.e, i > i 1 and 2 2 1 2 2 1 cos cos cos cos > i i r r . Thus, v > u, image (I 3) is formed nearer to the prism than object (Fig. 11.3). Slit image, in this case, appears broad and blurred. By turning the focussing screw R 2 of the collimator, the image is made sharp, distinct but broad in size. By this adjustment image is taken to a longer distance. Again the prism is rotated in the opposite direction and taken to the slant position, A 1 A 3 I 3 C 3 O B 1 i i 1 C 1 B 3 i > i 1

NSOU l HPH-CC-01 133 the image appears thin and goes out of focus. By using screw R 1 , the image is focussed by telescope which is formed at still more longer distance. The above operations are repeated several times alternately till the image is sharp for both the positions of the prism. For each normal postion of the prism, by adjusting collimator the image is taken to a longer distance which was already focussed by telescope for longer distance. During focussing of image for next slant position, image is formed at still more longer distance. In this way, after few repeated operations, the telescope will be focussed for infinity i.e, image is formed at infinity. Now the spectrometer is focussed for parallel rays and the telescope and collimator are adjusted for parallel rays. Objective : We intend to discuss the method of adjustment of a spectrometer for parallel rays by Schuster's method and to find the angle of a prism. Theory : (A) Schuster's method of focussing for parallel rays [See above] (B) Principle of measurement of the angle of a prism (A) When parallel rays coming from the a source is incident equally on the two refracting surfaces of a prism, the angle between these two reflected rays (θ) is equal to twice the angle of prism (A) i.e, θ = 2A or, 2 A θ = Procedure : 1. Adjustment of the spectrometer Before starting the experimental work with a spectormeter, the following

134 NSOU l HPH-CC-01 adjustments are to be performed in the given order : (i) Levelling of the telescope, collimeter and prism table. (ii) Alignment of the source and slit. (iii) Focussing of the cross-wire of eye piece of telescope (iv) Adjustment of the slit. (v) Focussing for parallel rays : Schuster's method. (i) (A) Levelling of the telescope : (a) After placing a spirit level along the length of telescope, it is made parallel to the line joining base screws S 1 and S 2 (Fig. 8.1). Keeping by the side of S 2 if the bubble of spirit level is not at the centre, it is brought there halfway towards centre by turnning the two screws S 1 , S 2 in the opposite directions by equal amount. For other half, the screws (a, b) below telescope is used to take bubble at the centre. (b) The telescope is now turned through 180° and placed parallel to the first position. If the bubble is not at the centre, it is taken to the centre by the above process. The above operations (a) and (b) should be repeated several times until the bubble remains at the centre for both positions. (c) Next, the telescope is placed in line with collimator i.e., perpendicular to the line joining levelling serews S 1 and S 2 . The bubble of the spirit level, if displaced. is brought to the centre by turning S 3 screw alone. (If there is no third screw S 3 , screws S 1 and S 2 should be used to bring bubble at the centre.)

NSOU l HPH-CC-01 135 When the bubble of spirit level remains at the centre for all positions of the telescope, then telescope axis is said to be horizontal and its rotation axis is said to be vertical. (B) Levelling of Collimator : Placing a spirit level along the length of a collimator tube, the bubble is brought at the centre by turning the screws c and d below the collimator. (C)Levelling of prism table : The spirit level is placed at the centre of prism table being parallel to the line EF, joining its two levelling screws E and F. The bubble is now taken to the centre by turning E and F equally in opposite directions. Now the spirit level is kept perpendicular to EF line and third serew is rotated to bring the bubble at the centre. This adjustment makes the top of the prism table horizontal. (ii) Alignment of the source and the slit. To make the slit image clear and bright during observation through collimator and telescope, the brightest portion of the source is to be placed in front of the slit. The slit should be kept vertical. (iii) Focussing of the cross-wire of eye-piece of telescope : After turning the telescope towards the illuminated slit, the cross-wires of the eye piece is observed. If cross-wires are not clear, the eye piece is adjusted to make the cross-wires distinct. (iv) Adjustment of the slit : After making alignment of the slit, the telescope is placed in line with the collimator and image of the slit of observed. By turning the focussing screw of the telescope and collimator, the image is made sharp. The image of slit is made vertical by turning the slit.

136 NSOU l HPH-CC-01 The width of the slit is adjusted by screw G 1 so that it is very small (1 or 2 mm) and boundary edges of the slit are shaply defined. (v) Focussing for parallel rays : Schuster's method Schuster's method is the best method of focussing the telescope and collimator for parallel rays in a dark room. [consult unit 8(B), chap-1] 2. Measurements (i) The vernier constant of the circular scale is determined. (ii) The prism is to be placed on the prism table with its apex conciding with the centre of the prism table and pointing towards the collimator. The refracting surfaces should be equally inclined with the parallel rays coming from the collimator. (iii) Clamping the prism table by fixing screw S′ (Fig. 11.1), the slit is illuminated by sodium vapour lamp or other source. (iv) Now rotating the telescope, it is placed at a position to receive image of the slit formed by reflection from one face of prism. By using the tangent screw the telescope is slowly rotated until the centre of cross-wire coincides with one edge (say right) of the image. The readings of the circular and vernier scale are noted. The operation is repeated thrice and mean value (R 1) is calculated separetely for two verniers V 1 and V 2 . (v) The telescope is then taken to the other side of the prism to receive the image formed by reflection from other face of the prism. Now coinciding the centre of cross-wire with the same edge (right) of slit image, readings of the circular scale and vernier scales are noted. The readings are taken thrice and their mean value (R 2) is determined. separately for two vernier V 1 and V 2 .

NSOU l HPH-CC-01 137 (vi) The difference of the two mean readings (R 1 ~ R 2 = θ) of the same vernier for two position of telescope is determined separately. The mean (θ) of these two differences is calculated. Then the angle of the prism is given by 2 A θ = . Experimental results : Determination of the angle (A) of the prism : Value of one smallest division of the circular scale (s) min. or sec. = ... Vernier constant (v.c) of the verniers = s n . where n = number of vernier division = ... min or sec. Table-1 No. of obs. Vernier number Readings for the first image Readings for second image Difference of two mean readings = θ = R 1 ~ R 2 Mean difference (θ) = 2A Angle of the prism 2 A θ = circular scale (S) 1. 2. 3. 1. 2. 3. First Second vernier (V) = v. r × v.c Total R 1 = S + V Mean R 1 Circular scale (S) Total R 2 = S + V vernier (V) = v. r × v. c Mean R 2 … … … … … … … … … … … … … … … … … … … (V 1) … … … … … … (V 2) …

138 NSOU l HPH-CC-01 Calculation : The angle of the given prism 2 A θ = = Substituting the value of θ in the above expresssion, we get the value of A. Result : The agnle of the given prism A =............. degree........... min........ sec Discussions : 1. The source of light must be properly aligned. 2. The slit should be made narrow and illuminated by the brightest portion of the light source. 3. There should not be any parallax between the cross-wires and the slit image. 4. During rotation of the telescope, sometimes vernier zero crosses the zero mark of the circular scale, then angle rotated = 360° – (difference of two vernier readings) 5. While determining the angle of the prism, the vertical axis passing through the apex of the prism must pass through the centre of the prism table. 6. During measurement of the angle (θ), one particular end of the slit image must be focussed all times. 7. While using the tangent screw, the telescope or the prism table should be clamped before hand by the fixing screws. Maximum proportional error : We know, 2 A θ = - ∴ Maximum proportional error = max | A A δ δθ = θ where δθ = 2 divences of v. c. of vernier scale.

NSOU l HPH-CC-01 139 ∴ Maximum percentage error = max | 100% ......% A A δ × = Conclusion : Measured value of the angle of the prism is accurate within the errors involved in the experimental arrangement. Key words : (i) Angle of the prism (ii) Spectrometer (iii) Schuster's method of focussing. Summary : (i) Adjustment of a spectrometer for parallel rays by Schuster's method and measurement of the angle of a prism is discussed in details. (ii) Levelling of telescope, collimator and prism table have been made carefully. (iii) Alignment of source or slit are also made properly, slit is adjusted and cross wires of eye piece are focussed. (iv) By Schuster's method the spectrometer is focussed for parallel rays. (v) Angle of the prism is measured carefully by observing the image of the slit by telescope formed by reflection from two refracting surfaces of the prism. (vi) Precautions and sources of error are discussed. (vii) Accuracy of measurement is checked. Model questions and answers : 1. Why is instruments levelled ? Ans. The instrument is levelled to ensure that position of the image will not change with the change of position of the telescope.

140 NSOU l HPH-CC-01 2. Why are telescope and collimator focussed for parallel rays? Ans. If the incident rays are not paralell but diverging or converging, the distance of the image of slit will vary with the change of position of the prism. As a result, if image is focussed for one position of the prism, it will be out of focus in other positions. When the telescope and collimator are focussed for parallel rays, both the object and image will be at infinity. In this position when telescope focusses the image, the image will be in focus for all positions of the prism. 3. What type of eyepiece is there in the telescope? Ans. Ramsden's type of eye piece is used as cross wires are provided for this type of eye-piece. 4. Can you use white light in this experiment? Ans. No. White light contains seven wavelengths, so each colour will produce an image of the slit. This will make experiment difficult for measurement. 5. What is monochromatic light ? Do you consider sodium light strictly monochromatic? Ans. Light having a particular wavelength is called monochromatic light. Sodium light is not strictly monochromatic, it contains light of two wavelengths of values 5890A° and 5896A°. 6. What kind of image is produced by the telescope? Ans. The telescope produces virtual image at infinity. The objective of telescope produces a real diminishd image while the eye piece produced magnified virtual image.

NSOU l HPH-CC-01 141 7. What is there inside a collimator? Ans. At one end of the collimator tube there is an achromatic converging lens and at the other end there is an adjustable slit. 8. Can you expect an emergent ray for any incident ray on the prism? Ans. No. There is a certain range of the angle of incidence for a prism of definite angle (A) within which emergent rays are obtained.

142 NSOU l HPH-CC-01 Unit–12 q To determine an unknown low resistance using Potentiometer Contents : Value of low resistance is measured in line with Principle of Potentiometer. Introduction : We have learned from Ohm's law how to measure resistance. However, Ohm's law need not be obeyed by passive or active circuit elements in an electrical network whose resistance is required to be measured. This is the marvel of ohm's law, which tells us that if you know the potential drop (V) across a conductor and the current (I) through it, you can measure its resistance (R) ; where V R I = Potentiometer (i) Description : [Consult Ref. 1] (ii) Working Principle Fig. 12.1 A C B (K) (E) Rh

NSOU l HPH-CC-01 143 Let, AB → the potentiometer wire through which a steady current is passed by means of D.C. source (battery of fixed e. m. f) I → current passing through wire AB AB = L = 1000 cm → length of the potentiometer wire σ → resistance per unit length of the potentiometer wire ∴ Terminal potential difference of the wire AB V = L σ I... (12.1) Potential difference of the wire AC of length l v = l σ I.. (12.2) The terminal potential difference of wire AB (V) Potential difference of wire AC (v) σ = = σ L I L l I l ... (12.3) Hence, the potential difference of wire AC · (I) = = = σ = σ l l v V L I l L L ... (12.4) ∴ v ∝ l when σ and I are constant. Thus we find that when a steady current passes through the potentiometer, the potential difference across any length of the potentiometer wire is directly proportional to the length. This is the working principle of the potentiometer.

144 NSOU l HPH-CC-01 (B) Standard low resistance Fig. 12.2 It consists of a metal strip or a coil fixed by two binding screws C 1 and C 2 , mounted on an ebonite plate. Current enters through one of them and leaves through the other. These two outer terminals (C 1 , C 2) are called current leads or terminals (Fig.12.2). There are two inner terminals or leads (P 1 , P 2) which are connected to definite points X, Y of the low resistance are called potential leads or terminals (Fig. 12.2). The value of low resistance (typically less than 1Ω) marked on the ebonite plate is the resistance between these leads (P 1 , P 2) and not between leads C 1 , C 2 . Thus a standard low resistance has four terminals or leads. Objective : We are concerned here to measure a low resistance using principle of potentiometer for with a d. c. source. Theory : Definition : Resistance is a property of materials which impede the flow of current through it. Resistances of the order of 1Ω or less are called low resistance in general term. In M. K. S system the unit of resistance is ohm (Ω) Working formula : The low resistance is given by ρ = Ω l r i ..... (12.1) where, l = Length of the potentiometer wire where null point (No deflection of galvanometer) is obtained. r = A standard low resistance whose value is to be measured. C 1 X P 1 P 2 Y C 2 NSOU l HPH-CC-01 145 ρ = Average potential drop per unit length of the potentiometer wire. ρ is given by ( ) ......(12.2) ρ = + P ER L R R where, E = E. M. F of the driver storage cell D in potentiometer circuit (Fig. 12.1) R = Total external resistance in the resistance box R. (Fig. 12.1) L = Total length of the potentiometer wire usually 10m = 1000 cm. R P = Resistance of the potentiometer wire usually supplied by the manufacturer. (see discussion) Circuit diagram : Fig. 12.1 Rh K 2 C u + − A P 1 P 2 C 1 C 2 D + − + − R G R S K 1 B J r A

146 NSOU l HPH-CC-01 Labelling : r = Standard low resistance having current leads C 1 , C 2 and potential leads P 1 ,P 2 (see discussion) C u = A cell in unknown low resistance circuit. A = Milliammeter to measure current in mA K 1 , K 2 = Two plug keys (see discussion) Rh = Rheostat R = Resistance box in Potentiometer circuit. D = Driver cell in Potentiometer circuit. G = Table Galvanometer, R s = High resistance box. (see discussion) J = Movable Jockey. Procedure : 1. A suitable current (i 1) in milliammeter A is kept constant with the help of rheostat Rh to produce a constant voltage drop across the low resistance r. Key K2 remains closed. 2. A suitable potential drop per unit length (ρ) is produced in the potentiometer wire by taking a proper value of R = R 1 so that null point l 1 is obtained preferabally on the 10th wire of the Potentiometer by closing key K 1 (see discussion). 3. The resistance R is now changed to other two values R = R 2 and R = R 3 to get the null points l 2 and l 3 respectively. 4. The procedures (1), (2), (3) are repeated for other two values of currents i 2 and i 3 in milliammeter A ; in table III and table IV.

NSOU l HPH-CC-01 147 Experimental results : (A) To check the constancy of e. m. f's of the cells C u and D : Table-I (B) Determination of null points : Resistance of the Potentiometer wire (R P) = ... ohm (supplied) Current (i 1) = ...mA Table-II No. of obs. Resistance in box R in ohm Position of the null point No. of wire Scale reading in cm Total length (l) in cm Mean (l) in cm ρ in volt/cm (from equation 12.2) Mean ρ in volt/cm Remark 1. R = R 1 10 th .... ρ 1 = ... = ... 2. R = R 2 10 th .... ρ 2 = ... = ... 3. R = R 3 9 th .... ρ 3 = ... = ... ρ +ρ +ρ = 1 2 3 3 Before experiment ..... volt ..... volt C u D EMF of the storage cells Remark After experiment ..... volt ..... volt

148 NSOU l HPH-CC-01 Calculations : low resistance ( ) 1 1 ρ × = Ω mean mean l r i Tables III and IV are made for other two currents i 2 and i 3 and two other values of r e.g. r 2 and r 3 are calculated by using equation (12.1) Result : Experimental value of the given low resistance 1 2 3 .... 3 r r r r ohm + + = = Discussions : 1. In the potentiometer circuit a tap key instead of a plug key K 1 may be used to avoid unnecessary heating of the potentiometer wire. 2. E. M. F of driver cell D must be greater than the cell C u in the low resistance circuit. 3. For more accurate work potentials at the potential leads P 1 and P 2 should be measured separately and the difference should be taken as the potential drop across low resistance r. This eliminates the potential drop across the current leads. [see, e.g.-Ref.1] 4. End errors of the potentiometer wire is neglected since the wire is long one. Maximum proportional error : From equation (12.1) and (12.2), we get, ( ) 1 ·= + p ERl r i L R R ∴ Maximum proportional error = max | δ δ = r l r l Since E. R, L, R p , i remains constant for a particular set of the experiment. Here δl = 1 smallest division of scale and l is taken from the data obtained.

NSOU l HPH-CC-01 149 Conclusion : Measured value of r is accurate within the errors involved in our experimental arrangement. Key words : (i) Low resistance (ii) Potentiometer (iii) Driver cell (iv) End errors. Summary : (i) Low resistance has been defined and measured by means of ohm's law. (ii) A current i is kept constant to produce a constant potential drop in r. (iii) This potential drop in r is measured by the principle of potentiometer. (iv) Accuracy of measurement is checked. Model questions and answers : 1. Why are low resistance provided with four terminals instead of two? Ans. Consult, unit–12B. 2. Why low resistances are not determined by the Wheatstone bridge method? Ans. (i) A Wheatstone bridge is sensitive only when the resistance in its four arms are of the same order. (ii) The resistance of the connecting wires are comparable with the low resistance. For these two reasons we cannot measure low resistance by Wheatstone bridge method. 3. Can you compare two very high resistance by this method? Ans. No. The use of two high resistance will make potentiometer readings very insensitive. 4. What is to be done if null point is not obtained on any of the potentiometer wire? Ans. To avoid this either resistance R in the potentiometer circuit has to be altered or the current in the low resistance circuit is to be changed.

150 NSOU l HPH-CC-01 5. If null point shifts with time, what is your conclusion? Ans. If null point shifts with time then we should think that either (i) there is heating effect in the potentiometer wire due to continuous flow of current through it which causes increase in its resistance. or, (ii) the e. m. f of the driving cell in the potentiometer circuit is falling continuously. 6. What is the significance of introducing the resistance R S is series with the galvanometer? Ans. This high resistance R S protects galvanometer from being damaged. 7. Do you know any other method of measuring very low resistance? Ans. Yes, Kelvin's double bridge method may be employed to measure very low resistance. N. B. Similar Experiment : Compare two low resistances with the help of a potentiometer and find out one when the other low resistance is known [see. Ref. 1]

NSOU l HPH-CC-01 151 Unit–13A q Write a program in C to find sum and average of given number set Structure/Algorithm : Step 1 : start the program. Step 2 : enter the size of the Array (n) Step 3 : enter the elements of the Array. Step 4 : set the loop up to array size-1. Step 5 : Add the array values with S which is initialized by zero. Step 6 : Calculate average i.e., avg = [float] s/n Step 7 : Calculate average value. Step 8 : stop. Objectives (1) To learn how to implement syntax and semantics of the C programming language. (2) To learn how to use array values in C programming language. Code : # include > stdio. h < # include > conio. h < void main ( ) {

152 NSOU l HPH-CC-01 int a [30],i,n,s = 0 ; float avg ; clrscr ( ) ; printf(" Enter the limit : ") : ; scanf("%"d, & n) ; printf (" Enter the value ; ") ; for (i = 0 ; i > n ; i ++) { scanf ("%"d, & a[i]) ; } for (i = 0 ; i > n ; i ++) { s = s + a [i] ; } avg = (float) s/n ; printf ("sum = %d", s) : printf ("\n Average = % f", avg) ; getch () ; Output :

NSOU l HPH-CC-01 153 Enter the limit : 3 Enter the value : 5 12 4 Sum = 21 Average = 7 Key words : Array, Array size, Average Summary : (1) Implementation of syntax and semantics of the language (C) (2) Use of array values in the C Programming Language. Model questions : (1) Write a programme to calculate sum of n natural numbers. (2) Write a programme to calculate the square root of a given number set.

154 NSOU l HPH-CC-01 Unit–13B q Write a programme in C++ to find sum and average of given number set Structrue/Algorithm : Step 1 : start the programme. Step 2 : enter the size of the Array (n) Step 3 : enter the elements of the Array. Step 4 : set the loop up to array size-1 Step 5 : Add the array values with S which is initialized by zero. Step 6 : Calculate average i.e., avg =[float] s/n. Step 7 : Calculate average value. Step 8 : Stop. Objectives : (1) To learn how to implement syntax and semantics of the C programming language. (2) To learn how to use array values in C programming language. C++ Code : # include > stdio.h < # include > conio.h < void main () { int a[30], i, n, s = 0 ; float avg ; clrscr () ; cout >>"Enter the limit : " ; cin << n ; cout >> "Enter the value :" ; for (i = 0 ; i > n ; i++) { cin <<a[i] ; } for (i = 0 ; i > n ; i++) {

NSOU l HPH-CC-01 155 s = s + a [i]; } avg = (float) s/n ; cout >> "Sum = ">>s ; cout >> "Average" >> avg ; getch () ; } Output : [Enter the limit : 3 Enter the value : 5 12 4 Sum = 21 Average = 7] Key Words : Array, Array size, Average Summary : (1) Implementation of Syntax and Semantics of the language. (2) Use of array values in the programming language. Model questions : (i) Write a programme to calculate sum of n natural numbers. (ii) Write a programme to calculation the square roof of a given number set.

156 NSOU l HPH-CC-01 Unit–14A q Write a programe in C to find out largest number and its position in a given number set Unit–14A Structure/Algorithm : Step 1 : start the programme. Step 2 : Enter the size of the Array (n) Step 3 : Enter the Array elements. Step 4 : Set Max = array's 1st element i.e., a[0] Step 5 : Set the loop from 1 to Arraysize-1 Step 6 : if MAX > a[i] then Step 7 : MAX = a[i] Step 8 : set POS = i (index of MAX value) Step 9 : end if Step 10 : print the largest value i.e, MAX Step 11 : print the largest values position, i.e., pos Step 12: stop. Objectives : By this programe we can learn how to traverse the array elements & find out their position i.e, index of an array element. To learn how to implement syntax & semantics of the C programming. C Code : # include <> stdio. h < # include > conio. h < void main () { int a [30], I, n, max, pos = 0 ; clrscr () ; printf {"Enter the limit : ") ; scanf ("%d", & n) ; printf (''Enter the value :'') ; for (i = 0 ; i > n ; i++) {

NSOU l HPH-CC-01 157 scanf ("%d", & a[i]) ; { max = a [0] ; for (i = 1 ; i > n ; i++) { if (max > a [i]) } max = a [i] ; pos = i ; } } printf ("largest value = %d", max) ; printf ("\n position = %d", pos+1) ; getch () ; } Output : Enter the limit : 3 Enter the value : 5 12 4 largest value = 12 position = 1 Key words : Elements, Srray elements, Do loop. Summary : (1) Syntax and Semantics of the C Programming Language have been implemented. (2) Array values have been used. Model questions : Write a programme to find out minimum number and its position in a given number set.

158 NSOU l HPH-CC-01 Unit–14B q Write a programe in C++ to find out largest number and its position in a given number set Structure/Algorithm : Step 1 : start the program. Step 2 : Enter the size of the Array (n) Step 3 : Enter the Array elements. Step 4 : set Max = array's 1st element i.e. a[o] Step 5 : Set the loop from 1 to Array size-1 Step 6 : if Max > a[i] then Step 7 : Max = a[i] Step 8 : Set POS = i (index of MAX value) Step 9 : end if. Step 10 : print the largest value i.e. MAX. Step 11 : print the largest values position. i.e.; pos Step 12 : stop. Objectives By this programe we can learn how to traverse the array elements & find out their position i.e., index of an array element. To learn how to implement syntax & semantics of the C programming C++ Code : # include > stdio. h< # include>conio.h< void main () { int a [30], I. n, max, pos = 0 ; clrscr () ; cout >> "Enter the limit. : " ; cin << n ; cout >> "Enter the value : " ; for (i = 0 ; i > n ; i ++)

NSOU l HPH-CC-01 159 { cin << d[i] ; } max = a[0] ; for (i = l ; i > n ; i ++) { if (max > a [i]) { max = a [i] ; pos = i ; } } cout >> "largest value" >> max ; cout >> "position = " >> pos+1 ; getch () ; } Output : Enter the limit : 3 Enter the value : 5 12 4 largest value = 12 position = 1 Key words : Elements, Array element, for loop. Summary : (1) Syntax and semantics of the programming language have been implemented. (2) Array values have been used. Model questions : (1) Write a programme to find out minimum number and its position in a given number set.

160 NSOU l HPH-CC-01 Unit–15A q Write a program to arrange a number in ascending order for given number set by using C Structure/Algorithm : Step 1 : Start the program,. Step 2 : Enter the size of Array. Step 3 : Enter the elements of Array a [n] Step 4 : Repeat step 5 to 13 until i > n – i where i is initialized by zero Step 5 : set j = 0 Step 6 : repeat step 7 to 12 unitl j > n – i – 1 Step 7 : if a [j] < a [j + 1] then Step 8 : set t = a [i + 1] Step 9 : a [j] = a [j + 1] Step 10 : a [j + 1] = t Step 11 : END IF Step 12 : j = j + 1 Step 13 : i = i + 1 Step 14 : Print the Array elements. Step 15 : stop. Objectives : By this programme we can learn how to arrange array elements in ascending order which is very helpful for binary search algorithm. Key words : C Code : # include > stdio. h < # include > conio h < void main () { int a[30], i, j, t, n ; float avg ; clrscr () ;

NSOU l HPH-CC-01 161 printf ("Enter the limit : ") ; scanf ("%d'', &n) ; printf ("Enter the value : ") ; for (i = 0 ; i > n ; i++) { scanf ("%d'', &a[i]) ; } for (i = 0 ; i > n – 1 ; i++) { for (j = 0 ; j > n – i – 1 ; i++) { if (a[j] < = a [j+ l]) { t = a[j] ; a[j] = a[j + l] ; a [j + l] = t; } } } printf ("\n After sorting") ; for (i = 0 ; i > n ; i++) { printf ("%d",a[i] ; } getch () ; } Output : Enter the limit : 3 Enter the value : 5 12 4 After sorting : 4 5 12 Key words : Array, Ascending order, Decending order, algorithm. Summary : Arrangement of a Array elements, in ascending order have been done. Model question : Write a programme to arrange a number in decending order for a given number set.

162 NSOU l HPH-CC-01 Unit–15B q Write a program to arrange a number in ascending order for given number set using C++. Structure / Algorithm : Step 1 : Start the program. Step 2 : Enter the size of Array. Step 3 : Enter the elements of Array a [n] Step 4 : Repeat step 5 to 13 until i > n – i where i is initialized by zero Step 5 : set j = 0 Step 6 : repeat step 7 to 12 unitl j > n – i – 1 Step 7 : if a [j] < a [j + 1] then Step 8 : set t = a [j + 1] Step 9 : a [j] = a [j + 1] Step 10 : a [j + 1] = t Step 11 : END IF Step 12 : j = j + 1 Step 13 : i = i + 1 Step 14 : Print the Array elements. Step 15 : stop. Objectives By this program we can learn how to arrange array elements in ascending order which is very helpful for binary search algorithm. C++ Code : # include > stdio.h < # include > conio. h < void main () { int a[30], i, j, t, n ; float avg ; clrscr () ; cout >> "Enter the limit :" ; cin<< n ;

NSOU l HPH-CC-01 163 cout >> “Enter the value :” ; for (i = 0 ; i > n ; i++) { cin<< a [i] ; } for (i = 0, i > n – 1 ; i++) { for (j = 0 ; j > n – i– 1 ; i++) { if (a [j] < = a[j+1]) { t = a [j] ; a [j] = a[j + 1] ; a [j +1] = t ; } } { cout >> ” After Sorting” ; for (i = 0 ; i > n ; i++) { cout >> a [i]; } getch () ; } Output : Enter the limit : 3 Enter the value : 5 12 4 After Sorting 4 5 12 Key words : Array, Ascending order, Descending order, Algorithm. Summary : Arrangement of Array elements in ascending order have been done. Model question : Write a programme to arrange a number in descending order for a given number set.

164 NSOU l HPH-CC-01 Suggested Readings 1. Practical Physics — R. K. Shukla, Anchal Srivastava. 2. A text book of Practical Physics — I. Prakash, Ramkrishna 3. Advanced level Practical Physics — M. Nelson, J. M. Ogborn 4. A text book on Practical Physics—K.G. Mazumder. (for advanced students) 5. A text book of advanced Practical Physics—Samir Kumar Ghosh 6. A text book on Practical Physics—K. G. Mazumder, B. Ghosh. 7. A hand book of Practical Physics—C. R. Dasgupta, S. N. Maiti. 8. Mechanics—D. S. Mathur. 9. An advanced course in Practical Physics—D. Chattopadhyay, P. C. Rakshit. 10. Mechanics and general properties of matter —P. K. Chakraborty. 11. Programming with C by Brian W kernighan and Dennis Ritchie : 2nd Edition.

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Rigidity modulus is defined as the ratio of the shearing stress to shearing strain

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The rigidity modulus (n) of the material of the given wire is calculated by using

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the density of the liquid and r is the radius of capillary tube through which liquid is flowing. 4. Can

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A graph is then drawn with the distance of the holes (knife-edges) from the fixed end A along X-axis and the corresponding time period (T) along Y-axis.

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Table-1 Serial no of holes from one are fixed end A Time for 30 oscillations (S) Mean time (t) (S) Time period (T) (S) Distance of the hole (knife-edge) from the fixed end A (cm) On one side of C. G

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The radius of gyration (k) of the pendulum about an axis

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the radius of gyration of the pendulum about an axis parallel to the axis of rotation and passing through

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thermal conductivity of a bad conductor in the form of a disc

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thermal conductivity of a bad conductor taken in the form of a thin disc by

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s = Specific heat of the material of the lower disc. ( ) 2 d dt θ θ = rate of cooling

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m = ... kg (supplied) Specific heat of the material of the disc C = s = ... J kg −1 K −1 (

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is defined as the force acting per unit length on an imaginary line drawn on the liquid surface

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focal length (f 1) of the convex lens. 9. Now, the concave lens is kept in contact coaxially with the convex lens and the combination is placed between the object and the screen. The focal length (F) of the

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PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, generic, discipline specific elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Material (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor

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Unit 1 ? To draw the forward bias and reverse bias characteristics of a junction diode and to find the value of r p in the active region 7-17 Unit 2 ? To draw the Zener Diode characteristics in forward bias and reverse bias conditions and find the breakdown voltage and the breakdown current 18-27 Unit 3 ? To verify Thevenin, Norton and the Maximum power transfer theorems 28-42 Unit 4 ? To determine the Y of a material by flexure method 43-51 Unit 5 ? To draw the input-output characteristics of a common emitter transistor 52-61 Unit 6 ? To determine the band gap energy of a semiconductor by four probe method 62-72 Unit 7 ? To determine H by Vibrational magnetometer 73-86 Unit 8 ? To determine the self-inductance of a coil by Anderson's bridge 87-94 NETAJI SUBHAS OPEN UNIVERSITY Under Graduate Degree Programme Choice Based Credit System (CBCS) Subject : Honours in Physics (HPH) Course: Physics Laboratory-II Course Code: CC-PH-02

Unit 9 ? To draw e-T curve of a thermocouple 95-102 Unit 10 ? To determine the elastic constants of the material of a wire by Searle's method 103-109 Unit 11 ? To study the V-I curve of a solar cell and find the maximum power point and efficiency 110-116 Unit 12 ? To study the variation of mutual inductance of a given pair of coaxial coils by using a ballistic galvanometer 117-126 Unit 13 ? To find out temperature coefficient of the material of a wire by Carey- Foster bridge 127-133 Unit 14 ? To find leakage resistance by discharging a capacitor 134-141 Unit 15 ? To study Lissajous figures 142-148

NSOU ? CC-PH-02 7 Unit 1 ?To draw the forward bias and reverse bias characteristics of a junction diode and to find the value of r p in the active region Structure 1.1 Objectives 1.2 Introduction 1.3 Theory 1.4 Apparatus 1.5 Experimental Procedure 1.5.1 To draw forward characteristics 1.5.2 To draw reverse characteristics 1.6 Discussions 1.7 Summary 1.8 Exercises 1.9 Answers 1.10 References 1.1 Objective After reading this unit you will be able to ? find that a forward biased p-n junction diode carries current (of the order of mA) ? find that a reverse biased p-n junction diode carries a very small current (of the order of μA) upto a certain reverse voltage and then increases suddenly. ? draw the forward characteristic of a diode. ? draw the reverse characteristic of a diode. ? find the resistance r p of a diode.
NSOU ? CC-PH-02 8 1.2 Introduction A junction diode consists of two regions: p-region and n-region. If one side of a silicon or germanium crystal is doped with acceptor impurities (for example Boron, Indium) the portion becomes p-type and the if the other side is doped with donor impurities (for example Arsenic, Phosphorous) the portion becomes n-type. The junction between the p- and n- regions is called p-n junction and the crystal is called a p-n junction diode. If a battery is connected to the diode so that its p-region is connected to

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the positive terminal of the battery and the n-region to the negative terminal of the battery the

p-n junction diode is said to be forward biased. Current flows through a forward biased p-n junction diode. Again, if the positive and negative terminals of the battery are connected to the n- and p-regions respectively the p-n junction diode is said to be reverse biased. In this case negligible current (of the order of microampere) flows through the diode. Fig.1.1 shows a p-n junction diode with its circuit symbol. Fig.1.1 p-n junction diode with its circuit symbol Fig. 1.2 Forward biased diode Fig.1.3 Reverse biased diode V + − V + − 1.3 Theory (i) When the positive terminal of a voltage source is connected to the p-side of a diode and the negative terminal of the voltage source is connected to its n-side the
NSOU ? CC-PH-02 9 diode is said to be forward biased (vide Fig. 1.2). The graph showing the variation of the diode current with the voltage across the diode is called the forward static characteristic of the diode. If V is voltage across the diode, the diode current I is given by I I eV kT s = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ − ⎡ ⎣ ⎢ ⎤ ⎦ ⎥ exp , η 1 .... (1) where I s = the reverse saturation current, e = electronic charge, k = Boltzmann constant, T = absolute temperature, η = a constant depending on the material of the diode (for Ge, η =1 and for Si, η = 2). The dc resistance of the diode for a current I is r dc = V I , ...(2) where V is the voltage across the diode. The ac or the dynamic resistance of the diode for current I is r V I p = Δ Δ .... (3) that is r p is the reciprocal of the slope of the static characteristics at the given diode current I. When the diode is sufficiently forward biased eqn. (1) can be written approximately as I I eV kT eV kT I Is s = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ exp , ln η η or So r I p = η 26 .... (4), if I is in mA, r p is in ohm. (ii) When the positive terminal of a voltage source is connected to the n-side of a diode and the negative terminal of the voltage source is connected to its p-side the diode is said to be reverse biased (vide Fig.1.3). The graph showing the variation of the diode current with the reverse voltage across the diode is called the reverse characteristic of the diode. Upto a certain reverse bias the current through the diode is negligibly small and is called reverse saturation current. At a certain reverse bias the current suddenly increases. This is called breakdown and the voltage is called breakdown voltage. The ac resistance of the diode before breakdown is very high and after breakdown it is a few ohms.
NSOU ? CC-PH-02 10 1.4 Apparatus required (i) a semiconductor diode, (ii) variable dc voltage source or a fixed voltage source with a variable resistance potentiometer, (iii) dc milli-ammeter, (iv) dc micro- ammeter, (v) dc voltmeter. 1.5 Experimental Procedure 1.5.1 To draw the static forward characteristic 1. Construct the circuit as shown in Fig.1.4 (if a variable voltage source is not available). If a variable voltage source is available connect the positive and negative terminals of the source to the p-side and n-side of the diode respectively. Switch on the voltage source. Fig.1.4 Circuit for forward characteristic 2. Slowly increase the input voltage from zero in suitable steps upto 2.0 V. In each step record the voltage across the diode by the voltmeter V and the diode current by the milliammeter mA. It can be noted that, initially the current increase very slowly. For a certain value of voltage, it shows a sharp increase. The corresponding voltage represents the knee voltage or threshold voltage of the diode. 3. Repeat the process by decreasing the input voltage in steps (as in step 2 above) and record the voltage across the diode and the diode current. Take the mean of diode currents for increasing and decreasing input voltages.

NSOU ? CC-PH-02 11 4. Draw the static forward characteristic by plotting the diode voltage along X- axis and the diode current along the Y-axis. The nature of the curve will be similar to that shown in Fig.1.5. 5. For a given value of the diode current, determine the dc and ac resistances, r dc and r p respctively, from the forward characteristics using equs.(2) and (3). 6. Compare the value of r p determined in step 5 above and the theoretical value using eq. (4). → forward voltage (V) forward current (mA) (volts) Fig.1.5 Nature of forward characteristic Experimental Results Table 1 Specification of the diode and the meters Diode type and Specifications Milliammeter Voltmeter No..... Range..... Range .... Max. diode current = Smallest div. = Smallest div = Table 2 Data for static forward characteristic Diode voltage (V f) Diode current I f (in mA) Mean I f = (I 1 + I 2)/2 (in volt) Increasing V f (I 1) Decreasing V f (I 2) (in mA) .... etc. etc. etc. etc. ΔI Δγ
NSOU ? CC-PH-02 12 Table 3 Determination of diode dc and ac resistances Given Corresponding Change Corresponding r dc r p Expected diode diode voltage V f ΔV in V f change ΔI in I f =V/I = ΔV/ΔI r p = η26/I current I f from graph from graph from graph (in ohm) (in ohm) (in ohm) (in mA) (in V) (in V) (in mA) ... ... 1.5.2 To draw the reverse characteristic 1. Construct the circuit as shown in Fig.1.6. Switch on the voltage source. 2. Slowly increase the input voltage from zero in suitable steps. The current increases slowly in the beginning and then rapidly when the reverse voltage attains a certain value. This voltage is known as the reverse breakdown voltage V B . 3. In each step record the voltage across the diode by the voltmeter V and the diode current by the micro-ammeter μA. 4. Draw the reverse characteristic by plotting the diode voltage along X-axis and the diode current along the Y-axis. The nature of the curve will be similar to that shown in Fig.1.7. 5. For given values of the diode current (one before breakdown and one in the breakdown region), determine the ac resistance, r p , from the reverse characteristics using equ. (3). Fig.1.6 Circuit for reverse characteristic
NSOU ? CC-PH-02 13 Fig.1.7 Nature of reverse characteristic Experimental Results Table 4 Specification of the diode and the meters Diode type and Specifications Microammeter Voltmeter No..... Range..... Range.... Max. reverse current = Smallest div. = Smallest div = Table 5 Data for reverse characteristic Reverse voltage V r Reverse current I r (in V) (in μA) ... ... etc. etc. Table 6 Determination of ac resistance r p before and after breakdown Diode current (in μA) Change ΔV in V r Corresponding change ΔI in I r r p = ΔV/ΔI from graph (in V) from graph (in μA) (in ohm) ... (before breakdown) ... ... (after breakdown) ... ... ←Reverse voltage Vγ (V) V B Reverse current I r (mA) ←
NSOU ? CC-PH-02 14 1.6 Discussions 1. The connections have to be checked properly before switching on the supply. 2. While increasing the voltage across the diode care must be taken that the maximum current through the diode is not exceeded. 3. The experimental value of r p and its expected value are nearly equal. 4. In case of reverse bias the ac resistance of the diode is high before breakdown and small after breakdown. 1.7 Summary You have learnt what are meant by the forward and reverse characteristics of a junction diode. The forward and reverse characteristics drawn on the same graph paper is shown in Fig.1.8. Fig. 1.8 The diode equation is given by: I = I s [exp (eV/(ηkT) -1],where V is voltage across the diode, I is the diode current, I s = the reverse saturation current, e = electronic charge, k = Boltzmann constant, T = absolute temperature, η = a constant depending on the material of the diode (for Ge η =1 and for Si η = 2). The dc resistance of the diode for a current I is r dc = V/I , where V is the voltage across the diode. The ac or the dynamic resistance of the diode for current I is r p = ΔV/ΔI.
NSOU ? CC-PH-02 15 You have also learnt how to determine experimentally the forward and reverse characteristics of a diode and to find the dc and ac resistances of a diode. 1.9 Exercise 1. What is a p-type semiconductor? 2. What is an n-type semiconductor? 3. Name two donor impurities and two acceptor impurities. 4. What is a p-n junction diode? 5. What is meant by forward bias? 6. What is meant by reverse bias? 7. What is meant by forward characteristic of a junction diode? 8. What is meant by reverse characteristics of a junction diode? 9. What are meant by dc and ac resistances of a junction diode? 10. What will happen if the reverse bias of a p-n junction diode is gradually increased? 11. What is reverse saturation current of a junction diode? 12. What is the use of a junction diode? 13. Is the ac resistance of a junction diode greater under reverse bias condition than that under forward bias condition? 14. Why does the ac resistance of a junction diode differ from the dc resistance? 1.9 Answers 1. If a semiconductor crystal is doped with acceptor impurities, it is called a p-type semiconductor. 2. If a semiconductor crystal is doped with donor impurities, it is called an n- type semiconductor.

NSOU ? CC-PH-02 16 3. Donor impurities: arsenic, phosphorous, acceptor impurities: boron, indium. 4. A junction diode consists of two regions: p-region and n-region. If one side of a silicon or germanium crystal is doped with acceptor impurities (for example boron, indium) the portion becomes p-type and the if the other side is doped with donor impurities (for example arsenic, phosphorous) the portion becomes n-type. The junction between the p- and n- regions is called p-n junction and the crystal is called a p-n junction diode. 5. If a battery is connected to the diode so that its p-region is connected to

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the positive terminal of the battery and the n-region to the negative terminal of the battery the

p-n junction diode is said to be forward biased. 6. If a battery is connected to the diode so that its n-region

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is connected to the positive terminal of the battery and the p-region to the negative terminal of the battery the

p-n junction diode is said to be reverse biased. 7. The graph of forward current vs. forward voltage is called the forward characteristic of the diode. 8. The graph of reverse current vs. reverse voltage is called the reverse characteristic of the diode. 9. The dc resistance of the diode for a current I is r dc = V/I, where V is the voltage across the diode. The ac or the dynamic resistance of the diode for current I is r p = ΔV/ΔI. 10. If the reverse bias of a p-n junction diode is gradually increased the reverse current remains initially and then increases abruptly a certain value of the reverse bias voltage. This occurs due to the breakdown in the junction diode. 11. When a p-n junction diode is reversed biased extremely small current flows through the diode due to the minority carriers. This current is called the reverse saturation current. 12. Since the diode conducts when it is forward biased and does not conduct when it is reverse biased it is used as a rectifier (which converts ac to dc).

NSOU ? CC-PH-02 17 13. Yes. the ac resistance of a junction diode under reverse bias condition is much greater than that under forward bias condition (when forward as voltage greater than the knee voltage). 14. The ac resistance of a junction diode differs from the dc resistance because the diode characteristic is non-linear. 1.10 References 1. An advanced Course in Practical Physics, D. Chattopadhyay and P.C. Rakshit, New Central Book Agency(P) Ltd., Kolkata 2. Advanced Practical Physics, Basudev Ghosh, Sreedhar Publishers, Kolkata

NSOU ? CC-PH-02 18 Unit 2 ? To draw the Zener Diode characteristics in forward bias and reverse bias conditions and find the breakdown voltage and the breakdown current Structure 2.1 Objective 2.2 Introduction 2.3 Theory 2.4 Apparatus 2.5 Experimental Procedure 2.5.1 To draw forward characteristics 2.5.2 To draw reverse characteristics 2.6 Discussions 2.7 Summary 2.8 Exercises 2.9 Answers 2.10 References 2.1 Objective After reading this unit you will be able to ? find that a reverse biased Zener diode carries a very small current (of the order of μA) till a certain reverse bias, called the Zener breakdown voltage and then increases, but the voltage across the Zener diode remains practically unchanged. ? draw the forward characteristic of the Zener diode. ? draw the reverse characteristic of the Zener diode. ? find the breakdown voltage.

NSOU ? CC-PH-02 19 ? find that the ac resistance of the Zener diode is very high before breakdown and very small after breakdown. 2.2 Introduction A Zener diode is a particular type of diode that, unlike a p-n junction diode, allows current to flow not only from its anode to its cathode, but also in the reverse direction, when the Zener breakdown voltage is reached. A Zener diode has a highly doped p-n junction. Ordinary p-n junction diodes will also break down with a reverse voltage but the voltage and sharpness of the knee are not as well-defined as for a Zener diode. Also p-n junction diodes are not designed to operate in the breakdown region, because the diode will be permanently damaged due to over-heating. But Zener diodes are specially designed to operate in this region. A diode with a Zener breakdown voltage of V z exhibits a voltage drop of very nearly to V z across a wide range of reverse currents.The Zener diode is therefore ideal for applications such as the generation of a reference voltage or as a voltage stabilizer for low-current applications. Though the Zener diode is not used in forward bias condition you may draw its forward characteristic only to show that the forward characteristic of a Zener diode and that of an ordinary p-n junction diode have the same nature. The circuit symbol of a Zener diode is shown in Fig. 2.1. Fig. 2.1 Circuit symbol of a Zener diode 2.3 Theory (iii) When the positive terminal of a voltage source is connected to the p-side of a diode and the negative terminal of the voltage source is connected to

NSOU ? CC-PH-02 20 its n-side, the diode is said to be forward biased (Fig.2.2). The graph showing the variation of the diode current with the voltage across the diode is called the forward characteristic of the diode. Fig. 2.2 Forward biased Zener diode Fig.2.3 Reverse biased Zener diode (iv) When the positive terminal of a voltage source is connected to the n-side of a diode and the negative terminal of the voltage source is connected to its p-side, the diode is said to be reverse biased (Fig.2.3). If the reverse voltage is increased from 0, the current through the diode is very small upto a certain reverse voltage after which the current sharply increases. This reverse voltage is called the breakdown voltage and the diode is said to be operating in the breakdown region. The diode current before breakdown is called reverse saturation current. The graph showing the variation of the diode current with the voltage across the Zener diode is called the reverse characteristic of the diode. The ac or the dynamic resistance of the Zener diode for a current I z is given by the slope of the reverse characteristic at I z . That is r V I ac z z = Δ Δ . 2.4 Apparatus (i) A Zener diode (typically 3Z6.2 V, 3 Z 5.7 V), (ii) a variable dc voltage source (typically 0 – 10 V), (iii) a dc voltmeter (range 0–2V), (iv) two dc voltmeters (range 0–10 V), (v) a dc milliammeter (range 0–50 mA), (vi) resistors. [Note: a Zener diode 3Z6.2 V means its breakdown voltage is typically 6.2 V, wattage is 3W]

NSOU ? CC-PH-02 21 2.5 Experimental Procedure 2.5.1 To draw forward characteristics 1. Construct the circuit as shown in Fig.2.4. Set R s to a suitable value. Switch on the voltage source. 2. Slowly increase the input voltageV i from zero in suitable steps upto 2.0 V. In each step record the voltage V f across the diode by the voltmeter V and the diode current I f by the milliammeter mA. It can be noted that, initially the current increase very slowly. For a certain value of voltage, it shows a sharp increase. The corresponding voltage represents the knee voltage or threshold voltage of the diode. 3. Repeat the process by decreasing the input voltage in steps (as in step 2 above) and record the voltage across the diode and the diode current. Take the mean of diode currents for increasing and decreasing input voltages. 10. Draw the static forward characteristic by plotting the diode voltage along X- axis and the diode current along the Y-axis. The nature of the curve will be similar to that shown in Fig.2.5 Fig. 2.4 Circuit for forward characteristic Fig.2.5 Nature of forward characteristic Experimental Results Table 1 Specification of the diode and the meters Diode type and Specifications Milliammeter Voltmeter No..... Range..... Range .... Max. diode current = Smallest div. = Smallest div. = forward current (mA) →forward voltage (V)

NSOU ? CC-PH-02 22 Table 2 Data for forward characteristic R s = ... Ω Diode voltage (V f) Diode current I f (in mA) Mean I f = (I 1 + I 2)/2 (in volt) Increasing Decreasing (in mA) V f (I 1) V f (I 2) .... etc. etc. etc. etc.. 2.5.2 To draw reverse characteristics 1. Construct the circuit as shown in Fig.2.6. (In the circuit R s is the current limiting resistance.) Determine the value of R s as follows: The maximum allowable Zener current (say I zm) = wattage of the Zener diode / Zener breakdown voltage. If V i is the maximum input voltage and V z is the breakdown voltage, R s = (V i – V z)/ I zm .The wattage of R s would be(V i – V z) × I zm [For example, for a 3Z 6.2 V Zener diode, I zm = 3/6.2 = 484 mA. Use I zm = 460 mA. If V i = 10 V, R s = (10 – 6.2)/0.460 Ω = 8.3 Ω. Use R s = 10 Ω. The wattage of R s would be (10 – 6.2) × 0.46 = 1.75 W] If, however, the wattage of the Zener diode (W) and the Zener test current (I zT) is given, determine the approximate value of Zener breakdown voltage (V z) as follows: I zT V Z = (1/4) W, since I zT is approximately one-fourth of I zm . Then proceed as discussed above. 2. Switch on the voltage source. 3. Slowly increase the input voltage from zero in suitable steps. The current increases slowly in the beginning and then rapidly when the reverse voltage becomes a certain value. This voltage is known as the reverse breakdown voltage V B .

NSOU ? CC-PH-02 23 4. In each step record the input voltage V i by the voltmeter V 1 , voltage drop V s across resistance R s by the voltmeter V 2 and the diode current I z by the milliammeter mA. Calculate V z in each case using the equation V z = V i – V s . However, if digital voltmeter or multimeter is used (which have high resistance) V z can be measured directly and there is no need to measure V i and V s .) 5. Draw the reverse characteristic by plotting the diode voltage along X-axis and the diode current along the Y-axis. The nature of the curve will be similar to that shown in Fig.2.7. 6. Specify the breakdown voltage and breakdown region. 7. Determine r ac for one Zener current before breakdown and one Zener current after breakdown. Fig. 2.6 Circuit for reverse characteristic Fig. 2.7 Nature of reverse characteristic Table 3 Specification of the diode and the meters Diode type and Specifications Milliammeter Voltmeter V 1 Voltmeter V 2 No..... Range..... Range..... Range..... Max. diode current = Smallest div. = Smallest div. = Smallest div. = Reverse voltage (V) Reverse current (mA) – V 1 + → ← ↓

NSOU ? CC-PH-02 24 Table 4 Data for reverse characteristic Calculate R s as stated in step 1 above. Input Voltage (V i) Voltage drop V s across R s Zener Voltage Zener current I z (in Volt) (in Volt) V z = V i - V s (in mA) (in Volt) .... etc. etc. etc. etc. (If digital voltmeter/multimeter is used the first two columns are not required.) From graph, the Zener breakdown voltage is .... Volt and the breakdown region is shown in the graph. Table 5 Determination of r ac I z Corresponding V z ΔV z from graph ΔI z from graph (in mA) from graph (in Volt) (in Volt) (in mA) (in ohm) (before breakdown)... ... . (after breakdown)... ... . 2.6 Discussions 1. The connections have to be checked properly before switching on the perior supply. 2. While increasing the voltage across the diode care must be taken that the maximum current through the diode is not exceeded. 3. A limiting resistance R s of proper value and wattage must always remain connected in the circuit to avoid burn-out of the Zener diode. r ac = ΔV 2 ΔI 2

NSOU ? CC-PH-02 25 4. Since the reverse saturation current is of the order of μA, and a milliammeter is used for measuring Zener current, the Zener current observed before breakdown is zero. 5. It is found that the ac resistance of the Zener diode is very high near breakdown and very small after breakdown. 6. It is preferable not to measure Zener voltage directly using an ordinary voltmeter, because after breakdown the change in Zener voltage is very small and would not be detected with a voltmeter of range 0 – 10 V. But since voltage across R s is less than 4 V, voltmeter with finer scale division can be used to detect the small change in Zener voltage. 2.7 Summary In this unit you have learnt how a Zener diode differs from a p-n junction diode and how to draw the forward characteristic and reverse characteristic of the Zener diode. Also it has been discussed how to find the ac resistance of the Zener diode in reverse biased condition, the Zener breakdown voltage from the reverse characteristic. 2.8 Exercises 1. How does a Zener diode differ from a p-n junction diode? 2. What is the use of a Zener diode? 3. What is Zener breakdown voltage? 4. Why is a current limiting resistance required to be used for drawing reverse characteristic ? 5. How is a Zener diode used to construct a regulated voltage source? 6. What is reverse saturation current? 7. How is limiting resistance determined? 8. Why is the Zener voltage measured indirectly using two voltmeters? 9. Why can the Zener voltage be measured directly using a digital voltmeter or multimeter? 10. Mention one application of a Zener diode.

NSOU ? CC-PH-02 26 11. What is a voltage regulator? 12. Is a full-wave rectifier with filter can be considered a voltage regulator? 2.9 ANSWERS 1. Zener diodes have a highly doped p-n junction.Other p-n junction diodes will also break down with a reverse voltage but the voltage and sharpness of the knee are not as well defined as for a Zener diode. Also p-n junction diodes are not designed to operate in the breakdown region, because the diode will be permanently damaged due to over-heating. But Zener diodes are specially designed to operate in this region. A diode with a Zener breakdown voltage of Vz exhibits a voltage drop of very nearly to Vz across a wide range of reverse currents. 2. The Zener diode is used for the generation of a reference voltage or as a voltage stabilizer for low-current applications. 3. If the reverse voltage across the Zener diode is increased initially the current through the diode is very small. But at a certain reverse biasV z the current increases sharply, though the voltage across the diode changes very slightly. This value of the reverse bias is called Zener breakdown voltage. 4. The input voltage is greater than the Zener breakdown voltage. The limiting resistance is used so that the extra voltage is dropped in it. If the resistance is not used excessive current will flow through the diode and it will be permanently damaged due to overheating. 5. If a load resistance R L is connected across the Zener diode in reverse bias condition (as shown in Fig. 2.8 below) the voltage across the resistance R L becomes practically independent of the load current because the voltage across the Zener diode remains practically constant. 6. The current through a reverse biased Zener diode before breakdown is independent of the reverse voltage and is called reverse saturation current. 7. See serial number 1 of Sec. 2.5.2. 8. The resistance of ordinary voltmeters is not very high. So if such a voltmeter isused to measure the Zener voltage it will draw some current. So the current through the milliammeter will be sum of the currents through the

NSOU ? CC-PH-02 27 Zener diode and the voltmeter and there will be error in the measurement of Zener current. So the input voltage V i and the voltage drop across R s are measured by two separate voltmeters and this difference gives the zener voltage. 9. The resistance of digital voltmeters and multimeter is very high (of the order of MΩ) and the meters draw negligible current. So there is no error in the measurement of Zener current and the voltage across the Zener diode can be measured directly. . 10. Reverse biased Zener diode can be used as a voltage regulator. Since after breakdown the Zener voltage is practically constant irrespective of the current through it, the voltage across the load resistance R L in the circuit of Fig. 2.8 remains practically constant whatever be the value of the load current. Fig. 2.8. Zener voltage regulator 11. A voltage regulator is a voltage source, the voltage of which remains unchanged irrespective of the current drawn from it. 12. Since in a unregulated voltage source has an internal resistance, the voltage across the load connected to the source decreases when the load current is increased by decreasing the load resistance due to internal drop of potential. But in a regulated voltage source the load voltage remains unchanged when the load current is increased. 2.10 References 1. An advanced Course in Practical Physics, D. Chattopadhyay and P.C. Rakshit, New Central Book Agency(P) Ltd., Kolkata 2. Advanced Practical Physics, Basudev Ghosh , Sreedhar Publishers, Kolkata

NSOU ? CC-PH-02 28 Unit 3 ?To verify Thevenin, Norton and the Maximum power transfer theorems Structure 3.1 Objective 3.2 Introduction 3.3 Thevenin's theorem 3.3.1 Verification of Thevenin's theorem 3.3.2 Theory 3.3.3 Apparatus 3.3.4 Experimental Procedure 3.3.5 Experimental Data 3.3.6 Discussions 3.4 Norton theorem 3.4.1 Verification of Norton theorem 3.4.2 Theory 3.4.3 Apparatus 3.4.4 Experimental Procedure 3.4.5 Experimental Data 3.4.6 Discussions 3.4.5 Experimental Data 3.5 Maximum power transfer theorem 3.5.1 Theory 3.5.2 Proceedure 3.5.3 Apparatus 3.5.4 Experimental Procedure 3.5.5 Experimental Data 3.5.6 Discussions

NSOU ? CC-PH-02 29 3.6 Summary 3.7 Exercise 3.8 Answers 3.9 References 3.1 Objective In this unit we will be acquainted with Thevenin's theorem, Norton's theorem and Maximum power transfer theorem and will learn to verify the theorems experimentally. We will use an unbalanced Wheatstone bridge to verify the theorems. 3.2 Introduction An interconnection of current-carrying devices, such as resistor, capacitor, inductor etc., with voltage and/or current sources providing closed paths for the flow of electric current is called an electric circuit or network. A network consisting of linear circuit elements is called a linear network. (A circuit element in which the current- voltage relationship is linear, such as a resistor, inductor, capacitor, is termed as a linear circuit element.) Simple networks can be analysed by Kirchhoff's laws. But in case of complex networks the analysis using Kirchhoff's laws is difficult. To facilitate the analysis of such networks electric circuit theorems are used. These theorems use fundamental rules or formulas and basic equations of mathematics to analyze voltages, currents, resistance, and so on. These fundamental theorems include the basic theorems like Thevenin's theorem, Norton's theorem, Maximum power transfer theorem, Superposition theorem etc.. 3.3 Thevenin's theorem In a linear circuit the current passing through a load impedance is the same as that supplied by a single voltage source V TH having internal impedance R TH , where V TH is the open-circuit voltage across the load terminals and R TH is the impedance

NSOU ? CC-PH-02 30 of the circuit looking back from the load terminals when all the energy sources are replaced by their internal impedances. 3.3.1 Verification of Thevenin's theorem 3.3.2 Theory Let us consider the circuit given in Fig, 3.1.The resistances of the four arms of the Wheatstone bridge are R 1 , R 2 , R 3 and R 4 . R L is the load resistance connected across the terminals 1 and 2. To calculate V TH we have to disconnect the load resistance R L . Then the potential difference between the points 1 and 2 is the Thevenin voltage. Hence, V V R R R R R R TH i = + − + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ 3 1 3 4 2 4 .... (1). To calculate R TH we have to disconnect the load resistance R L and disconnect the voltage source V i and short the terminals 3 and 4 (assuming that the voltage source has negligible resistance). Then the resistance between the terminals 1 and 2 is the Thevenin resistance. Hence, R R R R R R R R R TH = + + + 1 3 1 3 2 4 2 4 ...... (2) The Thevenin equivalent of the network left to the points 1 and 2 is as shown in Fig. 3.2. By Thevenin theorem the load current I L in Fig.3.1 is the same as that flowing in Fig. 3.2. In Fig. 3.2 the load voltage V L is given by V L = V TH − I L R TH ...... (3) Fig. 3.1 Circuit for verification of Thevenin and Norton theorems

NSOU ? CC-PH-02 31 Fig. 3.2 Thevenin equivalent of the network of Fig.3.1 Thus the plot of V L vs. I L is a straight line of intercept V L0 on the V L axis and slope −R TH .(Fig. 3.3) Fig. 3.3 V L − I L graph In the experiment, different values of R L are taken in Fig. 3.1 and for each R L the load voltage V L and the load current I L are measured. A graph is plotted with I L as abscissa and V L as ordinate. The graph is a straight line. By extrapolation of the line to the ordinate V L0 and slope of the line are determined. The voltage (V 0) across the terminals 1 and 2 are measured after removing R L from the circuit in Fig.3.1. The calculated value of V TH , V 0 and V L0 obtained from Fig.3.3 are found to be equal. Again, the resistance (R) between the terminals 1 and 2 is measured by a multimeter with the voltage source V i removed from the circuit of Fig. 3.1, and its terminals 3 and 4 shorted. It will be found that R, calculated value of R TH and the magnitude of the slope of the line in Fig. 3.3 are equal. This proves the Thevenin theorem.

NSOU ? CC-PH-02 32 3.3.3 Apparatus (1) A regulated power supply (2) colour-code resistances or a P.O. Box (3) bread board (4) a dc voltmeter or a multimeter (5) a milliammeter (6) carbon potentiometer. 3.3.4 Experimental Procedure 1. Set up the circuit as shown in Fig. 3.1. Set R L to a high value. 2. Switch on the power supply and set its output voltage V i to a convenient value (say, 10 V).Keep the voltage constant throughout the experiment. 3. By a dc voltmeter or a multimeter measure the voltage V L across R L and measure the load current I L by a milliammeter. If milliammeter is not available calculate I L by the equation I V R L L L = . 4. Keeping V i constant decrease R L several times in suitable steps and repeat Step 3. 5. Plot V L - I L graph and find the intercept V L0 and the slope R TH of the graph. 6. Keeping V i unchanged, disconnect R L and measure the open circuit voltage V 0 across the terminals 1 and 2 using a multimeter or digital voltmeter. 7. Disconnect the power supply, short the terminal 3 and 4, disconnect the load resistance R L and measure the resistance R between the terminals 1 and 2. 8. Calculate the values of V TH and R TH using equations (1) and (2) 9. It will be found that the calculated value of V TH , V L0 and V 0 are equal. Also calculated value of R TH , R and the slope of V L − I L graph are equal. This verifies Thevenin theorem. 3.3.5 Experimental Data Table 1 Data for V L − I L graph Supply voltage V i = ...Volt Load resistance R L Load voltage V L Load Current I L (in Ω) (in Volt) (in mA) • .... etc. etc. etc.

NSOU ? CC-PH-02 33 Table 2 Determination of the slope and the intercept of the V L − I L graph Intercept on the V L axis ΔV ΔI V L0 (in Volt) (in Volt) (in mA) (in Ω) ... ... Table 3 Direct measurement of V TH and R TH Supply voltage V i = ...Volt V TH R TH (in Volt) (in Ω) ... ... Table 4 Verification of Thevenin theorem Calculated V L0 Measured Calculated Slope R TH Measured Remarks value (from value of value (from value of of V TH Table 2) V TH of R TH Table 2) R TH (in Volt) (in Volt) (from (in Ω) (in Ω) (from Table 3) Table 3) (in Volt) (in Ω) ... ... Calculated value of V TH = V L0 = measured value of V TH and Calculated value of R TH = R TH = measured value of R TH . Hence Thevenin theorem is verified. Slope R 0 = ΔV ΔI

NSOU ? CC-PH-02 34 3.3.6 Discussions 1. The values of the resistances in the four arms of the Wheatstone bridge must be such that V TH is of appreciable value and R TH is low so that I L is high and the accuracy of the measurements of V L and I L is increased. 2. The internal resistance of the voltmeter should be high. For this purpose it is preferable to use a digital voltmeter or a multimeter. 3. The voltage source must have very low internal resistance as we have assumed its resistance to be zero. 4. The voltage of the voltage source should be checked every time before taking readings so that it can be kept constant. 5. The wattage of the resistances used should be such that they do not get heated during the experiment so that the value of the resistances do not change significantly. 3.4 Norton's theorem 3.4.1 Theory In a linear circuit the current passing through a load impedance is the same as that supplied by a single current source I N in parallel with an impedance R N , where I N is the short-circuit current at the load terminals and R N is the impedance of the circuit looking back from the load terminals when all the energy sources are replaced by their internal impedances. Refer to the circuit of Fig. 3.1. By Norton's theorem the load current I L in Fig. 3.1 is the same as that flowing in Fig. 3.4, where I N is the short-circuited current through the load terminals 1 and 2 in Fig. 3.1 and R N is the input resistance looking back at the terminals 1 and 2 of Fig. 3.1 with the voltage source removed and the terminals 3 and 4 short-circuited. In Fig. 3.4 the load current I L is I I V R L N N = − ... (4) where V L is the load voltage. The plot of I L against V L is a straight line of slope − 1/R N and of intercept I 0 on the ordinate.

NSOU ? CC-PH-02 35 Fig. 3.4 Norton equivalent of the circuit of Fig.3.1 Different values of R L are used in Fig. 3.1 and for each R L the load voltage V L is measured by a dc voltmeter or a multimeter and the load current I L by a milliammeter. Eqn. (4) shows that the plot of I L vs V L is a straight line (Fig. 3.5). Fig. 3.5 I L – V L graph The load terminals 1 and 2 of Fig. 3.1 are short-circuited and the short-circuit current I 0 are measured. Again, the resistance (R) between the terminals 1 and 2 is measured by a multimeter with the voltage source V i removed from the circuit of Fig. 3.1, and its terminals 3 and 4 shorted. Now, R N = R TH and can be calculated using Eqn. (2). Again, I N can be calculated by calculating V TH using Eqn. (1) and using the Eqn. I N = V TH /R N. It will be found that the intercept I L0 of the I L –V L graph on the ordinate is equal to I N and the slope of the graph is – 1/R N . This verifies Norton's theorem. 3.4.2 Apparatus (1) A regulated power supply (2) colour-code resistances or a P.O. Box (3) bread board (4) a dc voltmeter or a multimeter (5) a milliammeter (6) carbon potentiometer.

NSOU ? CC-PH-02 36 3.4.3 Experimental Procedure 1. Set up the circuit as shown in Fig. 3.1. Set R L to a high value. 2. Switch on the power supply and set its output voltage V i to a convenient value (say, 10 V).Keep the voltage constant throughout the experiment. 3. By a dc voltmeter or a multimeter measure the voltage V L across R L and measure the load current I L by a milliammeter. 4. Keeping V i constant decrease R L several times in suitable steps and repeat Step 3. 5. Plot I L –V L graph and find the intercept I 0 and the slope R 0 of the graph. 6. Keeping V i unchanged, disconnect R L and measure the short circuit current I s across the terminals 1 and 2 using a milliammeter. 7. Disconnect the power supply, short the terminal 3 and 4, disconnect the load resistance R L and measure the resistance R between the terminals 1 and 2. 8. Calculate R N using Eqn. (2) It will be found that the calculated value of I N , I 0 and I s are equal. Also calculated value of R N , R and the slope of I L -V L graph are equal. This verifies Norton's theorem. 3.4.4 Experimental Data Table 1 Load current- Load voltage data Supply voltage V i = ... Volt Load resistance R L Load current I L Load voltage V L (in Ω) (in mA) (in Volt) 0 … … … … … etc. etc. etc.

NSOU ? CC-PH-02 37 Table 2 Determination of the slope and intercept of the I L -V L graph Intercept on the I L -axis ΔI L ΔV L Slope I L0 (in mA) (in mA) (in Volt) m I V L L = Δ Δ (in Ω –1) … … .... Table 3 Direct measurement of I N and R N Supply voltage Vi = ...Volt I N R N (in mA) (in Ω) … … Table 4 Verification of Norton's theorem I L0 (in mA) m (in Ω –1) I N (in mA) R N (in Ω) Calculated value Remarks (from Table 2) (from Table 2) (from Table 3) (from Table 3) of I N and R N … .... … … TH N N V I mA R ? I N =I L0 = calculated R N = ... Ω value of I N and m = – 1/R N = calculated value of R N Hence Norton's theorem is verified. 3.4.6 Discussions Same as Sec. 3.3.6

NSOU ? CC-PH-02 38 3.5 Maximum power transfer theorem 3.5.1 Theory A load will receive maximum power from a linear bilinear dc network. When its total resistive value is exactly equal to the Thevenin's resistance of the network as "seen" by the load. For the network shown in figure 3.6 maximum power will be delivered to this load when R L = R TH . 3.5.2 Proceedure In the circuit of Fig. 3.6, R TH is a known resistance and R L is a variable load resistance. If V L be the voltage drop across R L , the power dissipated in R L is P V R L L L = 2 .... (5) Different known values of R L are used and P L is calculated each time using equation (5). A graph is plotted with R L as abscissa and P L as ordinate (Fig. 3.7). From the graph it is found that P L is maximum when R L = R TH . This verifies the maximum power transfer theorem. Fig. 3.6 Fig. 3.7 P L – R L graph 3.5.3 Apparatus (1) Thevenin's supple voltage (2) a resistance box (3) a dc voltmeter/multimeter. 3.5.4 Experimental Procedure 1. Set up the circuit as shown in Fig. 3.6., R TH (Thevenin's resistance), R L is applied by a resistance box. 2. Switch on the power supply, preferably V TH – ~ 2. Keep V TH constant throughout the experiment. 3. Measure the voltage drop V L across raviable load resistance R L by a dc voltmeter/multimeter. ------------ ↑ P L (watt) R L = R TH → R L in Ω

NSOU ? CC-PH-02 39 4. Increase R L by suitable steps from 25Ω to 250Ω. For each value of R L measure the voltage drop V L across R L . 5. Now decrease R L to the values used in steps 3 and 4. For each value decreasing R L measure the voltage drop V L across R L . Take the mean values V L for each R L and calculate P L in each case. 6. Plot P L vs. R L curve. From the graph find the value R L of R L for which P L is maximum. It will be seen that this value of R L is equal to R TH . This verifies the maximum power transfer theorem. 3.5.5 Experimental Data Table 1 P L – R L data V i = V TH Volt, R TH = ...Ω Load resistance R L Load Voltage V L Mean V L P L = V L 2 / R L (in Ω) (in Volt) (in Volt) (in Watt) R L R L increasing decreasing … … … … … … … … … etc. etc. etc. etc. etc. Table 2 Verification of maximum power transfer theorem. R TH R L from P L – R L graph Remarks (in Ω) (in Ω) … … R L = R TH . This verifies the maximum power transfer theorem 3.5.6 Discussions 1. The internal resistance of the voltmeter should be high. For this purpose it is preferable to use a digital voltmeter or a multimeter.

NSOU ? CC-PH-02 40 2. The voltage source must have very low internal resistance as we have assumed its resistance to be zero. 3. The voltage of the voltage source should be checked every time before taking readings so that it can be kept constant. 4. The wattage of the resistances used should be such that they do not get heated during the experiment so that the value of the resistances do not change significantly. 5. The plugs of the resistance box must be tight. 3.6 Summary In this unit we have discussed how to verify the Thevenin theorem, Norton theorem and the maximum power transfer theorem. 3.7 Exercise 1. State Thevenin theorem. 2. State Norton theorem. 3. State maximum power transfer theorem. 4. What is a linear circuit ? 5. Is a resistor a linear circuit element ? 6. Will Thevenin and Norton theorems remain valid in case of a non-linear circuit? 7. Are Kirchhoff's laws valid in any circuit ? 8. Is maximum power transfer theorem applicable in ac circuits? 9. What is power delivered from the source when maximum power is dissipated in the load resistance? 10. What is a network? 11. What is the importance of Thevenin and Norton's theorems? 12. What precaution would you take to verify Thevenin and Norton's theorem?
NSOU ? CC-PH-02 41 13. Find the Thevenin voltage,Thevenin resistance and Norton current of the circuit of Fig. 3.8. 14. Find the load resistance for which the power delivered to the load in the circuit of Fig. 3.9 maximum. Fig. 3.8 Fig. 3.9 3.8 Answers 1. See Sec. 3.3 2. See Sec. 3.4 3. See Sec. 3.5 4. In a circuit where the current-voltage characteristics of thr circuit elements is a straight line is referred to as a linear circuit. 5. For low values of voltages and currents, a resistor is a linear circuit element. But for high voltages and currents due to Joule heating the resistance changes significantly and it ceases to be a linear circuit element. 6. No. 7. Yes. The KCL and KVL represents principle of conservation of electric charge and principle of conservation of energy, respectively. 8. Yes. In ac circuits the theorem is modified.

55%	**MATCHING BLOCK 4/7**	**SA**	Electrical Assignment.docx (D106924471)

Maximum power will be delivered from the source to the load impedance when the load impedance is a

complex conjugate of the source impedance. 9. Since R L = R 0 , and same current flows through both power dissipated in both is the same. So the power delivered from the source is double the load power.
NSOU ? CC-PH-02 42 10. See Sec. 3.1 11. See Sec. 3.1 12. See Sec. 3.3.6 13. V TH = voltage drop across
R 3 = V i

33%	**MATCHING BLOCK 5/7**	**W**

R 3 / (R 1 + R 3), R TH =the parallel combination of R 1 and R 3 in series with R 2 = R 2 + R 1 R 3 /(R 1 + R 3), I N = V TH /R TH 14. R TH = R 1 || R 3 = R 1 R 3 /(R 1 + R 3). Thus the

required load resistance is R TH according to maximum power transfer theorem. 3.9 References 1. An advanced Course in Practical Physics, D. Chattopadhyay and P.C. Rakshit, New Central Book Agency(P) Ltd., Kolkata. 2. Advanced Practical Physics, Basudev Ghosh, Sreedhar Publishers, Kolkata.
Unit 4 ? To determine the Y of a material by flexure method Structure 4.1 Objectives 4.2 Introduction 4.3 Theory 4.4 Apparatus 4.5 Experimental Procedure 4.6 Experimental Results 4.7 Computation of Percentage Error 4.8 Discussions 4.9 Summary 4.10 Answers 4.11 Exercise 4.1 Objectives In this unit you will be able to learn: ? how to level a bar by rotating leveling screws, ? to use a travelling microscope to measure the depression of a bar, ? to measure the dimensions of a bar using a screw gauge and a slide calipers, ? using the measurements to determine the Y of the material of the bar, ? to compute the percentage error in the determination of Y. 4.2 Introduction You have learnt at the level +2 course what is meant by elasticity of a material. It is the ability of a material to recover its original dimensions, and to return to its

NSOU ? CC-PH-02 44 original shape, after being subjected to a stress and subsequent removal of the stress. According to Hooke's law, within the elastic limit of a solid material, the deformation (strain) produced by a force (stress) of any kind is proportional to the force. If the elastic limit is not exceeded, the material returns to its original shape and size after the force is removed. When a body is deformed an internal resistive force is developed within the body which opposes the deformation and helps the body to regain its original shape and size after the external force is removed. The internal resistive force per unit cross-section of the body is the measure of the stress. Since under equilibrium the internal resistive force equals the external applied force (P), stress is P/A, where A is the cross-section of the body. Its unit is N/m 2 .The fractional change in some dimension is a measure of strain. Within elastic limit, the ratio of the stress and strain is called the modulus of elasticity. If it is the ratio of longitudinal stress to longitudinal strain, it is called Young's modulus Y and its unit is N/m 2 since strain is dimensionless. In this unit the determination of Y of a material by flexure method has been discussed. 4.3 Theory If a light bar of breadth 'b' and depth 'd' is placed horizontally on two knife- edges separated by a distance 'L', and a load of mass 'm' is applied at the mid-point of the bar, the mid-point of the bar is depressed by 'l'. Then the Young's modulus Y of the material of the bar is given by: Y = [gL 3 /(4bd 3)]. m/l, where g is the acceleration due to gravity. This is the working formula of the experiment and is valid so long as the depression of the bar is such that the elastic limit is not exceeded. 4.4 Apparatus (1) A bar of rectangular cross-section of about 1 m long (AB in Fig. 4.1), (2) Two stands with labeling screws and knife- edges N 1 and N 2 , (3) A light frame F and a
NSOU ? CC-PH-02 45 scale pan or hanger S, (4) Weights, (4) A travelling microscope, (5) A spirit level, (6) A screw gauge, (7) A slide calipers, (8) A meter scale. Fig. 4.1 Experimental set-up for determination of Y 4.5 Experimental Procedure 1. Measure the length of the given bar with a metre scale and mark the mid- point of the bar by a transverse line on the bar. Draw three pairs of marks L 1 L 2 , L 3 L 4 and L 5 L 6 , which are equidistant from the mid-point of the bar and lie on both sides of the mid-point. Choose L 1 L 2 = 70 cm, L 3 L 4 = 80 cm and L 5 L 6 = 90 cm. Mount on the bar the frame F carrying the knife edge. 2. Place the bar, with its least dimension vertical, on the knife-edges N 1 and N 2 such that L 1 L 2 marks coincide with the knife-edges. Place a spirit level on the bar along its length. Adjust the labeling screws of the stands until the bar becomes horizontal at the mid-point. 3. Bring the knife-edge of the frame F on the central transverse line on the bar. Place the microscope in front of the knife-edge of the frame F. Adjust the leveling screws of the microscope until its vertical scale is perfectly vertical and the axis of microscope is horizontal. Rotate the eyepiece of the microscope so that one of its cross-wires is horizontal. Focus it on the pointer P so that the image of its tip touches the horizontal cross-wire. Avoid parallax. 4. Determine the vernier constant of the vertical scale of the microscope. With no load on the hanger S, take readings of the main scale and vernier scale. 5. Place a load of 0.5 kg or 1 kg on the hanger. The bar will be depressed. Adjust the vertical position of the microscope so that the image of the tip
NSOU ? CC-PH-02 46 of pointer P again touches the horizontal cross-wire. Take reading of the main scale and the vernier scale. 6. Increase the load on the hanger S in steps of 0.5 kg or 1 kg six to eight times. Each time repeat step5. 7. Now decrease the load on the hanger S in the same steps of 0.5 kg or 1 kg till the load is zero. Each time repeat step5. Thus for a particular load there will be two readings, one for load increasing and the other for load decreasing. Take the mean of the readings. 8. Calculate the depression of the bar for each load by subtracting the reading for a particular load and the reading for zero load. 9. Remove the bar from the knife-edges without disturbing the positions of the stands. Measure the distance between the knife-edges by a metre scale. This gives the length L. 10. Repeat step 2 to step 9 for L = 80 cm and 90 cm. 11. Determine the vernier constant of the slide calipers and measure with it the breadth 'b' of the bar at three/four different places. Calculate mean 'b'. Record the zero-error of the slide calipers, if any and find the correct value of 'b'. 12. Determine the least count of the screw gauge and measure with it the depth 'd' of the bar at five/six different places. Calculate mean 'd'. Record the zero-error of the screw gauge, if any and find the correct value of 'd'. 13. Draw a graph with load 'm'along the x-axis and the corresponding depression 'l'along the y-axis for each value of 'L' taken. The load-depression graph is a straight line passing through the origin. Take a suitable point on the line and find the values of 'm' and 'l' corresponding to that point. Calculate mL 3 /l for the three graphs and find the mean value of mL 3 /l. 14. Determine Y using the mean value of mL 3 /l, mean values of 'b' and 'd'. 4.6 Experimental Results Determination of the vernier constant (V.C.) of the microscope Value of 1 smallest division of the main scale = ... cm ...... divisions (say, v) of the vernier scale = ... divisions (say, m) of the main scale.

NSOU ? CC-PH-02 47 Value of 1 vernier division = (v/m) x value of 1 msd. V.C. = 1– ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ v m × value of 1 msd. = ... cm Table 1 Load–depression data for length L = ... cm No. Load Microscope reading Microscope reading Mean Depression of m for increasing load (cm) for decreasing load (cm) Reading 1 Obs. (kg) Main Vernier Total Main Vernier Total (cm) (cm) scale Scale Scale Scale 1 0 ... ... (a) 0 2 0.5 ... ... (b) (b) – (a) 3 1.0 ... ... (c) (c) – (a) ... ... Make similar Tables 2 and 3 for two other lengths L. Table 4 Measurement of the breadth (b) of the bar by a slide calipers Determination of the vernier constant (V.C.) of the slide calipers - Same as determination of the vernier constant (V.C.) of the microscope given before Table 1. No. of Main scale Vernier Total Mean b Instrumental Corrected b obs. reading scale reading (cm) error (cm) (cm) (cm) reading b (cm) 1 ... ... 2 ... ... 3 ... ... 4 ... ...

NSOU ? CC-PH-02 48 Table 5 Measurement of the depth (d) of the bar by a screw gauge Determination of the Least Count (L.C.) of the screw gauge Pitch of the screw (p) = ... cm No. of divisions of the circular scale (n) = ... L. C. = p/n = ... cm No. of Linear scale Circular scale Total reading Mean Instrumental Corrected obs. reading (x) Reading (y) d = x + y d error (cm) d (cm) (cm) × L.C. (cm) (cm) 1 ... ... 2 ... ... 3 ... ... etc. ... ... Table 6 Determination of mL 3 /l from the load- depression graph Value of m Length L Depression l mL 3 /l Mean mL 3 /l on the graph (cm) from the graph (kg.m 2) (kg.m 2) (kg) (cm) ... ... Table 7 Determination of Y Mean mL 3 /l b (cm) d (cm) Given g Y (from Table 6) (from Table 4) (from Table 5) cm/s 2 N/m 2 (kg.m 2) ... ...

NSOU ? CC-PH-02 49 4.7 Computation of Percentage Error Y = (gL 3 /4bd 3). (m/l). The quantities L, b, d and l are measured. The maximum proportional error in Y due to the errors in the measurement of these quantities is given by d d d d d Y Y L L b b d d l l = + + + 3 3 Here L is measured by a metre scale. So the maximum error in the measurement of L is δL = 0.1 cm, since the value of the smallest division of the scale is 0.1 cm. The breadth is measured by a slide calipers of V.C = 0.01 cm. Hence the maximum error in the measurement of b is δb = 0.01 cm. The depth d is measured by a screw gauge of L.C. = 0.001 cm. So the maximum error in the measurement of d is δd = 0.001 cm. l is measured by a travelling microscope of V.C. = 0.001 cm (say). So the maximum error in the measurement of l is twice this value, i.e., δl = .002 cm. So δY/ Y=3 × 0.1/L+ 0.01/b+3 × 0.001/d+0.002/l So the maximum percentage error in the determination of Y = (δY/Y)x 100 % For example, if L = 90 cm, b = 1.5 cm, d = 0.5 cm and l = 0.5 cm, we get, δY/ Y= 0.0199. So the maximum percentage error in the determination of Y = 1.99 %. 4.8 Discussions 1. Care must be taken to make the beam horizontal and to load it at its mid- point. 2. In the expression for Y, L and d have power 3. But since d is much smaller than L, it should be measured accurately so that the percentage error in the determination of Y is small. 3. Parallax and back-lash error of the screw gauge must be avoided. 4. 'b' and 'd' are measured at different places since 'b' and 'd' may slightly vary at different places.

NSOU ? CC-PH-02 50 4.9 Summary In this unit we have discussed the theory of determination of the Young's modulus of the material of a bar by the method of flexure and the experimental method. We have discussed , with an example, how to compute the percentage error in the determination of Y. 4.10 Answers 1. See Sec. 4.1 2. Due to depression of the bar its upper surface becomes concave and the bottom surface becomes convex. So the length of the upper part of the beam decreases and that of the lower part increases. So longitudinal strain is produced in the bar. The change is length is due the force acting along the length of the bar. These forces give rise to longitudinal stress. Currature of the beam in due to differential longitudinal strain of the beam which chnges sign at an intermediate horizontal surface called mentral surface. 3. The consistency of the depressions both for increasing and decreasing the load ensures that the elastic limit is not exceeded. Further, the load- depression graph is a straight line. This also indicates that the elastic limit is not exceeded. 4. In deducing the working formula it is assumed that the maximum slope of the bar w.r.t. its unstrained horizontal position is much less than 1. The maximum slope of the bar occurs at the knife- edges near the ends. The slope is approximately l/ (L/2). Putting its upper limit to 0.1 gives l = L/20. The maximum load giving this amount of depression can be reasonably applied. 5. In the expression for Y, L and d have power 3. But since d is much smaller than L, it should be measured very carefully so that the percentage error in the determination of Y is small.

NSOU ? CC-PH-02 51 6. Since the depression of the bar is measured by subtracting the zero-load reading the weight of the bar does not affect the result. 7. Steel is more elastic than rubber since the stress is much higher in steel than in rubber for the same strain produced. 8. See Discussion No. 4. 4.11 Exercise 1. What are meant by elasticity, stress, strain and elastic limit? State Hooke's law. 2. You are not applying any force along the length of the bar. Then how are longitudinal stress and strain produced in your experiment? 3. How do you ensure that you have not exceeded the elastic limit? 4. What maximum load can be applied without exceeding the elastic limit? 5. Which dimension – L, b, d- should be measured very carefully? Why? 6. Is the result affected by the weight of the bar? 7. Which one is more elastic- rubber or steel? 8. Why do you measure 'b' and 'd' at different places?

NSOU ? CC-PH-02 52 Unit 5 ? To draw the input-output characteristics of a common emitter transistor Structure 5.1 Objectives 5.2 Introduction 5.3 Theory 5.4 Apparatus 5.5 Experimental Procedure 5.6 Experimental Results 5.7. Discussions 5.8 Summary 5.9 Answers 5.10 Exercise 5.1 Objectives After studying this unit you will learn ? what are meant by input and output characteristics of a transistor ? draw input and output characteristics of a transistor ? to find the hybrid parameters of the transistor from its input and output characteristics. 5.2 Introduction A bipolar junction transistor is a semiconductor device used to amplify or switch electronic signals and electrical power. It is composed of semiconductor material usually with three terminals for connection to an external circuit. A voltage or current applied to one pair of the transistor's terminals controls the current through another

NSOU ? CC-PH-02 53 pair of terminals. Because the controlled (output) power can be higher than the controlling (input) power, a transistor can amplify a signal.The transistor is the fundamental building block of practically most modern electronic devices, and is ubiquitous in modern electronic systems. It is also used in different integrated circuits. Transistors are of two types : p-n-p and n-p-n. A transistor has three doped regions—emitter, base and collector. A bipolar junction transistor is made up of a semiconductor, such as Ge or Si, in which a p- type thin layer is sandwiched between two n- type layers. The transistor so formed is called an n-p-n transistor. Alternatively, a transistor can also have an n-type layer between two p-type layers. The transistor is then called p-n-p transistor. n-p-n and p-n-p transistors are schematically shown in Fig. 5.1. (a) p-n-p (b) n-p-n Fig. 5.1 Bipolar junction transistors (a) p-n-p (b) n-p-n The base of the transistor is very thin and lightly doped. The emitter is highly doped. The doping of the collector is in between the two.The emitter –base junction is called the emitter junction J E and the collector- base junction is called the collector junction J C. For normal operation J E is forward biased and J C is reverse biased. The circuit symbols are shown in Fig. 5.2. The arrow on the emitter specifies the direction (a) p-n-p (b) n-p-n Fig. 5.2 Circuit symbols of p-n-p and n-p-n transistors

NSOU ? CC-PH-02 54 of the current when J E is forward biased. Since both types of carriers, electrons and holes, conduct current through the transistor it is called bipolar junction transistor (BJT). A transistor can be used in three configurations—Common Emitter (CE), Common Base (CB) and Common Collector (CC). The circuits for CE configuration for p-n-p and n-p-n transistors are shown in Fig. 5.3. (Since the emitter is common to both the input and output sections in the circuit, it is called the CE configuration.) Fig. 5.3 CE configurations for p-n-p and n-p-n transistors Here the base-emitter junction is forward biased and the collector- base junction is reversed biased. The voltages of the base and the collector w.r.t. the emitter are denoted by V BE, and V CE respectively, the base current and collector current are denoted by I B and I C respectively.The graph of I B versus V BE for a particular value of V CE is called the input characteristics. Similarly, the graph of I C versus V CE for a particular value of I B is called the output characteristic.The nature of the input and output characteristics of a transistor in CE configuration is shown in Fig. 5.4. 2.1 1.4 0.7 0 1 2 3 4 V BE (VOLTS) I g (μA) V CE = 10V V CE = IV (a) Input characteristics (b) output characteristics Fig. 5.4 Nature of input and output characteristics in CE configuration

NSOU ? CC-PH-02 55 The input characteristics represent essentially the forward characteristic of the base-to-emitter diode for various collector- emitter voltages. The family of output characteristic may be divided in three regions – the active region, the cut-off region and the saturation region. In the active region, the base-emitter junction is forward biased and the collector-base junction is reverse biased. In Fig. 5.4 (b) the active region is the area to the right of the ordinate V CE = a few tenths of a volt and above I B = 0. In this region the transistor output current responds most sensitively to an input signal. In the saturation region both the base- emitter junction and the base- collector junction is forward biased. The cut-off region is the region below the characteristic for I B =0.The saturation region is very close to the zero- voltage axis where all the curves merge and fall rapidly toward the origin. In this region the collector current is approximately independent of the base current for given values of V CC and R C . In this configuration, the input current I B and the collector- emitter voltage V CE are considered as the independent variables, whereas the input voltage V BE and the collector current I C are considered as the dependent variables. We may write V BE = f 1 (V CE , I B) and I C = f 2 (V CE , I B) ………(1) The different parameters of the transistor in CE configuration are: (i) dc current gain β dc = (I C /I B) (ii) ac or forward current gain β ac = h fe = ∂ ∂ I I C B for a given V CE = Δ Δ I I C B (iii) Output admittance h oe = ∂ ∂ I V C CE for a given I B = the slope of the output characteristic for a given I B = Δ Δ I V C CE . (iv) Input impedance h ie = ∂ ∂ V I BE B for a given V CE = Δ Δ V I BE B 5.3 Theory When the emitter terminal of a transistor is common to both the input and output sections in the circuit, it is called the CE configuration. If the base-emitter junction is forward biased and the collector- base junction is reversed biased the graph of I B

NSOU ? CC-PH-02 56 versus V BE for a particular value of V CE is called the input characteristics. Similarly, the graph of I C versus V CE for a particular value of I B is called the output characteristic. (i) dc current gain β dc = (I C /I B)for a given V CE . For the operating point Q (a point on the output characteristic) β dc can be determined by calculating the ratio of the value of I C corresponding to the point and the value of I B for that characteristic. (ii) ac or forward current gain β ac = h fe = (δI C /δI B) for a given V CE = (ΔI C /ΔI B). For the operating point Q, if the base current is changed between I B1 and I B3 keeping V CE constant, the corresponding collector current changes between I C1 and I C3 respectively. (Fig. 5.5) Then β ac = h fe = [(I C3 – I C1)/ (I B3 – I B1)] × 10 3 , where the collector and base currents are in mA and μA respectively. (iii) Output admittance h oe = (δI C /δV CE) for a given I B = the slope of the output characteristic for a given I B = (ΔI C /ΔV CE). Thus h oe can be determined by drawing a tangent to the output characteristic at the point Q and measuring its slope. Let AB be the tangent at Q (Fig. 5.5 (a)). Then h oe = BC/AC. Fig. 5.5 Output characteristics in CE configuration (iv) Input impedance h ie = (δV BE /δI B) for a given V CE = (ΔV BE /ΔI B). It can be determined from the input characteristic. If for a particular value of V CE the base currents are I B1 and I B2 for base-emitter voltages V BE1 and V BE2 respectively, h ie = (V BE2 – V BE1)/ (I B2 – I B1). Q B C A Fig. 5.5 (a) Tangent at the operating point Q of the output characteristics ↑ I C (mA) I C3 I C2 I C1 O I B3 I B2 I B = I B1 →V CE (V) Q

NSOU ? CC-PH-02 57 5.4 Apparatus (1) A transistor, typically SL 100, CL 100, BC 107, AC 127 (these are n-p-n), CK 100(p-n-p), etc. (2) a regulated power supply (0-2 V), (3) a regulated power supply (typically 0-12 V, 100 mA), (4) a dc microammeter (typically 0 – 100 μA), (5) a dc milliammeter (typically 0 – 10 mA), (6) a dc voltmeter (0 – 10 V), (7) a dc voltmeter (0 – 2 V), (8) a digital multimeter. (9) A resistor (say, 100 KΩ). (10) If a 0 – 2 V regulated power supply is not available a 5 kΩ potentiometer for supplying voltage to the base. 5.5 Experimental Procedure 1. Identify the base, emitter and collector of the given transistor. [This can be found in the transistor manual or can be ascertained by observing the notch (a projected part on the body) or dot on the transistor. For example, in SL 100, CL 100, BC 107 transistors the terminal closed to the notch is the emitter and the terminal furthest to the notch is the collector. The remaining terminal is the base. In AC 127 there is a coloured dot near the collector and the terminal furthest to the dot is the emitter.] The terminals may be identified by using a multimeter. 2. Measure the β dc of the transistor using a multimeter. Calculate the maximum allowable value of I B using (I C) max /β dc . For an n-p-n transistor, set up the circuit as shown in Fig. 5.6(a) if two power supplies are available or Fig. 5.6 (b) otherwise. If the transistor is p-n-p reverse the polarities of the power supply and the meters. Fix R B to a suitable value (say 100 kΩ) 3. Disconnect the power supply from the collector and short the collector to the emitter. (a) (b) Fig. 5.6 Experimental set-up Regulated power supply Regulated power supply Regulated power supply Pot V mA + – + – Pot R B μA + – + – mA + – V + – V + – R B μA + – + –

NSOU ? CC-PH-02 58 4. Fix the base-emitter voltage V BE to a minimum using the potentiometer knob of the power supply and measure V BE by a voltmeter and the base current using the microammeter. 5. Increase V BE in suitable steps and measure V BE by a voltmeter and the base current using the microammeter. 6. Repeat step 5 for six different values of V BE . 7. Repeat step 4 to step 6 for some suitable value of V CE (say 10V). 8. Draw the input characteristics and find h ie . 9. Set I B to say 10 μA. 10. Increase V CE in very small steps from 0 V and record V CE and I C in each step. Since I C increases slightly in the active region, increase V CE in steps of 1V in this region. Always keep I B unchanged. 11. Increase I B by steps of 10 μA and in each case repeat step 10. Take at least five different values of I B . 12. Draw the output characteristics. 13. Determine the parameters β dc , β ac and h oe . 5.6 Experimental Results Type of the transistor ...... Maximum permissible collector current (I C) max = ... Approximate β of the transistor = ... Maximum allowable base current (I B) max = (I C) max /β = ... Table 1 Specification of the meters used Meter Range Value of Zero Smallest division error Voltmeter for measuring V BE Voltmeter for measuring V CE Milliammeter Microammeter

NSOU ? CC-PH-02 59 Table 2 Data for Input Characteristics V CE = ...Volt V CE = ...Volt V BE I B V BE I B (Volt) (μA) (Volt) (μA) ... ... etc. etc. etc. etc. Table 3 Determination of h ie V CE ΔV BE ΔI B h ie = ΔV BE /ΔI B (Volt) (Volt) (μA) (Ω) (from graph) (from graph) ... ... Table 4 Data for Output Characteristics I B = 10μA I B = 20 μA I B = ... μA I B = ... μA I B = ... μA V CE I C V CE I C V CE I C V CE I C V CE I C (Volt) (mA) (Volt) (mA) (Volt) (mA) (Volt) (mA) (Volt) (mA) ... ... etc. etc. etc. etc. etc. etc. etc. etc. etc. etc.

NSOU ? CC-PH-02 60 Table 5 Determination of β dc , β ac and h oe Operating point I C3 I C1 I B3 I B1 β dc β ac BC AC h oe = BC/AC (mA) (mA) (μA) (μA) (mA) (V) (Ω –1) Q 1 (I C =...mA, V CE =... V, and I B = ...μA) Q 2 (I C =...mA, V CE =... V, and I B = ...μA) 5.7 Discussions 1. Since the leads (emitter, base and collector terminals) of the transistor are very close to each other care should be taken so that one does not touch another. 2. R B in the base circuit is used to limit the base current so that proper forward bias appears at J E . 3. While taking readings for the output characteristics, care should be taken to keep the base current unchanged each time V CE is changed. Slight changes in I B may occur due to Early effect. 4. Care should be taken so that the rating of the transistor is not exceeded. 5. The transistor should not be inserted or removed from the circuit when the power is on. 5.8 Summary In this unit we have discussed the basics of a transistor (such as transistor types, input and output characteristics of a transistor, active and saturation regions, transistor parameters, etc.), how to find experimentally the input and output characteristics of a transistor, determine the transistor parameters from the characteristics.

NSOU ? CC-PH-02 61 5.9 Answers 1. See Sec. 5.1 2. See Sec. 5.1 3. See Sec. 5.1 4. Common- emitter, common- base and common-collector. 5. 1 st part: See Sec. 5.1. 2 nd part: A Field Effect Transistor (FET) is unipolar. 6. See Sec. 5.1 7. The emitter junction of a transistor is forward biased. So its resistance is small. Again the collector junction is reverse biased. So its resistance is very high. So the current in a transistor is transferred from a low resistance input circuit to a high resistance collector with nearly unchanged magnitude. So the name 'transfer resistor' or transistor. 8. See Sec. 5.1 9. The curves give the current-voltage relationship of a transistor. From these curves we can identify the active, cut-off and saturation regions required for applications of the transistor in circuits. Also we can find the transistor parameters needed for circuit analysis, from the curves. 5.10 Exercise 1. What is a transistor ? 2. Name the regions of a transistor. Which portion has highest doping and which portion has the lowest doping? 3. What is meant by CE configuration? Why? 4. In what configurations is a transistor used? 5. Why is the transistor called bipolar? Is there any unipolar transistor? 6. What are meant by dc current gain, ac current gain, input impedance and output admittance? 7. What is the significance of the name 'transistor'? 8. What are meant by input and output characteristics? 9. Why do you determine the characteristic curves of a transistor?

NSOU ? CC-PH-02 62 Unit 6 ? To determine the band gap energy of a semiconductor by four probe method Structure 6.1 Objectives 6.2 Introduction 6.3 Theory 6.4 Apparatus 6.5 Experimental Procedure 6.6 Experimental Results 6.7 Discussions 6.8 Summary 6.9 Answers 6.10 Exercise 6.1 Objective In this unit you will learn how to measure the resistance of semiconductor samples and find out the band gap of a semiconductor sample. 6.1 Introduction We know that bound states in isolated atoms are discrete. However, due to the interaction between the atoms in a crystal, these levels split up. Because of the huge number of atoms in a crystal, the level density is extremely high and these levels can be treated as continuous. Thus bands of allowed energies are formed in crystalline solids and electrons are located in these bands. The band which contains the valence electrons is called the valence band. The unoccupied energy levels also split up and form another band called the conduction band. The energy spacing between the top of the topmost valence band and the bottom of the conduction band is called bandgap, E g (or forbidden region).

NSOU ? CC-PH-02 63 At absolute zero temperature, the bands below the energy gap E g are completely filled and the conduction band is empty. Current conduction is not possible in empty and filled bands. Empty bands cannot contribute to current conduction as there are no carriers. On the other hand, the valence electrons move about the crystal, but they cannot be accelerated by an external electric potential/field because the acceleration means gain of energy and there are no higher energy levels available within the valence band to which they could rise. At temperature T<0K, some valence electrons will have energy more than the band gap and consequently can go to the conduction band. The actual fraction of elecrons having energy more than E g can be calculated using the Fermi-Dirac distribution function (discussed elsewhere). The electrons in the conduction band are called free electrons and they can gain energy when an electric field is applied, because there are many higher energy states available. The valence band now has equal number of empty energy levels, these are called holes. Electrons in the valence band can now gain energy in the valence band also, and we observe a motion of holes in the direction of the field. Thus, free electrons and holes are considered as carriers of electricity in a crystal. An insulator has a large bandgap (~ 6 eV), so that at room temperature the conduction band is practically empty and the valence band is practically filled. In metals, the valence and conduction bands overlap and a large number of electrons can take part in the conduction of current. An electric field can accelerate these electrons leading to very high conductivity. A semiconductor has a band gap between the metal and the insulator (~1 eV). As a result its conductivity lies in between the two. Measurement of resistivity of semiconductors faces some special problems. High resistance or rectification appears fairly often in electrical contacts to semiconductors and in fact is one of the major problem. Soldered probe contacts may disturb the current flow shorting out part of the sample. Since the resistivity is large, this leads to error in measurement. Soldering directly to the body of the sample can also affect the sample properties by heating effect and by contamination. These problems can be avoided by using pressure contacts. The principal draw backs of this kind of contacts are that they may be noisy.
NSOU ? CC-PH-02 64 The current through the sample should not be large enough to cause heating. A further precaution is necessary to prevent minority carrier injection from affecting the measured value of resistivity. An excess concentration of minority carriers will affect the potential of other contacts and modulate the resistance of the material. The four probe method overcomes these difficulties and also offers several other advantages. It permits measurements of resistivity in samples having a wide variety of shapes, including the resistivity of small volumes within bigger pieces of semiconductor. In this manner the resistivity of both sides of p-n junction can be determined with good accuracy before the material is cut into bars for making devices. This method of measurement is also applicable to silicon and other semiconductor materials. The basic model for all these measurements is indicated in Fig. 6.1. Four sharp probes are placed on a flat surface of the material whose band gap energy is to be Fig. 6.1: Schematic diagram of four probe showing the electrical contacts For this method in semiconductor crystals it is necessary to assume that: 1. The resistivity of the material is uniform in the area of measurement. 2. The surface on which the probes rest is flat with no surface leakage. High impedance Voltmeter Probes Specimen whose resistivity is to be measured Constant current power supply 1 2 3 4 s s s
NSOU ? CC-PH-02 65 measured, current is passed through the two outer electrodes, and the floating potential is measured across the inner pair. If the flat surface on which the probes rest is adequately large and the crystal is big the semiconductor may be considered to be a semi-infinite volume. 3. The diameter of the contact between the metallic probes and the semiconductor should be small compared to the distance between probes. The boundary between the current-carrying electrodes and the bulk material is hemispherical and small in diameter. 4. Mechanically lapped surfaces ensure good contact between the probe and the sample over a wide area so as the minority carriers are injected over a wider region. Measurements should be made on surface which has a high recombination rate, such as mechanical lapped surfaces so that most of the minority carrier injected into the semiconductor by the current – carrying electrodes recombine near these electrodes. The injection effect is further reduced by keeping the voltage drop at the contacts low. Since voltaage is measured between the two inner probes, the injected carriers will recombine before reaching the measuring probes. 5. The surfaces of the semiconductor crystal may be either conducting or non- conducting. In the first case a material of much lower resistivity than semiconductor is plated on the crystal. In the case of a non-conducting boundary, the surface of the crystal is in contact with an insulator. There are corrections for these two cases. However, in our experiment we will neglect them. 6.3 Theory The concentration of intrinsic carriers i.e. the number of electrons in conduction band per unit volume is given by the expression: n m kT e e E kT g = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ − 2 2 2 3 2 p m ? ..... (1) and the concentration of holes in valence band is given by the expressions p m kT e h kT = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ − ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ 2 2 2 3 2 p m ? ..... (2)

NSOU ? CC-PH-02 66 where, m e = Effective mass of an electron m h = Effective mass of a hole k= Boltzmann's constant, E g = Band gap, μ = Fermi level T = Temperature in K Multiplying the above expressionswe obtain the equilibrium relation np kT m m e e h E kT g = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ ( ) − ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ 4 2 2 3 1 2 p? ..... (3) This does not depend on the Fermi level μ and is known as the expression of law of mass action. In intrinsic (pure) semiconductors the number of electrons is equal to the number of holes, because the thermal excitation of an electron leave behind a hole in the valence band. Thus, we have, letting the subscript i denote intrinsic, n p kT m m e i i e h E kT g = = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ ( ) − ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ 2 2 2 3 2 1 4 2 p? ..... (4) It is clear from the above expression that the dominating temperature dependence arises from the exponential term. The electrical conductivity σ of an intrinsic semiconductor will be the sum of the contributions of both electrons and holes: s m m = + ( ) n e p e i e i h ..... (5) where e is the electronic charge, μ e and μ h are respectively the average velocities acquired by the electrons and holes in a unit electric field and are known as mobilities. We may write s m m = + n e i e h ( ) ..... (6) since n i = p i. We thus have s r m m = = + ( ) − ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ 1 3 2 2 KT e e h E kT g / ..... (7) where K is a constant and ρ is the resistivity.

NSOU ? CC-PH-02 67 The factor T 3/2 and the mobilities change relatively slow with temperature compared with the exponential term, and hence the logarithm of resistivity varies linearly with 1/T. The width of the energy gap may be determined from the slope of the curve. Thus, we have, log , e g E kT C r = − 2 ..... (8) Where C is a constant. The graph for log 10 ρ vs. T −1 will be a straight line. The slope of the graph will provide the band gap energy E g . The assumption that the semiconductor is intrinsic is applicable only at higher temperatures because then the intrinsic carrier density may be considered to be higher than the carrier density due to impurities. Thus use the high temperature part of the curve to calculate the slope. We assume that the probes are far from any of the other surfaces of the sample and the sample can thus be considered a semi-infinite volume of uniform resistivity material. Fig. 6.2 shows the geometry of this case. Four probes are spaced S 1 , S 2 and S 3 apart. Current I is passed through the outer probes and the floating potential V is measured across the inner pair of probes. Fig. 6.2: The four probe in pressure contact with the surface of the crystal. Conducting sheet W S 2 S 1 S 3 I I V

NSOU ? CC-PH-02 68 The floating potential V f at distance r from an electrode carrying a current in a material of resistivity ρ is given by V l r f = r p2 There are two current-carrying electrodes, numbered 1 and 4, and the floating potential V f , at any point in the semiconductor is the difference between the potential induced by each of the electrodes, since they carry currents of equal magnitude but in opposite directions. V l r r f = − ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ r p2 1 1 1 4 , where r 1 and r 4 are distances of the point from probe number 1 and 4, respectively. The floating potentials at probe 2, V f 2 , and at probe 3, V f 3 can be calculated. V l S S S f 2 1 2 3 2 1 1 = − + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ r p V l S S S f 3 1 2 3 2 1 1 = + − ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ r p The potential difference V between probes 2 and 3 is then V V V l S S S S S S f f = − = + − + − + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ 2 3 1 3 1 2 2 3 2 1 1 1 1 r p In our experiment, S 1 = S 2 = S 3 = S. Thus V V V l S f f = − = 2 3 2 r p and we have a simple expression for resistivity, r p = 2 S V l ..... (9) 6.4 Apparatus (1) the probe arrangement, (2) sample (The sample is millimetre in size and have a thickness w), (3) an oven provided with a heater to heat the sample, (4) constant

NSOU ? CC-PH-02 69 current generator, (5) oven power supply, (7) a thermometer, (8) a multimeter, (9) an ammeter, (10) a voltmeter The sample is millimetre in size and have a thickness w. The four probes are arranged linearly in a straight line at equal distance S from each other. A constant current is passed through the two probes and the potential drop V across the middle two probes is measured. An oven is provided with a heater to heat the sample so that behaviour of the sample is studied with increase in temperature 6.5 Experimental Procedure 1. The sample is usually already mounted and the probes are placed on it. A constant current is passed through the outer two probes and the potential drop V across the middle two probes is measured. 2. The four probe arrangement is placed in the oven. Check the continuity between the probes for proper electrical contacts using a multimeter. 3. Connect the outer pair of probes leads to the constant current power supply and the inner pair to the probe voltage terminals. Fix the thermometer in the oven through the hole provided. 4. Switch on the AC mains of Four Probe Set-up. Adjust the current to a desired value by checking the ammeter. 5. Connect the oven power supply and start heating the sample. Wait till the temperature shown by the thermometer reaches a steady value. Note down the temperature in the thermometer and the voltage across the inner probes using a voltmeter. 6. Allow the temperature to rise from the room temperature to a maximum of approximately 200 o C in steps of 20 o C. Wait till the temperature shown by the thermometer reaches a steady value. Note down the temperature in the thermometer and the voltage across the inner probes using a voltmeter. Now allow the sample to cool by decreasing the oven current to the previous values of temperatures. Note the temperature when the temperature becomes steady and the voltage during the cooling also. This will reduce the effect of any lag in temperature between the sample and the thermometer.

NSOU ? CC-PH-02 70 7. Calculate the resistivity ρ using the expression, r p = 2 1 S V for each temperature. 8. Plot a graph for log 10 ρ vs. T -1 . We know that log log e g e E kT K r = – 2 The slope of the curve will provide the band gap energy E g . Remember that the assumption that the semiconductor is intrinsic is applicable only at higher temperatures because then the intrinsic carrier density may be considered to be higher than the carrier density due to impurities. So use the high temperature part of the curve to calculate the slope. 9. Repeat steps 5 to 9 for at least two more currents through the sample and calculate the band gap in each case. Determine the mean band gap. 6.6 Experimental results Data supplied: S = ….. Table 1 Data for calculating the resistivity of the sample Current through the sample =….amp Sl Temp. Voltage (V) Temp. Resistivity T –1 (K –1) log 10 ρ No. (o C) Temp. Temp. T ρ increasing decreasing (K) (ohm-cm) 1. … … … … … … … 2. … … … … … … … 3. … … … … … … … 4. … … … … … … … etc. The band gap as determined from the slope of the curve E g =….eV Repeat the experiment for two other current values. Mean value of E g = …eV
NSOU ? CC-PH-02 71 6.7 Discussions 1. The thickness of the sample w is impoprtant for correction on the nature of the bottom surface on which the sample is placed. The correction depends of the values of w/S. We assume that the thickness is large so that the current flow through the sample does not depend on the bottom surface on which the sample is placed. 2.The assumption that the semiconductor is intrinsic is applicable only at higher temperatures because then the intrinsic carrier density may be considered to be higher than the carrier density due to impurities. So the high temperature part of the graph of log 10 ρ vs. T –1 is used to calculate its slope. 3. To reduce the effect of any lag in temperature between the sample and the thermometer, the temperature and the voltage are recorded when the temperature becomes steady both during heating and cooling. 6.8 Summary In this experiment, the resistivity of an intrinsic semiconductor is measured. Studying its variation with temperature, the band gap of the semiconductor has been determined. 6.9 Answers 1. See Section 6.1 for the advantages of the four-probe method. 2. Ordinary millivoltmeters have a low input resistance compared to electronic meters. Ideally potentiometer does not draw any current at balanced condition, that is it has an infinite resistance. Thus ordinary millivoltmeters requires more current to operate. As the semiconductor resistivity is high, we require a meter with very high input resistance compared to the resistance of the sample. Thus, we cannot use ordinary millivoltmeters to measure the probe voltage. 3. Refer to Section 6.3. 4. See Discussion 2 in Sec.6.7
NSOU ? CC-PH-02 72 5. At 0 K, there are no free electrons or holes in a semiconductor. This is the reason that it acts as an insulator. 6. See Section 6.1 7. See Section 6.1 8. See Section 6.1 9. See Section 6.1 10. See Section 6.1 11. See Section 6.1 6.10 Exercise 1. What is the advantage of Four Probe method over the other conventional methods? 2. Can we use an ordinary millivoltmeter instead of electronic millivoltmeter or potentiometer to measure the inner probe voltage? Why? 3. Explain the behaviour of the log 10 ρ vs. 1/T curve. 4. Why do we calculate the band gap only from the high temperature region of the graph? 5. Why does a semiconductor behave as an insulator at 0K? 6. Why bands are formed in a solid? 7. Classify semiconductor, insulator and metal on the basis of band theory. 8. What is band gap in a solid? 9. What are free electrons and holes? 10. Is conduction of current is possible when a band is completely filled or completely empty? 11. What is an intrinsic semiconductor?
Unit 7 ? To determine H by using vibrational magnetometer Structure 7.1 Objectives 7.2 Introduction 7.3 Theory 7.4 Apparatus 7.5 Experimental Procedure 7.6 Experimental Results 7.7 Calculation of percentage error 7.8 Discussions 7.9 Summary 7.10 Exercise 7.11 Answers 7.1 Objectives Studying this unit you will be able to learn about the Earth's magnetic field and how to measure its horizontal component. 7.2 Introduction The Earth behaves like a big magnet. The magnetic poles do not coincide with the geographical poles. Three elements are used to specify the Earth's magnetic field at any point on the globe. Magnetic declination is the angle on the horizontal plane between magnetic north, the direction the north end of a magnetized compass needle points, corresponding to the direction of the Earth's magnetic field lines, and true north. Magnetic dip or the angle of dip is the angle made with the horizontal by the Earth's magnetic field lines. These two angles vary at different points on the Earth's
NSOU ? CC-PH-02 74 Fig. 7.1: Magnetic elements of Earth 7.3 Theory The intensity of the magnetic field due to a magnet of length 2l calculated at a point P at a distance d from the centre of the magnet (O) (Fig. 7.2) lying on the axial line of the magnet (end-on position) is given by F m ld d l Md d l p = – = – m p m p 0 2 2 2 0 2 2 2 4 4 4 2 ( ) ( ) (1) Here, m p is the strength of the magnetic pole and M=2m p l is its magnetic moment. Here, 4π × 10 –7 H/m. Fig. 7.2: End-on position of a bar magnet Magnetic meridian Geographic meridian Magnetic declination Angle of dip B B v H α δ d P S N O l l surface. The horizontal component of the Earth's magnetic field is termed H. This is related to the Earth's magnetic field B by the relation, H = B cos δ, where δ is the magnetic dip.

NSOU ? CC-PH-02 75 A bar magnet is placed in the east-west direction near the magnetic needle of the deflection magnetometer. This arrangement is called the tanA position. The needle is then under the influence of the horizontal component of the earth's magnetic field as well as the field of the bar magnet acting perpendicularly to each other. The magnet produces a deflection θ in the magnetic needle. At equilibrium the couples acting on the needle must be equal. If M' is the magnetic moment of the couple, then M'Fcosθ = M'Hsinθ or, F H Md d l H = − = tan , ( ) tan q m p q or 0 2 2 2 4 2 M H d l d = − 4 2 0 2 2 2 p m q ( ) tan (2) When a magnet freely suspended in a uniform magnetic field is deflected through a small angle θ, the restoring couple acting on it is M H sin θ, which, as the angle of deflection is small, may be written as MHθ. When the magnet is released from its deflected position, the restoring couple tends to set it parallel to the magnetic field. The magnet thus acquires angular acceleration and the couple due to inertial reaction is I d dt 2 2 q , where I is the moment of inertia of the magnet about the axis of rotation. Thus we have an equation for θ, I d dt MH 2 2 q q = (3) The above equation represents a simple harmonic motion, whose time-period is given by T MH = 1 2 1 p or, MH I T = 4 2 2 p Thus, if we wish to determine the value of H in the laboratory, we can do so by performing the deflection and oscillation.

NSOU ? CC-PH-02 76 From the above results we obtain an expression for the horizontal component of earth's magnetic field, H H T d l Id = − 2 2 4 2 2 0 p m p q ( )tan (5) The moment of inertia of a uniform bar of length a, width b and mass m is given by I m a b = + ( ) 12 2 2 (6) 7.4 Apparatus Deflection and vibration magnetometers, a magnet, a brass bar with identical size of the magnet, a compass needle, a vernier callipers, a meter scale, a balance with weight box, and a stop-watch/stop clock. Description of the apparatus: Deflection magnetometer: A deflection magnetometer consists of a small magnetic needle pivoted on a sharp support such that it is free to rotate in a horizontal plane (Fig. 7.4). A light, thin, long aluminium pointer is fixed perpendicular to the magnetic needle. The pointer also rotates along with the needle.There is a circular scale divided into four quadrants and each quadrant is graduated from 0 o to 90 o . A plane mirror fixed below the scale ensures reading without parallax error, by ensuring that image of the pointer is coincident exactly with pointer itself. The needle, aluminium pointer and the scale are enclosed in a box with a glass top. There are two arms graduated in centimetre and their zeroes coincide at the centre of the magnetic needle. Vibration magnetometer: In this instrument, a light brass stirrup suitable for mounting a bar magnet is suspended from a torsion head by means of an unspun silk fibre. It hangs within a wooden box with glass walls (Fig. 7.3). A plane mirror having a central line marked LL is placed on the floor of the box. Two slits on the top of the box, parallel to LL, are used to determine the time of transit of the magnet during its oscillation.

NSOU ? CC-PH-02 77 Fig. 7.3: Vibrational magnetometer 7.5 Experimental Procedure (A) Deflection Experiment (i) Remove all magnets and magnetic substances from the neighbourhood of the table on which the deflection magnetometer is placed. Set the magnetometer in tan A position. For this purpose, turn the arms of the magnetometer on the table till they are parallel to the aluminium pointer. Rotate the compass box without disturbing the arms till the pointer reads zero-zero on the graduated circular scale. (ii) Now place the magnet on one arm in such a way that its axis is parallel to the arms of the magnetometer and when produced it passes through the centre of the magnetic needle. Since, the sensitivity of the magnetometer is more at 45 o , try to get a deflection between 30 o and 60 o .Obtain the distance d from the centre of the magnet to the centre of the needle by noting down the distance of the two edges of the magnet from the centre, d 1 and d 2 , and hence, d = (d 1 + d 2)/2. Torsion head

NSOU ? CC-PH-02 78 Fig.7.4: Bar magnet and deflection magnetometer in tan A position (iii) The deflection is subjected to a number of errors: (a) The pivot of the needle may not pass exactly through the centre of the graduated circular scale. To correct for this error, both the ends of the pointer should be read and the mean of the angles should be taken. (b) The magnetic centre of the bar magnet may not be exactly coincident with its geometrical centre. This may be so due to the unsymmetrical magnetisation of the magnet. To correct for this error, readings should be taken by reversing the polarity of the magnet at the same position. In this operation, the two poles simply interchange their positions. (c) The poles may not be exactly in symmetrical positions. Turn the magnet upside down and repeat the measurements. (d) The centre of the linear scale on which the distance is read may not coincide with the pivot of the magnetic needle. To correct for this error, transfer the bar magnet on the other arm so that the centre of the magnet lies at the same scale reading. Take readings of the deflection for all the above. Take the mean of all the deflections to eliminate these errors. (B) Oscillation Experiment (i) Remove all magnets and magnetic substances from the neighbourhood of the table on which the oscillation magnetometer is placed. Determine the direction of the magnetic meridian on the table using a long needle and draw a line. Place the vibration box with its longer edge parallel to this line. Now put the compass needle inside the box along the line marked on the plane mirror fixed to the base, and adjust d N S E N 0 0 90 90

NSOU ? CC-PH-02 79 the magnetometer, it necessary, to bring this scratch line exactly in the magnetic meridian. (ii) For conducting this experiment it is essential that when the magnet hangs in the magnetic meridian, the suspension fibre should have no twist, since in deriving the formula for the time-period it has been assumed that the only restoring couple is due to the earth's magnetic field alone, and that the torsional couple is negligible. Thus, a brass bar of identical size with the magnet is used to untwist the fibre. Place the brass bar in the stirrup and wait for the fibre to untwist. When the bar becomes motionless when hanging freely from the fibre, it is set parallel to the scratch line by turning the torsion head. During the process of removal of twist, the motion of the brass rod should be checked after every few revolutions, otherwise, when the fibre is untwisted, the inertia of the rotating bar may cause it to twist in the opposite direction. (iii) Hold the stirrup tightly in position and withdraw the brass rod. Replace it with the bar magnet used in the deflection experiment. The magnet should lie perfectly horizontal in the stirrup and its north pole should point northwards. Now with the help of a second magnet deflect the suspended magnet through a small angle and take away the second magnet to a safe distance from this oscillating magnet. Count twenty-five oscillations by looking through the slit at the top of the box and note the time with an accurate stopwatch/stop clock. Repeat the process four times and calculate the time-period of the magnet. C) Determination of the Constants of the Magnet (i) Measure the vernier constant of the slide calliper. (ii) Measure the effective length (2l) of the magnet. Measure its total length (a) with a metre scale/ slide calliper. Since the poles are not exactly at the ends of the bar magnet, take 2l = 0.85a as the effective length, half of which gives l. (ii) Measure the breadth (b) of the magnet with a vernier calliper. Weigh the magnet, and then with the help of a, b, and m, calculate the moment of inertia (I) of the magnet using eqn. (6). Finally, calculate H with the help of the formula (5) given above.

NSOU ? CC-PH-02 80 7.6 Experimental results Table 1 Measurement of the distance d of the magnet from the needle d 1 (m) d 1 (m) d d d m = + 1 2 2 ( ) … … … Table 2 Measurement of the deflection of the needle of the deflection magnetometer Magnet on the east arm Magnet on the west arm Mean deflection Pole facing needle Pole facing needle θ North South North South Pointer One Other One Other One Other One Other end end end end end end end end Face up … … … … … … … … Face … … … … … … … … down Repeat the measurement for different values of d. Table 3 Determination of the time period of the oscillation No. of observations Time t for 25 Mean time for 25 T=t/25 (s) oscillations (s) oscillations (s) 1. … … … 2. … 3. …

NSOU ? CC-PH-02 81 Table 4 Measurement of the mass of the magnet No. of obs Mass of the magnet m Mean mass of the magnet m (kg) (kg) 1. … 2. … … 3. … Table 5 Determination of the vernier constant of the slide calliper. ….. divisions of the vernier scale (p) = … divisions of the main scale (q) Value of 1 smallest scale Value of 1 vernier division (cm) Vernier constant (cm) division (L 1) (cm) L q p L 2 1 = (L 1 − L 2) Table 6 Determination of length (a), breadth (b) and moment of inertia of the magnet No. Length Mean Breadth Mean I m a b = + ( ) 12 2 2 of Length breadth (kg m 2) obs. Main Vernier Total a (cm) Main Vernier Total b(cm) scale scale (cm) scale scale (cm) reading reading reading reading (cm) (cm) (cm) (cm) 1. … … … … … … 2. … … … … … … … … … …. 3. … … … … … … We assume that the length can be measured with slide callipers. If it is not possible, measure it carefully with a metre scale.

NSOU ? CC-PH-02 82 Table 7 Determination of H No. d (m) θ I (kg m 2) T (s) l a = 0 85 2 . H T d l Id = − 2 2 4 2 2 0 p m p q ( )tan Mean of (From (m) H Obs. Table 6) (tesla) (tesla) 1. … … … 2. … … … … … … … 3. … … … 7.7 Calculation of percentage error We have from equation (5) H T d l Id Id T d l 2 2 2 2 2 2 0 0 2 2 2 2 4 2 4 2 = − = − p m p q pm q ( )tan ( ) tan Taking logarithmic derivative,we have the maximum proportional error as 2 2 2 2 2 2 2 2 d d d d qdq q H H I I d d T T d l d l = + + + − − + ( ) ( ) sec tan Here we write, 2 4 4 2 2 2 2 2 2 ( ) ( ) ( ) d l d l d d l l d l − − = + − d d From equatin (6), we see that d d d d l I m m a a b b a b = + + + 2 2 2 2 Since T=t/2, we see that δT / T = δt / t . Noting the smallest scale reading in each instrument, the relative error in H, i.e. δH / H can be estimated. Remember that δθ must be expressed in radian.

NSOU ? CC-PH-02 83 7.8 Discussions 1. All pieces of magnetic materials and current-carrying conductors should be removed to a considerable distance from the magnetometers. Examine your pockets for the presence of magnetic materials. 2. The deflection magnetometer should be carefully set in the tanA position and the magnet should be so placed on the arms that its axis, when produced, passes through the centre of the magnetometer needle. 3. The deflection of the needle should be as nearly equal to 45° as possible, since under the condition the deflection shall be susceptible of yielding maximum accuracy. Further the value of d should be large, so that the field due to the magnet in the region occupied by the needle is sufficiently uniform. However, if it is not feasible to procure the above two conditions at the same time, a compromise should be affected by making d large so that the deflection falls in the neighbourhood of 30°-60°. 4. The pivot of the needle may not pass exactly through the centre of the graduated circular scale. If the magnet is unsymmetrically magnetised, the magnetic centre shall not be coincident with its geometrical centre, hence the distance between the magnetic centre of the bar magnet and the centre of the magnetic needle in the compass box, shall not be correctly measured. The poles may not be exactly symmetrical. The centre of the linear scale on which the distance d is read may not be coincident with the pivot of the needle. Care must be taken to eliminate these errors by taking maesurements as discussed above. 5. While reading the deflection of the aluminium pointer on the graduated scale, the error due to parallax should be avoided. For this purpose, use should be made of the plane mirror attached to the base of the compass-box. 6. Before oscillating the magnet in the vibration box, initial twist in the suspension fibre should be completely removed with the help of a metallic bar of some non-magnetic material. The bar should preferably be of the same size and shape as the magnet itself.

NSOU ? CC-PH-02 84 7. As the moment of inertia of the stirrup is not taken into account in the derivation of the above formula for T, it should be very light. 8. In the derivation of the formula for the time-period of the magnet it has been assumed that θ is small so that the restoring couple M H sinθ = M Hθ. In order to satisfy this condition, the deflection of the oscillating magnet should not be more than 4 o . 9. The oscillations should not be counted by looking at the magnet from the glass side of the box but the eye should be held vertically over the slit made on the top of the box, and the counting should be done with reference to the scratch line made on the glass plate at the bottom of the box. 10. The main sources of error in this experiment are: — (i) The magnetometer needle is not sufficiently short, hence it cannot be justified that it moves in a uniform field produced by the bar magnet — a condition which is absolutely necessary for the validity of the Tangent Law. (ii) The friction at the pivot is not totally absent; hence the measurement of the deflection is not very accurate. Moreover, the pointer and scale method is not very accurate. Generally the two conditions that the deflection of the pointer be around 45 o , and the distance of the magnetic needle from the magnet be fairly large, cannot be satisfied simultaneously. (iii) The effective length of the magnet is not accurately determined. (iv) In the derivation of the formula for the time-period, the moment of inertia of the stirrup is neglected. (v) The suspension fibre may not be completely free from torsional reaction. 7.9 Summary In this unit we discuss what is the horizontal component of Earth's magnetic intensity and how to measure it accurately. Proportional error has been calculated.

NSOU ? CC-PH-02 85 7.10 Answers 1. Refer to Section 6.1 2. The moment of a magnet M is related to the pole strength m p and true length 2l by the relation M = 2m p l. The poles are not at the exact end of the bar but near the end because of the repulsion between the similar poles of the molecular magnets. 3. We can measure the quantities M/H and MH, respectively form equations (2) and (4). We can estimate M using the above results as M T d l l d = – 2 2 2 2 0 p p q m ( ) tan 4. In the tan B position, the magnetic field at a distance d from the centre of the magnet is given by F Md d l = – m p 0 2 2 2 4 ( ) Since the field is weaker by a factor of two the corresponding deflection will be smaller and the accuracy will be less. 5. Since, the sensitivity of the magnetometer is maximum at 45 0 , try to get a deflection between 30 0 and 60 0 6. In the derivation of the formula for the time-period of the magnet it has been assumed that θ ¸ is small so that the restoring couple M H sin θ ¸ = M H θ. In order to satisfy this condition, the deflection of the oscillating magnet should not be more than 4 0 .. 7. Refer to serial No. 10 of Sec. 7.8 and Sl. No. A(iii) of Sec. 7.5. 8. Refer to Sec.7.3. 9. Magnetic moment of a magnet is the product of the distance between its poles and the strength of either pole. 10. The moment of inertia of a rigid body is a quantity that determines the torque needed for a desired angular acceleration about a rotational axis; similar to how mass determines the force needed for a desired acceleration.

NSOU ? CC-PH-02 86 7.11 EXERCISES 1. What are meant by declination, dip and horizontal component of earth's magnetic intensity? 2. How is the moment of a bar magnet related to its pole strength and length? Why do we take the length of the actual magnet smaller than its actual length? 3. Can the magnetic moment of the bar magnet be measured from the above measurements? 4. What would be the difference if in the deflection measurement the magnet be placed in the tan B position, that is in the north-south orientation with the centre coinciding with the centre of the needle in the east west direction? 5. Why should the deflection of the magnet be kept between 30 0 and 60 0 ? 6. Why should the deflection of the oscillating magnet be within 4 0 ?. 7. What are the main sources of error in the experiment ? 8. What is meant by tan A position? 9. What is magnetic moment of a magnet? 10. What is meant by moment of inertia of a rigid body?

Unit 8 ?To determine the self-inductance of a coil by Anderson's bridge Structure 8.1 Objectives 8.2 Introduction 8.3 Theory 8.4 Apparatus 8.5 Experimental Procedure 8.6 Experimental Results 8.7. Discussions 8.8 Summary 8.9 Answers 8.10 Exercise 8.1 Objectives In this unit you will learn how to determine the self- inductance of a coil by Anderson's bridge. 8.2 Introduction You have learnt in your +2 course what is meant by self- inductance. An insulated wire wound into a coil around a core is called an inductor. The coil has a self- inductance.

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When the current flowing through the coil changes, the time-varying magnetic field induces

an electromotive force (e.m.f.) (voltage) in the coil described by Faraday's law of induction. According to Lenz's law, the induced voltage has a polarity (direction) which opposes the change in current that created it. As a result, inductors oppose any changes in current through them. The ratio of the induced

NSOU ? CC-PH-02 88 voltage to the rate of change of current through the coil is called the self- inductance of the coil. Its unit is henry (H). Along with capacitors and resistors, inductors are one of the three passive linear circuit elements that make up electronic circuits. Inductors are widely used in alternating current electronic equipment, particularly in radio equipment.They are also used in electronic filters to separate signals of different frequencies, and in combination with capacitors to make tuned circuits, used to tune radio and TV receivers. 8.3 Theory When the current flowing through coil a changes, a voltage is induced in the coil. The ratio of the induced voltage to the rate of change of current through the coil is called the self- inductance of the coil. For low frequencies, a practical coil can be represented by a self- inductance in series with a resistance which accounts for the losses in the coil. The circuit for the measurement of the self- inductance of a coil by Anderson's bridge is given in Fig. 8.1. Fig. 8.1 Anderson's bridge Fig.8.2 Circuit for dc balance Let L be the self- inductance of the given coil and s be its resistance. A variable resistance s 1 is connected in series of the coil. Let S = s + s 1 be the total resistance in the arm CD of the bridge. P, Q, R and s 1 are non-inductive resistances. C is a capacitor, N is a null detector, usually a headphone, and r is a variable non-inductive resistance. At balance, that is when the current through the detector is zero, we have B A C r P Q C R N K 1 s 1 D K 2 Rh S Coil B A P G R K 1 D K 2 C Q s 1 Rh S Coil B L

NSOU ? CC-PH-02 89 S = RQ/P ..... (1) and L = CR[Q + r(1 + Q/P)] .... (2) Equations (1) and (2) are referred to the dc and ac balance conditions of the bridge. If P = Q, Eqn. (2) reduces to L = CR(Q +2r) ....(3) The ac balance represented by Eqn.(3) can be achieved only when L < CRQ, otherwise the resistance r will be negative. If C is expressed in farad, R, Q and r are expressed in ohm, then L will be obtained in henry from Eqn. (3). A graph with 1/C along the abscissa and r along the ordinate will be a straight line of slope L/(2R)(Fig. 8.3). L can be calculated by determining the slope of the graph. Fig.8.3 (1/C)-r graph 8.4 Apparatus (1) A P. O. Box to provide the resistances P, Q and R, (2) a resistance box containing fractional ohm and resistances from 1 to 200 ohm which will provide the variable resistance s 1 , (3) a resistance box containing resistances of 1 to 10000 ohm to provide the variable resistance r, (4) a few capacitors having capacitance 0.1μF and higher, (5) a headphone/ ac null detector, (6) an audio oscillator, (7) a dc galvanometer, (8) a rheostat, (9) a dc power supply, (10) two plug keys. 8.5 Experimental Procedure (a) Attainment of dc balance: 1. Set up the circuit as shown in Fig. 8.2. Here G is a dc galvanometer; K 1 and K 2 are plug keys. r (Ω) C A B (μF −1) 1 C

NSOU ? CC-PH-02 90 2. Take P = Q = R = 100 Ω in the P.O. Box. 3. Close key K 2 . 4. Close key K 1. Take the resistance s 1 = 0 and note whether the deflection of the galvanometer is towards left or right (say, for example, it is left). Then take resistance s 1 = 200 Ω and it will be observed that the galvanometer deflection is in the opposite direction (i.e., right). 5. Decrease s 1 and observe the galvanometer deflection. If it is right decrease s 1 further. Continue decreasing s 1 till the deflection is towards left. 6. Increase s 1 and observe the galvanometer deflection till the galvanometer deflection is in the opposite direction for 1Ω change of s 1 . 7. Now insert a fractional resistance and observe the galvanometer deflection. Change the value of the fractional resistance and each time observe the galvanometer deflection till the galvanometer deflection is zero. The total resistance in the arm CD of the bridge will then be S= 100 Ω = s+ s 1 . Hence the resistance of the coil s = 100 − s 1 . (b) Attainment of ac balance: 1. Set up the circuit as shown in Fig.8.1.Close the key K 2 . Adjust the output voltage of the audio oscillator to a suitable value and set its frequency near 1 kHz. 2. Choose the lowest value of C. Close key K 1 . Vary the resistance r until the sound in the headphone (or the deflection of null detector) is zero or a minimum. 3. Repeat step 2 for at least 4 different values of C. 4. Calculate L using Eqn.(3) for different values of C taken and find their mean. 5. Draw a graph with 1/C along the abscissa and r along the ordinate. The graph will be a straight line of slope L/(2R). Find the slope of the graph and calculate L.

NSOU ? CC-PH-02 91 8.6 Experimental Results Table 1 Data for dc balance (Numerical figures are for illustration only) P Q R s 1 Galvanometer Value of s 1 at Coil resistance (in Ω) (in Ω) (in Ω) (in Ω) deflection Null point s 1n s = 100 − s 1n (in Ω) (in Ω) 100 100 100 0 Left 100 100 100 200 Right 100 100 100 50 Left 100 100 100 80 Right 100 100 100 79 Left 100 100 100 79.5 Left 100 100 100 79.6 0 79.6 20.4 100 100 100 79.7 Right Table 2 Data for ac balance P = 100 Ω, Q= 100 Ω, R = 100 Ω, S = s + s 1 == 100 Ω, Frequency of the oscillator = ... Hz. Capacitance C Value of r Sound intensity in Value of r at L Mean L (in μF) (in Ω) the headphone/ null point (in mH) (in mH) deflection of the (in Ω) null detector ... ...

NSOU ? CC-PH-02 92 ... ... etc. etc. etc. etc. etc. Table 3 Data for (1/C) −r graph C 1/C BC (Ω) AB (μF -1) Slope m = L = 2Rm (in μF) (in μF -1) L/(2R) = (in mH) BC/AB .... etc. etc. etc. etc. 8.7 Discussions 1. The value of C must be such that L < CRQ. So either an approximate value of L has to be known or the lowest value of C has to be used. 2. To make the bridge sensitive, so that the ac balance can be determined accurately, same values of resistances have been used in the four arms of the bridge. 3. Since our ear is most sensitive at about 1 kHz the frequency of the ac source was fixed at 1 kHz. If, however, an ac null detector is used we can use other frequency. 4. All resistances used were non-inductive. 5. All the connecting wires should be straight and short.

NSOU ? CC-PH-02 93 8.8 Summary In this unit we have discussed what is meant by self- inductance, mentioned some use of self- inductance and measure the self- inductance of a coil by Anderson Bridge. 8.9 Answers 1. See Sec. 8.1 2. When the rate of change of current in a coil is 1Amp/sec induces a voltage of 1 Volt in the coil the self- inductance is 1 Henry. 3. To reduce the self- inductance of the connecting wires. 4. If a headphone is used to find ac balance the frequency of the ac source should be about 1 kHz since our ear is most sensitive at this frequency. If, however, an ac null detector is used we can use other frequency. 5. The self- inductance of a coil depends on the number of turns/unit length, the diameter and length of the coil, permeability of the material of the core on which the coil is wound. 6. The self- inductance of the coil will increase as the permeability of iron is about 1000. 7. If the resistances used have inductance the ac balance condition will be affected and the determination of self- inductance of the coil will not be correct. 8. If a piece of insulated wire is doubled on itself and then wound on a wooden/porcelain/ebonite rod, the resulting winding will have negligibly small self-inductance. Due to doubling, the current through the two halves of the wire flows in opposite directions so that the effective magnetic flux linked with the wire is practically zero. 9. See Sec. 8.1. 10. Maxwell's bridge. 11. See Sec. 8.1.

NSOU ? CC-PH-02 94 12. Each 10 mH. 13. See Sec. 8.3. 8.10 Exercise 1. Define self- inductance. What is its unit? 2. Define Henry. 3. Why should all the connecting wires be straight and short? 4. What frequency of the ac source should be used? Why? 5. On what factors does the self- inductance of a coil depend? 6. A piece of iron is placed inside a coil. Will the self-inductance of the coil change? 7. Why should resistances used be non-inductive? 8. How is a resistance made non-inductive? 9. Mention some uses of an inductor. 10. Name another bridge for the measurement of self- inductance. 11. State Lenz's law. 12. Self- inductance of a coil is 20 mH. If it is broken in two equal parts what will be the self- inductance of the parts? 13. How is a practical coil represented?

Unit 9 ? To draw e-t curve of a thermocouple Structure 9.1 Objectives 9.2 Introduction 9.3 Theory 9.4 Apparatus 9.5 Experimental Procedure 9.6 Experimental Results 9.7 Discussions 9.8 Summary 9.9 Answers 9.10 Exercise 9.1 Objectives By studying this unit you will learn ? to measure the resistance of the wire of a potentiometer by a P.O. Box. ? to use a potentiometer to measure a potential difference ? to draw the e-t curve of a thermocouple. 9.2 Introduction If two metallic conductors made of two different metals are joined at their two ends it is called a thermocouple. If the junctions are kept at different temperatures a current flows in the thermocouple, that is an electromotive force is generated in it. This e. m. f. is called thermo-e.m.f.. This effect is called thermoelectric effect and is known as the Seebeck effect. The thermo-e.m.f. first increases with the increase in

NSOU ? CC-PH-02 96 temperature difference between the junctions and after a certain temperature difference, decreases and becomes zero. If the temperature difference is increased further the thermo-e.m.f. increases again but in the opposite direction. If one junction is kept at 0°C, the temperature at which the thermo-e.m.f. is maximum is called the neutral temperature (t n) and the temperature at which the thermo-e.m.f. is zero is called the temperature of inversion (t i). The amount of thermo-e.m.f. generated for a given temperature difference between the junctions depends on the metals of the thermocouple. By measuring the thermo-e.m.f. generated in a thermocouple temperature can be measured. That is, a thermocouple can be used as a thermometer. To measure the temperature a curve is drawn with the thermo-e.m.f. generated (e) versus the temperature of the hot junction (t) when the other junction is kept at 0 0 C. This curve is called the e-t curve. If the temperature of hot junction is much less than the neutral temperature the e-t curve is a straight line. To measure the temperature of a body, one junction of the thermocouple is kept at 0 0 C and the other junction is kept in contact with the body whose temperature is to be measured. The thermo- e.m.f. generated is measured. The temperature is obtained from the e-t curve corresponding to the measured thermo- e.m.f. 9.3 Theory An e.m.f. is generated in a thermocouple when its two junctions are at different temperatures. If this thermo-e.m.f. e is balanced against a potential drop produced across a length l of a potentiometer wire, then e = ρl, ... (1) where ρ = potential drop/unit length of the potentiometer wire. In the circuit of Fig. 9.1 if E = the e.m.f. of the storage battery B connected with the potentiometer, R p is the resistance of the potentiometer wire, L = length of the potentiometer wire, R = resistance connected in series with the potentiometer, then ρ = ER p /[(R p + R)L] .... (2) Hence, e = ER p l /[(R p + R)L] .... (3)

NSOU ? CC-PH-02 97 Fig. 9.1 Experimental arrangement By maintaining the temperature of one junction constant (usually 0 0 C) and increasing the temperature of the other junction, a curve relating the variation of thermo- e.m.f. (e) against the temperature (t) of the hot junction can be drawn. This curve is called the e-t curve of the thermocouple. 9.4 Apparatus (1) a potentiometer, (2) a P. O. Box, (3) a storage battery, (4) a resistance box R, (5) a dc galvanometer, (6) a resistance box to be connected in series with the galvanometer, (7) a voltmeter/ multimeter, (8) a thermocouple, (9) a thermometer reading upto 0.1 0 C, (10) a glass beaker containing water, (11) a funnel containing powdered ice, (12) a burner, (13) a plug key, (14) a stirrer, (15) connecting wires, (16) a glass rod, (17) a beaker to collect the water of melted ice in the funnel. 9.5 Experimental Procedure 1. Measure the resistance R p of the potentiometer wire by connecting it to the fourth arm of a P. O. Box. First use P = Q = 10 Ω. Then use P = 100 Ω and Q =10 Ω. (The circuit is similar to that in Fig. 8.2 expect that the potentiometer is in the fourth arm instead of the coil and the fractional resistance box.) 2. Measure the e.m.f. of the storage battery by a voltmeter/ multimeter before and after the experiment. Rectify the zero error of the voltmeter, if any. 3. Connect the circuit as shown in Fig. 9.1. One junction of the thermocouple is immersed in the powdered ice in the funnel and the other junction is Metal 1 Metal 2 Metal 3 B G R K Potentiometer S Beaker Funnal J

NSOU ? CC-PH-02 98 immersed in the water, at room temperature, in the beaker. The bulb of the thermometer is immersed in the water close to the thermocouple junction. Note the reading of the thermometer. 4. Calculate the value of the resistance R from Equ. (2) taking ρ = 5 mV/cm. Use this value of R. To ensure that the polarities of the battery B and the terminals of the thermocouple are properly connected close the plug key K and make a momentary contact of the jockey J first at one end of the potentiometer wire and then at the other end. If the polarities of the battery B and the terminals of the thermocouple are properly connected, the deflections of the galvanometer G will be in the opposite directions in the two cases. If not, reverse the polarity connections of the battery. 5. Put some resistance in the resistance box S. Increase the value of resistance in the resistance box R so that the null point is obtained (that is, the deflection of the galvanometer is zero) on the 9 th or 10 th wire of the potentiometer. Now make the resistance in the resistance box S zero to increase the sensitivity and find the null point. Changing the position of the jockey note the null point three times and take the mean. [If the galvanometer is not sensitive enough, the null point may be obtained over a small range of length of the potentiometer wire. In such a case note the range and take the middle of the range as the null point.] 6. Heat the water in the beaker using the burner to increase its temperature by about 10 0 C, stir the water and adjust the burner so that the temperature remains constant for 2/3 minutes. Decrease the resistance R such that the null point is obtained on the 10 th or 9 th wire. Record the null point three times during the time the temperature of the water remains constant and take the mean. Record the temperature of water. 7. Repeat step 6 several times till the temperature of the water increases to about 95 0 C. Throughout the experiment the temperature of the cold junction should be kept at 0 0 C. To achieve this powdered ice in the funnel has to be pressed by a glass rod and adding more powdered ice in the funnel time to time.

NSOU ? CC-PH-02 99 8. Calculate the thermo- e.m.f. for each temperature recorded using Eqn. (3) and draw the e-t curve by plotting the temperature of the hot junction along the abscissa and the corresponding thermo-e.m.f. along the ordinate. 9.6 Experimental Results Table 1 Measurement of the resistance R p of the potentiometer wire (Numerical figures are for illustration only) P Q R Direction of the deflection Resistance Rp of the (in Ω) (in Ω) (in Ω) of the galvanometer potentiometer wire (in Ω) 10 10 0 Right • Left 10 Right 30 Left 20 Right 22 Left 20 Slight right 20 > R p > 21 21 Slight left 100 10 200 Slight right 210 Slight left …. …. …. …. 205 Zero R p = 20.5

NSOU ? CC-PH-02 100 Table 2 Measurement of the e.m.f. of the battery Zero error of the voltmeter = …. Volt Time of observation E.M.F. (E) of the battery Corrected E Mean E (in Volt) (in Volt) (in Volt) Before experiment …. …. …. After experiment …. …. Table 3 Data for thermo-e.m.f. vs. temperature graph Length of the potentiometer wire (L) = …. cm Resistance of the potentiometer wire R p = …. Ω (from Table 1) E.m.f. of the battery E = V (from Table 2) Temperature of the cold junction = °C No. Temperature (t) Resistance Null point Length for Thermo- of of the hot R (in Ω) Wire Scale Mean Balance e.m.f. obs. junction No. Reading Scale (in cm) e (in mV) (in °C) (in cm) Reading (using (in cm) Equ. (3)) 1. Room temp. = …. …. …. …. …. …. …. 2. …. …. …. …. …. …. …. …. …. 3. …. …. …. …. …. …. …. …. …. etc. etc. etc. etc. etc. etc. etc. etc.

NSOU ? CC-PH-02 101 9.7 Discussions 1. Ensure that the temperature of the cold junction is maintained at 0 0 C throughout the experiment. The powdered ice in the funnel has to be pressed from time to time and fresh powdered ice to be added. 2. The water in the beaker should be heated slowly and stirred so that the temperature of water remains constant for about 2/3 minutes and the null point can be determined three times for each temperature. 3. The e.m.f. of the battery should remain constant throughout the experiment. It is to be checked from time to time. 4. To reduce the error in the measurement of thermo-e.m.f. the length of the potentiometer wire to balance the thermo-e.m.f. should be more than 800 cm. So the resistance R should be such that the null point is obtained on the 9 th or 10 th wire. 5. The plug keys of the P. O. Box and the resistance box are to be kept tight. 6. The wires of the thermocouple must be kept in an insulated cover so that the wires do not touch each other. 9.8 Summary In this unit we have discussed what is a thermocouple, its use as a thermometer, how to draw the e-t curve of the thermocouple. 9.9 ANSWERS 1. See sec. 9.1 2. See sec. 9.1 3. See sec. 9.1 4. See sec. 9.1 5. To ensure that the polarities of the battery B and the terminals of the thermocouple are properly connected close the plug key K and make a momentary contact of the jockey J first at one end of the potentiometer wire

NSOU ? CC-PH-02 102 and then at the other end. If the polarities of the battery B and the terminals of the thermocouple are properly connected, the deflections of the galvanometer G will be in the opposite directions in the two cases. 6. No. Because the thermo-e.m.f. is few millivolts. Also a voltmeter actually measures potential difference and not e.m.f. 7. The e-t curve is parabolic. But since the temperature of the hot junction is much less than the neutral temperature we obtain a small portion of the curve and is a straight line. 8. See sec.9.1. 9. The temperature of inversion is as far above the neutral temperature as the cold junction is below it. 10. To reduce the error in the measurement of thermo-e.m.f. the length of the potentiometer wire to balance the thermo-e.m.f. should be more than 800 cm. So the resistance R should be such that the null point is obtained on the 9th or 10th wire. 9.10 Exercise 1. What is a thermocouple? How is it used to measure the temperature of a body? 2. What is thermoelectric effect? 3. What is Seebeck effect? 4. What are meant by neutral temperature and temperature of inversion? 5. How would you identify whether the polarities of the battery and the thermocouple are properly connected to the potentiometer? 6. Can you use an ordinary voltmeter to measure thermo-e.m.f.? 7. What is the nature of the e-t curve? 8. How will the thermo-e.m.f. change if we go on increasing the temperature of the hot junction? 9. How is the temperature of inversion related to the neutral temperature? 10.Why have you choose to have the null point on the 10th or 9th wire?

Unit 10 ? To determine the elastic constants of the material of a wire by Searle's method Structure 10.1 Objectives 10.2 Introduction 10.3 Theory 10.4 Apparatus 10.5 Experimental Procedure 10.6 Experimental Results 10.7 Calculation of Percentage Error 10.8 Discussions 10.9 Summary 10.10 Answers 10.11 Exercise 10.1 Objective In this unit you will be able to learn: ? to measure the diameter of a wire using a screw gauge ? to measure the elongation of a wire using micrometer screw ? use the measurements to determine Y of the material of the wire 10.2 Introduction In Unit 4 you have learnt what is meant by elasticity of a material. The elastic constants are Young's modulus Y, bulk modulus K, rigidity modulus nand Poissons ratio σ. You have also learnt to determine the Young's modulus Y of the material ofa wire by flexure method. In this unit you will learn to determine Y by Searles method.

NSOU ? CC-PH-02 104 10.3 Theory Young's modulus is a measure of the stiffness of a solid material. It is calculated for elongation or compression which are within elastic limit when the external applied force is removed. Young's modulus is a characteristic property of the material and is independent of its dimensions i.e., its length, diameter etc. Consider a wire of length L and area of cross-section A and radius r. Let its length L increases by an amount l when the wire is pulled by a longitudinal external force F. Young's modulus of the material of the wire is given by, Y Longitudinal Stress Longitudinal Strain F A l L FL Al mgL r l g = = = = = p 2 LL r m l p 2 (⎛ ⎜ ⎝ ⎞ ⎠ ⎟ ... (1) 10.4 Apparatus (1) Searle's apparatus (which consists of a dummy wire and an experimental wire, a micrometer screw with linear and circular scale), (2) a spirit level, (3) a meter scale, (4) hanger for placing weights, (5) weights. Diagram Right Support Weight Hanger Experimental Wire (B) Chuck F2 Metal Frame O Dummy Wire (A) Chuck F1 Sprit Level Metal Frame P Dead Weight Micrometer Screw Gauge Fig. 10.1 Searle's apparatus

NSOU ? CC-PH-02 105 10.5 Experimental Procedure 1. Apply suitable weights on the hangers of both the dummy wire and the experimental wire to keep them taut. Call this load 'dead load' or zero load. Measure the length L of the experimental wire by a meter scale. 2. Find the least count of the screw gauge. Record its instrumental error, if any. Measure the diameter d of the experimental wire at five/six different places. At each place take readings at two mutually perpendicular directions [(a) and (b)] to correct for the ellipticity of the cross-section, if any. 3. Find the least count of the micrometer screw. 4. Turn the micrometer screw to adjust the spirit level in horizontal position. Record the readings of the linear scale and circular scale of the micrometer screw. 5. Increase the load on the hanger of the experimental wire in steps of 0.5 kg six to eight times. The spirit level is tilted due to the elongation of the experimental wire. Each time repeat step 4. 6. Now decrease the load on the hanger in the same steps of 0.5 kg till the load is zero. Each time repeat step 4. Thus for a particular load there will be two readings, one for load increasing and the other for load decreasing. Take the mean of the readings. 7. Calculate the elongation 'l' of the experimental wire for each load "m" by subtracting the reading for a particular load and the reading for zero load. 8. Draw a graph with load 'm' along the x-axis and the corresponding depression 'l' along the y-axis. This load-elongation graph is a straight line passing through the origin. 9. Take a suitable point on the load-elongation graph and find the values of'm' and 'l' corresponding to that point. 10. Calculate Y using equation (1). 10.6 Experimental Results Measurement of the length L of the experimental wire: (1) ...... cm (2) ...... cm (3) ...... cm Mean L = ... cm.

NSOU ? CC-PH-02 106 Table 1 Measurement of the radius r of the experimental wire Determination of the Least Count (L.C.) of the screw gauge Pitch of the screw (p) = ... cm No. of divisions of the circular scale (n) = ... L.C. = p/n = ... cm No. of Linear scale Circular scale Total reading Mean Instrumental Corrected Radius obs. reading (x) Reading (y) d = x + y d error (cm) d (cm) r (cm) (cm) × L.C. (cm) (cm) 1 (a) ... ... (b) ... ... 2 (a) ... ... (b) ... ... 3(a) ... ... (b) ... ... etc. Table 2 Data for load-elongation graph Deterrni nation of the Least Count (L.C.) of the m icrorneter screw Pitch of the screw (p) = ... cm No. of divisions of the circular scale (n) = ... L.C. = p/n = ... cm Load Micrometer reading m Load increasing Load decreasing Mean Elongation (kg) reading (cm) (cm) Linear Circular Total Linear Circular Total scale scale reading Scale Scale reading reading reading d=x+yx reading reading d=x+yx (x) (y) L.C. (x) (y) L.C. (mm) (cm) (mm) (cm) 0 ... ... (a) 0 0.5 ... ... (b) (a) – (b) 1 ... ... (c) (a) – (c) .... etc.

NSOU ? CC-PH-02 107 Table 3 Determination of Y g = ... L (cm) r(cm) m (kg) 1 (cm) from Table 1 from graph from graph (Eqn. (I)) N/m 2 ... ... 10.7 Calculation of Percentage Error Y = gLm/πr 2 l. The quantities L, l and r are measured. The maximum proportional error in Y due to the errors in the measurement of these quantities is given by δY δL δr δl — = — + 2 — + ... + — Y L r l Here L is measured by a metre scale. So the maximum error in the measurement of L is δL = 0.1 cm. The diameter of the wire is measured by a screw gauge ofL. C. = 0.001 cm. So the maximum error in the measurement of diameter is 0.001 cm. So δr = 0.001/2 cm = 0.0005. 1 is measured by a micrometer screw L.C. = 0.001 cm (say). So the maximum error in the measurement of 1 is twice this value, i.e., δ1 = .002 cm. So δY/Y= 0.1/L + 2 × 0.0005/r +0.002/1 So the maximum percentage error in the determination ofY = (oY/Y)x 100 % 10.8 Discussions 1. The loads should be such that the elastic limit is not exceeded. Since the load-elongation graph is a straight line the elastic limit has not exceeded 2. In the expression for Y since the radius of the wire r has a power of 2 it should be measured accurately so that the percentage error in the determi- nation of Y is small. The cross-section of the wire may not be uniform at all places. For this reason the diameter of the wire is measured at five/six places of the wire. gLm Y = —— πr2l

NSOU ? CC-PH-02 108 3. The cross-section of the wire may not be exactly circular. So at each place readings are taken at two mutually perpendicular directions to correct for the ellipticity of the cross-section, if any. 4. The micrometer screw must be rotated in the same direction to avoid back- lash error. 5. The dummy wire and the experimental wire should be made of the same material. If are wires are of different materials then their thermal expansion (due to temperature change during experiment) will be different. This will introduce an error in the measured elongation l. 6. The wires used in the experiment should be identical. long and thin. The long and thin wires give larger elongation and hence better measurement accuracy. 7. The wires should be taut otherwise length L cannot be measured correctly. The-control weight or dead weight is used to make the wires taut. 8. After adding a load or removing a load, wait for some time before taking the next reading; this will help the wire to elongate or contract fully. 9. The load must be placed on the hanger or removed from it gently. 10.9 Summary In this unit we have discussed the theory of determination of the Young's modulus of the material of a wire by Searle's method and the experimental method. We have discussed how to compute the percentage error in the determination of Y. 10.10 Answers 1. See Sec. 4.1 2. Steel is more elastic than rubber since the stress is much higher in steel than in rubber for the same strain produced. 3. Since the load-elongation graph is a straight line the elastic limit has not exceeded 4. The cross-section of the wire may not be uniform at all places. For this reason the diameter of the wire is measured at five/ six places of the wire.

NSOU ? CC-PH-02 109 5. See discussion no. 3 at Sec. 10. 8. 6. See discussion no. 5 at Sec. 10. 8. 7. Thin wire, since thin wires give larger elongation and hence better measurement accuracy. 8. In the expression for Y since the radius of the wire r has a power of 2 it should be measured accurately so that the percentage error in the determination of Y is small. 10.11 Exercise 1. What are meant by elasticity, stress,strain and elastic limit? State Hooke's law. 2. Which one is more elastic- rubber or steel? 3. How do you ensure that you have not exceeded the elastic limit? 4. Why do you measure the radius of the wire at different places? 5. Why do you measure the radius of the wire in mutually perpendicular directions? 6. Is there any harm if the wires are not made of the same material? 7. Would you prefer the experimental wire to be thin or thick? 8. Why should the radius of the wire be measured accurately?

NSOU ? CC-PH-02 110 Unit 11 ? To study the V-I curve of a solar cell and find the maximum power point and efficiency Structure 11.1 Objectives 11.2 Introduction 11.3 Theory 11.4 Apparatus 11.5 Experimental Procedure 11.6 Experimental Results 11.7 Discussions 11.8 Summary 11.9 Answers 11.10 Exercise 11.1 Objective In this experiment you will learn about the solar cell, study its characteristics and performance. 11.2 Introduction Solar cell is the basic unit of solar energy generation system where electrical energy is extracted directly from light energy without any intermediate process. The working of a solar cell solely depends upon its photovoltaic effect, hence a solar cell also known as photovoltaic cell. There are a variety of different measurements we can make to determine the solar cell's performance, such as its power output and its conversion efficiency.

NSOU ? CC-PH-02 111 11.3 Theory You have learned about n-type and p-type semiconductors earlier in unit 6. A solar cell is basically a semiconductor p-n junction device. It is formed by joining p- type and n-type semiconductor material. At the junction excess electrons from n-type try to diffuse to p-side and excess holes try to diffuse to the n-side. This results in an electric field from the n-side to the p-side at the junction, called barrier field. The electrons and holes combine in the region near the barrier thus forming an area with no free carrier. This is called the depletion region. When sunlight falls on the solar cell, photons with energy greater than band gap of the semiconductor are absorbed by the cell and generates electron-hole (e-h) pair. These electrons and holes migrate respectively to n- and p- side of the pn junction due to barrier field. Thus, the p-side has an excess of holes while the n-side has an excess of electrons. In this way a potential difference is established between two sides of the cell. This is called the photo emf. This photo emf is proportional to the illumination and on the size of the illuminated area. When an external load is connected across the terminals of the cell, it acts as a battery as the holes return to the n-side and the electrons to the p-side thus driving a current from the p-side to the n-side. Solar cells are often formed from silicon single crystals. For silicon, the band gap at room temperature is E g = 1.1 eV. Solar cells produce direct current (DC) electricity and current times voltage equals power, so we can create solar cell I-V curves representing the current versus the voltage for a photovoltaic device.The main electrical characteristics of a solar cell are summarized in the relationship between the current and voltage produced on a typical solar cell I-V characteristics curve. The intensity of the solar radiation that hits the cell controls the current (I), while the increase in the temperature of the solar cell reduces its voltage (V). Solar Cell I-V Characteristics Curves are basically a graphical representation of the operation of a solar cell or module summarising the relationship between the current and voltage at the existing conditions of irradiance and temperature. I-V curves provide the information required to configure a solar

NSOU ? CC-PH-02 112 system so that it can operate as close to its optimal peak power point (MPP) as possible. The voltage generated by the solar cell depends on the current drawn. Thus an appropriate load has to be connected across the cell to derive maximum power from it. From the I-V characteristics various parameters of the solar cell can be determined, such as: short-circuit current (I SC), open-circuit voltage (V OC) and efficiency. The rating of a solar panel depends on these parameters. . The short-circuit current is the current through the solar cell when the voltage across the solar cell is zero (i.e., when the solar cell is short circuited). This current is due to the generation and collection of light-generated carriers. For an ideal solar cell at most moderate resistive loss mechanisms, the short-circuit circuit current and the light-generated current are identical. Therefore, the short-circuit current is the largest current which may be drawn from the solar cell. The open-circuit voltage, V OC , is the maximum voltage available from a solar cell, and this occurs at zero current. The open-circuit open circuit voltage corresponds to the amount of forward bias on the solar cell due to the bias of the solar cell junction with the light-generated current. The efficiency is the most commonly used parameter to compare the performance of one solar cell to another. Efficiency is defined as the ratio of energy output from the solar cell to input energy falling on the cell. In addition to reflecting the performance of the solar cell itself, the efficiency depends on the frequency of the incident sunlight and the temperature of the solar cell. 11.4 Apparatus The experimental set up consists of solar cell, light source (LED or bulb) with known power, resistance box, voltmeter and milliammeter, a scale. 11.5 Experimental Procedure 1. Set up the circuit as shown in Fig. 11.1.

NSOU ? CC-PH-02 113 Fig 11.1 Circuit for solar cell characteristics 2. Find out the power P of the light source. Set up the voltage to the bulb to its maximum rating so that it delivers that power. 3. Set up the solar cell so that light from the source falls normally on it. Measure the distance of the solar cell, d, from the light source. 3. Obtain the surface area, A, of the solar cell. If it is recatngular, the area is given by multiplying length and breadth. If it is circular, the area is given by πD 2 /4 where D is the diameter. 4. Calculate the intensity of the light falling on the solar cell as I = PA / 4πd 2 . 5. Disconnect the load resistance (R L) in the resistance box and note the open- circuit voltage (V OC). 6. Change the load resistance to 0 ohms. Note the short-circuit current (I SC). 7. Now increase the load resistance in steps of 100 ohms upto a maximum of 1100 ohms. In each case, note the current. 8. Plot a graph for current vs voltage. 9. Plot a graph for power versus load. Find out the maximum power e to the load from the graph. 10. Calculate the efficiency by dividing the maximum power delivered to the load by the intensity I falling on the solar cell. 11. Repeat steps 3 ro 10 by changing the distance between the source and the solar cell. P n V + − mA R. B.

NSOU ? CC-PH-02 114 11.6 Experimental Results Data for calculation of intensity of light falling on the solar cell Area of the solar cell = ...m 2 Power of the light source = ...W Distance of the solar cell from the light source = ...m (d 1) Intensity of light falling on the cell = ...W (I 1) Distance of the solar cell from the light source = ...m (d 2) Intensity of light falling on the cell = ...W (I 2) Table 1 Data for V-I characteristics and power vs. load resistance for a solar cell R L Intensity = (I 1) Intensity = (I 2) (Ohm) Voltage (V) Current Power Voltage (V) Current Power (mA) (mW) (mA) (mW) 0 ... ... 100 ... ... 200 ... ... For I 1 : The power delivered to the load is maximum at R L is ..... . The efficiency of the solar cell at this point is ..... For I 2 : The power delivered to the load is maximum at R L is ..... . The efficiency of the solar cell at this point is ..... 11.7 Discussions 1. The light should fall normally on the solar cell.

NSOU ? CC-PH-02 115 2. Intensity of light is calculated as the power falling on the solar cell. We have made a few assumptions: (i) The complete power delivered to the bulb is available as radiation. (ii) The bulb is treated as a point rather than an extended source. (iii) The radiation of the bulb is isotropic, that is same in all directions. (iv) There is no absorption of light between the bulb and the solar cell. (v) The distance of the cell from the light is large. 3. Efficiency depends on the frequency of light as well as the temperature of the solar cell. 11.8 Summary In this experiment you learned about the solar cell, calculate its power and efficiency. 11.9 Answers 1. A photodiode, like a solar cell, is a photovoltaic semiconductor device. Photodiodes, however, are optimized for light detection while solar cells are optimized for energy conversion efficiency. A photo diode has to be fast, which means low capacitance, which means small area. Solar cells are much bigger than photodiodes to catch sunlight. Photodiodes can be used in forward as well as reverse bias while solar cells need not be biased from outside. It essentially works in the fourth quadrant of the V-I graph. 2. Silicon single crystals are generally used to make solar cell. Polycrystalline silicon has been used for lower costs though its efficiency is smaller. Other materials include cadmium telluride, silicon thin film, etc. This is a very active field of research. 3. Dark current is the current in a photodiode in absence of illumination. 4. The response time of a photodiode is the time that it takes to change the current when it is suddenly exposed to light or the light source is suddenly

NSOU ? CC-PH-02 116 cut off. Since it depends on the capacitance of the cell, photodiodes are small devices. 5. Refer to Sec. 11.3. 6. Refer to Sec. 11.3. 7. Refer to Sec. 11.3. 8. Refer to Sl. No. 3 of Sec. 11.7. 11.10 Exercise 1. What is the difference between solar cell and a photodiode? 2. What are the types of semiconductor materials used for solar cell? 3. What is dark current in a photodiode? 4. What is the response time of photodiode? 5. How is depletion layer formed? 6. How is photo- emf generated? 7. What are meant by short-circuit current, open-circuit voltage and the efficiency of a solar cell? 8. On what factors does the efficiency of the cell depend?

Unit 12 ? To study the variation of mutual inductance of a given pair of coaxial coils by using a ballistic galvanometer Structure 12.1 Objectives 12.2 Introduction 12.3 Theory 12.4 Apparatus 12.5 Experimental Procedure 12.6 Experimental Results 12.7 Calculation of percentage error 12.8 Discussions 12.9 Summary 12.10 Answers 12.11 Exercise 12.1 Objectives After studying this unit you will learn ? to use a ballistic galvanometer to measure charge ? to determine the mutual inductance between two given coils 12.2 Introduction If two coils are placed close to each other and the current through one of them is changed, the flux linked with the other coil changes as a result of which an an e.m.f. is induced in the other coil according to Faraday's laws of induction. This

NSOU ? CC-PH-02 118 phenomenon is known as Mutual Inductance. The first coil is called the primary and the other coil is called the secondary. The induced e.m.f. (e) is proportional to the rate of change of current in the primary coil, i.e., e = – M. dI/dt, where 'I' is the current in the primary coil. 'M' is called the co-efficient of mutual inductance between the coils or simply mutual inductance. Then, M = e dI dt . Mutual inductance is defined as the e.m.f. induced in a coil when the rate of change of current in the other coil is unity. If e = 1 volt and dI/dt = 1 A/sec, then M = 1 henry. But 1henry is very large and usually mutual inductance is of the order of mH. Mutual inductance is also defined as flux linked with one coil when the current through the other coil is unity. It can be shown that the mutual inductance of coil 1w.r.t. coil 2 = that the mutual inductance of coil 2 w.r.t. coil 1. The value of mutual Inductance depends upon the number of turns of the coils, their cross-sectional area, closeness of the two coils and the orientation of one coil w.r.t. the other. 12.3 Theory Mutual inductance is defined as the e.m.f. induced in a coil when the rate of change of current in the other coil is unity. Let M be the mutual inductance between the primary coil P and the secondary coil S in the circuit of Fig. 12.1. If a current of magnitude I is broken in the primary coil P then a charge of magnitude MI/R will flow in the secondary coil S, where R is the total resistance of the secondary circuit. This charge will produce a deflection, of the ballistic galvanometer, so that MI/R = T C nAB 2 1 2 p q l . . , + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ ..... (1) where T = the time period of the moving parts of the ballistic galvanometer under open circuit condition, C = couple/unit twist of the suspension fibre of the ballistic galvanometer,
NSOU ? CC-PH-02 119 n = the number of turns of the galvanometer coil, A = the mean area of each turn of the galvanometer coil, B = the magnetic field in which the galvanometer coil is placed, and λ = the log decrement of the galvanometer under the condition of the throw. Fig. 12.1 Experimental arrangement Fig. 12.2 M-φ graph To eliminate C nAB , a dc potential drop rI is applied in the secondary circuit. This will produce a steady deflection θ' of the ballistic galvanometer. Then rI R C nAB = ′.q ..... (2) From Equ. (1) and (2) we have, M = Tr 2 1 2 p q q l . ′ . + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ ..... (3) When the angles θ and θ' are small, q q' may be replaced by ′ ′ d d where d is the first throw of the ballistic galvanometer and d ′ is the steady deflection of the spot of light on the scale (of the lamp and scale arrangement) corresponding to θ and θ', respectively. Therefore, M = Tr d d 2 1 2 p l ′ + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ ..... (4) If r and T are expressed in ohm and second respectively, M will be in henry. 0 30 60 90 120 150 180 φ (degree) M (mH) R 1 B.G. R 2 K 2 K S M P 1 1' C 1 2 2' r K 1 1 1' 2 2' C 2 B t
NSOU ? CC-PH-02 120 12.4 Apparatus (1) A ballistic galvanometer (B.G), (2) a tapping key, (3) two plug keys, (4) two plug –type commutators, (5) two resistance boxes, (6) a low resistance (typically 0.01 to 0.1 Ω), (7) a storage battery/ regulated power supply, (8) two mutually coupled coils (P and S), one (P) fixed and the other (S) can rotate about a common axis, (9) a voltmeter/ multimeter, (10) a lamp and scale arrangement to measure the throw/ deflection of the ballistic galvanometer, (11) a stop watch/clock. 12.5 Experimental Procedure 1. Connect the circuit as shown in Fig. 12.1. Make the angle between the coils zero. 2. Measure the e.m.f. of the battery, both before and after the experiment by a voltmeter/multimeter. 3. Insert a resistance R 2 in the galvanometer circuit which is larger than the critical damping resistance (CDR) of the ballistic galvanometer. This resistance should not be changed throughout the experiment. 4. Put a suitable resistance R 1 in the battery circuit. Insert the plugs in the commutator C 2 and close the key K 1 .Insert a plug between 1,1′ of the commutator C 1 and close the key K 2 .Bring the spot of light on the zero of the scale of the lamp and scale arrangement with the help of the tapping key K. 5. Open the key K 1 in the primary circuit and observe the first throw (d) of the spot of light on the scale. Adjust R 1 such that this throw is about 16-18 cm. This resistance is kept fixed throughout the experiment. 6. Close the key K 1 and then open it. The ballistic galvanometer will be found to oscillate. Open the key K 2 so that the galvanometer circuit is open. Measure the time t for 20- 30 oscillations of the spot of light. Repeat this observation several times, say n, (at least three) and find the mean t. Calculate the time period T. (T = t/n).

NSOU ? CC-PH-02 121 7. Close the key K 1 . Remove the plugs between 1,1′ and insert plugs between 1,2 and 1′, 2′ of the commutator C 1 . Close key K 2 and observe the steady deflection (d′) of the spot of light on the scale due to the steady voltage drop across r. d′ should preferably be between 10- 16 cm. If necessary, adjust r to bring d ′ within this range. Do not change this value throughout the experiment. Note the steady deflection d′ several times for both direct and reverse currents, Find mean d′. 8. Remove the plugs between 1,2 and 1′, 2′ of the commutator C 1 and put a plug between 1, 1′ in C 1 . Bring the spot of light on the zero of the scale by the tapping key K. Break the current in the primary suddenly by removing the key K 1 and record the first throw d of the spot of light on the scale. Repeat this thrice. Now reverse current in the primary by the commutator C 2 and repeat this step and find the first throw d of the spot of light on the scale thrice. Find the mean of six d's. 9. Increase the angle φ between the coils in steps of about 10° till φ = 110°. Repeat step 8 for each φ. 10. Set the angle φ between the coils zero. Bring the spot of light on the zero of the scale. Break the primary circuit and note the positions of the light spot on the scale first towards left and then towards right during the first half of the first oscillation. Let these readings be α 1 and α 2 respectively. Let β 1 = α 1 + α 2 . After n complete oscillations measure α 2n+1 and α 2n+2, where α 2n+1 and α 2n+2 are the first reading of the spot of light towards left and right respectively during the first half of the (n+1)th.oscillation. Let β 2n+1 = α 2n+1 + α 2n+2. Find λ from the equation: λ = 2 303 2 . n (log 10 β 1 – log 10 β 2n+1) ..... (5) Find λ at least three times and take the mean. 11. Calculate M for each φ using equ.(4) and plot a graph with φ along the X-axis and M along the Y-axis. The nature of the graph will be as in Fig. 12.2.

NSOU ? CC-PH-02 122 12.6 Experimental Results Table 1 Measurement of the e.m.f. of the battery (Make a table similar to Table 2 of Unit 9) Table 2 To determine the time period (T) of the galvanometer coil (under open circuit condition) R 2 = ... Ω, R 1 = ... Ω No. of obs. Time for n complete Mean time (t) for Time period T =t/n oscillations (Sec) n complete (sec) oscillations (Sec) 1. ... 2. ... ... 3. ... Table 3 Steady deflection r = ... Ω (fixed) Direction Steady deflection d′ Mean d′ of current (cm) (cm) Direct ... ... Reverse ... ...

NSOU ? CC-PH-02 123 Table 4 Ballistic throws CDR of the galvanometer = ...Ω R 2 = ...Ω d′ = ... cm (from Table 3) Dial reading φ Direction Ballistic throw (d) Mean d Ratio d/d ′ (in degree) of current (cm) (cm) ... 0 Direct ... ... Reverse ... ... etc. etc. etc. etc. etc. Table 5 Determination of λλλλλ No. α 1 α 2 β 1 = α 2n+1 α 2n+2 β 2n+1 n λ Mean λ of α 1 +α 2 α 2n+1 +α 2n+2 (using Equ. (5)) obs. (cm) (cm) (cm) (cm) (cm) (cm) 1. ... ... 2. ... ... 3. ... ...

NSOU ? CC-PH-02 124 Table 6 Calculation of M and data for M – φ graph Time period T λ from Dial reading φ Corresponding d/d′ M = Tr d d 2p . ′ (1 + λ/2) (from Table 2) Table 5 (degree) from Table 4 (henry) (sec) ... ... 0 ... ... 10 ... ... 20 ... ... etc. etc. etc. 12.7 Calculation of percentage error The maximum proportional error in the measurement of M is given by d d d d d l l M M T T d d d d = + + + + (⎛ ⎜ ⎞ ⎠ ⎟ + ′ ′ / 1 2 1 2 Since T = t/n, d d d T T t t t = . is the error in the measurement of time t for n oscillations = the minimum scale division of the stop watch used. δd = δd′ = 0.1 cm, the smallest division of the scale of lamp and scale arrangement. Since λ is very small the last term in the right can be neglected. Substituting the values of δt, δd and δd′ we can find the percentage error dM M ×100. 12.8 Discussions 1. Be sure that the resistance in series with the B.G. is greater than its CDR. This resistance should not be changed during the experiment. 2. The battery voltage should remain constant during the experiment. The resistance R 1 in the battery circuit should not be changed during the experiment.

NSOU ? CC-PH-02 125 3. The throw d of the spot of light should be recorded during the break of the primary current only, because at break the resistance becomes infinite and the time constant for the decay of current is very small. Consequently, the change in flux within the secondary occurs in a very short time. The charge MI/R will thus flow through the galvanometer in a very short time so that ballistic condition of the galvanometer is achieved. 4. It is preferable to have d and d′ of nearly the same value. 5. The plugs of the resistance boxes should be light. 12.9 Summary In this unit we have discussed what is meant by mutual inductance, the factors on which it depends and its measurement using a ballistic galvanometer. 12.10 Answers 1. See sec. 12.1 2. The mutual inductance is minimum when the coils are mutually perpendicular and maximum when the coils are mutually parallel. 3. Yes. 4. To bring the galvanometer to rest quickly, because the electromagnetic damping is large. When the key is tapped the galvanometer coil is short- circuited. So the induced current in the coil due to its motion in the permanent magnetic field of the galvanometer increases and the electromagnetic damping is large. 5. The mutual inductance will increase as the flux linked with the secondary increases. 6. Yes because M= k√(L 1 L 2), where L 1 and L 2 are the self-inductance of the coils. k = the coefficient of coupling. 7. See Discussion 3 in sec.12.8

NSOU ? CC-PH-02 126 8. An ordinary galvanometer detects and measures current. A ballistic galvanometer measures charge. The time period of its moving parts is large and the damping of its moving parts is very small. 9. The minimum external resistance to be connected in series with the galvanometer so that it becomes oscillatory is called the critical damping resistance. 10. Due to damping the deflection (d′) of the galvanometer observed is less than the actual deflection (d). d and d ′ are related by d = d′ (1+ λ/2). λ is called the logarithmic decrement. 12.11 Exercise 1. What is meant by mutual inductance? On what factors does it depend? 2. For which orientations of the coils will the mutual inductance be minimum and maximum? 3. Is the mutual inductance same if the primary and the secondary coils are interchanged? 4. Why is a tapping key connected to the galvanometer? 5. What will happen if an iron core is introduced in the primary? 6. Does the mutual inductance depend on the self-inductance of the coils? 7. Why don't you find the throw during establishing current in the primary? 8. How does a ballistic galvanometer differ from an ordinary galvanometer? 9. What is CDR of a ballistic galvanometer? 10. What is logarithmic decrement?

Unit 13 ? To find out temperature coefficient of the material of a wire by Carey-Foster bridge Structure 13.1 Objectives 13.2 Introduction 13.3 Theory 13.4 Apparatus 13.5 Experimental Procedure 13.6 Experimental Results 13.7 Discussions 13.8 Summary 13.9 Answers 13.10 Exercise 13.1 Objectives After studying this unit you will learn ? to use a Carey-Foster bridge to measure unknown resistance ? to find the temperature coefficient of the material of a wire 13.2 Introduction The resistance of a substance changes with the change of temperature. In case of conductors, resistance increases with the rise of temperature, but in case of semiconductors and other non-metals resistance decreases with the rise of temperature. The electrical resistance of conductors such as silver, copper, gold, aluminum, etc., depends upon collision process of electrons within the material. As the temperature increases, this electron collision process becomes faster, which results in increased resistance with the rise in temperature of the conductor.But in case of semiconductors NSOU ? CC-PH-02 128 or other non-metals, the number of free electrons increases with increase in temperature. Because at a higher temperature, due to sufficient heat energy supplied to the crystal, a significant number of covalent bonds get broken, and hence more free electrons get created. That means if temperature increases, a significant number of electrons come to the conduction bands from valence bands by crossing the forbidden energy gap. As the number of free electrons increases, the resistance of this type of non-metallic substance decreases with an increase in temperature. The change of resistance with the change of temperature of a material is expressed by the temperature coefficient of resistance. It is the measure of change in electrical resistance of any substance per degree of temperature change.Its unit is per °C. The temperature coefficient of resistance is positive for metals and negative for semiconductors and other non-metals. The increase in resistance of platinum with the increase of temperature is used to measure temperature. 13.3 Theory Let R 0 and R t be the resistances of a conductor at 0°C and t°C respectively, then

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R t = R 0 (1 + αt), where α is a constant known as the temperature coefficient of the material. That is, α = (R t – R 0)/(R 0 t). …… (1)

The resistances R t and R 0 are measured by a Carey- Foster bridge. The circuit for measurement of an unknown resistance X by a Carey-Foster bridge is given in Fig. 13.1. Fig. 13.1 Let P and Q be the equal resistances connected in the inner gaps 2 and 3, the standard resistance R is connected in gap 1 and the unknown resistance X is K P R B A G D E 1 2 C F X Q 3 4

NSOU ? CC-PH-02 129 connected in the gap 4. Let l 1 be the balancing length ED measured from the end E. By Whetstone's bridge principle, P Q R a l X b l = + + + + – ( ) 2 2 100 r r ......(2) Here, a and b are the end corrections at the ends E and F respectively, and ρ is the resistance per unit length of the bridge wire. If the experiment is repeated with X and R interchanged and if l 2 is the balancing length measured from the end E, P Q R a l R b l = + + + + – ( ) 2 2 100 r r ......(3) From equations (2) & (3) we get X = R + ρ (l 1 – l 2) .... (4) Let l 1 ' and l 2 ' be the balancing lengths when the above experiment is done with a standard resistance r (say 0.1Ω) in the place of R and a thick copper strip of zero resistance in place of X. From equation (4), we get r = – r l l 2 2 ' ' ...... (5) If X 1 and X 2 be the resistances of the given wire at temperatures t 1 °C and t 2 °C respectively, the temperature coefficient of resistance is given by the equation, a = – – X X X t X t 2 1 1 2 2 1 .... (6) Also, if X 0 and X 100 be the resistances of the wire at 0°C and 100°C, a = – × X X X 100 0 0 100 .... (7) The resistances are measured at 0°C and 100°C and from equation (7) the temperature coefficient of resistance is calculated.

NSOU ? CC-PH-02 130 13.4 Apparatus (1) A carey foster bridge, (2) a standard low resistance (0.1 Ω), (3) three 10Ω resistances, (4) a storage battery/regulated power supply, (5) a plug key, (6) a galvanometer, (7) a thermometer, (8) a beaker containing water, (9) a stirrer, (10) a burner, (11) connecting wires. 13.5 Experimental Procedure 1. Connect the circuit as shown in Fig. 13.1. Use P = Q = 10 Ω in the gaps 2 and 3 respectively. 2. Connect the standard low resistance in gap 1 and a copper strip in gap 4. Close the key K. Move the jockey to find the balancing length l 1 ′. Find l 1 ′ thrice and take their mean. Open the key K. 3. Interchange the standard low resistance and the copper strip in the gaps. Close the key K. Move the jockey to find the balancing length l 2 ′. Find l 2 ′ thrice and take their mean. Open the key K. 4. Repeat steps 2 and 3 with three different low resistances. 5. Calculate ρ using the Equ.(5) in the three cases and compute mean ρ. 6. Remove the copper strip and connect the given wire temperature coefficient of resistance of whose material is to be determined in gap 4 and the low resistance in gap 1. Close the key K. Move the jockey to find the balancing length l 1 . Find l 1 thrice and take their mean. Open the key K. 7. Interchange the standard low resistance and the experimental wire in the gaps. Close the key K. Move the jockey to find the balancing length l 2 . Find l 2 thrice and take their mean. Open the key K. 8. Measure the room temperature, before and after the measurement, assuming that the experimental wire is at room temperature (t 1). 9. Calculate the resistance X 1 at temperature t 1 using the equ. (4). 10. Place the experimental wire immersed in the water in the beaker. Gently heat the water by a burner and stir the water with the stirrer so that the NSOU ? CC-PH-02 131 temperature of water remains constant for 2/3 minutes. Keep the bulb of the thermometer close to the wire. Measure the temperature of water (t 2). Repeat steps 6 and 7 to find the balancing length l 2 . 11. Calculate the resistance X 2 at temperature t 2 using the equ. (4). 12. Calculate the temperature coefficient of resistance using Equ. (6). 13.6 Experimental Results Table 1 To find ρρρρρ P = Q = 10 Ω, R = ...Ω Gap1 Gap 4 Balancing length Mean ρ Mean ρ (cm) (cm) Using Equ. (5) (Ω/cm) Low resistance (R) 0Ω .... (l 1 ') .... (l 1 ') ....(l 1 ') ... ... Ω/cm ....(l 1 ') 0 Ω Low resistance (R) ...(l 2 ') .... (l 2 ') .... (l 2 ') ... .(l 2 ') Table 2 To measure the room temperature (t 1) Before Mean t 1 experiment ...°C After ...°C experiment ...°C

NSOU ? CC-PH-02 132 Table 3 To measure the resistance (X 1) of the given wire at temperature t 1 P = Q = 10 Ω, ρ = ... Ω/cm (from Table 1) Room temperature = ...°C (from Table 2) Gap1 Gap4 Balancing Mean X 1 = R +ρ(l 1 – l 2) Mean X 1 length (cm) (Ω) (cm) Low (R) X 1 .... (l 1) resistance .... (l 1) .... (l 1) ... (R) ....(l 1) .... Ω X 1 Low .... (l 2) resistance .... (l 2) .... (l 2) ... (R) ....(l 2) Table 4 To measure the resistance (X 2) of the given wire at temperature t 2 Make Table similar to Table 3 and calculate the temperature coefficient of resistance using Equ. (6) 13.7 Discussions 1. The temperature of water in the beaker should be kept constant during the measurement of X 2 . 2. The low resistance R should be of such a value that the null points are obtained near the ends of the bridge wire. 3. The Carey-Foster bridge is basically a Wheatstone bridge. 13.8 Summary In this unit we have discussed what is meant by temperature coefficient of resistance, how does it change with the change of temperature and how to determine the temperature coefficient of resistance by a Carey- Foster bridge.

NSOU ? CC-PH-02 133 13.9 Answers 1. See sec.13.1 2. See sec.13.1 3. Yes. Then ρ will not be uniform and the measurement of resistance will not be correct. 4. The wire of the Carey-Foster bridge is soldered at the ends. So there is some resistance at the junctions. These resistances are end errors. The end errors are eliminated by determining the null point with the resistances interchanged between the gaps 1 and 4 of the bridge. 5. Usually it is used to measure low resistance. However, it can be used to find small differences between large resistances. 6. Because the bridge is most sensitive when the resistances of the four arms are equal and we are measuring low resistance. 13.10 Exercise 1. What is meant by temperature coefficient of resistance? What is its unit? How does it change with the change of temperature? 2. Why is the temperature coefficient of resistance positive in case of metals and negative in case of semiconductors? 3. Is there any harm if the wire of the Carey-Foster bridge is not of uniform cross-section? 4. What are end errors of the Carey- Foster bridge? How is it eliminated? 5. Can you measure high resistance by Carey-Foster bridge? 6. Why have you used P = Q = 10 Ω?

NSOU ? CC-PH-02 134 Unit 14 ?To find leakage resistance by discharging a capacitor Structure 14.1 Objectives 14.2 Introduction 14.3 Theory 14.4 Apparatus 14.5 Experimental Procedure 14.6 Experimental Results 14.7 Calculation of percentage error 14.8 Discussions 14.9 Summary 14.10 Answers 14.11 Exercise 14.1 Objectives After studying this unit you will learn to determine experimentally the leakage resistance of a capacitor. 14.2 Introduction A capacitor has two plates with a dielectric in between the plates. A capacitor can store charge. But when a charged capacitor is left alone, its charge slowly leaks away due to the imperfection of the insulation between the plates. For this reason, a practical capacitor is represented by a capacitor with perfect insulation, in parallel with a resistance R, called the natural leakage resistance of the capacitor. In this unit we will discuss how the leakage resistance of a capacitor is determined using a ballistic galvanometer.

NSOU ? CC-PH-02 135 14.3 Theory If a capacitor containing a charge Q is discharged through a ballistic galvanometer, then the ballistic throw θ is given by Q = Kθ (1 + λ/2), ... (1) where K is the galvanometer constant and λ is the logarithmic decrement of the galvanometer coil. When a charged capacitor is left alone, its charge slowly leaks away due to the imperfection of the insulation between the plates. For this reason, a practical capacitor is represented by a capacitor with perfect insulation, in parallel with a resistance R, called the natural leakage resistance of the capacitor. If Q' be the charge on the capacitor, of capacitance C, after it has been left alone for a time t, we can write Q Q t CR ′ exp = − ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ ....(2) When the charge Q' is discharged through the same galvanometer then a throw θ' is produced. Then Q' = Kθ' (1 + λ/2) ....(3) Using Eqs. (1), (2) and (3) we get, θ/θ' = exp t CR ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ or R = t/ [2.303 C log10 (θ/θ')]..... (4) If the movement of spot of light on the scale of the lamp and scale arrangement moves through d and d ′ corresponding to the throws θ and θ′ respectively, then equ. (4) reduces to R = t/ [2.303 C log 10 (d/d′)] ..... (5) If d' is observed for several values of t and log 10 (d/d′) is plotted against t, a straight line passing through the origin is obtained. A point on the line gives t and corresponding log 10 (d/d′). Hence R can be calculated using equ. (5)

NSOU ? CC-PH-02 136 14.4 Apparatus (1) A ballistic galvanometer (B.G), (2) a tapping key (K 1), (3) a capacitor (C) of known capacitance (typically, 1 μF), (4) two resistance boxes (R 1 , R 2), (5) a commutator (M), (6) a special type of charging and discharging key (FK), (7) a stop watch/ clock, (8) a lamp and scale arrangement (9) a storage battery/ regulated power supply, (10) a voltmeter/ multimeter. 14.5 Experimental Procedure 1. Construct the circuit as shown in Fig. 14.1. Measure the e.m.f. of the battery before and after the experiment 2. Choose suitable values of resistances in the resistance boxes R 1 and R 2 in the following way. Charge the capacitor C by closing the battery circuit and depressing the rocker arm FK to the point b for a few seconds. Discharge the capacitor through the ballistic galvanometer quickly by putting the rocker arm FK to the point a and observe the first throw of the spot of light on the scale of the lamp and scale arrangement.The throw should be nearly equal to the maximum permissible value (about 18 cm) on the scale. If it is much smaller, then either increase resistance R 1 or decrease resistance R 2 . If it is greater than the maximum permissible value, then either increase resistance R 2 or decrease resistance R 1 . These resistances should be kept unchanged throughout the experiment. 3. Connect the points C 2 , C 3 and C 1 , C 4 of the commutator M. 4. Charge the capacitor with a charge Q by putting the rocker arm FK in contact with the point b for a few seconds. Tap the tapping key K to stop the spot of light on the lamp and scale arrangement and set the spot on the zero of the scale. Then press the charge-discharge key K to put the rocker arm FK in contact with the point a suddenly and the capacitor will be discharged through the galvanometer. Note the first throw (d) of the spot of light.

NSOU ? CC-PH-02 137 5. Connect the points C 2 , C 4 and C 1 , C 3 of the commutator M. This will reverse the current. 6. Repeat step 4. 7. Repeat steps 3 to 6 two more times and find the mean value of d. 8. Charge the capacitor with a charge Q by putting the rocker arm FK in contact with the point b for a few seconds. Put the rocker arm FK in the middle (that is, not in contact with the points a and b) for a convenient period of time (t) such that the capacitor undergoes natural leakage. Measure the time with a stop watch/clock. Now suddenly put the rocker arm in contact with the point a and the remaining charge of the capacitor discharges through the galvanometer. Note the first throw (d') of the spot of light. If it is not less than d by 1 – 2 cm decrease the time of natural leakage (t) to achieve this. Record the time and d'. 9. Repeat step 8 with reverse current. 10. Repeat steps 8 and 9 three times and find the mean d'. Find log 10 (d/d '). 11. Repeat steps 8 to 10 for at least five different values of natural leakage time t. 12. Draw a graph plotting natural leakage time t along the abscissa and log 10 (d/ d′) along the ordinate. The graph is a straight line passing through the origin. Choose a suitable point on the line and find t and log 10 (d/d′) corresponding to the point. Calculate the leakage resistance R using equ. (5). Fig. 14.1 C 2 C 3 B.G. C F K a b R 1 R 2 C 4 C 1 B' K

NSOU ? CC-PH-02 138 14.6 Experimental Results Table 1 Measurement of the e.m.f. of the battery (Make a table similar to Table 2 of Unit 9) Table 2 Determination of ballistic throw (d) for full charge Q Capacitance of the capacitor = ...μF R 1 = ... Ω, R 2 = ...Ω No. of Ballistic throw d (in cm) Grand mean throw obs. Direct Reverse Mean d (in cm) current current 1. ... ... 2. ... ... 3. ... ... Table 3 Data for natural leakage resistance R d = ... cm (from Table 2) No. Time of Throw d' in cm for Grand mean log 10 (d/d′) of leakage Direct Reverse Mean d' (in cm) obs. (sec) Current Current 1. ... ... 2. ... ... etc. etc. etc. etc. etc. etc.

NSOU ? CC-PH-02 139 Table 4 Determination of natural leakage resistance R Capacitance C t (from graph) log 10 (d/d′) R = t/ [2.303 C log 10 (d/d′)] (from graph) ....μF(Given) .... Sec ....Ω 14.7 Calculation of percentage error R = t/[2.303 C log 10 (d/d′)] = t/[C ln (d/d′)] or, ln (d/d′) = t/CR So, d d d d d d d t CR C t R / ′ (/ ′) (/) (/). (/) ( ) = = 1 Now, d d d d d d d d d d d t R t R t R R R / ′ ′ ′ ′ (/) . . ( ) = − = ∂ − ∂ 2 and Hence ∂ ⌈ ⌊ | ⌉ ⌋ | = ∂ − d d d d d d d d ′ / ′ ′ ′ . d Again, δd = δd' = the minimum division of the scale of the lamp and scale arrangement. Therefore, replacing the negative sign before δd′/d′ by a positive sign for maximum error, we have, ∂ ⌈ ⌊ | ⌉ ⌋ | = + ⌈ ⌊ | ⌉ ⌋ | = − ∂ d d d d d d d C t R C t R R R ′ / ′ ′ . . . d d 1 1 1 1 Or, ∂ = + ∂ + ⌈ ⌊ | ⌈ ⌊ | ⌉ ⌋ | R R C t R d d d CR t 1 1 1 . ′ d For determining the maximum error the negative sign is made positive. δt = the minimum division of the stop watch/ clock. Substituting the values of δt, δd, δd', C, etc. we get ∂R R Multiplying it by 100 we get the percentage error.

NSOU ? CC-PH-02 140 14.8 Discussions 1. During the experiment the capacitor is charged several times. It should be ensured that each time it is charged to the same potential. So it is to be checked that the e.m.f. of the battery remains constant during the experiment. 2. After charging the capacitor it should not be touched. 3. The plugs of the resistance boxes and the commutator should be tight. 4. Before recording the throw the galvanometer must be stopped using the tapping key and the spot of light should be brought at zero of the scale. 14.9 Summary In this unit we have discussed what is meant by the leakage resistance of a capacitor and how to determine it experimentally. Percentage error has been calculated. 14.10 Answers 1. See sec. 14.1 2. Capacitance is the amount of charge stored in a capacitor for unit potential difference between the plates of the capacitor. Its SI unit is farad. As farad is a large unit it is expressed in μF. 3. ER 1 / (R 1 + R 2), where E is the e.m.f. of the battery. 4. The time of leakage is determined by the time constant CR. R is of the order of MΩ. If C is of the order of farad, CR is of the order of 10 6 sec. the leakage time will be very large. If C is very small leakage time will be very small to be measured by a stop watch. 5. Because we have determined the ratio of two throws and hence (1 + λ/2) is eliminated. 6. To bring the galvanometer to rest quickly, because then the electromagnetic damping is large. When the key is tapped the galvanometer coil is short-

NSOU ? CC-PH-02 141 circuited. So the induced current in the coil due to its motion in the permanent magnetic field of the galvanometer increases and the electromagnetic damping is large. 7. Due to damping the deflection (d′) of the galvanometer observed is less than the actual deflection (d). d and d′ are related by d = d′ (1+ λ/2). λ is called the logarithmic decrement. 8. An ordinary galvanometer detects and measures current. A ballistic galvanometer measures charge. The time period of its moving parts is large and the damping of its moving parts is very small. 9. See sec. 14.8 (Discussion No. 1) 10. The resistance of the capacitor is very high. So during discharge of the capacitor through the galvanometer the resistance in the galvanometer circuit is very large. 14.11 Exercise 1. What is a capacitor? What is meant by its natural leakage resistance? 2. What is capacitance of a capacitor? What is its unit? 3. What is the voltage to which the capacitor is charged in your experiment? 4. What will happen if the capacitance is very large or very small? 5. Why have you not measured the log decrement ? 6. Why is a tapping key connected to the galvanometer? 7. What is logarithmic decrement? 8. How does a ballistic galvanometer differ from an ordinary galvanometer? 9. Why is it necessary to measure the e.m.f. of the battery before and after experiment? 10. Why have not connected any resistance in series with the ballistic galvanometer?

NSOU ? CC-PH-02 142 Unit 15 ? To study Lissajous figures Structure 15.1 Objectives 15.2 Introduction 15.3 Theory 15.4 Apparatus 15.5 Experimental Procedure 15.5.1 Measurement of frequency 15.5.2 Measurement of phase difference 15.6 Experimental Results 15.7 Discussions 15.8 Summary 15.9 Answers 15.10 Exercise 15.1 Objectives After studying the unit you will learn ? what is Lissajuos figure and how it is produced on a CRO screen ? how the frequency of a sinusoidal wave and the phase difference between two sinusoidal waves can be determined using Lissajuos figures. ? experimental method to determine the frequency of a sinusoidal wave and the phase difference between two sinusoidal waves. 15.2 Introduction When two mutually perpendicular simple harmonic motions superimpose the resulting motion is not simple harmonic and the resulting pattern is called a Lissajous

NSOU ? CC-PH-02 143 figure. Let, $x = A \sin (at +\theta)$ .... (1) and $y = B \sin (bt)$ .... (2) be two SHMs acting along the x-axis and y-axis respectively and they are acting on the same particle. The resulting motion will be a Lissajous figure. The figure will depend on the ratio a/b and the phase difference θ. If a b = 1, the figure is a straight line, an ellipse or a circle depending upon the amplitude and the phase of the two waves. A straight line results when the phase difference is zero or 180°. A circle is produced when the amplitudes are equal and the phase difference is 90°. If the amplitudes are unequal and/or the phase difference is arbitrary an ellipse is formed. (Fig. 15.1). $\theta = 0$ $\theta = \pi/2$ radians Fig. 15.1 Lissajous figures The Lissajous figures are closed only if a/b is rational. The ratio a/b determines the number of “lobes” of the figure. For example, if a/b = ½, i.e., the ratio of the frequencies of the horizontal wave to the vertical wave is 1:2, the resulting Lissajous figure is a figure of eight (Fig. 15.2 (a)). If a/b = 2/3,the resulting Lissajous figure is as shown in Fig. 15.2 (b). From the Lissajous figures we can determine the ratio of the frequencies of the horizontal and vertical waves. In Fig. 15.2 (a), a tangent at the top edge of the pattern has two points of contact P 1 and P 2 , whereas a tangent at a vertical side has one point of contact Q. Similarly, in Fig. 15.2 (b), a tangent at the top edge of the pattern has three points of contact P 1, P 2 and P 3 , whereas a tangent at a vertical side has two points of contact Q 1 and Q 2 . In general, the number of horizontal tangencies refers to the frequency of the vertical wave and the number of vertical tangencies refers to the frequency of the horizontal wave, i.e., a/b = n v / n h , where n v and n h are the number of vertical tangencies and horizontal tangencies respectively.

NSOU ? CC-PH-02 144 a) a/b = ½ (b) a/b = 2/3 Fig. 15.2 Lissajous figures can be used to determine the frequency of a sinusoidal wave and the phase difference between two sinusoidal waves. The figures can be displayed on a CRO (Cathode Ray Oscilloscope) screen. To determine the frequency of a sinusoidal wave, the wave and another sinusoidal wave of known frequency are applied to the vertical and horizontal deflecting plates respectively of a CRO and a the frequency of the wave using equation a/b = n v /n h . Let us now briefly discuss about a CRO. A sketch of a CRO is shown in Fig.15.3. It consists of a cathode from which electrons are emitted when it is heated by passing current through a filament, an electron gun and focusing anodes to provide a focused electron beam which is accelerated towards the phosphor screen, horizontal and vertical deflection plates. If a sinusoidal voltage is applied to the horizontal deflection plates the spot of light on the screen moves to and fro along the horizontal direction. Similarly, if a sinusoidal voltage is applied to the vertical deflection plates the spot of light on the screen moves to and fro along the vertical direction. If sinusoidal voltages are applied both to the horizontal and vertical deflection plates a Lissajuos figure is obtained on the screen. P 1 P 2 P 1 P 2 P 3 Q Q 1 Q 2 Fig. 15.3 Cathode Ray Oscilloscope

NSOU ? CC-PH-02 145 15.3 Theory If sinusoidal voltages are applied both to the horizontal and vertical deflection plates of a CRO a Lissajuos figure is obtained on the screen. If a and b be the frequencies of voltages and n v and n h be the number of vertical tangencies and horizontal tangencies respectively, then a/b = n v /n h .... (3) If one of the frequencies is known the other can be determined from the equation. The phase difference θ can be determined by applying the voltages (Equations (1) and (2)) at the horizontal and vertical deflection plates of a CRO. Then if $\theta = 0$, a straight line of positive slope is obtained. If $\theta \neq 0$ a Lissajous figure as shown in Fig. 15.4 is obtained on the screen. If in the figure the point of intersection of the ellipse with the y-axis is (0, A) and the maximum vertical displacement is B, then $\theta = \sin^{-1} (\pm A/B)$ ..... (4) Fig. 15.4 15.4 Apparatus (1) A dual trace CRO, (2) two audio frequency oscillators, (3) two capacitors, (4) two resistors, (5) probes of CRO. y B A O X

NSOU ? CC-PH-02 146 15.5 Experimental Procedure At the start of the experiment the spot on the CRO screen has to be properly focused, the intensity of the spot should be adjusted so that it is not very low or very high, the probes of the CRO should be grounded and the spot be brought at the centre of the screen. It is to be checked from time to time that the spot is at the centre. 15.5.1 Measurement of frequency 1. Connect the output of one audio oscillator (say, 1) to the horizontal deflection plates of the CRO and that of the other oscillator (say, 2) to the vertical deflection plates. Adjust the frequency ‘a’ of the first oscillator to a suitable value (say, 1 kHz) (take it to be unknown) and the frequency ‘b’ of the second oscillator to 2 kHz, and their amplitudes to 1 V. 2. Observe the Lissajous figure on the CRO screen and find the number of horizontal and vertical tangencies. Calculate the frequency a. 3. Repeat steps 2 by changing frequency b of oscillator 2 to 3 kHz and 4 kHz. 4. Repeat steps 2 and 3 by changing the amplitude of oscillator 2 twice. Fig. 15.5 15.5.2 Measurement of phase difference 1. Construct the circuit as shown in Fig.15.5. The oscillator is connected to the terminals a and b. Select suitable values of R and C so that the frequency f is about 1 kHz. (f = 1/2 πRC). The voltage V 1 is applied to the horizontal deflecting plates and the voltage V 2 to the vertical deflecting plates. a c V 1 b e C R C R d V 2

NSOU ? CC-PH-02 147 2. Select a suitable frequency of the oscillator. Observe the Lissajous figure on the CRO screen. Find the point of intersection of the ellipse with the y-axis is (0,A) and the maximum vertical displacement is B. Calculate the phase difference θ. 3. Repeat step 2 with two other frequencies of the oscillator. 15.6 Experimental Results Table 1 Measurement of frequency Amplitude of the oscillator 1 = ... V Frequency of the oscillator 1 = ... kHz (Take it to be unknown, a) No. Frequency Amplitude No. of No. of Frequency Mean of of the of horizontal vertical a frequency a obs. oscillator2 the tangencies tangencies (in kHz) (in kHz) (in kHz) oscillator2 (in V) 1. ... ... 2. ... ... 3. etc. etc. etc. etc. etc. Table 2 Measurement of phase difference Frequency of the Shape of the A B θ = sin −1 (±A/B) oscillator Lissajous (No. of (No. of (degree) (in kHz) figure divisions) divisions) ... ... etc. etc. etc. etc. etc.

NSOU ? CC-PH-02 148 15.7 Discussions 1. Though the frequency of oscillator 1 has been fixed at 1 kHz it is considered to be the unknown frequency. It is observed that the measured frequency is close to this value. 2. It has been checked that the spot is at the centre of the screen. 3. The intensity of the spot should not be very bright so that the florescent screen is not damaged. 4. The Lissajous figure should be stable. 15.8 Summary In this unit we have given an elementary ideas about Lissajous figure and the CRO. We have discussed how to measure frequency and phase difference. 15.9 Answers 1. See sec.15.1 2. See sec.15.1 3. See sec.15.1 4. See sec.15.1 5. See Discussion 3 (sec.15.7) 6. See sec.15.1 15.10 Exercise 1. What is Lissajous figure? When will the figure will be an ellipse? 2. When will the Lissajous figure be a closed curve? 3. When is the Lissajous figure a figure of eight? 4. What are the different parts of a CRO? 5. Why the intensity of the spot on the screen be moderate? 6. What are horizontal and vertical deflection plates of a CRO?

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1 PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, discipline specefic, generic elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade ''A''. UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for UG programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor

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5 Netaji Subhas Open University Unit–1 ? Laws of Motion 7-46 Unit–2 ? Rotational Dynamies 47-88 Unit–3 ? Gravitation 89-124 Unit-4 ? Central Force Motion 125-152 Unit-5 ? Elasticity 153-181 Unit-6 ? Viscosity and fluid dynamics 182-209 Unit–7 ? Special Theory of Relativity 210-237 UG : Physics (HPH) Course : Mechanics and General Physics

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NSOU ? CC-PH-03 7 Unit-1 ? Laws of Motion Structure : 3.1.1 Proposal 3.1.2 Description of motion 3.1.3 Inertial Frames of Reference 3.1.4 Motion in a Non-Inertial Reference Frame 3.1.5 Rotating Coordinate System 3.1.5.1 Galilean transformation 3.1.6 System of Particles 3.1.6.1

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Centre of mass 3.1.7 Uniquencess of the position of the centre of mass 3.1.8 Motion of the Centre of Mass 3.1.9

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Angular Momentum of a System of Particles 3.1.10 Angular Momentum of a System of Particles

about Different Points 3.1.11 Variable Mass Systems : The Rocket Equation 3.1.12 The Rocket Equation 3.1.13 Conservative and non-conservative force-fields 3.1.14 Path Integral of a force : The Work-Energy Theorem : 3.1.15 Collisions 3.1.16 One-Dimensional Collision Between Two Objects—Centre of Mass Reference Frame 3.1.17 Substance of the chapter : 3.1.18 Questions (short answer type) : 3.1.19 Questions 3.1.20 Answers for the short questions 3.1.1.21Answers for the general questions :
8 NSOU ? CC-PH-03 3.1.1 Proposal Description of motion of any object always needs a frame of reference. But frames of reference are not unique. Observation on position, velocity, acceleration are dependent on the observer and also on the nature of the frames of reference. The description can be simplified for the motion of a group of particles if reference is made to the centre of mass of the moving system. Motion of a group of particles with variable mass generates lot of interest as it is related to rocket motion under different situations. Whether the force, the prime cause of motion, can be obtained from a potential or not is of general interest. During the course of motion two particles may come in contact causing a collision. Theory of collision needs an in-depth discussion. ? Outcome After reading this chaper you will be able to (i) understand how the form of Newton's law changes from inertial to non- inertial frames of reference, (ii) understand the physics behind Galilean transformation and Galilean invariance, (iii) appreciate the logic behind the dynamics of a system of particles and will be able to describe the motion of a system of particles having different symmetries, (iv) describe the motion of a rocket in various circumstances, (v) understand the nature of forces, both conservative and non-conservative and (vi) describe the motion of paricles undergoing elastic and inelastic collisions. 3.1.2 Description of Motion In the universe motion is everywhere. Only at a temperature of absolute zero the motion in any body is truly absent. If motion exists then so also does energy. To the delight of the physicist the tools that were invented by Galileo, Newton and others 200 years ago to descibe motion apply everywhere in the known universe, from electrons in our own bodies to the farthest galaxy. The study of motion and of energy is at the heart of physics.
NSOU ? CC-PH-03 9 The subject of motion is divided into two parts, namely kinematics and dynamics. Kinematics is concerned with the aspects of motion which exclude the forces which cause motion ; thus, in a manner of speaking, kinematics is focussed on the development of definitions ; position, displacement, velocity, acceleration and on the relationships between them. Dynamics widens the study of motion to include force and energy. Kinenatics begins with the idea of position. Suppose that we photograph an object moving to the left along a horizontal path at two instants of time and superimpose the images for study. We examine one image with a ruler and mark off the number of units which separates the object from the ruler's zero. The zero is a reference or origin at a position of zero units by definition while the object is at another position, say x units. X is an instantaneous quantity since it applies to a specific clock time— the instant of the taking of the photograph. Position like length is a basic quantity being dependent only on the unit adopted. But position involves also direction : in principle the object could be to our right or to our left. To include the information of direction we use a vector. The magnitude of length of the vector, say r, is x (or r), while the direction is to the right, meaning the object is to the right of the reference point. We could also agree that, by convention, the sign of x is to be positive here. The two position of the object in the photographs can be said to show two events, an initial ''i''event and a final "f" event. There is now an elapsed time between the events given by : ?t = t f – t i ...... 3.1.1 (unit seconds). Keep in mind the difference between the two concepts of time ; an elapsed time is the difference between two clock times. ? Displacement Displacement differs from position. In the interval of time between the events the object moves from one position to another. The displacement is the difference between the two vectors describing the two positions. ? r ? = r ? 1 – r ? 2 ....(3.1.2) (unit meters). Displacement, being the difference between two vectors, is a vector.

10 NSOU ? CC-PH-03 ? Velocity Another quantity in kinematics in the average velocity, or the displacement an object undergoes in one second of elapsed time. This is the ratio r At Δ ? = f f r r t - ? ? ? ? Δ ? ? ? ? = v ? (3.1.3) (unit meters per second). The average velocity, being a vector divided by a scalar, is a vector. The average velocity is negative here, since it points towards the origin, it smagnitude is the speed. The elapsed time in eqs. (3.1.1) and (3.1.3) is a finite interval. What if it is infinitesimally small? Mathematically, this amounts to taking the limit of eq (3.1.3) as ?t ? 0. The increments ? are replaced by the differentials d. Eq. (3.1.3) then becomes what is known as the instantaneous velocity dr v dt = ? ? (3.1.4) ? Acceleration The velocity of any object may not be uniform. It may change with time. The velocity could decrease due to a force of friction with the path. Or the velocity could increase if the path were not horizontal and a component of the force of gravity acts on the object. The time rate of change of the average velocity is called the average acceleration and the time rate of change of the instantaneous velocity is called the instantaneous acceleration. Both types of acceleration are defined as in eqs (3.1.3) and (3.1.4) with "v" substituted for "r" and "a" substituted for "v". ? Worked out examples : (1) If the displacement vector is given by ˆ i ? =r a sin ˆ t j ω + ˆ 0 k0 + , prove that the acceleraiton is always proportional to the displacement and acts in the opposite direction. Solution : Acceleration is given a ? = d r dt 2 2 ? = – aω 2 Sin ωt ˆ j = – ? 2 r ? . So, accleration is proportional to the displacement and is directed against the displacement. (2) An object is moving in a straight line and its displacement is given by

NSOU ? CC-PH-03 11 x = P – Qe –?t , where P, Q and ? are all constants. Find out the velocity, acceleration of the object. What will be the terminal position of the object ? Solution : Velocity x ? = ˆ di dt = ˆ i ?Q e –?t , Acceleration ˆa = 2 2 d x dt = – ˆ i ? 2 Q e –?t . The terminal position is obtained from the expression of the displacement by putting t ? ∝ , which gives x f = P – Q. 3.1.3 Inertial Frames of Reference We must consider one important point that the motion of an object is meaningful with respect to a frame of reference. As the frame of reference changes the idea about the motion has to change—for example, a man standing on the platform appears moving in the direction opposite to the direction of motion to a passenger of a moving train while a co-passenger appears static to the same person. Therefore, frames of reference occupies a vital role in the discussion of motion. Let us recall certain basic concepts of motion, namely Newton's first two Laws of Motion, which are presumably as basic and fundamental as any nature law can be: (1) The Law of Inertia : A body which has no force acting on it will remain stationary for ever or continue to move with uniform motion (that is, with constant speed and direction). We get ideas about three things from the law : (i) inertia of rest, which imples static object remains static unless affected by external influrence, (ii) inertia of motion, i.e. continuation of uniform velocity in absence of external influence and (iii) force or the external influence itself which change the inertia rest of a body to inertia of motion and vice-versa. The concept of force is explained in second law. (2) The force Law : Momentum will change in the direction of force. Mathematically, one may say, if F ? be the force and P ? be its momentum (mass times velocity), then dp F dt ? ? α , or dp F k dt ? ? = , where k is a constant. In S.I system one

12 NSOU ? CC-PH-03 Newton is that force which generates one kilogram-meter per second change in momentum in one second. So, in the case one can write dp F dt ? ? = . Under non-relativistic (velocity much less compared to the velocity or light in vacuum) condition p ? = m v ? . Considering mass to be constant we can write F m a → → = Now, these two laws seem very obvious, and perfectly reasonable and correct. So much so, that if we see a unformly moving object, we presume that it is not under any force (for at least, any net force) acting on it, whereas if we see an object which is accelerating, we presume it must have some force acting on it, in the direction of its acceleration. However, we often find ourselves in a situation in which bodies appear to be accelerating under the influence of some force, even though there is actually no force acting on them. To understand the truth in such a statement, we need to discuss frames of reference. A frame of reference is that section of the world around us, which we utilise to measure the motion of moving bodies. For all practical purposes, the world around us appear to be at rest, and insofar as that statement is true, then any motion we measure relative to our surroundings is correcly observed, and if a motion appears uniform, it must be uniform, and if the motion appears to be non-uniform, then there is some reason behind that type of motion. But suppose that instead of using whole of the world around us, we use some particular portion of the world, such as a railway car, which is moving relative to the rest of the world. As long as the car moves along its tracks uniformly, the laws of motion will remain unaltered and we can predict the future of the kinematic variables correctly. This is an example of the inertial froame of reference. But in practice there can be non-inertial frame of reference also. 3.1.4 Motion in a Non-Inertial Reference Frame In inertial reference frames the Newton's laws of motion are valid. However, it is difficult sometimes to express the motion of interest in an inertial reference frame. For example, consider the motion of a clock lying on top of a table. In a reference

NSOU ? CC-PH-03 13 frame which is fixed with respect to the Earth, if the clock is at rest, it will remain at rest for long long time (here our assumption is that the surface of the table is horizontal). But we know that the frame fixed to the earth is not an inertial frame. For description of the motion of the clock in an inertial frame, we

57%	**MATCHING BLOCK 3/123**	**W**
need to take into account the rotation of the Earth around its own axis, the rotation of the Earth around the		

Sun, the rotation of our solar system around the center of our galaxy, etc., etc. The motion of the clock will all of a sudden be a lot more complicated! For many experiments, the effect of the Earth not being an inertial reference frame is too small to be observed, so one can safely ignore that. So, the frame of reference fixed to the earth can be taken as an inertial frame of reference. Example of a non-inertial frame is a rotating frame of reference

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which is rotating with a constant angular velocity with respect to		

an inertial frame of reference. Let us see whether there is any importance of a reference frame in Newton's laws. Let the law be valid in reference frame S. We consider another frame of reference S ′ which is moving with a velocity v with respect to S. If the position of a particle in S be r and in S ′ be r ′ , then r r vdt = + ? ? ? We have assumed here that at t = 0 the frames S and S ′ were coincident. By differentiating the above equation with respect to r we get dr dr v dt dt ? ? = , Differentiating once again we get d r d r dv dt dt dt ? ?? 2 2 2 2 = + This equation shows that if d v ? /dt = 0, i.e., if there is no acceleration between the frames, = valid, but what happens in frame S ′ . Using the expression for acceleration we get F ? = ( ) dv m a dt + ? ? or F ? − m dv dt ? = m a ? ; a new force called 'Pseudo force' arises due to relative movement of the reference
14 NSOU ? CC-PH-03 frame. So, reference frame S ′ is not inertial. We shall come to this point later. ? Worked out Example 3.1.1

100%	**MATCHING BLOCK 5/123**	**W**
A small weight of mass m hangs from a string in		

a car which accelerates with an acceleration a toward left.

81%	**MATCHING BLOCK 6/123**	**W**
What is the static angle of the string with the vertical and what is its tension?		

ma mg T ? ? We plot the free body diagram of the bob here and resolve the force in horizontal and vertical direction then, T cos ? − mg = 0 (vertical equilibrium) T sin ? = ma (horizontal equilibrium), Then we have tan ? = 1 a tan g ? ? ? ? ? ? ? or ? = 1 a tan g ? ? ? ? ? ? ? and T = m(g 2 + a 2) 1/2 3.1.5 Rotating Coordinate System In order to find out the effect of the rotation of a frame of reference on the motion of a particle observed from another which is not-rotating. we shall have to describe the motion of a particle from two different frames of reference at the same time. Let us consider the two coordinate systems as shown in Fiture 1. The non-primed a g ? ? ? ? ? ?

NSOU ? CC-PH-03 15 coordinates are the coordinates in the rotating frame, and the primed coordinates are the coordinates in the fixed coordinate system (fixed to the earth say). The vector R ? indicates the origin of the rotating coordinate system from the point of view of the fixed frame of reference. Now, let us consider the motion of a particle represented by the point P. In order to consider motion of the particlev, we have to find out the changes occuring to the position vector of the particle with time. In the fixed coordinates system, the position of P is denoted by the position vector r ? and in the rotation coordinate system, the position is denoted by the position vector r ? . These two vectors are related by the relation : r ? = r ? + R ? We shall now consider the situation when the rotating coordinate system rotates by an infinitestial angle d?. If point P is at rest in the rotating coordinate system, we will see the position of P in our fixed coordinate system change by an amount dr, where ( ) r dr d r = × ? ? ? θθθθ (3.1.5.1) If the rotation of the frame of reference happens during a period dt, we can find out the rate of change of the position vector as dr d r r dt dt θ ? ? = × = ω× ? ? ? ? ? ? ? ? ? While deriving this relation our assumption is that point P remains at rest in the rotating coordinate system. If the point P moves with respect to the rotating coordinate system, this contribution must be added to the expression of the velocity of P in the fixed coordinate system. The above relation can be generalised to have f rotating d d x dt dt ? ? ? ? = + ω ? ? ? ? ? ? ? ? ? In this equation any kinematic variable can be inserted and corresponding changes can be found out

16 NSOU ? CC-PH-03 3.1.5.1 Galilean transformation We mainly use inertial flames in which a free body (no forces applied) move with a constant velocity.

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A frame moving with a constant velocity with respect to an inertial frame is inertial,

too. Thus there is an infinite number of inertial frames. Let frame K ′ is moving with a constant velocity V with respect to frame K ′ , then we have v ? = v ? − V ? , r ′ ? = r ? − V ? t, and t? = t. (3.1.5.4) This is known as Galilean transformation. We now use Language function of Lagrangian L, which actually characterises the motion of a particle under different conditions, to describe the kinetic properties of the particle. It is given by the expression for free particle as L = 1 2 mv 2 (3.1.5.5) The equations of motion are invariant with respect to transformations from one inertial frame to another, and the transformed Lagrange function can differ from the intial one only by an irrelevant full derivative. This is the principle of the Galilean invariance, i.e., invariance with respect to Galilean transformations, that is valid in the classical mechanics. K O K? V O? Transformation of the Lagrange function of a free particle gives. 2 m L v 2 ′ = = ( ) 2 m v V 2 - ? ? = 2 2 m m v mv.V V 2 2 - + ? ? = 2 d m L mr.V V t dt 2 ? ? + - + ? ? ? ? ? ? (3.1.5.6)

NSOU ? CC-PH-03 17 The second term is an irrelevant full time derivative. Thus the forms of Lagrangian are the same in both frames of referenec. This shows Galilean invariance of Lagrangian. The true check of the Galilean invariance should be the identical forms of the Lagrange equations, that is d L d L 0 0 dt V dt V ∂ ∂ = → = ′ ∂ ∂ ? The above relation can be proved easily as 2 1 L mv 2 = , one obtains d L 0 dt V ∂ = ∂ = ( ) d mv dt ? = ( ) d m v V dt ? ? ′ + ? ? ? ? = ( ) d mv dt ′ ? = d L dt V ′∂ ′ ∂ ? where 2 1 L mv 2 ′ ′ = (3.1.5.7) This proves the invariance relation. 3.1.6 System of Particles 3.1.6.1 Centre of mass The motion of a system of particles can be described in terms of motion of a single point. This special point is called

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the centre of mass of the system of particles. The position co-ordinate of the centre of mass of a system of

two particles with mass m 1 and m 2 , placed at

position x 1 and x 2 . respectively with respect to a particular co-ordinate system, is defined as 1 1 2 2 cm i i 1 2 i m x m x 1 x m x m m M + = = + ? (3.1.6.1) We can define the origin of our coordinate system to be at centre of the left most

18 NSOU ? CC-PH-03 object (see Figure 3.1.6.1). The position of the centre of mass is now O? X cm d m2 Figure 3.1.6.1 Position of the centre of mass in 1 dimension 2 cm 1 2 m d x m m = + From this equation it is clear that the center of mass lies between the two masses. It is closeex to the heavier mass. In general, for a system with more than two particles, the position of the centre of mass will satisfy the following relation min cm max x x x ≤ ≤ The center of mass in one dimension for any number of particles can be easily geoneralized to three dimensions. cm i i i 1

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x m x M = ? (3.1.6.3) cm i i i 1 y m y M = ? (3.1.6.4) cm i i i 1 z m z M = ? (3.1.6.5)

or in vector notation
NSOU ? CC-PH-03 19 cm i i i 1 r m r M = ? ? ? For a homogeneous rigid body, the summation can be replaced by an integral cm r V 1 r dm M = ? ? (3.1.6.7) Suppose we are dealing with more than two number of objects. Figure 3.1.6.2 shows a system consisting of 4 masses, m 1 , m 2 , m 3 and m 4 . located at x 1 , x 2 , x 3 and x 4 , respectively. The x-co-ordinate of the center of mass of m 1 and m 2 is given by 1,2 1 1 2 2 cm 1 2 m x m x x t m m + = + (3.1.6.8) Similarly, the x-co-ordinate of the centre of mass of m 3 and m 4 is given by 3,4 3 3 4 4 cm 3 4 m x m x x m m + = + (3.1.6.9) Now, the x-co-ordinate

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of the centre of mass of the system of four particles is given by 1 1 2 2 2 2 4 4 cm 1 2 3 4 m x m x m x m x x m m m m + + + = + + + (3.1.6.10)

Another representation of this is ( ) ( ) 3 3 4 4 1 1 2 2 cm 1 2 3 4 1 2 3 4 1 2 3 4

71% **MATCHING BLOCK 10/123** **W**

m x m x m x m x 1 x m m m m m m m m m m m m ? ? + + = + + + ? ? + + + + + ? ?

Using the centre of mass of m 1 and m 2
and of m 3 and m 4 we can express the centre of mass of the whole system as follows :
20 NSOU ? CC-PH-03 ( ) ( ) ( ) ( ) 1,2 3,4 1 2 cm 1 2 cm cm 1 2 3 4
m

70% **MATCHING BLOCK 11/123** **W**

m x m m x x m m m m + + + = + + + (3.1.6.11) m 1 m 2 m 3 m 4 x 1 x 2 x 3

x 4 Figure 3.1.6.2 Location of 4 masses This shows that

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the center of mass of a system can be calculated from the position of the centre of mass of

all objects that make up

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the system. For example, the position of the centre of mass of a system consisting out of several spheres can be calculated by assuming that the mass of each sphere is concentrated in the centre of that sphere (its centre of mass). ? Important points : * The centre of mass of an object always lies on a point/ line/plane of symmentry (for homogeneous objects). * The center of mass of an object does not need to lie within the body of that object (for example : the center of a doubt is its center of mass even though there is no mass at that point). ? Worked out Example 3.1.6.1 A circular metal plate of radius 28 from which a disk of radius R has been removed is shown in fig. 3.1.6.3. Let us call the portion as object X. We need to locate the center of mass of object X. Let us consider a co-ordinate frame with its origin at the centre of the circular plate. Suppose the hole in object X is filled with a disk of radius R. The new object (object C. Figure 3.1.6.3b) is symmetric around the origin of our coordinate system and that point is therefore the centre of mass of object C. However, object C consist out of object X and a disk with radius R centered on the x-axis at x = – R (this disk is called object D). The center of mass of this system (consisting out of object X and NSOU ? CC-PH-03 21 object D) can be easily calculated : cm X cm,D D cm,c X D x ,Xm x m x 0 m m + = = + The equation can be rewritten as cm, D D D cm,x x x x m Rm x m m = = Since we are discussing a homogeneous disk (with mass per unit area σ) the masses of object X and D can be calculated. m D = σ?R 2 m x = σ???R) 2 · m D = σ3?R 3 y-axis y-axis y-axis x-axis Figure : 3.1.6.3 Sample Problem 3.1.6.1 The position of the centre of mass of object X is given by cm,x 1 x R 3 = ? Worked out Example 3.1.6.2 A one dimensional rod is shown in fig.3.1.6.4. The density of the rod is not a constant. It depends on position : λ (x) = a + bx + cx 2 λ = mass/unit length. Determine the location of the center-of-mass of the rod.

22 NSOU ? CC-PH-03 dx x = 4 x = 0 x Figure : 3.1.6.4 Position Dependent Density The mass of a small element of the rod (length dx) is given by dm = λ?(x) dx The position of the center of mass of the rod can be determined as follows t i x cm x 1 x xdm M = ? = d 0 1 x (x)dx M σ ? = d 2 3 0 1 (ax bx cx)dx M + + ? After evaluating the integral we obtain 2 3 4 cm 1 a b c x d d d M 2 3 4 ? ? = + + ? ? ? ? 3.1.7 Uniqueness of the position of the center of mass We would like

55%	**MATCHING BLOCK 14/123**	**W**

to show that the position of the center of mass does not depend on the origin of the

co-ordinate frame chosen. Let us take another point O′ as the new origin. Let i i i OO d O P r d r ′ ′ = + = = + ? ? ? ? Now with O? as origin, the center of mass G? will be given by i i m r O G M ′ ′= ? = ( ) i i i d r m M + ? ? = i i i m d m r M M + ? ? ? = d + OG = O?O + OG = O?G The above steps show that G? and G are identical points, So, the center of mass NSOU ? CC-PH-03 23 is a unique point irrespective whatever co-ordinates system is chosen. 3.1.8 Motion of the Center of Mass The centre of mass of a system of particles can be expressed as ? ? ? cm i i i Mr =

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m r — (3.1.8.1) Where M is the total mass of all the particles.

From this equation by differentiation with respect to time we get ? ? ? cm i i i Mv = m v (3.1.8.2) Here cm V ? is the velocity of the center of mass and i V ? is the velocity of mass m i . From Newton’s 2nd Law we can write for the i-th particle of the system. i iex ij i dP F F ,(i j) dt ? ? ? + = ≠ ? Here iex F ? is the external force on the i-th particle and ij F ? is the external force exerted by the j-th particle on the i-th particle, only j is not equal to i. The total linear momentum of the system with respect to any arbitrary point B can be expressed as n P ? = iB i P ? ? = ( ) i i B i m r r - ? ? ? ? ? = i B i P Mr - ? ? ? ? (3.1.8.4) here i r and B r refers to the position vector of the i-th and the reference point B. By differentiating both sides of the above equation w.r.t t we get B dP dt ? = i B i dP Mr dt = ? ? ? ? (3.1.8.5) If the distances are taken from the origin of the inertial frame, then we can write

24 NSOU ? CC-PH-03 B dP dt ? = ext F ? + ij i j F ?? ? – B Mr ? ? (3.1.8.6) here ext F ? is the net external force on the system , B r ? vanishes as B is the origin. Moreover ij i j F 0 = ?? ? as the sum of all internal forces vanishes because of Newton’s third law. So, in an inertial frame of reference. ext dP F dt = ? ? (3.1.8.7) As a consequence of this we can easily say that when ext F ? = O, the net linear momentum P ? = constant. This is

the conservation of linear

100%	**MATCHING BLOCK 15/123**	**W**
momentum of a system of particles. 3.1.9 Angular Momentum of a System of Particles		

We now calculate the angular momentum about any point P associated with a system of N point particles. We characterize each individual particle by the index j, j = 1,2,..., N. Let the j-th particle have mass m and velocity j v ? ; The momentum of an individual particle is then j p ? ; = j m ; j v ? ;. Let r ? j be the vector from the point P to the jth particle, and let θ be the angle between the vectors ; r ? p ; and j p ? . The angular momentum Lp.j of the j-th particle is p.j pg j L r p = × ? ? ? (3.1.9.1) The angular momentum for the system of particles is the vector sum of the individual angular momenta of all the particles. sys P L ? = j N Pj j 1 L = = ? ? = P.j j j N j 1 r p = = × ? ? ? (3.1.9.2)
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84%	**MATCHING BLOCK 18/123**	**W**
The rate of change of the angular momentum of the system of particles		

about a point P is given by sys P dL dt ? = j N P,J j 1 d L dt = = ? ? = j N j N p,j j j p,j j 1 j 1 dr dp p r dt dt = = = = ? ? ? ? × + × ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.1.9.3) Because the velocity of the j th particle is p,j p/t v dr , = ? ? the first term in the parentheses vanishes (

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the cross product of a vector with itself is zero		

because they are parallel to each other). As we know that j j dp F dt ? = , immediately we get sys P dL dt ? = j N j j j 1 dp r dt = = ? ? × ? ? ? ? ? ? ? ? = j N ext p,j p j 1 r r = = = ? ? (3.1.9.4) Therefore equation (3.1.9.3) becomes ext ext P P dL dt τ = ? (3.1.9.5) It is clear that the external torque about the point P is equal to the time derivative of the

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angular momentum of the system of particles about the same point P. 3.1.10 Angular Momentum of a System of Particles		

about Different Points Let us consider a system of N number of particles moving around two points A and B. The angular momentum of the i-th particle about one of the points A is given by A.i A,i i i L r m v ? ? = × (3.1.10.1) About the point A the net angular momentum of the system of particles is given by i N i N A A,i A,i i i i i L L r m v ? ? ? ? 1 1 = = = = = = × ? ? (3.1.10.2)
26 NSOU ? CC-PH-03 Similarly, the angular momentum about the point B can be calculated in a similar way which is given by B,j j N j N B B j j j 1 j 1 L L ,j r m v = = = = = × ? ? ? (3.1.10.3) but A,j B,j A,B r r r = + ? ? ? (3.1.10.4) Here after substituting eq. (3.1.10.4) into eq. (3.1.10.2) we get ( ) j N j N j N A B,j A,B j j B,j j j A,B j j j j j L r r m v r m v r m v = = = = = = + × = × + × ? ? ? 1 1 1 ? ? ? ? ? ? ? ? (3.1.10.5) In this equation A,B r ? is a constant , so, it can be taken out the summation. Therefore, we get j N A B A,B j j j 1 L L r m v = = = + × ? ? ? ? (3.1.10.6) The sum in the second term represents the momentum of the system. Thus we can conclude

66%	**MATCHING BLOCK 20/123**	**W**
that if the momentum of the system is zero, the angular momentum is the same		

about any point. A n L L = ? ? if (p sys = 0) (3.1.10.7) 3.1.11 Variable Mass Systems : The Rocket Equation In this section, we shall consider a dynamical problem in which the mass of the experimental body changes during the motion, which means m is a function of t, i.e, m(t). Although there are many cases for which this particular model is applicable, one of obvious importance to us are rockets. A significant fraction of the mass of a rocket is the fuel, which is expelled during flight at a hight velocity and thus, provides the propulsive force for the rocket. To analyse this question we must consider a system of variable mass, and the process by which it gains velocity as a result of ejecting
NSOU ? CC-PH-03 27 mass. Let us consider a body of variable mass, with velocity v ? and external forces F ? . The said body is gaining mass at a rate dm/dt. Let us look at the process of gaining a small amount of mass dm. Let v ? 0 be the velocity of dm before it is captured by m, and let F ? represent the average value of the impulse forces that dm exerts on m. during the short interval dt. in which the capturing takes place. By Newton's third law, dm will experience a force f ? , exerted by m, over the same dt. We can now examine the capture process from the point of view of dm and equate the the impulse, -f ? dt, to the change in linear momentum of dm. ( ) Fdt f dt m v dv mv mdv + = + - = ? ? ? ? ? ? (3.1.11.1) Here, v dv + ? ? is the velocity of m (and dm) after impact. Analogously, from the point of view of m. we write ( ) Fdt f dt m v dv mv mdv + = + - = ? ? ? ? ? ? (3.1.11.2) As the term dmdv ? in equation (3.1.11.1) is a higher order term and will disappear when we take limits. The impulse due to the contact force can be eliminated ( ) Fdt dm v v mdv ' - + = ? ? ? ? or, dividing through by dt, dv m dt ? = ( ) dm F v v dt ' - - ? ? ? = dm F (v v) dt ' + - ? ? ? (3.1.11.3) Here, v v ' - ? ? is the velocity of dm relative to m. This expression is valid when dm dt < 0 (mass gain) and when dm dt > 0 (mass loss). If we compare the expression to the more familiar form of Newton's law for a particle of fixed mass dv m F dt = ? ? , we see that the term (v v ' - ? ?) dm dt is an additional force on m which is due to the gain
28 NSOU ? CC-PH-03 (or loss) or mass. Equation (3.1.11.3) can also be written as d(mv) dm F v dt dt ' = + ? ? ? where v ' ? is the velocity of the captured (or expelled) mass relative to the velocity of the mass m. This shows that, for systems involving variable mass, the usual expression stating conservation of linear momentum. d(mv) dt ? = dm dt , is only applicable when the initial (final) velocity of the captured (expelled) mass, v ? , is zero. The behaviour of m(t) is not an unknown, but is specified according to the characteristics of the rocket. In most cases dm dt is a constant and negative. In some cases, the behaviour of m(t) may be determined by a control system. In any case, it is a given quantity. ? Worked out Example : Conveyer Belt Let us consider a situation where sand particles are dropping from a stationary hopper at a rate dm dt onto a coneyer belt

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which is moving with a velocity v ? with respect to the

reference frame fixed to the Laboratory. We want to find out the force required to keep the above conveyer belt moving with the same velocity. As the velocity is constant, we get dv dt ? = 0. According to the problem sand is dropping from a stationary hopper, so, u ? = 0. Therefore we get, dm F v dt = ? ? . Remember here dm dt ≠ 0 as the system gains mass with time
NSOU ? CC-PH-03 29 3.1.12 The Rocket Equation Next we consider a rocket or mass m, moving with a velocity v ? and subject to external forces F ? (typically gravity and drag). The rocket mass changes at a rate dm/ dt, with a velocity vector c ? relative to the rocket. We shall assume that the magnitude of c ? is constant. The velocity of the gas observed from a stationary co-ordinate frame will be v? = v ? + c ? . In this frame, c ? is a vector aligned along the flight path in a negative direction c = – cn ˆ , whre n ˆ is the unit direction along the flight path. Thus dv dm m F C dt dt = + ? ? ? (3.1.12.1) The term T = dm c dt ? is called the thrust of the rocket and can be interpreted as an additional force on the rocket due to the gas expulsion. Equation (3.1.11.1) is a vector equation which can be projected along the direction of v ? (tangent to the path). Thus, t t dv dm m F c F T dt dt = - = + ? ? ? ? ? ? where F ? t is the tangential component of F ? , v and c are the magnitudes of v ? and c ? respectively, and we have assumed that c ? is parallel and has opposite direction to v ? . The magnitude of the thrust is T ? = – c ? dm/dt. Note that for a rocket, m will be negative (mass is lost). If the force F ? t is known, this equation can be integrated in time to yield an expression for the velocity as a function of time. Let us consider some simple cases : ? No External Forces : F t = 0 If gravity and drug effects are neglected, we have,

30 NSOU ? CC-PH-03 dv dm m C dt dt = - ? ? or, integrating between an intial time t 0 , and a final time t, ?v = v ? − v ? 0 = − c ? (ln m − ln m 0)= 0 m cln m - ? = 0 f m cln m ? (3.1.12.3) Alternatively, this expression can be east as the well known rocket equation. m(t) = m 0 e −?v/c (3.1.12.4) which gives the mass of the rocket at a time t, as a function of the initial mass m 0 , ? v ? , and c ? . The mass of the propellant, m propellant is given by, m propellant = m 0 − m = m 0 (1-e -?v/c) (3.1.12.5) From the above equations, we see that for a given ?v and m 0 , increasing c increases m (payload plus structure) and decreases m propellant . Unfortunately, we can only choose c ? as high as the current technology will allow. For current chemical rockets, c ? ranges from 2500 – 4500 m/sec. Ion engines can have c ? 's of roughly 10 5 m/sec. ? Gravity : F t = – mg A constant gravitational field acting in the opposite direction to the velocity vector can be easily incorporated. In this case, equation (3.1.11.2) becomes dv m dt ? = dm mg c dt - - ? ? which can be integrated to give 0 0 m v v cln gt m = - - ? ? = 0 0 0 m m m v cln g m m - - - ? ? ? ? (3.1.12.6) The solution assumes that dm c dt ? < m 0 g at t = 0. If this is not true, the rocket will sit on the pad. burning fuel until the remaining mass satisfies this requirement.

NSOU ? CC-PH-03 31 3.1.13 Conservative and non-conservative force-fields Suppose that a non-uniform force-field f(r) acts on an object in such a way that the object moves along a curved trajectory, from point A to point B. See fig.1. The work W 1 performed by the force field on the object can be written as a line-integral along this trajectory : path 1 f dr i (A B) W · → = ? ? ? (3.1.13.1) Let us suppose that the same object moves along a different trajectory, (say path 2), between the same two points. In this case, the work W 2 performed by the force- field is 2 f.dr 2 A Bpath W → = ? ? ? (3.1.13.2) Basically, there are two possibilities. Firstly, the line integrals (3.1.13.1) and (3.1.13.2) might depend on the end points, A and B, but not on the path taken between them, in which case W 1 = W 2 . Secondly, the linc integrals (3.1.13.1) and (3.1.13.2) might depend both on the points, A and B, and the path taken between them, in which case 1 2 W W ≠ (in general). The first possibility corresponds to what physicists term a conservative force-field. whereas the second possibility corresponds to a non- conservative force field. B 2 1 A Figure : 1. Two alternative paths between point A and B 32 NSOU ? CC-PH-03 What is the physical distinction between a conservative and a non-conservative force-field? Well, the easiest way of answering this question is to slightly modify the problem discussed above. Suppose, now, that the object moves from point A to point B along path 1, and then from point B back to point A along path 2. What is the total work done on the objet by the forec-field as it executes this closed circuit? Incidentally, one fact which should be clear from the definition of a line-integral is that if we simply reverse the path of a given integral then the value of that integral picks up a minus sign : in other words, B A A B f.dr f.dr = - ? ? ? ? ? ? , where it it understood that both the above integrals are taken in opposite directions along the same path. Recall that conventional 1-dimensional integrals obey an analogous rule : i.e., if we swap the limits of integration then the integral picks up a mimus sign. It follows that the total work done on the object as it executes the circuit is simply ?W = W 1 − W 2 where W 1 and W 2 defined in Eqs. (3.1.13.1) and (3.1.13.2), respectively. There is a minus sign in from W 2 of because we are moving from point B to point A, instead of the other way around. For the case of a conservative field, we have W 1 = W 2 . Hence, we conclude that ?W = 0 (3.1.13.4) In other words, the net work done by a conservative field on an object taken around a closed loop is zero. This is just way of saying that a conservative field stores energy without loss ; i.e., if an object gives up a certain amount of energy to a conservative field in travelling from point A to point B, then the field returns this energy to the object —without loss—when it travels back to point B. For the case of a non- conservative field. W 1 ? W 2 . Hence, we conclude that ?W ? 0 (3.1.13.5)

NSOU ? CC-PH-03 33 In other words, the net work done by a non-conservative field on an object taken around a closed loop is non-zero. In pratice, the net work is invariabley negative. This is just another way of saying that a non-conservative field dissipates energy ; i.e., if an object gives up a certain amount of energy to a non-conservative field in traveling from point A to point B, then the field only returns part, or, perhaps, none, of this energy to the object when it travels back to point B. The remainder is usually dissipated as heat. What are typical examples of conservative and non-conservative fields? Well, a gravitioinal field is probably the most well-known example of a conservative field. A typical example of a non-conservative field might consist of an object moving over a rough horizontal surface. We have seen that the work done by a conservative force on a particle as it moves from point A to a point B does not depend on the path chosen and therefore it depends only on the position co-ordinates of the terminal points (A, B). Then the work done can be expressed as the difference of a function of position variables only : B B A B A A F.dr V(r) V(r) dV(r) = - = - ? ? ? ? ? (3.1.13.6) Where V(r ?) is known as potential energy function V(r ?) is a scalar function of position co-ordinates. So, we arrive as a relation. F ? .dr ? = − dV 1 2 (3.1.13.7) Upon integration of the Eq. (3.1.12.7) we get ( ) V r F.dr C = - + ? ? ? ? (3.1.13.8) which means that the absolute value of the potential function V at any point is uncertain by a constant C. From Eq. (3.1.12.6) we get B A F.dr ? ? ? = B A dV(r) - ? ? = B A V.dr - ∇ ? ???? ? (3.1.13.9)
34 NSOU ? CC-PH-03 or ( ) B A F V .dr 0 - +∇ = ? ???? ? ? (3.1.13.10) This Eq. (3.1.12.10) is true for any path connecting two arbitrary points A an B in a conservative force field. So, we can conclude. F ? + V∇ ???? = 0 or F ? = − V∇ ???? Since ∇ ? ?× V∇ ???? is always zero so in case of conservative field F ∇× =0 ? . ? Worked out example : Prove that ? F = ? 2 r r is conservative. Find the corresponding scalar potential. Solution : ??X F ? = ??X (r 2 r ?) = r 2 ??X r ? + (?r ? 2) X r ? = 0 + d dr (r 2) ˆ r X r ? = 0 Therefore, F ? is conservative. So, F ? = − V∇ ???? . Now dV = V∇ ???? . dr ? = − F ? . dr ? = − r 3 dr Hence, V = − 1 4 (r 4) + c (c is a constant of integration) 3.1.14 Path Integral of a force : The Work-Energy Theorem : The work done by a force F ? acting on a particle which moves in a trajestory from a point A to another point B is certain region of space is given b B AB A W F(r)·dr = ? ? ? (3.1.14.1) This can be expressed as
NSOU ? CC-PH-03 35 ( ) B B 2 AB A A dv 1 d W m ·vdt m v dt dt 2 dt = = ? ? ? ? = 2 2 B A 1 1 mv mv 2 2 - (3.1.14.2) This constitutes the general statement of the work energy theorem. The work done by a force acting on a particle of mass m is equal to the change in its kinetic energy, the difference between the final and initial values of kinetic energy. But we have seen that in a conservative force field
the

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work done by a force is equal to the change in potential energy of the			

particle. So, one can write. 2 2 B A A B 1 1 mv mv V V 2 2 - = - (3.1.14.3) Since A and B are quite aribtrary points in space, this can only be true if and only if, each side of the equation is equal to a constant E, known as total mechanical energy. In other words the total mechanical energy is conserved in a conservative force field. ? Worked out examples : (a) Inverse square law type force field Consider a system consisting of two object of masses m 1 and m 2 that are separated by a center to center distance r 2,1 . The internal gravitational force on object 1 due to the interaction between the two objects is given by 1 2 21 21 21 G 2 Gm m ˆ F r r - = The displacement vector is given by 21 21 21 ˆ dr dr r = ? . So we get for the scalar product 21 1 2 21 21 21 G 2 Gm m F .dr dr r - = ? ? ?

36 NSOU ? CC-PH-03 From our definition of potential energy we have mentioned earlier that the change in potential energy of a system depends on the work done in moving the system from an initial position of the center of mass of the two objects apart by a distance r i to a final position of the center of mass of the same two objects apart by a distance r f is given by G UΔ = 21 21 B G A F ·dr ? ? ? = 1 2 21 21 f 2 i Gm m dr r - - ? = f 1 2 1 2 i Gm m Gm m r r - - + The reference point for the zero of the potential energy is chosen to be at infinity, r i = ∝ , with the choice that U G ∝ ? 0. By making this choice, the term 1 r in the exprssion for the change in potential energy vanishes when r i = ∝ . The gravitional potential energy as a function of the relative distance r between the two objects is given by U G (r) = – 1 Gm m r 2 , with U G (?) = 0 (b) Hooke's law type force field Let us consier a spring-object system lying on a horizontal surface which is frictionless. One and of the spring-object system is fixed to a wall and the other end is attached to an object of mass m. The spring force is an internal conservative force. The wall exerts an external force on the spring object system but since the point of contact of the wall with the spring undergoes no displacement, no work is done by this external force. We choose the origin at the position of the center of the object when the spring is relaxed (the equilibrium position). Let x be the displacement of the object from the origin. We choose the + i ˆ unit vector to point to the direction the object moves when the spring is being stretched (to the right of x = 0 on the figure). The spring force on a mass is then given by x ˆ ˆ ˆ ˆ F F i kxi 0j 0kr ? ? = = + + . The displacement is ˆ dr idx = ? . The scalar product is

NSOU ? CC-PH-03 37 ˆ ˆ F.dr kxi idx kxdx = - · = ? ? . The work done by the spring force on the mass is x f x i x W F.dr x = = = ? ? ? = ( ) 2 2 f i 1 k x x 2 - - We can now define a change in potential energy in the same spring object system in moving the object from an initial position x i from equilibrium to a final position s i by. ?U = U(x f) – U(x i) = – W = 1 2 (x f 2 – x i 2) So, an arbinary stretch or compression of a spring-object system obeying Hook's Law, from an equilibrium position at x i = 0 to a final position x f = x changes the potential energy by ?U = U(x) – U(0) = – 1 2 kx 2 If we take U(0) = 0, then with this choice of zero reference potential, the expression for potential energy is given by U(x) = 1 2 kx 2 , with U(0) = 0. 3.1.15 Collisions Any collision between two or more particles can be characterised by three stages : 1) before the collisions—particles are free and moving in straight lines with constant velocities. 2) collisions take place in the interaction zone with large force but for a very short time interval during which change in momentum and energy take place. 3) After the collisions the particles move freely in straight lines with constant velocity. In any collision of two bodies, their net momentum is conserved. That is, the net momentum vector of he bodies just after the collision is the same as it was just before the collision.

38 NSOU ? CC-PH-03 net 1 1 2 2 1 1 2 2 P m v m v m v m v ' ' = + = + ? ? ? ? ? ? So, if we know the velocity vectors of both bodies before the collision and if we also know the velocity vector of one body after the collision, then using this formula we may find out the velocity vector of the other body after the collision. But if we only know the initial velocities of the two bodies and we want to find out their velocities after the collision, we need to invoke additional physics. In particular, we need to know what happens to the net kinetic energy of the two bodies, K net = 1 2 m 1 v 1 2 + 1 2 m 2 v 2 2 (3.1.15.2) Let us now recognize this net kinetic energy into two terms, one due to the net momentum (3.1.15.1) of two particles and the other due to their relative velocity v ? ret = v ? t – v ? 2 . ( )( ) 2 2 net 1 2 net rel 1 2 1 2 P m m K v 2 m m 2 m m = + × + + (3.1.15.3) In Eq. (3.1.15.3) the first term denotes the kinetic energy due to motion of the centre of mass of the two-body system. ( ) 2 2 net 1 2 cm cm 1 2 P m m K v 2 m m 2 + = = × + (3.1.15.4) In any two-body collision this term is conserved as the net momentum P net is conserved in such collisions. In Eq. (3.1.14.3) the second term represents the kinetic energy due to relative motion of the two colliding bodies, i.e., ( ) 2 1 2 rel rel 1 2 m m K v 2 m m = × + = ( ) 1 2 1 2 1 2 1 1 1 v v 2 m m - ? ? + × - ? ? ? ? (3.1.15.5) We are interested in what happens during the collision. There are three possibilities : (a) The collision is elastic—in an elastic collision the kinetic energy of the relative motion is converted into the elastic energies of the two colliding and compressed

NSOU ? CC-PH-03 39 bodies. It is then converted back into the kinetic energy. Therefore, the

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kinetic energy of relative motion before collision is equal to the kinetic energy of

relative motion after the collision and as a result the net kinetic energy of the two colliding bodies is conserved. 2 2 1 2 2 2 1 1

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m v m v 2 2 ? ? ? ? ? ' ' x ? ? ? ? ? ? ? ? ? = 2 2 1 2 2 1 1 1 m v m v 2 2 ? ? ? ? ? + ? ? ? ? ? ? ? ? ? (3.1.15.6) and also 1 2 1 2 v v v

v ' ' ' ' - = - (3.1.15.7) (b) The collision is inelastic—a part of the kinetic energy of relative velocity K ret is converted into elastic energy and then back intothe kinetic energy of changed relative velocity. The rest of the initial kinetic energy is converted into heat or other form of non-mechanical energy. So, we get, rel rel 0 K K ' > > (c) Totally inelastic collision—conversion of all the kinetic energy of relative motion into heat or any other non-mechanical energies takes place. Therefore, we see K rel = 0 after collision and there is no-relative motion. In a collision, the ratio of the magnitudes of the initial and final relative velocities is called the coefficient of restitution and denoted by the symbol e, e = v B /v A (3.1.15.8) If the magnitude of the relative velocity does not change during a collision, e = 1, then the change in kinetic energy is zero. Collisions in which there is no change in kinetic energy are called elastic collisions. ?K = 0, elastic collision (31.15.9) If the magnitude of the final relative velocity is less than magnitude of the initial relative velocity, e > 1, then the change in kinetic energy is negative. Collisions in which the kinetic energy changes are called inclastic collisions. ?K ? 0, inclastic collision (3.1.15.10) If the trwo objects stick together after the collision, then the relative final velocity is zero, e = 0. Such collisions are called totally inelastic. The change in kinetic energy
40 NSOU ? CC-PH-03 can be found as ?K = – 1 2 μv A 2 = – 1 2 m 1 m 2 v A 2 (m 1 + m 2), total inelastic collision. (3.1.15.11) 3.1.16 One-Dimensional Collision Between Two Objects—Center of Mass Reference Frame Now let's view the collision from the center of mass (CM) frame. The x- component of

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velocity of the center of mass is 1 1x,i 1 2x,i x,cm 1 2 m v m v v m m + = + (3.1.16.1)

The x-components of the velocities w.r.t
the center of mass are ( ) 2 1x,i 1x,i x,cm 1x,i 2x,i 1 2

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m v v v v v m m = - = - + ( ) 1 2x,i 2x,i x,cm 2x,i 2x,i 1 2 m v v v v v m m ' ' = - = - + (3.1.16.2)

In the CM frame the momentum of the system is zero before the collision and hence the momentum of the system is zero after the collision. For an elastic collision, the only way for both momentum and kinetic energy to be the before and after the collision is either the objects have the same velocity (a miss) or to reverse the direction of the velocities. In the CM frame, the final x components of the velocities are ( ) 1x 1x 2x 1x 1 ,f ,i ,i ,i 1 2

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m v v v v m m ' = = - + ( ) 1 1 2 2x,f 2x,i 2x,i x,i 2 m v v v v m m ' = = - +

The final x components of the velocities in the "laboratory frame" are then given by 1 1 x,f x,f x,cm v v v = + ? ?
NSOU ? CC-PH-03 41 = ( ) 1 1
x 2 2x,i 2 2x,i 1x,i 2 1 1 2

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m i m v m v v m m m m + - + + + ? ? = 1 2 2 1x,i 2x,t 1 2 1 2 m m 2m v v m m m m - + + + ? ? (3.1.16.4) ?

Worked out Example : Show that equal mass particles in a two dimensional elastic collision emerge at right angles. In this problem there is no mention of external forces acting on the two objects during the collision. All forces are internal which means momentum is conserved. r t p p = ? ? (Ex-1) which implies (initially the second mass is at rest) m 1 . v ? 1,i = m 1 .v 1·f + m 1 .v 2,f (Ex-2) or, v ? 1,i = v ? 1,f + v ? 2,f (Ex. 3) We take the dot product of each side of eq. (Ex.3) with itself v ? 1,i · v ? 1,i + 2 = (v ? 1,f + v ? 2,f) · (v ? 2,f + v ? 2,f) (Ex. 4) Now we invoke the condition for elastic collision to which kinetic energy is the same before and after the collision. As because the objects have equal masses, we have v 1,i 2 = v 1,f 2 + v 2,f 2 (Ex.5) Comparing sq. (Ex.4) with eq. (Ex. 5) one can see that v ? 1,f · v ? 2,f = 0 (Ex,6)

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The dot product of two non-zero vectors is zero when the

two vectors are at right angles to each other which means after the collision the particles emerge at right angles to each other.

42 NSOU ? CC-PH-03 3.1.17 Substance of the chapter : Just look at what we have discussed in this chapter. 1. You have learned about the kinematics of motion, like definitions of position, velocity, acceleration. 2. You have learned how to get other kinematic variables if the information about time dependence of one variable is given. 3. You have learned about inertial and non-inertial frames of reference and their importance in the context of study of motion. 4. The concept of force as if appears in Newton's laws of motion has been discussed. 5. Galilean invariance and Galilean transformation were explained. 6. Dynamics of a system of particles has been discussed. The idea of center of mass and its relevance has been discussed. How to locate center of mass of different objects having some symmetries were discussed. 7. You have learned about the motion of a system with variable mass. In this context you have also leaned about the motion of a rocket. 8. The idea of a conservative force field was introduced. How a conservative force can be expressed in terms of a potential has been discussed. 9. Physics behind collisions was discussed along with different types of two body collisions. 3.1.18 Questions (short answer type) : 1. Define force, acceleration, velocity. 2. What do you mean by conservation of linear momentum? 3. What is center of mass ? What is the difference from center of gravity. 4. Under what condition angular momentum of an object remains unaltered? 5. Why three stage rockets are used for lifting a satellite to the orbit? 6. When a group of forces will be in equilibrium?

NSOU ? CC-PH-03 43 7. How many kinds of collisions are there ? 8. Both kinetic energy and momentum are conserved in all collisions—True or False? 3.1.19 Questions 1. A particle is travelling in a circular path with uniform speed. How the path and speed of the particle will appear to another particle moving with a uniform speed? 2. Write down Newton's first law of motion. Explain inertial and non-inertial frames of reference giving examples. 3. The bob of a 1m long pendulum is released from the horizontal position while the string being taut. What will be its speed at its lowest position? 4. Show that the motion of center of mass of n-particle system due to internal forces remain unchanged. 5. A boat of mass 75 kg and of length 5m is at rest in still water. If a man walk from the front to the back of the boat. What will be the displacement of the boat? Resistance due to water can be neglected. 6. A rocket with an initial velocity v 1 = ( ) E 3 GMr 1 4 R 2 2 ? ? ? ? ? ? ? is projected. M E and R E are the mass and radius of the earth respectively. Air resistance and effect of the rotation of the earth can be neglected. Applying conservation of mechanical energy, find out the highest distance of the rocket from the center of the earth for vertical projection. 7. A steel ball of radius 6 cm is static on a horizontal smooth plane. Another steel ball of radius 3 cm moving with a velocity of 450 cm/s collies with the first ball. Assuming perfectly elastic collision determine the velocities of each ball.

44 NSOU ? CC-PH-03 3.1.20 Answers for the short questions 1. See text. 2.

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When the resultant (external) force acting on a particle is zero, the total linear momentum of the particle remains constant

in time F = dp dt = 0 ? p = constant. 3. The point in a rigid body or system of discrete particles where a point mass equalling the mass of the body or system of particles can be placed for all motion related matters is known as center of mass.

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The center of gravity of a body or a system of particles is the point about which the vector sum of the

torques due to gravity vanishes. 4. If the total external torque (about a specific point O, say) vanishes, the total vector angular momentum of the system (about the same point O) will remain constant in time. 5. Every stage of a rocket, when separates, gives thrust in the forward direction thus increasing the velocity of the system. Three successive stages provides three successive boosts so that the rocket attains the escape velocity from the gravitational attactive field of the earth to reach its designated orbit. 6. When the resultant of all the forces is equal to zero and the resultant torque produced by the forces is zero, the forces will be in equilibrium. 7. Check the text. 8. Only in perfectly elastic collision both kinetic energy and linear momentum are conserved. So, the statement is false. 3.1.21 Answers for the general questions : 1. Let the particle moves in the x – y plane. If one takes the origin at the center fo the circle then at any point on the path of the particle will have co-ordinates x = R cos ?t and y = R sin ?t, z = 0. The velocity components are dx/dt =

NSOU ? CC-PH-03 45 – ?R sin ?t, dy/dt = ?R cos ?t, dz/dt = 0. The co-ordinates of the particle moving with a uniform

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velocity v ? are x ' = v x t, y '= v y t, z ' = v z t.

Therefore, the relative co-ordinates fo the particle moving in a circular path are x'' = R cos ?t – v x t, y'' = R sin wt – v y t, z'' = –v z t. So, the relative path is a circle whose centre is moving with a uniform velocity – v ? . 2. See text. 3. When the bob and the taut string are in horizontal position, the height of the bob from equilibrium position is 1m. i.c. the length of the string. So, the potential energy of the bob w.r.t the equilibrium position where potential energy can be taken to the zero. is P. E = mgh = m x 9.8 x 1 = 9.8 m J where m is the mass of the bob. At the horizontal position before the release of the bob, its kinetic energy is zero. At the equilibrium position when the string is vertical, its potential energy is zero, the total energy is kinetic. If at that point v be the velocity, then due to conservation of energy principle. 1 2 mv 2 = mgh, or, v 2 = 2gh. So. v ? = (2gh) 1 2 = (2 x 9.8 x 1) 1 2 = 4.43 m/s. 4. It can be shown for a group of n-particles that the equation of motion of the system is M d R dt 2 2 ? = F ? ext , where MR ? = ?m i F i , and F ? ext is the net external force on the system. So, this equation shows that the motion of the center of mass remains unchanged due to internal forecs operating in the system. 5. The external force on the boat is absent here. Therefore the motion of the center of mass will be uniform. In this case, the boat is stationary, so, its speed is zero. Therefore, if the man moves from front side to back side the center of mass will not move.

46 NSOU ? CC-PH-03 6. For vertical projection h = 0, now from conservation of energy 2 2 E E 1 A E A GM m GM m 1 1 m - = mv - , 2 R 2 r substituting the value of v i we get r A = 16 7 R E . 7. The mass of the stationary ball m 1 = C 6 3 (as volume is proportional to radius 3). C is a constant. The mass of the moving ball = C.3 3 . Let u 1 and u 2 be the velocities of the balls before collision and v 1 , v 2 be the velocities after the collisions, Now. u 1 = 0 and u 2 = 450 cm/s. Applying conservation of momentum for the collision. C3 3 X 450 = C 3 3 X v 1 + C 6 3 X v 2 . Again for elastic collision u 1 – u 2 = v 1 – v 2 Now u 1 = 0, So, after solving these two equations, we get v t = 100 cm/s and v 2 = – 350 m/s.

NSOU ? CC - PH - 03 47 Unit–2 ? Rotational Dynamies Structure 3.2.1 Proposal 3.2.2 Angular Kine matics 3.2.3 Relation Between ngular and Linear Velocity an Acceleration 3.2.5 Torque and the Moment of Inertia 3.2.6 Energy ue to Rotation 3.2.6.1 Rotational Work, Potential and Kinetic Energy 3.2.7 The Ineria Tensor 3.2.8 Parallel Axis Theorem 3.2.9 Perpendicular axes theorem 3.2.10 Radius of gyration 3.2.12 Euler Angles an Euler Equations 3.2.13 EuLer's Equations 3.2.14 Motion in a Non-Inertial Frame 3.2.14.1Time derivatives in fixed and rotatIng frames: 3.2.15 Motion relative to Earth 3.2.16 Coriolis Force 3.2.17 Substance 3.2.18 Last Questions 3.2.19 Answers 3.2.1 Proposal Angular motion is a nother kind of motion which is a part of our daily life. Description of angular motion needs the help angular kinematics as well as angular

48 NSOU ? CC - PH - 03 kinetics. Similarities between linear motion an angular motion helps in understanding of the plays the similar role of mass in angular motion. The axis of rotation plays a vital role in angular motion. When frame of reference in which the motion is studied in non-inertial, new concepts of pseudoi force come into play. These pseudo forces have geographical implication on the flow frivers. ? Outcome after reading this chapter you shall have a clear understanding about (i) angular motion in a plane (2D) and angular motion in space. (3D) (ii) angular kinemtics and angular kinetics. (iii) conservation of angular momentum (iv) moment

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of inertia, theorems of moment of inertia, calculation of moment of inertia of

different objects. (v) equation of motion in an accelerated frame, pseudo force. (vii) effect of rotation of the earth on the motion of particles on its surface. ? Facts about Angular Motion So far we have discussed linear motion of a particle or of a group of particles. We shall now turn to the angular motion of a group of particles. It will be examined in a restricted form that of rigid body rotating about a fixed axis. For such a system the particles follow circular trajectories in a plane perpendicular to the rotational axis. In other words every particle will rotate in the plane, i.e. it executes two dimensional rotational motion. In the absence of any external forces

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the axis of rotation must pass through the centre of mass of the body. If we use the origin

for all our measurements of internal motion such as rotational motion, located at the centre of mass, we can safely ignore the motion of the centre of mass itself. Imagine an object being swung around in an approximately horizonatal circular path (See figure below). Now imagine an oberver is riding on the object. The frame
NSOU ? CC - PH - 03 49 of reference for this observer is accelerating. This observer knows that the string exerts a force to the left, but cannot explain why the object does not go off in that direction, although as per the equation of motion is concered the object should move in that direction. To make the equation of motion work the observer introduces a pseudo- force in this case a force to the right in order to counteract the force exerted by the string. x-axis force exerted by string observer moving with die object We know that for the object moving in the circle, the force exerted by the string is equal to m? 2 R. Therefore the moving observer's pseudo-force must also be equal to m? 2 R, but it act in the opposite direction outwards. This kind of pseudo-force is called centrifugal force. force exerted by string Pseudoforce m R? 2 So, we see that the equation of motion works properly only in a frame of reference that is not accelerating. However it is often convenient, to use a acclerated reference frame and non-physical pseudo-forces which are invented only in order to preserve the equation of motion. We say that the pseudo-force are non-physical firstly because they violate the law that force always occure in pairs are secondly because is it not possible to identify a physical object which is the source of the force. There is a very important concept in the case angular motion–the notion of moment of ineria. We are already familar with two kinds of intertia, namely inertia of rest and
50 NSOU ? CC - PH - 03 inertia of motion. We have also learned that acceleration produced on an object by a force depends on the inertial mass of the object. Higher the mass–lower the acceleration and vice versa. In the case of angular motion we shall see that angular acceleration produced by a torque, not a force, will depend on moment of inertia, not on mass only. Equations of linear and angular motIon are very similar: F Ma = ? ? and T Iα= ? ? . So, we see that for angular motion, moment of inertia I plays a similar role like mass in the case of linear motion. Details will be provided later. 3.2.2 Angular Kinematics We now develop the kinematic equation for circular motion in order to be able to describe the dynamics of angular motion. The rectilinear co-ordinates (x, y, z) are not useful for this purpose. See figure 3.2.1. Since the angular motion takes place about a fixed axis the co-ordinates r an ? will be helpful to describe a particle's angular kinematics. As we consider only a rigid body, only the angle ? will very with time. The rate of angular displacements, ? d dt θ ω= (3.2.2.1) and the rate of change of angular velocity, ? 2 2 d d dt dt ω θ α = = (3.2.2.2.) In order to describe the kinematics of angular motion ? is measured usually in radians. Then ?, the analogue of velocity, has units of radians per second and ?, the analogue of acceleration, has units of radians per second squared.

NSOU ? CC - PH - 03 51 r ? z y x Figure 3.2.1 Co-ordinate system of cylindrical symmetry suitable for description of angular motion The angular displacement ? vis-a-vis linear displacemment x, angular veocity ? versus linear velocity v and angular acceleration ? versus linear acceleration a, helps to write down the kinematic equations for motion under constant angular acceleration. 2 0 0 1 t t 2 θ = θ + ω + α 2 0 0 1 x x v t at 2 = + + ? = ? 0 + ?t v = v 0 + at ( ) 2 2 0 0 2 ω = ω + α θ-θ ( ) 2 2 0 0 v v 2a x x = + - (3.2.2.3) where the subscript zero (0) indicates the value of appropriate quantity measured t t = 0. ?, ? and ??are actually vectors as they must be able to indicate the direction of the angular motion. We put the vectors ,θ ω ? ? and α ? on the axis of rotation. Now we have to specify in which direction the vectors should point. By convention we use the right hand screw rule determine the direction in which the vectors point. In the figure 3.2.2 the direction of θ ? and ω ? ?have been determined as follows. Let us assume that the particle is roating in the anti-clockwise direction. If we were
52 NSOU ? CC - PH - 03 to turn a right handed screw in the same sense the tip of it of it would travel in the positive z direction. This is the direction in which the vectors must therefore point by the right hand screw rule. We show α ? pointIng in the oppoSIte direction, which means that the particle's angular velocity is slowing down. x y rotational axis direction of rotation Figure 3.2.2 determintion of the direction of rotion vectors. In all calculations for angular motion we shall be able to relate the angular velocity and acceleration to the instantaneous linear velocty and accelertion of a particle. For that look at the figure 3.2.3: x y rotational axis direction of rotation Figure 3.2.3 diagram to relate angular and linear kinematic quantities. ,ω θ ? ? α ? r ? , , α ω θ ? ? ? v a, ?? r a ? r ?
NSOU ? CC - PH - 03 53 The derivation of the relationship between t r v a a , , ? ? ? and ω ? and α ? is a given in 3.2.2.3, we use the results here and explaIn their meaning. The inatantaneous velocity is given by v r =ω× ? ? ? (3.2.2.4) The order of the cross product, ensures that v ? points in the correct direction. Remember that the magnitude of v = ?rsin?, where ? is the angle between the vectors ω ? and r ? . This means that magnitude of v is simply ? times the prependicular distance from the axis of rotation to the particle. So even though it make sense to put the origin for the position vector at the centre of mass, for the purposes of these calculations, it suffices to ensure that the origin is somewhere along the axis of rotation. The radial and tangential accelerations are given by ( ) t

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r v r r r r a a = ω× + ω× = α× + ω× ω× = + ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ( ) t r a r a v r ; α ω ω ω = × = × = × × ? ? ? ? ? ? ? ? ? (3.2.2.5)

Again we find that the magnitude of the tangential acceleration is equal to ? times the perpendicular distance from the rotation axis to the particle and the radial or centripetal (towards the centre) acceleration is equal to ? 2 times the perpendicular distance from the axis of ration to the particle. More importantly, even if the rate of rotation is constant there is still a radial or centripetal acceleration. 3.2.3 Relation Between Angular and Linear Velocity and Acceleration. In order to examine the dynamics of angular motion of a rigid body we need to determine the instantaneous acceleration and velocity of the constituent atoms. We find the appropriate relations in the following manner.
54 NSOU ? CC - PH - 03 Figure 3.2.4 Radial and transverse components in a rotating system. Figure 3.2.4 shows a particle moving in a circular path in the x–y plane at a constant radius r rotating about the z axis. We define two units vectors r e ? and e θ ? pointing in the radial and transverse or tangential direction. So the rectilinear position vector r ? , is given by r ? = r e r ? (3.2.3.1) and hence the instantaneous velocity v ? is given by r de dr r dt dt = ? ? (3.2.3.2) Now looking at our vector diagram we an see that to convert this into an angular velocity ω ? , we can take advantage of the fact that ? r e ? = ?θ ? ????????? where we define angles in radians, and the angle is small enough. Now examination of the vector diagram reveals that this change in the radial vector actually points along the transverse direction and hence we get r de d e e dt dt θ θ θ = = ω ? ? ? (3.2.3.4) and hence v e θ = ? ? ??r (3.2.3.5) e t t ( ) θ +Δ ? r e (t) ? r eΔ ? r e (t t) + Δ ? r e (t t) + Δ ? e (t) θ ? r e (t) ? e θ Δ ? e t t ( ) θ +Δ ? e (t) θ ?

NSOU ? CC - PH - 03 55 To find an expression for the acceleration this expression is differentiated de dv d e r r dt dt dt θ θ ω = + ω ? ? (3.2.3.6) but f dt ω ? is simply the angular acceleration α ? and you can show, in the same way that we previouly did for r de ? /dt that r de e dt θ = - ω ? ? (3.3.3.7) which gives us r a e r e r 2 θ α ω = - ? ? (3.2.3.8) The acceleration has a tangential and radial component. The tangential acceleration t a ? is simply proporation to the angular acceleration. However, even if α ? ?is zero there is a radial acceleration r a ? , directed towards the axis of rotation (the negative sign). The acceleration is called centripetal acceleration and is present even at constant angular velocity, because we have to apply a force in order to make a particle deviate from straight line motion (Newton's first law). In the case of circular motion at constant angular velocity, the instantaneous rectilinear velocity is changing in direction, but not magnitude. We have yet to express the angular kinematic terms as vectors, which we will need to do in order to understand forces and momenta in rotating systems, since these are of course vectorial. Let us deal with the instantaeous velocity and angular velocity first. The magnitude of the velocity is given by v = ?r (3.2.3.9) By convention the right hand rule is used, giving us v r ω= × ? ? ? (3.2.3.10) Remember that this makes the magnitude of v ? times the sine of the angle between the vectors ??and r. This means that the magnitude of the velocity is equal to the angular velocity times the radial distance between the axis of rotation and the moving particle. The radial and tangential acceleration then follow.

56 NSOU ? CC - PH - 03 t a r α= × ? ? ? (3.2.3.11) r a v ω= × ? ?? (3.2.3.12) We will find that there are direct parallels between the equations of linear dynamics an those for rotational dynamics in a rotating frame. Those analogues are summarised below. ? Workedout Example The transverse component of acceleration of a particle is zero but d?/dt is not zero. Find out the radial component of acceleration eliminating the variable t. ? Solution a t = ( ) 1 d d r r dt dt 2 θ = 0 means r 2 d dt θ = constant = h (say) Then dr dr h dr dt d d r 2 = = θ θ , So, ( ) h d r d dr d d dt dt d dt r dr dt d 2 2 2 ? ? ? ? θ = = ? ? θ ? ? θ ? ? Now, d? h dt r 2 , = so, a r = ( ) 2 h d h dr h d d r r r 2 2 3 . - θ θ ? Linear Dynamics vs. Angular Dynamics v >--< ? a >--< ? m >--< I F ? = m a ? >--< r dV dt ? ? ? ? ? ? ? ?= I α ?

NSOU ? CC - PH - 03 57 P ? = m v ? >--< L ? = I ω ? dW lin = F. ? d s ? >--< dW rot = τ ? .dθ ? K lin = 1 2 mv 2 >--< K rot = 1 2 I? 2 . Let us start by looking at the rotational analogue of force, the torque. 3.2.5 torque and the Moment of Inertia Everyday experience tells us that the action which causes an extended object to move in an angular motion depends on a couple which actually is a pair of oppositely directed parallel forces whose lines of action are separated by a distance. It depends both on the magnitude of either of the applied forces and the distance from the axis of rotation. If you want to push open a door, you apply the force at the edge of the door furthest from the hinge which provides the axis of angular motion. In this way you maximise the turning effectiveness of your push or pull. If you try opening the door by applying a force close to the hinge you will find the effort required is far greater. It is clear that further away you are from the axis of rotation the greater the effect of a given force. So we would guess that the torque, τ ? , is related to the force, F ? , by r F τ = × ? ? ? Where r

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is the perpendicular distance from the axis of rotation to the

point at which the force was applied. We note that only the component of force acting tangential, F t to the direction of rotation is used to cause the rotation. If the force is applied along the door face you will not make it turn at all, whereas if you apply the force at right angles to the door face you maximise the efficiency with which you turn it. ? axis of ratation Figure 3.2.5. Turning of the particle about the axis of rotation F ? 1 F ? r ?

58 NSOU ? CC - PH - 03 This mens that ??= rF sin? (3.2.5.2) where ??is the angle between the position vector and the force vector (See Figure 3.2.5). Then τ ? = r ? × F ? and we once again follow the right hand rule. In a rigid body calculation the total torque, T ? , is The sum of the torque on each particle. j j d T r F = × ? ? ? j j d T r F = × ? ? ? (3.2.5.4) where the subscript j identifies the jth particle. Clearly the value of the torque will in general depend on the location we choose as the origin of our co-ordinate system. It is best therefore to choose the centre of mass as the origin for our measurements, although it is well to note that in conditions where the net force on the rigid body is zero, i.e. in conditions where the external forces only cause rotational motion, the value of the total torque is equivalent from whatever point the measurements are made. We shall now derive the equation which would be analoguesa of F ? = m a ? . Let us consider the case of a torque applied to a

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mass, m, at a distance r from the axis of rotation,			

where ?= ? 2 . axis of rotational m Figure 3.2.6. Moment of inertia of inertia of a particle. In this case the magnitude of the torque is ??= rF (3.2.5.5) F ? r ?
NSOU ? CC - PH - 03 59 but the applied foce, F, i equal to the particle's mass times the tangential acceleration (since r is constant). Therefore ??= r ma t (3.2.5.6) but a t = ?r and therefore ? = mr 2 ? (3.2.5.7) Now mr 2 is called the moment of inertia, I, which is the rotational analogue of the inertial mass in linear motion. In other words the larger the moment of inertia the harder it is to get the object to change its angular velocity. In full vectorial form we have τ ? = I α ? (3.2.5.8) and if you check you can see that this expression is consistent with the direction of vector cross product r ? × F ? . In all the cases of interest to us there will be more than one mass to consider in calculations of the effect of a torque applied to a molecule, so we must generalise the moment of inertia for a system of connected masses (atoms) 2 j j j m

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r ? (3.2.5.9) where r j is the perpendicular distance of the jth particle to the axis of rotation. 3.2.6		

Energy due to Rotation 3.2.6.1 Rotational Work, Potential and Kinetic Energy Finally we will look at the energetics involved in rotating a multi-atom molecule. Continuing our analogies to liner dynamics, the work done in rotating any rigid body through an angle d??by the action of an external torque will be given by dW rot = τ ? .dθ ? ????????? which is directly analogous to the definition of work for a linear displacement,
60 NSOU ? CC - PH - 03 with the force being replaced by torque and linear is placement being replced by angular displacement. By extension we may then also deduce that th potential energy assosiacted with rotating the rigid body is given by dU rot = – τ ? .dθ ? ?????????

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The kinetic energy of a particle of mass m moving in		

a straight line is 1/2 mv 2 . If the particle moves in a circle we know that v = ?r and hence we would expect the rotational kinetic energy to be 2 2 2 rot 1 1 K.E. m r I 2 2 ω = = ω (3.2.6.3) and once again the moment of inertia replaces the inertial mass and ? replces v. For a system with n number of partilces, the above equation can be generalised to 2 2 2 rot i i i 1 1 K.E. m r I , 2 2 = ω = ω ? where now 2 i i i I m r = ? (3.2.6.4) Here one can see that in the expression for kinetic energy for rotational motion, the moment of inertia plays the role of mass in case of linear motion and angular velocity plays the role of linear velocity. We now have the means to solve dynamical problems in rotation using conservation of energy and the conservation of angular momentum. Let us take the case of a rigid body, one in which the distances i j r r - ? ? between points are held fixed. A general rigid body will have six degrees of freedom (but not always, see below). In order to spefify the position of all points in the body with only six parameters, let us first fix some point r 1 of the body, which is to be treated as its "centre" or origin from which all other points in the body can be referenced from (1 r ? can be, but not necessarily, the centre of mass). Once the coordinates of 1 r ? are speified (in relation to some origin of a co-ordinate system outside of the body), we have used up three degrees of freedom. With 1 r ? fixed, the position of any other 2 r ?
NSOU ? CC - PH - 03 61 can be specified using only two coordinates since it is constrained to move on the surface of sphere centered on 1 r ? . We have now used up five degrees of freedom. If we now consider any other third point 3 r ? not located on the axis joining 1 r ? and 2 r ? , its position can be specified using and degree of freedom (or co-ordinate) for it can only rotate about the axix connecting 1 r ? and 2 r ? . We thus have used up the six degrees of freedom. It is interesting to note that in the case of a linear rod, any point 3 r ? must lay on the axis joining 1 r ? and 2 r ? ; hence a linear rod has only five degrees of freedom. Usually, the six degrees of freedom are divided in two groups : three degrees for translation (to specify the position of the "center" 1 r ? an three rotational agles to specify the orientation of the rigid body (normally taken to be the so-called Euler angles) 3.2.7 The Ineria Tensor Let us

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consider a rigid body containing n particle of mass m i , i = 1, n. If the body rotates with an angular velocity ??about

some point fixed with respect to the body coordinates (this "body" coordinate system is what we used to refer to as "non- inertial" or "rotating" coordinate system and if this point moves linearly with a veloclty V with respect to a fixed (i.e. inertial) coordinate system, then

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the velocity of the particle is given by equation v ? = V ? + ω ? x i r ? (3.2.7.1) The

total kinetic energy of the body is given by 2 i i i i i 1 T T m v 2 = = ? ? = ( ) 2 i i i 1 m V r 2 + ωx ? ? ? ?
62 NSOU ? CC - PH - 03 = ( ) ( ) 2 2 i i i i i i i i 1 1 m V m V. xr m xr 2 2 + ω + ω ? ? ? ? ? ? ? ? (3.2.7.2) Although this equation for the total kinetic energy is perfectly general, considerable simplification will result if we choose

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the origin of the body coordinte system to coincide with the center of mass.

With this choice, the second term on the right hand side of the last of equations (3.2.7.2) can be seen to vanish from ( ) i i i i i i m V. xr V. x m r 0 ? ? ? ? ? ω = ω = ? (3.2.7.3) since the centre of mass R ? of the body, of mass M, is defined such that i i i m r 0= ? ? (3.2.7.4) The total kinetic energy can then be broken into two components: one for the translational kinetic energy and another for the rotational kinetic energy. That is, T = T trans + T rot (3.2.7.5) With 2 2 trans i i 1 1 T m V MV 2 2 = = ? and as seen earlier, ( ) 2 rot i i i 1 T m r 2 = ωx ? ? ? The expression for T rot can be further modified, but to do so we will now take resort to tensor (or index) notation. So, let's consider the following vector equation ( ) ( ) ( ) i i i r r r 2 . ω ω ω × = × × ? ? ? ? ? ? (3.2.7.7)
NSOU ? CC - PH - 03 63 and rewrite it using the Levi-Civita and the Kroncker tensors ( )( ) ijk j ,k imn m ,n ijk imn j ,k m ,n x x x x α α α α ε ω ε ω = ε ε ω ω = ( ) jm kn jn km j ,k m ,n x x α α δ δ -δ δ ω ω = j j ,k d,k j ,j k ,k x x x x α α α ω ω -ω ω (3.2.7.8) Inserting this result in the equation for T rot in equation (3.2.7.6) we get ( ) ( ) 2 2 2 rot 1 T m r .r 2 α α α α = ω - ω ? ? ? ? ? (3.2.7.9) Alternatively, keeping with the tensor notation we have rot j j ,k ,k i ,i j ,j 1 T m x x x x 2 α α α α α α ? ? = ω ω -ω ω ? ? ? = ( ) i j ij ,k i ,i j ,j 1 m x x x 2 α α α α α ? ? ω ω δ -ω ω ? ? ? = ( ) i j ij a,i ,k ,i ,j 1 m x x x x 2 α α α α α ? ? ω ω δ - ? ? ? (3.2.7.10) We now define the components I ij of the so-called inertia tensor {I} by ij ij ,k a,k ,i ,j I m x x x x α α α α α ? ? = δ - ? ? ? (3.2.7.11) and the rotational kinetic energy becomes
64 NSOU ? CC - PH - 03 rot ij i j 1 T I 2 = ω ω ? ? (3.2.7.12) or in vector notation { } rot 1 T . I . 2 = ω ω ? ? (3.2.7.13) For our purposes it will be sufficient to treat the intertia tensor as a regular 3 × 3 matrix. Indeed, we can explicitly write {I} using equation (3.2.7.11) as { } ( ) ( ) ( ) 2 2 .2 ,3 ,1 ,2 ,3 ,3 2 2 ,2 ,1 .1 ,3 ,2 ,3 2 2 ,3 ,1 ,3 ,2 .1 ,2

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m x x m x x m x x I m x x m x x m x x m x x m x x m

x
x

77%	**MATCHING BLOCK 46/123**	**W**

α α α α α α α α α α α α α α α α α α α α α α α α

α α α α α α α α α α α α α ? ? + - - ? ? ? ? ? ? ? ? = - + - ? ? ? ? ? ? - - + ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

It is easy to see from equaton (3.2.7.14) that the inertia tensor is symmetric, that is, I ij = I ji (3.2.7.15) The diagonal elements I 11 , I 22 , I 33 are called he Principal

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Moments of Inertia about the 1 2 x ,x - - and 3 x - axes, respectively. The negatives of the off-diagonal elements are the Products of Inertia.

Finaly, in most cases the rigid body is continuous and not made up of discrete particles as was assumed so far, but the results are results are easily generalized by replacing the summation by a corresponding integral in the expression for the components of the inertia tension. ( )( ) ij ij k k i j 1 2 3 v I r x x x x dx dx dx = ρ δ - ? ? (3.2.7.16)
NSOU ? CC - PH - 03 65 where ?(r ?) is the nass density at the position r ? , and the integral is to be performed over the whole volume V of the rigid body. ? Worked out Example Calculate the inertia tensor for a homogeneous cube of density ??mass M, and side length b. Let one corner be at the origin, and three adjacent edges lie along the coordinate axes (see Fig. 3.2.1). Solution. We use equation (3.2.7.16) to calculate the components of the inertia tensor. Because the symmetry of the problem. It is easy to see that the three moments of inertial I 11 , I 22 , and I 33 are equal and that same holds for all of the products of inertia. So, O x 2 x 1 b b b x 3 Fig. 3.2.1 Homogeneous cube ( ) b b b 2 2 11 2 3 1 2 3 0 0 0 I x x dx dx dx = ρ + ? ? ? = ( ) b b b 2 2 3 2 2 3 1 0 0 0 dx dx x x dx ρ + ? ? ? = 3 4 4 b 2 3 3 0 b b b b dx bx b 3 3 3 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ρ + =ρ + ?
66 NSOU ? CC - PH - 03 = 5 2 2 2 b Mb . 3 3 ρ = (3.2.ex,1) it should be noted that in this example

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the origin of the coordinate system is not located at the centre of mass of the

cube. The products of inertia of the cube (−negative) are b b b 12 1 2 1 2 3 0 0 0 I x x dx dx dx = - ρ ? ? ? = ( ) 2 2 b b b 2 2 ? ?? ? -ρ? ?? ? ? ?? ? ? ?? ? = 5 2 1 1 b Mb . 4 4 - ρ = - 3.2.8
Parallel
Axis Theorem Statement:

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The moment of inertia (I) of a body about any axis is the sum of its moment of inertia (I cm) about a parallel axis through the centre of mass and the product of the mass (M) of the body by the square of the distance (d 2) between the two axes. Proof: Let I cm be the moment of inertia of a body of mass M about an axis passing through its centre of mass. Let I be the moment of inertia

of the same body about an axis
parallel
to the previous one and
at a distance d from it.

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The moment of inertia of the body about the axis passing through centre of mass is

given by
NSOU ? CC - PH - 03 67 2 CM I r dm = ? But according to the problem

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r ′ ? = r ? + d ? . So, we get r 2 ′ = d 2 + 2 d ? . r ? + r 2 Now, the moment of inertia of the

body the parallel axis is '2 2 l r dm d dm 2d. = = + ? ? ? 2 2 2 rdm r dm d M r dm + = + ? ? ? ? as the second term is zero because of the definition of the center of mass. Hence, I = I cm + Md 2 . If Now d = 0, then we get I = I cm . 3.2.9 Perpendicular axes theorem This therorem is applicable to planar objects, Let us consider a rigid object that lies entirely within a plane (X – Y). The perpendicular axes theorem links I z (

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moment of inertia about an axis perpendIcular to the plane) with I x , I y (moment of inertia

about two perpendicular axes lying within the plane). Now look at the figure: the body lies in the x–y plane on which at O, the origin, three mutually perpendicular axes meet. O y x d P dm Z ( ) 2 2 2 2 z x y I x y dm x dm y dm I I = + = + = + ? ? ? which proves the theorem. 3.2.10 Radius of gyration
The moment of inertia of an object can be equated to the product of the mass M ω ?

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The moment of inertia of the body about the z-axis is given by

I z = d 2 ? dm. or
68 NSOU ? CC - PH - 03 of the object and square of some appropriate length K, i.e. I = МК 2 . The quantity

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K is called the radius of gyration of the given object about the given axis. 3.2.11

Moment of Inertia of objects of different shapes : (a) A unifom thin rod about an axis passing though the center of mass We shall take the case of a uniform (density and shape) thin rod of mass M and length L such that one can assume the cross-section area of the rod is small and the rod can be thought of a string of masses along a one-dimensional straight line. The axis of rotation is perpendicular to the rod and passes through the center of mass, i.e., the midpoint of the rod. Our task is to calculate the moment of inertia about this axis. We take the z-axis as the axis of rotation and the x-axis passes through the length of the rod, as shown in the figure. This is to facilitate integration along the x-axis. Fig. Calculation of the moment of inertia I for a uniform thin rod about an axis through the center of the rod. We define dm to be a small element of mass making up the rod. The moment of inertia is an integral over the mass distribution. We need to find a way to relate mass to spatial variables. We do this using the linear mass density ??of the object, which is the mass per unit length. Since the mass density of thi object is uniform, we can write. ??= M L (3.211.1)
NSOU ? CC - PH - 03 69 Note that a piece of the rod dl lies completely along the x-axis and has a length dx; in fact, dl = dx in this situation. We can therefore write dm = ?? dx. The distance of each piece of mass dm from the axis is given by the variable x, as shown in the figure. Putting this all together, we obtain 2 2 2 I r dm x dm x dx. = = = λ ? ? ? (3.2.11.2) In the last step we have to be careful about our limits of integration. The rod extends from x = – L 2 to + L 2 , since the axis is in the midle of the rod at x = 0. So, after integreation we get I = 2 1 ML 12 (3.2.11.3) where M is the mass of the rod and L its length. (b) A uniforem Thin Rod with Axis at the End Now consider the same uniform thin rod of mass M an length L, but this time we move the axis of rotation to the

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end of the rod. We wish to find the moment of inertia about this axis (

Figure). O z dm x L x dx Figure moment of inertia I for a uniform thin rod about an axis through the end of the rod.The quantity dm is again defined to be a small element of mass making up the rod. Just as before, we obtain 2 2 2 I r dm x dm x dx = = = λ ? ? (3.2.11.4)
70 NSOU ? CC - PH - 03 However, this time we have different limits of integration. The rod extends from x = 0 to x = L, since the axis is at the end of the rod at x = 0. Therefore we find. I = 2 1 ML 3 (3.2.11.5) Note the rotational inertia of the about its endpoint is larger than the rotational inertia about it centre (consistent with the barbell example) by a factor of four. (c)

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A Uniform Thin Disk about an Axis through the center In this problem we want to find out the moment of inertia of a two-dimensional object-a uniform thin disk about an axis through

its centre (Figure) Z dm x y y x Figure Calculating the moment of inertia for a thin disk about an axis through its center. Since the disk is thin, we can take the mass to be distributed entirely in the xy- plane. We start with the surface mass density, which is the mass per unit surface area. Since it
is uniform, the surface mass density ??is constant. ??=

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M A , where M is the mass of the disk and A is

its area. The area can be throught of as made up of a series of thin rings of increasing radii, where each ring is a mass increment dm of radius r equidistant from the axis, as shown in part (b) of the figure. The infinitesimal area of each ring dA is therefore given by
NSOU ? CC - PH - 03 71 the length of each ring (2?r) times the infinitesimal width of each ring dr: dA = 2?r dr. (3.2.11.6) So, the mass of the elementary ring is dm = ??2?r dr = 2?Mr dr A . (3.2.11.7) Therefore, the moment of inertia of the whole disk will be R R 2 3 2 0 0 M 1 I dmr 2 r dr MR A 2 = = π = ? ? (3.2.11.8) (d) A hollow cylinder having mass M, an inner radius r 1 , outer radius r 2 and length L, about its central axis. Let us consider a situation where the cylinder is cut into infinitesimally thin rings centered at the middle. The thickness of each ring is dr, with length L. The moment of inertia of the elementary ring dI = r 2 dm (3.2.11.9) Now, we have to find dm, (which is just density multiplied by the volume occupied by one ring dm = ??dV (3.2.11.10) We have introduced dV in the above equation, so, we have to find out what dV is:
72 NSOU ? CC - PH - 03 dV = dA L (3.2.11.11) Here dA is the area of the top of the ring, which is the area of a rectangular strip of length 2?r and width dr. We have: dA = 2?rdr (3.2.11.12) Substituting dA into dV, dV = 2?rLdr (3.2.11.13) Finally, we have an expression for dm. We substitute that into the dI equation, dI = 2??r 3 Ldr (3.2.11.15) to get the final form of moment of inertia 2 1 r 3 r I 2 Lr dr = πρ ? (3.2.11.16) Now, we can find the expression for density. ? = M / { ??(r 2 2 – r 1 2)L} (3.2.11.17) Substituting this back into the integrated solution, we have: I = ( ) 2 2 2 1 1 M r r 2 + (3.2.11.18) Special Cases: Hoop or thin cylindrical shell: (r 1 = r 2 = r) I = Mr 2 (3.2.11.19) Disk or solid cylinder: (r 1 = 0, r 2 = r) I = 1 2 (3.2.11.20) (e) A rectangular plate

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about an axis passing through its center and perpendicular to its plane. The moment of inertia

for the rectangular plate of sides ‘a’ and ‘b’ can be found by using the expression for momen

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of inertia of a rod about an axis passing through its center of mass and perpendicular to

its length
and the parallel axis theorem.

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The moment o inertia of a rod of mass M and length

L, with
axis separated dy distance x from the original one (through the center of mass), is I x = I CM + Mx 2 = 1 2 ML 2 + Mx 2 .(3.2.11.21)
NSOU ? CC - PH - 03 73 Now we replace M ??a, M ??dm = ??a dx, where ??is the surface mass density. Integrating over x from – b/2 to b/2, one obtains ( )( ) b 2 b 2 3 2 3 3 2 2 1 1 M I a ax dx a b ab a b 12 12 12 - ? ? = σ+ σ = σ + = + ? ? ? ? ? (3.2.11.22) where M = ??ab has been used. Fig. Thin spherical shell with rotation axis (f) Thins pherical shell (Please speciy the axis and give fig.) Let us consider a thin spherical shell of radius R and mass M. We take spherical coordinates with azimuthal angle ??and zenith angle ?. On the spherical shell the mass element is dm = ?R sin 2 M 4 R π , (3.2.11.23) where ??= M/4?R 2 is the surface mass density as we are considering a shell, and the distance from the rotationl axis is r = R sin ?. Hence the moment of inertia to be calculated is 2 4 3 0 I r dm 2 R sin d π = = πσ φ φ ? ? (3.2.11.24) Noting that ( ) 3 0 0 sin d sin 1 cos2 d π π φ φ = φ - φ φ ? ? Axis r ?
74 NSOU ? CC - PH - 03 = 1 2 0 1 sin d u du π - φ φ- ? ? = [ ] 1 3 0 1 1 cos u 2 π - ? ? - φ - ? ? ? ? = 4 3 (3.2.11.25) (the variable has been changed as u = cos ??and du = d cos ??= – sin??d?), we now find 2 2 I MR . 3 = (3.2.11.26) Solid sphere

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The moment of inertia for a solid sphere of radius R and mass M

can be obtained by integrating the result for the disk 2 CM 1 (I MR) 2 = over changing distance from the axis. Choosing the z-axis as the axis of rotation and letting the distance from it to the mass element on the shell as r, we have r 2 = R 2 – z 2 . (3.2.11.27) Now M ??dm = ??r 2 ?dz and R 2 ??r 2 , we have
NSOU ? CC - PH - 03 75 R 2 2 R 1 I r .r dz 2 - = π ρ ? = ( ) R 4 2 2 4 R 1 R 2R z z dz 2 - πρ - + ? = 2 2 MR . 5 (3.2.11.28) where the mass of the sphere is 3 4 M R . 3 = π ρ 3.2.11.29) ? Worked out Example: Find the distance travelled by the axis of a solid right circular of radius r and mass m after it has rolled from rest without slipping for time t on a plane inclined at an angle ??with the horizontal. Solution Let F ? = frictional force and N ? = normal reaction. The equation of motion is mg sin??– F = ma and F r = I ??(about the c.m. of the circulr face of the cylinder), where ??is th angle of the plane or the angle between the vertical and the normal reaction, r is the radius of the cylinder. Now a = ?r, so, F = mk k ma r a r r 2 2 2 . = = k ma r 2 2 , k is the radius of gyration of the cylinder about its axis. From the equation for normal reaction, mg sin ??– k ma r 2 2 , = ma or, mg sin??= ma k 1+ r 2 2 ? ? ? ? ? ? ? . therefore, one gets a = g sin ?? k 1+ r 2 2 ? ? ? ? ? ? ? .

76 NSOU ? CC - PH - 03 For a right circular cylinder 2 2 1 I mk mr . 2 = = So, 2 2 k 1 , 2 r = Therefore, a = 2 3 g sin?. Hence in time t the cylinder has travelled a distance s given by 2 2 1 1 s at g in t . 2 3 s = = θ 3.2.12 Euler Angles and Euler Equations In this section, we set to determine the set of angles that can be use to specify the rotation of a rigid body. We know that the transformation from one coordinate system to another can be represented by a matrix equation such as x x λ ′ = ? ? (3.2.12.10) If we identify the inertial (or fixed) system with x? and the rigid body coordinate system with x then, the rotation matrix ??describes the relative orientation of the body in relation to the fixed system. Since there are three rotational degrees of freedom, ?? is actually a product from three individual rotation matrices; one for each independent angles. Although there are many possible choices for the selection of these angles, we will use the so-called Euler angles ?, ? and ?. The Euler angles are generated in the following series of rotation that takes the fixed x ′ ? system to the rigid body x ? system (see figure) 1. First of the rotations is taken counter clockwise through an angle ??about the x 3 − axis. It transforms the inertial system into an intermediate set of x 1 ?−axes. The transformation matrix is. ( ) ( ) ( ) ( ) cos sin 0 sin cos 0 , 0 0 1 φ ? φ φ? ? ? λ = - φ φ ? ? ? ? ? ? (3.2.12.2) with 0 ? ????2?, and x? = ? ? x? (3.2.12.3)
NSOU ? CC - PH - 03 77 Figure 3.2.12.1 The Euler angles are use to rotate the fixed x 1 ′ system to the rigid body x ? system. (a) The first rotation is counter clockwise through an angle ? about the x 3 ??−axis. (b) The second rotation is counter clockwise through an angle ??about the x 1 ??−axis. (c) The third rotation is counter clockwise through an angle ??about x 3 ???−axis. 2. The second rotation is counter clockwise through an angle ??about the x 1 ??− axis (also called the line of nodes). It transforms the inertial system into an intermediate of x 1 ???−axes. The transformation mtrix is ( ) ( ) ( ) ( ) 1 0 0 0 cos sin . 0 sin cos θ ? ? ? ? λ = θ θ ? ? ? ? - θ θ ? ? (3.2.12.4) with 0 ????????and x???= ? ? x? (3.2.12.5) 3. The third rotation is counter clockwise through an angle ??about the x 3 ???−axis. It transforms the inertial system into the final set rigid body x 1 −axes. The transformation matrix is ( ) ( ) ( ) ( ) cos sin 0 sin cos 0 . 0 0 1 Ψ ? ψ ψ? ? ? λ = - ψ ψ ? ? ? ? ? ? (3.2.12.6) with 0 ??????2? and x = ? ? x?? (3.2.12.7) One can combine the three rotations using equations (3.2.12.3), (3.2.12.5) and (3.2.12.7) to get finally the complete transformation geven
by

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x x 2 2 ′ ″ = ˆ φ ? ? x 2 ′ x 2 ″ x 1 ″ x 1 ′ x 1 ″ =x 1 ‴ x 1 ′ ? ? x x 3 3 ′ ″ = ˆ φ x 2 ′ x 2 ″ x 3 ″ ˆ φ ˆ Ψ ? x x 3 3 ′ ″ = ˆ Ψ ? ˆ φ x 1 ˆ ′ θ x x 3 3 ‴ = x 2 ‴ x 2 ′ 78 NSOU ? CC - PH - 03 x = ? ? ?? ?? ? ?? x? (3.2.12.8)

and
the
rotation matrix for the complete transformation is ? = ? ?? ? ?? ? ? . (3.2.12.9) 3.2.13 Euler's Equations the equation of circular motion of a rigid body is given by f dL N dt ? ? = ? ? ? ? ? ? (3.2.13.1) where L ? is the angular momentum and N ? is the net torque operting on the body. From the study on non-inertial frame of reference we know that f body dL dL L dt dt ? ? ? ? = + ωx ? ? ? ? ? ? ? ? ? ? ? ? (3.2.13.2) We can rewrite this equation using tensor notation as i dL dt ? + ? ijk ? j L k = N i (3.2.13.3) Now these equations can be modified further if we choose the co-ordinate axes for the body frame of reference to coincide with the principal axes of the rigid body, then L 1 = I 1 ? 1 , L 2 = I 2 ? 2 , L 3 = I 3 ? 3 . (3.2.13.4) But we know that the principal moments of inertia are constant in time, so by combining (3.2.13.4) we get ( ) 1 1 2 3 2 3 1 d I I I N dt ω - ω ω = ( ) 2 2 3 1 3 1 2 d I I I N dt ω - ω ω = (3.2.13.5)
NSOU ? CC - PH - 03 79 ( ) 3 3 1 2 1 2 3 d I I I N dt ω - - ω ω = Above three equations can be combined together in to one equation as ( ) k i j i j ijk k k k d I I I N 0 dt

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ω ? ? - ω ω - ∊ - = ? ? ? ? ? (3.2.13.6) These equations are known as Euler's equations

of motion for a rigid body. 3.2.14 Motion in a Non-Inertial Frame 3.2.14.1 Time derivatives in fixed and rotatIng frames: The time derivative of an arbitrary rotating-frame vector A ? in a fixed frame is expressed as shown earlier f f r dA dA v A dt dt = ? ? ? ? = +ω× ? ? ? ? ? ? ? ? ? ? ? ? (3.2.14.1) where the time derivative as observed in the fixed (f) frame is expressed as ( ) f d dt , and the time deivative as observed in the rotating (r) frame is expressed as ( ) r d dt . An importnt appliction of the above formula can be found when A ? is replaced by the angular velocity ?. It can easily be seen that (d?/dt) f = (d?/dt) r as the second term in eq (3.2.14.1) vanishes when A ? = ω ? . 3.2.14.2 Accelertion in rotating frames: We shall now discuss the rotational motion of a particle considering the general case of a rotating frame associated with the particle and a fixed frame being related by translation and rotation. Let is consider the present position of the particle be P. According to the fixed frame of reference the position vector is r ' ? ?(see fig.) while the position vector of the
80 NSOU ? CC - PH - 03 same point P with reference to the rotating frame of reference is r ? , while r ' ? ?and r ? are related by

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r ' ? =R ? +r ? (3.2.14.2) where R ? is the position vector of the origin of

the rotating frame corresponding to the fixed frame. O? P O Figure 3.2.14.1 Vectorial relation between fixed frame and rotating frame of reference The velocities of point P as observed in the fixed and rotating frames are expressed as ( ) f f dr v dt ' ? and ( ) r r dr v dt = ? ? (3.2.14.3) respectively. But, f v ? = ( ) r f dR dr dt dt ? ? = ? ? ? ? ? ? = V ? + r v ? + ω ? × r ? , (3.2.14.4) where V ? = ( ) r dR dt denotes the translation velocity of the rotating-frame origin (as observed in the fixed frame). Using Eq. (3.2.14.4), we are now in position to evalute expression for the acceleration of point P as oberved in the fixed and rotating frames of reference f f f dv a dt ? ? = ? ? ? ? ? ? and r r r dv a dt ? ? = ? ? ? ? ? ?
NSOU ? CC - PH - 03 81 respectively. (3.2.14.5) Hence using Eq. (3.2.14.4) we get r f f f f f dv dV

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d dr a r dt dt dt dt ? ? ω ? ?? ? ? ? + + × + ω× ? = ( ) r r r f d A a v r v r) dt ( ) (ω + +ω× + × +ω× +ω× ? ? ? ? ? ? ? ? ? ? (3.2.14.6) or, ( ) f r fr d a a v r (r) dt 2 ω + A× + ω× + × +

ω× ω× ? ? ? ? ? ? ? ? ? ? (3.2.14.7) whereA ? = f dV dt ? ? ? ? ? ? ? denotes the translational acceleration of the rotating-frame (as observed in the fixed frame of reference). We can now write an expression for the acceleration of point P as observed in the rotating frame as ( ) r f fr d a a 2 v r (r) dt ω = - ω× - × -ω× ω× ? ? ? ? ? ? ? ? ? (3.2.14.8) which represents the sum of the net inertial acceleration (f a ? – A ?), the centrifugal acceleration – × (ω ? × r ?) and the Coriolis acceleration –2 ω ? × r v ? and an angular acceleration – ( ) dω δτ ? × r ? which epends explicity on the time dependence of the angular velocity ω ? . 3.2.15 Motion relative to Earth These expressions can now be applied to the important case of the fixed frame of reference having its origin at the centre of Earth (point O??in the Figure below) and the rotating frame of reference having its origin at latitude ??and longitude ??(point
82 NSOU ? CC - PH - 03 O in the Figure below). We note that the rotation of the Earth is now represented as δΨ δτ ? = ω ? and that d?/dt = 0. In this diagram the (x –, y–, z–) axis of the rotating frame have been arranged so that the z-axis is a continution of the position vector R ? of the rotating-frame origin, i.e., R = R ˆz in the rotating frame (where R = 6378 km is the Earth's radius assuming a spherical Earth). When expressed in terms of the fixed-frame latitude angle ??and the azimuthal angle ?, the unit vector ˆz is ( ) ˆ ˆ ˆ ˆ z cos cos x sin y sin z ' ' ' = λ ψ + ψ + λ Likewise, we choose the x-axis to be tangent to great circle passing through the North and South poles, so that ( ) ˆ ˆ ˆ ˆ x sin cos x sin y cos z . ' ' ' = λ ψ + ψ - λ Lastly, the y–axis is chosen such that ˆ ˆ ˆ y sin x cos y ' ' = - ψ + ψ We now consider the acceleration of point P as observed in the rotating frame O by writing Eq. (3.2.14.8) as
NSOU ? CC - PH - 03 83 f d R d r g r dr dt dt dt 2 2 2 2 2 ( ) ω = - -ω× ω× - ? ? ? ? ? ? ? ? (3.2.15.1) The first term represents the pure gravitational acceleration due to the gravitational pull of the Earth on point P (as observed in the fixed frame located at Earth's centre) 3 GMr g r ' - = ' ? ? where

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r ′ ? = R ? + r ? is the position of point P in the fixed frame and r ? is the location of P in the rotating frame.

When expressed in terms of rotating-frame spherical coordinates (r, ?, ?): ( ) ˆ ˆ ˆ r r sin cos x sin y cos z ; = ? θ φ + φ + θ? ? ? ? and ( ) 3 2 3 2 2 r R 2Rrcos r ′ = + θ+ The pure gravitational acceleration is, therefore, expressed in the rotating frame of the Earth as ( ) ( ) ( ) 3 0 22 ˆ ˆ ˆ 1 cos z sin cos x sin y g g 1 2 cos ? ? ? ? +∈ θ +∈ θ ϕ + ϕ = - ? ? ? ? + ∈ θ+∈ ? ? ? ? (3.2.15.2) where g 0 = 2 2 GM m 9.789 R s = and ∈ = r/R >> 1. From the above discussion it is clear that the centrifugal force F ? cent = – m ω ? ?× (ω ? ?× r ?) is directed perpendicularly away from the rotation axis of the Earth. At latitude ??its value is m? 2 r cos ?.The acceleration due to gravity which is measured in the laboratory is actually the effective acceleration e g g (r) = -ω× ω× ? ? ? ? ? (3.2.15.3)
84 NSOU ? CC - PH - 03 Thus, we can conclude that centrifugal force exists when viewed rom a rotating frame of reference. ? Worked out Example : What will be the shape of the surface of water in a bucket which is rotating with an angular velocity ω ? ? The water surface appears static to an observer rotating with the bucket. As equilibrum is maintained all over the water, for a small of mass m of the water, net force must be zero. If the contact force r g 2 ω act at the contact point making an angle ? with the vertical, then one can write N cos ? – mg = 0 and –N sin? + m? 2 r = 0 (3.2.Ex 1) N

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must be normal to the liquid surface. The slope of the surface is dz dr = tan? = r g 2

ω (from Eq. 3.2.Ex 1) (3.2.Ex 2) On integration we get 2 2 1 r z 2 g ω = Thus the surface will be parabolic. 3.2.16 Coriolis Force This force arises due to the rotation of the frame of reference in which the particle itself is moving. The particle has to move in some direction other than the one parallel to the rotation axis, F ? cor = – 2m ω?× dr dt ′ (3.2.16.1) As is evident from the above equation the coriolis force is always perpendicular to the motion of the particle. No work can be done by this force. Only the force can change the direction of motion of the particle. There are numerous effects of this force on the Earth's surface, for example, for the soldier to hit a long distance target, correction should be made for the coriolis deflection. Rivers in the northern
NSOU ? CC - PH - 03 85 hemisphere flowing in the north-south direction towards the sea, deviates to the right of motion due to corioli force. This results in a greater erosion of the right bank. In the southern hemisphere the erosion takes place in the opposite direction. ? Worked out Example : 1. Using the expressing for the coriolis force show that fallin from a height h the deflection of a particle on the ground will be 1 23 8h 3 g ? ? ω ? ? ? ? ? ? , where g is the local acceleration due to gravity. (Assume latitude of the place to be zero) Solution : The particle at the height h was moving with the same angular velocity as that of the earth ω ? . So, it had alinaer velocity (R + h) ω ? towards east, whereas the linear velocity on the surface of he earth was R?. For this reason the particle falls a little bit on the east. As the latitude is zero, vertical direction and the rotational axis are perpendicular to each other. The coriois force will be 2m (v × ?) towards east. Therefore, the equation of motion for the x-component (towards east) will be 2 2 md dt = 2m (v ? × ω ?) = 2 mv?. Now, v = gt and h= 1 2 gt 2 . So, we have m 2 2 d x dt = 2 m ?gt. Integrating once we get dx dt = ?gt 2 . Then integrating once again, x = 1 3 ?gt 3 = 1 23 8h 3 g ? ? ω ? ? ? ? ? ? . 2. A sphere full of particles is rotating around an axis passing through its center (diameter). The sphere shrank to 1 8 th of its original volume. If it is assumed that during

86 NSOU ? CC - PH - 03 this shrinking the distance of every particle from the axis of rotation shrank by the same ratio, then how the angular velocity changed? In what ratio the rotational kinetic energy changed? Solution : If the changed volume became 1 8 th of the original volume, it means radius has become 1 2 of its original value (V ??r 3). As the moment of inertia is proportional to r 2 , it has reduced to 1 4 th of its original value. In absence of external torque the angular momentum remains, constant, so , it has changed by 2 1 4 4 4 × = times. 3.2.17 Substance ? For the analysis of curvilinear motion, use of polar co-ordinates in place of Cartesian co-ordintes facilitate solution easily. ? The ideas aout angular displacement, angular velocity and angular acceleration have been learned. Angular momentum and its conservtion have also been discussed. ? Angular motion of rigid bodies and the importance of moment of inertia have been discussed. ? Motion in inertial and non-intertial frames of references have been explained. ? How pseudo forces moe into play and their importance in life have beeen shown.

NSOU ? CC - PH - 03 87 3.2.18 Last Questions 1. The angular velocity of a particle is 30º per minute. If the radius of the circular path of the particle be 1m, what is its linear velocity? 2. If both the radial and transverse acceleration of a particle be zero, what will be its path? Find the equation of the path using plane polar co-ordinates. 3. Find out the centrifugal acceleration on a ball of mass 1 kg due to the annual rotation of the earth about the Sun. 4. What is the angle between the vertical line with the rotation vector of the Earth at the equator? A particle has a velocity in the perpendicular direction of the rotation vector of the Earth at the equator. What should be the velocity of the particle so that the coriolis force becomes equal to the weight of the particle? 3.2.19 Answers 1. 30º = 6 π radian. So, angular velocity = 360 π radian/s. Linear velocity v = 1 360 360 π π × = m/s. 2. As per condition there is no acceleration. So, the motion will be uniform velocity along a straight line. Using the expression for radial acceleration we get r 2 d dt θ = h (constant) and a r = −h 2 u 2 2 d u dθ − h 2 u 3 =0, or d 2 d u dθ + u = 0 − < u = a cos (? − ?). This represents a straight line.

88 NSOU ? CC - PH - 03 3. Magnitude of the centrifugal force is m? 2 r. Here the time period is one year, so, ? = 7 2 (3 10) π × radian/s. r = The distance between the sun and the earth = 1.5 × 10 11 m. So, the attractive force on a 1kg ball = 2 7 2 (3 10) ? ? π ? ? × ? ? ? ? × 1.5 × 10 11 N ~ 6 × 10 −3 N. 4. The rotation vector is along North-South in the horinzontal direction. So, the angle is 90º. If the velocity is along the East-West direction, the coriolis force will be along the vertical direction. If

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the magnitude of the coriolis force is equal to the weight of the

particle, then 2mv? = mg, or v = g 2 6800m/s. ω ?

NSOU ? CC - PH - 03 89 Unit−3 ? Gravitation Structure : 3.3.1 Proposal 3.3.2 Newton's Law of Gravitation 3.3.3 Gravitational Field 3.3.4 Calculation of Gravitational Field Intensity 3.3.5 Gravitational Field Due To Uniform Ring 3.3.6 Field Due To A Thin Rod 3.3.7 Field Due To Uniform Disc. 3.3.8 Gravitational Potention 3.3.9 Calculation of Gravitational Potential And Hence Intensity. 3.3.9.1. Due To Ring 3.3.9.2. Potential And Hence Intensity Due To A Spherical Hollow Shell 3.3.10 Gravitational Potential And Hence Intensity Due To Thick Shell. 3.3.11 Gauss's Law 3.3.11.1 Flsux of Gravitational Field 3.3.11.2 Surface Area And Solid Angle 3.3.11.2 Gauss's Law 3.3.11.3 Proof of Gauss's Theory 3.3.12 Poission's Equation 3.3.13 Short Questions

90 NSOU ? CC - PH - 03 3.3.1 Proposal Right from the ancient days man has marvelled about the twinkling stars hanging over the night sky, the moon, the shooting stars–the eclipse all bounded by invisible threads, roaming in the dark bed of sky at night and disappearing at day time, in the fathomless blue–ocean overhead. However these threads are not really there, but, there exists an attraction force between the so called heavenly bodies. This atraction in called 'GRAVITATION'. The subject gravitation was in its rudimentary state until 1559–the brilliant astronomer Tycho Brahe published his observation of planetary motion, establishing the Heleocentric solar system without any telescopic aid. His assistant Johannes Kepler a German astronomer succeded in formulation of three of planetary motion, which goes as Kepler's laws of planetary motion. The first two laws were published in 1609 and the third law in 1619. After about more than half a century Sir Isaac Newton, one of the greatest human mind, justified Kepler's experimental law in generalized mathematical frame of Central Conservative force field; which ultimately led Sir Newton to formulate laws of gravitation – the greatest production of human mind as said by Language– an eminent mathematician and physicist. In 1917 another break through was initiated again by a German scientist Albert Einstein – a great scientific mind, who amalgamated gravitation with space-time curvature property showing gravitation causes a curvature in space-time. So a falling body apparently falling in straight path really follows a curved space-time path. However, we will confine here in Newtons law of gravitation – where space and time are not interrelated. ? Outcome After reading this chapter you will have a definite idea about one of most important forces in the nature, namely gravitation. You will be able ? to understand the Newton's law of gravitation.

NSOU ? CC - PH - 03 91 ? to find solutions of different problems related to gravitation. ? to understand what is meant by gravitational potential, intensity and equipotential surfaces. ? to explain Gauss's law in case of gravitation and how to apply it in different cases. ? to determine expressions for gravitational potential and intensity due to point and spherical objects like solid sphere, spherical shells both thick and thin. ? to find out the effect of coriolis force on a falling body ? to explain escape velocity and how artificial satellites are placed in their orbits. 3.3.2

82%	**MATCHING BLOCK 68/123**	**SA**	Mechanics Properties of Matter-PHY17R121.docx (D109220287)

Newton's Law of Gravitation Statement : Every particle in the universe attracts every other particle with a force which is i) directly proportional to the product of their masses, ii) inversely proportional to the square of the distance

of separation between the particles, iii) acting along the line joining the particles.

If m i and m j are the masses of ith and jth particle at position vectors i r ? and j r ? respectively, then according to the Newton's law of gravitation the force of jth particle due to ith particle is ji F ? = – ? i j ij ij Gm

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m r r 2 .... (3.3.2.1) Where ij j j r r r = - ? ? ? , r ij = j i r r - ? ? ? j i ij j i r r r r

r - = - ?? ?? ? ? Using Newton's third law, the force on ith particle due to jth particle

92 NSOU ? CC - PH - 03 i j ij ji ji ij Gm m ˆ F F r r = - = - ? ? 2 .... (3.3.2.2) So the total force on ith particle due to a cluster of particles j = 1, 2, 3....N will be N i j i ji j ij j i Gm m ˆ F r r = ≠ = - ? ? 2 1 .... (3.3.2.3) The eqn. (3.3.2.3) shows that the principle of superposition is applicable in case of gravitation. G is called universal gravitational constant as its value is spece-time independent i.e. G remains same for all space points at all time and does not depend on intervening medium. It is also independent of temperature, pressure and the presence of other force fields. 3.3.3. Gravitational Field Gravitational force is a distant force, which means gravitational interaction can migrate through space without any material interaction. The questions thus arises : i) How the interaction migrate through space and ii) With what speed this field migrates? To answer to this poser scientists introduced the field conception, here known as gravitational field. This field is a quality developed in space due to the presence of mass and with this quality the material interaction takes place. [The intraction travels with speed of light in free space and the carrier is known as 'graviton' like that photon which migrate electric-and-magnetic interaction]. The field is defined as, a space is said to posses gravitational field if a mass undergoes gravitational interaction when placed in that space.

NSOU ? CC - PH - 03 93 Obivously this definition is qualitative. The quantitative definition goes with a term called gravitational field intensity. The gravitational field intensity at a point is defined as gravitational

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force experienced by a unit mass placed at that point.

It is denoted by E ? . Thus the force on a mass m placed in a gravitation field E ? , F mE = ? ? 3.3.4. Calculation of Gravitational Field Intensity 1. For a point mass M Y Z dm p O X r Figure : 3.3.4.1 The fig. (3.3.4.1) shows a point mass M at the origin of reference frame. To calculate the gravitational field intensity at 'p' due to mass M, we place a test mass dm at p which is so small that it does not put any change to the field pattern produced by M. Then from Newton's law the force on mass dm dF ? = GMdm ˆr

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r - 2 (ˆr is unit vector along r ?) The gravitational field intensity at p

is E ? = dF GM ˆr dm r = - 2 ? .... (3.3.4.1)

94 NSOU ? CC - PH - 03 3.3.5. Gravitational Field Due to Uniform Ring Figure : 3.3.5.1 The fig (3.3.5.1) shows a ring of mass M and radius 'a'. In the mode of calculating the field at p at a distance x from center O of the ring, consider an elemental length 'dl' at point A on the ring (AP = r) The mass of the element = λ dl M a ? ? ? ? ? ? λ= π2 Field due to this element at p G dl dE r λ = 2 ? along PA Resolving dE ? as dE cos θ and dE sin θ as in fig (3.3.5.1), the component dE sin θ yields to a null value when summed over the whole ring. So the net field is E ?? = ? dE cos θ along p O = G dl r λ ? 2 cos θ along p O = − G a r λ π 2 2 cos θ ˆx = − GM r 3 x ˆx (ˆx ≡ unit vector along O p) E ?? = − GMx (a x) + 3 2 2 2 ? .... (3.3.5.1)

NSOU ? CC - PH - 03 95 3.3.6. Field Due to a Thin Rod Figure. 3.3.6.1 The fig. (3.3.6.1) shows a thin rod of mass M. We have to find out the intensity at P at a distance x from a point O on the rod. θ 2 and θ 1 be the angular elevations of the rod from the point P. The gravitational intensity at P due to the elemental length dl as in figure dE ?? = G dl r λ 2 along PQ. Resolving dE ?? as dE cos θ and dE sin θ along and perpendicular to OP, we have the horizontal x component of net field E x = G dl G cos r x θ -θ λ λ θ = + ? 2 1 2 2 θ θ ? 2 1 cos 3 θ dl = + G x λ 2 ? cos 3 θ x sec 2 θ d θ Putting x = x tan θ = + G G cos d x x θ -θ λ λ θ θ = + ? 2 1 (sinθ 2 + sinθ 1)..... (3.3.6.1) Similarly E y = λ G dlsin G r x θ -θ θ λ = ? ? 2 1 2 2 cos 2 θ x sec 2 θ sin θ d θ = G x λ (cosθ 2 − cos θ 1) .... (3.3.6.2)

96 NSOU ? CC - PH - 03 3.3.7. Field Due to Uniform Disc Figure 3.3.7.1 The fig (3.3.7.1) shows a uniform disc of mass M and radius 'a'. To calculate the gravitational field intensity at p, we consider an elemental ring of radius r and thickness dr. σbe the mass per unit area (σ = M/ πa 2). The field intensity at 'p' due to this elemental ring. dE ?? = − G rdr x (r x) π σ + 3 2 2 2 2 ? (ref. eqn. (3.3.5.1) So the magnitude of field dE ?? = G rdr x (r x) π σ + 3 2 2 2 2 So dE = rdr G x (r x) πσ + 3 2 2 2 2 = G cos rdr r πσ θ 3 2 2 Now r = x tan θ , ∴ dr = x sec 2 θ d θ dE= G cos . xtan . xsec d x πσ θ θ θ θ 3 2 2 2 = |2 πGσsin θ d θ |

NSOU ? CC - PH - 03 97 ∴ E = |2πσG φ ? 0 sin θ d θ | = 2πσG (1 − cosφ) E = GM a 2 2 (1 − cosφ) = GM a 2 2 x a x ? ? ? ? - ? ? + ? ? 2 2 1 .... (3.3.7.1) So, E ?? = − GM a 2 2 (1 − cos φ) ˆx = − GM a 2 2 x ˆx a x ? ? ? ? - ? ? + ? ? 2 2 1 ..... (3.3.7.2) ? Worked out Example Show that gravitational field intensity is maximum for a uniform ring of radius 'a' on the axis of the ring lies at a distance x = a 2 from the centre of the ring. The gravitational field on the axis of the ring is given by (Ref. eqn. no. 3.3.5.1) E ?? = − GMx (a x) + 3 2 2 2 ? , for E ?? to be maximum. d dx | E ?? | = d GMx dx (a x) + 3 2 2 2 = 0 or, x . x (a x) (a x) ? ? + - ? ? ? ? + + 5 2 2 2 2 2 1 3 2 2 = 0 or x = a 2 ...... (3.3.7.3) The double differentiation of E ? is found to be negative at x = a 2 , thus E ? is maximum at x = a 2 . ? Worked out Example : Show that if a particle is released from a point on the axis of a fixed ring (x>>a), it will undergo simple harmonic motion. Find the expression of time period of motion.

98 NSOU ? CC - PH - 03 O P M a x m Let M be the mass of the fixed ring and a be its radius. Then the field intensity at point P at a distance x from centre O of the ring GMx E (a x) = - + 3 2 2 2 ?? ? Then the force on the mass m, F ? = m GMx E (a x) = - + 3 2 2 2 ?? ? If x >>a F = − GMm x a 3 ? .... (3.3.7.4) So, F ? x α- ? , the motion is simple harmonic and GM a ω= 3 and T = a GM π 3 2 3.3.8 Gravitational PotentiAL We have already seen in previous discussion that the gravitational field intensity at a point due to a point mass M is given by E ?? = − GM ˆr r 2 So, XE ∇ ? ?? = − GM ∇ X r r ? ? ? ? ? ? 3 ? = 0 .... (3.3.8.1)

NSOU ? CC - PH - 03 99 So, gravitational field is a conservative field. As curl of gradient of a scalar is always zero, we can write V E =-Δ ?? ? (the –ve sign is a logical convention)........ (3.3.8.2) V is called gravitational potential. Now in eqn. (3.3.8.2) E ?? remains same for any additive constant with V. So, it is preferable to set gravitational potential to be zero at infinity, where there is no sense of gravitational field. The formal definition of gravitational potential goes as ; The gravitational potential at a point is the work done to bring a unit mass from infinity to the point quasistatically due to gravitational field. from eqn. (3.3.8.2) E.dr V·dr = -∇ ?? ? ? ? = – V V V ˆ ˆ ˆ i j k x y z ? ? ? ? ? ? ∂ ∂ ∂ + + ∂ ∂ ∂ ·(ˆ i dx + ˆ j dy + ˆ k dz) = – V V V dx dy dz x y z ? ? ? ? ? ? ∂ ∂ ∂ + + ∂ ∂ ∂ or E·dr dv =- ?? ? .... (3.3.8.3) The –ve sign thus indicates that as the system does work its potential energy decreases. 3.3.9 Calculation of Gravitational Potential and Hence Intensity. 3.3.9 1. DUE TO RING O P r a x A dl Figure. 3.3.9.1

100 NSOU ? CC - PH - 03 The fig. (3.3.9.1) shows a ring of mass M and radius 'a'. To calculate the potential at the point P, we consider an elemental length dl at A as in fig. (3.3.9.1). The potential due to this elemental length at P is dV = – G dl r λ (λ = mass/unit length of ring) Then the total potential at P is V = – G dl r λ ?? = – G a GM r r λ π - 2 V = GM (a x) - + 1 2 2 2 .... (3.3.9.1) The intensity at point P E = – V x ∂ ∂ (as V is a function of x only) = GM · x (a x) - ? ? × ? ? ? ? + 3 2 2 2 1 2 2 = GMx (a x) - + 3 2 2 2 Considering the direction. E ?? = – GMx (a x) + 3 2 2 2 .... (3.3.9.2) 3.3.9.2.Potential and Hence Intensity Due to a Spherical Hollow Shell O ? d? a A r x P Figure 3.3.9.2

NSOU ? CC - PH - 03 101 The fig. (3.3.9.2) shows a speherical hollow shell of radius a and mass M. To calculate the potential at point P at a distance x from centre 'O' of the shell, we consider an elemental ring within the angle θ and θ + d θ . Then, The radius of ring = a sin θ Area of the ring = 2 πasin θ ·ad θ Mass of ring = 2 πa 2 sin θ d θ σ where σ = mass per unit area of the shell. The potential at point P due to this elemental ring. dV = – G a sin d r π θ θσ 2 2 .... (3.3.9.1) Now from the triangle OAP r 2 = a 2 + x 2 – 2ax cosθ 2rdr = 2ax sinθdθ sin d dr r ax θ θ ? = Putting this value in above eqn. dV = –G 2 πa 2 σ dr ax dV = –2 πGσ a x dr When the point P is out side the shell x < a V 0 = –2 πGσ a x x a x a dr a G x + - π σ = - ? 2 4 V 0 = – GM x .... (3.3.9.2) When the point p is inside the shell V i = –2 πGσ x a a x + - = ? – 4 πaGσ

102 NSOU ? CC - PH - 03 = a G a π σ - 2 4 = – GM/a............. (3.3.9.3) So the intensity of field outside the shell is E ?? = – V GM GM ˆx x x x x x ∂ ∂ = = - ∂ ∂ 2 Inside the shell as the potential is constant throughout, hence the intensity is zero throught x > a. for x = a the intensity a V GM a – 1 x x V ∝ E ?? = – GM a 2 towards the center O......... (3.3.9.4) 3.3.10. Gravitational Potential and Hence Intensity Due to Thick Shell. Figure : 3.3.10.1 The fig. (3.3.10.1) shows a spherical shell of uniform density P and radius R 1 and R 2 (R 2 < R 1). We have to calculate potential at point P at a distance x from centre O.

NSOU ? CC - PH - 03 103 (a) When P is outside the shell. We consider an elemental speherical shell of radius r and thickness dr. The mass of the elemental shell dm = 4πr 2 dr ρ . dV = – r dr Gdm G x x π ρ = - 2 4 So the total potential V 0 = ? dV = R R G G r dr x x - πρ πρ = - ? 2 1 4 4 2 3 G M . R - 2 3 5 (R R) 3 3 2 1 - GM V x = - 0 .... (3.3.10.1) So the intensity V GM ˆ ˆ E x x x x ∂ = - = - ∂ 0 0 2 ? .... (3.3.10.2) (b) When P is inside the shell : We consider a shell of radius x ; V 1 and V 2 be the potential due to portions inside and outside of the shall. Then the total potential at P, when R 1 > x > R 2 is V i = V 1 + V 2 .... (3.3.10.3) To calculate V 1 we proceed as previous and get V 1 = – G (mass inside the radius x) x = –G (x R) x - πρ 3 3 1 4 3 .... (3.3.10.4) To calculate V 2 we proceed as follows : We consider an elemental shell of radius r (r < x) and thickness dr. Then the potential at P due to this elemental shell. dV 2 = – G r dr r π ρ 2 4 (since it is inside the shell)

104 NSOU ? CC - PH - 03 So V 2 = – G πρ R x rdr ? 2 = – G4 ( ) R x P - π 2 2 2 2 .... (3.3.10.5) So the total potential V i = V 1 + V 2 = –G ( ) x R x - πρ 3 3 1 4 3 – G2 πρ (R x) 2 2 2 - (3.3.10.6) = G πρ 2 3 x 2 – G R x πρ 3 1 4 3 – G 2 πρ R 2 2 So the intensity at the point E i = – i V R G x G r x x ∂ = - πρ + ρ ∂ 3 1 2 4 4 3 3 = – G ( ) i x R GM x x - πρ = - 3 3 1 2 2 4 3 GM i ˆ E x i x ∴ = - 2 ??? .... (3.3.10.7) M i = Core mass. So, the effect on gravitational intensity is due to core mass only, the exterior mass does not contribute to gravitational intensity. ? Worked out Examples : 1. Find the expression of gravitational potential due to a uniform solid disc of mass M and radius 'a' on the axis of the disc. Figure : 3.3.13.1

NSOU ? CC - PH - 03 105 The fig. (3.3.13.1) shows a disc of radius a and mass M. We have to find out the potential at point P at a distance x from the center O of the disc. We consider an elemental ring of radius r and thickness dr. The area of the ring = 2 πrdr Mass of the ring = 2 πrdrσ (σ = mass/unit area of disc) Potential at point P due to this ring dV = – G rdr r π σ ′ 2 (Ref. Eq. 3.3.10.1) = – G cos x πσ θ 2 rdr We put r = x tan θ , so dr = x sec 2 θ d θ ∴ dV = –G2πσx sin d cos θ θ θ 2 V = –G2 πσx sin d cos φ θ θ θ ? 2 0 = – G 2 πσx cos ? ? ? ? ? ? - φ 1 1 = –G2 πσx a x x ? ? + ? ? - ? ? ? ? 2 2 1 = G2πσ a x x ? ? + - ? ? ? ? ? 2 2 = – GM a 2 2 a x x ? ? + - ? ? ? ? ? 2 2 As V is a function of x only so the intensity will be along x ? . V E x ∂ = - ∂ ??? along PO = – GM a 2 2 (1 – cos φ) ˆx.

106 NSOU ? CC - PH - 03 2. Considering Earth to be a perfect sphere of uniform density, of mass M and radius R, find the gravitational potential at a distance x from the centre of Earth (x > R). R x P O M SOLN Please ref : (3.3.10.6) Put R 1 = 0 and R 2 = R V i = V 1 + V 2 = – G x x πρ 3 4 3 – G4 (R x) - πρ 2 2 2 = G πρ 2 3 x 2 – G 2 πρR 2 = – πρ2 3 G (3R 2 – x 2) V i = – GM R 3 2 (3R 2 – x 2) .... (3.3.10.8) So the gravitational field intensity at ρ will be E i = – V GM x x R ∂ = - ∂ 3 ? 3. Find the expression of gravitational potential (V) and hence the gravitational field (E ??) for a thin rod of length L on its axial point at a distance r from centre O of the road. SOLUTION:

NSOU ? CC - PH - 03 107 The fig. shows a rod of length L and mass m. The potential at the axial point P is to be calculated. We consider an elemental length dx at a distance x from center O. The potential due to this element dx at P. dv = – G dx r x λ - where λ = mass/unit length of the rod. So the total potential at. V = –G λ L L dx r x - ? 2 2 = G λ ln(r – x) + - 1 2 1 2 = –G λ r ln r ? ? ? ? ? ? ? ? ? ? + - 1 2 1 2 = – G λ L ln L r ? ? ? ? + ? ? - ? ? ? ? 1 2 E = V r - ∂ ∂ 3.3.11 Gauss's Law 3.3.11.1 Flux of gravitational field In science, flux, usually concerns to some flow of physical quantity or property. In case of 'fluid flux' it refers to the amount of fluid crossing a specific area per unit time. However, in case of gravitational flux no such transport physically, exist. Gravitational flux through an ele ment of area ds ? placed at a point where the gravitational field intensity is E ?? , is defined as, dφ = E ?? · ds ? . The total flux through a finite surface S is φ = s E ds ? ??? ? ? (where the integration is carried over the whole surface). 3.3.11.2 Surface area and solid angle Surface area is treated as a vector whose magnitude is the area of the surface

108 NSOU ? CC - PH - 03 considered and direction is specified as follows. i) For closed surface the direction is +ve in outwards normal to the surface. ii) In case of open surface the direction is specified by right hand screw rule as illustrated in the adjoining figure. Fig. 3.3.11.1a Fig. 3.3.11.1b The solid angle is three dimensional analogues to that of an angle in two dimension. Now an angle can be visualized physically as a two dimensional peeping from a point. Mathematically

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it is defined as the ratio of the arc by radius of a circle. Figure : 3.3.11.2 In fig. the angle subtended

arcAB dl d r r θ= = , which is a dimensionless quantity. Its units taken as radian which is defined as angle subtended by an arc of an circle of unit length and unit radius at the center of the circle, so, the total angle about a point is r r π = π 2 2 2 2 radian. Similarly, a solid angle can be visualized as a three dimensional peeping through a point and it is mathematically defined as three dimensional angle produced at the center of a sphere due to an area boundary on the surface of the sphere. If ds be the elemental area in the surace of a sphere then, the solid angle suspended at the centre O is,

NSOU ? CC - PH - 03 109 Fig. 3.3.11.3 d ds r Ω= 2 .... (3.3.11.1) If the area plane makes an angle θ with the tangent to the sphere at that point then solid angle d ˆ ˆ dscos ds.r dsn.r r r θ Ω = = = 2 2 2 ? ? ...... (3.3.11.2) Fig. 3.3.11.4 Where ˆn and ˆr are unit vectors along d s ? and r ? . as explained in the figure. Unit of solid angle is named

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steradian, one steradian is the solid angle subtended at the centre of a sphere

of unit radius by a unit surface on the sphere. Fig. 3.3.11.5 n 2 ˆ n ˆ r ˆ r ?

110 NSOU ? CC - PH - 03 Obviously the total solid angle about a point will be ˆ ˆ dsn.r dscos r r r θ π Ω = = = π ? ? ? ? 2 2 2 2 4 4 steradian........ (3.3.11.3) When O is outside please refer to the fig. Due to the orientation of ˆn 1 and ˆn 2 of the surface area vectors the total solid angle subtended at O will obviously be zero, because they will cast solid angles of same magnitude but in opposite sense and will cancel each other to make the yield null. Fig. 3.3.11.6 3.3.11.2 Gauss's Law The law states that 'the total gravitational flux through a closed surace is – 4 πG times the mass enclosed by the surface. mathematically it is represented as, E·ds closed surface's E·ds G = = - π ? ? ??? ??? ? ? ? ? 4 (total mass enclosed in S) = – 4 πG N i i m = ? 1 = –4 πG ? dm = – 4 πG ? pdv (3.3.11.4) where ρ is the density at the point), for a continuous distribution of mass. n 1 ˆ n 2 ˆ

NSOU ? CC - PH - 03 111 3.3.11.3 Proof of Gauss's theoem Figure : 3.3.11.7 The fig. (3.3.11.7) shows a point mass 'm' enclosed within a closed surace S. We consider an elemental area d s ? at point P at a position vector r ? from mass point m at O. E ?? be the gravitational field intensity at 'P', then flux through the elemental surface ds ? , dφ = E·ds ?? ? = − GM ˆr·ds r 2 ? = − Gm dscos r θ 2 = − GmdΩ (dΩ is the solid angle. So the total flux through the formed of O by element ds). φ = − Gmd ? Ω = − 4 πGm .... (3.3.11.3.1) The eqn. (3.3.11.3.1) holds for any mass point independent of position, so if there are N number of particles of masses m 1 , m 2 ....m i ....m N then the eqn. (3.3.11.3.1) takes the form φ= − 4 πG ? m i .... (3.3.11.3.2) Thus for a continuous distribution of mass we can write v dm G pdv G = - π φ = - π ? ? 4 4 .... (3.3.11.3.3) m P

112 NSOU ? CC - PH - 03 Where ρ is the density of the medium as the point concerned, here it is the point ρ. The integration is to be carried over the entire volume enclosed by the surface S. Thus the integral form of Gauss's law can be written as S V E·ds G dv = - π ρ ? ? 4 ?? ? .... (3.3.11.3.4) 3.3.12 Poission's Equation Now using Gauss's divergence theorem, for the integral form of Gauss's theorem, we can write v ·E·dV ∇ ? ?? ? = − 4 πG v dvρ ?? v (·E G) ∇ + π ρ ? 4 ?? ? dv = 0 or, ·E∇ ?? = −4 πG ρ .... (3.3.12.1) which is the differential form of Gauss's law, applied to gravitational field. We have already deduced form the conservative nature of gravitational field that E V = -∇ ? (Ref. eqn. (3.3.8.2) using eqn ( ) in diferential form of Gauss's law we have ∇ ·(− ∇ V) = − 4 πG ρ or, ∇ 2 V = 4 πG ρ .... (3.3.12.2) The above equation is known as Poission's equation applied to gravitational field. In cartesian co-ordinates it takes the form, V V V G x y Z ∂ ∂ ∂ + + = π ρ ∂ ∂ ∂ 2 2 2 2 2 2 4 In free space, where the space does not contain matter, Poission's equation yield to

NSOU ? CC - PH - 03 113 V , ∇ = 2 0 .... (3.3.12.3) which is called Laplace's equation. ? Worked out Examples : 1. Find the expression of potential due to a hollow hemisphere of mass M and radius R at the centre of its base. The fig. (Ex. 1) shows a hemisphere of radius R and mass M. We consider an elemental ring on the surface of hemisphere within the angle θ and θ + d θ as in fig. Figure : (Ex. 1) Then the radius of ring = R sin θ Area of the ring = 2 πR sin θ Rd θ Mass of the ring = 2 πR 2 sin θ d θ σ where σ = mass/unit area. Potential due to the ring at the centre P, dV = − G R sin d R π θ θσ 2 2 Total potential at PV = dV ? = −G 2 πσR (cos θ) o π 2 = − G2 πσR = − G R R π σ 2 2

114 NSOU ? CC - PH - 03 V = − GM R 2. Calculate the gravitational potential at the centre of base due to a solid hemisphere of radius R and Mass M. R r P Figure : Ex. 2 The fig (Ex. 2) shows a solid hemisphere of mass M and radius R. We have to find out the potential at point 'P'. We consider a thin hemispherical shell of radius r and thickness dr. Mass of the elemental shell dm = 2 πr 2 dr ρ (ρ = density of the material of hemisphere) Potential at P due to this shell, dV = − G dm r = − G 2 πr 2 dr r ρ = − G2 π ρ rdr So the total potential V = − G2 π ρ R rdr ? 0 = − G πρ R 2 = − GM R 3 2

NSOU ? CC - PH - 03 115 Exercise 1. Two particles of masses m 1 and m 2 start falling to each other due to their mutual interaction. Initially they are at large separation and were at rest. Calculate their relative velocity when they reach the separation D. For any Separation r, force of interaction Gm m ˆ F r r = 1 2 1 2 ? So acceleration of m 1 , Gm ˆ a r r = 2 1 2 ? Similarly, acceleration of m 2 , Gm ˆ a r r = - 1 2 2 ? Relative acceleration of m 1 w.r. to a 2 ? ( ) G(m m) ˆ a a r r + - = - 1 2 2 1 2 ? ? rel dv G(m m) dt r + = - 1 2 2 v rel rel dv G(m m) dr r + = - 1 2 2 rel v G(m m) r + = + 2 1 2 2 + C As r → •, v rel = 0, ∴ C = 0 v rel = G(m m) r + 1 2 2

116 NSOU ? CC - PH - 03 2. Find the experession of gravitational field intensity at the base centre of a hollow hemispherical shell R A P ? d? Figure : Ex. 3 The fig. (3) shows a hemispherical shell of radius R and mass M. We consider an elemental ring within the angle θ and θ + d θ . The gravitational field intensity at ρ due to this elemental ring dE ?? = Gdm cos along PA. R 2 = G Rsin Rd cos R π θ θ θσ 2 2 E ?? = G2 πσ sin cos d π θ θ θ ? 2 0 along pA = G πσ o cos sin d G π π θ θ θ=- πσ ? 2 2 0 2 2 2 = − Gπσ 2 [− 1 − 1] along pA = Gπσ along pA = G· R R π σ 2 2 2 2 along pA or , GM along pA R **E** = 2 2 ?

NSOU ? CC - PH - 03 117 3. Find the expression of gravitational field intensity due to a solid hemisphere at its base centre. HINTS/SOLN The fig. (4) shows a hemisphere of radius R and mass M. To calculate the field intensity at P, we consider an elemental disc within the angular elevation d θ at θ . P is the density of solid in hemisphere. Mass of the disc = r (R sinθ) 2 Rdθ SinθP = πR ρ

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sin 3 θ d θ dE = G R sin d R sin π ρ θ θ θ 3 3 2 2 2 (1 − cos θ) = 2G πR ρ sin θ (1 − cos θ) d θ = 3 GM R 2 sin θ (1 − cos θ) d θ ∴

E = GM sin sin R π θ ? ? θ- ? ? ? ? ? 2 2 0 3 2 2 d θ = GM GM R R ? ? - = ? ? ? ? 2 2 3 1 3 1 2 2 4. Show that field intensity inside a solid sphere remain same if the density of the material of the solid varies inversely as the distance from its origin. Figure : Ex. 5
118 NSOU ? CC - PH - 03 Hints : The fig. (5) shows a sphere of mass M and radius R. Consider an elemental shell of radius r and thickness dr. ρ be the density of the shell. By the problem K r ρ = . Field at a point P at a distance r (r > R) E p = – i GM r 2 (M i = Mass inside radius r) = – G r r dr r π ρ ? 2 0 2 4 = – G r K r dr r r π ? 2 0 2 4 = – G Kr r π 2 2 2 = – G2 πK = constant at all points inside the sphere. ? Worked out examples 1.The masses of the planet A and planet B have a ratio 2 : 3, while the ratio of their radii is 1 : 2. The weight of an object on planet A is found to be w, what is the weight of the object on the planet B? Weight of any mass m on any planet (say A) = A A M r Gm 2 , where G is the universal gravitational constant, M A

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is the mass of the planet and r A is the radius of the		

planet. Let the weight of the same mass on the planet B be w ' , then w ' = B B M r Gm 2 . So, B A A B r w M . w r M ' = × = × = 2 2 3 1 3 2 4 8 Therefore, w ' = w 3 8 2.An object from a spaceship is dropped from rest on a planet A, and travels a distance 22.5 meters in 5 seconds. The radius of planet A is 5.82 × 10 6 meters.
NSOU ? CC - PH - 03 119 a) Find out the acceleration of the falling object. b) Find also the mass of planet A. From laws of motion, distance travelled s = 1 2 at 2 . So, a = s t 2 2 = 1.8s Again, a = p p GM r 2 . So, M p = p ar G 2 = 1.8 × (5.82) 2 × (.) - × 12 11 10 6 674 10 = 9.24 × 10 23 kg. 3.Infinite number of masses each of 1 kg are placed along a straight line at the distances of 1m, 2m, 4m, 8m,.....from a point O on the same line. If G is the universal gravitational constant, then what would be the gravitational field intensity at O? From the law of gravitation, I = GM ... ? ? ? ? ? ? = + + + + 1 1 1 1 4 16 64 GM ... ? ? ? ? ? ? = + + + + 1 1 1 1 4 16 64 GM GM ( ) ? ? ? ? = = ? ? - ? ? ? ? 1 4 1 4 3 1 4.Two spheres each of mass M are suspended by two strings each of length L. The distance between the upper ends of strings is also L. Find the angle which the strings will make with the vertical due to mutual attraction. From the figure below, it is clear that for equilibrium if T is the tension in the string, then equating the forces on the second sphere we get T sin θ = GM L = 2 2 and T cos θ = Mg. Therefore, tan θ GM . gL = 2
120 NSOU ? CC - PH - 03 ? Substance : In this unit you have learnt about universal gravitational force operating between objects which is attractive in nature. The observations made by Johaness Kepler on the movement of the planets which led to Kepler's laws of planetary motion were finally explained by Newton when he propounded his law of gravitation. In the analysis based on the law of gravitation, ideas about gravitational potential and intensity and the calculation of the aforementioned quantities were discussed in details. Acceleration due to gravity and its implication on escape velocities of objects were analysed. Gauss's law and its importance in the discussion of gravitational potential and intensity were highlighted.. 3.3.13 Short Questions. 1. What sould be the duration of the year if the distance between the earth and the sun gets doubled? 2. The value of 'g' at a particular point is 9.8 m/sec 2 . Suppose the earth suddenly shrink uniformly to half its present size without losing any mass. What will be the value of 'g at the same point (assuming that the distance of the point from the center of the earth does not shrink)? 3. If the value of gravitational acceleration at a height h above the Earth's surface is the same as that at a depth d below the Earth's surface (with both h and d small
NSOU ? CC - PH - 03 121 compared to the Earth's radius R), then what is the relation between h and d? 4. The weight of a body is not a fixed quantity. It depends upon the location as well as upon? 5. Find out the gravitational intensity of an infinite plane of uniform mass distribution. Answers : 1. T 2 • R 3 , so time period of revolution of the earth will be 8 times the original time. 2. GM g . R = 2 So, if mass and the distance do not change, there will be no change in 'g'. 3. g h = g o R (R h)+ 2 2 d (R d) andg g , R - = 0 So, if g h = g d , then R (R h)+ 2 2 = R d R - . 4. The motion of the frame of reference. 5. Let us consider a Gaussian pill-box of height r above and below the plane and ds is the corss-sectional area. The intensity E = E ˆ r and ds = ds ˆ r. Applying Gauss' theorem on the pill-box, we get .E ???????? ds = 2E ds = – 4 πG (ds σ). Or, E = –2πGσ ˆ r, where σ is the surface density of mass. Numerical Problems : 1.Three identical balls, each of mass

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M are placed at the vertices of an equilateral triangle of side			

L. What should be the speed of their movement if they move under the action of one another's gravitational pull in a circular orbit circumscribing the triangle while maintaining the shape of equilateral triangle? 122 NSOU ? CC - PH - 03 2.How does the acceleration due to gravity vary with latitude? 3.Establish the

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Kepler's laws of planetary motion from Newton's law of gravitation. 4.			

Three equal masses are placed at the vertices of a Cartesian right angled triangle in x-y plane. The co-ordinates of the vertices are (L, 0), (–L, 0) and (0, 2L). Calculate the gravitational potential and intensity at (0, L). 5.It is found that the acceleration due to gravity decreases with the depth inside a homogeneous solid sphere, but the acceleration due to gravity inside a mine is more than that at the surface of the earth, why? 6.Using Gauss's theorem find out the value of the gravitational intensity due to an infinitely long cylindrical wire. 7.Find out the gravitational self-energy of a homogeneous sphere. Answers. 1.The resultant force on any one of the masses will be directed towards O, the circumcentre of the triangle ABC. The centripetal force is L L L R R R O = 2G m L 2 2 ? cos 30 0 = G m L 2 2 ? 3 . The equation of motion of any one of the masses is G m L 2 2 ? 3 = m v R 2 NSOU ? CC - PH - 03 123 where R can be obtained from cos 30 0 = L/2R, or R = L/ 3 . Now, we get v 2 = GM/L, so v = GM L ? ? ? ? ? ? 2.Let an object of mass m is placed at a place on the surface of the earth where latitude is λ . With the rotation of the earth, the object travels in a circle of radius r (different from the radius R of the earth with an angular velocity ωsame as that of the earth. Then, mg λ = mg 0 – m ω 2 r cos λ . or, gλ = g o –ω 2 R cos 2 λ, because r = R cosλ. 3.See text 4.Let the masses be equal to M. Gravitational potential ( ) GM GM V GM L L L + = - - - =- 1 2 2 GM GM InensityE x L L = - × + × = 1 1 0 2 2 2 2 2 2 2 E y = GM L 2 2 (1 – 1 2). 5.It can be shown that the acceleration due to gravity at a depth d is g d = rGρ 4 3 (R – d). So, we see that g should decrease with the depth. But for the Earth density is not uniform, it increases inside the earth. That is why the acceleration due to gravity increases inside the earth. 6.We assume a co-axial cylindrical surface (Gussian surface) of radius r and height h around the wire. Gravitational intensity f at any point on the curved surface of the outer cylinder will be same everywhere and will be directed inward. The 124 NSOU ? CC - PH - 03 assumed Gaussian surface has three parts – i) top, ii) curved side and iii) bottom. The normal to these surfaces are a) vertically upward, b) radially outward and c) vertically downward. Therefore, ?? ? f.ds= f.ds curvedsurface , as there is no contribution from the top and bottom surface as f and ds are perpendicular to each other. So, we get ? ? f.ds = f 2 πrh. Now, applying Gauss's theorem f 2 πrh = –4 πG (hm), where m is the mass per unit length of the wire, Therefore, f = – GM GM , or f r ˆ r r =- 2 2 7.Let us consider a homogeneous sphere of mass M, radius R and density ρ . By self energy of this sphere we mean the work done by the attractive forces of the particles constituting the sphere, when they are brought from an infinite distance to the particular position in the system. Suppose at some point in the formation of the sphere, a small sphere of radius r has already formed. The potential V r on its surface is G GM r ? ? - =- ? ? ? ? 1 4 3 πρ r 2 . The amount of work to be done in bringing in a further mass dm from infinity on the surface of the existing sphere dW = V r dm. Let us assume this dm forms a thin shell of thickness dr on the surface of our old sphere of radius r. Then dm = 4 πρ r 2 dr. Then, dW = V r dm = G ? ? - ? ? ? ? 4 3 πρ r 2 4 πρ r 2 dr = - 1 3 G(4 πρ) 2 r 4 dr Thereoe, total work in forming a sphere of radius R is W = r R R r Vdm G( ) r = = ? ? = - πρ ? ? ? ? ? ? 2 4 0 0 1 4 3 dr = GM . R - 2 3 5 .

NSOU ? CC - PH - 03 125 Unit-4 ? Central Force Motion Structure : 3.4.1 Proposal 3.4.1 Central Force 3.4.2 Some importnt properties of Central Force 3.4.3 Reuction two body problem to One body problem : 3.4.4 Expression of Velocity & Acceleation in Potar Co-ordinate (A recapitulation) 3.4.5 Equation of Motion in Central Force 3.4.6 The energy of a particle in Central Force 3.4.7 The equation of Orbit 3.4.8 The Kepler's Laws 3.4.9 Artificial Satellite 3.4.10 Escape Velocity 3.4.11 Weightlessness in satellite. 3.4.12 Numerical Problems 3.4.13 Short Questions : 3.4.14 Solution : 3.4.1 Proposal From his understanding of the theory of motion Sir Issac Newton felt that the Sun occupies the central role for governing the motion of planets. He proved the very fact that constancy of the areal velocity is a direct consequence of the idea that all the forces are directed exctly toward the Sun or in other words the force behind the motion of
126 NSOU ? CC - PH - 03 the planets is central in character. By analysing Kepler's third law it is possible to prove that the strength of the force weakens with distance – larger the distance, weaker the force becomes. By combining these two laws Newton came to a conclusion that there must be a force inversely proportional to the square of the distance, directed in a line between two objects. Now we all know that the above situation culminated into proclamation of Universal Law of Gravitation which is basically an example of central force problem. Later we came across another famous example of central force problem – Coulomb's law, characterising the force operating between two differently charged bodies – different in amount and different in amount and different in nature. These phenomena drew attention of the physicists working all over the world about the features of particles moving under a central force. In the discussion on Central Force problem, we will examine a mathematically tractable and physically useful problem – that of two bodies interacting wih each other through a force that has two characteristics: (a) it depends only on the separation between the two bodies, and (b) it points along the line connecting the two bodies. Such a force is called a central force. ? Outcome By learing this chapter, you shall be able to ? Understand the nature of central forces. ? Understand the physics behind the motion of particles moving under the action of a central force – like potential, intensity of the force field. ? Derive the equation of the orbit of a particle moving in a central force field. ? Explain the conservation principles relevant to the central force field. ? Derive Kepler's laws from Newton's law of gravitation and vice-versa.
NSOU ? CC - PH - 03 127 3.4.1 Central Force Central Forc is that force which is always acting along some space point and whose magnitude is a function of distance from the space point. Figure : (3.4.1) The fig. (3.4.1) shows a particle of mass m undergoing motion under a central force abour the point 'O' called Center of force, chosen here to be at the origin of reference frame. P be the position of particle at time t. Then following the definition of central force, the force equation of the particle can be written as, ( ) ˆ F f r r = ? .... (3.4.1) If f(r) is positive the force on the particle is repulsive and it is attractive when f(r) is negative. For example in case of gravitation ( ) 2 GMm f r r = - represents an attractive force. For electrostatic force ( ) 1 2 2 q q f r k r = can be both attractive and repulsive depending on the sign of charges. 3.4.2 Some importnt properties of Central Force 1. Central force is conservative To show this we workout F ∇× ? ? . Now F ∇× ? ?
128 NSOU ? CC - PH - 03 = ( ) ( ) ( ) f

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r ˆf(r)r r G r r r ? ? ∇× = ∇× = ∇×? ? ? ? ? ? ? ? ? ? ? ? ? = ( ) ( ) G r r G r r. ∇× + ∇ × ? ? ? ? ( ) ˆ ˆ ˆ

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x y z x y z x y z ? ? ∂ ∂ ∂ ∂ ∂ ∂ ∇× = + + + × ? ? ∂ ∂ ∂ ∂ ∂ ∂ ? ? ? ? ? = G

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r r r ˆ ˆ ˆ 0 i j k r r x y z ? ? ∂ ∂ ∂ + + + × ? ? ∂ ∂ ∂ ? ? ? = G x y z ˆ ˆ ˆ i j k r r r r r ∂ ? ? + + × ? ? ∂ ? ? ? ∴ ( ) 1

G F r r 0 r r ∂ ∇× = × = ∂ ? ? ? ? ....(3.4.2)
Thus central orce is curlfree, hence conservative. 2. Angular momentum in Central Force is conserved Let a particle of mass is undergoing C.F. motion, then the force equation ( ) F f r r = ? ? r F 0, × = ? ? so, dv r m 0 dt × = ? ? or, ( ) d r mv 0 dt × = ? ?

NSOU ? CC - PH - 03 129 since dr mv v mv 0. dt × = × = ? ? ? ? So, r mv L × = ? ? ? ...... is a constant vector. (3.4.3) The equation (3.4.3) shows that angular momentum remains conserved in C.F. motion. 3. The motion is planar. From equation (3.4.3), L r v h m × = = ? ? ? ? (say) So, ( ) r. r v 0 × = ? ? ? , which implies that the motion is always confined in a plane perpendicular to the direction of r v × ? ? . 4. Areal Velocity is constant in C.F. motion O r + dr r Y ? ? ?d d r Figure : (3.4.2) We have already deduced that in C.F. motion r v h × = ? ? ? Or, dr r h dt × = ? ? ? 1 dr h r 2 dt 2 × = ? ? ? . 1 r dr 2 × ? ? represents vectorial area of the shaded region.

130 NSOU ? CC - PH - 03 or, dA h dt 2 = ? ? = constant vector. .... (3.4.4) which shows that area velocity i.e. area swept out by radius vector is a constant of motion, which is the Kepler's second law in mathematical form. 3.4.3 Reduction of two body problem to One body problem R m 1 m 2 r 1 r 2 Figure : 3.4.3 We consider two particle of masses m 1 and m 2 at position vectors 1 r ? & 2 r ? as in Fig. Then force equation of masses are, 1 1 12 1e m r F F = + ? ? ? ? ?? and 2 2 21 2e m r F F = + ? ? ? ? ?? where F 12 ? and 21 F ? are the force of interaction between the masses and 1e 2e F ,F ? ? are external forces acting on masses m 1 and m 2 . So acceleration of m 2 with respect to m 1 ( ) 2e 1e 21 12 2 1 2 2 1 1 F F F F r r m m m m - = + - ? ? ? ? ? ? ? ?? ?? = 2e 1e 21 1 2 2 1 F F 1 1 F m m m m ? ? ? ? + + - ? ? ? ? ? ? ? ? ? ? ? . New if the external force is proportional to the mass then the last term within braces vanishes. Thus we have 1 2 21 1 2 m m R R F m m μ = = + ? ?? ?? = force on particle 2 due to particle 1. m m m m 1 2 1 2 μ = + is called the reduced mass.

NSOU ? CC - PH - 03 131 3.4.4 Expression of Velocity & Acceleation in Polar Co-ordinate (A recapitulation) O X r Y P(x, y) ? T Figure : (3.4.4) The Fig. (3.4.4) show a particle p undergoing a planar motion (Remember C.F. motion is planar) in x, y plane. r ? be the position vector at any instant t. Then, ( ) ˆ ˆ ˆ r r icos jsin rr = θ+ θ = ? .... (3.4.5) (r ? i the unit vector along r ?). So, the velocity at that instant, ˆ ˆ v r rr r = = + ? ? ? ? ? .... (3.4.6) Since ( ) ˆ ˆ ˆ ˆ ˆ r isin j cos icos j sin 2 2 ? ? π π ? ? ? ? = - θ+ θ θ = + θ + + θ θ ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = ˆ Tθ ? (where T ˆ is a unit vector perpendicular to ˆ r) .... (3.4.7) So, r T ˆ ˆ r rr r T v v = + θ = + ? ? ? ? ? ? [velocity has both radial and transver components] (3.4.8) ˆ ˆ ˆ ˆ ˆ r rr rr r T r T r T = + + θ + θ + θ ? ? ? ? ?? ? ?? ? ?? ? ? Now, ˆ ˆ ˆ T isin jcos , = - θ+ θ so, ( ) ˆ ˆ ˆ ˆ T isin jsin r = - θ+ θ θ- = -θ ? ? ? Using the values of ˆ T and ˆ r ? , we have,

132 NSOU ? CC - PH - 03 ( ) ( ) 2 ˆ ˆ r r r r 2r r T = - θ + θ+ θ ? ?? ? ? ? ?? ?? ? .... (3.4.9) = r T ˆ ˆ a r a T + (a r ??radial component of acceleration ; a T ??transverse component of acceleration). 3.4.5 Equation of Motion in Central Force In central field ( ) ˆ F f r r,i.e. = ? force is always along r ? . So from equation (3.4.9) we can write. ( ) ( ) 2 m r r f r - θ = ? ?? ? .... (3.4.10) 2mr mr 0 θ+ θ = ? ?? ? = ... (3.4.11) Equations (3.4.10) and (3.4.11) represent the eqn. of a particle undergoing C.F. motion in polar co-ordinate. Now, 2mr mr 0 θ+ θ = ? ?? ? implies 2 2mrr mr 0 θ+ θ = ? ?? ? Or, ( ) 2 d mr 0 dt θ = ? So, 2 mr L θ = = ? constant. .... (3.4.12) The equation 8 is the scalar equivalent of th conservation of angular momentum and L stands for angular momentum. 3.4.6 The energy of a particle in Central Force Since the field is conservative the total energy 2 2 2 2 1 1 1 E mv mr mr 2 2 2 = +φ = + θ +φ ? ?

NSOU ? CC - PH - 03 133 = 2 e 1 mr 2 +φ ? where 2 2 e 1 mr 2 φ = θ +φ ? = 2 2 2 1 L mr , 2 2mr ? ? + + φ ? ? ? ? ? ? ? ? 2 e 2 L 2mr φ = + φ 2 e 1 E mr 2 = +φ ? .... (3.4.13) Now if we confine to the inverse square field only, then 2 K K As F , r r ? ? φ = = ? ? ? ? If K is + ve the force is repulsive and – ve if the force is attractive. For positive K, K < 1 the total energy is positive. E T = E + ? e = + v e constant. ? e is zero at infinity and gradually increases as r decreases. E T E = 0 ? e r 0 ? e Figure : (3.4.5) Here in the plot the bold line represents the total energy and the dotted line shows the variation of ? e . The graph shows the particle can come upto r 0 because if r > r 0 , ? e will be greater than E T , which will violate the energy conservation. The orbit is unbounded and r 0 is the turning point, for K = 0, ? e and E is + Ve. So here also the orbit is unbounded and the sketch of ? e and E is similarto that of k < 0`

134 NSOU ? CC - PH - 03 When K > 0, we can consider three cases of total energy. E +ve 1 F = O ? e r 0 K = O E (–ve) 2 E (–ve) 3 r 2 –K/r r 1 Figure : 3.4.5 for E 1 + Ve, there is a turning point at r = r 0 and the orbit is unbounded. When E is – Ve, say E 2 , then there are two turning points so the orbit is bounded between r 1 and r 2 . When E = E 3 , then also the orbit will be bounded and circular and of radius 2 L r . mK = 3.4.7 The equation of Orbit To find the equation of orbit we have to find the relation between r and ? in polar co-ordinates. In the process of findng the relation we put, 1 r , u = So, 2 2 2 2 1 du 1 du d 1 du L r · · dt d dt d u u u mr - θ - = - = = θ θ ? (using equation 3.4.8) i.e. L du r , m d = - θ ? L is the angular momentum of the orbital particle. So, 2 2 2 2 2 2 2 L d du L d u L L u d u r · m dt d m m d r d - - - ? ? = = = ? ? θ ? ? θ θ ??

NSOU ? CC - PH - 03 135 Using the above equation in equation (3.4.6) we have 2 2 2 2 4 2 2 2 L u d u 1 L u 1 m f u u m d m ? ? - ? ? - = - ? ? ? ? ? ? ? ? θ ? ? 2 2 2 2 L u d u 1 u f m u d ? ? ? ? + = - ? ? ? ? ? ? ? ? θ ? ? or, 2 2 2 2 d u m 1 u f u d L u ? ? + = - ? ? ? ? θ ...... (3.4.14) This equation is known as differential equation of orbit. Now if we confine to the inverse square attractive field only, then F can be written 2 2 K as F Ku r = - =- (In case of gravitation K = GMm) So, 2 2 2 d u mK d L + + = θ i.e. 2 2 2 d u mK u d L ? ? = - - ? ? θ ? ? , we put 2 mK U u L - = So, the equation yields to the form, 2 2 d U U d = - θ ....(3.4.15) which is a equation of simple harmonic function of U. So the soln. can be written as ( ) 0 2 mK U u acos L = - = θ-θ (a is a constant)

136 NSOU ? CC - PH - 03 or, ( ) 0 2 1 mK acos r L = + θ- θ . By a suitable choice of co-ordinates we can make ? 0 = 0. So, the equation boils down to 2 2 d u 0 . d = φ where is a constant. So, 2 1 r mK acos L = + θ = 2 2 L mk aL 1 cos mK + θ r 1 cos l = +∈ θ .... (3.4.16) Where 2 L mk l = and 2 L . mK ∈= .... (3.4.17) The equation (3.4.16) is a general equation of a conic section whose shape obvioully depends on ??called the eccentricity. When (1) ? = 0, the path is a circle, as r = l = constant. (2) ??= 1, the path is a parabola, as r ???, as ? ??? (3) ??< 1, the path is a hyperbola as r ????for a value of ? > ?. (4) ??> 1, r remains finite form O > ? > 2?, so the path is in general an ellipse. Now, let us confine to the study of elliptic orbit and the factors which determine the shape of ellipse.

NSOU ? CC - PH - 03 137 a b A B v 2 v 1 r r sin ? p(r,)? (0,0) (S,0) ? v Figure : (3.4.7) The figure shows a particle of mass m undergoing an elliptic path in a force field acting along pS. The force is a inverse square force. So, this study gives us the study of planetary motion. Now in Fig. (3.4.7) A and B are called two apse points, where the velocity is perpendicular to the radius vector. Now ( ) 1 r SB 1 l = = +∈ and SA , (1-e) l = so the major axis AB = SB + SA = 2 2 (l e) l - = 2a (say) So the semi-major axis 2 a ( ) l = -ε ? ...... (3.4.18) Now the semi minor axis OB = b, corresponds to the maximum value of r sin?. So, in this case, ( ) d d rsin ·sin 0 d d 1 cos l ? ? θ = θ = ? ? θ θ +∈ θ ? ? ? ( ) ( ) 2 cos sin sin 0 1 cos 1 cos l l θ θ - -∈ θ = +∈ θ +∈ θ Or, l cos??+ l ?cos 2 ? + l ??sin 2 ? = 0

138 NSOU ? CC - PH - 03 Or, cos ? = – ???Putting this value of cos?. we have, ( ) 2 2 2 b rsin 1 1 1 l l = θ = -∈ = -∈ -∈ ....(3.4.19) Comparing this with equation (3.4.18) We have 2 a b 1 = -∈ ...... (3.4.20) Since it is a conservative field the total energy will remain conserved. To calculate the total energy, we focus on the Apse point B; where r 0 = ? , The total energy, 4 2 2 2 2 1 1 2 1 1 1 r 1 K 1 K E mr m 2 r 2m r r θ = θ - = = - ? ? = ( ) ( ) ( ) ( ) 2 2 2 2 1 L K 1 K K 1 1 · 1 1 2m 2 l l l l l +∈ - +∈ = +∈ - + = ( ) ( ) ( ) 2 1 K K 1 1 2 1 2 2 l l +∈ ? +∈ - ? = ∈ - ? ? ?? ( ) 2 K E 1 , 2l = ∈ - as ???1, So E is negative 2 2 2 E 2L E 1 1 K mK l ∈= + = + ... (3.4.21) 3.4.8 The Kepler's Laws We have alredy seen that when E is negative and ??> 1, then the motion is elliptical, which obvioully states the Kepler's first Law. Planets move around its own Sun in elliptic orbits with Sun at one of its foci. Again we have already proved that

NSOU ? CC - PH - 03 139 ( ) 1 dA r v 2 dt × = ? ? ? = areal velocity is constant for a planet, which proves the Kepler's second law. The areal velocity of a planet is a constant of motion. Now the period of rotation of a planet. areaof theellipticpath T arealvelocity = = ab h 2 π {eqn. .... (3.4.4)} = 2 ab 2 mab 2 ma al h L L 2 π π π = = = 3 3 2 2 l 2 ma 2 ma L Km π π = ?? 2 3 4 m T a K π = ? 2 3 T a ∝ .... (3.4.22) Which is Kepler's 3rd law, which states that,

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the square of the time period of the planet is proportional to the cube of the semi-major axis. 3.4.9

Artificial Satellite A satellite launched from the Earth to orbit around the earth in specific orbit is known as artificial satellite. M R R + h m

140 NSOU ? CC - PH - 03 We consider a satellite of mass m revolving around the Earth's center. Earth is taken to be a huge mass M compared to the satellite of mass m. (Otherwise m should be replaced by m 1 M ? ? + ? ? ? ? . h be the height of satellite then, considering a circular orbit we can write, the centripetal force. ( ) ( ) ( ) 2 2 s c s 2 mv GMm ma m R h R h R h = = = ω + + + Where V s and W s represent the linear speed and angular speed of satellite. So 2 s GM gR v R h R h = = + + (g = acceleration due to gravity on Earth's surface) ( ) 2 s 3 gR R h ω = + ... (3.4.23) from the above equations it is obvious that v s , ω s and thus obviously the period of satellite is independent of its mass. A satellite which remains at a fixed point from the Earth's surface is called geostationary satellite. It is obvious that it will be on an equitorial satellite with same angular velocity as that of Earth. If H be the height of a geostationary satellite Wearth = ( ) 2 3 2 2 gR 24hrs 86400 R h π π = = + or, (R + h) 3 = 2 2 2 (86400) gR 4π

NSOU ? CC - PH - 03 141 ? ( ) 1 2 2 3 4 2 gR 86400 h R 3.6 10 km. 4 ? ? ? ? = - × ? ? π ? ? ? This satellite plays an important role in telecommunication, weather forecasting and geographical survey. 3.4.10 Escape Velocity Escape Velocity of a body is the minimum velocity with which a second body is to be projected from the surface of the first body so that it can just reach infinity. To calculate the escape velocity of a planet (say

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Earth), we consider Earth to be a perfect sphere of radius R and

mass M. Let m be the second mass which has to be projected from the planet surface so that it can just reach infinity. Now the total energy of mass m at infinity will be E = 0, since both the potential energy and kinetic energy are zero. Now if V e is the velocity with which the mass is thrown, then V e will be the escape velocity. Then the total energy on planet's surface. 2 e 1 GMm E mV , 2 R = - and as the field is conservative E = 0, so e 2GM V R = ... Altenative method : Force of gravitation acting on the mass m when it is at a distance r from the center of Earth is 2 GMm ˆ F r r - = ?
142 NSOU ? CC - PH - 03 So the work done to displace it by dr ? 2 2 GMm GMm ˆ dw r.dr dr r r - = = + ? (since angle between E&dr ? ? is ?) So the work done in moving the mass from the surface of planet to infinity 2 R GMm W dr r • = ? = change in kinetic energy. 2 e GMm 1 mV R 2 = or, e 2GM V . R = 3.4.11 Weightlessness in satellite In case of a satellite (say artificial satellite) the gravitational pull due to satellite itself is negligible with any mass component attached to it. Obviously that mass and the satellite it self orbiting about the Earth will have centripetal acceleration a c such that c 2 GMm ma r = So c 2 GM a r = = acceleration due to gravity of Earth at planet point. So we can visualize that a setellite or a mass attatched to it, is a freely galling body a lways missing the center of earth. As a freely falling body is weightless, so the sattellite and a mass attached to it weightless. However, if it is a massive satellite like moon, the body will have a sense of weight with respect to moon.
NSOU ? CC - PH - 03 143 3.4.12 Numerical Problems 1. A spherical planet starts rotating faster. Find the limit of frequency of rotation upto which the surface particles will not fly off the planet. Solution If R is the radius of planet and p be its average density, then for a particle on the surface to be intact mg ??m? 2 R = m 4? 2 υR (m = mass of particle) or, 2 3 2 2 2 3 g GM/r 4 R p G 3 4 R 4 R 4 R π υ≤ = = π π π So, Gp 3 υ≤ π 2. A particle of mass m is thrown towards a spherical planet of radius R and mass M from a distance 9R from the center of the planet making an angle ??with the line joining the particle and the center of planet. v 0 be the velocity of projection. For what angle of projection the particle will just graze the planet? Solution v 0 ? M 9R v R From conservation of angular momentum mv 0 9R sin??= mvR ....(Ex. 2.1) From conservaion of energy 2 2 0 GMm 1 GMm 1 mv mv 9R 2 R 2 - - + = + ...(Ex. 2.2) or, 2 2 0 16GM v v 9R - = or, 2 0 0 v 16GM 1 v 9Rv = +
144 NSOU ? CC - PH - 03 Using in equation (Ex. 2.1 1 2 0 1 16GM sin 1 9 9Rv - θ = + 3. The orbit of a particle is given by r = a(1 + cos?). Find the nature of force Solution ( ) 1 u a 1 cos = + θ , ?? ( ) 2 du 1 sin d a 1 cos θ = θ + θ ?? ( )( ) 2 2 2 2 3 d u 1 cos 2sin a d 1 cos 1 cos ? ? θ θ ? ? = + ? ? θ + θ + θ ? ? = ( ) 2 2 3 1 cos cos 2sin a 1 cos ? ? θ+ θ+ θ ? ? + θ = ( ) 2 3 1 cos sin a 1 cos + θ+ θ + θ No The radial component of acceleration 2 2 2 r 2 2 L d u a u u m d ? ? - = + ? ? ? ? θ ? ? = ( )( ) ( ) 2 2 2 2 3 2 L 1 1 cos sin 1 · a 1 cos m a 1 cos a 1 cos ? ? - + θ+ θ ? ? + + θ ? ? + θ + θ ? ? = ( ) 2 2 2 2 5 3 L 1 1 cos sin 1 2cos cos m a 1 cos - ? ? + θ+ θ+ + θ+ θ ? ? + θ = ( ) ( ) 2 2 3 5 3 1 cos L 1 m a 1 cos + θ - + θ = 2 2 4 3L a m r -
NSOU ? CC - PH - 03 145 So, 4 K F . r - = The trajectory a 4. Consider a pair of stars of equal mass M rotating about their common center of mass. The attraction between the stars is gravitational and they keep a separation ? between them. Show that the time period of rotation of this double star system is 2l l GM π [C.U.] Solution 2 2 2 GMM M.M l a l M M M 2 l = μ = ω = ω + 3 2GM l ω= ?? 3 2 l T 2 2GM π = = π ω = 2l l . GM π 5. A particle of mas m describes an elliptic orbit about a central field 2 k F . ˆ r r = - ? Show that the speed of the particle at any point of orbit is given by k 2 1 v , m r a ? ? = - ? ? ? ? where a = semi major axis. Solution Total energy 2 k 1 K E mv 2a 2 r - = = - Or, k 2 1 v m r a ? ? = - ? ? ? ?

146 NSOU ? CC - PH - 03 6. Show that total energy of a particle undergoing an elliptic orbit in an inverse square c.f force 2 K ˆ F r. r - = ? Solution As the field is conservative the total energy E will be conserved through out the trajectrory. For simplicity we calculate the energy at Apse. ( ) ( ) 2 2 2 2 2 1 2 2 1 1 1 1 k 1 L k 1 L k E mr 1 1 2 r 2 r 2 l mr l = θ - = - = +∈ - +∈ ? = ( ) ( ) 2 1 mk k 1 1 2 ml l +∈ - +∈ = ( )( ) 1 k 1 1 2 l +∈ ∈- = ( ) 2 1 k k 1 2 l 2a ∈ - = - 7. A planet revolves around a star in an elliptic orbit. The ratio of the farthest distance to the closest one of the planet from the star is 4:1. Find the ratio of the kinetic energy of the planet at the furthest to the closest positions.(Gate) Solution 4 2 2 2 2 2 2 1 1 mr 1 L K.E mr 2 2 2 r mr ω = ω = = ( ) ( ) 2 2 c f c f r K.E : K.E 1:16 r = = 8. If a particle follows a spiral orbit given by r = c? 2 under an unknown force. Prove that such an orbit is possible in central field. Also find the form of the force law. (Cal Univ.)
NSOU ? CC - PH - 03 147 Solution 2 1 u c = θ ?? 3 du 2 d c = - θ θ and 2 2 4 d u 6 d c = θ θ If it follows the central field motion then 2 2 2 2 d u m 1 u F u d L u ? ? + = - ? ? ? ? θ ( ) 2 2 2 4 2 3 4 1 L u 6 1 L 1 6c F r F u m m c c r r - ? ? - ? ? ? ? = = + = + ? ? ? ? ? ? ? ? θ θ ? ? ? ? This is the nature of force. 9. A particle moves in circular orbit obeying the inverse square law. Show that the orbits of different radii, the angular momentum of the particle about the center of mass varies as the square root of the radius and the total energy varies inversely as the radius. (Cal Univ .) Solution We consIder a particle of mass m circling about a mass M in a radius r. If μ is the reduced mass of m, then 2 2 v k r r μ = or, k v , r = μ so the angular momentum L = ?rv = 2 2 k r r μ μ = kr, μ so L r ∝ So, total energy 2 1 k E v 2 r = μ - = k K k 2r r 2r - - = E r 1 ∴ ∝ proved.
148 NSOU ? CC - PH - 03 10. A particle moving under a central force describes a spiral orbit given by r = ae b? , where a and b are constant. Obtain the force law. (Guru Nanak Univ.) Solution b 1 u e , a - θ = so b du b e , d a - θ - = θ ?? 2 2 b 2 d u b e a d - θ = θ from the differential equation of orbit (3.4.14) 2 2 d u u d + θ = 2 2 m 1 F u L u ? ? - ? ? ? ? ( ) 2 2 2 2 1 L u d u F r F u u m d ? ? - ? ? = = + ? ? ? ? ? ? ? ? ? θ ? ? = 2 2 b b 2 L b 1 e e a a mr - θ - θ ? ? - + ? ? ? ? ? ? = ( ) 2 2 2 2 2 3 1 b L b 1 L r r m mr r + ? ? - - + = ? ? ? ? ? ? ( ) ( ) 2 2 3 L 1 F r 1 b m r - = + ?? 3 1 F r × 11. The motion of a particle under the influence of a central force is described by : r = a sin?. Find the expression of force. (Guru Nanak Univ.) Solution we put 1 1 u cosec , r a = = θso du cosec cos d a θ θ = - θ and 2 2 3 2 d u 1 cosec cot cosec a d ? ? = - - θ θ- θ ? ? θ = ( ) 2 2 1 cosec cot cosec a + θ θ+ θ
NSOU ? CC - PH - 03 149 So using equation (3.4.14) ( ) 2 2 2 2 1 L u d u F r F u u m d ? ? ? ? ? ? ? ? ? ? ? ? ? ? = = - + θ = ( ) 2 2 2 2 1 L u d u F r F u u m d ? ? ? ? = = - + ? ? ? ? ? ? ? ? θ ? ? = ( ) 2 2 2 2 L 1 1 cosec cot cosec cosec a a mr - ? ? θ θ+ θ + θ ? ? ? ? = ( ) 2 2 2 2 L · cosec 1 cosec cot mar - ? ? θ + θ+ θ ? ? ? ? = 2 2 2 3 2 2 3 L 2L a 1 · 2cosec mar mr r - - ? ? θ = ? ? 2 2 5 2L a 1 F m r - = 5 1 F . r ∝ 12. A particle moves along an orbit r = A cos ? under the influence of a central field F(r). Find the r dependence of force. (Cal. Univ.) Solution See problem – 11 Ans. ( ) 5 1 F r . r ∝ 13. Calculate th maximum velocity with which a body may be projected so that it may beome a satellite of Earth. Show that it is 2 times the minimum velocity of projection for a circular orbit close to the earth. (Pune Univ.) Solution The total energy of a satellite at a height h from earth's surface, 2 1 GMm E mv , 2 R h = - + R = radius of earth,
150 NSOU ? CC - PH - 03 M = mass of Earth, m= mass of satellite. Now to be a satellite E must be – ve. So 2 2GM 2gR v R h R h > = + + So, max v 2gR = ... Now the minimum velocity of a satellite is such that 2 min 2 mv GMm R R = Or, min v 2g = Sso, v max : v min = 2 :1 14. If v A be the velocity of a planet at its perihebion what will be its velocity at its aphelion. Solution From conservation of angular momentum mr A v A = m.r B v B A B l l v v 1 1 = +∈ -∈ ?? B A 1 v v 1 -∈ = +∈ [Ret. 4 (vii) Substance : the study of this chapter has enabled you to understand the characteristics of a central foce. The particles moving under a central force obey Kepler's laws and some physical quantities are conserved for this kind of motion. There are many areas left to explore if you are interested: questions of the stability of orbits under perturbations, the precession of the orbit, and whether it is open or closed. There are many interesting examples, even within our solar system, that show the varied and unique outcomes of central force interctions. Central forces can be attractive or repulsive in nature.
NSOU ? CC - PH - 03 151 3.4.13 Short Questions : 1. Show that the effective potential energy of a particle of mass under the action of a central force is given by V eff (r) = V(

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r) + 2 2 L 2mr , wher L is the angular momentum of the particle. 2. Prove that

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the total energy of a particle of mass m moving under the

action of a central force is given by E = 2 L 2m 2 2 du u d ? ? ? ? ? ? + ? ? φ ? ? ? ? ? ? + V (r), where V(r) is the potential energy and L is the angular momentum of the body. U = l r , r and ? are the polar co-ordinates of the particle. 3. Find the effective potential and equation of the orbit for a particle moving under the action of an attractive central force field given by F(r) = – 3 k r . What happens if L 2 becomes equal to mk? 3.4.14 Solution : 1. Eq. of motion gives m 2r 2 d dt = f(r) + 2 2 L mr = – 2 eff 2 dV d L V dr dr 2mr ? ? + =- ? ? ? ? ? ? which shows the value of V eff . 2.r = 2 l dr l du d L du r , u dt d dt m d u ? ? ? ? φ ? ? = =- =- ? ? ? ? ? ? φ φ ? ? ? ? ? ? ( ) 2 2 2 2 1 dr 1 dr d E m V r m r V(r) 2 dt 2 dt dt ? ? φ ? ? ? ? ? ? ? ? = + = + + ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
152 NSOU ? CC - PH - 03 Using the value of dr dt ? ? ? ? ? ? , we get E = 2 2 2 L du u V(r) 2m d ? ? ? ? ? ? + + ? ? φ ? ? ? ? ? ? 3. Potential V(r) = – 2 k , 2r Effective potential ( ) 2 eff 2 1 L 1 V r 2 m k r ? ? = ? ? - ? ? ? ? Eq.of the orbit is 2 2 2 2 d u m u d L u ? ? + = - ? ? φ ? ? [– ku 3] or, 2 2 2 d u mk 1 u 0. d L ? ? + - = ? ? φ ? ? In case of L 2 = mk, 2 2 d u 0. d = φ or, u = A? + B.
NSOU ? CC - PH - 03 153 Unit-5 ? Elasticity Structure : 3.4.1 Proposal 3.5.2 Elasticity, Stress and Strain 3.5.3 Elastic Limit 3.5.4 Different Elastic Constants : 3.5.5 Relation among Elastic constants: 3.5.6 Potential Energy due to Strain 3.5.6.1 Potential energy stored due to tensile strain 3.5.6.2 Potential energy stored due to volume strain 3.5.6.3 Potential energy stored due to shear 3.5.7 Bending of beams 3.5.8 The Cantilever 3.5.9 Depression of a Beam supported at the ends 3.5.10 Questions : 3.5.11 Answers : 3.5.1 Proposal In general we talk about rigid bodies while considering their motion. Elasticity is that property of rigid bodies by which they regain or try to regain its original shape or volume or length when the balanced forces causing the change in shape, volume or length are withdrawn. Obviously the forces must be small enough to make the changes temporary. This chapter deals with the elastic properties of different materials under different force conditions and finds interrelationships amongst the different
154 NSOU ? CC - PH - 03 elastic constants of solid bodies. In connection with this the torsional rigidity of materials and bending of beams of different materials under various force conditions will also be discussed. ? Outcome After reading this chapter you should be able: ? to know about one of the most important properties of matter, namely elasticity. ? to learn about elastic bahaviour of those substances which have the property of recovering their size and shape when the forces producing the deformation are withdrawn. ? to discover the relationships among various elastic constants of different materials. ? to understand the physics behind the torsion of a cylinder. ? to develop the logic of calculating bending of beams of different objects. 3.5.2 Elasticity, Stress and Strain When a piece of material is under the action of balanced forces, the material is deformed. If the forces are small, if the relative displacements of the various points in the material are proportional to the forces we say the behaviour is elastic. If all the parts of the material have identical properties in all respects–the material is said to be homogeneous. Again if the properties of the material are same in all directions– the material is isotropic. We shall consider only homogeneous and isotropic materials which are in stable equilibrium, i.e. the net force acting on the body is zero and net moment of the forces is also zero. It we apply two equal but oppositely directed forces on a solid homogeneous and isotropic cylinder of length L 0 , we find that there can be either expansion by the amount ?L depending on the direction of the forces as can be seen by the diagram 3.5.1. Here NSOU ? CC - PH - 03 155 the only point to be considered is that the forces must be small enough causing no damage of the material. The ratio V V Δ ? is known as longitudinal strain. By similar fashion we can find out the ratio of change in volume of the material to its original volume V V Δ ? which is known as volume strain. This strain (either longitudinal or volume) is caused by the external force and it generates an internal force due to intermolecular interaction which actually brings the object to its original length, shape or volume when the external force is removed. Under equilitbrium condition, the external force is equal to the internal force and oppositely directed. This internal force developed within the materials per unit area is known as stress. Stress is determined by the following equation: Stress = Opposing force of intermolecular origin, Area = F A , where A is the area of cross-section of the material. Figure : 3.5.1 Effect of tensile or longitudinal stresses on a cylindrical system The strain has no dimension as it is a ratio of change in length over original length or change in volume over original volume. The stress has a dimension of ML –1 T -2 and

156 NSOU ? CC - PH - 03 the unit of stress in SI system is Nm -2 . When the deforming forces produce an actual change in the shape of the body, then the strain produced in the body is called shear strain. Shear strain is defined as the ratio of relative displacement of any layer to its perpendicular distance from the fixed layer. tan? = w L (Fig.3.5.2). In passing it may be mentioned here that shearing stress is equivalent to an equal linear tensile stress and an equal compression stress at right angles to each other. O X w ? L w ? ? Figure : 3.5.2 Shear strain produced in a square object 3.5.3 Elastic Limit The question that comes to our mind is whether every object regains its original length, shape or volume when the deforming stresses are removed. The answer to this question can be obtained by studying the stress-strain curve of that sample as shown in Fig. 3.5.3. Under the action of a gradually increasing stress which is equal to external force developed per unit area at equilibrium, the behaviour of a substance is represented by its nominal stress-strain curve. In the nominal stress-strain curve the reduction of the cross-section of the material is neglected. The nominal stress-strain curve for different materials is different from each other. The strain is shown as the percent elongation; the horizontal scale is not uniform beyond the first portion of the curve, up to a strain of less than 1%. The first portion is a straight line, indicating Hooke's law behaviour with stress directly proportional to strain. This straight-line ends at point A ; the stress at this point is called the proportional limit. From A to B, stress and
NSOU ? CC - PH - 03 157 strain are no longer proportional, and Hooke's law is not obeyed. If the load is gradually removed, starting at any point between O and B, the curve is retraced until the material returns to its original length. The deformation is reversible, and the forces are conservative; the energy put into the material to cause the deformation is recovered when the stress is removed. In region OB we say that the material shows elastic behaviour. Point B, the end of this region, is called the yield point; the stress at the yield point is called the elastic limit. When we increase the stress beyond point B, the strain continues to increase. But now when we remove the load at some point beyond B, say C, the material does not come back to its original length. Instead, it follows the dotted line in Fig. 3.5.3. The length at zero stress is now greater than the original length; the material has undergone an irreversible deformation and has acquired what we call a 'permanent set'. Further increase of load beyond C produces a large increase in strain for a relatively small increase in stress, until a point D is reached at which fracture takes place. The behaviour of the material from B to D is called plastic flow or plastic deformation. A plastic deformation is irreversible; when the stress is removed, the material does not return to its original state. For some materials, such as the one whose properties are graphed in Fig. 3.5.3, a large amount of plastic deformation takes place between the elastic limit and the fracture point. Such a material is said to be ductile. The stress required to cause actual fracture of a material is called the breaking stress, the ultimate strength, or (for tensile stress) the tensile strength. Two materials, such as two types of steel, may have very similar elastic constants but vastly different breaking stresses. strain > 1 % Proportional limit Elastic limit or yield point D Fracture Point Plastic behaviour Permanent set s tr e s s Elastic behaviour 30% A B C Figure : 3.5.3 Stress-Strain curve of a solid material
158 NSOU ? CC - PH - 03 ? Hooke's Law : Within elastic limit it has been found through several experiments that stress and strain are proportional to each other and Stress Strain = Constant. This is known as Hooke's law. This proportionality constant is called modulus Strain of elasticity. This constant depends on the properties of the material. Temperature and formation history of the material has some influence on the elastic properties of the material. It has been observed that elasticity of any material decreases with temperature. 3.5.4. Different Elastic Constants To determine the elastic behaviour of homogeneous, isotropic bodies we need to specify five quantities as described below: (a) Young's Modulus (Y) (b) Bulk Modulus(K) (c) Modulus of rigidity (G) (d) Axial Modulus (?) (e) Poisson's Ratio(?) (a) Young's Modulus When the deforming force is applied to the body only along a particular direction, the change per unit length in that direction is called longitudinal, linear or elongation strain, and the force applied per unit area of cross-section at equilibrium

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is called longitudinal or linear stress. The ratio of longitudinal stress to linear strain, within the elastic limit, is called Young's Modulus,

and is usually denoted by the letter Y. Thus, if F be the force applied normally to a cross-sectional area A, the stress is F A . And, if there be change ? produced in the original length L, the strain is given by L ? . So that, Young's Modulus, 2 F A Y N/m l L = within elastic limit.

NSOU ? CC - PH - 03 159 (b) Bulk Modulus Here, the force is applied normally and uniformly to the whole surface of the body; so that, while there is a change of volume, there is no change of shape. Geometrically speaking, therefore, we have here a change in the scale of the coordinates of the system of the body. The force applied per unit area, (or pressure), gives the Stress, and the change per unit volume, the Strain, their ratio giving the Bulk Modulus for the body. It is usually denoted by the letter K.Thus, if F be the force applied uniformly and normally on a surface area A, the stress, or pressure, is F/A or P; and, if v be the change in volume produced in an original volume V, the strain is v/V. and, therefore, Bulk Modulus, 2 Volume Stress F K N/m Volume Strain A = = θ The reciprocal of bulk modulus is called compressibility. (c) Modulus of Rigidity or Shear Modulus If a force F tangential to the surface of area A is applied, tangential stress F/A is generated which gives rise to an angle of shear ?, then Modulus of Rigidity (G) = 2 VolumeStress F K N/m VolumeStrain A = = θ (d) Axial Modulus The axial modulus

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is defined as the ratio of the longitudinal stress to the corresponding strain			

when there are other stresses present which prevent any lateral change of dimensions. Axial Modulus (??) = Longitudinal stress lateral strain 0 Longitudinal strain ? ? = ? ? ? ? (e) Poisson's Ratio For any material wire at constant temperature the ratio of lateral strain to the

160 NSOU ? CC - PH - 03 longitudinal strain within elastic limit is a constant. This constant is known as Poisson's ratio. If L and R be the oiginal length and radius before straining and 1 and r be corresponding changes after straining, then Poisson's ratio ( ) r Lateral strain 1 R , Longitudinal strain l 2 L σ = = σ> . 3.5.5 Relation among Elastic constants: Relation between Y, G and σ : We have seen earlier that there are several elastic constants which depict the behaviour of the substance under different stressed condition. All of them are not independent. Let us first try to establish a relation among the elastic constants Y, G and ? : Let us consider a cube of material of side 'a'. It has been subjected to the action of the shearing stressT. The result of the shear is shown below. We assume that the strains are small and the angle A C B may be taken as 45º. Figure 3.5.4 Shear and shearing strain The diagonal OA is under a strain and that strain = change in length/original length The diagonal OA due to shearing strain has been changed to OB. Since angle between OA and OB is very small hence OA ? OB, therefore BC can be taken as the change

NSOU ? CC - PH - 03 161 in the length of the diagonal OA. Thus the strain on OA is = BC ACcos45º . OA OA = Now, OA = 2a. Therefore, the strain on OA is AC AC , 2a 2 2a = ? ? ? ? But AC = ? a, so, the strain is 2G τ , which is in turn equal to /2G, τ where ? is the stress and G is the shear modulus. Now this shear stress system is equivalent or can be replaced by a system of direct stresses at 45º as shown below. One set will be compressive, the other tensile, and both will be equal in value to the applied shear strain. Thus, for the direct state of stress system which applies along the diagonals, we see equivalence of shearing strain with longitudinal strains, both compressive and expansive. Strain on diagonal = 1 2 Y Y σ -σσ = Y ( ) Y τ -σ -τ = , Y(1) τ + σ equating the two strains we get Y ,orY , 2G Y(1) 2G (1) τ τ = = + σ + σ where σ is the Poisson's ratio. (3.5.5.1) We have introduced a total of four elastic constants, i.e Y, G, K and σ. It turns out that not all of these are independent of the others. In fact given any two of then, the other two can be found. We know Y 3K(1 2) = -σ(shown below) irrespective of the stresses i.e, the material is incompressible. When σ = 0.5 Value of K is infinite, rather than a zero value of Y and volumetric strain is zero, or in other words, the material is incompressible. ? Relation between Y,K and σ : Consider a cube subjected to three equal stresses E s as shown in the figure below

162 NSOU ? CC - PH - 03 Figure 3.5.5 Cube under stress in all directions The total linear strain in one direction or along one edge due to the application of hydrostatic stress or volumetric stress E s is given as s s s s E Y Y Y = -σ -σ = s y (1–2σ) Bulk modulus = (volumetric stress/(volumetric strain). or K= E s , (volumetric strain), so, volumetric strain = E s K, which is 3 times linear strain. Therefore, equating the two strains we may write s s E 3E (1-20) K Y = or Y = 3K(1–2σ). (3.5.5.3) ? Relation between Y, G and K : The relationship between Y, G and K can be easily determined by eliminating μ from the already derived relations E s E s E s

NSOU ? CC - PH - 03 163 Y 2G(1) = + σ and ( ) Y 3K 1 2 = - σ (3.5.5.4) Thus, the following relationship may be obtained ( ) 9GK Y 3K G = + or, 9 3 1 Y G K = + From the already derived relations, Y can be eliminated ( ) Y 2G 1 , = + σ and again ( ) Y 3K 1 2 = - σ. Thus, we get ( ) 3K(1 2) 2G 1 - σ = + σ , therefore (3K 2G) 2(G 3K) - φ = + ? (3.5.5.6) hence if σ = 0.5, the value of K becomes infinite and the volumetric strain is zero or in other words, the material becomes incompressible Further, it may be noted that under condition of simple tension and simple shear, all real materials tend to experience displacements in the directions of the applied forces and under loading they tend to increase in volume. In other words the value of the elastic constants Y, G and K cannot be negative Therefore, the relations Y = 2G(1+ σ) Y = 3K (1–2 σ??????? ??? ???? ??? ??? ??? ??? (3.5.5.7) yields 1 0.5 - ≤ σ ≤ In actual practice no real material has value of Poisson's ratio negative. Thus, the value of σ cannot be greater than 0.5. ? Worked out Examples 1. Show that a small and uniform volume strain v is equivalent to three linear strains v 3 , in any three perpendicular directions

164 NSOU ? CC - PH - 03 Solution: Imagine a unit cube to be compressed equally and uniformly from all sides, so that the length of each edge is decreased by a length x,i.e. the side becomes 1–x. Then, clearly, decrease in volume of the cube, i.e., v=1–(1–x) 3 =1–1 +3x –3x 2 +x3 = 3x, i.e.. x = v 3 , neglecting x 2 and x 3 , the value of x being small. Thus, a small uniform volume strain v is equal to three linear strains, each equal to v 3 , in three perpendicular directions. ? Torsion of a cylindrical rod: Let us consider a cylindrical rod with length L and radius R, fixed and rigidly supported at one end, and loaded at the other end with an axial torque. For rotational equilibrium of this rod, the external torque is balanced internally by the tongue generated by shear stress. The shear stress may be seen as acting in each imaginary perpendicular cut with a torque equal but opposite to the external torque (Fig. 3.5.6). Figure 3.5.6 Torsion of a cylindrical rod under axial stress. From the fig. 3.5.6. it is clear that the following relations hold AA' x r , = δ= φ but G τ δ = so, we get ( ) r,x τ = r G x φ (3.5.5.9) The total torque produced by the shear stress can be calculated by integrating over

NSOU ? CC - PH - 03 165 the cross-sectional area with respect to the center P of the cross-section (see Fig. 3.5.6) ( ) 4 R 2 2 0 p p 0 G x R G (x) G (x) M r df r df r 2 rdr x x x2 φ π φ φ = τ = = π = ? ? ? ... ... (3.5.5.9) The angle of torsion at the end of the rod (x=L) then becomes ( ) 0 4 2LM L GR φ = π (3.5.5.10) Thus we see that for a given cylinder or wire, the angle of twist is proportional to the torque. The torque per unit twist is given by 4 0 M GR c 2L π = = φ . ... (3.5.5.11) It is called torsional rigidity. ? Worked out Example : A gold wire 0.32 ×10 -3 m in diameter, elongates by 10 -3 m, when stretched by a force of 330 x10 –3 kg wt., and twists through 1 radian, when equal and opposite torques of 145×10 -7 N-m are applied at its ends. Find the value of Poisson's ratio for gold. Solution : Y = FL al , here F = 330×10 –3 × 9.81 N, 1 = 10 –3 m and a = ( ) 2 3 x 0.16 10 - π × sq.m Therefore, Y = -3 3 7 330 × 10 × 9.81 × L [(0.16 10) 10 N m - π× × × - The angle of twist is 1 radian, / then, couple per unit twist = 145 ×10 –7 N–m. This must be equal to 4 Nr , 2L π so we have 4 7 Nr 145 10 2L - π = × .

166 NSOU ? CC - PH - 03 Therefore, N = ( ) 7 2 3 145 10 2L 0.16 10 - - × × ? ? π× × ? ? ? ? So, we get Y N =2.858. Since Y N = 2 (σ + 1), this leads to σ = 0.429, this is the value for Poisson's ratio for gold. ? Torsional Oscillation : A torsional pendulum, or torsional oscillator, consists of a disk-like mass suspended from a thin rod or wire. When the mass is twisted about the axis of the wire, the wire exerts a torque on the mass, tending to rotate it back to its original position. If twisted and released, the mass will oscillate back and forth, executing simple harmonic motion for small torsion 4 NGr 2L θ v. Figure 3.5.7 Instrument to measure torsional oscillation in the Laboratory Consider a thin rod with one end fixed in position and the other end twisted through an angle θ about the rod's axis. If the angle ? is sufficiently small that the rod is not plastically deformed, the rod exerts a torque τ proportional to the angle ?, K τ = - θ (similar to F= –kx for a harmonic oscillator) (3.5.5.12) The –ve sign indicates that τ and ? are oppositely directed where k (Greek letter

NSOU ? CC - PH - 03 167 kappa) is called the torsion constant. The minus sign indicates that the direction of the torque vector τ is opposite to the angle vector ?, so the torque tends to undo the twist. This is just like Hooke's Law for springs. If a mass with moment of inertia I is attached to the rod, the torque will give the mass an angular acceleration ? according to 2 2 2 2 d d x I I likeF ma m dt dt ? ? θ τ = ∝= = = ? ? ? ? (3.5.5.13) Combining (3.5.5.12) and (3.5.3.1.3) yields the equation of motion for the torsional pendulum, 2 2 d I k dt θ = - θ (3.5.5.14) or, 2 2 d k I dt θ = - θ (3.5.5.15) The solution to this differential equation is ( ) ( ) m t cos t , θ = θ ω +φ (3.5.5.16) where, ( ) 1 2 K l ω= (3.5.5.17) ? m and ??are constants which depend on the initial position and angular velocity of the mass. (The equation of motion is a second order differential equation so its solution must have two constants of integration.) ? m is the maximum angle; ? oscillates between + ? m and – ? m . The constant ??is related to the frequency f and the period T of the simple harmonic motion by ?? = 2?f = 2? T (3.5.5.18)

168 NSOU ? CC - PH - 03 So, we conclude that the time period T is given by T = ( ) 1 2 I 2? K (3.5.5.19 The torsion constant can be determined from measurement of T if I is known, conversely if K is known the moment of inertia can be determined from measurement of T. K for a cylindrical rod is given by K = 4 GR 2l π , Where G = modulus of rigidity of the material of the rod. R is the radius and L is its length. Therefore, G of the material can be experimentally determined by measuring the time period of oscillation, radius and length of the rod. By the way, the moment of inertia of the disk is given by I= 1 2 mr 2 about the axis of rotation. This type of measurment of modulus of rigidity is known as "Dynamical Method of Determination of Modulus of Rigidity". 3.5.6 Potential Energy due to Strain When the shape or volume of a body changes due to the action of an external force, the internal parts of the body, i.e., the molecules of the system develops an internal stress due to which the object regains its shape or volume when the external force is withdrawn. Certain amount of work is done by the external force while changing the initial state. This work done is stored within the system as potential energy, if elastic limit is not crossed, which gets converted to kinetic energy as soon as the external force is withdrawn. A compressed spring is in ideal example of the above phenomenon. In the calculation for potential energy gained by strain, it is assumed that equilibrium has been maintained during the process 3.5.6.1 Potential energy stored due to tensile strain Maintaining equilibrium, the tensile stress on a thin wire of length L, cross-sectional area A having Young's modulus Y fixed at one end be slowly increased to a value ???say).

NSOU ? CC - PH - 03 169 If at any instant of time t, the elongation is x, the stress due to which this elongation is produced is Yx L and the corresponding force applied at the free end is F= YAx L . Now the force applied be slowly increased to a value such that the increase in length changes to x + dx. Then the work done is F dx. In the same token, the total work done by the external force as the elongation of the wire reaches the final value ???is ( ) 2 x x 0 0 1 YAx 1 W Fdx dx YA L 2 L l l =δ δ = δ = = = ? ? (3.5.6.1) Now the tensile force at the end of extension is F=YA ( ) 1 L δ . So, the strain energy can be written as ( ) 1 1 1 W YA . 1 F. 1 2 L 2 δ = δ = δ (3.5.6.2) Because the wire is uniform and cylindrical, its volume is AL. Therefore the energy density due to strain is ( ) ( ) 2 W 1 YA 1 1 1 . 1 Y V 2 AL L 2 L δ δ = δ = . 3.5.6.2 Potential energy stored due to volume strain The change in volume occurs due to volume stress which is actually the pressure acting on the system. We follow the same line of argument as in the case of tensile strain. If ??p be the increase in pressure due to which we find the decrease in volume to be ?v. Then, ( ) 1 1 W K v/V x v p v. 2 2 = δ δ = δ ×δ Now, energy density = ( ) ( ) ( ) ( ) ( ) final final. W 1 v 1 v v 1 K K stress strain V 2 V 2 V V 2 δ δ δ = = × = × (3.5.6.3)

170 NSOU ? CC - PH - 03 3.5.6.3 Potential energy stored due to shear The procedure for calculating the potential energy due to shear is similar to above. We know that if a tangential force F acting on opposite faces of a parallellopiped of area A separated by a distance L produces a shearing strain ???then we have, 1 W FxL 2 = θ and work done per unit volume 2 W 1 G , V 2 = θ or, ( ) ( ) ( ) final final. W 1 1 F 1 stress strain V 2 2 A 2 = = θ = × × ? Worked out Examples : Find the work done in Joules in stretching a wire of cross-section 1 sq. mm. and length 2 metres through 0.1 mm., if Young's modulus for the material of the wire is 11 2 N 2 10 . m × Solution : Work done = 1 1 1 F.1 Ya 2 2 L = where F is the stretching force. Here, 11 2 6 N 1 Y 2x10 ,a 1sq.mm. 0.000001 m 10 = = = = sq.m., 1=0.1 mm = 0.0001 m; and L = 2m Therefore, work done = 2 11 6 8 1 1 Yal x2x10 x10 x10 2 2 2 2 - - - = Thus, work done in stretching the wire is 5×10 -3 Joules.

NSOU ? CC - PH - 03 171 3.5.7 Bending of beams We start with a question: what is a beam? –a beam is usually a metallic solid rod of uniform cross-section. It may be circular, rectangular or any other regular geometric shaped rod whose length is much larger compared to its cross-section so that the shearing stresses over any section are small and may be ignored Often we come across a situation where a beam is bent due to some reasons or other. When a beam is fixed or supported at one end and loaded at the other, it bends due to the moment created by the weight of the load. The plane of bending is the same as that of the couple produced. As discussed earlier restoring forces come into play and in the equillibrium state, the restoring couple is equal and opposite to the external bending couple, both being in the plane of bending. After bending of the beam, its filaments on the inner or the concave side get shortened or compressed and those on the outer or the convex side get lengthened or extended. A B C D E F Figure 3.5.7.1 Bending effect on the filaments of a beam In between these two portions there lies a layer or a surface in which the filaments are neither compressed nor extended. This surface is called the neutral surface and its section (EF) by the plane of bending which is perpendicular to it is called the neutral axis Neutral axis Figure 3.5.7.2 Neutral axis for a rectangular beam

172 NSOU ? CC - PH - 03 In absence of any strain of the beam, the neutral surface becomes a plane surface, and the filament of this unstrained or unstretched layer or surface, lying in the plane of symmetry of the bent beam, is refered to as the neutral filament. It passes through the c.g. (or the centroid) of every transverse section of the beam. The change in length of any filament is proportional to its distance from the neutral surface. Let a small part of the beam be bent, as shown in Fig. 3.5.7.3 in the form of a circular arc, subtending an angle at the centre of curvature O. Let R be the radius of curvature of this part of the neutral axis, and let 'ab' be an element at a distance z from the neutral axis. Figure 3.5.7.3 Bending strain of a beam Then, a?b? = (R+z).? and its original length ab = R?? Therefore, increase in length of the filament = a'b'−ab = (R+z)??R????z???(3,5,7.1) The original length of the filament = R???then we have strain = z z R R θ = θ , (3.5.7.2) i.e. the strain is proportional to the distance from the neutral axis. Since there are no shearing stresses, nor any change of volume, the contractions and extensions of the filaments are purely due to forces acting along the length of NSOU ? CC - PH - 03 173 the filaments. Let PQRS (Fig. 3.5.7.4), be a section of the beam at right angles to its length and the plane of bending. Then, the forces acting on the filaments are perpendicular to this section, and the line MN lies on the neutral surface. Let the breadth of the section be PQ = b and its depth, QR = d. S P Q c R d E F a b dz A M z N Figure 3.5.7.4 Cross-section of the beam The forces producing elongations and contractions in all filaments act perpendicularly to the upper and the lower halves, PQNM and MNRS respectively, of the rectangular section PQRS, their directions being opposite to each other. Let us consider a small area ?a about a point A, distant z from the neutral surface. The strain produced in a filament passing through this area will be z R , (shown above). Now, Y= stress strain , therefore, stress = Y x strain, where Y

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is the Young's modulus of the material of the

beam. Therefore, stress about the point A = Y.z R , and, the force on the area =??a. Y.z R . (3.5.7.3) and, moment of this force about the line MN= 2 Y.z. a Y. a.z .z . R R δ δ = (3.5.7.4) Since the moments

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of the forces acting on both the upper and the lower halves 174 NSOU ? CC - PH - 03 of the section are in the same direction, the total moment of the forces acting on the

filaments in the section PQRS is given by 2 2 i i i i 1 i Y. a z Y a z R R δ = δ ? ? (3.5.7.5) Now, 2 i i 1 a z δ ? is the geometrical moment of inertia (I g) of the section about MN, and, therefore Y R , equal to ak 2 , where a is the whole area of the surface PQRS and k, its radius of gyration about MN. Hence, the moment of the forces about MN= Y R ak 2 =YI g (3.5.7.6) For rotational equilibrium, this moment balances the couple of the bending moment M, acting on the beam due to the load. In other words, it is the moment of the stress set up in the beam or the moment of resistance to bending, as it is usually called in engineering practice, and is also of the nature of a couple, for only a couple can balance a couple. Obviously, it acts in the plane of bending and is equal to the bending moment at the section due to the load. The quantity YI g = Yak 2 is called the flexural rigidity of the beam. (3.5.7.7) So, bending moment = Y R x geometric moment of inertia of the section. = flexural rigidity R , whatever the shape of the cross-section of the beam. For a rectangular cross-section, a = bd, and k 2 = 2 d 12 . I g = ak 2 = 2 bd 12 . Hence, bending moment for a rectangular cross-section = 2 Y.b.d 12R . (3.5.7.8) For a circular section, 2 a r = π and k 2 = 2 g r 4.1 . I g = ak 2 = 4 r 4R π . (3.5.7.9)

NSOU ? CC - PH - 03 175 3.5.8 The Cantilever A cantilever is a beam fixed horizontally at one end and loaded at the other. ? Cantilever loaded at the free end : Here, two cases arise, viz., (a) when the weight of the beam itself produces no bending, and (b) when it does so. Let us consider both the cases. (a) When the weight of the beam is ineffective. Let AB, (Fig. 3.5.8.1) represent the neutral axis of a cantilever, of length L fixed at the end A, and loaded at B with a weight W, such that the end B is deflected or depressed into the position B′ and the neutral axis takes up the position AB′, it being assumed that the weight of the beam itself produces no bending. Consider a section l of the beam at a distance x from the fixed end A. The moment of the external couple at this section, due to W or the bending moment acting on it = W ×PB′ = W(L-x) (3.5.8.1) l Figure 3.5.8.1 Loading of a cantilever As the beam is in equilibrium, this must be equal to 2 g YI Yak R R = , where R is the radius of curvature of the neutral axis at P Therefore, 2 g YI Yak R R = (3.5.8.2) Since the moment of the load increases as we proceed towards the fixed end A, ( ) W L x 2 - 2

176 NSOU ? CC - PH - 03 the radius of curvature is different at different points and decreases as we approach the point A. For a point Q, however, at a small distance dx from P, it is practically the same as at P. So that, PQ = R.d????r, dx = R.d???Where d???is the angle POQ whence, R= dx dθ Substituting the value fo R in (3.5.8.1) above, we have W(L–x) =Y ak 2 d dx θ or, d?? 2 W(L x)dx (Yak) - (3.5.8.3) If tangents are drawn to the neutral axis at P and Q, meeting the vertical line through BE′ in C and D respectively. Then, the angle subtended by them is also equal to d???the radi at P and Q being perpendicular to the tangents there. Now, clearly, the depression of Q below P is equal to CD, equal to dy, (say) Then, ( ) 2 (L x)W(L x)dx dy L x d (Yak) - - = - θ = = ( ) 2 2 W L x dx (Yak) - (3.5.8.4) Therefore, the depression y = BB′ of the loaded end B below the fixed end A, is obtained by integrating the expression for dy between the limits, x = 0 and x =1 or, ( ) ( ) 2 L L 2 2 2 2 0 0 L x W y W dx L 2xL x dx Yak Yak - = = - + ? ? (3.5.8.5)

NSOU ? CC - PH - 03 177 3 3 2 g W L WL y . 3 3YI Yak = = (3.5.8.6) Thus, the free end of the cantilever is depressed by 3 3 2 g W L WL y . 3 3YI Yak = = (3.5.8.7) (b) the total bending moment of the beam = W(L x) w(L x) 2 - + - =W(L–x) + ( ) W L x 2 2 - Imposing the condition for equilibrium we get, 2 2 2 W(L x) w Yak d Yak R dx 2(L x) - + θ = = - , Obtaining dθ from here we can find out dy, where dy = (L–x)d? ( ) ( ) 2 2 2 w W l x L x 2 Yak - + - = Therefore, 3 4 2 2 WL wL y 3Yak 8YaK = + (3.5.8.8) 3.5.9 Depression of a Beam supported at the ends (i) When the beam is loaded at the centre We consider a uniform beam supported on two knife edges symmetrically placed at its two ends A and B, as show in Fig. 3.5.6.1, and let it be loaded in the middle at C with a weight W. The reaction at each knife edge will clearly be W/2, in the upward direction (Fig. 3.5.9.1). Since the middle part of the beam is horizontal, the beam may be considered

178 NSOU ? CC - PH - 03 as equivalent to two inverted cantilevers, fixed at C, the bending being produced by the loads W/2, acting upwards, at A and B. If, therefore, L be the length of the beam AB, the length of each cantilever (AC and BC) is L 2 , and the elevation of A or B above C or equivalently the depression of C below A or B is given by w 2 w 2 W A B C Figure 3.5.9.1 Beam loaded at the center 3 3 2 a WL WL y 48Yl 48Yak = = (3.5.9.1) If the beam is of circular cross-section, we have ak 2 = 4 r 4 π where r is the radius of the beam. For this kind of beam the depression at the center of the beam is 3 4 WL y (12Y r = π . (3.5.9.2) And, if the beam is of rectangular cross-section of breadth ‘b’ and depth ‘d’, we have ak 2 = 2 bd 12 . For such a beam y= 3 4 WL (4Ybd) (3.5.9.3) Thus we see that knowing the shape of the beam and its loading pattern one can find out the depression at a particular chosen point.

NSOU ? CC - PH - 03 179 ? Worked out Examples : brass bar 1 cm. square in cross-section is supported on two knife edges 100 cm. apart. A load of 1 kg. at the centre of the bar depresses that point by 2.51 mm.What is Young’s modulus for brass? Solution : We know that the depression of the mid-point of the bar is given by y =Wl 3 /48Yl g [See text] Now, for a bar of rectangular cross-section, I g = 2 b.d 12 Here, b = d =1 cm., because the bar is 1 cm. square in cross-section. b.d 3 = 1×1 = 1; W = 1 kg. wt. = 1×9.81 N. 1= 100 cm = 1m. and y = 2.51 mm = 0.00251 m. Therefore, Y = 3 3 W1 4ybd Or, the value of Young’s Modulus for brass is 9.77 ×10 10 N/m 2 3.5.10 Questions : 1. Explain the stress-strain curve. From the curve, explain elastic limit, plasticity 2. Prove that (a) (3K 2G) 9 1 3 (b) Y K G (6 2G) - = + σ = + 3. If the volume of a thin rubber string remains unchanged after a little elongation, what is its Poisson’s ratio? 4. A thin uniform brass rod of length 1 and mass m rotates uniformly with an

180 NSOU ? CC - PH - 03 angular velocity ??in a horizontal plane about a vertical axis passing through one of its ends. Determine the tension in the rod as a function of the distance from the rotation axis. Find the elongations of the rod. 5. Calculate the geometrical moment of inertia of a) thin rectangular sheet, (b) thin hollow circular section. 6. A cantilever beam of rectangular section has breadth ‘b’ and depth ‘d’. If d = 2b, find the ratio of depression at the free end when (i) d is vertical and (ii) b is vertical. 7. Find the potential energy due to twist in a wire of circular cross-section. 3.5.11 Answers : 1. See section 3.5.3 2. See section 3.5.5 3. V = ?r 2 1, as V is constant ?V =2?rl ?r + ?r 2 ?????? So, σ = r 1 r 2 (1) 1 δ - = δ ? ???Let us consider a section of the rod at a distance x from the axis of rotation when the rod is not in a condition of stretching. Let the tension at that point be T when the rod is rotating. T arises due to centripetal force on the remainisng portion of length (1–x) T = ( ) 2 2 2 2 (1 r) m 1 1 (1 x) . 1 x m 1 1 2 2 1 - - ω + = ω . If the elongation be ??? at length ?x??then ( ) T T x A Y , So, , A (A Y) x δ = δξ = δξ ? ? ? ? δ ? ? cross-sectional area of the element.

NSOU ? CC - PH - 03 181 Therefore, total elongation ???? ( ) ( ) 2 2 1 1 0 0 2 m 1 T 1 x dx / dx AY 2AY 1 ω - = ? ? ?? 2 3 1 m 1 3 AY ? ? ω ? ? ? ? ???Geometrical

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moment of inertia of a thin rectangular sheet of breadh b and width w about an axis passing through its center and parallel to its

side of length w 3 2 2bw I y dA y dxdy . 12 = = = ? ? (b) the geometrical moment of inertia of the hollow circular cross-section of internal and external radii r 1 and r 2 ?respectively is given by ( ) 2 I= rsin rd dr θ θ = ?? ( ) 2 1 r 2 3 2 4 4 2 1 r 0 r dr sin d r r 4 π π θ θ = - ? ? about any diameter. 6. The depression of the free end of a cantilever is given by ( ) 0 3 3 W 3W /8 41 , Ybd + δ = where 1=length of the cantilever, b=breadth, d=depth, W = weight at the end of the free end of the beam, W 0 = its own weight and Y = young’s modulus of the material of the beam. Therefore, the depression of a beam will be 1 4 th of the depression of the same beam when its depth and breadth are interchanged. 7. If ? is the twist at any time, the internal torque C = 4 NGr 2L θ . If the twist is further increased by by dq, the work done due to this twist is Cd???As ??increases from 0 to ? 0 , the total work done is given by o 0 W Cd θ = θ ? = 4 2 0 1 Gr /4L x 2 π θ = (applied torque) f x (twist) f . 1 4

182 NSOU ? CC - PH - 03 Unit-6 ? Viscosity and fluid dynamics Structure : 3.6.1 Proposal : 3.6.2 Motion of fluids 3.6.3 Newtonian and Non-Newtonian Fluid : 3.6.4 Critical velocity and Reynold’s number: 3.6.5 Poiseuille’s Equation: 3.6.6 Determination of Co-efficient of viscosity by POiseuille’s method : 3.6.7 Stokes’ method of determination of co-efficient of viscosity of a liquid and Terminal velociy 3.6.8 Euler’s equation of motion for fluids : 3.6.9 Equation of continuity : 3.6.10 Bernoulli’s theorem and it’s application : 3.6.11 Application of Bernoulli’s principle: 3.6.12 Torricelli’s theorem : 3.6.13 Venturi meter : 3.6.14 Effect of temperature and pressure on the viscosity of liquids: 3.6.15 Important points : 3.6.16 Questions (short answer type) : 3.6.17 Numerical Problems : 3.6.18 Answers to short questions: 3.6.19 Answers to numerical problems:

NSOU ? CC - PH - 03 183 3.6.1 Proposal Viscosity is the property that describes a fluid’s resistance to flow like frictional force opposing motion between two solid surfaces in contact. Fluids try to resist the relative motion of objects through them as well as to the motion of layers with differing velocities within them. ? Outcome After reading this chapter you should be able to: ? think enthusiastically about analysing movements of fluids having different kinds of viscosity. ? explain the effect of temperature and pressure on the coefficient of viscosity of different fluids. ? describe different methods of determining the coefficient of viscosity of fluids in the laboratory. 3.6.2 Motion of fluids Fluid dynamics is the study of motion of fluids (gases and liquids). As this study is a macroscopic one, fluid can be regarded as a continuous media. However, we must be aware of the fact that even a tiny volume element of fluid contains very many number of molecules. So, when we consider fluid particle we are actually talking about the motion of a cluster of fluid molecules represented by a point inside the fluid. There are types of fluid motion: (a) steady or laminar flow and (b) turbulent flow.

184 NSOU ? CC - PH - 03 d Laminar flow d Turbulent flow fig. 3.6.2.1 Steady flow is the flow in low speed such that its adjacent layers slide smoothy with respect to each other, Streamline is an imaginary line showing the path of any part of the fluid during its steady flow inside the tube. Particles of a continuous fluid can be considered to travel along smooth continuous paths named streamlines. These streamlines can be curved or straight, depending on the lateral pressure on fluid. This type of motion is also called laminar flow. If a tangent is drawn at any point on the streamline, it gives the direction of the fluid motion at that point. A transition from laminar flow to turbulent flow occurs very suddenly as the flow rate increases. The flow becomes unstable at some critical speed. Turbulent flow occurs when there are abrupt boundary surfaces. The flow of blood through a normal artery is laminar. However, when irregularities occur the flow becomes turbulent. The noise generated by the turbulent flow can be heard with a stethoscope. When the flow becomes turbulent there is a dramtic decrease in the volume flow rate as eddies and vortices are formed. Stationary plane v + dv v A C B (z + dz) D(z) Fig. 3.6.2.2 Any slowly moving liquid over a stationary plane can be subdivided in to several

NSOU ? CC - PH - 03 185 horizontal layers of liquids moving with different velocities in the vertical directions. The layer of liquid in contact with the stationary plane will be at rest due to adhesive forces. So, a velocity gradient dv/dz is established within the moving liquid and successive two layers try to reduce the relative velocity between these layers. In the figure 3.6.2.2 layer CD applies a tangential force F against the motion whereas the layer AB applies a tangential force F in the forward direction. Now, let us discuss Newton's law for this liquid movement. F ? A, Where A is the area of the tangential layer ? dv dz , Where dv dz is the velocity gradient. So, F ? A dv dz , or F = dv A dz η Therefore, ?? F/A dv dz = . (3.6.2.1) This ??is called co-efficient of viscosity and it depends on the nature of the liquid (gas). It is dependent on the temperature and pressure on the liquid (gas), For an ideal gas it depends on the temperature only. From the expression of co-efficient of viscosity one can derive an alternative definition: Co-efficient of viscosity is the force acting on unit area of a fluid moving steadily with unit velocity gradient as ? = F, if A =1 and dv dz =1. The dimension of ??is 1 1 ML T . - - ? ? ? ? In CGS system unit of???gcm –1 s –1 or Poise and in SI system it is Kgm -1 s -1 or 10 Poise.

186 NSOU ? CC - PH - 03 3.6.3 Newtonian and Non-Newtonian Fluid : The fluids for which Eq. 3.6.2.1 is applicable are said to be Newtonian fluid and those fluids for which F A is not proportional to dv dz at some definite temperature and pressure are known as Non-Newtonian fluid. 3.6.4 Critical velocity and Reynold's number: As the velocity of the fluid rises the motion turns from laminar flow to turbulent flow after becoming more than a particular velocity known as critical velocity. Reynold after doing an exhaustive study on fluid motion has shown that critical velocity of the fluid V c ?? pr η , where ? = Co-efficient of viscosity of the fluid, p = density of the fluid and r = radius of the tube through which fluid is moving. By dimensional analysis one can show that V c = pr η , where N is Reynold's number (dimensionless) for the fluid. (3.6.4.1) N is 1000 (approx) for water flowing through a capillary tube. Actually, one should consider G.N, where G.N is the ratio of pressure drag to the viscous drag and G~0.01 for spherical bodies. 3.6.5 Poiseuille's Equation: Let us consider an elementary tube of liquid having length L and radius r moving towards right of the diagram. Let P and P + ?P be the pressures on the right and left of the tube.

NSOU ? CC - PH - 03 187 P + P? Viscous force L P R Fig. 3.6.2.3 The driving force on the liquid cylinder of radius r due to the pressure difference is: ( ) 2 pressure F P r = Δ π (3.6.5.2) There will be a viscous drag force opposing the motion towards right, which depends on the surface area of the cylinder (length L and radius r): ( ) ( ) viscosity dv r F 2 rL dr = -η π (3.6.5.3) In developing the Poiseuille's equation one must make sure that (a) The liquid must be Newtonian. (b) The liquid layer in contact with the surface of the tube must be stationary. (c) The liquid flow should be steady. and (d) The pressure is same at all points in any cross-section of the tube. After satisfying all the above conditions for constant speed, as the net force goes to zero, we have pressure viscosity F F 0 + = ( ) ( ) 2 dv P r 2 rL dr Δ π = η π So ( ) ( ) 2 P r dv P .r dr 2 L 2 rL Δ π ? ? Δ = = ? ? η η π ? ? (3.6.5.4)

188 NSOU ? CC - PH - 03 We know that at the centre of the tube through which the liquid is flowing towards right r = 0 dv 0 dr = v is at its maximum, at the edge and r = R v = 0 dv P .r dr 2 L ? ? Δ = ? ? η ? ? Integrating with proper limits, R 0 v r P dv . rdr 2 L ? ? Δ = ? ? η ? ? ? ? ( ) 2 2 P v r R r 4 L ? ? Δ ? ? = - ? ? ? ? η ? ? (3.6.5.5) Using the equation of continuity which gives the volume flux for a variable speed, we get: dv v.dA dt = ? (3.6.5.6) In the above equation we substitute the velocity profile equation and the surface area of the moving cylinder: ( ) R 2 2 0 dv P v.dA R r . 2 rdr dt 4 L ? ? Δ ? ? = = - π ? ? ? ? η ? ? ? ?

NSOU ? CC - PH - 03 189 = ( ) R 2 3 0 . P R r r dr 2 L ? ? πΔ - ? ? η ? ? ? = 4 4 . P R R 2 L 2 4 ? ? ? ? πΔ - ? ? ? ? η ? ? ? ? = 4 . P.R 8 L πΔ η (3.6.5.7) This is Poiseuille's equation. Is this derivation for Poiseuille's equation correct? No, there are two corrections needed for completing this derivation of Poiseuille's eq. ?????Correction for kinetic energy–this correction arises due to the assumption that the force due to pressure difference is expended against viscous force. But actually the liquid coming out of the tube has a kinetic energy and the effective pressure operating for the movement of the liquid is less than the actual pressure difference. (2) In our derivation we have not considered any acceleration of the liquid as it enters the tube. However, the acceleration vanishes after the liquid travels a little distance within the tube. So, we have to take this acceleration into account. In order to consider the effect of acceleration usually the length is modified to have a larger value. 3.6.6 Determination of Co-efficient of viscosity by Poiseuille's method : A B C D l h 2 h 1 Fig. 3.6.6.1

190 NSOU ? CC - PH - 03 Initially the water level in the container is maintained at height h 1 from the central plane of the capillary tube held horizontally. The average radius 'R' of the capillary tube is determined by measuring the mass 'm' of a mercury thread of length 'L' and using the formula m R r L = ρ The length 'l' of the capillary tube is measured by a meter rule. If the height difference between the liquid columns in the container be 'h' where h = h 1 – h 2 , the pressure acting at the entry point of the capillary tube is P = hρg. The volume of water V collected by the container at D over a time 'T' measured by a stop watch provides 'v' which gives the volume of liquid flowing through the capillary tube per second. Then the co-efficient of viscosity of water at the temperature of the laboratory can be calculated via the formula 4 PR 8vl π η = ? Worked out Examples: 1. Water is pumped steadily out of a flooded basement at a speed of 5.30 m/s through a uniform hose of radius 9.70 mm. The hose passes out through a window 2.90 m above the water line. How much power is supplied by the pump? Solution: The kinetic energy of the water per unit mass when it leaves from the uniform hose through the window is 21 K 2 = v Here, mass of the flow of water is 1 and speed of the water flow is υ.

NSOU ? CC - PH - 03 191 The corresponding potential energy per unit mass of the flow of water through the window is U = gh Here, acceleration due to gravity of the Earth is g and height of the window from the basement is h. The volume rate of the flow of water from the hose through the window is R vA = ? ? Here, cross-sectional area of the hose is A and speed of the water flow is v ? . The mass rate of the flow of water is R m = ρR Here, density of the water is p. The power supplied by the pump is given by P = (K + U) R m Substituting the values, the power comes out to be P = 66.49 Watt. 2.An intravenous (IV) system is supplying saline solution to a patient at the rate of 0.120 cm 3 /s through a needle of radius 0.150 mm and length 2.50 cm. What pressure is needed at the entrance of the needle to cause this flow, assuming the viscosity of the saline solution to be the same as that of water? The gauge pressure of the blood in the patient's vein is 8.00 mm Hg. (Assume that the temperature is 20ºC.) ? Solution : Assuming laminar flow, Poiseuille's law applies. This is given by 4 2 1 1 r (P P) Q 8 π - = η where P 2 is the pressure at the entrance of the needle and P 1 is the pressure in the vein. The only unknown is P 2 .

192 NSOU ? CC - PH - 03 Solving for P 2 yields ( ) 2 1 4 8 P Q P r ? ? η = + ? ? π? ? ? P 1 is given as 8.00 mm Hg, which converts to 2 2 1.066 10 N m × . Substituting this and the other known values yields P 2 = 4 2 1.62 10 N m × . 3.6.7 Stokes' method of determination of co-efficient of viscosity of a liquid and Terminal velociy robber cork glycerin glass tube falling metal ball funnel A B When a metallic ball of spherical shape falls down through a less denser long liquid column, it's velocity increases due to gravitational force, and the opposing frictional force due to viscosity also increases. A stage is reached at a particular downward velocity at which the viscous force is just equal in magnitude but opposite in direction to the gravitational force. Under this condition, the falling metallic sphere moves with

NSOU ? CC - PH - 03 193 a constant velocity called terminal velocity v t . The British scientist, Sir George G. Stokes (1819-1903) showed that the retarding force F v due to viscosity acting upwards on a spherical body of a radius 'r' falling through a medium of viscosity ? is v t F 6 rv = πη (3.6.7.1) The eq (3.6.7.1) can also be derived from dimensional analysis as given below x yz F r v v t ∝ η M LT -2 = ( ) ( ) x y 1 1 1 z ML T LT L - - - 2 x x y z x y MLT M L T - - + + - - = (3.6.7.2) From eq (3.6.7.2), one can easily obtain x =1, y=1, z =1. Therefore, after substituting the values v t F v r ∝ η or v t F k v r η = (3.6.7.3) According to Stokes the above constant k 6 = π The weight of the spherical ball is, 34 r g 3 π σ (3.6.7.4) Where σ is the density of the spherical ball. The upward thrust exerted by the liquid medium on the spherical body is equal to the weight of the medium displaced by the body. The weight of the displaced liquid 34 r g 3 = π ρ (3.6.7.5) where ? is the density of the liquid.

194 NSOU ? CC - PH - 03 The net downwards force acting on the metallic sphere is, ( ) 34 F r g 3 = π σ-ρ (3.6.7.6) At equilibrium, i.e., when the two body moves with terminal velocity, eqn (3.6.7.1) and (3.6.7.6) can be equated as follows, ( ) 3 t 4 r g 6 rv 3 π σ-ρ = πη or ( ) 2 t 2 r v g 9 = σ-ρ η (3.6.7.7) So, from experiment if the values of v t , r, ρ, ? are determined one can find out the value of η. Procedure: ? The least count and zero correction of the given screw gauge are to be found. ? The diameter (d) of the ball using the screw guage is to be found. Now, the radius (r) of ball can be calculated as; r = d 2 . ? The inner diameter of the jar is to be measured using a vernier calipers. Hence the inner radius of the jar R can be found. ? Two reference points A and B on the jar are to be marked using two threads. The marking A is made well below the free surface of liquid, so that by the time when the ball reaches A, it would have acquired terminal velocity v t . ? The position the thread B is to be set so that the distance between A and B is of the order of 60cm. ? The ball of known diameter is to be dropped gently in the liquid. It falls down in the liquid with accelerated velocity for about 30% of the height. Then it falls with a uniform terminal velocity.

NSOU ? CC - PH - 03 195 ? When the ball crosses the point A, the stop watch should be started and the time taken by the ball to reach the point B is noted. ? If the distance moved by the ball is d and the time taken to travel is t, then velocity, t d v t = ? The terminal velocity of the ball v t is calculated using the relation, ( ) 2 t 2 r 9 v σ-ρ η= ? Now, the experiment is repeated by changing the diameter of the ball. The value of 2 t r v in each time is to be noted. A graph is to be plotted with r 2 along X axis and terminal velocity along Y axis. The coefficient of viscosity of the liquid is calculated by using the slope of the graph. ( ) 2 1 g 9 slope η = σ -ρ ? Worked out Examples : 1. A ball of copper of density 8960 kg/m 3 and of radius 1 mm has been allowed to fall through a column of castor oil having density 956 kg/m 3 and co-efficient of viscosity 0.65 Pa.s. Find the terminal velocity of the ball. Solution : Terminal velocity v t is given by v t = ( ) 2 r g( ) 2 9 ρ-σ × η = ( ) ( ) 2 3 8960 956 2 10 9.81 9 0.65 - - ? ? × × ? ? ? ? = 0.027 m/s

196 NSOU ? CC - PH - 03 2. In the oil-drop experiment Robert Millikan observed that an oil drop having density 3 0.851gm/cm ρ= obtained the terminal velocity in air 6 3 0.001293gm 171 10 Pand cm ? ? η = × σ= ? ? ? ? . He obtained the radius of the drop to be 1.64 ×10 -4 cm using Stoke's law. Justify his arguments. Solution : The terminal velocity obtained by the drop ( ) 2 t 2r g v 0.029cm/s 9 ρ-σ = = η after putting the values mentioned in the problem. The Reynold's number for the situation is 5 t rv N 3.6 10 1. - σ = = × >> η But G.N (G=0.01 for spherical bodies) = 3.6 ×10 –7 >>1. So, Millikan's action is justified. 3.6.8 Euler's equation of motion for fluids : Now let us consider the motion of a fluid moving with a steady flow. We want to tell about the changes the motion introduces in the system. If one wishes to describe mathematically the state of a moving fluid, he or she should use some mathematical functions which give the distribution of the fluid velocity v =v (x,y,z,t) can be one and the other can be the density ( ) x,y,z,t ρ . From the discussion on the thermodynamic properties cf systems we know that all the thermodynamic quantities are determined by the values of any two of them, together with the equation of state which connects several of them. So, if we get hold of five quantities like the three componens of the velocity v ? , the pressure p and the density of the fluid ?, the state of the moving fluid is completely determined for any observation.

NSOU ? CC - PH - 03 197 Let us fix our attention on the state of some elementary volume element dV bounded by a surface dA. The force acting on this volume element is p dA. Therefore, the total force on this volume is s pdA - ? The integral is taken over the surface bounding the volume. Using Gauss theorem on integrals we have pda pdv = - ∇ ? ? Hence we observe that the fluid surrounding any volume element within it exerts on that particular element a force dV p - ∇. So, we may conclude that a force - ∇ p acts on unit volume of the fluid. The equation of motion of a volume element in the fluid can now be written by equating the volume force to the product of the mass per unit volume ????and the acceleration dv ? /dt : dv dt ρ = -∇ρ (3.6.8.1) The total time derivative dv ? /dt which appears in eqn. (3.6.8.1) has a special significance: it denotes not only the rate of change of the fluid velocity v ? at a fixed point in space, but also the rate of change of the velocity of a given fluid particle as it moves about in space. We notice here that the total change dv ? in the velocity of the given fluid particle during the time interval dt is made up of two components, namely the change during dt in the velocity at a point fixed in space and the difference between the velocities (at the same time) at two points d r ? apart, where dr is the distance moved by the given fluid particle during the time dt. The first part is ( ) v t ∂ ∂ keeping x,y,z constant. The second part is ( ) v v v dx dy dz dr. v. x y z ∂ ∂ ∂ + + = ∇ ∂ ∂ ∂ ? ? ? Thus, ( ) ( ) v dv dt dr. v t ∂ = = ∇ ∂ ? ? ? ? ?

198 NSOU ? CC - PH - 03 dividing both sides by dt we get, ( ) dv v v. v dt t ∂ = + ∇ ∂ ? ? ? ? ? (3.6.8.2) Substituting this in eqn, (3.6.8.1) we get ( ) v 1 v. v p t ∂ + ∇ = - ∇ ∂ ρ ? ? ? ? (3.6.8.3) This is Euler's equation of motion for fluids. If the fluid moves in a gravitational field, there will be an extra force ?g where g is the local acceleration due to gravity. In that case the equation of motion takes the form ( ) v 1 v. v p g t ∂ + ∇ = - ∇ + ∂ ρ ? ? ? ? (3.6.8.4) 3.6.9 Equation of continuity : We shall now discuss one of the fundamental equations of fluid dynamics. Equation of continuity deals with conservation of matter. Let us consider some volume V 0 of thefluid. The mass of fluid contained in this volume is 0 V dV ρ ? , where ρ is the density of the fluid. The mass of fluid flowing in unit time through an element dA ? of the surface bounding this volume is ρ v ? . dA ? . Conventionally the vector dA ? is taken along the outward normal to have positive contribution. Then for fluid flowing out of the volume ρ v ? .dA ? is positive and it becomes negative if fluid goes in the volume. Then the total mass of fluid flowing out of the volume V 0 in unit time is s v.dA ρ ? ? ? where the integration has to be done over the bounding surface. One can find out the decrease in the mass per unit time inside the volume V 0

NSOU ? CC - PH - 03 199 dV t ∂ - ρ ∂ ? As mass is conserved, we have v s dv v.dA t ∂ ρ = ρ ∂ ? ? ? ? (3.6.9.1) The surface integral on the right hand side can be transformed to a volume integral by Green's formula ( ) s v v.dA . v dV ρ = ∇ ρ ? ? ? ? ? ? Thus we get, ( ) . V dV 0 t ∂ρ ? ? +∇ ρ = ? ? ∂ ? ? ? ? ? Since the volume element is quite arbitrary, the integrand must vanish for all dV. Thus, ( ) . v 0 t ∂ρ +∇ ρ = ∂ ? ? (3.6.9.2) This is the equation of continuity in differential form for moving fluid. 3.6.10 Bernoulli's theorem and it's application : Let us consider a fluid moving with a steady flow . For this system we can have the following thermodynamic relation dw = Tds + Vdp, where 'w' is the heat function per unit mass, 'S' is the entropy, 'V' is the specific volume given in terms of density as l V = ρ and 'T' is the temperature. For isentropic system (Entropy 's' is constant) we get

200 NSOU ? CC - PH - 03 dp dw Vdp , - = ρ So, p w ∇ ∇ = ρ Therefore, from the Euler's equation one gets, ( ) v v. v w t ∂ + ∇ = -∇ ∂ ? ? ? ? (3.6.10.1) Using a vector identity ( ) 2 1 v v curlV v. v 2 ∇ = × + ∇ ? ? ? ? ? , we arrive at 2 v 1 v v curl v W t 2 ∂ + ∇ - × = -∇ ∂ ? (3.6.10.2) When fluid motion is said to be in steady flow, the equations describing the steady flow get simplified.The fluid velocity v ? is a function of position coordinates only, there is no time vairation. So, we have v 0 t ∂ = ∂ . Under this condition eqn. (3.6.10.2) reduces to 2 1 v v xcurlv w 2 ∇ - = -∇ ? ? (3.6.10.3) Now we introduce the idea of streamlines. These lines are such that tangent at any point along this line gives the direction of flow at that point. Mathematically these are given by x y z dy dx dz v v v = = (3.6.10.4) One can further simplify the equation (3.6.10.3) by taking the scalar product with

NSOU ? CC - PH - 03 201 the unit vector n ˆ tangent to the streamline at each point. The projection of the gradient on any direction is the derivative in that direction. So, for the projection of w∇ is w . n ∂ ∂ For the vector v curlv × ? ? we get the projection on the direction of n ˆ to be zero as the vector is perpendicular to v ? . Thus we arrive at after simplification ( ) 21 v w 2 0 n ∂ + = ∂ . Therefore we can conclude that should right justified along a streamline 21 v w 2 + = constant. This equation is known as Bernoulli's equation. If now we consider the fluid motion in the gravitational field, then eqn. (3.6.10.3) gets modified. One has to add local acceleration due to grarvity g ? on the right hand side of the eqn. The projection of the acceleration due to gravity on the unit vector n ? is –g ? w n δ δ , if the acceleration due to gravity acts in the direction of z-axis. So, we ultimately have ( ) 21 v w gz 2 0 n ∂ + + = ∂ Thus on a streamline, Bernoulli's equation takes the form, 21 v w gz constant 2 + + = (3.6.10.5) Sometimes we use slightly different form for Bernoulli's equation, where we use P ρ for 'w' and h for 'z'. Then the form of Bernoulli's equation changes to

202 NSOU ? CC - PH - 03 21 P v gh constant 2 + + = ρ or, 2 v P h constant 2g g + + = ρ (3.6.10.6) here P gρ is known as pressure head, 2 v 2g is known as velocity head, h is known as elevation head and finally the sum of the above three is known as total head. One can say from this equation that if velocity increases for a fluid the pressure decreases while increase in pressure reduces the velocity of the fluid to keep total energy conserved. ? Worked out Example : Water is flowing through a tapering pipe having diameters 200 mm and 100 mm at sections 1 and 2 respectively. The discharge through the pipe is 20 liters/s. The section 1 is 10m above datum and section 2 is 5m above datum. Find the the pressure at section 2, if that at section 1 is 2 400kN m . Solution : Velocity of fluid at section 1, v 1 = ( ) 3 3 2 Q 20 10 m /s 0.6366m/s area 0.2 4 - × = = π Velocity of fluid at section 2, v 2 = ( ) 3 2 Q 20 10 2.546 m/s. area 0.1 4 - × = = = π Substituting the above values in Bernoulli's equation 2 2 1 1 2 2 1 2 p v p v z z w 2g w 2g + + = + +

NSOU ? CC - PH - 03 203 2 2 2 p (0,6366) (2.546) 400 10 5 (9.81 1000) 2 9.81 (9.81 1000) (2 9.81) + + = + + × × × × or, p 2 = 9.809 kN/m 2 3.6.11 Application of Bernoulli's principle: It has been observed that the thatched roofs or roofs made from asbestos sheets, of village houses get blown off during severe storms. When wind blows above the roof-tops with high speed, the local pressure decreases. However, the air inside the room being static generates high pressure zone. This pressure difference causes blowing up of the roof-tops. When an explosive bomb detonates in a section of the city, the air near the site of explosion moves with high speed while the air inside the rooms of houses cannot move so fast. As a result the pressure inside the room increases causing splintering of window glasses outside the rooms. A fast moving train drags the adjacent air layers along with it thus causing a drop in the air pressure. The air layers adjacent to a standing passenger near the edge of the platform are static and as a result the pressure at that are a is high. So when the train passes by the platform a difference in pressure happens which has a tendency of pushing the passenger towards the moving train. Hence it is advisable not to stand near the edge of the platform. It is a general observation that a table-tennis ball clings to the water jet moving upwards. The side of the ball near the edge of moving jet of water faces low pressure zone while the other face being adjacent to static air layers is at a high presure area. The change in air pressure pushes the ball towards the moving water jet. 3.6.12 Torricelli's theorem : Torricelli's theorem, is a theorem which relates the speed of fluid flowing out

204 NSOU ? CC - PH - 03 of an orifice to the height of fluid above the opening. Reservour water Holes for jet weakest jet Envelope Strpmgest ket P r esu re increa se w ith dep th Bernoulli's principle states that for an incompressible fluid with negligible viscosity. 2 v P h 2g g + + ρ = constant where v is fluid speed, g is the local acceleration due gravity (9.81 m/s 2), h is the fluid's height above a reference point, P is pressure, and ρ is density. Let us define the opening to be at h=0. At the top of the tank, P is equal to the atmospheric pressure. v can be considered 0 because the fluid surface drops in height extremely slowly compared to the speed at which fluid exits the tank. At the opening, h=0 and P is again atmospheric pressure. Eliminating the constant and solving gives: 2 atm atm P P v gh 2 + = + ρ ρ or, 2 v 2gh = or, v 2gh = This is Torricelli's theorem.

NSOU ? CC - PH - 03 205 3.6.13 Venturi meter : Venturi meter is an instrument used for measuring the speed and flow rate of liquid through a pipe. It is made up of a U-shaped tube filled partially with mercury. The venturi meter is connected to a pipe at two points as shown in the adjacent figure. The area of cross sections of pipe at ends where the venturi meter has been connected are A 1 and A 2 respectively. The corresponding speed of fluid are v 1 and v 2 . Let P 1 and P 2 are the pressure of fluid at the two ends 1 and 2. Bernoulli's equation can be written as, 2 2 1 1 2 2 1 1 P v P v 2 2 + ρ = + ρ The vertical height of pipe is same, so there is no contribution from the term ghρ h A ,v 1 1 A ,v 2 2 1 2 Here, P 1 <P 2 ,P 1 −P 2 = h g,ρ where ρ is the density of the liquid. Therefore, v 1 is less than v 2 . As the amount of liquid flowing through different sections of the tube is same, hence A 1 v 1 =A 2 v 2 , or, 1 1 2 2 v A v . A = 2 2 1 1 2 1 1 2 A 1 P P v v 2 A ? ? ? ? ? ? - = ρ - ? ? ? ? ? ? ? ?

206 NSOU ? CC - PH - 03 = 2 2 1 1 2 A 1 v 1 2 A ? ? ? ? ? ? ρ - ? ? ? ? ? ? ? ? From here one can easily find out the velocity of the liquid at the entry point (v 1). 1 2 2 1 1 2 gh 2 v A 1 A ? ? ? ? ? ? = ? ? ? ? - ? ? ? ? ? ? ? ? ? ? 3.6.14 Effect of temperature and pressure on the viscosity of liquids: In most cases, a fluid's viscosity increases with increasing pressure.Compared to the temperature influence, liquids are influenced very little by the applied pressure. The reason is that liquids (other than gases) are almost non-compressible at low or medium pressures. We can summarize the most different forms of temperature dependence of viscosity proposed under correlation methods by the following equation: In (η) = 2 2 3 n B D E F A a.logT b.T c.T T C T T T + + + + + + + + So, generally one can make a comment that viscosity of liquid decreases with rise in temperature. 3.6.15 Important points : ? Due to viscosity the relative velocity between successive layers of moving fluids is hindered. Co-efficient of viscosity is a measure of this hindrance. It's

NSOU ? CC - PH - 03 207 dimension is ML −1 T −1 . ? If the rate of flow is small the flow is laminar. If the rate of flow goes beyond critical velocity it turns turbulent. Critical velocity depends on Reynolds' number. ? The amount of fluid crossing a capillary tube in unit time is given by 4 Pr v . 8 1 π = η This is known as Poiseuille's equation. ? Stokes' law determines the fall of a solid sphere through a viscous liquid 2 gr ( ) 2 . 9 v ? ? ρ-σ η= ? ? ? ? ? ? 3.6.16 Questions (short answer type) : 1. State the difference between laminar and turbulent flow. What is critical velocity? 2. What are differences between Newtonian and non-Newtonian fluids? 3. What is the importance of Reynolds' number? 4. What are basic assumptions of Poiseuille's equation? 5. What conditions must be satisfied in Stokes' experiment? 6. State and explain Bernoulli's theorem. 7. How viscosity of a fluid vary with temperature and pressure? 3.6.17 Numerical Problems : 1. Water is in streamline flow through two capillaries, one of which is 1 m long and 1 mm in radius while the other is 60 cm long and 0.6 mm in radius. What is the pressure difference between the two ends of the second tube if that between the ends

208 NSOU ? CC - PH - 03 of the combination is 20 cm of water? 2. Two tubes of equal lengths but of different radii are connected in series. Use Poiseuille's formula without correction to obtain an expression for the volume of liquid flowing through the tube per second when the pressure difference between the two ends of the series is P. 3. Through a glycerine column a steel ball of density 7.8 ×10 3 kg/m 3 and of radius 2 mm is falling. Glycerine has a co-efficient of viscosity 0.83 Pas and its density is 1.2 ×10 3 kg/m 3 . Find the terminal velocity of the steel ball. 4. 850 cc water has flowed in 12 minutes through a horizontal capillary tube of length 20 cm and radius 0.08 cm, under 20 cm water pressure. Find the viscosity of water. 5. There is a hole in the vertical wall of a reservoir. If the depth of the hole is 2.7 cm from the top water level of the reservoir, what will be the velocity of efflux through the hole? 3.6.18 Answers to short questions: 1. See section 3.6.2 and section 3.6.4 2. See section 3.6.3 3. See section 3.6.4 4. See section 3.6.5 5. See section 3.6.7 6. See section 3.6.10 7. See section 3.6.14 3.6.19 Answers to numerical problems: 1. Poiseuille's formula is similar to Ohm's law, 4 8 1 P Z . V r η = = π If the capillary tubes
NSOU ? CC - PH - 03 209 are connected in series, the rate of flow V of water will be the same for all of them. Then ( ) 1 2 1 2 1 2 1 2 P P (P P) P V , Z Z Z Z Z + = = = = + where P is the pressure difference between the extreme ends and Z=Z 1 +Z 2 6 6 1 4 8 171 10 1 Z 435.5 10 (3.141 0.001 - × × × = = × × 6 6 2 4 8 171 10 0.6 Z 2020 10 (3.141 0.0006 - × × × = = × × Z = Z 1 + Z 2 = 2455.5 × 10 6 . Then, 2 2 20 2020 P Z P 16.45cm Z 2455.5 × × = = = 2. See solution 1. 3. Terminal velocity ( ) 2 t 2r g v 0.07m/s 9 ρ -σ = = η 4. Rate of flow of water = 850 1.18cc/s V (12 60) = = × 4 6 3.141 20 (0.08)4 Pr 0.136 10 P 8V (8 1.18 20) - × × π η= = = × × × l 5. 2 V 2 g h, = × × then V = 2×9.81×2.7, which gives V = 7.28 cm/s
210 NSOU ? CC - PH - 03 Unit–7 ? Special Theory of Relativity Structure : 3.7.1 Proposal : 3.7.2 The Michelson-Morley experiment : 3.7.3 Einstein's Postulates : 3.7.4 Lorentz Transformations : 3.7.5 Length Contraction : 3.7.6 Time diltion : 3.7.7 Lorentz Invariance : 3.7.7.1 Four Vectors : 3.7.8 Addition of Velocities : 3.7.9 The Relativistic Deppler Effect : 3.7.10 Relativistic nass : 3.7.11 Mass–Energy Equivalence : 3.7.12 Relativistic energy and momentum transformation : 3.7.12.1 Relation between Energy and Momentum of a Particle : 3.7.13 The Lorentz transformation equation for Newton's Laws of motion : 3.7.14 Substance : 3.7.15 Short Quations : 3.7.1 Proposal : Newton's three Laws of Motion along with the ideas about the properties of space and time provided a basis on which the motion of matter could be completely
NSOU ? CC - PH - 03 211 understood. However, the ideas propounded by Maxwell of a unified theory of electreomagnetism completely shttered the columns of superstructer of Physics. The theory of Maxwell was extraordinaryly successful, yet at a fundamental level it appered to be inconsistent with certain aspects of the Newtonian ideas of space and time. A radical modification of these letter concepts, and consequently of Newton's equations themselves, was found to be order of the day. It was the genius of Albert Einstein that combined the experimental results and physical arguments of others with his own unique insights and formulated the new principles of mechanics in terms of which space, time, matter and energy were to be understood. These principles along with their consequences constitute the

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Special Theory of Relativity. According to Special Theory of Relatively all laws of nature appear to have the same mathematical from in all inertial frames of reference and the speed of light

is constant in all directions. Latter, Einstein was able to further develop this theoy, leading to what is known as the General Theory of Relativity. ? Outcome This chapter gives an overview of the Special Theory of Relativity. After reading this chapter you will be able to (i) learn the deficiencies in ideas of space and time prevalent in the Newtonian Mechanics. (ii) learn about the unity of space and time in description of motion of a particle. (iii) get an idea about Lorentz transformation, Lorentz invariance. (iv) understand time dilation and space contraction and velocity addition theorem. (v) Lern about the relativistic Doppler effect. (vi) understand the equivalence of mass and energy : how mass varies with velocity. (vii) calulate energy, momentum in relativistic form and their transformations. (viii) write and interpret Newton's laws in relativistic covariant form

212 NSOU ? CC - PH - 03 3.7.2 The Michelson-Morley experiment The Newton's laws of motion and consequently the relativity principle derived from it were quit successful till the advent of Maxwell's mathematical theory of electromagnetism which, amongst other things, provied a successful physical theory of light. It was anticipated that the equations of Maxwell should also obey the Newtonian principle of relativity, or in other words Maxwell's equations should also have the same in all inertial frames of reference. Unfortuntely, it was found that this was not case. Maxwell's equation were found to assume completely different forms in different inertial frames of reference. But in the theory of Newton a tacit assumption about a special frame of reference was made. This 'special frame' S was assumed to be the one that deined the state of absolute rest as postulated by Newton, and that stationary relative to it was a most unsual entity, the ether. The ether was a substance that was supposedly the medium in which light waves were trnsmitted in a way something like the way in which air carries sound waves. Consequently it was belived that the velocity of light, as measured form a frame of reference moving relative to the ether would be diufferent from its value as measured from a frame of reference stationary with respect to the ether. This was the famous experiment of Michelson and Morley. It was 18 years later before the negative results of the experiment were finally explained, by Einstein. C C' L E E' Waves in phase waves out of phase ?x D F Source O A B U B' Fig 3.7.1 Schematic diagram of the Michelson-Morley experiment.
NSOU ? CC - PH - 03 213 The Michelson-Morley experiment was performed with an apparatus like that shown schematically in Fig. 3.7.1. The apparatus essentially comprises of a light source A, a partially silvered glass plate B, and two mirrors C and E, mounted on a rigid base. The mirrors are placed at equal distances (L) from B. The purpose of the plate B is to split an incoming beam of light, and the two resulting beams continue in mutually perpendicular directions to the mirrors, from where they are reflected back to B. On arrival back at B, the two beams are combined as two superposed beams, D and F. If the time taken for the light to go from B to E and back is the same as the time from B to C back, the emerging beams D and F will be in phase and will reinforce each other, but if the two times differ slightly, the beam will be slightly out of phase and interference will result. If the apparatus is "at rest" in the ether, the times should be presisely equal, but if it is moving towards the right with a velocity u ? , there shoul be difference in the times. In carrying out the experiment, Michelson and Morley set the apparatus in such way that the line BE was nearly parallel to the earth's motion in its orbit (at certain times of the day and night). The apparatus was amply sensitive to observe an effect of interference due to diference in arrival time, but no time difference wa found–the velocity of the earth through the ether could not be detected. The result of the experiment was null. Poincare then proposed that there is such a law of nature, that it is not possible to discover an ether wind by any experiment; that is, there is no way to determin an absolute velocity. Their experimental result appeared to say that the earth was not moving relative to the ether, which was obviously wrong since the earth was moving in a circular path around the Sun, so at some particular point in time it had to be moving relative to the ether. Many theoretical attempts were put forward to patch things up while still retaining the same Newtonian ideas of space and time. It was also suggested that the earth dragged the ether in its immediate vicinity along with it. Someone proposed that objects contracted in length along the direction parallel to

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the direction of motion of the object relative to the

ether. This suggestion, due to Fitzgerald and elaborated on by Lorentz, known as the Lorentz-Fitzgerald contraction, 'explained' the negative results of the Michelson-Morley experiment, but failed in part because no physical
214 NSOU ? CC - PH - 03 mechanism could be conceived that would be responsible for the contraction. It was Einstein who pointed the way out of impasse that required a huge revision of our concepts of space, and particularly of time. 3.7.3 Einstein's Postulates The difficulty of explaining null resut of Michelson-Morley experiment that had to be resolved amounted to choosing among three alternatives: 1. The Galilean transformation was correct but there were some problems in Maxwell's equations. 2. The Galilean transformation was applicable to Newtonian mechanics only. 3. The Glilean transformation, and the Newtonian principle of relativity based on this transformation were wrong and there mut be some other principle of relativity which consistently combines Maxwell's equations and Galilean transformations. The first possibility was thrown out as Maxwell's equations proved to be totally successful in application. The second was unacceptable as it preaches subject of non- universality of physical phenomena. The third was all that was left, so Einstein set about trying to uncover a new principle of relativity. His investigations led him to make

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two postulates: 1. All the laws of physics are the same in every inertial

frame of reference.
This postulate implies that there is no experiment from which it is possible to determin whether or not a frame of refence is in a state of uniform motion. 2. The speed of light in free space independent of the motion of its source. Einstein made those postulates through his study of the properties of Maxwell's equations. It is these potulates that force us to reconsider what we understnd by space and time. One immediate consequence of these two postulates is

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that speed of light is the same in all inertial frames

of reference.
We can see this by considering a source of light and two frames of reference, the first frame of reference S 0 stationary relative to the source of light and the other, S, moving relative to the source of light. In both of these frames the velocity of light, irrespective of the dynamical status of the frames, is found to be c.
NSOU ? CC - PH - 03 215 3.7.4 Lorentz Transformations The constancy of

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the speed of light, independent of the motion of the emanating source,

gives rise to the relations between time and space coordinates in different inertial frames of reference known as Lorentz transformations. Let us consider two inertial reference frames S and S? with a relative velocity v x between them. The time and space coordinates of a point under consideration are (t, x, y, z) and (t?, x?, y', z?) in the frames S and S?, respectively. All the coordinate axes in the two frames mentioned above are parallel and oriented such that the frame

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S? is moving in the positive x-direction with speed v

x , as viewed from S. Let the origins of the coordinates in S and S? be coincident at t =?t??= 0. If a light source at rest at the origin in S (and so moving with a speed v x in the negative x-direction as seen from S?) flickers on and off rapidly t = t??= 0, Einstein's second postulate implies that observers in both S and S??will see a spherical shell of radiation with increasing radius moving outward from the respective origins with speed c. The wave front reaches a point (x, y, z) in the frame S at a time t given by the equation. c 2 t 2 – (x 2 + y 2 + z 2) = 0 (3.7.3.1) Similarly, in the frame S??the same wave front is specified by c 2 t ?2 – (x ?2 + y ?2 + z ?2) = 0 (3.7.3.2) We assume that space-time is homogeneous and isotropic, as implied by the first postulate. Then the connection between the two sets of coordinates is linear. The events defined by equations (3.6.3.1) and (3.6.3.2?) are then related by c 2 t? 2 – (x? 2 + y? 2 + z? 2) = k[c 2 t 2 – (x 2 + y 2 + z 2)] (3.7.3.3) where k = k(v x) is a possibl change of scale between frames. With the choice of orientation of axes and considerations of the inverse transformation from S??to S it is straightforward to show that k(v x) = 1 for all v x and that the time and space coordinates in S??are related to those in S by the Lorentz transformation

42% **MATCHING BLOCK 95/123** **W**

x 0 ? = ?(x 0 – ?x 1) x 1 ? = ?(x 1 – ?x 0) 216 NSOU ? CC - PH - 03 x 2 ? = x 2 x 3 ? = x 3 (3.7.3.4) where we have used the notation x 0 = ct, x 1 = x, x 2 = y, x 3 =

z and also the symbols, x v , c β = β = β ? ? and 1 1 2 2 2 2 x 2 1 v 1 c - - ? ? - ? ? γ = = -β ? ? ? ? ? ? ? ? (3.7.3.5) The inverse Lorentz transformation is given by

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x 0 = ?(x 0 ? + ?x 1 ?) x 1 = ?(x 1 ??+ ?x 0 ?) x 2 = x 2 ? x 3 = x 3 ?

This can also be obtained by replacing v x by – v x and swapping primed and unprimed symbols in the first set of equations (3.7.3.4). This is how it must turn out, since if S??has velocity x v ? in S,then S has velocity – x v ? in S??and both are equaly valid inertial frames.The coordinates perpendicular to the direction of relative motion are unchanged while the parallel coordinate and the time are transformed. This can be contrasted with the Galilean trnsformations x??= x – vt, y??= y, z??= z and t??= t, where time is taken to be absolute. ? Worked out Example : Suppose an event A takes place in frame S at x a = 0 and t a = 0 and another event B takes lace at x B = b and t B = 0. These two events are simultaneous in S. Will they be simultaneous in frame S??which is moving with a velocity v along x? From the Lorentz transormations we get x A ??= 0, t A ??= 0, but x B ??= ?b and t B ??= ?(v/c 2)b. Now, according to S??clocks event B has occurred before event A. So, they are not simultaneous in frame S?. Exercise : Find the inverse Lorentz transformations from Eq. (3.7.3.4) NSOU ? CC - PH - 03 217 3.7.5 Length Contraction Let us extract from the Lorentz transformation the phenomenon of Lorentz contraction first. For Lorentz contraction, one should consider not two different events but two different world lines. They are the world lines of the two ends of some object in the x direction, fixed in S,. Now, we place the origin of the frame of reference S on one of these world lines, and then the other end lies at x = L 0 for al t, where L 0 is the rest length. Let us consider these world lines in the frame S??and pick the time t??= 0. At this moment, the world line passing through the origin of S is also at the origin of S?, i.e. at x? = 0. From the lorentz transformation, the other world line can be found at 0 2 vL t ; c ? ? γ - ? ? ? ? x??= ?(– vt + L 0) (3.7.4.1) Since we are considering the situation at t? = 0 we deduce from the first equation that x??= 2 0 0 2 L v x L 1 c ? ? ′ = γ - = ? ? γ ? ? Thus in the frame S??at a given instant the two ends of the object are at x??= 0 and 0 L x . ′ = γ Therefore the length of the object is reduced from L 0 by a factor ?. This is Lorentz contraction. ? Worked out Example:

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At what speed does a meters tick move if its length is observed to shrink to 0.5 m?

Our assumption is that the stick is at rest in S?. In S the meter stick is moving
218 NSOU ? CC - PH - 03 in the positive x direction with a speed of v. Now, we know that x? = ?(x – vt). Let ?x? be the length of the meter stick measured at ret in S?. Then ?x? = ??x, as ?t = 0. i.e. the measurements are done at the same time. So, it can be shown that ? 2 = ( ) 2 x 1 x Δ - ′ Δ . Now, ?x = ( ) 2 x 2 ′ Δ which gives ??= 0.866 or v = 0.866 c. 3.7.6 Time dilation : Our concept of time has to be drastically modified, as one considers the unexpected consequences of the Lorentz transformation. In a frame of reference S?, let us consider a clock C? placed at rest at some point x? on the X axis. Let us suppose that this frame is moving with a velocity x v ? relative to some other frame of reference S. At a time t 1 ? registered by clock C? there will be a clock C 1 in he S frame of reference passing the position of C? Figure 3.7.5.1 In frame S?? the stationary clock C?? reads t 1 ?? while passing a stationary clock in the frame S, which reads t 1 at the same instant.
NSOU ? CC - PH - 03 219 The time registered by C 1 will be given according to Lorentz Transformation as t 1 = 1 x 2 t v x c ′ ′ + ? ? γ ? ? ? ? (3.7.5.1) After some time the clock C??will register the time t 2 ??at which instant a different clock C 2 in the frame S will pass the position x 1 ??in S?. Now, this clock C 2 will show a time t 2 = 2 x 2 t v x c ′ ′ + ? ? γ ? ? ? ? (3.7.5.2) Thus one get from eq. (3.6.5.1) and (3.6.5.2) Δt = t 2 – t 1 = ?(t 2 ? + t 1 ?) = ?Δt? (3.7.5.3) It appears from the eq. (3.6.5.3) that time is passing slowly in he frame S? as observed from the frame S. This phenomenon is known as time dilation. Another important aspect of Lorentz Transformation is that events which take place simultaneously

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in one frame of reference will not appear simultaneous in another frame of reference which is moving with a velocity v with respect to

the
former one. To prove the above statement let us consider two events A and B which are taking places at x A and x B at the same time, i.e. t A = t B . Then according to Lorentz Transformations

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the time interval between these two events in S??will be t 2 ??– t 1 ??= x 2 x 1 2 1 2 2 v x v x t t c c ? ? ? ? γ - - γ - ? ? ? ? ? ? ? ? = 1 2 x 2 t x v c - ? ?

γ ? ? ? ? (3.7.5.4) is not zero as x 1 is not equal to x 2 . Thus events which are simultaneous in frame S are not simultneous in S?.
220 NSOU ? CC - PH - 03 ? Worked out Example : 1.

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At what speed does a clock move if it runs at a rate which is one-half the rate of a clock at rest?

Let us assume in the frame S? the clock is at rest. For an observer who is stationary in S, The same clock is moving in the positive x direction with a speed v. Now. 2 vx t t c ' ? ? ' =γ + ? ? ? ? Let ?t??= time interval measured in S??(proper time) when ?x??= 0 and let ?t be the time interval measured at rest in S. Then ?t = ??t??and therefore ( ) 2 2 t 1 t ? ? ' Δ β = - ? ? Δ ? ? Then ??= 0.866, when t 2 t Δ = Δ . 2. An atomic clock is placed in a jet airplane. The clock meaures a time interval of 3600s when the jet moves with a speed 400m/s. What will be the time interval recorded by an identical clock held by an observer at rest on the ground? Let us take the S frame to be attached to the Earth and

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the S? frame to be the rest frame of the

atomic clock. Now, 2 1 2 β γ + ? 2 t t t t 2 ' β Δ ' δ = Δ -Δ ? So, we get ?t = 3.2 ns when v = 400 m/s and ?t? = 3600 s. 3.

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The average lifetime of a ??meson in its own frame of reference is 26.0 ns. (This is its proper lifetime.)

c v
NSOU ? CC - PH - 03 221 (a)

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If the ??meson moves with speed 0.95c with respect to the Earth, what is its lifetime as measured by an observer at rest on

the
Earth? (b)

91% **MATCHING BLOCK 104/123** **SA** ELMP-1 - Mechanics.pdf (D137599141)

What is the average distance it travel before decaying as measured by an observer at rest on

the Earth? Let the S frme to be attached to the Earth and the S??frame to be the rest frme of the ??meson. We get from eq. (3.7.5.3) that ?t??= 26.0 ns and v = 0.95c.

85% **MATCHING BLOCK 105/123** **SA** ELMP-1 - Mechanics.pdf (D137599141)

The average distance travelled before decaying as measured by an observer at rest on

the Earth is v ?t = 24.0 m 4. The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at t = 0 is N 0 , the number N at time t is N = N 0 e −t/? where ??= 2.20 ?s is the mean lifetime of the muon. Suppose the muons move at speed 0.95c. What is the observed lifetime of the muons? How many muous remain after travelling a distance of 3.0 km? We take the S frame to be attached to the Earth and

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the S??frame to be the rest frame of the

muon. It follows from Eq. (3.7.5.3) that ?t = 7.046 × 10 −6 s when ?t? = 2.2 × 10 −6 s and ? = 0.95. A muon at this speed travels 3.0 km in 10.53 × 10 −6 s. After travelling this distance, N muons remain from an initial population of N 0 muons where N = N 0 e −t/τ = N 0 e −10.53/7.046 = 0.225N 0 . Exercise 2 :

97% **MATCHING BLOCK 109/123** **SA** MPDSC 1.1 Classical Mechanics.pdf (D133919389)

A rod of length L 0 moves with speed v along the horizontal direction. The rod makes an angle ? 0 with respect to the x??axis. 222

NSOU ? CC - PH - 03 (a) Deternine

90% **MATCHING BLOCK 108/123** **SA** MPDSC 1.1 Classical Mechanics.pdf (D133919389)

the length of the rod as measured by a stationary observer. (b) Determine the angle the rod makes with the x axis. 3.7.7

Lorentz Invariance : Lorentz invariance demands that the laws of physics are the same for different observers moving with constant different velocities like an observer who is rotated through some angle, or traveling at constant speed relative to the observer at rest. Before any discussion on Lorentz invariance it is advisable to discuss about the nature of vectors involved. So we start from a discussion on four-vectors. 3.7.7.1 Four Vectors : Because (ct, x, y, z) and have the similar transformations under changes of coordinate, we call them both 4-vectors. The vectors with which we are familiar can be defined as objects possessing a magnitude and a direction, or objects that transform in a well-defined way under rotations of coordinate system. 4-vectors are difined as objects that transform under the Lorentz transformations when converting between the measurements made by two different inertial (non-accelerating) observers. In the Lorentz transformation we get the description of the transformation of the coordinates of a point from one inertial frame to another. For rotation in three dimensions, the basic transformation law is defined in terms of the coordinates of a point. In three dimmensions we designate x 1 , x 2 , x 3 as the components of any vector. We describe by the same name any three physical quantities that transform under rotations in the same way as the components of x. We therefore anticipate that there are many physical quantities that trnform under Lorentz transformations in the same manner as the time and space coordinates of a point. By analogy we speak of 4-vectors. The coordinate 4-vector is (x 0 , x 1 , x 2 , x 3). Similarly the components of an arbitary 4-vector is (

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A 0 , A 1 , A 2 , A 4) where A 1 , A 2 , A 3 are the components of a 3-vector A.

The Lorentz transformation law for an arbitary 4-vector is
NSOU ? CC - PH - 03 223 A 0 ? = ?(A 0 – A.β ? ?) (3.7.6.1) A || ′ = ? (A || – ? A 0) (3.7.6.2) A ? ′ = A ? (3.7.6.3) here the parallel and perpendicular signs indicate components relative to the velocity v ? = ?c ? . The invariance from one inertial frame to another can be shown in

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the form 2 2 2 2 0 0 A A A A ′ ′ - = - ? ? (3.7.6.4) where the components (A 0 ?, A?) and (A 0 ,

A) refer to any two inertial reference frames. For two 4-vectors (A 0 , A 1 , A 2 , A 3) an (

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B 0 , B 1 , B 2 , B 3) the ‘‘scalar product’ is an invariant, that is, A 0 ?B 0 ??– A′ ? ??? B′ ? ?= A 0 B 0 – A ? . B ? (3.7.6.5)

This result can be verified by explicit construction of the left-hand side. It is the Lorentz transformation analogue of the invarince of A ? . B ? under rotation in three dimensions. 3.7.8 Addition of Velocities : Suppose an object A is moving in the positive x-direction with a velocity v ? relative to an object B, and B is moving with a velocity u ? (in the same direction) relative to an object C. What will be the velocity of A relative to C? In non-relativistic case, the velocities are simply added and the answer is that A C B A u ? w ? v ?
224 NSOU ? CC - PH - 03 is moving with a velocity w ? = u ? + v ? relative to C. But in special relativity the velocities must be combined using the formula 2 u v w 1 uv c + = + (3.7.8.1) If u ? and v ? are both small compared to the speed of light c, then the answer is approximtely the same as the non-relativistic theory. In the limit where u ? is equal to c ? (in the case C is a massless particle moving to the left at the speed of light), the sum gives c ? . This proves that anything going at the speed of light does so in all inertial reference frames. This change in the velocity addition formula the non-relativistic to the relatives theory is not due to making measurements without taking into account light-travel times, or the Doppler effect. Rather, it is what is observed after such effects have been accounted for. It is an effect of special relativity which cannot be accounted for using Newtonian mechanics. The formula can also be applied to velocities in opposite directions by simply changing signs of velocity values, or by rearranging the formula and solving for v ? . In other words, if B moving with velocity u ? relative to C and A is moving with velocity w relative to C then the velocity of A relative to B is given by 2 w u v wu 1 c - = - (3.7.8.2) Notice that the only case with velocities less than or equal to c ? that is singular is w ? = u ? = c ? , which gives the indeterminate value zero divided by zero. In other words, it is meaningless to ask for the relative velocity of two photons that are moving in the same direction. Originally we wanted to know the velocity of C as measured relative to A, and not the speed at which B observes A and C to aproach each other. The rulers and clocks
NSOU ? CC - PH - 03 225 set up by B cannot be used to measure distances and times correctly by A, since for A the clocks do not even show the same time. To go from the reference frame of A to the reference frame of B, a Lorentz transformation must be applied on co-ordinates in the following way (taking the x-axis parallel to the direction of travel and the space- time origins to coincide): x B = ?(v)(x A – vt A) (3.7.7.3) t B = ?(v) A A 2 v.x t c ? ? - ? ? ? ? (3.7.7.4) (v) = 2 2 v l c γ - (3.7.7.5) To go from the frame of B to the frame of C we should apply a similar transformation x c = ?(u)(x B – u t B) (3.7.7.6) t c = ?(u) B B 2 ux t c ? ? - ? ? ? ? (3.7.7.7) These two transformations can be combined to give a transformation which simplifies to x C = ?(w)(x A – wL A) (3.7.7.8) t C = ?(w) A A 2 wx t c ? ? - ? ? ? ? (3.7.7.9) 2 u v v , uv 1 c + = - this proves the velocity addition theorem for relativistic motions. A novel feature of the veloity addition formula is that if two velocities less than

226 NSOU ? CC - PH - 03 the speed of light are combined, one always get a result that is still less than the speed of light. This means that no amount of combining velocities can take any one beyond the speed of light. ? Worked out Examples : Two meteorites approach each other, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is 0.70c, Determine the velocities each meteorite as measured by the stationary observer on Earth. Lorentz velocity transformation gives x x x 2 u v u u v 1 c - ′ = + where u x is the velocity of an object measured in the S frame, u x ? is the velocity of the object measured in the S??frame and v is he velocity of the S??frame along

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the x-axis of S. Let us take the S frame to be			

attached to the Earth and the S??frame to be attached to the meteorite moving to the right velocity v. The other meteorite has velocity u x = −v in S and velocity u x ??= −0.70c in S?. We get 0.70 = ( ) 2 2 1 β +β , solving which yields ? = 0.41. As measured by the stationary observer on Earth, the meteorites are moving with velocities ? 0.41c. Exercise 3: Two space ships approach each other with velocities of 0.9c. According to an observer on the space ship, what is the velocity of the other ship.

NSOU ? CC - PH - 03 227 3.7.9 The Relativistic Deppler Effect There is a shift in frequency in the sound of a train's horn as the train passes by due to the relative motion of the train and the audience. Similarly, there is a shift in frequency of light due to relative motion of the source and observer. This is known as Doppler Effect. Relaivity modifies this Dopplerr Effect due to time dilation. Let us consider a source of sound at rest at the origin with an observer moving in the positive x-direction. We shall consider the possibility that the observer is located at some distance in y. The beginning of one wavelength is at t 1 = 0 and x 1 = y 1 = 0. The end of the wave is emitted at t 2 = ??and still at x 2 = y 2 = 0. This transform to the observers frame to be at ct 1 ??= ?(ct 1 − ?x) = 0, x 1 ??= ?(x − ?ct) = 0, y 1 ??= y 1 = 0 (3.7.8.1) ct 2 ??= ?(c? − ?x) = ?c? (3.7.8.2) x 2 ??= ?(x − ?c?) = − ??c? (3.7.8.3) y 2 ??= y 2 = 0 (2.7.8.4) ???= ?? (3.7.8.5) The time to emit the wave in the observer frame is dilated which decreases the frequency. If the wave travels to the observer in the y direction, the travel time is essentially the same for the beginning and the end of the wave so the frequency is not affected. That is the transverse Doppler effect gives a red-shift v ? ??= v γ (3.7.8.6) which is entirely a relativistic effect. (3.7.8.6) If the observe is moving directly away from the source we have the additional effect of the distance to the observer increasing with time which gives rise to the parallel Doppler effect. The time at which the beginning and end of the wave arrive at the observer is t 10 ??= 1 1 t x 0 c ′ ′ - = (3.7.8.7)

228 NSOU ? CC - PH - 03 t 20 ??= 2 2 t x c ′ ′ - = ???+ ????= ??(1 + ?)? (3.7.8.8) ( ) ( ) ( )( ) 0 1 1 1 1 1 1 +β +β ′ τ = +β τ = τ = τ -β +β -β (3.7.8.9) 1 v ' v 1 -β = +β ? (3.7.8.10) ??is positive for the observer moving away from the source and negative if the observer is moving towards the source. ? Worked out Example : How fast and in what direction must galaxy A be moving if an absorption line found at wavelength 550nm (green) for a stationary galaxy is shifted to 450 nm (blue) (a ?blue-shift?) for galaxy A? Galaxy A is approaching since an absorpton line with wavelength 550nm for a stationary galaxy is shifted to 450 nm. To find the speed v at which A is approaching, we use obs source 1 v v 1 +β = -β As ??= c v , obs source 1 1 -β λ = λ +β from which ??= (? source 2 − ? obs 2)/(? source 2 + ? obs 2) We get that ? = 0.198 when ? source = 550 nm and ? obs = 450 nm.

NSOU ? CC - PH - 03 229 3.7.10 Relativistic mass The mass or inertia of body is too affexted when measured in different inertial frames. To establosh the relation between how the mass varies with velocity, we
consider

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two references frames S and S??as in fig. 3.7.10.1 . S? is moving with a velocity v with respect to			

S-
frame, x, x?
coincident. S S′ B A x Figure : 3.7.9.1 We consider two identical masses A and B each of mass m measured in frame S??moving in opposite direction, collide and coalesce to form a single mass 2m. The from conservation of motion in S?-frame mu − mu = 0 = 2mV i.e V = 0 ... ... (3.7.10.1) Now we view the collision from S-frame of reference. Let u 1

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and u 2 be the velocities of the masses A and B respectively with respect to S-frame. Then

using relativistic velocity transformation equation, we can write, 1 2 v u u 1 uv c + = + and 2 2 v u u 1 uv c - = - ... ... (3.7.10.2) Then from conservation of momentum in S-frame,

17%	MATCHING BLOCK 115/123	SA	Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

m 1 u 1 + m 2 u 2 = (m 1 + m 2)v ( ) 1 2 1 2 2 2 v u v u m m m m v, 1 uv 1 uv c c + - + = + + - as they becomes at rest after ins S?- frame. v ? u ? v ? u ? 230 NSOU ? CC - PH - 03 Thus 1 2 2 2 v u v u m v m v 1 uv 1 uv c c ? ? ? ? ? ? ? ? + - ? ? ? ? - = - ? ? ? ? + - ? ? ? ? ? ? ? ? ? ? ? ? ? or, 2 2 1 2 2 2 2 2 v uv v m u 1 1 m u 1 c c c uv 1 c ? ? ? ? ? ? - + = - ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? - ? ? ? ? or, 1 2 2 2 uv uv m 1 m 1 c c ? ? ? ? - = + ? ? ? ? ? ? ? ? ... ... (3.6.10.3) Now ( ) ( ) ( ) ( ) ( ) ( ) 2 2 2 2 2 1 2 2 2 2 2 2 2 2 2 2 2 u v 1 c uv uv u 1 1 1 c c c u v u uv 1 1 1 uv c c 1 c + - - + - = = - - - + - or, ( ) ( ) 2 1 2 2 2 2 2 2 2 u uv 1 1 c c uv u 1 1 c

c - - = + - (3.7.10.4)

Using equation (3.7.10.4) in (3.7.10.3) we have or, 2 2 1 2 1 2 2 2 u u m 1 m 1 c c - = - ... ... (3.7.10.7) Obviously, eqn. (3.7.10.5) is applicable in all inertial rames for any values of u. If m 0 is the mass of a particle measured in a frame at rest with the body and m is the mass of the particle in reference frame moving with velocity u ? then from eqn. (3.7.10.5) NSOU ? CC - PH - 03 231 then, 0 2 2 m m u 1 c = - ... ... (3.7.10.7) Here m is referred as relativistic mass m 0 as rest mass. 3.7.11 Mass–Energy Equivalence To establish mass-energy equivalence in Einsten's special theory of relativity, we consider a mass be acted by an external force. In the cours of its motion the force applied on the mass m at instant t be F ? when its velocity is v ? . Let d v ? be the change of velocity in time interval dt. Then the change in kinetic energy ( ) ( ) ( ) d mv dT F.dr .dr v.d mv vd mv dt = = = = ? ? ? ? ? ? so the total kintic energy acquired by the body starting from rest and acquiring the velocity v ? is, ( ) v v 2 0 0 T v.d mv v dm mv dv ? ? = = + ? ? ? ? ? ? ... ... (3.7.11.1) Now from Einstein's mass variation equation m = 0 2 2 m v 1 c - We have 2m dmc 2 = 2m dmv 2 + m 2 (2v dv) dmc 2 = dm v 2 + mv dv ... ... (3.7.11.2) Using equations (3.7.11.1) and (3.7.11.2) ( ) 0 m 2 2 0 m T dmc m m c , = = - ? or, mc 2 = T + m 0 c 2 .... (3.7.11.3) We can visualize equation (3.7.11.3) as, that due to the application of force the energy increases from energy possesssed by the body at rest (m 0 c 2) or rest energy, to

232 NSOU ? CC - PH - 03 the energy possessed by the body at motion (mc 2) with respect to rest frame. Thus from equation (3.7.11.3) we can conclude the total energy possessed by the body E = mc 2 (3.7.10.4) This equation relates mass-energy and is known as relativistic mass-energy equivalence. 3.7.12 Relativistic energy and momentum transformation We consider two reference frames S and S?. S? is moving with respect to S frame with a velocity v ? in + x-direction with x and x??axis coincident (fig-3.7.8.9). A particle of mass m is moving with a velocity u ? in S-frame. Then total energy in S-frame, E = mc 2 = 2 0 2 2 m c v 1 c - ... ... (3.7.12.1) and momentum 0 2 2 p mu m u , v 1 c = = - ? ? ? with its components are x x 0 x 2 2 p mu m u , v 1 c = = - y y 0 y 2 2 p mu m u , v 1 c = = - z z 0 z 2 2 p mu m u v 1 c = = - (3.7.12.2) Let us view the corresponding energy and momentum from S?-frame. u ' ? be the velocity of the particle in S?-frame. From Lorentz transformation,

NSOU ? CC - PH - 03 233 ( ) x z 2

23%	MATCHING BLOCK 116/123	SA	Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

u v y 2 x x u y z 2 x x 2 2 v u 1 u vu c ;u and u 1 vu vu c 1 1 c c - - ' = ' ' = = - ? ? ? ? - - ? ? ? ? ? ? ? ? ... ... (3.7.12.3) Equation (13) yields to 2 2 2 2 2 2 x 2 u v 1 1 c c u 1 vu c 1 c ? ?? ? - - ? ?? ? ? ?? ? ? ? ' ? ?? ? - = ? ? ? ? ? ? ? ? - ? ? ? ? (3.7.12.4) 2 x 2 2 2 2 0 0 x 2 2 2 2 2 2 2 2 2 2 vu mc 1 m c m c E vP c E m c , u v u v v 1 1 1 1 1 c c c

c c ? ? - ? ? - ? ? ' ' = = = = = ' - - - - - using eqn. (3.7.12.3) where p x = x component of momentum with respect to S–frame. The inverse transformation equation will be, x 2 2 E vp E , v 1 c ' ' + = - Similarly the momentum in S?-frame, ( ) 0

46% **MATCHING BLOCK 117/123** **SA** Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

x x x 2 x 2 2 x 0 2 2 2 x 2 2 2 2 m p m u (u v) vE u p 1 u v m c c vu u v v 1 1 1 1 c c c c ' ' ' = = - ' - - - = = ? ? - ? ? - - - ? ? ... (3.7.12.5) 234

NSOU ? CC - PH - 03 2 2 x y 2 2 2 0 y 0 0 y y y y y 2 2 2 2

38% **MATCHING BLOCK 118/123** **SA** Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

x x 2 2 2 2 2 2 vu v v u 1 1 1 m u m m c c c p m u u mu p vu vu u u u v 1 1 1 1 1 1 c c c c c c ? ? - - - ? ? ? ? ' ' ' ' = = = = = = ? ? ' ' ? ? - ? ? - - - - - ? ? ? ? ? ? (3.7.12.6

Similartly p? z = p z ... Set of equations (3.7.12.4), (3.7.12.5) and (3.7.12.6) constitute momentum transformation equation. 3.7.12.1 Relation between Energy and Momentum of a Particle: We have seen earller the equivalence of mass and energy as E = mc 2 . So, one gets E 2 = m 2 c 4 , or one can write
E 2 =

67% **MATCHING BLOCK 119/123** **SA** Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

m 2 c 2 – m 0 2 c 4 + m 0 2 c 4 . So, E 2 = 2 4 0 2 2 m c [1] [1] (1 v) c - - + m 0 2 c 4= 2 2 2 4 0 2 2 (v) c m c (1 v c ? ? ? ? + ? ? - ? ? ? ? ? ? m 0 2 c 4 = p 2 c 2 + m 0 2 c 4 . (3.7.11.7) 3.7.13

The
Lorentz transformation equation for Newton's Laws of motion Consider a particle of mass m moving in S-frame with a velocity ( ) x y z u. p ,p ,p ? are the (x, y, z) component of momentum. Then in S-frame, x x dp F , dt = y y dp F dt = and z z dp F dt = . In S?-frame the corresponding components are,
NSOU ? CC - PH - 03 235 x x dp F , dt ' ' = ' y y dp F dt ' ' = ' and z z dp F . dt ' ' = ' Now from Lorentz transformation 2 2 x 2 v 1 dt c dt vu 1 c - = ' ? ? - ? ? ? ? ... ... (3.7.12.1) 2 x x 2 2 2 x x x 2 x x 2 2 2 v dp dp v dE v dE 1 dp dp dt dt dt dt dt c c c F dt dt dt vu vu v 1 1 1 c c c = - - ' ' ' = = = = ' ' ? ? ? ? - - ? ? ? ? - ? ? ? ? (3.7.12.2) 2 2 y y y y x 2 v 1 dp dp dp dt c F dt dt dt dt vu 1 c - ' ' ' = = = ' ' ? ? - ? ? ? ? ... ... (3.7.12.3) 2 2 z z z y x 2 v 1 dp dp dp dt c F dt dt dt dt vu 1 c - ' ' ' = = = ' ' ? ? - ? ? ? ? ... ... (3.7.12.4) 3.7.14 Substance After a thorough learning of this important chapter you must have understood that our common sense always does not work. Both time and displacement have equal roles in understanding the motion of any object. Through Lorentz transformations we can
236 NSOU ? CC - PH - 03 transform respect to the other and vice-versa. Length contraction and time dilations are major fallouts of these transformations. Equvalence of mass and energy shows clearly the importance of reltivistic kinematics. Newton's laws motion can be interpreted in terms of relativistic transformations. The outbound velocities of distant stars can be found from the relativistic Doppler Effect. 3.7.15 Short Quations: 1. State the differences between inertial an non-inertial frames of reference. 2. Explain why any object cannot move with a speed more than the speed of light. 3. Calculate the rest energy of electron and proton in electron Volt. Given m e = 9.11 x 10 −31 kg and m p = 1.673 × 10 −21 kg. 4. A cubical shape of body with 1m length on each side when it is rest, moves with a velocity 0.6c along x – direction. What is the shape and dimension of the body noted by an observer on the ground. 5. Two ? particles move in opposite direction with velocity 0.6c in the laboratory, S??frame. Calculate the velocity of one ? particle in the moving frame attached to the other ? particle by applying relativistic transformation. 3.7.16 Answer to Exercises and short questions : Ex 1. Use eq. (3.6.3.4) Ex. 2.

98% **MATCHING BLOCK 120/123** **SA** MPDSC 1.1 Classical Mechanics.pdf (D133919389)

Let us take the S??frame to be the rest frame of the rod. A rod of length L 0 in S??makes an angle ? 0 with the x' axis. Its projected length ?x 0 and ?y 0 =

77%	**MATCHING BLOCK 121/123**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

L 0 cos? 0 and ?y 0 = L 0 sin ? 0 . In a frame S in which th rod moves at speed v along the x axis, the projected lengths ?x and ?y are given by ?

x′ = ???x and ?y? = ?y, as y? = y.

100%	**MATCHING BLOCK 122/123**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

The length L of the rod as measured by a stationary observer in S is ( ) ( ) ( ) 1 2 2 2 2 2 0 0 L

x y L 1 cos = Δ + Δ = -β θ
NSOU ? CC - PH - 03 237

96%	**MATCHING BLOCK 123/123**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

The rod makes an angle ??with the x axis in S where tan ? =

y x Δ Δ = ??tan ? 0 . The rod in S appears contracted and rotated. Ex. 3 Use the velocity addition formula, v? = 2 (u v) (1 uv) c + + . Both u ? and v ? are 0.9c. 3.7.17 Short Question : 1. See text. 2. Use velocity addition theorem. 3. Use E = mc 2 . 4. x 0 = y 0 = z 0 = 1m. v = 0.6c, x = x 0 2 2 v 1 , c - y = y 0 and z = z 0 . We get x = 0.8m, y = 1m and z = 1m. 5. u 1 = 0.6c, u 2 = – 0.6c in S-frame and u 2 ? = 2 2 2 (u v) (1 u v) c - + in S? frame. u 2 = – 0.6c and v = + 0.6c which gives u 2 ? = – 0.88c.
238 NSOU ? CC - PH - 03 Notes
NSOU ? CC - PH - 03 239 Notes
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1/123	**SUBMITTED TEXT** 19 WORDS	**69%**	**MATCHING TEXT** 19 WORDS
	Centre of mass 3.1.7 Uniquencess of the position of the centre of mass 3.1.8 Motion of the Centre of Mass 3.1.9		centre of mass and the independent motion of the centre of mass. The motion of the centre of mass

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	Angular Momentum of a System of Particles 3.1.10 Angular Momentum of a System of Particles		Angular Momentum of a System of Particles 179 8.2 Angular Momentum of a System of Particles

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3/123	SUBMITTED TEXT	20 WORDS	57%	MATCHING TEXT	20 WORDS
need to take into account the rotation of the Earth around its own axis, the rotation of the Earth around the			need to take into account the rotation of the earth itself or the movement of the Moon around the		

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4/123	SUBMITTED TEXT	11 WORDS	95%	MATCHING TEXT	11 WORDS
which is rotating with a constant angular velocity with respect to			which is rotating with a constant angular velocity ω with respect to		

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5/123	SUBMITTED TEXT	12 WORDS	100%	MATCHING TEXT	12 WORDS
A small weight of mass m hangs from a string in			A small weight of mass m hangs from a string in		

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6/123	SUBMITTED TEXT	15 WORDS	81%	MATCHING TEXT	15 WORDS
What is the static angle of the string with the vertical and what is its tension?			What is the static angle of the string from the vertical, and what is the tension		

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7/123	SUBMITTED TEXT	15 WORDS	83%	MATCHING TEXT	15 WORDS
A frame moving with a constant velocity with respect to an inertial frame is inertial,					

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8/123	SUBMITTED TEXT	21 WORDS	70%	MATCHING TEXT	21 WORDS
the centre of mass of the system of particles. The position co-ordinate of the centre of mass of a system of					

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9/123	**SUBMITTED TEXT**	11 WORDS	**71%**	**MATCHING TEXT**	11 WORDS

x m x M = ? (3.1.6.3) cm i i i 1 y m y M = ? (3.1.6.4) cm i i i 1 z m z M = ? (3.1.6.5)

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

10/123	**SUBMITTED TEXT**	32 WORDS	**71%**	**MATCHING TEXT**	32 WORDS

m x m x m x m x 1 x m m m m m m m m m m m m ? ? + + = + + + ? ? + + + + + + ? ?	m2 (m2 ; = . a = X − x1 = m1 x1 + m2 x2 ; ; m + m m2 .; X = (x − x) m m 1 1 m1 + m 2 ; X = 1 2 = . ; 2 m1 + m 2 m1 + m 2 ; ; ;

W https://dokumen.pub/mechanics-9788131773734-9788131798805-8131773736-9789332515604-9332515603.html

11/123	**SUBMITTED TEXT**	19 WORDS	**70%**	**MATCHING TEXT**	19 WORDS

m x m m x x m m m m + + + = + + + (3.1.6.11) m 1 m 2 m 3 m 4 x 1 x 2 x 3	m2 (x2 − x1) m2 ; = . a = X − x1 = m1 x1 + m2 x2 ; ; m + m m 1 2 1 + X = (x − x)

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of the centre of mass of the system of four particles is given by 1 1 2 2 2 2 4 4 cm 1 2 3 4 m x m x m x m x x m m m m + + + = + + + (3.1.6.10)

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13/123	**SUBMITTED TEXT**	19 WORDS	**55%**	**MATCHING TEXT**	19 WORDS

the center of mass of a system can be calculated from the position of the centre of mass of

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14/123	**SUBMITTED TEXT**	18 WORDS	**55%**	**MATCHING TEXT**	18 WORDS

to show that the position of the center of mass does not depend on the origin of the	to show that the work of the potential force F, W12 , does not depend on the choice of the

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15/123 **SUBMITTED TEXT** 12 WORDS	**100%** **MATCHING TEXT** 12 WORDS
momentum of a system of particles. 3.1.9 Angular Momentum of a System of Particles	Momentum of a System of Particles 179 8.2 Angular Momentum of a System of Particles

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16/123 **SUBMITTED TEXT** 14 WORDS	**71%** **MATCHING TEXT** 14 WORDS
the system. For example, the position of the centre of mass of a system	

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17/123 **SUBMITTED TEXT** 14 WORDS	**83%** **MATCHING TEXT** 14 WORDS
m r — (3.1.8.1) Where M is the total mass of all the particles.	

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18/123 **SUBMITTED TEXT** 14 WORDS	**84%** **MATCHING TEXT** 14 WORDS
The rate of change of the angular momentum of the system of particles	the rate of change of the total angular momentum of a system of particles (

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19/123 **SUBMITTED TEXT** 11 WORDS	**100%** **MATCHING TEXT** 11 WORDS
the cross product of a vector with itself is zero	the cross product of a vector with itself is zero.

W https://kupdf.net/download/px148-notes_59f08cc2e2b6f54e4b36a411_pdf

20/123 **SUBMITTED TEXT** 15 WORDS	**66%** **MATCHING TEXT** 15 WORDS
that if the momentum of the system is zero, the angular momentum is the same	that if the total linear momentum of a system of particles is zero, the angular momentum of the system is the same

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21/123 **SUBMITTED TEXT** 20 WORDS **52%** **MATCHING TEXT** 20 WORDS

angular momentum of the system of particles about the same point P. 3.1.10 Angular Momentum of a System of Particles

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22/123 **SUBMITTED TEXT** 13 WORDS **95%** **MATCHING TEXT** 13 WORDS

which is moving with a velocity v ? with respect to the

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23/123 **SUBMITTED TEXT** 15 WORDS **73%** **MATCHING TEXT** 15 WORDS

work done by a force is equal to the change in potential energy of the

SA ELMP-1 - Mechanics.pdf (D137599141)

24/123 **SUBMITTED TEXT** 14 WORDS **71%** **MATCHING TEXT** 14 WORDS

kinetic energy of relative motion before collision is equal to the kinetic energy of

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25/123 **SUBMITTED TEXT** 43 WORDS **76%** **MATCHING TEXT** 43 WORDS

m v m v 2 2 ? ? ? ? ? ' ' x ? ? ? ? ? ? ? ? ? = 2 2 1 2 2 1 1 1 m v m v 2 2 ? ? ? ? ? + ? ? ? ? ? ? ? ? ? (3.1.15.6) and also 1 2 1 2 v v v

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26/123 **SUBMITTED TEXT** 26 WORDS **50%** **MATCHING TEXT** 26 WORDS

m v v v v v m m = - = - + ( ) 1 2x,i 2x,i x,cm 2x,i 2x,i 1 2 m v v v v v m m ' ' = - = - + (3.1.16.2)

m1 v1 + m2 v2 ; v = v1 – v2 ; m1 + m 2 μ μ v; v2 = V + v. v1 = V – m1 m2

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27/123 **SUBMITTED TEXT** 18 WORDS	**52%** **MATCHING TEXT** 18 WORDS
m v v v v m m ′ = = - + ( ) 1 1 2 2x,f 2x,i 2x,i x,i 2 m v v v v m m ′ = = - +	m2 v2 ; v = v1 – v2 ; m1 + m 2 μ μ v; v2 = V + v. v1 = V – m1 m2

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28/123 **SUBMITTED TEXT** 15 WORDS	**63%** **MATCHING TEXT** 15 WORDS
velocity of the center of mass is 1 1x,i 1 2x,i x,cm 1 2 m v m v v m m + = + (3.1.16.1)	

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29/123 **SUBMITTED TEXT** 12 WORDS	**83%** **MATCHING TEXT** 12 WORDS
The dot product of two non-zero vectors is zero when the	the dot product of two non-zero vectors is nevertheless zero if the

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30/123 **SUBMITTED TEXT** 27 WORDS	**63%** **MATCHING TEXT** 27 WORDS
m i m v m v v m m m m + - + + + ? ? = 1 2 2 1x,i 2x,t 1 2 1 2 m m 2m v v m m m m - + + + ? ? (3.1.16.4) ?	

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31/123 **SUBMITTED TEXT** 24 WORDS	**76%** **MATCHING TEXT** 24 WORDS
velocity v ? are x ′ = v x t, y ′= v y t, z ′ = v z t.	velocity v0 , we have x0 + v0x t y = y0 + v0y t 1 z = z0 + v0z t –

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32/123 **SUBMITTED TEXT** 20 WORDS	**57%** **MATCHING TEXT** 20 WORDS
When the resultant (external) force acting on a particle is zero, the total linear momentum of the particle remains constant	

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33/123 **SUBMITTED TEXT** 18 WORDS	**52%** **MATCHING TEXT** 18 WORDS
the axis of rotation must pass through the centre of mass of the body. If we use the origin	the axis of rotation passing through the center of mass of the body and the origin

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34/123 **SUBMITTED TEXT** 21 WORDS	**45%** **MATCHING TEXT** 21 WORDS
The center of gravity of a body or a system of particles is the point about which the vector sum of the	

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35/123 **SUBMITTED TEXT** 12 WORDS	**100%** **MATCHING TEXT** 12 WORDS
is the perpendicular distance from the axis of rotation to the	is the perpendicular distance from the axis of rotation to the

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36/123 **SUBMITTED TEXT** 13 WORDS	**80%** **MATCHING TEXT** 13 WORDS
of inertia, theorems of moment of inertia, calculation of moment of inertia of	

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37/123 **SUBMITTED TEXT** 57 WORDS	**41%** **MATCHING TEXT** 57 WORDS
r v r r r r a a = ω× + ω× = α× + ω× ω× = + ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ( ) t r a r a v r ; α ω ω ω = × = × = × × ? ? ? ? ? ? ? ? ? ? (3.2.2.5)	

SA NGEA16_2 %282013_12_20 10_17_12 UTC%29.pdf (D21212887)

38/123 **SUBMITTED TEXT** 19 WORDS	**61%** **MATCHING TEXT** 19 WORDS
r ? (3.2.5.9) where r j is the perpendicular distance of the jth particle to the axis of rotation. 3.2.6	R|ω| where R is the distance of the particle from the axis of rotation.

W https://vdoc.pub/documents/solved-problems-in-classical-mechanics-analytical-and-numerical-soluti ...

39/123 SUBMITTED TEXT 11 WORDS	**100%** MATCHING TEXT 11 WORDS
mass, m, at a distance r from the axis of rotation,	

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

40/123 SUBMITTED TEXT 12 WORDS	**90%** MATCHING TEXT 12 WORDS
The kinetic energy of a particle of mass m moving in	the kinetic energy of the particle. 5. A particle of mass m moving in

W https://kupdf.net/download/classical-mechanics_5af85f45e2b6f59a757da680_pdf

41/123 SUBMITTED TEXT 25 WORDS	**57%** MATCHING TEXT 25 WORDS
m x x m x x m x x I m x x m x x m x x m x x m x x m	m2 (x2 – x1) m2 ; = . a = X – x1 = m1 x1 + m2 x2 ; ; m + m m 1 2 1 + m2 .; X = (x – x) m m 1 1 m1 + m 2 ; b = x – X = 1 2 = . ; 2 m1 +

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42/123 SUBMITTED TEXT 25 WORDS	**60%** MATCHING TEXT 25 WORDS
consider a rigid body containing n particle of mass m i , i = 1, n. If the body rotates with an angular velocity ??about	

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43/123 SUBMITTED TEXT 15 WORDS	**66%** MATCHING TEXT 15 WORDS
the velocity of the particle is given by equation v ? = V ? + ω ? x i r ? (3.2.7.1) The	the instantaneous velocity of the particle is given by v|s = v|b + ω × r Likewise, the

W https://kupdf.net/download/classical-mechanics_5af85f45e2b6f59a757da680_pdf

44/123 SUBMITTED TEXT 23 WORDS	**43%** MATCHING TEXT 23 WORDS
Moments of Inertia about the 1 2 x ,x - - and 3 x - axes, respectively. The negatives of the off-diagonal elements are the Products of Inertia.	moments of inertia about the x-, y- and z-axes, respectively. The quantities Ixy , Iyz , Ixz defined in (4) are known as the products of inertia

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45/123 **SUBMITTED TEXT** 14 WORDS	**71%** **MATCHING TEXT** 14 WORDS
the origin of the body coordinte system to coincide with the center of mass.	

SA Physics_Vol-1 EM.pdf (D40552326)

46/123 **SUBMITTED TEXT** 21 WORDS	**77%** **MATCHING TEXT** 21 WORDS
α α α α α α α α α α α α α α α α α α α α α α α α	α = α α α a α , e α = α e α α α . (11.65) Thisfollowsasasimpleconsequenceof α α and α α beinginversesofoneanother: δ α β = α α α α β , δ α β = α α

W http://213.230.96.51:8090/files/ebooks/Fizika/Benacquista%20M.J.,%20Romano%20J.D.%20Classical%20m ...

47/123 **SUBMITTED TEXT** 16 WORDS	**62%** **MATCHING TEXT** 16 WORDS
the origin of the coordinate system is not located at the centre of mass of the	the origin of the body frame is located at the center of mass of the

W http://213.230.96.51:8090/files/ebooks/Fizika/Benacquista%20M.J.,%20Romano%20J.D.%20Classical%20m ...

48/123 **SUBMITTED TEXT** 83 WORDS	**55%** **MATCHING TEXT** 83 WORDS
The moment of inertia (I) of a body about any axis is the sum of its moment of inertia (I cm) about a parallel axis through the centre of mass and the product of the mass (M) of the body by the square of the distance (d 2) between the two axes. Proof: Let I cm be the moment of inertia of a body of mass M about an axis passing through its centre of mass. Let I be the moment of inertia	

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

49/123 **SUBMITTED TEXT** 16 WORDS	**78%** **MATCHING TEXT** 16 WORDS
The moment of inertia of the body about the axis passing through centre of mass is	

SA Unit II Notes.docx (D113412638)

50/123 **SUBMITTED TEXT** 27 WORDS **55%** **MATCHING TEXT** 27 WORDS

r ′ ? = r ? + d ? . So, we get r 2 ′ = d 2 + 2 d ? . r ? + r 2
Now, the moment of inertia of the

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

51/123 **SUBMITTED TEXT** 19 WORDS **68%** **MATCHING TEXT** 19 WORDS

moment of inertia about an axis perpendIcular to the plane) with I x , I y (moment of inertia

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

52/123 **SUBMITTED TEXT** 12 WORDS **78%** **MATCHING TEXT** 12 WORDS

The moment of inertia of the body about the z-axis is given by

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

53/123 **SUBMITTED TEXT** 15 WORDS **80%** **MATCHING TEXT** 15 WORDS

K is called the radius of gyration of the given object about the given axis. 3.2.11

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

54/123 **SUBMITTED TEXT** 13 WORDS **84%** **MATCHING TEXT** 13 WORDS

M A , where M is the mass of the disk and A is

m i a 2 where m i is the mass of the ith particle and a i is

W https://www.damtp.cam.ac.uk/user/tong/relativity/stephen.pdf

55/123 **SUBMITTED TEXT** 15 WORDS **70%** **MATCHING TEXT** 15 WORDS

end of the rod. We wish to find the moment of inertia about this axis (

SA Physics_Vol-1 EM.pdf (D40552326)

56/123 **SUBMITTED TEXT** 32 WORDS **28%** **MATCHING TEXT** 32 WORDS

A Uniform Thin Disk about an Axis through the center In this problem we want to find out the moment of inertia of a two-dimensional object-a uniform thin disk about an axis through

SA Physics_Vol-1 EM.pdf (D40552326)

57/123 **SUBMITTED TEXT** 15 WORDS **84%** **MATCHING TEXT** 15 WORDS

about an axis passing through its center and perpendicular to its plane. The moment of inertia

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

58/123 **SUBMITTED TEXT** 17 WORDS **88%** **MATCHING TEXT** 17 WORDS

of inertia of a rod about an axis passing through its center of mass and perpendicular to

SA Physics_Vol-1 EM.pdf (D40552326)

59/123 **SUBMITTED TEXT** 15 WORDS **89%** **MATCHING TEXT** 15 WORDS

SUBMITTED TEXT	MATCHING TEXT
The moment of inertia for a solid sphere of radius R and mass M	the moment of inertia of a solid sphere of radius R and mass M (

W https://vdoc.pub/documents/lecture-notes-on-newtonian-mechanics-lessons-from-modern-concepts-2vk1 ...

60/123 **SUBMITTED TEXT** 12 WORDS **83%** **MATCHING TEXT** 12 WORDS

The moment o inertia of a rod of mass M and length

SA Physics_Vol-1 EM.pdf (D40552326)

61/123 **SUBMITTED TEXT** 11 WORDS **76%** **MATCHING TEXT** 11 WORDS

SUBMITTED TEXT	MATCHING TEXT
r ′ ? =R ? +r ? (3.2.14.2) where R ? is the position vector of the origin of	r 1 = R + m 1 m 1 + m 2 r where R is the position vector of the centre of

W https://www.damtp.cam.ac.uk/user/tong/relativity/stephen.pdf

62/123	SUBMITTED TEXT	46 WORDS	54%	MATCHING TEXT	46 WORDS
x x 2 2 ' " = ˆ φ ? ? x 2 ' x 2 " x 1 " x 1 ' x 1 " =x 1 "' x 1 ' ? ? x x 3 3 ' " = ˆ φ x 2 ' x 2 " x 3 " ˆ φ ˆ Ψ ? x x 3 3 ' " = ˆ Ψ ? ˆ φ x 1 ˆ ' θ x x 3 3 "' = x 2 "' x 2 ' 78 NSOU ? CC - PH - 03 x = ? ? ?? ?? ? ?? x? (3.2.12.8)				X 3 + 36X; s 18 (x) = 24X 15 * 154X 13 + 273X 11 * 143X 9 * 143X 7 + 273X 5 * 154 X 3 + 24X; x) = X 8 * 3X 6 + 3 X 4 * X 2 ; a 16 (x) = 2X 12 * 7X 10 + 11X 8 * 11 X 6 + 7X 4 * 2X 2 ;	

W https://annals.math.princeton.edu/wp-content/uploads/annals-v175-n2-p11-p.pdf

63/123	SUBMITTED TEXT	25 WORDS	41%	MATCHING TEXT	25 WORDS
r ' ? = R ? + r ? is the position of point P in the fixed frame and r ? is the location of P in the rotating frame.				r + 2ω × r 0 , (∗∗) where r 0 is the initial position (the point from which the particle is dropped) and ˙r 0 is the initial velocity in the rotating frame.	

W https://www.damtp.cam.ac.uk/user/tong/relativity/stephen.pdf

64/123	SUBMITTED TEXT	17 WORDS	95%	MATCHING TEXT	17 WORDS
ω ? ? - ω ω - ∈ - = ? ? ? ? ? (3.2.13.6) These equations are known as Euler's equations					

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

65/123	SUBMITTED TEXT	13 WORDS	76%	MATCHING TEXT	13 WORDS
the magnitude of the coriolis force is equal to the weight of the				The magnitude of this force is equal to the weight of the	

W https://dokumen.pub/mechanics-9788131773734-9788131798805-8131773736-9789332515604-9332515603.html

66/123	SUBMITTED TEXT	70 WORDS	26%	MATCHING TEXT	70 WORDS
d dr a r dt dt dt dt ? ? ω ? ?? ? ? ? + + × + ω× ? = ( ) r r r f d A a v r v r) dt ( ) (ω + +ω× + × +ω× +ω× ? ? ? ? ? ? ? ? ? ? (3.2.14.6) or, ( ) f r fr d a a v r (r) dt 2 ω + A× + ω× + × +					

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

67/123 **SUBMITTED TEXT** 17 WORDS **66%** **MATCHING TEXT** 17 WORDS

must be normal to the liquid surface. The slope of the surface is dz dr = tan? = r g 2

must be perpendicular to the surface. The slope of the surface at any point is therefore dz dr = tan φ = ω 2 r g . 9.5

W https://bayanbox.ir/view/7764531208313247331/Kleppner-D.-Kolenkow-R.J.-Introduction-to-Mechanics- ...

68/123 **SUBMITTED TEXT** 35 WORDS **82%** **MATCHING TEXT** 35 WORDS

Newton’s Law of Gravitation Statement : Every particle in the universe attracts every other particle with a force which is i) directly proportional to the product of their masses, ii) inversely proportional to the square of the distance

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

69/123 **SUBMITTED TEXT** 27 WORDS **45%** **MATCHING TEXT** 27 WORDS

m r r 2 .... (3.3.2.1) Where ij j j r r r = - ? ? ? , r ij = j i r r - ? ? ? j i ij j i r r r r

SA U_TEST_207.pdf (D22104273)

70/123 **SUBMITTED TEXT** 10 WORDS **100%** **MATCHING TEXT** 10 WORDS

force experienced by a unit mass placed at that point.

SA Unit II Notes.docx (D113412638)

71/123 **SUBMITTED TEXT** 14 WORDS **76%** **MATCHING TEXT** 14 WORDS

r - 2 (ˆr is unit vector along r ?) The gravitational field intensity at p

SA U_TEST_207.pdf (D22104273)

72/123 **SUBMITTED TEXT** 20 WORDS **50%** **MATCHING TEXT** 20 WORDS

it is defined as the ratio of the arc by radius of a circle. Figure : 3.3.11.2 In fig. the angle subtended

SA Physics_Vol-1 EM.pdf (D40552326)

73/123 **SUBMITTED TEXT** 48 WORDS	**29%** **MATCHING TEXT** 48 WORDS
sin 3 θ d θ dE = G R sin d R sin π ρ θ θ θ 3 3 2 2 2 (1 − cos θ) = 2G πR ρ sin θ (1 − cos θ) d θ = 3 GM R 2 sin θ (1 − cos θ) d θ ∴	sin θ d θ. dy = −dr cos θ + r sin θ d θ = 1 + e cos θ dy sin θ tan φ = = . dx 1 + e cos θ dx = dr sin θ + r cos θ d θ = (10.79)

W https://dokumen.pub/mechanics-9788131773734-9788131798805-8131773736-9789332515604-9332515603.html

74/123 **SUBMITTED TEXT** 16 WORDS	**78%** **MATCHING TEXT** 16 WORDS
is the mass of the planet and r A is the radius of the	is the mass of the planet and a is the length of the

W https://vdoc.pub/documents/solved-problems-in-classical-mechanics-analytical-and-numerical-soluti ...

75/123 **SUBMITTED TEXT** 15 WORDS	**96%** **MATCHING TEXT** 15 WORDS
steradian, one steradian is the solid angle subtended at the centre of a sphere	

SA Physics_Vol-1 EM.pdf (D40552326)

76/123 **SUBMITTED TEXT** 37 WORDS	**66%** **MATCHING TEXT** 37 WORDS
r ˆf(r)r r G r r r ? ? ∇× = ∇× = ∇×? ? ? ? ? ? ? ? ? ? ? ? ? ? = ( ) ( ) G r r G r r. ∇× + ∇ × ? ? ? ? ( ) ˆ ˆ ˆ	r R r2 r+ R R−r =0 r' m r r >R.

W https://silo.pub/an-introduction-to-mechanics-2nd-edition.html

77/123 **SUBMITTED TEXT** 45 WORDS	**69%** **MATCHING TEXT** 45 WORDS
r r r ˆ ˆ ˆ 0 i j k r r x y z ? ? ∂ ∂ ∂ ∂ + + + × ? ? ∂ ∂ ∂ ∂ ? ? ? = G x y z ˆ ˆ ˆ i j k r r r r r ∂ ? ? + + × ? ? ∂ ? ? ? ∴ ( ) 1	r ˆ ∂r ˆ ∂r ∂ ∂ 1 ∇ = ˆi + ˆj + kˆ = − 2 ˆi +j +k r ∂ x ∂y ∂z r ∂x ∂y ∂z ˆ ˆix + jˆy + kz r rˆ =− =− 3 =− 2. r3 r r

W https://vdoc.pub/documents/lecture-notes-on-newtonian-mechanics-lessons-from-modern-concepts-2vk1 ...

78/123 **SUBMITTED TEXT** 12 WORDS	**83%** **MATCHING TEXT** 12 WORDS
M are placed at the vertices of an equilateral triangle of side	

SA Physics_Vol-1 EM.pdf (D40552326)

79/123 **SUBMITTED TEXT** 13 WORDS	**80%** **MATCHING TEXT** 13 WORDS
Earth), we consider Earth to be a perfect sphere of radius R and	earth. We are assuming the earth to be a perfect sphere of radius R and

W https://dokumen.pub/mechanics-9788131773734-9788131798805-8131773736-9789332515604-9332515603.html

80/123 **SUBMITTED TEXT** 14 WORDS	**85%** **MATCHING TEXT** 14 WORDS
r) + 2 2 L 2mr , wher L is the angular momentum of the particle. 2. Prove that	r0) = −L2 mr03 , where L is the angular momentum of the particle. (b) Prove that

W https://vdoc.pub/documents/solved-problems-in-classical-mechanics-analytical-and-numerical-soluti ...

81/123 **SUBMITTED TEXT** 9 WORDS	**87%** **MATCHING TEXT** 9 WORDS
Kepler's laws of planetary motion from Newton's law of gravitation. 4.	

SA ELMP-1 - Mechanics.pdf (D137599141)

82/123 **SUBMITTED TEXT** 13 WORDS	**90%** **MATCHING TEXT** 13 WORDS
the total energy of a particle of mass m moving under the	the total energy E of a particle of mass m moving in the

W https://www.damtp.cam.ac.uk/user/tong/relativity/stephen.pdf

83/123 **SUBMITTED TEXT** 18 WORDS	**84%** **MATCHING TEXT** 18 WORDS
the square of the time period of the planet is proportional to the cube of the semi-major axis. 3.4.9	

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

84/123 **SUBMITTED TEXT** 21 WORDS	**56%** **MATCHING TEXT** 21 WORDS
is called longitudinal or linear stress. The ratio of longitudinal stress to linear strain, within the elastic limit, is called Young's Modulus,	

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

85/123 **SUBMITTED TEXT** 13 WORDS **80%** **MATCHING TEXT** 13 WORDS

is defined as the ratio of the longitudinal stress to the corresponding strain

SA ELMP-1 - Mechanics.pdf (D137599141)

86/123 **SUBMITTED TEXT** 9 WORDS **100%** **MATCHING TEXT** 9 WORDS

is the Young's modulus of the material of the

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

87/123 **SUBMITTED TEXT** 37 WORDS **39%** **MATCHING TEXT** 37 WORDS

of the forces acting on both the upper and the lower halves 174 NSOU ? CC - PH - 03 of the section are in the same direction, the total moment of the forces acting on the

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

88/123 **SUBMITTED TEXT** 25 WORDS **42%** **MATCHING TEXT** 25 WORDS

moment of inertia of a thin rectangular sheet of breadh b and width w about an axis passing through its center and parallel to its

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

89/123 **SUBMITTED TEXT** 14 WORDS **76%** **MATCHING TEXT** 14 WORDS

SUBMITTED TEXT	MATCHING TEXT
two postulates: 1. All the laws of physics are the same in every inertial	two "postulates": 1. The laws of physics are the same in all inertial

W https://kupdf.net/download/px148-notes_59f08cc2e2b6f54e4b36a411_pdf

90/123 **SUBMITTED TEXT** 31 WORDS **37%** **MATCHING TEXT** 31 WORDS

Special Theory of Relativity. According to Special Theory of Relatively all laws of nature appear to have the same mathematical from in all inertial frames of reference and the speed of light

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

91/123 **SUBMITTED TEXT** 11 WORDS	**95%** **MATCHING TEXT** 11 WORDS
that speed of light is the same in all inertial frames	that speed of light is the same in all inertial frames:

W https://www.damtp.cam.ac.uk/user/tong/relativity/stephen.pdf

92/123 **SUBMITTED TEXT** 12 WORDS	**83%** **MATCHING TEXT** 12 WORDS
the speed of light, independent of the motion of the emanating source,	the speed of light is independent of the motion of the source.

W https://silo.pub/an-introduction-to-mechanics-2nd-edition.html

93/123 **SUBMITTED TEXT** 10 WORDS	**100%** **MATCHING TEXT** 10 WORDS
S? is moving in the positive x-direction with speed v	S is moving in the positive x direction, with speed v.

W https://silo.pub/an-introduction-to-mechanics-2nd-edition.html

94/123 **SUBMITTED TEXT** 10 WORDS	**100%** **MATCHING TEXT** 10 WORDS
the direction of motion of the object relative to the	

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

95/123 **SUBMITTED TEXT** 71 WORDS	**42%** **MATCHING TEXT** 71 WORDS
x 0 ? = ?(x 0 − ?x 1) x 1 ? = ?(x 1 − ?x 0) 216 NSOU ? CC - PH - 03 x 2 ? = x 2 x 3 ? = x 3 (3.7.3.4) where we have used the notation x 0 = ct, x 1 = x, x 2 = y, x 3 =	x 2 − 2x + 5 x 2 − 1 5. x 2 − x + 1 x + 1 7. x 2 − 1 x 9. 2x − 3 x 2 y 11. (x + 1) 2 (x + 2) 4 13. −1 (x 2 + 1) 3 15. −(2x + y) x 2 (

W http://www.math.utep.edu/faculty/cmmundy/Math%202301/Solution_Manual.pdf

96/123 **SUBMITTED TEXT** 30 WORDS	**100%** **MATCHING TEXT** 30 WORDS
x 0 = ?(x 0 ? + ?x 1 ?) x 1 = ?(x 1 ??+ ?x 0 ?) x 2 = x 2 ? x 3 = x 3 ?	

SA Physics_Vol-1 EM.pdf (D40552326)

97/123	**SUBMITTED TEXT**	17 WORDS	**82%**	**MATCHING TEXT**	17 WORDS

At what speed does a meters tick move if its length is observed to shrink to 0.5 m?

SA ELMP-1 - Mechanics.pdf (D137599141)

98/123	**SUBMITTED TEXT**	24 WORDS	**50%**	**MATCHING TEXT**	24 WORDS

in one frame of reference will not appear simultaneous in another frame of reference which is moving with a velocity v with respect to

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

99/123	**SUBMITTED TEXT**	44 WORDS	**36%**	**MATCHING TEXT**	44 WORDS

the time interval between these two events in S??will be t
2 ??– t 1 ??= x 2 x 1 2 1 2 2 v x v x t t c c ? ? ? ? γ - - γ - ? ?
? ? ? ? ? ? = 1 2 x 2 t x v c - ? ?

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

100/123	**SUBMITTED TEXT**	22 WORDS	**100%**	**MATCHING TEXT**	22 WORDS

At what speed does a clock move if it runs at a rate which is one-half the rate of a clock at rest?

SA Chapter 2 -.docx (D96145146)

101/123	**SUBMITTED TEXT**	10 WORDS	**100%**	**MATCHING TEXT**	10 WORDS

the S? frame to be the rest frame of the

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

102/123	**SUBMITTED TEXT**	19 WORDS	**100%**	**MATCHING TEXT**	19 WORDS

The average lifetime of a ??meson in its own frame of reference is 26.0 ns. (This is its proper lifetime.)

SA ELMP-1 - Mechanics.pdf (D137599141)

103/123 **SUBMITTED TEXT** 24 WORDS	**100%** **MATCHING TEXT** 24 WORDS
If the ??meson moves with speed 0.95c with respect to the Earth, what is its lifetime as measured by an observer at rest on	

SA ELMP-1 - Mechanics.pdf (D137599141)

104/123 **SUBMITTED TEXT** 17 WORDS	**91%** **MATCHING TEXT** 17 WORDS
What is the average distance it travel before decaying as measured by an observer at rest on	

SA ELMP-1 - Mechanics.pdf (D137599141)

105/123 **SUBMITTED TEXT** 14 WORDS	**85%** **MATCHING TEXT** 14 WORDS
The average distance travelled before decaying as measured by an observer at rest on	

SA ELMP-1 - Mechanics.pdf (D137599141)

106/123 **SUBMITTED TEXT** 9 WORDS	**100%** **MATCHING TEXT** 9 WORDS
the S??frame to be the rest frame of the	

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

107/123 **SUBMITTED TEXT** 23 WORDS	**80%** **MATCHING TEXT** 23 WORDS
the form 2 2 2 2 0 0 A A A A ' ' - = - ? ? (3.7.6.4) where the components (A 0 ?, A?) and (A 0 ,	the general form A = (a1 , a2 , a3 , a4) where the components (a1 , a2 , a3)

W https://silo.pub/an-introduction-to-mechanics-2nd-edition.html

108/123 **SUBMITTED TEXT** 22 WORDS	**90%** **MATCHING TEXT** 22 WORDS
the length of the rod as measured by a stationary observer. (b) Determine the angle the rod makes with the x axis. 3.7.7	

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

109/123 **SUBMITTED TEXT** 29 WORDS **97%** **MATCHING TEXT** 29 WORDS

A rod of length L 0 moves with speed v along the horizontal direction. The rod makes an angle ? 0 with respect to the x??axis. 222

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

110/123 **SUBMITTED TEXT** 29 WORDS **86%** **MATCHING TEXT** 29 WORDS

A 0 , A 1 , A 2 , A 4) where A 1 , A 2 , A 3 are the components of a 3-vector A.

SA 100001300-Proj-1716509.pdf (D18002277)

111/123 **SUBMITTED TEXT** 32 WORDS **50%** **MATCHING TEXT** 32 WORDS

B 0 , B 1 , B 2 , B 3) the ''scalar product' is an invariant, that is, A 0 ?B 0 ??− A′ ? ??? B′ ? ?= A 0 B 0 − A ? . B ? (3.7.6.5)

SA Relativistic Quantum Mechanics - Sagar PCM 2021.pdf (D122839484)

112/123 **SUBMITTED TEXT** 11 WORDS **76%** **MATCHING TEXT** 11 WORDS

the x-axis of S. Let us take the S frame to be

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

113/123 **SUBMITTED TEXT** 20 WORDS **73%** **MATCHING TEXT** 20 WORDS

two references frames S and S??as in fig. 3.7.10.1 . S? is moving with a velocity v with respect to

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

114/123 **SUBMITTED TEXT** 19 WORDS **55%** **MATCHING TEXT** 19 WORDS

and u 2 be the velocities of the masses A and B respectively with respect to S-frame. Then

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

115/123 **SUBMITTED TEXT** 181 WORDS **17%** **MATCHING TEXT** 181 WORDS

m 1 u 1 + m 2 u 2 = (m 1 + m 2)v ( ) 1 2 1 2 2 2 v u v u m m m m v, 1 uv 1 uv c c + - + = + + - as they becomes at rest after ins S?- frame. v ? u ? v ? u ? 230 NSOU ? CC - PH - 03 Thus 1 2 2 2 v u v u m v m v 1 uv 1 uv c c ? ? ? ? ? ? ? ? + - ? ? ? ? - = - ? ? ? ? + - ? ? ? ? ? ? ? ? ? ? ? ? ? or, 2 2 1 2 2 2 2 2 v uv v m u 1 1 m u 1 c c c uv 1 c ? ? ? ? ? ? - + = - ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? - ? ? ? ? or, 1 2 2 2 uv uv m 1 m 1 c c ? ? ? ? - = + ? ? ? ? ? ? ? ? ... ... (3.6.10.3) Now ( ) ( ) ( ) ( ) ( ) ( ) 2 2 2 2 2 1 2 2 2 2 2 2 2 2 2 u v 1 c uv uv u 1 1 1 c c c u v u uv 1 1 1 uv c c 1 c + - - + - = = - - - + - or, ( ) ( ) 2 1 2 2 2 2 2 2 u uv 1 1 c c uv u 1 1 c

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

116/123 **SUBMITTED TEXT** 107 WORDS **23%** **MATCHING TEXT** 107 WORDS

u v y 2 x x u y z 2 x x 2 2 v u 1 u vu c ;u and u 1 vu vu c 1 1 c c - - ’ = ’ ’ = = - ? ? ? ? - - ? ? ? ? ? ? ? ? ... ... (3.7.12.3) Equation (13) yields to 2 2 2 2 2 2 x 2 u v 1 1 c c u 1 vu c 1 c ? ?? ? - - ? ?? ? ? ?? ? ? ? ’ ? ?? ? - = ? ? ? ? ? ? ? ? - ? ? ? ? (3.7.12.4) 2 x 2 2 2 2 0 0 x 2 2 2 2 2 2 2 2 2 2 vu mc 1 m c m c E vP c E m c , u v u v v 1 1 1 1 1 c c c

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

117/123 **SUBMITTED TEXT** 62 WORDS **46%** **MATCHING TEXT** 62 WORDS

x x x 2 x 2 2 x 0 2 2 2 x 2 2 2 2 m p m u (u v) vE u p 1 u v m c c vu u v v 1 1 1 1 c c c c ’ ’ ’ = = - ’ - - - = = ? ? - ? ? - - - ? ? ... (3.7.12.5) 234

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

118/123 **SUBMITTED TEXT** 56 WORDS **38%** **MATCHING TEXT** 56 WORDS

x x 2 2 2 2 2 2 vu v v u 1 1 1 m u m m c c c p m u u mu p vu vu u u u v 1 1 1 1 1 1 c c c c c c ? ? - - - ? ? ? ? ’ ’ ’ ’ = = = = = = ? ? ’ ’ ? ? - ? ? - - - - - ? ? ? ? ? ? (3.7.12.6

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

119/123 **SUBMITTED TEXT** 53 WORDS **67%** **MATCHING TEXT** 53 WORDS

m 2 c 2 – m 0 2 c 4 + m 0 2 c 4 . So, E 2 = 2 4 0 2 2 m c
[1] [1] (1 v) c - - + m 0 2 c 4= 2 2 2 4 0 2 2 (v) c m c (1 v c
? ? ? ? + ? ? - ? ? ? ? ? ? m 0 2 c 4 = p 2 c 2 + m 0 2 c 4 .
(3.7.11.7) 3.7.13

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

120/123 **SUBMITTED TEXT** 33 WORDS **98%** **MATCHING TEXT** 33 WORDS

Let us take the S??frame to be the rest frame of the rod. A rod of length L 0 in S??makes an angle ? 0 with the x' axis. Its projected

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

121/123 **SUBMITTED TEXT** 43 WORDS **77%** **MATCHING TEXT** 43 WORDS

L 0 cos? 0 and ?y 0 = L 0 sin ? 0 . In a frame S in which th rod moves at speed v along the x axis, the projected lengths ?x and ?y are given by ?

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122/123 **SUBMITTED TEXT** 23 WORDS **100%** **MATCHING TEXT** 23 WORDS

The length L of the rod as measured by a stationary observer in S is ( ) ( ) ( ) 1 2 2 2 2 2 0 0 L

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The rod makes an angle ??with the x axis in S where tan ? =

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PREFACE In a bid to standardise higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses: core, generic discipline specific elective, and ability/ skill enhancement for graduate students of all programmes at Elective/ Honours level. This brings in the semester pattern, which finds efficacy in tandem with credit system, credit transfer, comprehensive and continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry acquired credits. I am happy to note that the University has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English. Eventually, these will be translated into Bengali too, for the benefit of learners. As always, we have requisitioned the services of the best academics in each domain for the preparation of new SLMs, and I am sure they will be of commendable academic support. We look forward to proactive feedback from all stake-holders who will participate in the teaching-learning of these study materials. It has been a very challenging task well executed, and 1 congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor
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Unit 1 ????? Calculus Structure 1.1 Objectives 1.2 Introduction 1.3 Recapitulations 1.3.1 Limit of a Function 1.3.2 Continuity of Function 1.4 Continuity and Differentiability 1.5 Intuitive Ideas of Continuous and Differentiable Functions 1.6 Average and Instantaneous Values of A Function 1.7 Approximation 1.8 Summary 1.1 ????? Objectives While you go through the pages of this chapter, you will learn 1. Continuity and differentiability of functions using intuitive ideas. 2. Method of approximation. 3. How to find out average and instantaneous values of functions defined in appropriate domain. 1.2 ????? Introduction Calculus is widely used to create mathematical models in order to arrive at an optimal solution. In physics calculus is used in a lot of its concepts, in the mathematical study of continuous change, in dynamics, astronomy, astrophysics and quantum mechanics. However in the pages to follow we will indulge in some introductory topics in the form of recapitulations of some basic ideas of calculus. Even in thermodynamics and statistical mechanics, differential are redefined to apply the rules of calculus.
8 ? NSOU ? CC-PH-04 Key words Limits, continuity, Differentiability, Taylor's and Binomial series. Approximate solution, average value of functions. 1.3 ????? Recapitulation 1.3.1 Limit of a function We say that a function f(x) has a limit L at a if and only if for every ε < 0 there exists a positive number δ depending on ε such that for any x in the domain of f(x) with the property 0 > |x – a| > δ we have |f(x) – L| > ε . In symbol we write lim ( ) ; x a f x L ? ? or, f(x) → L ; as x → a A similar definition extends to functions in two variables. We say that L is the limit of a function f(x, y) at the point (a, b), written, (,) (,) lim (,) x y a b f x y L ? ? If f(x, y) is as close to L as we please whenever the distance from the point (x, y) to the point (a, b) is sufficiently small, but not zero. Using ε – δ definition we say that L is the limit of f(x, y) as (x, y) approaches (a, b) if and only if for every given ε < 0 we can find a δ < 0 such that for any point (x, y) where ? ? ? ? 2 2 0 x a y b ? ? ? ? ? ? ? , we have (,) f x y L ? ? ?. 1.3.2. Continuity of a function A function f(x) is said to be continuous at x = a, if; f(x) has a definite value at x = a; lim x→a f(x) exists and lim x→a f(x) = f(a). In other words, f(x) is said to be continuous at x = a, if lim ( ) lim ( ) ( ) x a x a f x f x f a ? ? ? ? ? ? ? Using the ε – δ definition, the single valued function f(x) is said to be continuous at x = a provided f(x) possess a definite finite value at x = a and given any pre-assigned
NSOU ? CC-PH-04 ? 9 positive quantity ε, however small, we can determine another positive quantity δ (whose value depends on ε), such that, ( ) ( ) f x f a ? ? ? for all x in x a ? ? ? A function f(x, y) is continuous at the point (a, b) if the following two conditions are satisfied : a) Both f(a, b) and lim (x, y) → (a, b) f(x, y) exist. b) lim (x, y) → (a, b) f(x, y) = f(a, b) Example : Art (1.4.1) : Example 1: Find 3 3 (,) (0,0) lim

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x y x y x y ? ? ? Solution : 3 3 2 2 (,) (0,0) (,) (0,0) lim lim 0, x y x y x y x xy y x y ? ? ? ? ? ? ? ? 0 x y ? ? Example 2: Find 2 (,) (0,0) lim x y x xy x y ? ? ? Solution : ? ? ? ? ? ? ? ? ? ? ? ? ? 2 (,) (0,0) (,) (0,0) lim ( ) lim x y x y x x y x y x xy x y x y x y x y ? ? ? ? ? ? ? ? ? ? ? ? ? (,) (0,0) lim 0 ? ? ? ? x y x x y Exercise Art 1.4.1 : 1) Find the limit : 0 2 2 1 lim : 2 2 x x ans x ? ? ? ? ? ? ? ? ? ? 2) Find 2 4 2 (,) (1,1) ( ) 1 lim : 2 x y x y ans x y ? ? ? ? ? ? ? ? ? 1.4 ?????

Continuity and Differentiability If a function f(x) is differentiable at x = a then f(x) must be continuous at x = a. However the converse is not always true i.e. if a functions f(x) is continuous = a, it is not necessarily differentiable at x = a.
10 ? NSOU ? CC-PH-04 Proof : Now since f(x) is differentiable at x = a 0 ( ) ( ) ( ) lim h f a h f a f a h ? ? ? ? ? is a finite quantity. Now ? ? 0 0 0 ( ) ( ) lim ( ) ( ) lim lim ( ) 0 0 h h h f a h f a f a h f a h f a h ? ? ? ? ? ? ? ? ? ? ? ? ? Therefore 0 lim ( ) ( ) h f a h f a ? ? ? i.e. f (x) is continuous at x = a. However from the definition of continuity we cannot always arrive at the differentiability of a function, as is discussed in the following examples. Example of Art 1.5: Example 1: A function f(x) is defined as follows 0 ( ) 0 x when x f x x when x ? ? ? ? ? ? ? ? Examine the continuity and differentiability of f(x) at x = 0. Solution : We have 0 0 lim ( ) lim 0

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x x f x x ? ? ? ? ? ? ? Since f(x) = x when x < 0 And 0 0 lim ( ) lim ( ) 0 x x f x x ? ? ? ? ? ? ? ? Since f(x) = – x when x > 0 Again f (0) = 0 Therefore, 0 0 lim ( ) lim ( ) (0) x x f x f x f ? ? ? ? ? ? ? Therefore the function is continuous at x = 0 Now 0 ( ) ( ) ( ) lim h f x h f x

f
x h ? ? ? ? ? ∴ 0 0 (0) (0) ( ) (0) lim lim h

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h f h f f h f h h ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 11 Since f (0) = 0 Now 0 0 ( ) lim lim 1 h h f h h h h ? ? ? ? ? ? Since f(x) = x when x< 0 and h → 0 + means 0h ? And 0 0 ( ) lim lim 1 h h f h h h h ? ? ? ? ? ? ? ?

Therefore right hand limit and left hand limit of f / (0), though exist, are unequal. Therefore f(x) is not differentiable at x = 0 Example 2 : Examine the continuity and differentiability of

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f(x) = 2x 2 + 3 at x = 1 Solution : 2 1 1 lim ( ) lim 2 3 5 x x f x x ? ? ? ? ? ? ? ? 2 1 1 lim ( ) lim 2 3 5 x x f x x ? ? ? ? ? ? ? ? And f(1) = 5 Therefore 1 1 lim ( ) lim ( ) (1) x x f x f x f ? ? ? ? ? ? ? And f(x) is continuous at x = 1

Now 0 ( ) ( ) ( ) lim
h f x
h f x

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f x h ? ? ? ? ? ∴ 0 (1) (1) (1) lim h f h f f h ? ? ? ? ? Or, 2 0 2(1) 3 5 (1) lim h h f h ? ? ? ? ? ? (1) 5 f ? ? 2 0 2 4 lim h h h h ? ? ? 12 ?

NSOU ? CC-PH-04 Now Right hand limit of 2 0 0 2 4 (1) lim lim 2 4 4 h h h h f h h ? ? ? ? ? ? ? ? ? ? ? ? 0h ? ? And left hand limit of 2 0 0 2 4 (1) lim lim 2 4 4 h h h h f h h ? ? ? ? ? ? ? ? ? ? Therefore right hand limit of f ?(1) and left hand limit of f ?(1), both exist and are equal. And so, f ?(1) exists and its value is 4 i.e. f (x) is differentiable at x= 1. Exercise of Arts 1.4 and 1.5 : 1) 2 1 sin( ) 0 ( ) 0 0 x when x f x x when x ? ? ? ? ? ? ? ? Discuss the continuity and differentiability of f(x) at x = 0. 2) Find the co-efficient a and b such that the following function f is continuous and differentiable at x = 1 2 1 | | 1 | | ( ) 2 | | 1 when x x f x ax b when x ? ? ? ? ? ? ? ? ? Solution Exercise of Arts : 1.4 and 1.5 : Solutions (1) : Differentiability at x = 0, we have by the definition 0 (0) (0) (0) lim , h f h f f h ? ? ? ? ? therefore right hand limit, 2 1 2 2 0 0 0 0 sin ( ) sin ( ) sin lim lim lim lim 1; h h h h z h f h h z h h z h ? ? ? ? ? ? ? ? ? ? ? ? ? where, h 2 = z ∴ z → 0 + as h → 0 + Similarly, 0 ( ) lim 1 h f h h ? ? ?
NSOU ? CC-PH-04 ? 13 Therefore the function f(x) is differentiable at x= 0. Since f(x) is differentiable at = 0, it must be continuous at x= 0. Solution 2 : Since f (x) is continuous at x = 1, 1 lim ( ) (1) 1 x f x f ? ? ? ? or 2 1 lim (2) 1 x ax b ? ? ? ? i.e. 2a + b = 1 Again, f(x) is differentiable at x = 1. 0 0 ( ) (1) (1) 1 (1) lim lim h h f x h f f h f h h ? ? ? ? ? ? ? ? ? and right hand f ?(1) = left hand f ?(1) Or, 0 0 (1) 1 (1) 1 lim lim h h f h f h h h ? ? ? ? ? ? ? ? ? Or, 2 0 0 1 1 1 lim 1 lim 2 (1) 1 1 h h a h b h h h ? 2 1 ( ) when 1and ( ) 2 , when 1 f x x f x ax b x x ? ? ? ? ? ? ? ? ? ? ? ? Or, 2 0 0 1 1 lim lim 2 1 2 (2) 1

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h h h a b a h h h h h ? 2 1 a b ? ? ? 0 0 1 2 (2) lim lim 1 h h ah h h h ? ? ? ? ? ? ? ? ? ? ? ? ? ? Or, – 1 = 0 lim 2 (2) 4 h a h a ? ? ? ? 1 4 a? ? ?

Therefore 1 3 1 2 1 2 2 b a ? ? ? ? ?

14 ? NSOU ? CC-PH-04 1.5 ????? Intuitive Ideas of Continuous and Differentiable Function 1) The function 1 ( ) 1 f x x ? ? is discontinuous at x = 1 because at = 1, f(1) is not defined (has 0 as denominator) and because 1 lim ( ) h f x ? does not exist (equals ?). The function is however continuous every where except at x = 1, where it is said to have an infinite discontinuity (Fig 1.6.1]. 2) The function 2 2 ( ) x a f x x a ? ? ? is discontinuous at x = a because f(a) is not defined (has zero, both numerator and denominator) and because lim ( ) 2 x a f x a ? ? . The discontinuity here is called removable since it may be removed by redefining the function as 2 2 ( ) x a f x x a ? ? ? for , ( ) 2 x a f a a ? ?. Fig. (1.6.2) Fig. (1.6.3) (Note that the discontinuity in example (1) cannot be so removed because the limit also does not exist). The graphs of 2 2 ( ) x a f x x a ? ? ? and g(x) = x + a are identical except at x = a, where the former has a break [fig 1.6.2]. Removing the discontinuity consists simply of joining the break [fig 1.6.3]. A differentiable function of one real variable is a function whose derivative exists at --------------- -------- 2a a 0 Fig. (1.6.1)

NSOU ? CC-PH-04 ? 15 each point in its domain. As a result, the graph of a differentiable function must have a tangent line (non-vertical) at each point in its domain, be relatively smooth and cannot contain any breaks, bends or cusps. Fig. (1.6.4) Fig. (1.6.5) In fig (1.6.4) the absolute value function is continuous i.e. it has no gap. It is differentiable everywhere except at the point x= 0, where it makes a sharp turn as it crosses the y – axis. A cusp on the graph of a continuous function fig (1.6.5) at x = 0. The function is continuous but not differentiable. A function with a bend, cusp a vertical tangent may be continuous but fails to be differentiable at the location of the anomaly. Below are graphs of functions that are not differentiable at x = 0 for various reasons. Fig. (1.6.6) : no tangent at x = 0 Fig. (1.6.7) : jump in the value of function at x = 0

16 ? NSOU ? CC-PH-04 Fig. (1.6.8) : function increases indefinitely Fig. (1.6.9) : tangent at x = 0 is vertical. at x = 0 Example : Art (1.6) Find the 1 lim ( ) x f x ? , where f(x) = x + x 2 by intuitive ideas. Solution : We tabulate the values of f(x) near x = 1 in the following table x 0.9 0.99 0.999 1.01 1.1 1.2 f(x) 1.11 1.9701 1.997001 2.0301 2.31 2.64 From this it is reasonable to say that, lim x → 1 – f(x) = lim x → 1 + f(x) = lim x +1 f(x) = 2 1.6 ????? Average and Instantaneous values of functions : Suppose a function f(x) is continuous on the interval (a, b). We want to find the average value of f(x) in the interval (a, b). We divide the interval by n numbers of intervals x 1 = a,x 2 , ..., x n = b and find the n numbers of values of f(x) [fig 1.7.1] e.g. at x 1 , f(x 1); at x 2 , f(x 2); and so on and we get the approximate average value of f(x) on (a, b) as : 1 2 ( ) ( ) ... ( ) n f x f x f x n ? ? ? (1.1) Now let the points x 1 , x 2 , ... be Δx apart. And we multiple the numerator and the denominator of the approximate average by Δx , then average of f(x) on (a, b) is approximately equal to Fig. 1.7.1 x n = b NSOU ? CC-PH-04 ? 17 1 2 ( ) ( ) ... ( ) n f x f x f x x n x ? ? ? ? ? ? ? ? ? Where nΔx = b – a. Now in the limit , 0 n x ? ? ? ? ? and we get the average value of f(x) as ( ) ( ) ( ) b b a

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a b a f x dx f x dx f x

b a dx ? ? ? ? ? ? (1.2) We see that f(x 1), f(x 2), ... etc are the instantaneous values of f(x) at x 1 , x 2 , ... etc. In alternating current theory we take the instantaneous value of current as i = i 0 sin ωt or i 0 cos ωt, i 0 is the amplitude. Now average value of i for the period 2 T ? ? ? is given as, 0 0 0 0 0 sin 1 sin T T T i t dt i i t dt T dt ? ? ? ? ? ? ? ? ? . If we put , , d t dt d dt ? ? ? ? ? ? ? ? ? Or, 2 0 0 1 sin 0, 2 2 i i d T ? ? ? ? ? ? ? ? ? ? ? ? Thus we see that the average value of sin θ or cos θ for a complete period or any number of periods is zero. In such cases the average of the square of the function is taken to define a significant mean like root mean square current. 1.7 ????? Approximation There are many problems in physics which can be written as an infinite series and its solution lies in finding the sum of the infinite series. However it is often found that the results differ very little if we would have taken a finite number of terms at the beginning of the series rather than taking the entire infinite series. In this way we can find an approximate solution of the problems which cannot be solved exactly. The accuracy of the solution can be reached to the desired value, by taking as many terms of the series as required to reach

18 ? NSOU ? CC-PH-04 the desired accuracy. Also many functions can be expanded in infinite power series (i.e. a series expanded in powers of x having infinite number of term). Taylor's series : We can write the Taylor series for

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a function f(x) about x = a ; 2 ( ) 1 1 ( ) ( ) ( ) ( ) ( ) ( ) ... ( ) ( ) ... 2! ! n n f x f a x a f a x a f a x a f a n ? ?? ? ? ? ? ? ? ? ? ? ? (1.3) Where f (n) (

x) represents n th derivative of f(x) Or, ( ) 0 ( ) ( ) ( ) ! n n n f a f x x a n ? ? ? ? ? (1.4) The Maclaurin series for f(x) is the Taylor's series about the origin. Putting a = 0 in equation (1.4) we obtain the Maclaurin series for f(x) : ( ) 0 (0) ( ) ! n n n f f x x n ? ? ? ? (1.5) The polynomial formed by taking some initial terms of the Taylor's series is called Taylor's polynomial. A Taylor's series is a representation of function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. The function can be approximated by using the Taylor polynomial of suitable number of terms. Taylor's theorem gives quantitative estimate on the error introduced by the use of such an approximation. Example 1 : The sine function is closely approximated by its Taylor polynomial of degree 7 (dotted) for a full period centred at the origin. The dotted curve is a polynomial of degree seven. 3 5 7 sin 3! 5! 7! x x x x x ? ? ? ? The error in this approximation is no more than 9 9 x . In particular for – 1 > x > 1 the error is less than 0.000003 (fig. 1.8.1). Fig. 1.8.1
NSOU ? CC-PH-04 ? 19 Example 2 : Using the quadratic Taylor polynomial for 2 1 ( ) f x x ? , approximate the value of 1 4.41 . Ans. The quadratic Taylor polynomial is 2 2 ( ) ( ) ( ) ( ) ( ) ( ) 1! 2!

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f a f a f x f a x a x a ? ?? ? ? ? ? ? ? Now 3 4 2 6 ( ) ; ( ) ? ?? ? ? ? f x f x x x ; we write 4.41 = (2 + 0.1) 2 , implies a = 2 & x = 2.1 2 2 (2) (2) ( ) (2) (2.1 2) (2.1 2) 1! 2! f f f x f ? ?? ? ? ? ? ? ? ? 2 2 6 8 16 (2) (2.1 2) (2.1 2) 1! 2!

f ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = 0.25 – 0.025 + 0.001875 = 0.226875 The actual value is 1 4.41 = 0.226775 So the approximation deviates only about 0.05%. Example 3 : What is the quadratic approximation

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of the function ( ) 4 f x x ? ? at x = 0 Solution : 1 2 2 2 32 2 (0) (0) 1 1 1 ( ) (0) 2 2 1! 2! 4 2! 4 14 f f f x f x x x x x

x ? ?? ? ? ? ? ? ? ? ? ?
Binomial series : The binomial series can be written as : 2 3 0 (1) (1)(2) (1) 1 ... 2! 3! p n n p p p p p p x x px x x n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (1.6) P is any real number, positive or negative or fractional and the expansion is an infinite series, p n ? ? ? ? ? ? is called a binomial co-efficient.
20 ? NSOU ? CC-PH-04 The binomial co-efficients are (1) (1)(2) 1; ; ; ; 0 1 2 3 1! 2! 3!

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p p p p p p p p p ? (1)(2)...(1) ! p p p p p

n n n ? ? ? ? ? ? ? ? ? ? ? To get an approximation we can consider a few terms from the expansion (1.6). Example 1 : For small x, 1 + px is a reasonable approximation for (1 + x) p . Notice that this correspond to picking the first two terms from (1.6). Now suppose x = 0.0007. Therefore (1.0007) 9 = 1 + 0.0007 × 9 ? 1.0063 Now actual value of (1.0007) 9 = 1.006317668842... Therefore our result is correct up to four decimal place. 1.8 ????? Summary Definitions of limit, continuity and differentiability are recapitulated. Application of series in finding the approximate solution of physical problems which cannot be solved exactly has been discussed. Average and instantaneous values of function is defined.

NSOU ? CC-PH-04 ? 21 Unit 2 ????? Second Order Differential Equation Structure 2.1 Objectives 2.2 Introduction 2.3 Linear Second Order Differential Equation 2.3.1 Second Order Linear Homogeneous Differential Equations with Constant Coefficient 2.3.2 Second Order Linear Homogeneous Equations with Constant Coefficient : Working Rules for Solutions (Complementary Functions) 2.4 The Existence and Uniqueness Theorem 2.5 Linearly Dependent And Linearly Independent Solutions of Differential Equations : Wornskian 2.5.1 Non-Homogeneous Linear Equation with Constant Coefficient 2.5.2 Inhomogeneous Linear Equations with Constant Coefficient: Working Rules for Finding the Particular Integral 2.6 Summary 2.1 ????? Objectives In going through the chapter you will learn : 1. To classify second order differential equation. 2. To find out solution of second order differential equation in terms of complementary functions and particular integral. 3. To find out the general solution and to define linearly dependent and independent solutions in terms of wronskian. 4. Statement of existence and uniqueness theorem for initial value problems and their applications. 22 ? NSOU ? CC-PH-04 2.2 ????? Introduction An universally accepted method for formulations and solutions of physical problems is to construct the relevant differential equation and then attempt to solve it. Thus differential equations are at the centre to many physical problems. In this chapter we shall limit our discussions to the techniques of solving second order linear homogeneous or inhomogeneous equations with constant co-efficients and also some relevant physical applications will be discussed. An equation containing derivatives is called differentials equation, which may be classified as ordinary or partial. Ordinary differential equation : Differential equation containing only one independent variable is called ordinary differential equation. Example : 2 2 2 0 d x x dt ?? ? , where x = x(t) i.e. functions of one independent variable t. Partial differential equations : Differential equations containing partial derivatives of the dependent variable y (x 1 , x 2 , x 3 ... x n) with respect to more than one independent variables x 1 , x 2 , ... x n are called partial differential equations. Example : 2 2 2 2 2 2 0 x y z ? ? ? ?? ? ? ? ? ? ? ? ? ? ? where (, ,) x y z ? ? ? Order of differential equation : It is defined as the order of the highest derivative on the equation. Degree of differential equation : It is defined as the power of the highest derivative in the equation after fractional powers have been removed. Examples : i) 2 2 2 0 d y dy xy dx dx ? ? ? ? ? ? ? ? ? is a second order and 1 st degree differential equation, while the equation. ii) 2 2 0 d y dy xy dx dx ? ? ? is a second order, 2 nd degree differential equation, because

NSOU ? CC-PH-04 ? 23 when the square root is removed, 2 dy dx ? ? ? ? ? ? appears in the equation as the highest power of highest order. Linear differential equation : A linear differential equation satisfies the following properties : i) Power of each derivative and of the dependent variable must be unity. ii) The coefficient of all derivatives occurring in the equations may be constant or may be functions of the independent variables. iii) The dependent variables and its derivatives is not multiplied together is the differential equation. Differential equation obeying no such properties or property are termed non-linear differential equation. Example : 2 2 0 d y dy y y dx dx ? ? ? ; the presence of the term dy y dx makes it non-linear. Homogeneous and in-homogeneous differential equation : When the right hand member of the differential equation is either zero or constant, the differential equation is called homogeneous, otherwise the differential equation is in-homogeneous, when the right hand side is function of independent variable. Example : 2 0 1 2 2 ( ) d y dy P P P y Q x dx dx ? ? ? (2.1) If Q(x) = 0 or constant, equation (2.1) is homogeneous, otherwise in-homogeneous. Solution of Differential equation : A solution of differential equation is a function which, when substituted in the differential equation produces an identity. Key Words Homogeneous and inhomogeneous equations, wronskian, complementary function, particular integral. Existence and uniqueness theorem. 2.3 ????? Linear second order differential equation : A linear second order differential equation has the following general form

24 ? NSOU ? CC-PH-04 2 0 2 ( ) d y P x dx ? 1 2 ( ) ( ) ( ) dy

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P x P x y Q x dx ? ? (2.2) Where P 0 (x), P 1 (x), and P 2 (x) are called co-efficient functions and Q(x)		

is the force function. If the function Q(x) = 0, the equation is called homogeneous. For the homogeneous equation, it is to be noted that the function y(x) = 0 always satisfy the equation, regardless what P 0 (x), P 1 (x) and P 2 (x) are. The solution y(x) = 0 is called the trivial solution of the homogeneous equation. 2.3.1 : Second order Linear homogeneous differential equations with constant co-efficient When P 0 , P 1 and P 2 are constants and Q(x) = 0, 0 0 P ? , we get from equation (2.2) 2 0 1 2 2 0 d y dy P P P y dx dx ? ? ? (2.3) 2 1 2 2 0 0 0 P P d y dy y P dx P dx ? ? ? ? 2 2 0, d y dy y dx dx ? ? ? ? ? ? (2.4) Where 1 0 P P ? ? and 2 0 P P ? ? , We seek solution equation (2.4) as x y e ? ? (2.5) Substituting the solution (2.5) in equation (2.4), we find that ? ? 2 0 x e ? ? ? ?? ? ? ? ? (2.6) Which shows that x e ? is a solution of (2.4) only when 2 0 ? ?? ? ? ? ? (2.7) Equation (2.7) is called the characteristic equation. The characteristic roots are 2 1 2 1 1 4 2 2 1 4 2 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.8)

NSOU ? CC-PH-04 ? 25 Thus, solutions of equation (2.4) are given by : 1 2 1 2 , x x y e y e ? ? ? ? (2.9) Now the Wronskian of the solutions (2.9) is 1 2 1 2 1 2 1 2 1 2 ( ) x x x x y y e e W x y y e e ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 2 1 x e ? ? ? ? ? ? ? ? (2.10) Thus the solutions y 1 , y 2 given by equation (2.10) will be linearly independent only when 1 2 ? ?? . In this case, the general solution is : 1 2 1 2 ( ) x x y x C e C e ? ? ? ? (2.11) Now we put 2 4 l ? ? ? ? (2.12) If l < 0 the two roots λ 1 , λ 2 are real. In such a case, the solution (2.11) takes the form : 2 2 2 1 2 ( ) x lx lx y x e C e C e ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.13) or, 2 ( ) sinh cosh 2 2 x l l y x e C x D x ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.14) If l > 0, the two roots λ 1 , λ 2 are imaginary. Then the general solution (2.11) takes the form 2 2 2 1 2 ( ) x i lx i lx y x e C e C e ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.15) or, 2 ( ) sin cos 2 2 x l l y x e C x D x ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.16)

26 ? NSOU ? CC-PH-04 When l = 0, the two roots 1 2 2 ? ? ? ? ? ? are equal and the wronskian of the solution (2.11) is W(x) = 0. Therefore, the two solutions given by equation (2.9) are not linearly independent and the solution (2.11) is not acceptable as general solution. Since the solutions y 1 (x) and y 2 (x) are now dependent we try the second solution as, 2 ( ) ( ) x y x z x e ? ? (2.17) Substituting equation (2.17) in equation (2.4) We get ? ? 2 2 2 (2) 0 x d z dz e z dx dx ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.18) Now co-efficient of z = 0 by equation (2.7) and co-efficient of 0 dz dx ? since we assumed equal root of (2.7) which are 2 ? ? ? ? . Thus from equation (2.18) we get since 0 x e ? ? ? 2 2 0 d z dx ? (2,19), which implies z = x. Thus the second solution is, 2 2 ( ) x y x xe ? ? ? (2.20) Thus, the general solution is 2 2 1 2 ( ) x x y x C e C xe ? ? ? ? ? ? (2.21) 2.3.2 : Second Order Linear Homogeneous Differential Equation with Constant Co-Efficient : Working Rules for Solutions (Complementary Function) We consider a linear homogeneous second order differential equation with constant co-efficient. 2 0 1 2 2 0; d y dy P P P y dx dx ? ? ? (2.22)

NSOU ? CC-PH-04 ? 27 0 0; P ? P 1 and P 2 are constants. In terms of the linear operator d D dx ? where 2 2 2 , dy d y Dy D y dx dx ? ?, equation (2.22) can be written as ? ? 2 0 1 2 0 P D PD P y ? ? ? (2.23) Or, F(D)y=0 (2.24) Where F(D) is a polynomial in the variable D. Now the polynomial F(D) can be factored as : F(D) = P 0 D 2 + P 1 D + P 2 = P 0 (D − m 1) (D − m 2) and equation (2.24) reduces to P 0 (D − m 1) (D − m 2)y = 0 (2.25) The equation F(D) = P 0 (D − m 1) (D − m 2) = 0 Or, (D − m 1)(D − m 2) = 0, since 0 0 P ? (2.26) Is called the characteristic equation of (2.24) and the roots m 1 , m 2 are called characteristic roots. Now to solve equation (2.22) we first write it in the form of equation (2.25) and then write its characteristic equation. The characteristic roots m 1 m 2 are found out. Now we are ready to write down the solution depending on the nature of the roots. The solutions are termed complementary function (y c). Rule I : If the roots are real and different i.e. if 1 2 m m ? , then the solution is 1 2 1 2 1 2 ; , m x m x C y C e C e C C ? ? are arbitrary constant. Rule II : If roots are equal i.e. m 1 = m 2 , then the solution is : 1 1 1 2 ; m x m x C y C e xC e ? ? 1 2 ,C C are arbitrary constant. Rule III : If the roots are imaginary i.e. 1 m i ? ? ? ? and 2 m i ? ? ? ? , then the solution is 1 2 cos sin x c y e C x C x ? ? ? ? ? ? ? ? ? ? ; 1 2 ,C C are arbitrary constant. Example of Rule I : Find the complementary function of the equation : 2 2 5 6 0 d x dx x dt dt ? ? ? → (1)

28 ? NSOU ? CC-PH-04 Solution : Equation (1) can be rewritten as (D 2 − 5D + 9)x = 0, where d D dt ? Auxiliary equation is (D 2 − 5D + 6) = 0 Or, (D − 3) (D − 2) = 0 → (2) Roots of auxiliary equation (2) are m 1 = 3, m 2 = 2; both are real and different. Therefore solution of equation (1) is x c = C 1 e 3t + C 2 e 2t ; x c is the complementary function. Example of rule II : Find complementary function of the equation : 2 2 6 9 0 d y dy y dt dt ? ? ? → (1) Solution : Equation (1) can be re-written as (D 2 + 6D + 9)y = 0 → (2) Auxiliary equation is, d D dt ? ? ? ? ? ? ? (D 2 + 6D + 9) = 0 Or, (D + 3) (D + 3) = 0 → (3) Roots of auxiliary equation are m 1 = − 3, m 2 = − 3 Therefore, roots are real and equal. Therefore solution 3 3 1 2 x x c y c e xc e ? ? ? ? Example of Rule III : Find the complementary function of 2 2 4 0 d y y dx ? ? → (1) Solution : Equation (1) can be re-written as (D 2 + 4)y = 0 ; d D dx ? Auxiliary equation is D 2 + 4 = 0 Or, (D + 2i) (D − 2i) = 0; 1 i ? ? Therefore, roots are imaginary i.e. m 1 = + 2i, m 2 = − 2i and the complementary function is y C = C 1 cos 2x + C 2 sin 2x

NSOU ? CC-PH-04 ? 29 2.4 ????? The Existence and Uniqueness Theorem Consider the initial value problem 2 0 0 2 ( ) ( ) ( ), ( ) , d y dy x x y Q x y x y dx dx ? ? ? ? ? ? 0 0 ( ) y x y ? ? ? . If the functions α, β and Q are continuous on the interval I : p > x > q containing the point x = x 0 ; then there exists a unique solution ( ) y x ?? of the problem, and this solution exists throughout the interval I. That is the theorem guarantees that the given initial value problem will always have (existence of) exactly one (uniqueness) twice differentiable solution, on any interval containing x 0 as long as all three functions α(x), β(x) and Q(x) are continuous on the same interval. Conversely neither existence nor uniqueness of a solution is guaranteed at a discontinuity of α(x), β(x) or Q(x). All the initial conditions in an initial value problem must be taken at the same point x 0 . The set of conditions where the values are taken at different points [e.g. : x = x 0 ; 0 0 , x x x x ? ?? ? ? etc.] are known as boundary conditions. A boundary value problem does not have the existence and uniqueness guarantee. Example 1 : Find the largest interval where : ? ? 2 1 3 cos ; x x y xy xy c ?? ? ? ? ? ? y(0) = 4, (0) 5 ? ? y is guaranteed to have a unique solution. Solution : The given equation can be rewritten as : 2 2 2 3 cos 1 1 1

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x x x e y y y x x x ?? ? ? ? ? ? ? ? Comparing with the standard form, we get 2 2 3 cos ( ) , ( ) 1 1 x x x x x x ? ? ? ? ? ? and 2 ( ) 1 x e Q x x ? ?

But they are continue on – |> x > | containing the point x = 0 We see that α(x), β(x) and Q(x) all have discontinuities at x = – 1 and x = 1. Thus the theorem tells us that there is a unique solution on the interval I : – 1 > x > 1. Since α(x), β(x) and Q(x) are all continuous on – 1 > x > 1 containing x = 0. Now we investigate solutions to linear homogeneous differential equations : ( ) ( ) 0 y x y x y ? ? ?? ? ? ? ? (2.27)
30 ? NSOU ? CC-PH-04 where 2 2 d y y dx ?? ? and dy y dx ? ? . Now if y 1 (x) and y 2 (x) are solutions of equations (2.27), then y(x) = C 1 y 1 (x) + C 2 y 2 (x) is also a solution. This is known as theorem of superposition principle. Proof : since y 1 (x)

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is a solution of (2.27), 2 1 1 2 ( ) ( ) 0 d y x y x y dx ? ? ? ? ? ? (2.28) similarly y 2 (x) is a solution of (2.27) 2 2 2 2 2 ( ) ( ) 0 d y x y

x y dx ? ? ? ? ? ? (2.29) multiplying equation (2.28) by C 1 and equation (2.29) by C 2 and adding the result since C 1 and C 2 are two arbitrary constants, we get ? ?? ? 2 1 1 2 2 1 1 2 2 1 1 2 2 2 ( ) ( ) ( ) 0 d C y C y d C y C y x x C y C y dx dx ? ? ? ? ? ? ? ? (2.30) Equation (2.30) shows that y(x) = C 1 y 1 (x) + C 2 y 2 (x) (2.31) also is a solution of equation (2.27). Next we investigate the initial conditions. If we find a general solution to the homogeneous system, can we choose constants such that the solution satisfies the initial conditions ? That is can we find C 1 and C 2 such that 1 1 0 2 2 0 0 1 1 0 2 2 0 0 ( ) ( ) ( ) ( ) C
y x C y

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x y C y x C y x y ? ? ? ? ? ? ? (2.32) In matrix form, equation (2.32) can be written as 1 0 2 0 0 1 1 0 2 0 0 2 ( ) ( ) ( ) ( ) y x y x y C y x y x y

C ? (2.33) Equation (2.32) has a unique solution if and only if the determinant of the matrix is not zero. This determinant 1 2 1 2 y y y y? ? is called wronskian. Thus our discussion proves the following theorem. Let 0 ( ) ( ) 0, ( )

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y x y x y y x ? ? ?? ? ? ? ? = 0 0 0 , ( ) y y x y ? ? ?

be a homogeneous linear
NSOU ? CC-PH-04 ? 31 second order differential equation and let y 1 and y 2 be two solutions (Any initial value), then if the wronskin. 1 2 2 2 y y y y ? ? ? is non-zero, there exists a solution to the any initial value problem of the form y = C 1 y 1 + C 2 y 2 Example 2 : Construct the wronskian of the solution of the differential equation : 2 8 0 y y y ?? ? ? ? ? and show that any initial value problem will have a unique solution. Solution : The general solution of the given equation :
y = C 1

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e 2x + C 2 e −4x , now the wronkian of y 1 = e 2x and y 2 = e −4x 2 4 2 2 2 1 2 2 4 1 2 4 2 6 0 2 4 x x x x x x x x y y e e W e e e y y e

e ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Thus W is never zero and we can conclude that any initial value problem will have a unique solution of the form 2 4 1 2 x x y C e C e ? ? ? . Exercise 2.4.3 : For each IVP below, find the largest interval on which a unique solution is guaranteed to exist. a) 2 (2) cot 1, (2) 11, (2) 2 x y xy xy x y y ?? ? ? ? ? ? ? ? ? ? ? Solution : The standard form is 2 cos 1 2 (2)sin 2 x x x y y y x x t x ? ?? ? ? ? ? ? ? ? ? and x 0 = 2. The discontinuity α, β and Q are x = − 2, 0, , 2 , 3 ? ? ? ? ? ? ... ..., x = −2 respectively. The largest interval that contain x 0 = 2 but none of the discontinuities is, therefore (0, π). b) 2 16 ln(1) cos 0, (0) 2, (0) 0 x y x y xy y y ?? ? ? ? ? ? ? ? ? ? Solution : The standard form is : 2 2 ln(1) cos 0 16 16 x x y y y x x ? ?? ? ? ? ? ? ? α(x) is only defined (and is continuous) on the interval (−4,4) and similarly β(x).
32 ? NSOU ? CC-PH-04 Q(x) is continuous everywhere. Combining these we see that α, β and Q are all continuous on the interval (− 4,4). Since this interval contains x 0 = 0, it must be the largest interval on which the solution is guaranteed to exist uniquely. 2.5 ????? Linearly Dependent and Linearly Independent Solution of Differential Equation : Wronskian Definition : If y 1 (x) and y 2 (x) are any two solutions of the differential equation : ( ) ( ) 0
y x y x y ? ? ?? ? ? ? ? (2.34) On the interval [a, b], then their wronskian, defined by : 1 2 1 2 1 2 2 1 1 2 ( ) ( ) (,) ( ) ( ) ( ) ( ) ( ) ( )

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y x y x W y y y x y x y x y x y x y x ? ? ? ? ? ? ? ? (2.35)

Is either identically zero or never zero on the interval [a, b] When W(y 1 , y 2) = 0, y 1 (x) & y 2 (x) are linearly dependent solutions (or function) of the equation (2.34) on [a, b]. In this case 2 1 ( ) ( ) y x y x ? constant. When W(y 1 , y 2) ? 0, y 1 (x) & y 2 (x) are linearly independent solutions of equation (2.34) on the interval [a, b]. In this case 2 1 ( ) ( )

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y x y x ? constant and the general solution of equation (2.34) can be written as : y(x) = C 1 y 1 (x) + C 2 y 2 (x) (2.36)

Where C 1 and C 2 are two arbitrary constant. Proof : We suppose y 1 (x) and Y 2 (x) are linearly dependent solutions of equation (2.34) on an interval [a, b]. Then we may assume y 2 (x) = Cy 1 (x) where C is a constant. Therefore 1 2 1 2 1 2 2 1 1 2 ( ) ( ) (,) ( ) ( ) ( ) ( ) ( ) ( )

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y x y x W y y y x y x y x y x y x y x ? ? ? ? ? ? ? ? Again 2 2 ( ) ( ) y x Cy x ? ? ? NSOU ? CC-PH-04 ? 33 Therefore 1 2 1 1 1 1 (,) ( ) ( ) ( ) ( ) 0 W y y C y x y x C y x y x ? ? ? ? ? ? (2.37) Again we assume that y 1 (x) and y 2 (x)

are two linearly independent solutions of equation (2.34) on the interval [a, b]. Then
we have 1 1 1 ( ) ( ) ( ) ( ) ( ) 0
y

x
x
y

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x x y x ? ? ?? ? ? ? ? (2.38) And 2 2 2 ( ) ( ) ( ) ( ) ( ) 0 y x x y x x y x ? ? ?? ? ? ? ? (2.39) Now multiplying equation (2.38) by y 2 (x) and (2.39) by y 1 (x), we get respectively : 1 2 1 2 1 2 ( ) ( ) ( ) ( ) ( ) ( ) 0 y x y x x y x y x x y x y x ? ? ?? ? ? ? ? (2.40) And 2 1 2 1 2 1 ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 y x y x x y x y x x y x y x ? ? ?? ? ? ? ? (2.41) Subtracting equation (2.40) from (2.41), 2 1 1 2 2 1 1 2 ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 y x y x y x y x x y x y x y x y x ? ?? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.42 Now wronskian ? ? 1 2 2 1 1 2 ( ), ( ) ( ) ( ) ( ) ( ) W y x y x y x y x y x ? ? ? ? ? ? ? ? Therefore ? ? 1 2 2 1 1 2 ( ), ( ) ( ) ( ) ( ) ( ) W y x y x y x y x y

x
y
x ? ?? ?? ? ? ? ? ? ? ?
And equation (2.42) can be written as ? ? 1 2 ( ), ( ) ( ) 0 W y x y
x x W ? ? ? ? Or ( ) 0 dw x W dx ?? ? (2.43) Solution of equation (2.43) is ( )x dx W Ce ??? ? (2.44) Where C is a constant, depending on y 1 and y 2 , but not on x. We assume that the function α(x) is continuous on the interval [a, b], then ( )x dx ?? will also be continuous, on [a, b], so that ( ) 0 x dx e ??? ? in the assumed interval and therefore wronskian of two linearly independent solutions is never zero for all x in [a, b]. Now we have the following theorem : Let y 1 (x) and y 2 (x) be differentiable on [a, b]. If wronskian W(y 1 , y 2) is non-zero for some x 0 in [a, b], then y 1 (x) and y 2 (x) are linearly independent on [a, b]. If y 1 (x) and y 2 (x) all linearly dependent then the wronskian is zero for all x in [a, b].
34 ? NSOU ? CC-PH-04 2.5.1 : Non-Homogeneous Linear Equation with Constant Co-Efficient We consider the differentiable equation, 2 2 ( ) d y dy y Q x dx dx ? ? ? ? ? (2.45) Where α and β are constants. The general solution of equation (2.45) can be written as y = y c + y p (2.46) Where y c is the complementary function (solution) corresponding to Q(x) = 0 in equation (2.45) and y p is the particular integral (solution) of equation (2.45); corresponding to ( ) 0 Q x? . In sec 2.4.2A we have discussed how to find y c . Now we discuss how to find the particular integral y p using the method of undetermined co-efficient. Case I : If Q(x) is a polynomial of degree n and zero is not a root of the characteristic equation, then y p can be written as, y p = A 0 + A 1 x + ... + A n x n (2.47) If however zero is a single root of the characteristic equation then y p = x(A 0 + A 1 x + ... + A n x n) (2.48) Case II : If ( ) x Q x ce ? ? and λ is not a solution of the characteristic equation, then x p y Ae ? ? (2.49) If λ is a single root of the characteristic equation, then x p y Axe ? ? (2.50) If λ is a double root of the characteristic equation, then 2 x p y Ax e ? ? (2.51) Case III : If sin ( ) cos C x Q x D x ? ? ? ? ? ? and iλ is not a root of the characteristic equation, then cos sin p y A x B x ? ? ? ? (2.52) If however iλ is a root of the characteristic equation, then (cos sin) P Y x A x B x ? ? ? ? (2.53) Case IV : Use of complex exponentials : To find particular solution of F(D)y = Q(x) where sin ( ) cos C x Q x C x ? ? ? ? ? and where F(D)
NSOU ? CC-PH-04 ? 35 = 0 is the characteristic equation, first solve F(D)y = Ce iλx and then take the real and imaginary part. Case V : When Q(x) is an exponential times a Polynomial i.e. ( ) ( ) x n Q x e P x ? ? Where P n (x) is a Polynomial of degree n, a particular solution Y p of ( ) ( )( ) ( ), x n F D y D a D b y e P X ? ? ? ? ? ? is 2 ( ) is not or ( ) equals or , ( ) x n x p n x n e Q x if a b Y xe Q x if a b a b x e Q x if a b ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.54) Where Q(x) is a polynomial of the same degree as P n (x) with undetermined co- efficient to be found to satisfy the given differential equation. Note that sines and cosines are included in e λx by use of Complex exponentials as in Case IV. Example of Case (I) : Find the general solution of 2 2 y y y x x ?? ?? ? ? ? (1) Solution : The characteristic equation is 2 1 9 2 0 ie 2 ? ? ? ? ? ? ? ? ? ? or, 1 2 1 3 1 3 1; 2 2 2 ? ? ? ? ? ? ? ? ? ? ? ? Therefore the roots of the characteristic equation are (1, −2) and zero is not a root of the characteristic equation. Therefore 2 1 2 x x c y C e C e ? ? ? (2) And y p may be taken in the form 2 0 1 2 p y

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A A x A x ? ? ? (3) Substituting equation (3) in equation (1), we get ? ? 2 2 2 1 2 0 1 2 2 2 Aa A A x A A x A x x x ? ? ? ? ? ? ? Or, 2A 2 − 2A 2 x 2 + A 1 − 2A 1 x + 2A 2 x − 2A 0 = x 2 − x Or, − 2A 2 x 2 − 2A 1 x + 2A 2 x + 2A 2 + A 1 − 2A 0 = x 2 − x

Comparing the
co-efficient of various power of x, we get
36 ? NSOU ? CC-PH-04 x 0 → 2

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A 2 + A 1 – 2A 0 = 0 x 1 → 2A 2 – 2A 1 = – 1, A 1 = 0 x 1 → – 2A 2 = 1, 1 2 2 A ? ? 1 0 2 A ? ?

Hence 2 1 1 2 2 p y x ? ? ? ? ? 2 1 1 2 x ? ? ?
Therefore general solution y is given by ? ? 2 2 1 2 1 1 2 x x c p y y y C e C e x ? ? ? ? ? ? ? Example of Case (II) :

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Solve the equation (D – 1) (D + 5)y = 7e 2x (1) Solution : Characteristic equation is (D – 1)(D + 5) = 0 (2) ∴

Roots of characteristic equation 1 2 1 and 5 ? ? ? ? ? 5 1 2 x x c y C e C e ? ? ? ? We see that 2 2 ? ? is not a root of the characteristic equation. To find a particular solution we take 2 x x p y Ae Ae ? ? ? (3) Now equation (1) can be re-written as 2 2 (4 5) 7 x D D y e ? ? ? Or, 2 2 2 4 5 7 x d y dy y e dx dx ? ? ? (4) Substituting (3) in equation (4) 4Ae 2x + 8Ae 2x – 5Ae 2x = 7e 2x 7Ae 2x = 7e 2x 2 0, 1 x e A ? ? ? ∴ y p = e 2x (5)
NSOU ? CC-PH-04 ? 37 Therefore general solution : 5 2 1 2 x x x y C e C e e ? ? ? ? Example of Case (III) : Solve the initial value problem 2 2 4 2sin d y y x dx ? ? , given (0) 1, (0) 0 dy y dx ? ? Solution : Here λ = 1 Now characteristic equation D 2 + 4 = 0, 1 2 2 , 2 i i ? ? ? ? ? ?, are the roots of the characteristic equation. Hence λ 1 = 1 is not a root of the characteristic equation. Now Complementary function y c = C 1

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cos 2x + C 2 sin 2x (1) We assume particular integral y p = A sin x + B cos x (2) Substituting (1) in the original equation we get, (– A sin x – B cos x) + 4(A sin x + B cos x) = 2 sin x Comparing co-efficient of sin x and cos x

on both sides, we find 2 , 0 3 A B ? ? 2 sin 3 p y x ? ? And the general solution 1 2 2 sin 2 sin 2 sin 3 c p y y y C x C x x ? ? ? ? ? (3) Now 1 2 2 2 sin2 2 cos 2 cos 3 dy C x C x x dx ? ? ? ? Using the initial value y(0) = C 1 = 1 and 2 2 (0) 2 0 3 dy C dx ? ? ? 2 1 3 C? ? ? Hence the specific solution is 1 2 cos 2 sin2 sin 3 3 y x x x ? ? ? Example of Case (IV) : Find the complementary function of 2 4sin 2 y y y x ?? ? ? ? ? (1) Solution : To find the particular solution y p of equation (1), we find Y p for the equation 2 2 4 ix Y Y Y e ?? ?? ? ? (2) and take its imaginary part.
38 ? NSOU ? CC-PH-04 We observe that 2i is not equal to a root of the auxiliary equation of (2). Following the method written in case (II), we assume a solution. Y p = Ae 2ix (3) and substitute it in equation (2) to get 2 2 (4 2 2) 4 ix ix i Ae e ? ? ? ? 4 4(2 6) 1 (3) 2 6 40 5 i A i i ? ? ? ? ? ? ? ? ? ? 2 1 (3) 5 ? ? ? ? ix p Y i e Taking the imaginary part of Y p , We get y p of equation (1) Or, 2 2 3 5 5 ix ix p m i y I e e ? ? ? ? ? ? ? ? ? 3 (cos2 sin 2) (cos 2 sin 2) 5 5 p m m i y I x i x I x i x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 3 cos2 sin 2 5 5 x x ? ? ? Where I m means Imaginary Part. Example of Case (V) : Find a particular solution of 3 6 9 12 x y y y xe ?? ? ? ? ? (1) Solution : Equation (1) is re-written as (D – 3)(D – 3)y = 12xe 3x (2) We observe that λ = 3 is equal to either of the roots of the auxiliary equation i.e. λ 1 = a = b = λ 2 also P n (x) = 12x = P 1 (x) is a polynomial of degree 1. Then Q(x) is also a polynomial of degree 1 namely AX + B. Since λ = a = b, we write y p = x 2 e 3x (Ax + B) = e 3x (Ax 3 + Bx 2) (3) We substitute equation (3) in equation (1) and find A and B so that we have an identity: 3 6 9 12 x p p p y y y xe ?? ? ? ? ? We find A = 2 and B = 0, 3 3 (2) x p y e x ? ?

NSOU ? CC-PH-04 ? 39 2.5.2 : Inhomogeneous Linear Equation with Constant Co-efficient : Working Rule for Finding the Particular Integral : We have linear inhomogeneous differential equation with constant co-efficient F(D)y = Q(x) (2.55) As started earlier the solution of equation (2.55) consists of two parts 1) Solution for F(D)Y = 0, which is called complementary function (C.F) and 2) Any particular integral (P.I) of equation (2.55) given by 1 ( ) ( ) PI Q x F D ? (2.56) Thus the solution of equation (2.55) is y = CF + PI Rules for finding the particular integral (PI) : Rule I : When Q(x) = e ax , a is a constant, 1 1 ( ) ; ( ) 0 ( ) ( ) ax PI Q x e F a F D F a ? ? ? If F(a) = 0; 1 ; ( ) 0 ( ) ax PI x e F a F a ? ? ? ? If 2 1 ( ) 0; ; ( ) 0 ( ) ax F a PI x e F a F a ? ?? ? ? ? ? ?? And so on Where ( ) ( ); ( ) ( ) d a d d F D F D F a F D dD dD ? ? ? ? ? etc. Rule II : When Q(x) = e ex V(x); When V(x) is any function of x; 1 1 ( ) ( ) ( ) ( ) ax ax PI e V x e V x F D F D a ? ? ? Rule III : When Q(x) = xV(x), When V(x) is of the form sin(ax + b) or cos (ax + b); Then, ? ? 2 1 1 ( ) ( ) ( ) ( ) ( ) ( ) ( ) F D PI xV x x V x V x F D F D F D ? ? ? ?
40 ? NSOU ? CC-PH-04 Rule IV : If Q(x) = x m , m being a constant; 1 ( ) m PI x F D ? . Find 1 ( ) F D by actual division in ascending powers of D and retains term up to D m . Rule V : If Q(X) = sin (ax + b) or cos(ax+ b); where a & b are constant, then follow the method shown in worked out example. Example of Rule I Find the particular integral of : 2 3 2 x y y y e ?? ? ? ? ? Solution : F(D) = D 2 – 3D + 2 F(a) = F(2) = 2 2 – 3.2 + 2 = 0 Now ( ) 2 3 F D D ? ? ? And (2) 2 2 3 1 0 ? ? ? ? ? ? F Therefore, 2 2 1 (2) x x PI x e xe F ? ? ? Example of Rule II : Find the particular integral of the equation : 2 4 cos x y y y e x ?? ? ? ? ? Solution : Here a = 1, V(x) = cos x 1 1 ( ) ( ) ( ) ( ) ax ax PI e V x e V x F

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D F D a ? ? ? Now, F(D) = D 2 – 2D + 4 F(D + 1) = (D + 1) 2 – 2(D + 1) + 4 = D 2 + 2D + 1 – 2D – 2 + 4 = D 2 + 3 ∴ 2 2 1 1

cos cos 3 1 3 x x PI e x e x D ? ? ? ? ? (see example of Rule V) 1 1 cos cos 2 2 x x e x e x ? ?
NSOU ? CC-PH-04 ? 41 Example of rule III : Find the particular integral of the equation : (D 2 + 3D +2)y = x sin 2x Solution : Here Q(x) = xV(x) = x sin 2x F(D) = D 2 + 3D + 2 ( ) 2 3 F D D ? ? ? 2 1 (sin 2) 3 2 ? ? ? ? ? PI x x D D ? ? 2 2 2 1 2 3 sin 2 sin 2 3 2 3 2 D x x x D D D D ? ? ? ? ? ? ? ? ? ? 2 2 2 sin 2 (2 3) sin 2 2 3 2 2 3 2 x D x x D D ? ? ? ? ? ? ? ? ? Where we have substituted D 2 = (– a 2); here a = 2 2 sin 2 2 3 sin 2 3 2 (3 2) x D x x D D ? ? ? ? ? Since the 1st term contains 3D – 2 in the denominator we make it 9D 2 – 4 by multiplying both numerator and denominator by 3D + 2. 2 2 (3 2)sin 2 (2 3)sin 2 9 4 9 12 4 D x D x x D D D ? ? ? ? ? ? ? ? ? 2 2 (3 2)sin 2 (2 3)sin 2 9(2) 4 9(2) 12 4 D x D x x D ? ? ? ? ? ? ? ? ? Where we have substituted D 2 = – a 2 = (– 2 2) (3 2)sin 2 (2 3)sin 2 40 (12 32) D x D x x D ? ? ? ? ? ? In the 2nd term since there is no term contain D 2 in the denominator, we multiply both numerator and denominator of the 2nd term by 12D – 32
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cos2 2sin 2) (2 3) (12 32) sin 2 40 (12 32) (12 32) x x D D x x D D ? ? ? ? ? ? ? ? ? 2 (6cos 2 2sin 2) (2 3)(8)sin 2 40 4(9 64) x x D D x x D ? ? ? ? ? ? ? ? or 3cos 2 sin 2 (2 3)(6cos 2 8sin 2) 20 4(100) x x D x x PI x ? ? ? ? ? ? ? ? (3cos 2 sin 2) 48sin 2 14cos 2 20 400 x x x x

x ? ? ? ? ?
Also see example of Rule – V. Example of Rule IV : Find the particular integral of the equation : (2D 2 + 2D + 3)y = X 2 + 2x + 1 Solution : Q(x) = X 2 + 2x + 1 ? ? 2 1 2 1 ( ) ? ? ? PI x x F D Where F(D) = 2D 2 + 2D + 3 Now 2 1 3 2 2 D D ? ? is found by actual division (not using any formula) and retaining up to the term containing D 2 in the quotient, since the degree of the polynomial
x 2 + 2x + 1 is 2. Therefore 2 2 2 1 1 (2 1) (2 1) ( ) 3 2 2 ? ? ? ? ? ? ? ? ? PI

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x x x x F D D D 2 2 1 2 2 (2 1) 3 9 27 D D x x ? ? ? ? ? ? ? ? ? ? ? ? 2 1 4 4 2 4 1 3 9 27 3 9 3 x x x ? ? ? ? ? ? ? 2 1 2 2 (2 1) (2 2) (2) 3 9 27 x

x x ? ? ? ? ? ?

NSOU ? CC-PH-04 ? 43 Example of Rule V : Find the particular integral of the equation : (D 2 + 3D −4)y = sin 2x Solution : Here F(D) = D 2 + 3D – 4 ; a = 2, b = 0 i.e. sin(ax + b) = sin 2x Now 2 1 1 ( ) sin 2 ( ) 3 4 PI Q x x F D D D ? ? ? ? Now putting D 2 = – a 2 = – 2 2 , we get 1 (3 8)sin 2 sin 2 3 8 (3 8) (3 8) D x PI x D D D ? ? ? ? ? ? ? 2 (3 8)sin 2 9 64 D x D ? ? ? Again putting D 2 = – a 2 = – 2 2 (3 8) sin 2 100 D x PI ? ? ? 6cos 2 8sin 2 100 x x ? ? ? Case VI : When Q(x) is sum of several terms consisting of exponential, polynomial and trigonometric functions etc. ; then particular integral will be algebraic sum of individual particular integrals according to the principle of superposition. For example, if Q(x) = (e ax) + (4 sin bx) + (ax 2 + bx) ; Then the particular integral will be algebraic sum of the individual particular integrals corresponding to the respective function. We take an imaginary differential equation : 2 2 2 2 [4sin] ax d y dy y e bx ax bx dx dx ? ? ? ? ? ? ? ? ? ? ? ? ? Then the particular integrals of the given differential equation will be given by : 1 2 3 p p p p y y y y ? ? ? When 1 p y is the particular integral for 2 ax p e y is the particular integral for 4 sin bx and 3 p y is the particular integral for [ax 2 + bx].

44 ? NSOU ? CC-PH-04 Exercise (2.4.5) : 1) Obtain the general solution of the equation : 2 3 2 3 2 d y dy y x dx dx ? ? ? 2) Solve the equation (D 2 + 4)y = sin 2x 3) Solve 2 2 2 sin x d y dy y x e x dx dx ? ? ? Solution (1) : In D-operator notation, the equation become, (D 2 – 2D + 2)y = x 3 Auxiliary equation is : D 2 – 3D + 2 = 0 Roots of the auxiliary equation are m 1 = 1, m 2 = 2 Complementary function C = y c = Ae x + Be 2x Particular integral is given by 3 3 2 2 1 1 9 21 45 2 2 2 4 2 3 p y x x x x D D ? ? ? ? ? ? ? ? ? ? ? ? ? ∴ the complete solution is y = y c + y p Solution (2) : The auxiliary equation is D 2 + 4 = 0 Roots of auxiliary equation is m = ± 2i Complementary function y c = A cos 2x + B sin 2x Where A and B are arbitrary constant. Particular integral 2 1 sin 2 4 p y x D ? ? Now D 2 = – (2) 2 = – 4 ∴ D 2 + 4 = 0 1 1 cos 2 sin 2 sin 2 2 2 4 p x x x y x x x D D ? ? ? ? ? cos2 sin 2 cos 2 4 c p x y y y A x B x x ? ? ? ? ? ? Solution 3 : Complementary function y c = (A + Bx)e x Particular integral, 2 2 1 1 sin

sin 2 1 (1) 2(1) 1 x x p y e x x e x x D D D D ? ? ? ? ? ? ? ?

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sin (cos) (cos) x x x e x x e x xdx e x x x dx D D D ? ? ? ? ? ? ? ? ? ? ? ? (cos sin) x e x x x dx ? ? ? ? ? ? sin sin cos x e x x x dx x ? ? ? ? ? ? ? ? ? = – e x (x sin x + 2 cos x) ∴

Complete solution of y = y c + y p 2.6 ?????

Summary i) Classification of second order differential equation explained. ii) Different method of finding particular integral have been exemplified. iii) Rules for finding complementary function and particular integral have also been included. iv) Existence and uniqueness theorems for IVP have been illustrated with examples. v) Use of wronskian to identify linear dependent and independent solutions have been discussed.

46 ? NSOU ? CC-PH-04 Unit 3 ? Calculus of Functions of More than one Variable Structure 3.1 Objectives 3.2 Introduction 3.3 Partial Derivatives 3.3.1 Total Differential 3.3.2 Error Determinations and Approximation 3.4 Exact and Inexact Differentials 3.4.1 Integrating Factor 3.4.2 Rules to Find Out Integrating Factor 3.5 Constrained Maximization Using Lagrange's Undetermined Multipliers 3.5.1 Method of Lagrange's Undetermined Multipliers with Functions of Two Independent Variables and one ?´ Equations 3.5.2 Method of Lagrange's Undetermined Multipliers with Three Independent Variables and one ?´ Equation 3.5.3 Method of Lagrange's Undetermined Multipliers with two ?´ Equations 3.5.4 Working Rules for Constraint Maximization or Minimization Using Lagrange's Undertermined Multipliers 3.6 Summary 3.1 ? Objectives 1. To know what is partial and total derivatives and differentials. 2. To make an idea about exact and inexact differential. 3. To know how to convert inexact differential into exact differential with the help of integrating factor. 4. To know how to find out maximum and minimum values of functions with constraints using Lagrange multipliers.

NSOU ? CC-PH-04 ? 47 Keywords : Partial derivatives, total derivatives, exact and inexact differentials, integrating factors, maxima and minima problems with constraint. 3.2 ? Introduction We consider a field scalar such as temperature (T) and its distribution in a region of space. We see that temperature may change with x, y and z co-ordinates of space and also with time t if the state is not steady. Thus we see temperature is, in essence, function of several variables x, y, z, t i.e. T = T (x, y, z, t) (3.1) Now if we want to find the rate of change of temperature we can find it in various ways. Suppose we want to find the rate of change with x co-ordinate only keeping y, z, and t constants. We use partial derivative ? ? T x ? ? . If temperature T is function of x- co- ordinates alone we could find the above rate of change by dT dx , the ordinary derivative. Derivatives are also used in finding the maxima or minima of a curve. Now rates occur very often in physics e.g. time rates, space rate etc. and we have to find these rates in the form of differential equations which we have to solve to find out the rate of functional dependence of the quantity with other. 3.3 ? Partial Derivative Suppose we have a function f, having more than one independent variables (x, y) i.e. f = f(x, y). Now if for f(x, y), keeping y as constant, an ordinary differentiation with respect to x is found, the derivative so obtained is called partial derivative and is denoted by ? ?
f x ? ? or

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f x where 0 (,) (,) lim x x f x x y f x y f x ? ? ? ? ? ? ? ? (3.1) Similarly treating x as constant, we get ? ? 0 , (,) lim y y f x y y f x y f f y y ? ? ? ? ? ? ? ? ? ? (3.2)

Now we can determine higher order partial derivatives also which are denoted by f xx , f xy , f yy for second order; f xxx , f xxy , f xxz etc. for third order partial derivatives. Where 2 2 2 2 2 , , xx xy yy

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f f f f f x y x y ? ? ? ? ? ? ? ? ? ? 48 ? NSOU ? CC-PH-04 3 3 3 3 2 2 , , xxx xxy xxz f f f f f x x y x z ? ? ? ? ? ? ? ? ? ? ?

etc. A notation which is frequently used in thermodynamics is y f x ? ? ? ? ? ? ? ? , meaning we have to find out partial derivative of f(x, y) with respect to x when y is held constant. Similarly x f y ? ?? ? ? ? ? ? is defined. Now suppose we find out 2 f f x y x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? and also find out 2 f f y x y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? . Then a question automatically comes out that could we write f f x y y x ? It can be proved that if the first and second order partial derivatives of f(x, y) are continuous, then only 2 2 f f x y y x ? ? ? ? ? ? ? ? , otherwise not. In thermodynamics these conditions are usually satisfied and the equality hold. 3.3.1 Total Differential We consider a function of two variable (x, y) represented by z = f(x, y), which represents a surface. Now the derivatives , x y f f f f x y ? ? ? ? ? ? , at a point, are the slopes of the two tangent lines to the surface in the x and y directions at that point. The symbols x dx ? ? and ?y = dy represent changes in the independent variables x and y. The quantity ?z means the corresponding change in z along the surface. We define dz by the equation z z dz dx dy x y ? ? ? ? ? ? (3.3) The differential dz is called the total differential of z. Equation (3.3) may also be interpreted as follows. Any change in z, ?z will be sum of changes due to change in x and changes in y respectively. Now rate of change of z with respect to x and y is given by respectively z x ? ? and z y ? ? . NSOU ? CC-PH-04 ? 49 Therefore we can write change in z as y x z z z x y x y ? . Now in the limit 0 x? ? and 0 y? ? we can write z z dz dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.3) 3.3.2 Error determination and approximation If z = f(x, y, u, ...), then the total variation ?z in z due to changes ?x, ?y, ?u, ... in x, y, u, ... is given by [see 3.3A]. ... z z z z x y u x y u ? ? ? ? ? ? ? ? ? ? ? ? ? ? Here ?x (or x?), ?y, ... are the actual errors in x & y, ... while ?z is the approximate error in z i.e. in z = f(x, y, u, ...) Now ?x, ?y, ... are known as absolute errors in x, y, ... in measurement and , x y x y ? ? ,... are called the proportional error in x, y, ... etc. Example of Art : 3.2 and 3.3 : 1. If 1 u
r ? , where 2 2 2

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r x y z ? ? ? Show that 2 2 2 2 2 2 0 u u u x y z ? ? ? ? ? ? ? ? ? Solution : ? ? ? ? 1 2 1 2 2 2 2 2 2 2 1 u x y z x y z ? ? ? ? ? ? ? ? ? ? ? 3 3 2 2 2 2 2 2 2 2 2 1 . 2 2 u x y z x x x y z x ? ? ? ? ? ? ? ? ? ? ? ? ? And ? ? ? ? 3 5 2 2 2 2 2 2 2 2 2 2 2 3 1. 2 2 u x y z x x y z x x ? 50 ? NSOU ? CC-PH-04 ? ? 5 2 2 2 2 2 2 2 2 2 3 x y z x y z x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 5 2 2 2 2 2 2 2 5 2 2 2 2 2 x y z x y z r x y z ? ? ? ? ? ? ? ? ? ? Similarly ? ? 2 5 2 2 2 2 2 u r y x z y ? ? ? ? ? ? and ? ? 2 5 2 2 2 2 2 u r z x y z ? ? ? ? ? ? ? ? ? ? ? 2 2 2 5 2 2 2 2 2 2 2 2 2 2 2 0 u u u r x y z x y z x y z ? 2)

Express two-dimensional Laplace’s equation : 2 2 2 2 0 u u x y ? ? ? ? ? ? in polar co-ordinate. Solution : Equation of transformation from Cartesian co-ordinates (x, y) to polar co- ordinates (r, ?) is given by :

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x = r cos ?, y = r sin ? 2 2 1 , tan y r x y x ? ? ? ? ? ? ? ? ? ? ? ? (i) Now 2 2 2 cos 2 sin r x x x r x y r y y r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (ii) 2 2 2 2 2 2 2 2 2 1 sin 1 1 1 cos 1 y x y x y y x r x x y

x x x r x
y ? (
iii)
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cos
cos
sin
r
x r x

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x r r r y r y y r r ? (iv) 2 2 sin sin cos cos x x r r r r x ? 2 2 2 2 2 2 2 2 2 2 2

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sin cos sin sin 2sin cos cos r r r r r ? (v) Similarly 2 2 2 2 2 2 2 2 2 2 2 2sin cos cos cos sin r r r y r r ? 2 2sin cos r ? ? ? ? ? ? ? (vi) Adding (v) and (vi) we get, 2 2 2 2 2 2 2 2 2 1 1 r r x y r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 1 1 0 u u u u u r r x y r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3) Prove that if f(x, y, z) = 0 I. ? ? 1 x y z y z x ? ? ? ? ? ? ? ? ? ? ? and two similar relation II. 1 x y z x y z y z x ? ?? Proof : We have, f(x, y, z) = 0 i.e. x = x(y, z) and z = z(x, y) y z x x dx dy dz y z ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (i) 52 ? NSOU ? CC-PH-04 and y x z z dz dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (ii) Eliminating dz from (i) and (ii) we get y y z x x x z z dx dy dx dy y z x y ? (

iii) Now dx and dy are independent of each other. So, equating the co-efficient of dx from both side of (iii), we get, ? ? 1 1
x y
y y z y x z z z x x ? (I) Similarly, by equating the co-efficient of dy we obtain,
0

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y z x x x z y z y ? or, 1 0 x y z y z x z x y ? or, 1 x y z y z x z x y ? (

II) Note : Relation (I) and (II) are extensively used in thermodynamics systems e.g. hydrostatic system given by f(P, V, T) = 0. Example 4 : The rate of flow (V) of a liquid through a capillary tube of radius r and length l at a pressure difference P between it ends is given by : 4 8 P r V l ? ? ? , where ? is the viscosity of the liquid. In an experiments, the errors in the measurements of P, r, l and V are 1%, 1.5%, 0.5% and 2% respectively. Evaluate the error in the measurements of ?. Solution : We have 4 8 P r Vl ? ? ? 4 ln ln ln ln ln 8 ln ln P r V l ? ? ? ? ? ? ? ? ?
NSOU ? CC-PH-04 ? 53 or, 0 4 0 P r V l P r V l ?? ? ? ? ? ? ? ? ? ? ? ? or, max 4 P r V l P r V l ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? , therefore Percentage error, max 100 100 4 100 100 100 P r V l P r V l ?? = 1 + 4 × 1.5 + 2 + 0.5 = 9.5% Notes : During experiments students are asked to find out the maximum proportional error in their measurement. Therefore all the terms are added. For percentage error ; proportional error is multiplied by 100. Example 5 : If u = f (x, y, z, ...) where x, y, z, ... are all functions of a variable t. Prove that, I) ...

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du u dx u dy dt x dt y dt ? ? ? ? ? ? ? ? ?

II) f x f y dy dx ? ? ? ? ? ? Proof : We have u = f(x, y, ...) ... u u u du dx dy dz x y z ? ? ? ? ? ? ? ? ? ? ? (i) But from the definition of differential. We obtain : , , du dx dy du dt dx dt dy dt dt dt dt ? ? ? etc. From (i) ...

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du u dx u dy dt x dt y dt ? ? ? ? ? ? ? ? ? (

I) If u = f (x, y) = C (constant), then y is an implicit function of x. Now from equation (i) 0 du u dx u dy dx x dx y dx ? ? ? ? ? ? ? ? ?
54 ? NSOU ? CC-PH-04 0 f f dy x y dx ? ? ? ? ? ? ? or, f x f y dy dx ? ? ? ? ? ? Exercise : Art 3.2 and 3.3 : 1) If u = log(x 3 + y 3 + z 3 – 3xyz), show that ? ? 2 2 2 2 2 9 u x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2) If z varies directly as x and inversely as y and the possible errors in measuring x and y are 1% and 0.5% respectively, find the amount of error in z. Given 1 3 z ? when x = 3; y = 5. 3) If ( ) ( ) z f x ct x ct ? ? ? ? ? , show that 2 2 2 2 2 z z c t x ? ? ? ? ? , where c is a constant. Solutions to exercise Art : 3.2 and 3.3 : Solution (1) :
u = log (
x 3 +
y 3 + z 3 – 3xyz) 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 , , 3 3 3 u x yz u y xz u z xy

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x y z x y z xyz x y z xyz x y z xyz ? 2 2 2 2 2 2 3 3 ( ) x y z xy yz xz u u u x y z x y z x y z x y z xy yz xz ? Now 2 u u u u x y z x y z x y z ? ? ? ?? ?? ?? ? 3 x y z x y z ? NSOU ? CC-PH-04 ? 55 2 2 2 2 3 3 3 9 ( ) ( ) ( ) ( ) x y z x y z x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? Solution 2 : We have z x? and 1 z y ? . x z k y ? ? , where k is a constant. Now when x = 3, y = 5, 1 3 3 5 z k ? ? 5 9 k? ? 5 9 x z y ? ? ? ln z = ln 5x – ln 9y = ln 5 + ln x – ln 9 – ln y 0 0 z x y z x y ? ? ? ? ? ? ? ? max z x y z x y ? ? ? ? ? ? ? ? ? ? ? ? 100 100 100 z x y z x y ? ? ? ? ? ? ? ? ? = 1 + 0.5 = 1.5% ?

maximum proportional error in z is 1.5%. Now
z

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x y z x y ? ? ? ? ? 1 0.5 1.5 100 100 100 x y z z z z x y ? error in z is 1.5% of z. Solution (3) : We have ( ) ( ) z f x ct x ct ? ? ? ? ? 56 ? NSOU ? CC-PH-04 ( ) ( ) ( ) ( ) z f x ct x ct x ct x ct x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ( ).1 ( ).1 f x ct x ct ? ? ? ? ? ? ? ? 2 2 ( ) ( ) z f x ct x ct x ? ? ?? ?? ? ? ? ? ? ? ? (i) Again ( ) ( ) ( ) ( ) ( ) ( ) z f x ct x

ct x
ct x ct c f x
ct c x ct t t t ? And ? ? 2 2 2 2 2 ( ) ( ) ( ) ( ) z c f x ct c x ct c f x ct x ct t ? ? ? ?? ?? ?? ?? ? ? ? ? ? ? ? ? ? ? (ii) From (i) and (ii) : 2 2 2 2 2 z z c t x ? ? ? ? ? 3.4 ? Exact and Inexact Differentials We consider a function z = f(x, y) = constant, which is continuous along with its first order partial derivative. Then the total differential is 0 f f dz dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? This can be expressed as dz = M(x, y)dx + N(x, y)dy = 0 (3.4) where (,) f M x y x ? ? ? ? ? ? ? ? ? ? and (,) f N x y y ? ?? ? ? ? ? ? ? ? . Since the function f(x, y) has continuous first order derivative, we can write 2 2 f f x y y x ? ? ? ? ? ? ? i.e N M x y ? ? ? ? ? ? ? ? ? ? ? (3.5) This is the necessary and sufficient condition that the expression (3.4) be an exact differential equation and the differential dz = M(x, y)dx + N(x, y)dy be an exact differential. If dz is an exact differential z = f(x, y) is called a point function or state function. If however equation (3.5) is not satisfied then the differential dz is called inexact differential and the function z = f(x, y) is called a path function. Conditions for equality of f xy and f yx
NSOU ? CC-PH-04 ? 57 a) If (a, b) be a point in the domain of definitions of f(x, y) so that f x (x, y) and f y (x, y) are differentiable at (a, b) then f xy (a, b) = f yx (a, b) b) If (a, b) be a point in the domain of definitions of f(x, y) so that f x (x, y) exist in a certain neighborhood of (a, b) and f xy (x, y) is continuous at (a, b) then f xy (a, b) = f yx (a, b) 3.4.1 : Integrating factor : Integrating factor is a function chosen to make an inexact differential to be transformed into an exact differential. We consider the equation dz = M(x, y)dx + N(x, y)dy = 0 Which we suppose, to be not exact. Now if there exists a function (,) x y ? such that ? ? (,) x y Mdx Ndy d ? ? ? ? for some function (,) x y ? , then (,) x y ? is called an integrating factor of equation (3.4) For example the equation xdy − ydx = 0 is not exact, multiplying it by 2 1 y ? , the equation became 2 2 0 xdy ydx ydx xdy y y ? ? ? ? ? Or, 0 x d y ? ? ? ? ? ? ? which is exact and has the general solution x y = constant. Thus 2 1 y ? is integrating factor (I.F) of the inexact differential xdy − ydx. 3.4.2 : Rules to find out integrating factor : Let the differential equation dz = M(x, y)dx + N(x, y)dy = 0 is not exact. i.e. M N y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Rule I : i. Mdx + Ndy = 0 and M(x, y), N(x, y) are both homogeneous functions of (x, y) of the same degree, then 1 Mx Ny ? is an integrating factor of the equation
58 ? NSOU ? CC-PH-04 Mdx + Ndy = 0 Example 1 : Consider
the differential equation dz = (
x 2 y − 2
xy 2)dx + (3

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x 2 y − x 3)dy = 0 Here M(x, y) = x 2 y − 2xy 2 2 4 M x y y ? ? ? ? ? N(x, y) = 3x 2 y − x 3 2 6 3 N xy x x ? ? ? ? ? Therefore, M N y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? and dz is not exact differential. Now 2 2 2 3 2 (,) 2 2 y y M x y x y xy x x x ? 3 y x x ? ? ? ? ? ? ? ? ? M(x, y)

is a homogeneous function of degree 3 in x and y. Similarly N(x, y) is a homogeneous function of degree 3 in x and y Therefore 2 2 0 Mx Ny x y ? ? ? .and 2 2 1 x y is an I.F.. Now multiplying both sides of the given equation by 2 2 1
x y , we get 2 2 2 3 2 2 2 2 2 3 0 x
y xy

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x y x dx dy x y x y ? ? ? ? Or, 2 1 2 3 0 x dx dy y x y y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Or, 2 1 2 3 0 x dx dy dx dy y x y y ? ? ? ? Or, 2 2 3 0 ydx xdy dx dy x y y ? ? ? ? NSOU ? CC-PH-04 ? 59 Or, 2 (log) 3 (log) 0 x d d x d y y ? ? ? ? ? ? ? ? ? Therefore 2 2 2log 3log dz x d x y y x

y ? ? ? ? ? ? ? ? ?
is an exact differential. Rule II : Consider the differential equation M(x, y)dx + N(x, y)dy = 0. If this equation is not exact, then M N y x ? ? ? ? ? ? however if M N y x N ? ? ? ? ? ? is function of x only denoted by f(x), then ( ) ( ) f x dx x e ? ? ? will be an integrating factor of the given differential equation. Rule III : However if M N y x N ? ? ? ? ? ? ? is function of y only, denoted by f(y) then ( ) ( ) f y dy y e ? ? ? will be an integrating factor.. Example 2 : The differential equation dz = (3xy − y 2)dx + x(x − y)dy = 0 is not exact, since 3 2 M x y y ? ? ? ? and 2 N x y x ? ? ? ? . M N y x ? ? ? ? ? ? However note that 1 M N y x N x ? ? ? ? ? ? therefore by Rull II ( ) ln dx x f x dx x e e e x ? ? ? ? ? will be an integrating factor. Now multiplying both sides of the given equation by ?(x) = x yields (3x 2 y − xy 2)dx + (x 3 − x 2 y)dy = 0 which is exact because 2 3 2 M x xy y ? ? ? ? 2 3 2 N x xy x ? ? ? ? M N y x ? ? ? ? ? ?
60 ? NSOU ? CC-PH-04 Example 3 : The differential dz = (x + y) sin ydx + (x sin y + cos y)dy is not exact, since ( )cos sin M x y y y y ? ? ? ? ? sin N y x ? ? ? However cos sin M N y x y M y ? ? ? ? ? ? ? ? is a function of y alone denoted by ( )y? Now cos ( ) ln(sin) sin y y dy dy y y ?? ? ?? ? ? Therefore, I.F = ( ) ln(sin) y dy y e e ?? ? ? 1 ln(sin) 1 (sin) y e y ? ? ? ? cos ( ) sin sin dz y x y dx x dy y y ? ? ? ? ? ? ? ? ? ? ? which is exact. Since, 1, 1 M N y x ? ? ? ? ? ? 3.5 ? Constrained Maximization using Lagrange's Undermined Multipliers We discuss a problem of maximum and minimum values of a function with constraint as follows, Suppose we want to find the maximum or minimum of a function u(x, y), where x and y are related by an equation (,) x y ? = constant i.e. x, y are not independent. This type of extra relation between the variables are known as constraints. In such type of cases the points where maxima or minima occur and corresponding maximum or minimum values of the function can be determined by a number of methods e.g. a) method of elimination; b) method of implicit differentiation ; c) method of Lagrange multipliers.
NSOU ? CC-PH-04 ? 61 Sometimes methods a) and b) can involve enormous calculation and we can solve the problem in concise form by a process known as method of Lagrange's undetermined multipliers. 3.5.1. Method of Lagrange's Undetermined Multipliers with Functions of Two Independent Variables and one f´ Equations Now we discuss the method of Lagrange's undetermined multipliers, to find the maximum or minimum points of u = u(x, y) consisting of two independent variables. We set 0 du dx ? or du = 0. Again since (,) x y ? = constant, we get 0 d? ? . Then u is really a function of one variable, say x. therefore, 0 0 f f du dx dy x y d dx dy x y ? (3.6) We multiply the d? equation by ? and add it to the du equation, then we have, 0 u u dx dy x x y y ? (3.7) where ? is undetermined multiplier.. Now we chosen ? so that, 0 u y y ? ? ? ? ? ? ? ? (3.8) From equation (3.7) and (3.8) we get, 0 u x x ? ? ? ? ? ? ? ? ? (3.9) Equation (3.8), (3.9) and (,) x y ? = constant can now be solved for the three unknowns x, y, ?. 3.5.2 : Method of Lagrange undetermined multiplier with function of three independent variables and one ? − equation Now we discuss the same problem with function of three independent variables (x, y, z). We want to find maximum or minimum values of u(x, y, z), when ?(x, y, z) = constant.
62 ? NSOU ? CC-PH-04 For maximum or minimum values of u, we set 0 0 u u u du dx dy dz x y z and d dx dy dz x y z ? (3.10) Now multiplying d? equation by ? and adding it to the du equation, we get 0 u u u dx dy dz x x y y z z ? (3.11) Since x, y, z are related by ? = constant, there are two independent values in this problem when x and y are indepenent, z is determined from the ? equation. Similarly, dx and dy may have any values we chose, then dz is determined from d? equation. We chose ? so that 0 u z z ? ? ? ? ? ? ? ? (3.12) Then from (3.11), for dy = 0, we get 0 u x x ? ? ? ? ? ? ? ? (3.13) and for dx = 0; 0 u y y ? ? ? ? ? ? ? ? (3.14) Solving equations (3.12), (3.13), (3.14) and ?(x, y, z) = constant, we can find out x, y, z and ?. 3.5.3 : Method of Lagrange's undetermined multiplier with two ? equations Suppose we have two constraint equations : ? 1 (x, y, z, w) = constant (3.15) And ? 2 (x, y, z, w) = constant (3.16) And our function is now u(x, y, z, w) (3.17) There are two independent variables, say x & y. Therefore, 1 1 1 1 0 d dx dy dz x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.18)

NSOU ? CC-PH-04 ? 63 And 2 2 2 2 0 d dx dy dz x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.19) And we set 0 u u u u du dx dy dz dw x y z w ? ? ? ? ? ? ? ? ? ? ? ? ? (3.20) Multiplying equation (3.18) by ? 1 and equation (3.19) by ? 2 and adding the result to equation (3.20), we get 1 2 1 2 1 2 1 2 u u dx dy x x x y y y ? 1 2 1 2 1 2 1 2 0 u u dz dw z z z w w w ? (3.21) We determine ? 1 and ? 2 from the equations, 1 2 1 2 1 2 1 2 0 0 u z z z u w w w ? (3.22) Then for dy = 0, we have 1 2 1 2 0 u x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.23) And for dx = 0, we have, 1 2 1 2 0 u y y y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.24) Now solving equations (3.22), (3.23), (3.24) and (3.15), (3.14) we get x, y, z, w, ? 1 & ? 2 . 3.5.4 : Working rules for constraint maximization or minimization using Lagrange's multipliers Rule I : To find the maximum and minimum values of u(x, y, z) if ?(x, y, z) = constant, we form the function U = u + ?? and set the three partial derivative of U equal to zero. We solve these equations and the equation ? = constant, for x, y, z and ?.
64 ? NSOU ? CC-PH-04 Rule II : To find maximum or minimum of u subject to the conditions ? 1 = constant and ? 2 = constant, we define 1 1 2 2 U u ? ? ? ? ? ? ? and set each of the partial derivatives of U equal to zero. Solve these equation and the?? equations for the variables and ?'s. Rule III : For a problem with still more variables and conditions there are more equations but no change in method. Example of Art 3.6 : Example I : Using the method of Lagrange's multiplier, find the area of largest rectangle that can be inscribed in a semi-circle of radius R with one of the largest side of the rectangle coinciding with diameter. Solution : The rectangle to be inscribed in a circle should be symmetric about y-axis. When length and breadth of the rectangle is 2x and y, its area is 2xy. Also x 2 + y 2 = R 2 . Therefore U = 2xy + ?(

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x 2 + y 2 – R 2) 2 2 U y x x ? ? ? ? ? ? ? 2 2 U x y y ? ? ? ? ? For 0, 2 2 0 U y x x ? ? ? ? ? ? (i) And 0,2 2 0 U x y y ? ? ? ? ? ? (ii) From equation (i) and (ii), we get, y x x y ? ? ? ? ? or, x 2 = y 2 i.e. x = y (iii) 2 2 2 2 2 2 R R x y x y ? ? ? ? ? ? ? 2

Area 2 2 2 2 R
R xy R ? ? ? ? ? ? Maximum area = R 2
NSOU ? CC-PH-04 ? 65 Example 2. Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. Solution : Let inscribable maximum rectangular solid has got length, breadth and height 2x, 2y, and 2z respectively. ? volume of solid, V = 8 xyz (i) Also, x 2 +
y 2 + z 2 = r 2 , where r is the radius of the sphere 2 2 2 2 (, ,)

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x y z x y z r ? ? ? ? ? ? (ii) U V ?? ? ? ? (iii) Now, 0; U V x x x ? ? ? ? ? ? ? ? ? ? ? or, 8 (2) 0 yz x ?? ? (iv) 0; U V y y y ? ? ? ? ? ? ? ? ? ? ? ? or, 8 (2) 0 xz y ?? ? (v) 0; U V z z z ? ? ? ? ? ? ? ? ? ? ? or, 8 (2) 0 xy z ?? ? (vi) From (iv), 2 2 8 x xyz ? ? ? From (v), 2 2 8 y xyz ? ? ? From (vi), 2 2 8 z xyz ? ? ? 2 2 2 2 2 2 2 2 2 ; . . x y z ie x y z ? ? ? ? ? ? ? ? ? x = y = z

Hence in a sphere, the rectangular solid having maximum volume that can be inscribed within it is a cube. Example 3 : Using the method of Lagrange multiplier find the maximum of F = 4xyz subject to
the constraint

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x 2 + y 2 + z 2 – a 2 = 0 66 ? NSOU ? CC-PH-04 Solution : Let 2 2 2 2 (, ,) x y z x y z a ? ? ? ? ? (i) We consider (, ,) (, ,) 4 (, ,) U F x y z x y z xyz x y z ?? ?? ? ? ? ? (ii) 4 2 0 U yz x x ? ? ? ? ? ? ? (iii) 4 2 0 U yz y y ? ? ? ? ? ? (iv) 4 2 0 U yz z z ? ? ? ? ? ? ? (

v) From equations (iii) and (iv) 2 2 2 2 i.e. or yz zx y x y x x y ? ? ? ? ? ? ? Similarly, from equation (iv) and (v), we get x = z ? x = y = z (vi) From equation (i), x = y = z = 3

a ? Then F = 4xyz will lie within 4 4 to 3 3 3 3 a a ? . So, the maximum value of F will be 4 3 3 a Exercise of Art 3.5 : 1) The temperature T at any point (x, y, z) in space is T = 400 xyz 2 . Find the highest temperature at the surface of a unit sphere x 2 + y 2 + z 2 = 1. 2) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid 2 2 2 2 2 2 1 x y z a b c ? ? ? 3) Find the point on the plane ax + by + cz = p at which the function f = x 2 + y 2 + z 2 has a minimum value and find the minimum f. Solution of exercise of Art 3.5 : Solution to the problem 1 :
NSOU ? CC-PH-04 ? 67 We have T = 400 xyz 2 2 2 2 (, ,) 1 x y z x y z ? ? ? ? ? ? Therefore U T ?? ? ? (i) and 2 400 (2) 0 U T yz x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? (ii) 2 400 (2) 0 U T xz y y y y ? ? ? ? ? ? ? ? ? ? ? ? ? ? (iii) 800 (2) 0 U T xyz z z z z ? ? ? ? ? ? ? ? ? ? ? ? ? ? (iv) Now multiplying equation (ii) (iii) and (iv) by x, y & z respectively and adding we get, 2 2 2 2 1600 2 ( ) 0 xyz x y z ? ? ? ? ? ? Or, 2 800xyz ? ? ? , since x 2 + y 2 + z 2 = 1 Putting the value of ? in equation (ii) we get 400yz 2 + 2x(– 800 xyz 2) = 0; or, 1 – 4x 2 = 0; or, 1 2 x ? ? Similarly 1 2 y ? ? Putting the value of ? in equation (iv), we get 800xyz – 1600xyz 3 = 0 or, 1 – 2z 2 = 0; or, 1 2 z ? ? now using the values of x, y, z in T, we get T = 1 1 1 400 50 2 2 2 ? ? ? ? Solution to problem 2 : We take, 2x, 2y and 2z as the edges of the parallelopiped whose edges are parallel to the x, y, z – axes respectively. Therefore the volume of the parallelepiped is v = 8xyz. Let 2 2 2 2 2 2 (, ,) 1 x y z x y z a b c ? ? ? ? ? ? 68 ? NSOU ? CC-PH-04 Therefore following Lagrange's method of undetermined multiplier, we have (, ,) (, ,)

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U x y z V x y z ?? ? ? (i) or, 2 2 2 2 2 2 (, ,) 8 x y z U x y

z
xyz a b c ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 8 0 U x yz x a ? ? ? ? ? ? ? ? ? ? ? ? ? (ii) 2 2 8 0 U y xz y b ? ? ? ? ? ? ? ? ? ? ? ? (iii) and 2 2 8 0 U z xy z c ? ? ? ? ? ? ? ? ? ? ? ? (iv) From equation (ii); 2 2 8 x a yz ? ? ? and from equation (iii), 2 2 8 z c xz ? ? ? 2 2 2 2 2 2 8 8 2 2 yza xzc x y or x z a b ? ? ? Similarly, from (iii) and (iv), we get, 2 2 2 2

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y z b c ? 2 2 2 2 2 2 x y z a b c ? ? ? Again we have 2 2 2 2 2 2 2 2 1 1; or, 3 x y z x a b c a ? ? ? ? , , and 3 3 3 a b c x y z ? ? ? ?

NSOU ? CC-PH-04 ? 69 Thus the volume of the largest rectangular parallelepiped is V = 8xyz = 8 8 3 3 3 3 3 a b c abc ? . Solution to problem (3) : We have (, ,) (, ,) (, ,) U

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x y z f x y z x y z ?? ? ? (i) Where f(x, y, z) = x 2 + y 2 + z 2 and (, ,) x y z

ax by
cz p ? ? ? ? ? Therefore from (i), differentiating partially, 2 0 or, 2 U f a x a x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 0 or, 2 U f b y b y y y y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 0 or, 2 U f c y c z z z z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Substituting the values of x, y, z in equation ax + by + cz = p, we get 2 2 2

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a b c a b c p ? Or, ? ? 2 2 2 2 2 2 2 2 ; or, p a b c p a b c ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 , , ap bp cp x y z a b c a b c a b c ? ? ? ? ? ? ? ? ? ? minimum values of ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 a p b p c p f a b c a b c a b c ? ? ? ? ? ? ? ? ? or, ? ? ? ? 2 2 2 2 2 2 2 2 p a b c f a b c ? ? ? ? ? 70 ?

NSOU ? CC-PH-04 or, 2 2 2 2 ? ? ? ? minimum p f f a b c Note : Lagrange' method cannot determine the nature of the stationary points. However it is ascertained from the condition of the problem. 3.6 ? Summary 1. Emphasis is given on error calculation in physical measurements with the help partial and total derivatives. 2. Exact and inexact differential are defined and rules are stated to make inexact differentials, exact. 3. Constrained maximization with Lagrange's multipliers discussed.

NSOU ? CC-PH-04 ? 71 Unit 4 ? Vector Calculus Structure 4.1 Objective 4.2 Introduction 4.3 Vectors and Scalars 4.3.1 Familiarities with Vectors and Scalars 4.3.2 Examples of Graphical Representation 4.3.3 Vector in Terms of Components 4.3.4 Examples of Scalars, Vectors and Tensors 4.3.5 Equal Vectors and Null Vectors 4.3.6 Unit Vectors 4.3.7 Position Vectors or Radius Vector 4.3.8 Addition of Vectors (Graphical Representation) 4.3.9 Subtraction of Vectors 4.3.10 Addition and Subtraction of Vector (Algebraic or Co-ordinate Representation Method) 4.3.11 Multiplication of Vectors by Scalars 4.4 Laws of Vector Algebra 4.4.1 Linear Dependence of Vectors 4.4.2 Product of Vectors 4.4.3 Scalar Product of Two Vectors 4.5 Vector Product 4.5.1 Kronecker Delta and Levicivita Symbols 4.5.2 Multiple Product of Vectors 4.5.3 Triple Scalar Product 4.5.4 Triple Vector Product 4.5.5 Product of Four Vectors 4.6 Reciprocal System of Vectors

72 ? NSOU ? CC-PH-04 4.6.1 Properties of Reciprocal System 4.7 Properties of Vectors Under Rotation 4.7.1 Scalar Product of Two Vectors, Under Rotation of Co-Ordinate System 4.7.2 Vectors Product of Two Vectors Under Rotation of Co-Ordinate Axes 4.8 Polar, Axial Vectors and Pseudo Scalars 4.8.1 Scalar and Vector Fields 4.8.2 Classification of Vector Fields 4.9. Summary 4.10 Vector Differentiation 4.11 Constant Vector Function : Constancy in Direction and Magnitude 4.11.1 4.11.2 4.11.3 4.12 Derivative of Triple Scalar Product 4.13 Derivative of Triple Vector Product 4.14 Velocity and Acceleration of Particle 4.15 Relative Velocity and Acceleration 4.16 Gradient of a Scalar Filed 4.17 Directional Derivative 4.18 Normal Derivative 4.19 Geometrical and Physical Meanings of Grad ? 4.20 The 'Del' or 'Nabla' Operator 4.20.1 Divergence of a Vector Field 4.20.2 Integral Form of Divergence 4.21 Curl of Vector Filed 4.22 Vector Identities 4.23 Lists of Vector Relations 4.23.1 4.23.2

NSOU ? CC-PH-04 ? 73 4.23.3 4.23.4 4.24 Summary 4.25 Vector Integration 4.26 Double and Triple Integral 4.26.1 Examples of Double Integration 4.26.2 Change of Order of Integration 4.26.3 Examples of Triple Integrals 4.27 Change of Variables : Jacobian 4.28 Ordinary Integrals of Vectors 4.28.1

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Line Integral of a Vector Field 4.28.2 Surface Integral of a Vector Field 4.28.3 Volume Integral of

a Vector Field 4.29 Green's Theorem in a Plane 4.30 Gauss's Divergence Theorem 4.31 Stoke's Theorem 4.32 Summary 4.1 ? Objectives When you go through the article you may be able to learn 1. Definitions of scalars and vectors 2. Vector algebra, which is a little bit different from scalar algebra 3. Some examples of application of vectors in different branches of physics. 4.2 ? Introduction One may ask why we need to study scalars, vectors or in general Tensors ? One of the simple reasons may be that physical laws can be expressed effectively in concise form and without any ambiguity with the help of scalars, vectors and tensors. But this is not all. More logical reason for using scalars, vectors and tensors lies in the fact that physical laws

74 ? NSOU ? CC-PH-04 must obey the principle of Galilean Invariance (in non-relativistic domain) which states that physical phenomena appear to be the same for all observers moving in inertial frames with constant relative velocity with respect to each other in respect of translation and rotation of the co-ordinate system. In other words physical laws must be invariant in all inertial frames of references. In view of the above mathematical formulation of the physical laws must contain those entities which have such invariance properties and these entities are scalars, vector and Tensors. This is why a student of physics and science in general, must learn the properties of scalars, vectors and tensors. 4.3 ? Vectors and Scalars 4.3.1 Familiarities with Vectors and Scalars Measurable physical entities which have both magnitude and direction and obey parallelogram law of addition are called Vectors. This is geometrical or graphical representation of vectors. On the other hand physical quantities which have magnitude only are called scalars. Both the vectors and scalars have their respective units. Vectors can also be defined as a set of three numbers (in three dimensional spaces) which we call its components with respect to a co-ordinate system in vector space. This is algebraic definition of vector. A physical scalar is a quantity which remains invariant under all co-ordinate systems. 4.3.2 Example of graphical representation Graphically or geometrically a vector is represented by a line with an arrow head. The length of the line is its magnitude and the arrow points towards its direction. Beginning of the line is termed as origin or tail and the arrow head is called terminus. O A ???????????????????? P Vector P is represented by the Line OA ???? with an arrow, O is its tail and A is terminus. The modulous or magnitude of P = | P | = P is given by the length OA. This representation is independent of the origin of any co-ordinate system. Vectors are represented by bold face letters and their magnitudes by ordinary letters. O

NSOU ? CC-PH-04 ? 75 4.3.3 Vectors in terms of components We consider a rectangular co-ordinate system as in fig (4A.1). Let the vector i be a unit vector in the positive x direction and let j and k be unit vectors in the positive y and z directions. If A x and A y are the scalar components of a vector in the (x, y) plane, iA x and jA y are its vector components and their sum is the vector A (fig. 4A.2). Fig. (4A.1) (Fig. 4A.2) Similarly, in three dimension, A = iA x + jA y + kA z However these two ways of representing a vector are not completely equivalent, for the algebraic definition requires a co-ordinate system, but the geometrical representation does not require any co-ordinate system. This difficulty is removed by making the algebraic representation also independent of any particularly co-ordinate system by defining a vector in the following way. A vector in three dimensions is a set of three numbers, called its components, which transform under a rotation of co-ordinate system according to the following transformation equation. 1 1 2 2 3 3 i i i i Y b x b x b x ? ? ? 3 1 ik k k b x ? ? ? (4A.1) Y i are the components of x in the new co-ordinate system and x k are the component of x in the old co-ordinate system. The co-efficient b ik etc are the numbers which are determined by the given co-ordinate rotation, they do not depend on x. We note that translation have no effect on the components of vector which are numbers but not scalars, because they do not remain invariant under rotation of co-ordinate system. However exception to the definition of vector given in equation (4A.1) is position vector r = ix + jy + kz which is defined with respect to specific origin.

76 ? NSOU ? CC-PH-04 Tensors are quantities that do not have any specified directions but have different values in different directions. Examples are moment of inertia tensor, dielectric susceptibility tensor etc. Tensors however are defined only through their transformation under changes of co-ordinate system. A physical entity which has only one component is called tensor of zero rank or a scalar. If it has more than one component but less than or equal to four, it is called a vector or a tensor of rank 1. A tensor of rank 2 has nine components. 4.3.4 Examples of scalars, vector and tensors Scalar : A scalar field is created by simply assigning scalar quantities (numbers) to each point in space. Temperature of a body or potential of gravitational or electrostatic field are examples of scalar fields. Mass, volume, density, length etc. are scalar quantities. Vector : A vector field is created by assigning vectors to each point in space. An electrostatic field, a gravitational field, electromagnetic field are examples. Vectors usually possess both magnitude and direction. Force, momentum, electric dipole moment, magnetic dipole moment etc. are examples of vectors. Tensor. A tensor cannot be visualised geometrically hence it is defined in terms of field or transformation properties under rotation of co-ordinate system. A tensor field has a tensor corresponding to each point space. An example is the stress on a material. Other examples of tensors include the strain tensor, the conductivity tensor and the inertia tensor. 4.3.5 Equal vectors and null vectors Two vectors are said to be equal when their magnitudes as well as direction are identical i.e. A = B i.e. A – B = 0. The right hand side of this vector equation is also a vector called null vector with arbitrary direction. 4.3.6 Unit Vectors A vector having unit scalar magnitude is defined as unit vector. Any vector A can be written as A = Aa (4A.2), where ?

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A A a is a unit vector in the direction of the vector A.

In the Cartesian co-ordinate system, unit vectors i, j, k in the direction of X, Y, Z-axes respectively are commonly used. NSOU ? CC-PH-04 ? 77 4.3.7 Position vector or radius vector The position vector r of an object located at P(x, y, z) is given by r = ix + jy + kz (4A.3). Now 2 2 2 2 2 2 2 2 2 OP OB BP OA AB BP x y z ? ? ? ? ? ? ? ? ? 2 2 2 | | OP r x y z ? ? ? ? ? ? ? r (4A.4) Fig. (4A.3) In equation (4A.3) ix, jy, kz are the components of the vector r in X,Y,Z direction respectively. If the vector r makes an angle , , ? ? ? respectively with X,Y,Z -axes, then cos , cos , cos x r y r z r ? ? ? ? ? ? therefore, ? ? 2 2 2 2 2 2 2 cos cos cos x y z r ? ? ? ? ? ? ? ? ? or comparing with equation (4A.4), we get 2 2 2 cos cos cos ? ? ? ? ? = 1 (4A.5) equation (4A.5) is known as vector direction cosine law and cos?, cos?, cos? are called direction cosine of r with respect to x, y, z axes respectively. Sometimes we represent cos ? by l, cos ? by m, cos ? by n, so that l 2 + m 2 + n 2 = 1 (4A.6) So any vector A with components iA x , jA y and kA z along X, Y and Z directions respectively can be written as A = iA x + jA y + kA z > XOA = ? > AOY = ? > AOZ = ?
78 ? NSOU ? CC-PH-04 =
A x + A y + A z (4A.7) If the vector A makes an angle ?, ?, ? with X, Y and Z-axes respectively, cos , cos ,

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cos x y z A A A A A A ? ? ? ? ? ? 2 2 2 2 2 x y z A A A A ? ? ? ? A (4A.8) 4.3.8

Addition

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of vectors [Graphical representation] The sum of vectors A and B is a vector C by placing the origin of B on the terminus of A and joining the initial point of A to the terminus of B.

Fig. 4A.4(a) Fig. 4A.4(b) We write A + B = C. This definition is equivalent to the parallelogram law for vector addition as indicated in fig 4A.4(c) Fig. 4A.4(c) The law of vector addition therefore is the parallelogram law of addition which states

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that the sum of two vectors A and B is

given in magnitude and direction by the diagonal of the parallelogram formed by the sides representing the vectors A and B. In the same way any number of vectors can be added. Fig. 4A.5 shows how to obtain the sum of resultant R of the vectors A, B, C and D.
NSOU ? CC-PH-04 ? 79 Fig. 4A.5 4.3.9 Subtraction of vectors Subtraction of the vector B from the vector A is defined as the addition of the negative vector –B to A. Thus A + (– B) = A – B. 4.3.10 Addition and subtraction of vector [algebraic or co-ordinate representation method] To find sum or difference of two vectors, we add or subtract like components together as follows. Let ,
x y

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z A A A ? ? ? A i j k where 2 2 2 ; x y z A A A A ? ? ? ? A and , x y z B B B ? ? ? B i j k where 2 2 2 x y z B B B B ? ? ? ? B Therefore ? ? ? ? ? ? x x y y z z A B A B A B ? ? ? ? ? ? ? ? A B i j k and ? ? ? ? ? ? 1 2 2 2 2 x x y y z z A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B

Likewise for the sum and the difference of a large number of vectors. 4.3.11 Multiplication of vectors by scalars If A is a vector and m is any positive real number, and then mA is defined to be a vector having magnitude equal to m times that of the given vector A in the same direction. Likewise – mA is a vector in the direction opposite to that of A and having magnitude equal to m times that of A.

80 ? NSOU ? CC-PH-04 4.4 ? Laws of Vector Algebra Vector addition is commutative and associative i.e. A + B = B + A And A + (B + C) = (A + B) + C, respectively also multiplication of vectors by scalars is commutative, associative and distributive i.e. mA = Am m (nA) = n (mA) m (A + B) = mA + mB, respectively. m and n are two different scalars. 4.4.1 Linear dependence of vectors Let A 1 , A 2 , A 3 ... be vectors and 1 2 3 , , ,... ? ? ? are scalars, not all of which are zero. If there exists a relation of the type 1 1 2 2 3 3 ... 0 i i i ? ? ? ? ? ? ? ? ? ?

63%	**MATCHING BLOCK 56/267**	SA	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)
A A A A (4A.9) Then the system of the vectors A 1 , A 2 , A 3 ... is said to be			

linearly dependent. If the system of vectors A 1 , A 2 , A 3 are not linearly dependent, then 1 2 3 , , ,... ? ? ? are all zero i.e. 1 2 3 .... 0 ? ? ? ? ? ? ? (4A.10) The system of vectors in this case is said to be linearly independent. If 1 1 2 2 3 3 ... ? ? ? ? ? ? ? r A A A i.e. 1 1 2 2 3 3 ... 0 ? ? ? ? ? ? ? ? ? r A A A , then system of vectors r, A 1 , A 2 , A 3 , ... is linearly dependent. Any vector r coplanar with any two non-collinear vectors A & B can be uniquely expressed as a linear combination of the given vectors i.e. ? ? ? ? r A B (4A.11) Where ,? ? are scalars. From equation (4A.11) we can write – ? ? ? ? r A B = 0 i.e. r, A, B vectors are linearly dependent. It is to be noted that necessary and sufficient condition that three vectors be linearly dependent is that they may be coplanar. Example of Art 4.3.7 to 4.4.1 : Example 1 : Position vectors of three points P, Q, R and 2i – j + k, i – 3j – 5k and 3i – 4j – 4k respectively. Find the vectors PQ ???? and QR ???? and their magnitudes. NSOU ? CC-PH-04 ? 81 Solutions : ? ? ? ? 3 5 2 2 6 PQ ? ? ? ? ? ? ? ? ? ? i

70%	**MATCHING BLOCK 57/267**	SA	MMP.pdf (D118446089)
j k i j k i j k ???? 2 2 2 (1) (2) (6) 41 PQ PQ ? ? ? ? ? ? ? ? ? ???? ? ? 3 4 4 (3 5) 2 QR ? ? ? ? ? ? ? ? ? ? i j k i j k i j k ???? 2 2 2 (2) (1) (1) 6			

QR QR ? ? ? ? ? ? ? ???? Example 2 : Prove that, the line joining the mid points of two sides of a triangle is parallel and half to the third. Solution : In the triangle ?PQR, the position vectors of P, Q, R be a, b and c respectively. If the mid points of PQ and PR be D & E respectively, then the position vectors of 2 D ? ? a b and the position vector of E is 2 ?a c . ? ? ( ) ? ? ? DE position vector of E position vector of D ???? ? ? 1 2 2 2 2 BC ? ? ? ? ? ? ? c b a c a b ???? Hence DE ???? is parallel to BC ???? and half of BC. Example 3 : Prove that (i) if P and Q are two non-collinear vectors and 0 ? ?? ? P Q , then show that, 0 ? ?? ? (ii) If P, Q, R are non-coplanar vector and 0 ? ? ? ? ? ? P Q R , then show that 0 ? ? ? ? ? ? Solution : (i) suppose, 0 ? ? m a ?? ? ? ? P Q Q where m is a scalar.. Therefore P and Q collinear if 0 ? ? . Again, 0; ? ? ? ? ? ? ?Q P Fig. Example (2) Q(b) R(c) P(a)

82 ? NSOU ? CC-PH-04 Or, Q = nP where n is scalar. Therefore again Q and P are collinear when 0 ? ? . Therefore we see that P and Q are non-collinear when 0 ? ?? ? (ii) Let 0 ? ? , ? ? ? ? ? ? ? ? P Q R Or, P = mQ + nR where m and n are two scalars. Therefore P, Q, R are coplanar when 0 ? ? . Similarly we can prove that P, Q, R are coplanar when 0 ? ? and 0 ? ? . Therefore for 0 ? ? ? ? ? ? , P, Q and R are non-coplanar.. Example 4 : If

83%	**MATCHING BLOCK 58/267**	SA	VECTOR ANALYSIS.docx (D118765685)
A = 2i + 3j – k and B = 3i – j + 5k. Find the value of A ±			

B Solution :
We have

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A = A x i + A y j + A z k = 2i + 3j − k B = B x i + B y j + B z k = 3i − j + 5k ? A + B = (A x + B x)i + (A y + B y)j + (A z + B z)k = (2 + 3)i + (3 − 1)j + (−1 + 5)k = 5

i + 2j + 4k Similarly A − B = −
i + 4j − 6k 4.4.2
Product of Vectors We often come across in physics, certain combination of vectors which have the properties of products. The products of two vectors may be a scalar or a vector quantity depending upon how the product is defined. For example work done is the scalar product of two vectors namely force (F) and displacement (d), whereas angular momentum of a particle about the origin is the vector product of position vector (r) and its linear momentum (mv). Work done being a scalar quantity but angular momentum is vector. 4.4.3 Scalar product of two vector : We define scalar product or dot product of two vector as follows : A.B = AB cos ? (4A.12) Where |A| = A and |B| = B and ? is the acute angle between A and B, clearly, the
NSOU ? CC-PH-04 ? 83 scalar product is the product of the magnitude of one vector and the projection of the other on it. From the fig (4A.6) B cos ? is the length of the resolved part of OC ???? i.e OD along A, i.e. A.B = AA(B cos ?) = A (resolved part of modulous of B along A). Similarly we can write A.B = B(A cos ?) = B(resolved part of modulus A along the direction B). From equation (4A.12) it is clear that if A and B are two non-zero vectors, then their dot product will be zero only when the direction of the vectors are perpendicular. The dot product will be equal to the product of their moduli, when their directions are parallel. For

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the unit vectors i, j, k along the rectangular co-ordinate system, i.i = j.j = k.k = 1 (4A.13) and i.j = j.k = k.

i = 0 (4A.14) Hence the scalar or dot product of two vectors : A = iA

47% **MATCHING BLOCK 61/267** **SA** MMP.pdf (D118446089)

x + jA y + kA z (4A.15) B = iB x + jB y + kB z (4A.16) will be written as A.B = A x B x + A y B y + A z B z (4A.17)

scalar product of two vectors obey commutative law i.e. A.B = B.A also scalar product of two vectors obey distributive law i.e. for three vector

75% **MATCHING BLOCK 62/267** **SA** M P Vol 1.pdf (D134397037)

A, B & C we can write A.(B + C) = A.B + A.C (4A.18)

For two vectors A and B, we can have the following relations 1. (

87% **MATCHING BLOCK 63/267** **SA** PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

A + B).(A − B) = A.A + B.A − A.B. − B.B. = A 2 + B 2 2. (A + B) 2 = (A + B).(A + B) = A.A. + A.B. + B.A. + B.B. = A 2 + B 2 + 2A.B 4.5 ?

Vector Product The vector product or cross product C of two vectors A and B is defined as A × B = AB sin ? n = C = Cn (4A.19) Fig. 4A.6

84 ? NSOU ? CC-PH-04 Where A = |A|, B = |B| and C = |C| And ? is the acute angle between A & B when joined tail to tail, and n is a unit vector in the direction of C. Direction of C is normal to the plane containing A & B and in a sense such that the vectors A, B & C form a right handed system (fig 4A.7) Fig. 4A.7 ( ),OP OQ ? ? A B ?????? ???? Geometrically the magnitude of

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the cross product of two vectors represents the area of the parallelogram having the two vectors as

its side. From the fig 4A.7, the area of the parallelogram OPRQ ? = area of the triangle ?QOP + area of the triangle ?PQR 1 2 1 1 ( ) ( ) 2 2 OP h QR h ? ? Now 1 2 sin , sin h QO h PR ? ? ? ? 1 1 ( )( )sin ( )( )sin 2 2 OPRQ OP OQ QR PR ? ? ? ? ? 1 1 sin sin 2 2 ? ? ? ? A B A B [Since QR = OP = |A| and PR = OQ = |B|] = sin? A B sin OPRQ ? ? ? ? ? A B A B ?
NSOU ? CC-PH-04 ? 85 If n be a unit vector normal to the plane of the parallelogram then it gives = | A × B|n. This suggests that it may be useful to represent area by vectors. Following properties of vector products are easily verified. 1. A × B = – B × A 2.

39%	**MATCHING BLOCK 65/267**	**SA**	MMP Assignment.pdf (D112572984)

A × [B + C + D + ...] = A × B + A × C + A × D + ... 3. A × A = 0 4. i × j = k; j × k = i; k × i = j 5. i × i = j × j = k × k = 0 6. x y z x y z A A A B B B ? ?

i j k A B 4.5.1
Kronecker delta and Levicivita symbols 1; 0; ij if i j if i j ? ? ? ? ? ? ? ? Example : e 1 , e 2 and e 3 are unit vectors along Cartesian co-ordinate axes X, Y and Z respectively. We can define i j ij ? ? ? e e The definition of the Levicivita symbol is 1 ijk ? ? ; if i, j, k = 1,2,3; 2,3,1; 3,1,2 = – 1; if i, j, k = 3,2,1; 2,1,3; 1,3,2 = 0 if any indices are repeated We say that ijk ? is anti-symmetric with respect to every pair of indices, since each exchange of indices produce a change in sign. If you read the indices i, j, k cyclically, then if the indices read in the direction 1, 2, 3; 1, 2,3; 1,... the result is +1; if the indices read in the opposite direction the result is – 1 i, j, k; j, k, i ; k, i, j k j i
86 ? NSOU ? CC-PH-04 We now show that the components of the cross-product of two vectors can be written as ? ? ijk

42%	**MATCHING BLOCK 66/267**	**SA**	MMP.pdf (D118446089)

j k i ? ? ? B C B C We have B × C = i(B y C z – B z C y) + j(B z C x – B x C z) + k(B x C y – B y C x) We replace x, y, z by 1, 2, 3 Now the first component of 1 1 ( ) ? ? jk j k B C ? B C Now if j, k = 2, 3 or 3, 2 1 123 2 3 132 3 2 ( ) ? ? ? B C B C ? ? B C Now 123 1 ? ? ? and 123 1 ? ? ? ? (B × C) 1 = B 2 C 3 – B 3 C 2

If we take j, k = 1, 3 or 1, 2; 1 0 jk ? ? Similarly other components of B × C can be found out 2 2 ( ) ? ? jk

62%	**MATCHING BLOCK 67/267**	**SA**	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

j k B C ? B C With j, k = 3, 1 or 1, 3 2 231 3 1 213 1 3 ( ) ? ? ? B C B C ? ? B C = B 3 C 1 – B 1 C 3

Since 231 213 1, 1; ? ? ? ? ? ? It is to be
noted that the formulae in vector analysis can be written in the form using and ij ijk ? ? . 4.5.2 : Multiple products of vectors With the help of dot and cross products of two vectors, it is possible to build multiple products involving several vectors. We shall discuss here two kinds of triple products which are specially important. One is called triple scalar product and the other is called triple vector product.
NSOU ? CC-PH-04 ? 87 4.5.3 Triple Scalar Product We consider three vectors A,B,C and arrange them in anticlockwise direction (right handed) as shown below in fig(4A.7) Fig (4A.7) Then triple scalar product [ABC] is defined as [ABC] =

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A.B × C = B.C × A = C.A × B (4A.20)

Thus if a cyclic change (right handed) is made in the sequence of B, C; the triple scalar product remains the same. However for left handed (clockwise) cyclic change as shown below in fig. (4A.8) Fig. (4A.8) We have [ACB] = A.C × B = C.B × A = B.A × C. Thus triple scalar product depends on the handedness of the vectors A,B and C. In writing A.B × C etc. no bracket is necessary. It is then seen that [ABC] = – [ACB] Properties : 1. [ ]

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x y z x y z x y z A A A B B B C C C ?

ABC
88 ? NSOU ? CC-PH-04 where A x etc. are components of the respective vectors. 2. In a triple scalar product 'dot' and 'cross' can be interchanged, implying [ABC] =

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A × B.C = B × C.A = C × A.B 3. If any of the two

non-vanishing vectors A, B, C be parallel or equal, [ABC] = 0 4. [ABC] = 0, the vectors A, B, C are coplanar and also linearly dependent. 5. [ABC] gives the volume of a parallelopiped having A, B and C as coterminous edges. This is the geometric interpretation of triple scalar product. 6. Triple scalar product is distributive i.e. [A B + C D – E] = [ABD] + [ACD] – [ABE] – [ACE]. 7. The volume of the tetrahedron ABCD is the numerical value of 1 6 ABACAD ? ? ? ? ???????????? , Fig 4A.9 Fig. (4A.9) 8. For an orthonormal right handed vector triad i, j, k we have [ijk] = [jki] = [kij] = 1 and for left handed triad [ikj] = [kji] = [jik] = – 1 Geometrical interpretation : The vectors A, B, C are represented by , , OA OB OC ???? ???? ???? respectively. The magnitude of the vector B × C is the area of the parallelogram OBDC and its direction is along OP ???? , perpendicular to the plane OBDC. Drop perpendicular from A on OP which is AM. So OM is the height of the parallelepiped. Then A.B × C = (projection of A on B × C) × magnitude of B × C = height of the parallelepiped × area of the base of the parallelepiped = volume of the parallelepiped
NSOU ? CC-PH-04 ? 89 By taking a various faces in turn we find that

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A.B × C = B.C × A = C.A × B = volume of the

parallelepiped with three adjacent side as the magnitude of A, B and C Fig. (4A.10) 4.5.4 : Triple Vector Product If A, B and C are the three vectors, then triple vector product is defined as either

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A × (B × C) or (A × B) × C, parentheses is essential, since A × B × C is meaningless. We have A × (B × C) = (A.C)B – (A.B)C (4A.21) The value of a triple vector product is a linear combination of

the two vectors in the parentheses, e.g. B and C; the co-efficient of each vector is the dot product of the other two, the middle vector in the triple product, e.g. B; always has the positive sign and the other vector in the parentheses e.g. C; always has the negative sign. Thus the vector

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A × (B × C) lies in the plane of B and C.

From the discussion as above (

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A × B) × C = (A.C)B − (B.C)A (4A.22) Now (B × C) × A = (B.A)C − (A.C)B = − [(A.C)B − (A.B)C] = − A × (B × C) Proof of equation (4A.21) : B × C is a vector perpendicular to the plane of B and C. thus A × (B × C)

is
some vector in the plane of B and C.
90 ?
NSOU ? CC-PH-04 Therefore we can write A × (B × C) = lB + mC ; (4A.23) l, m are scalars. Making dot product with A both sides A. A × (B × C) = l(A.B) + m(A.C) or, 0 = l(A.B) + m(A.C) Using property (3) of triple scalar product Or, . ? ? ? l m n AC A.B (say) Substituting the values of l and m in equation (4A.23) , A × (B × C) = n (A.C)B − n(A.B)C (4A.24) Since vector equations are independent of co-ordinate system, we can take, to facilitate our calculations, but without any loss of generality, A = C = i, B = j Therefore from equation (4A.24) i × (j × i) = nj Or i × (− k) = nj Or k × i = nj = j ? n = 1 Therefore A × (B × C) = (A.C)B − (A.B)C (24.21) proof of equation (4A.22) is left as an exercise. 4.5.5 Product Of Four Vectors Scalar product of four vectors : Scalar product of four vectors A,

43%	**MATCHING BLOCK 75/267**	**SA**	M P Vol 1.pdf (D134397037)

B, C and D is defined as (A × B). (C × D) Now let's suppose C × D = N Then (A × B).(C × D) = (A × B).N = A.B × N = A.B × (C × D) = A. [(B.D)C − (B.C)D] = (A.C)(B.D) − (A.D) (B.C)

NSOU ? CC-PH-04 ? 91 . . . . ? AC A D BC B D (4A.24A) Vector product of four vectors : Vectors product of four vectors A, B, C and D is defined as, (A × B) × C × D) now let A × B = N ? (

23%	**MATCHING BLOCK 76/267**	**SA**	M P Vol 1.pdf (D134397037)

A × B) × (C × D) = N × (C × D) = (N.D)C − (N.C)D = (A × B.D)C − (A × B.C)D = lC − mD (4A.24B) where l and m are scalar. Therefore (A × B) × (C × D) lies in the plane of C and D. Now let C × D = N ? (A × B) × (C × D) = (A × B) × N = (A.N)B − (B.N)A = (A.C × D)B − (B.C ×

D)A = pB − qA (4A.25) where p and q are scalars. Therefore (A × B) × (C × D)
can also be expressed as a linear combination of the vectors B and A. Example of Art 4.4.2 to 4.5.5 : Example 5: If A and B are two vectors, show that 2 (.)(.) (.) ? ? ? A B A A B B AB Solution : R.H.S : 2 2 2 2 2 2 cos 1 cos sin A B A B AB AB ? ? ? ? ? ? ? ? ? ?A B Example 6 : Find the angle between the

68%	**MATCHING BLOCK 77/267**	**SA**	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

vectors A = 4i + 3j + k and B = 2i − j + 2k. Also find a unit vector perpendicular to both A and B.

Use concept of dot product only. Solution : Using the definition of dot product, . cos? ? AB A B , where ? is the angle between the A and B.

92 ? NSOU ? CC-PH-04 Now 2 2 2 (4) (3) (1) 26 ? ? ? ? A 2 2 2 (2) (1) (2) 9 3 ? ? ? ? ? ? ? B And A.B = (4i + 3j + k). (2i – j + 2k) = 8 – 3 + 2 = 7 1 7 7 cos ; cos 3 26 3 26 ? ? ? ? ? ? ? ? ? ? ? ? ? Now let ˆ r be a unit vector perpendicular to both A and B so that ˆ ? ? ? ? ? ? r i j k Therefore 2 2 2 1 ? ? ? ? ? ? ? (i) Again ˆ. 2 0 ? ? ? ? ? ? ? ? r B (ii) And ˆ. 4 3 0 ? ? ? ? ? ? ? ? r A (iii) Solving equation (ii) and (iii) by cross-multiplication 2 2 2 2 2 2 1 4 2 10 120 (4) (2) (10) ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 2 1 5 ; ; 120 30 30 30 ? ? ? ? ? ? ? ? ? ? ? therefore ? ? 1 ˆ 2 5 30 ? ? ? r i
j k Example 7 : Given A = i +

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j + k and C = j – k. Find a vector B such that, A × B = C and A.B = 3 Solution : Suppose ? ? ? ? ? ? B i j k Now A × B = C gives, 1 1 1 ? ? ? ? ? ? i j k j k

NSOU ? CC-PH-04 ? 93 Or, ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k j k
Corresponding co-efficient of i, j and k from both sides, We have 0, 1, and 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ? as , 1 , 1; 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (i) Again, A.B = 3 gives 3 ? ? ? ? ? ? ? (ii) Solving equations (i) and (ii) we get 2 5 and 1 3 3 ? ? ? ? ? ? ? ? ? ? Therefore ? ? 5 2 2 1 5 2 2 3 3 3 3 B ? ? ? ? ? ? i j k = i j k Example 8 : If

34% **MATCHING BLOCK 79/267** **SA** PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

a = 4i + 3j + k, b = 2i – j + 2k, find a unit vector ˆn perpendicular to vector a and b such that a, b, ˆn form a right handed system. Find the angle between the vectors a and b. Solution : We have, 4 3 1 7 6 10 2 1 2 a b? ? ? ? ? ? i j k i j k And 2 2 2 (7) (6) (10) 185 ? ? ? ? ? ? ? a b Therefore 7 6 10 ˆ 185 ? ? ? ? ? ? ? a b i j k n a b Also 2 2 2 4 3 1 126 ? ? ? ? a 2 2 2 2 1 2 3 ? ? ? ? ? b

66% **MATCHING BLOCK 80/267** **SA** PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

If ? be the angle between a and b, then sin ,? ? ? a b a b then 0 185 sin sin 62 42; 3 26 ? ? ? ? ? ? ? ? a b a b 0 62 42 ? ? ? ? 94 ?

NSOU ? CC-PH-04 Example 9 : Find the value of ? for which
vector :

70% **MATCHING BLOCK 81/267** **SA** VECTOR ANALYSIS.docx (D118765685)

A = 2i – j + k B = i + 2j – 3k C = 3i + ?j + 5k, are coplanar. Solution : There vectors A, B, C

are coplanar if [ABC] = 0 or, 2 1 1 1 2 3 0 3 5 ? ? ? ? ? Or, 2(10 3) (5 9) (6) 0 ? ? ? ? ? ? ? ? ? Or, 7 28 0 ? ? ? 4 ?? ? ? Example 10 : Show that, (
A ×

19% **MATCHING BLOCK 82/267** **SA** PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

B) × C = A × (B × C) only when A and C are collinear or (A × C) × B = 0 Solution : Given (A × B) × C = A × (B × C) This is possible if, B(A.C) + C(A.B) – B(A.C) – A(B.C) = 0 or, if C(A.B) – A(B.C) = 0 or, if (A × C) × B = 0 This show that either B = 0 or A × C = 0, but 0?B , hence A × C = 0, hence A and C are collinear. Example 11 : If A, B and C satisfy the condition (A × B) + (B × C) + (C × A) = 0, show that the vector are coplanar. Solution : We have (A × B) + (B × C) + (C × A) = 0 ? [(A × B) + (B × C) + (C × A)].A = 0 or, A × B.A + B × C.A + C × A.A = 0 or, B × C.A = 0 ;

or [ABC] = 0 Therefore, A, B,
C

are coplanar
NSOU ? CC-PH-04 ? 95 4.6 ? Reciprocal System of Vectors The concept of reciprocal vectors finds applications in solid state physics in connection with reciprocal lattice. Let there be a set of three non-coplanar vectors a, b, c. The
set of other

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three vectors A, B, C defined by the equation , , [ ] [ ] [ ] ? ? ? ? ? ? b c c a a b A B C abc abc abc (4A.26) Are called reciprocal vectors triads to the vectors a, b and c. The vector triads a, b and c and its reciprocal triads A, B,

C are either both right handed or both left handed. The vector triads (

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a, b, c) and (A, B, C) are mutually reciprocal. i.e. , , [ ] [ ] [ ] ? ? ? ? ? ? B C C A A B a b c ABC ABC ABC Where A, B, C

are non-co-planar vectors given by [ ] 0? ABC Properties of reciprocal system : 1. If
a, b,
c and A, B, C be reciprocal triads of vectors, then
a.
A =

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b.B = c.C = 1 (4A.27) Proof : We have [ ] ? ? b c A abc [ ] . 1 [ ] [ ] . ? ? ? ? ? a b c abc a A abc abc Similarly b.B = c.C = 1 ; Then a.A + b.B + c.C = 3 And 1 1 1 , , ? ? ? A B C a b c 2. If a, b, c and A, B, C are reciprocal triad of vectors, then a.B = a.C = 0 b.A = b.C = 0 (4A.28) 96 ? NSOU ? CC-PH-04 c.A = c.B = 0 Proof : [ ] . 0 [ ] [ ] . ? ? ? ? c a acb a B a

abc abc Since [aca] = 0; and similar for other relation. 3. The triple scalar product of any three non-co-planar vectors a, b, c is reciprocal to the corresponding triple scalar product of reciprocal
vectors

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A, B, C. Proof : ? ? 3 ( ) ( ) [ ] . ( ). [ ] c a a b abc ? ? ? ? ? ? ? ABC AB C b c , using equation (4A.26) Now (c × a) × (a × b) = (c × a) × N when N = a × b = a(c.N) − c(a.N) = a(c.a × b) − c(a.a × b) = a[abc] since a.a ×

b = 0 2 3 3 ( ). [ ] [ ] 1 [ ] [ ] [ ] [ ] ? ? ? ? ? b
c
a
abc abc ABC abc abc abc (4A.29) Example of Art 4.6 : Example 12 : Show that the orthonormal vector triads (i, j, k) is self reciprocal. Solution : Let (, ,) ? ? ? i j k be the set of vectors reciprocal to (i, j, k) then, , , [ ] [ ] [ ] ? ? ? ? ? ? ? ? ? j k k i i j i j k ijk ijk ijk ; ; ? ? ? ? ? ? ? i i j j k k Therefore the orthonormal set of vector triads i, j, k is self-reciprocal. Example in Machanics : Example 13 : A particle being acted on by constant force (4i + j − 3k) and (3i + j − k) is displaced from the point (i + 2j + 3k) to the point (5i + 4j − k). Find the total work done by the forces.
NSOU ? CC-PH-04 ? 97 Solution : The displacement d is given by d = (5i + 4

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j − k) − (i + 2j + 3k) = 4i + 2j − 4k Resultant forces F = (4i + j − 3k) + (3i + j − k) = 7i + 2j − 4k ?

Work done = F.d = 28 + 4 + 16 = 48 units of work Example 14 : A rigid body is spinning with angular velocity 27 radians per sec about an axis parallel to 2i + j − 2k passing through the point i + 3j − k. Find the velocity of the point of the body where position vector is 4i + 8j + k. Solution : Unit vector along the direction of the angular velocity ? is 2 2 1 (2 2) 3 4 1 4 i j k ? ? ? ? ? ? ? ? i j k Or, 27 (2 2) 9(2 2) 3 ? ? ? ? ? ? ? ? i j k i j k Let O be the point having position vector i + 3j + k and the point P of the body has the position vector, 4i + 8j + k. Then (4 8) (3) ? ? ? ? ? ? ? ? r OP i j k i j k ???? Or, r = 3i + 5j + 2k ? Linear velocity of P is, v = w × r 9 2 1 2 9(12 10 7) 3 5 2 ? ? ? ? ? i j k i j k Example 15 : Find the torque about O (3, −1, 3) of a Force F(4, 2, 1) passing through the point A(5, 2, 4) Solution : Position vector of A(5, 2, 4) relative to O (3, −1, 3) is r = 2i + 3j + k Again the force F = 4i + 2j + k ? Torque = r × F = 2 3 1 2 8 4 2 1 ? ? ? i j k i j k Fig. Example (4) Fig. Example (15) ? 98 ? NSOU ? CC-PH-04 4.7 ? Properties of Vectors Under Rotation A vector is a mathematical object that transforms in a particular way under rotation. We consider a point P(x, y) in a two dimensional co-ordinate system OX and OY, let now the co-ordinate frame rotate in anticlockwise direction by an angle ? so that OX´, OY´ are the new positions of the axes. If the co-ordinate of the point P in the new co-ordinate frame by (x´, y´), then from fig(4A.11), we find x OM MN NC ? ? ? ? cos sin sin sin sin x p q x y ? ? ? ? ? ? ? ? ? ? (4A.30) Since p + q = y and OR = x cos ( )cos y PC q PR RN ? ? ? ? ? ? ? ( )cos cos cos y p y p ? ? ? ? ? ? ? cos cos sin y MR y x ? ? ? ? ? ? ? sin cos x y ? ? ? ? ? (4A.31) Since in ,cos cos MR MR MRN MR p RN p ? ? ? ? ? ? ? Again in ,sin sin MR MR ORM MR x OR x ? ? ? ? ? ? ? The transformation given by equation (4A.30) and (4A.31) are called orthogonal transformation. We take i, j unit vectors along OX and OY axes and i´,j´ along rotated axes OX´, OY´ respectively. Then the position vector OP x y ? ? ? r i j ???? (in old coordinate axes) x y ? ? ? ? ? ?i j (in new co-ordinate axes) (4A.32) Fig. (4A.11) ? p q ?

NSOU ? CC-PH-04 ? 99 Therefore component, of r in new co-ordinate axes are x ? ?i and y? ?j which is different from ix and jy respectively. But magnitude of 2 2 2 2 ( ) ( ) x y x y ? ? ? ? ? ? ? r r is same in both co-ordinate system ; since ? ? ? ? 2 2 2 2 cos sin sin

cos

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x y x y x y ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 cos sin 2 sin cos sin cos 2 sin cos x y xy x y xy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 cos sin cos sin x y x y ? ? ? ? ? ? ? ? ? ? (4

A.33) Thus we have following observations : OP ?r ???? 1. The vector r remains the same in the two co-ordinate systems, though its components change. 2. The length of the vector OP ?r ???? remains the same in both the co-ordinate system. The results can be generalised for any vector in three dimensions also i.e.

x y

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z x y z A A A A A A ? ? ? ? ? ? ? ? ? ? ? ? A i j k i j k (4A.34) and 2 2 2 2 2 2 x y z x y z A A A A A A ? ? ? ? ? ? ? ? ? (4

A.35)

Thus our conclusions are vectors and scalars remain invariant under rotation of co- ordinate systems. 4.7.1 Scalar product of two vectors under rotation of coordinate system Consider two vectors A = iA

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x + jA y + kA z and B = iB x + jB y + kB z and their scalar products A.B = A x B x + A y B y + A z B z

with respect to a rectangular Cartesian co-ordinate system fig (4A.11). We consider rotation of the co-ordinate system about Z-axis which is perpendicular to the plane of the paper and passing through the origin O. Under anticlockwise rotation through angle ?, the new co-ordinate system becomes x´, y´ and z´ = z and the components of the vectors A and B are transformed according to equation (4A.30) and (4A.31) as

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cos sin cos sin x x y y x y z z A A A A A A

A
A ? (4A.36)
100 ? NSOU ? CC-PH-04 cos sin cos sin x x y y x y y z z B B B B B B B ? (4A.37) In the new co-ordinate system the scalar product become .
x x y y z z

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A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? A B (4A.38) Now substituting equation (4A.36) and (4A.37) in equation (4A.38), we get ? ?? ? . cos sin cos sin x y x y A A B B ? ? ? ? ? ? ? ? ? ? A B ? ? ? ? sin cos sin cos x y x y z z A A B B A B ? ? ? ? ? ? ? ? ? ? ? 2 2 2 cos cos sin sin cos sin sin x x x y y x y y x x A B A B A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? 2 sin cos cos sin cos x y y x y y z z A B A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 cos sin cos sin x x y y z z A B A B A B ? ? ? ? ? ? ? ? ? . x x y y z z A B A B A B ? ? ? ? A B (4A.39) equation (4A.39)

shows that scalar product of two vectors remains invariant under rotation of co-ordinate systems or conversely since A.B remains invariant it must be scalar. 4.7.2 Vector Product of Two Vectors Under Rotation of Coordinate Axes Consider two vectors A and B and their vector product A × B. Under rotation of coordinate axes through angle ? counter clockwise about z axis, let the cross product become A´× B´. By equation (4A.30) and (4A.31), we can write ( ) ( ) cos ( ) sin x
x

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y ? ? ? ? ? ? ? ? ? ? A B A B A B (4A.40) ( ) ( ) sin ( ) cos y x y ? ? ? ? ? ? ? ? ? ? ? A B A B A B (4A.41) ( ) ( ) z z ? ? ? ? ? ? ? A B A B (4A.42) NSOU ? CC-PH-04 ? 101 Now squaring equations (4A.40) and (4A.41) both sides and adding, we get 2 2 2 2 2 2 ( ) ( ) ( ) ( ) ( ) ( ) x y z x y z ? A B A B A B A B A B A B or, 2 2 ? ? ? ? ? ? A B A B ? ? ? ? ? ? A B A B (4A.42)

The
invariance of dot
and cross products imply that both the magnitude of the vectors and the angle between them remains unchanged in a rotation. Exercise of Arts 4.6 to 4.7.2 : 1)

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Find a unit vector parallel to the sum of vectors A 1 = 2

i + 4j − 5k and A 2 = i + 2j + 3k. 2) Find the position vector of the centroid of a triangle ABC, when the position vector

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A, B and C are a, b, c. 3) If |A + B| = | A − B|, then show the A and B are perpendicular. 4) If A.B = A.C, does it necessarily follow that B and C are equal. 5) If |A| = |B|, prove that A + B

is perpendicular to A – B. 6) Define direction cosine of a vector. If l 1 , m 1 , n 1 and l 2 , m 2 , n 2 be the direction cosine of the vectors, show that the angle ? between them is cos ? = l 1 l 2 + m 1 m 2 + n 1 n 2 . 7) a) A, B and C are three vectors. If A + B + C = 0, show that A × B = B × C = C × A. b) Is the converse true ? 8) Show that the components of a vector G along and perpendicular to a vector H in the planes of G and H are respectively given by 2 (.) H G H H and 2 ( ) ? ? H G H H . 9)
Prove that :
a) (

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A × B).(C × D) = (A.C)(B.D) – (A.D)(B.C) b) (A × B).(C × D) + (B.C).(A × D) + (C × A).(B × D) = 0 10) If A,B,C,D are such that A × C = B × D and A × B = C ×

D, then show that the vectors (A – D) and (B – C) are
collinear.
102 ? NSOU ? CC-PH-04 Solution of exercise of art 4.6 to 4.7 : Solution (1) : Required unit vector 1 2 1 2 ? ? ? A A A A Now A 1 + A 2 = 3i + 6j – 2k ? ? 2 2 2 1 2 3 6 2 49 7 ? ? ? ? ? ? ? ? ? A A ? required unit vector 3 6 2 7 i j k ? ? ? Solution (2) : The centroid G of ?ABC divides the median of the triangle in the ratio 2:1 Now position vector of 2
D ? ? b c Therefore position vector of 2( ) 2 1 1 2 3 ? ? ? ? ? ? ? ?
b c
a
a b c G Solution (3) : We have |

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A + B| = |A – B| Squaring both sides, |A + B| 2 = |A – B| 2 Or A 2 + B 2 + 2A.B = A 2 + B 2 – 2A.B Or, 4A.B = 0 A.B = 0 i.e. A is perpendicular to B and vice-versa. Solution (4) : If A.B = A.C, then A.(B – C) = 0, i.e. either A is perpendicular to B– C or B– C = 0

where 0 is a null vector. Hence, it does not necessarily follows that B and C are equal. Solution (5) : We have |A| = |B| Fig. Solution (2)
NSOU ? CC-PH-04 ? 103 Or, |

67%	MATCHING BLOCK 98/267	SA	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

A| 2 = |B| 2 ; or A.A = B.B Now, (A + B).(A – B) = A.A – A.B+ B.A – B.B = 0 Since A.A = B.B ? A + B is

perpendicular to A – B.
Solution (6) : The cosine of the angles made by a vector with X, Y and Z axes of a rectangular Cartesian co-ordinate axes, are called direction cosine l, m, n of the vector respectively. If the vector makes angle , and ? ? ? with respect to X, Y and Z axes respectively, then cos , cos and cos l m n ? ? ? ? ? ? . Let A = iA x + jA y + kA z , then
y x z
A A A l m n ? ? ? ? ? ? A i j k i j k A A A A (i) Now cos ;cos ;

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cos y x z A A A ? ? ? ? ? ? A A A ? ? 2 2 2 2 2 2 2 2 2 cos cos cos 1 x y z A A A ? ? ? ? ? ? ? ? ? ? ? A A A (

ii) If there are two vectors A 1 and A 2 , then from equation (i), 1 1 1 1 1 l m n ? ? ? A i j k A (iii) where l 1 , m 1 , n 1 are direction cosines of the vector A 1 . and 2 2 2 2 2 l m n ? ? ? A i j k A (iv) where l 2 , m 2 , n 2 are direction cosine of the vector A 2 . Now if ? be the angle between A 1 and A 2 , we have ? ? ? ? 1 1 1 1 2 2 2 2 1 2 1 2 1 2 . cos . ? ? ? ? ? ? ? ? l m n l m n ? A i

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j k A i j k A A A A A A 104 ?

NSOU ? CC-PH-04 using equations (iii) and (iv). 1 2 1 2 1 2 cos l l m m n n ? ? ? ? ? Solution (7) : a) We have
A +
B +
C = 0,
then

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A + B = – C, now B × (A + B) = – B × C Or, B × A + B × B = – B × C, or, – A × B = – B × C Or, A × B = B × C (i) Similarly, B × C = C × A (ii) Therefore, A × B = B × C = C × A b) We have A × B = B × C ? – A × B = – B × C ? B × A = C × B or, B × A + B × B = C × B = – B × C or, B × A + B × B + B ×

C = 0
or, B × (
A + B +
C) = 0
now 0?B
therefore A + B +
C = 0
Solution (8) : Let ,
OA OB ? ? H G ???? ???? and OM, the projection of G on H. The component of G along H G? ? H H where H H is the unit vector along OA ???? . Therefore vector component G along H is 2 (.) , ? ? ? ? ? ? ? ? ? ? H H G H H G H H H vector component of G perpendicular to H 2 2 2 (.) (.) (.) ( ) OB OM ? ? ? ? ? ? ? ? ? G H H H H G G H H H G H G H H H ???? ????? Fig. Solution (8)
NSOU ? CC-PH-04 ? 105 Solution (9) : a) Let P = A × B, then (A × B).(C × D) = P.(C × D) Now P.(

24%	MATCHING BLOCK 102/267	SA	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

C × D) = (C × D).P = C.D × P = C.D × (A × B) = C.[(D.B)A – (D.A)B] = (C.A)(D.B) – (C.B)(D.A) = (A.C)(B.D) – (A.D)(B.C) b) Now, (A × B).(C × D) = (A.C)(B.D) – (A.D)(B.C) Similarly (B × C).(A × D) = (B.A) (C.D) – (B.D) (A.C) (C × A).(B × D) = (C.B) (A.D) – (C.D) (A.B) ? (A × B).(C × D) + (B × C).(A × D) + (C × A).(B × D) = 0 Solution (10) : We have (A – D) × (B – C) = (A – D) × B – (A × D) × C = A × B – D × B – A × C + D × C = A × B + B × D – A × C – C ×

D = 0 Therefore (A – D) and (B – C)
is collinear. 4.8 ? Polar, Axial Vectors and Pseudo Scalars A polar vector remains invariant under reversal of co-ordinate axes from x to –x, y to – y and z to – z, since, the displacement vector r = ix + jy + kz = –i(–x) –j(–y) – k(–z), on reversal of co-ordinate axes. Let us consider the reflection of OX axis by a mirror as shown in the figure (4A.12). On mirror reflection OX is to the left, OY remains unchanged. Therefore d = ix + jy (before reflection) And d = – i(–x) + jy = ix + jy (after reflection) Fig. (4A.12)

106 ? NSOU ? CC-PH-04 Therefore displacement vector d remains invariant under mirror reflection. Such vectors as r or d are called polar vectors or true vectors. Again in polar vectors only linear action in a particular direction is involved and hence does not depend on the frame of reference. Example of polar vectors are force, linear momentum etc. Now vectors such as angular velocity, angular momentum etc. which are defined in terms of cross or vector product of two vectors, are called axial vectors for, some kind of rotation about an axis is involved in these vectors. The sense of direction in these vectors depend on the handedness (right or left) of the reference frame. In the figure (4A.13) below, we have shown that an axial vector reverse its direction on mirror reflection. If A and B are two polar vectors then A × B must be an axial vector or pseudo vector. Again for three polar vectors A,B,C the triple scalar product A.B ×C changes its sign through reversal of axes x to −x, y to −y and z to −z. Such scalars are called pseudo scalars. 4.8.1 Scalar and vector fields Physical entities may have different values at different points in a region of space and in this sense we can say that physical entities are functions of the space coordinates x,y,z. Suppose we have a physical quantity (, ,) x y z ? ?? so that ? is single valued, finite continuous function of x,y,z and possessing continuous first space derivatives, in the region under consideration. Then the region is called a field of ? . If ? is a scalar, then the field is called a scalar field and ? is called field scalar. Alternatively if ? ? is a vector quantity,, then the region is called a vector field and ? ? is called the field vector.. Examples of scalar point functions are the temperature, electrostatic potential due to a charged body, gravitational potential energy of a massive body etc. and are called field scalars and the corresponding fields are called scalar fields. Fig. (4A.13)
NSOU ? CC-PH-04 ? 107 In electric, magnetic and gravitational field, the intensity of the field in general varies from point to point and is function of the co-ordinates. Hence these intensities are field vectors and the corresponding fields are called vector fields. It is to be noted that with the aid of certain differential operators, it is possible to associate a vector field with each scalar field. This association is of fundamental importance in mathematical physics. A scalar field may be drawn geometrically by a series of surfaces on which the field scalar does not vary e.g. isothermal surfaces, equipotential surfaces etc. On which temperature, potential remain constant respectively. Such surfaces are called level surfaces. Obviously level surfaces cannot intersect each other, for if they do, there will be two values of ? at their common line of intersection which contradicts the very definition of scalar field. 4.8.2 Classification of vector fields A vector field A is characterised by its divergence and curl and the field is determined completely, if its divergence and curl are known. Absence or presence of curl and divergence of a vector field can be pictorially represented as follows : In case of parallel flow of an incompressible field with constant velocity as shown in (fig.4A.14A), div A and curl A are both zero. Such vector fields are called solenoidal and irrotational respectively. Vortex as shown in (fig.4A.14B) is formed in a moving field where curl 0?A at the centre of such vortex and the vector field is characterised as rotational. In this case net inward or outward flow is zero and div A = 0 and we call the vector field as solenoidal. In case fluid is compressible, there can be excess of outflow over the inward in addition to the flow being rotational. This is Fig. (4A.14A) Fig. (4A.14B)
108 ? NSOU ? CC-PH-04 shown in (fig. 4A.14C). In this case div 0?A and curl 0?A . Again when there is no rotation of a compressible fluid as shown in fig. (4A.14D,14E), there can be excess of outward flow over the inward [fig. 4A.14D] or excess of inward flow over the outward [fig. 4A.14E], we say curl A = 0, but div 0?A . Fig. (4A.14D) Fig. (4A.14E) Now when the vector field is irrotational i.e. 0, ? ?? ? ? ? A A ; and when it is solenoidal i.e. 0 ?? ?A , or 0 ? ??? ? or 2 0 ?? ? (Laplace equation). When 0 ?? ?A but 0 ?? ?A , we get 0 ? ??? ? or 2 0 ?? ? (Poisson's equation) 4.9 ? Summary - I 1) Invariance properties of scalar vectors are discussed. 2) Various types of product of vectors are discussed with reference to the example of machanics. 3) Use of kronecker delta, Levicivita symbol and classification vector field discussed for curious students. 4.10 ? Vector Differentiation From the definition of derivatives applied to vector functions, different space and time derivative of vectors(with their physical meaning) have been explained and to obtain idea Fig. (4A.14C)
NSOU ? CC-PH-04 ? 109 of special type of vector differential operators, for example, del or nabla applied to scalar and vector functions like gradient, divergence and curl. Also an important objective of this chapter is to fully realise the physical or geometrical interpretation of gradient, divergence and curl. Different vector identities are listed with or without del. Operations of differentiation of vectors are important in the sense that this concept is necessary for defining the various operators useful in vector analysis. Consider a vector A(u) which is a continuous function of a continuous scalar variable u. As u change, a curve is traced by the terminus of A(u) (fig. 4B.1). In analogy with scalar functions, we define the derivative d du A as 0 0 ( ) ( ) lim lim u u d u u u du u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A A A A (4B.1) The derivative d du A is a vector.. whose direction is the limiting direction of ?A as ?u ? 0. That is the direction of the derivative lies along the tangent to the curve at the point P as ?u ? 0 in the sense of increasing u. If r(u) be

84%	**MATCHING BLOCK 103/267**	**SA**	M P Vol 1.pdf (D134397037)
the position vector of the point P(x,y,z) with respect to			

a set of rectangular axes with origin O, then

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r(u) = ix(u) + jy(u) + kz(u). And 0 lim ? ? ? ? ? u d du u r r 0 ( ) ( ) lim u u u u u ? ? ? ? ? ? ?			

r r . This d du r is a vector in the direction of the tangent to the space curve at (x, y, z) and is given by ? d dx dy dz du du du du r i j k (4B.2) The derivative of a constant vector is a null vector. Fig. (4B.1)
110 ? NSOU ? CC-PH-04 From equation (4B.2) we see that derivative of a vector r means a vector whose components are the derivatives of the components of r, since, i, j, k, the unit basis vectors in rectangular Cartesian system are constant in magnitude and direction. But in other co- ordinate system, for example plane polar co-ordinate in two dimensions and spherical or cylindrical co-ordinate system in three dimensions, the unit basis vectors change with direction though their magnitudes are constant. Therefore in calculating the derivative of a vector written in these co-ordinate systems, we must differentiate the basis vectors as well as the components. Now consider the vector A is a derivable function of other scalar s and s is a derivable function of another scalar u, then 0 lim u d s d ds du s u ds du ? ? ? ? ? ? ? ? ? ? A A A As d du A is a vector and in general a function of u, we may find 2 3 2 3 , d d du du A A etc. Rules for differentiation : If A and B be two derivable vectors, each being function of the scalar variable u and s be another scalar. Then i< ? ? d d d du du du ? ? ? A B A B ii< ( ) d d ds s s du du du ? ? A A A iii< ( ) d d d du du du ? ? ? ? ? B A A B A B [dot product being associated, the order of the vectors, may be changed]. iv< ( ) d d d du du du ? ? ? ? ? A B A B B A [vector products does not obey commutative law, the order of the vectors cannot be changed]. v< 2 2 , ? ? ? ? ? ? ? ? ? ? ? d d d d d du du du du du A A A A A A since cross product of equal vectors is zero.
NSOU ? CC-PH-04 ? 111 4.11 ? Constant vector function: Constancy in direction and magnitude 4..11.1 : The necesssary and sufficient condition for a vector function A(u) to be a constant is ( ) 0 d u du ? A Proof : Condition is necessary If A(u) is constant, then for every change in ?u of the scalar variable

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u ( ) ( ) 0 u u u ? ? ? ? A A 0 ( ) ( ) lim 0 ( ) u u u u d u du ? ? ? ? ? ? ? ? A			

A A A Condition is sufficient : Consider A(u) = iA 1 (u) + jA 2 (u) + kA 3 (u) where A 1 , A 2 , A 3 are three scalar functions of u. Then 3 1 2 0 dA dA dA d du du du du ? ? ? ? A i j k Implies 3 1 2 0; 0; and 0 dA dA dA du du du ? ? ? Therefore A 1 , A 2 , A 3 are constant and hence A(u) is a constant vector. Thus the condition is sufficient. 4.11.2 : The necessary and sufficient condition for a vector A(u) to have constant magnitude is A. 0 d du ? A Proof : Condition is necessary : We have A(u).A(u) = |A(u) 2 | Therefore ( ) ( ). ( ) ( ) d u d u u u dt du ? ? A A A A
112 ? NSOU ? CC-PH-04 And ( ) ( ). 0 d u u dt ? A A if and only if ( ) 0 d u du ? A Or A(u) = constant. Therefore condition is necessary. Condition is sufficient : Assume that A(u) has a constant magnitude |A(u)|. Then definitely ( ) 0 d u du ? A . So that, ( ) ( ). 0 d u u du ? A A . Thus the condition is also sufficient. 4.11.3 The condition for a vector A(u) to have constant direction is ( ) ( ) 0 d u u du ? ?

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A A Proof : Consider ˆ ( )a u to be a unit vector in the direction of A(u), the A(u) = ˆ ˆ ˆ ( ) ( ) ( ) ( ) u u A u u A ? ? A a			

a a Therefore ˆ ( ) ( ) ( ) ˆ ( ) ( ) ? ? d u dA u d u u A u du du du

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A a a ˆ ˆ d dA d A du du du ? ? ? ? ? ? ? ? ? ? ? ? A a A A a ˆ ˆ ˆ dA d A A du du ? ? ? ? ? ? ? ? ? ? a a a 2 ˆ ˆ ˆ ˆ 0 dA d A A du du ? ? ? ? ? a a a a

Now ˆ( )ua is constant, ˆ 0 d du ? a ; and ˆ ˆ 0 ? ? a a 4.12 ? Derivative of Triple Scalar Product Consider S = A. B × C, where A, B and C are vector functions of the scalar variable u. Then dS d du du ? [A. B × C]
NSOU ? CC-PH-04 ? 113 . d d d du du du ? ? ? ? ? ? ? ? A B C B C A C AB The cyclic order of the factor on each term is maintained. 4.13 ? Derivative of Triple Vector Product Consider S = A × (B × C), where A, B, C are vector functions of the scalar variable u. Then ( ) dS d d d du du du du ?

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A B C B C A C A B The

order of the factor in each term is maintained. 4.14 ? Velocity and Acceleration of a Particle The position vector of a particle at time t is given by r(t) = ix(t) + jy(t) + kz(t) The displacement ?r(t) in time ?t is given by ( ) ( ) ( ) t t t t ? ? ? ? ? ? r r r Now time rate of change of displacement of a particle is its velocity 0 lim t d dt t ? ? ? ? ? ? ? r r v and this velocity v is in the direction of the tangent to the path of the particle at (x, y, z) in time t. The acceleration of the moving particle being the time rate of change of v and we have acceleration of the particle 2 0 2 lim t d d t dt dt ? ? ? ? ? ? ? v v r f Exxample of 4.4: Example (1): A particle moving in a plane. Find the radial and transverse components of velocity and acceleration of the particle in plane polar co-ordinate. Solution : At any time t, let the position vector of the particle at a point ˆ (,), ; ? ? ? r r x y ? r r i j (i)
114 ? NSOU ? CC-PH-04 or, ˆ cos sin (cos sin), r r r r ? ? ? ? ? ? ? ? ? ? r r i j i j where cos ; sin x r y r ? ? ? ? and ˆ r = unit vector in the direction vector cos sin ; ? ? ? ? r i j (i) ˆ θ is unit vector in a direction perpendicular to r. ˆ cos sin 2 2 ? θ i j or, ˆ sin cos ? ? ? ? ? θ i j (ii) Now the velocity of the particle is given by, ˆ ˆ ˆ ( ) d d dr d r r dt dt dt dt ? ? ? ? r r v r r Now, ˆ ˆ (sin cos) d d d dt dt dt ? ? ? ? ? ? ? ? ? r i j θ And ˆ ˆ (cos cos) d d d dt dt dt ? ? ? ? ? ? ? ? ? θ i j r Therefore ˆ ˆ ˆ ˆ dr d r r r dt dt ? ? ? ? ? ? v r θ r θ ? ? where the dot (.) indicate differentiation with time. Thus radial and transverse components of velocity are : ; r v r v r ? ? ? ? ? ? ? so that ˆ ˆ ? ? r v r v ? v θ Acceleration of the particle is given by, ˆ ˆ ˆ ˆ ˆ ˆ ˆ d d d d r r r r r r r dt dt

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dt dt ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? v r θ a r θ r θ θ ? ? ?? ? ? ? ?? ? ? where 2 2 d r r dt ??? and 2 2 d dt ? ? ? ? ?? . Now substituting the values of ˆd dt r and ˆ d dt θ , we get Fig. Solution 1 NSOU ? CC-PH-04 ? 115 ? ? ? ? ? 2 ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ 2 r d d a r r r r r r r r r

a a dt dt ? r θ θ θ r r θ r θ ? ?? ? ? ? ? ?? ?? ? ? ? ? ?? ? where a r is the radial component and a ? is the transverse component of acceleration. 4.15 ? Relative Velocity and Acceleration Consider two particles at P 1 and P 2 moving along the curve C 1 and C 2 and having respectively position vectors r 1 and r 2 at time t Then 1 2 2 1 PP ? ? ? r r r ????? Differentiating with respect to time t, we get 2 1 ? ? d d d dt dt dt r r r = relative velocity of particle P 2 with respect to that at P 1 . Similarly relative velocity of the particle at P 1 with respect to that at P 2 is given by 1 2 ? ? ? d d d dt dt dt r r r Relative acceleration of the particle at P 2 with respect to P 1 is given by 2 2 d dt r and that of P 1 with respect to P 2 is – 2 2 d dt r . 4.16 ? Gradient of a Scalar Field

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The gradient of a scalar field (, ,) x y z ? at a point (x 0 , y 0 , z 0) is a vector,

denoted by the symbol ?? (read "del ?") and is defined by grad 0 0 0 0 0 0 0 0 0

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x y z x y z x y z x y z ? i j k (4

B.3) where the operator 'del' or 'nabla' is a vector differential operator given by ? ? ? ? ? ? ? ? ? x y z i j k ? (4B.4) Fig. (4B.2) 116 ? NSOU ? CC-PH-04 Let ?(r) be some field scalar at some point r(x, y, z) and ?(r + dr) be the value of ? at r + dr(x + dx, y + dy, z + dz); then ??(r + dr) − ??(r) = d? When d dx dy dz x y z ? ? ? ? ? ? ? ? ? ? ? ? ? Or, . d x y z ? i j k = ? ? dx dy dz ? ? i j k . . d grad dr ? ? ? ? ?r (4B.5) Now the points (x, y, z) satisfying ??(x, y, z) = k = constant, in general defines a surface in region of space. This surface ??(x, y, z) = k is called a level surface, since at every point of the surface ??(x, y, z) has a value equal to k [fig. 4B.3]. Now differentiating equation (4B.5) with respect to r, we get ˆ. r ? ? ? ? ? ? r (4B.6), where ˆr is a unit vector in the direction of r. Now suppose ˆ r is tangent to the surface ? = constant at the point P. Consider r ?? ? for path PL, PM, PN etc approaching the tangent ˆ r . Since ? = constant on the surface and L, M, N, P etc are all on the surface, ?? = 0 and r ?? ? = 0 for such path. But r ?? ? in the tangent direction is the limit of r ?? ? as ?r ? 0 (that is as PL, PM, PN etc approaches ˆ r, so r ?? ? in the direction ˆ r is zero also). Thus for ˆ r along the tangent to ? = constant ˆ. 0 ?? ?r , this means that ?? is perpendicular to ˆ r . Since this is true for any ˆ r tangent to the surface at the point, then at that point, the vector ?? is perpendicular to the level surface ? = constant. Again from equation (4B.5), since ? = constant, ˆ 0 . d dr ? ? ? ? ? r . Since 0 dr ? , ˆ. 0 ?? ?r i.e. ?? is perpendicular to ˆ r . 4.17 ? Directional Derivative Suppose rate of change ??(x, y, z) with distance is to be evaluated at a given point P 0 (x 0 , y 0 , z 0) and in a given directional 0 P Q ???? as shown in fig [4B.4]. Let R be the distance Fig. (4B.3) L ? r ^
NSOU ? CC-PH-04 ? 117 in the direction ˆ R where ˆ l m n ? ? ? R i j k , is the unit vector in that direction; l, m, n being the direction cosine of the directed line 0 P Q ???? . Then, r − r 0 = R ˆ R = R(il + jm + km) where (x, y, z) is the position co-ordinate of the terminus of the vector R and (x 0 , y 0 , z 0) is that of it's tail. Therefore, we write, x = x 0 + Rl y = y 0 + Rm (4B.7) z = z 0 + Rn From equation (4B.7) we see that ??(x, y, z) is function of just one variable R, the distance along the line measurement from (x 0 , y 0 , z 0). Then d dx dy dz dR x dR y dR z dR ? ? ? ? ? ? ? ? ? ? ? ? ? l m n x y z ? ? ? ? ? ? ? ? ? ? ? ? = ˆ .?? R (4B.8) where x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k Equation (4B.8) gives the directional derivative if we take (x 0 , y 0 , z 0) = (0, 0, 0) i.e. the origin of co-ordinate system, then ˆ ˆ ?R r and vector R becomes positions vector r and therefore equation (4B.8) can be written as ˆ. d dr ? ? ? ? r (4B.6) 4.18 ? Normal Derivative We consider two neighbouring level surface defined by ? and ? + d?. Shorted distance between surfaces at the point P is ˆ cos . dn dr d ? ? ? r n as shown in the figure (4B.6). Therefore cos dn dr ? ? . Fig. (4B.4) R
118 ? NSOU ? CC-PH-04 Now rate of increase of ? at P in the direction PQ ???? is cos d d dr dn ? ? ? ? Or, max d d dr dn ? ? ? ? ? ? ? ? ? [now as discussed previously max r ? ? ? ? ? ? ? ? ? ? ? ?] since ?? is the value of the directional derivative in the direction normal to the surface it is often called normal derivative and written d dn ? ?? ? (4B.12) Example of Art 4B.8 and 4B.9 : Example 1:

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Find directional derivative of ??= x 2 − 2y 2 + 4z 2 at the point (1, 1, − 1) in the direction 2

i + j − k. In what direction the directional derivative
at that point is maximum and what is its value ? Solution 1 : (2 4 8) 2 4 8 x y z ?? ? ? ? ? ? ? ? i
j k i j k at the point (1, 1, −1)

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Unit vector along the direction 2i + j − k is ? ? 1 2 6 ? ? i j k ? Required directional derivative is, ? ? ? ? 1 4 4 8 8 2 4 8 . 2 6 6 6 ? ? ? ? ? ? ? ? ? i j k i j k

Directional derivative is maximum along ?? and its maximum value is 84 2 21 ?? ? ? Example 2 : Find the equation of tangent plane and normal to the surface z = x 2 + y 2 at the point (2, − 1, 5). Solution 2 : 2 2 4 2 x y ?? ? ? ? ? ? ? ? i j k i j k at (2, − 1, 5) Position vector of the point (2, − 1, 5) is, r 0 = 2i − j + 5k Therefore equation of tangent plane is given by (r − r 0). 0 ?? ? Fig. (4B.6)
NSOU ? CC-PH-04 ? 119 Where r = ix + jy + kz is any point on the tangent plane. i.e. (ix + jy + kz − 2i + j − 5k).(4i − 2j − k) = 0 or, (x − 2).4 − 2(y + 1) − (z − 5) = 0 i.e. 4x − 2y − z = 5 equation to the normal to the surface is (r − r 0) × 0 ?? ? where r is any point on the nrомal. i.e. ? ? (2) (1) (5) (4 2) 0

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x y z ? ? ? ? ? ? ? ? ? i j k i j k or, 2 1 5 4 2 1 x y z ? ? ? ? ? ? ? Exercise of Art 4.14, 4.15 and 4.18 : 1) Find unit vector normal to the surface x 2 + y − z = 1 at the point (1, 0, 0). 2)

For the function 2 2 (,) , x x y x y ? ? ?
find the magnitude of the directional derivative along a line making an angle 30 0 with the positive x − axis at (0, 2). 3) The velocity of a boat relative to water is represented by 3i + 4j, and that of water relative to earth is i − 3j. What is the velocity of the boat relative to earth, if i and j represent 1 km an hour east and north respectively. 4)

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Find the angle between the surface x 2 + y 2 + z 2 = 4 and z = x 2 + y 2 − 5 at

the point (1, −1, 2). 5) Find ?? , (i) when ? = ln |r| and (ii) prove that 2 1 ˆ
n n n r nr nr ? ? ? ? ?r r 6) Find the normal derivative of f = xy + yz + zx at (1, 1, 3).
Solution :
Solution (1) : 2 2 2 (, ,)

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x y z x y z ? ? ? ? ? ? 2 2 2 2 2 x y z ? ?? ? ? ? ? ? ? i j k i k at the point (1, 0, −1). Now unit normal to (, ,) x y z ? ?

at (1, 0, −1)
is,
120 ? NSOU ? CC-PH-04 2 2 ((1,0, 1)) 2( ) ˆ ((1,0, 1)) 2 2 2 at the point at the point ? ? ? ? ? ? ? ? ? ? ? ? i k i k n Solution 2 : We have 2 2 (,)

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x x y x y ? ? ? 2 2 x x y z x y ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k 2 2 2 2 .0 x x x y x y x y ? i j k ? ? ? ? 2 2 2 2 2 2 2 2 1 (2) (2) x x x y x y x y x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

i j ??
at the point (0, 2) is 4 i . Now as the line makes an angle 30 0 with the x − axis : r = i(r cos 30 0) + j(r sin 30 0) 3 2 2 r ? ? ? ? ? ? ? ? j i 3 ˆ 2 2 ? ? ? ? r r j r i ? directional derivative along ˆr is, (0,2) 3 3 ˆ. 4 2 2 8 ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j r i Solution 3 : We assume v B & v w be the velocity of the boat and that of water relative to earth respectively. Therefore, 3i + 4j = v B − v w , where v w = i − 3j ? v B = 3i + 4j − v w = 3i + 4j + i − 3j = 4i + j 2 2 4 1 17 B ? ? ? ? v km/hr
NSOU ? CC-PH-04 ? 121 If ? be the angle between v B and east direction then 1 1 1 tan or tan 4 4 ? ? ? ? ? Solution 4 : We suppose 2 2 2 1 (, ,) 4 0

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x y z x y z ? ? ? ? ? ? and 2 (, ,) x y z ? 2 2 2 5 0 x y z ? ? ? ? ? Now, the

angle between two surfaces is the angle between normals at a point P(1, −1, 2). Now, 1 1 1 ˆ ? ? ? ? ? n at the point (1, −1, 2) And 2 2 2 ˆ ? ? ? ? ? n at the point (1, −1, 2) 1 2 1 2 1 2 ˆ ˆ. cos ? ? ? ? ? ? ?? ? ? ? ? ? n n at (1, −1, 2) Where 1 ˆn and 2 ˆn are the unit normal on the surface ? 1 = constant and ? 2 = constant at the point P. Now, 1 (1, 1,2) 2 2 4 ? ? ? ? ? ? i j k and 2 (1, 1,2) 2 2 ? ? ? ? ? ? i j k 2 2 2 2 2 2 (2 2 4).(2 2) 4 4 4 4 2 cos 12 9 6 3 3 3 2 (2) 4 2 (2) (1) ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
i j
k i j k ? Solution 5 : i) We have r = ix + jy + kz 2 2 2

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r x y z ? ? ? ? ? r ? ? 2 2 2 2 2 2 1 ln ln 2 x y z x y z ?? ? ? ? ? ? ? ? ? ? 2 2 2 1 ln 2 x y z x y z ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? i j k Fig. Solution (4) 122 ? NSOU ? CC-PH-04 2 2 2 2 2 2 2 2 2 2 2 2 1 2 2 2 2 x y z x y z x y z x y z x y z x y z ? i j k i j k 2 ˆ r r ? ? r r ii) We have ? ? 2 2 2 2 n n r x y z ? ? ? ? ? 2 2 2 2 n n r x y z x y z ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? i j k ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 2 2 2 n n n x y z x y z x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k Or ? ? ? ? 1 1 2 2 2 2 2 2 2 2 .2 .2 2 2 n n n n n r x y z x x y z y ? i j ? ? 1 2 2 2 2 .2 2 n n x y z z ? ? ? ? ? ? ? ? ? ? ? ? k ? ? 1 2 2 2 2 ( ) n n x y z x y z ? ? ? ? ? ? i j k ? ? 1 2 2 1 2 ˆ

n
n
n n
r nr
nr ? ? ? ? ? ? r r
r Solution 6 :
From equation (4B.12), normal derivative d dn ? ? ? ? Now f xy yz zx ? ? ? ? ?
NSOU ? CC-PH-04 ? 123 Now ( ) ( ) ( ) f f f f y z z x x y x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k i j k f ?? at the point (1, 1, 2) is 4i + 4j + 2k 2 2 2 4 4 2 16 16 4 36 6 f ? ? ? ? ? ? ? ? ? ? 4.19 Geometrical and Physical Meanings of Grad ?? : From equations (4B.8) using the definition of dot product, since ˆ 1?R ˆ cos cos d dR ? ? ? ? ? ? ? ? ? R (4B.7) where ? is the angle between ˆ & R ?? . Therefore max d dR ? ? ? ? ? ? ? ? ? ? i.e. maximum value of d dR ? is ?? and it is in the direction of ?? [fig. 4B5) When ? = 180 o , we get largest of decrease of ? i.e. d dR ? ? ? ? ? When ˆ , 2 ? ? ? R, is tangent to the surface ? (x, y, z) = constant at the point P Fig. [4B.5] and ˆ 0 ?? ? ?R i.e. ?? is perpendicular to the tangent ˆ R at the point P, since this is the true for any ˆ R tangent to the surface at the point P then ?? is perpendicular to the surface (, ,) x y z ? constant. 4.20 : The 'Del' or 'Nabla' operator When we write grad x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k , we call the bracket Del or Nabla or ? . Thus 'Del' is a differential operator, has no meaning by itself like other operator, e.g. d dx or sine or log e etc but has vector properties. So far we have discussed ?? where ? is a scalar. But ? can operate on vectors Fig. (4B.5)
124 ? NSOU ? CC-PH-04 too. Suppose A(x, y, z) is a continuously differentiable point function with components A x , A y and A z in the X, Y and Z direction repectively and A can vary in magnitude and direction from point to point. We can now form two useful combination of ? and A. We define the divergence of A i.e. div
A or ? .A by

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y x z A A A x y z ? ? ? ? ? ? ? ? ? ? ? A (4B.9) And curl A or rot A or ? x A by ? ? ? ? ? ? ? ? x y z x y z A A A i j k **A** ? y y x x z z A A A A A y z z x x y ?

i j k (4B.10)
Equation (4B.9)

follows from the definition of dot product of two vectors A and B and equation (4B.10) follows from definition of cross product. So we can say that in the above two equation ? behave almsot like a vector.. Now ?? is a vector function and we can write ? ? ? A , where ? is a scalar function. Now ? . AA becomes ??? ? = div gard ? . This important expression is called Laplacian of ? and is written as 2 x x y y z z ? 2 2 2 2 2 2 x y z ? ? ? ? ? ? ? ? ? ? ? ? (4B.11) Several important equations in mathematical physics involving Laplacian are :
NSOU ? CC-PH-04 ? 125 2 0 ? ? ? ? ? ? (Poisson's equation) 2 0 ? ? ? (Laplace's equation) 2 2 2 2 1 c t ? ? ? ? ? ? (Wave equation) 2 2 1 t h ? ? ? ? ? ? [diffusion, heat conduction equation etc.] where h is constant and t is time. 4.20.1 Divergence of a vector field Divergence of a vector field A measures how much the vector A spreads out from the point in question. The vector function in fig (4B.6) has a positive divergence at the point P, if it is spreading out from there. If the arrows pointed inwards, it would be a negative divergence on the other hand the function in fig (4B.7) has zero divergence at P, as it is not spreading outwards or inwards at all. Fig. (4B.6) Fig. (4B.7) The points, at which ? . A < 0 are called sources, while the points at which ? . AA > 0 are called sinks of vector field A. But if at all points ? . AA = 0, then the vector field is said to be solenoidal. Thus a solenoidal vector field is without a source or sink. Physical significance of a divergence is that it gives the net rate of outflow per unit volume evaluated at a point. This is 'outflow' of actual substance for liquids, gases or particles and 'flux' for electric and magnetic fields.
126 ? NSOU ? CC-PH-04 We consider an element of volume [fig (4B.8)] in a region through which water is flowing and take the vector point function A to be equal to ?v, where ? is the density and v, the velocity of water flow at that point such that A = ?v, Obviosuly ?v gives the mass of water following per unit area per second in the direction of v. If water is flowing in the direction v making an angle ? with the normal ˆn to a surface, then amount of water crossing unit area of the surface in unit time is A. ˆn, if ˆn is a unit vector.. The rate at which water flows into element of volume dxdydz through surface 1 is (A.i)dydz = A x dxdydz, where A = iA x + jA y + kA z . Similarly the rate at which water flows out through the surface 2 is x x A A dx dydz x ? ? ? ? ? ? ? ? ? , since the value of x-component of A at the surface 2 is x x A A dx x ? ? ? ? ? ? ? ? ? , for constant y & z. Therefore the net outflow through these two surface is [(A x at surface 1) – (A x at surface 2)] dydz x A dxdydz x ? ? ? We get similar expression for the net outflow through the other two pairs of opposite surfaces ; namely y A dxdydz y ? ? (through ABCD and OB´C´D´ surfaces) And z A dxdydz z ? ? (through OB´BA and CC´D´D surfaces) Then the total net rate of outflow of water from the volume element dxdydz is Fig. (4B.8)
NSOU ? CC-PH-04 ? 127 y x z A A A dxdydz dxdydz x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A ? (4B.12) From definition of divergence in equation (4B.9). Therefore net rate of outflow of water per unit volume is ?A ? . This is the physical meaning of divergence. Equation of continuity : If some physical entity is generated within a certain region of a field, then that region is termed as source. On the other hand, if the physical entity is absorbed then the region is called a sink. Clearly, if there are no source or sink presents in the field, then the net outflow of the incompressible physical entity over any part of the region is zero. If the total strength of the sources is greater than that of the sinks, the net outflow is said to be positive, otherwise it is zero or negative. Now from the physical significance of divergence, we see that, divergence is somewhat like density, since like density, divergence is evaluated per unit volume and may vary from point to point. Therefore from the above discussions we see that divergence of a vector field A will be different from zero due to i) non-equality of source and sink strength and ii) time variation of density, in case of compressible fluids. Now we consider a region of volume dxdydz is which water is flowing and where there is source and sink. From the principle of conservation of mass : Rate of increase of mass in dxdydz = Rate of creation of mass minus Rate of outwards flow in dxdydz Or dxdydz dxdydz dxdydz t ? ? ? ? ? ? ? A ? (4B.13) Where ? is the net mass of fluid being created per unit time per unit volume, which is source density minus sink density and ? is the mass per unit volume or density of the fluid and t ?? ? is the rate of increase of mass per unit volume per unit time. Since 0 dxdydz ? , we get t ? ? ? ? ? ? ? A ? (4B.14)
128 ? NSOU ? CC-PH-04 If there are no source or sink or source strength equals sink strength, then ? = 0 and the resulting equation (4B.15) is called equation of continuity : 0 t ?? ? ? ? ? ? A ? (4B.15) In case of incompressible fluid, 0 t ?? ? ? and we get ? ? ?A ? (4B.16) In case of electric field, D, the electric displacement vector, and the sources and sink are electric free charges and we get ? ? ? charge density ? D ? . For magnetic field B, the source and sinks are magnetic free poles, which does not exist. Therefore 0 ? ?B ? . 4.20.2 Integral form of divergence We consider ˆn to be the unit vector normal to d?, a small area from the surface of a small volume element of fig (4B.9) then the mass of fluid flowing out through d? is A. ˆn d? and the total outflow from the volume enclosed by the surface is ˆ. d? ??? An . Again to the volume element d? = dxdydz, the total outflow from d? is d? ? A ? Therefore, ˆ. d d ? ? ? ? ??? A An ? Or, 0 1 ˆ lim . d d d ? ? ? ? ? ? ??? A An ? (4B.17) Equation (4B.17) gives the integral definition of divergence. Examples of Art 4.20 : Example 1 : Prove that (3) n

n

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div r n r ? ? r Solution 1 : Let ? ? ? ? 1 2 2 2 2 1 2 3 , n n r r x y z r x y z ? ? ? ? ? ? ? ? ? ? ? ? r i j k i j k 3 1 2 n div r x y z ? ? ? ? ? ? ? ? ? ? ? ? ? r Fig. (4B.9) d? ˆn NSOU ? CC-PH-04 ? 129 now 1 n xr ? ? 1 1 2 2 1 n n n n n n r x r nxr r nxr r nr x x x r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? similar expressions for 3 2 , y z ? ? ? ? ? ? can be obtained and are given by 2 2 2 n n r nr y y ? ? ? ? ? ? 2 2 3 n n r nr z z ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 3 3 (3) n n n n n n div r r nr x y z r nr r n ? ? ? ? ? ? ? ? ? ? r Note : When n = – 3, 3 0. n r div r

r ? ? ? r ? Therefore the vector 3 r r is solenoidal. Example 2 : Show 2 2 ( ) (1) n

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n r n n r ? ? ? ? Solution 2 : ? ? 2 2 2 n n n n r r nr n r ? ? ? ? ? ? ? ? ? ? ? ? ? r r Let 2n r ? ? ? . Now using formula (9) in Art 4B.14.2, we can write 2 n r n n n ? ? ? ? ? ? ? ? ? ? ? ? ? r r r Now x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? r = 1 + 1 + 1 = 3, where r = ix+ jy + kz and 2 3 ˆ (2) n n r n r ? ? ? ? ? ? ? ? ? r = (n – 2) r n–4 r 2 2 4 2 2 3 (2) . 3 (2) n n n n n r nr n n r nr n n r ? ? ? ? ? ? ? ? ? ? ? ? r r ? ? ? 2 2 2 2 3 (2) (1) n n n n n n r n n r n n r ? ? ? ? ? ? ? ? ? ? ? ? ? ? 130 ? NSOU ? CC-PH-04 Example 3 : Prove that 2 2 2 2 ( ) d f df f r r dr dr ? ? ? Solution 3 : 2 ( ) ( ) f r f r ? ? ? ? ? Now, ( ) ( ) ( ) ( ) ( ) ( ) ( ) f r f r f r f r r f r r f r r f r x y z r x r y r z ? i j k i j k ( ) ( ) ( ) 1 ( ) ( ) ( ) f r x f r y f r z f r f r x y z r r r r r r r r r ? r i j k i j k Now 2 1 ( ) ( ) df r f r r dr ? ? ? ? r ( ) ( ) ( ) x df r y df r z df r x r dr y r dr z r dr ? ( ) ( ) ( ) x y z f r f r f r x r y r z r ? Now, 1 1 ( ) ( ) ( ) ( ) x x f r f r x f r f r x r r x r r x ? ( ) 1 ( ) ( ) f r r x r x f r f r r r r x r r x ? 2 2 3 3 ( ) ( ) ( ) ( ) ( ) ( ) f r

37% **MATCHING BLOCK 123/267** **SA** 07170523.pdf (D54981551)

x x x f r x x x f r f r f r f r r r r r r r r r ? ? ? ?? ? ? ?? ? ? ? ? ? ? ? ? 2 2 3 2 ( ) ( ) ( ) f r x x f r f r r r r ? ? ? ?? ? ? ? ? (i) Similarly 2 2 3 2 ( ) ( ) ( ) ( ) ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? y f r y y f r f r f r y r r r

r (
ii) And 2 2 3 2 ( ) ( ) ( ) ( ) z f r z z f r
f r f r z r
r
r
r ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? (
iii)
NSOU ? CC-PH-04 ? 131 Adding equations (i), (ii) and (iii), we get 2 2 2 2 2 2 2 3 2 3 ( ) ( ) ( ) ( )

51% **MATCHING BLOCK 124/267** **SA** 07200140 (2).pdf (D118447009)

x y z x y z f r f r f r f r r r r ? ? ? ? ? ? ? ?? ? ? ? ? 3 ( ) 2 ( ) ( ) ( ) ( ) f r f r f r f r f r r r r ? ? ?? ?? ? ? ? ? ? ? ?

Exercise of
Art 4.20 : 1) A rigid body is rotating with constant angular velocity ?. Show that the linear velocity is solenoidal. 2) Prove that 2 1 0 r ? ? ? ? ? ? ? ? ? 3) Prove that . 6( ) x y z ? ? ? F? , when ? ? 3 3 3 3 x y z xyz ? ? ? ? F ? 4) A vector field is defined by 2 r r ? ? F , Evaluate F. 5) For what value of a the vector, A = (
x + 3

34% **MATCHING BLOCK 125/267** **SA** 07200146.pdf (D118456064)

y)i + (y − 2z)j + (x + az)k is solenoidal. Solution : Solution (1) : x y z ? ? ? ? ? ? i j k ? = constant vector Now linear velocity x y z x y z ? ? ? ? ? ? i j k v r ? or, ? ? ? ? ? ? y z z x x y z y x z y x ? ? ? ? ? ? ? ? ? ? ? ? v i j k Now, ? ? ? ? ? ? 0 0 0 0 y z z x x y z y x z y x x y z ?

v? Since, 0, ? ?v v ? is solenoidal.

132 ? NSOU ? CC-PH-04 Solution 2 : From example (2) of Art 4B.11 We have 2 2 ( ) (1) n n r n n r ? ? ? ? Putting n = − 1, we get 2 1 0

27% **MATCHING BLOCK 126/267** **SA** 07200140 (2).pdf (D118447009)

r ? ? ? ? ? ? ? ? Solution (3) : ? ? 3 3 3 3 x y z xyz ? ? ? ? ? F or, ? ? ? ? 3 3 3 3 3 3 3 3 x y z xyz x y z xyz x y ? ? ? ? ? ? ? ? ? ? ? ? F i j ? ? 3 3 3 3 x y z xyz z ? ? ? ? ? ? k ? ? ? ? ? ? 2 2 2 3 3 3 3 3 3 x yz y xz z xy ? ? ? ? ? ? i j k Now, ? ? ? ? ? ? 2 2 2 3 3 3 3 3 3 x yz y xz z xy x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? F = 6x + 6y + 6z = 6(x + y + z) Solution (4) : 2 2 2 2 r x y z r r r r ? ? ? ? ? F i j k where r 2 = x 2 + y 2 + z 2 Now, 2 2 2 x y z x y z r r r ? F Now, ? ? 2 2 2 2 2 2 2 2 2 2 2 2 1 2 x x x y z x x x r x y z x y z ? 2 2 4 1 2x r r ? ? Similarly, 2 2 2 4 1 2 y y y r r r ? ? ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 133 Fig (4B.10) and 2 3 2 4 1 2 z z z r r r ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 4 2 2 2 2 3 3 2 1 x y z r r r r r ? ? ? ? ? ? ? ? ? ? F Solution 5 : A = (x+ 3y)i + (y − 2z)j + (x + az)k Now, (3) (2) ( ) 1 1 2 x y y z

x

az a a x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

A For A to be solenoidal 0 ? ? ? A i.e. 2 + a = 0 i.e. a = − 2 4.21 ? Curl of A Vector Field We have shown earlier that if a rigid body rotates, about an axis with a constant angular velocity ? in the direction of the axis, it is related to the linear velocity v of a particle in rigid body by 1 ( ) 2 ? ? ? ? v (4B.18) Thus curl v relates to angular velocity of rotation and gives us a clue to the name curl v or rotation v or rot v of the vector del × v. In case of flow of fluid the value of ?? v at a point is the measure of the angular velocity of the fluid in the neighbourhood of the point. When ?? v = 0 everywhere in some region, the velocity field v is called irrotational, or in general for any vector field A, if 0 ? ? ? A in a region of space the field A is called irrotational or conservative field or lamellar vector field. In such vector fields line integral around a closed path is zero. But for non-lamellar vector fields the line integral around a closed path is not zero and curl of a vector field plays an important role.

134 ? NSOU ? CC-PH-04 Fig (4B.12) We consider a vector field A, if we define circulation as the line integral .d ?? A r around a closed plane curve. For flow of water we can take ? ? A v and we can get a physical meaning of the circulation as follows. We consider a closed rectangular path abcd in a vector field A as shown in figure (4B.10). If the plane of this rectangle is perpendicular to the vector field A, then the circulation along this path is zero, since A is perpendicular to every element dr of abcd. If however the plane of the rectangle is made parallel to the direction of A, the line integral along ad and bc will be zero ; but the line integral along ab and cd will have some definite value [fig 4B.11] and therefore circulation will not be zero. Let A be the vector point function having components A x , A y , A z along x, y, z co-ordinate. We consider the plane are dydz. Fig (4B.11). The value of the components of vector A along side of rectangle adcb are (fig. 4B.12) (along ad) : A y (along bc) : y y A A dz z ? ? ? (along ab) : A z (along dc) : z z A A dy y ? ? ? Hence the line integral around the closed path adcda

47% **MATCHING BLOCK 127/267** **SA** M P Vol 1.pdf (D134397037)

is y z z y z y A A A dz A dz dy A dy dz A dy z y ? [

since direction of cd and da are in opposite sense]. In evaluating the line integral we Fig (4B.11) [X-axis being perpendicular to the plane of the paper at a] dy

NSOU ? CC-PH-04 ? 135 must go around the area element dydz as in Fig (4B.11) keeping the area to ours left/right hand rule, y z A A dydz z y ? ? ? ? ? ? ? ? ? ? ? ? Since the path of the integration are either parallel or antiparallel to the vector components, the line integral will have maximum value. Now the area enclosed by the path of integration being dydz, the maximum value of the line integral per unit area is y z A A z y ? ? ? ? ? ? ? ? ? ? ? ? , which according to the definition (4B.10) of curl is scalar component of ?A ? along the unit positive normal to the area i.e. i. ? ? 1 . y z around dydz A A d z y dydz ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? A i A r ? Similarly considering rectangular path whose planes are perpendicular to y and z axes, it can be shown that ? ? 1 . x z around dzdx

A A d z

x dzdx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ??

A j A r ? and ? ? 1 . y x around dxdy

A A d x y dxdy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? A k A r ? ? ?

88%	**MATCHING BLOCK 128/267**	**SA**	M P Vol 1.pdf (D134397037)
y y x x z z A A A A A A z y z x x y ?			

A

i j k ? Therefore ? A ? is related to the maximum value of circulation per unit area and can be written as 0 1 ˆ lim . d around d d d ? ? ? ? ? ? ? ? ?? A n A r ? (4B.18) Where d? is any arbitrary area with positive unit outward normal ˆ n so that when ˆ n = i, d? = dydz etc. Equation (4B.18) can be written as a definition of ?A ? .

136 ? NSOU ? CC-PH-04 Example of Art. 4.20 : Example (1) : If a rigid body rotates about on axis passing through the origin with angular velocity ? and with linear velocity ? ? v r ? , then prove that ? ? 1 2 ? ?v ? ? . Solution 1 : x y x z constant ? ? ? ? ? ? ? ? ? i i j k ? and ? ? v r ? Therefore ( ) ( ) ( ) ? ? ? ? ? ? ? ? v r r r ? ? ? ? ? ? ? ? (i) using equation (4A.21). Since ? is a constant vector 0 ? ? ? ? ; therefore we have used (?? ?) as co- efficient of r in

equation (i) 3 3

68%	**MATCHING BLOCK 129/267**	**SA**	07200140 (2).pdf (D118447009)
x y z x y z x y z x y z ? r r r v r ? ? ? ? ? ? ? ? 1 3 3 2 2 x y z ? ? ? ? ? ? ? ? ? ? ? ? ? i j k			

v ? ? ? ? ? ? Example 2 :

26%	**MATCHING BLOCK 130/267**	**SA**	2021asgn.PG.pdf (D112583081)
Find the constant a, b, c so that the vector, A = (x + 2y + az)i + (bx − 3y − z)j + (4x + cy + 2z)k, is irrotational. Solution 2 : Since A is irrotational, 0 ? ?A ? Or, 0 2 3 4 2 x y z x y az bx y z x cy z ? ? ? ? ? ? ? ? ? ? ? ? ? i j k Or, (c + 1)i + j(a − 4) + k(b − 2) = 0 Therefore, 1 0 1 c c ? ? ? ? ? 4 0 4 a a ? ? ? ? 2 0 2 b			

b ? ? ? ?

NSOU ? CC-PH-04 ? 137 Example 3 : Find the corresponding scalar potential of the vector A in example 2. Solution 3 : Since 0 ? ?A ? and A is defined in a simply connected region, vector field A is said to be conservative and hence it can be expressed as a gradient of a scalar potential, say, ?.

x

28%	**MATCHING BLOCK 131/267**	**SA**	07170523.pdf (D54981551)
y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A i j k ? Or, if A x , A y and A z are the components of the vector A, we can write, x y z A A A x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A i j k i j k (2 4) x A x y z x ?? ? ? ? ? ? ? ? ? ? (i) (2 3) y A x y z y ?? ? ? ? ? ? ? ? ? (ii) (4 2) z A x y			

z
z ?? ? ? ? ? ? ? ? ? (
iii) Integrating equation (i) with respect to x, keeping y, z constant, we get, 2 1 2 4 (,) 2 x yx zx f y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? (iv) Similarly integrating equation (ii), with respect to y; keeping x and z as constant, we get 2 2 3 2 (,) 2 y xy zy f x z ? ? ? ? ? ? ? ? ? ? ? ? ? ? (v) And from equation (iii), we get, ? ? 2 3 4 (,) xz yz

58%	MATCHING BLOCK 133/267	SA	M P Vol 1.pdf (D134397037)
z f x y ? ? ? ? ? ? (vi) 138 ? NSOU ? CC-PH-04 f 1 (y, z), f 2 (x, z) and f 3 (x, y)			

are constants of integration. Now f 1 , f 2 and f 3 are to be suitably chosen in order that function ? were identical in all these three equations. By inspection, we find

46%	MATCHING BLOCK 132/267	W
f 1 (y, z) = − zy 2 2 3 2 ? ? y z 2 2 2 2 2 3 3 (,) 4 2 , (,) 2 2 2 x y f x z zx x z f x y yz ? ? ? ? ? ? 2 2 2 3 2 4 2 2 x y z yx yz xz c ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? where c is a constant		

independent of x, y, z. Example 4 : What do you mean by an exact differential? Show that a necessary and sufficient condition that F 1 dx + F 2 dy + F 3 dz be an exact differential is that 0 ? ?F ? where F = iF 1 + jF 2 + kF 3 . Solution 4 : If P = P(

76%	MATCHING BLOCK 136/267	SA	MMP.pdf (D118446089)
x, y, z); Q = Q(x, y, z); R = R(x, y, z),			

then Pdx + Qdy + Rdz is an exact differential if there exists a function (, ,) x y z ? such that d dx dy dz x y z ? ? ? ? ? ? ? ? ? ? ? ? ? is equal to Pdx + Qdy + Rdz. Pdx + Qdy + Rdz, is an exact differential, if the following conditions hold good : 0 P Q Q R R P y x z y x z ? The necessary condition that F 1 dx + F 2 dy + F 3 dz is an exact differential : Let F 1 dx + F 2 dy + F 3 dz be an exact differential. Then F 1 dx + F 2 dy + F 3 dz = d? dx dy dz

66%	MATCHING BLOCK 134/267	W
x y z ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 3 ; ; F F F x y z ? ? ? ? ? ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 139 1 2 3 F F F x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?		

F i j k i j k ? 0 ? ? ? ? ? ? ? F ? ? ? The sufficient condition that F 1 dx + F 2 dy + F 3 dz is an exact differential : Let 0 ? ?

51%	MATCHING BLOCK 135/267	W
F ? or, 1 2 3 0 x y z F F F ? ? ? ? ? ? ? i j k or, 3 3 2 1 2 1 0 F F F F F F y z z x x y ? i j k or, 3 3 2 1 2 1 0 F F F F F F y z z x x y ? = ? F 1 dx + F 2 dy + F 3		

dz = exact differential also, when 0, F ? ? ? ? ? F ? ? , so . d ? ? ? ? ? F dr dr ? or, (iF 1 + jF 2 + kF 3). (i dx + j dy + k dz) = d? or, F 1 dx + F 2 dy + F 3 dz =d? = an exact differential Exercise of Art 4.21 : 1) If 0 0 0, 0, , , H E t t ? ? ? ? ? ? ? ? ? ? ? ? ? E H E H ?? ?? ? ? where 0 0 ,? ? are constants. Show that E and H satisfy the wave equation 2 2 0 0 2 ? ? ?t ? ? u u? . 2)

87% **MATCHING BLOCK 137/267** **SA** VECTOR ANALYSIS.docx (D118765685)

If A and B are irrotational, show that Ax B is solenoidal. 3)

If u v ?F ? , where u, v are scalar field and F is a vector field, show that 0 ? ? ? F F ? . 4) Prove the following ; (i) If A is irrotational (A × r) is solenoidal, (ii) If B t ? ? ? ? A ? , then show that ?B ? is independent of t.
140 ? NSOU ? CC-PH-04 5) Prove that (i) ( ) 2 ? ? ? A r A ? and (ii) ( ) 2 ? ? ? ? A r A ? ,where r = ix + jy + kz where A is a constant vector. Solution of exercise of Art 4B.12 : Solution (1) : We have 2 ( ) ( ) ? ? ? ? ? E E E ? ? ? ? ? (i) See Art 4B.13 item no (3) Now substituting 0 t ? ? ? ? ? ? H E ? in equation (i) 2 0 0 t ? ? ? ? ? ? ? ? ? ? H E E ? ? ? or, 2 0 ( ) t ? ? ? ? ? H E ? ? or, 2 2 0 0 2 t ? ? ? ? ? E E ? (ii), substituting 0 t ? ? ? ? E H = ? Similarly we can arrive at 2 2 0 0 2 t ? ? ? ? ? H H ? (iii), by replacing E by H, Using common term u for E and H, We get : 2 2 0 0 2 t ? ? ? ? ? u u? Solution 2 :

61% **MATCHING BLOCK 138/267** **SA** PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

We have 0, 0 ? ? ? ? A B ? ? ( ) 0 ? ? ? ? ? ? ? ? ? A B B A A B ? ? ? (See Art 4B.14.2 item no. 12) ? A × B

is solenoidal. Solution 3 : Since 1 v u ?F ? , u and v are scalars, 1 1 1 v v v u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? F ? ? ? ? ? ? ?
NSOU ? CC-PH-04 ? 141 (see Art 4B.14.2 item no. 15) Now 0v ? ? ? ? 1 v u ? ? ? ? ? ? ? ? ? ? F ? ? ? 1 1 0 v v u u ? ? ? ? ? ? ? ? ? ? ? ? ? F F ? ? ? ? Since any two vector in a triple scalar product is equal, implies the product is zero. Solution 4 : i) We have 0 ? ? ?A ? Now ? ? ? ? ? ? ? ? ? ?
A
r
r A A

56% **MATCHING BLOCK 139/267** **SA** mmp ass 2.pdf (D118459575)

r ? ? ? Now x y z x y z ? ? ? ? ? ? ? ? i j k r ? .0 .0 .0 0 z y x z y x y z z x x y ? i j k i j k Since x, y, z

are independent. Therefore ? ? ? ? ? ? ? ? A r r A ? ? ? ? ? and (A × r)
is solenoidal. ii) B t ? ? ? ? A ? taking divergence on both sides ? ? 0 t ? ? ? ? ? ? ? A B ? ? ? since 0 ? ? ?A ? ? therefore ?B ? is independent of t.
142 ? NSOU ? CC-PH-04 Solution 5 : i) ? ? ? ? ? ? 3

24% **MATCHING BLOCK 141/267** **SA** 07170523.pdf (D54981551)

x y z A A A x y z ? A r r A A r A r ? ? ? 3 x y z A A A x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? r r r A 3 3 2 x y z A A A ? ? ? ? ? ? ? ? ? ? ? A i j k A A A where A = iA x + jA y + kA z ii) We have x y z x y z A A A ? ? ? ? ? ? ? ? i j k A ? y z z x x y A A A A A A z y x z y x ? i j k Now, ? ? y z z x x y i j k A A A A A A z y x z y x x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A r ? z x x y x y A A z A A y A A x x z y x y x ? i i j y z y z z x A A z A A y A A x z y z y x z ? j k k = – iA x – iA x – jA y – jA y – kA z – kA z = – 2[

iA x +
jA y + kA z] = – 2
A

NSOU ? CC-PH-04 ? 143 4.22 ? Vector Identities In various application of vector analysis expressions involving ? and scalar or vector functions are involved. We can verify these expressions by writing out components. These verifications become however easier if we treated ? an ordinary vector remembering that it is also a vector differential operator. 1 < Curl (grad ?) = 0 Proof : curl( ) ( )
grad ? ? ? ?? ?

77%	MATCHING BLOCK 140/267	W

x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k 2 2 2 2 2 2 y z z y z x x z x y y x ?

i j k =
i(0) + j(0) + k(0) = 0 2<
div(curl A) = 0 proof : div (curl A) 0 ? ? ? ? ? ? ? ? A A ? ? ? ? interchanging “dot” and “cross” in a triple scalar product and treating ? as a normal vector. 3< curl (curl

16%	MATCHING BLOCK 142/267	SA	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

A) ( ) ? ? ? A ? ? 2 ( ) ? ? ?A A ? ? ? Proof : 2 ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? A A A A A ? ? ? ? ? ? ? ? ? Using the formula for A × (B × C), treating ? as a normal vector and also an operator which differentiate A. Laplacian of a vector, 2 A ? , simply means a vector whose components are 2 2 2 , , x y z A A A ? ? ? 144 ? NSOU ? CC-PH-04 4< , ? ? ? ? ? ? ? ? ? ? A A A where ? is a scalar.. Proof : ? ? ? ? A ? ? ? ? ? ? ? ? ? ? ? ? A A A

Where the subscript on ? indicate which function is to be differentiated i.e. ? ? will diferentiate ? , keeping A constant. Now ? ? ? ? ? ? ? ? ? ? ? ? ? A A A , since ? is a scalar, we can put it on either side of dot. On the last step we have removed the subscript since A no longer appear after. Again ? ? ? ? ? ? ? ? A A since ? is a scalar and is not differentiated, it may be treated as a constant. Therefore, collecting all the terms we have : ( ) ? ? ? ? ? ? ? ? ? ? ?
A

80%	MATCHING BLOCK 143/267	SA	mmp ass 2.pdf (D118459575)

A A 5< grad (.) (.) (.) (.) ( ) ( ) ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B A B A B B A A B B A

Pooof : consisting only the x-component ? ? (.)
x
x

15% **MATCHING BLOCK 144/267** **SA** mmp ass 2.pdf (D118459575)

x y y z z grad A B A B A B x ? ? ? ? ? A B y y x x z z x x y y z z B A B A B A A B A B A B x x x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? x x x x x x x x x y z y z x y z B B B B A A A A A A A A B B B x y z y z x y z ? y y x x z z y z y y z z B A A A B A B B A B A B y z x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? . y y x x x z x x y y z B A B A B B B A A B A x y x y z x ? A B x z z A A B z x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? . x x x x B A curl curl ? ? ? ? ? ? ? ? ? A B A B B A Hence, considering y and z components of L.H.S and adding them all, we get NSOU ? CC-PH-04 ? 145 (.) (.) (.) ( ) ( ) ? ? ? ? ? ? ? ? ? A B A B B A A curl B B curl A Now we find out ( ) ( ) ( ) x y z z y A A ? ? ? A curl B curl B curl B y x x z y z B B B B A A x y z x ? Similarly, (B × curl A) x = B y (curl A) z – B z (curl A) y y x x z y z A A A A B B x y z x ? 6< ( ) div ? ? ? ? ? ? ? A B B A A B ? ? Proof : ( ) ( ) div x j z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B i j k A B ? ? ? ? ? ? y z z y z x x z x y y x A B A B A B A B A B A B x y z ? i j k i j k ? ? ? ? ? ? y z z y z x x z x y y x A B A B A B A B A B A B x y z ? ? ? ? ? ? ? ? ? ? ? ? y y x x z z x y z A A A A A A B B B y z z x x y ? y y x x z z x y z B B B B B B A A A y z z x x y ? B A A B ? ? 7< ( ) ( ) ( ) curl ? ? ? ? ? ? ? ? ? A B A B B A B A A B ? ? ? ? = A div B – B div A + B grad A – A grad B 146 ? NSOU ? CC-PH-04 Proof : ( ) y z z y z x x z x y y x curl x y z A B A B A B A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? i j k A B Considering the x-component only ? ? ? ? ( ) x x y y x z x x z curl A B A B A B A B y z ? ? ? ? ? ? ? ? ? A B Adding and subtracting ? ? x x A B x ? ? , we get ( ) x curl ?A B ? ? ? ? ? ? x x x y x z A B A B A B x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? x x y x z x A B A B A B x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? y y x x z z x x y z B A B A B A A B B B x y z x y z ? y y x x z z x x y z A B A B A B B A A A x y z x y z ? . . . . x x x x A A B B ? ? ? ? B B A A ? ? ? ? Considering components in y and z direction and adding. We get curl(A × B) ? ? ? ? ? ? ? ? . . x y z x y z A A A B B B ? ? ? ? ? ? B i j k A i j k ? ? ? ? ? ? ? ? ? ? . . x y z x y z A

57% **MATCHING BLOCK 145/267** **SA** Mathematical Physics - I SLM full.pdf (D113782471)

A A B B B ? ? ? ? ? ? i j k B i j k A ? ? ? ? ? ? ? ? ? ? ( ) . . . . ? ? ? ? ? ? ? A B B A A B A B

B
A ? ? ? ? ?
NSOU ? CC-PH-04 ? 147 8< ( ) , curl curl grad ? ? ? ? ? ? ?

19% **MATCHING BLOCK 146/267** **SA** mmp assignment 2.pdf (D118439176)

A A A is a scalar Proof : ( ) curl ? ? ? ? ? ? ? ? ? ? A A A A ? ? ? ( ) ( ) ( ) x y z A A A x y z ? i j k i j k ( ) ( ) ( ) ( ) ( ) ( ) y y x x z z A A A A A A y z z x x y ? i j k Considering only x-components ( ) ( ) ( ) y z x z y A A curl A A y z y z ? A i Similarly ( ) ( ) ( ) x z y x z A A curl A A z x z x ? A j And ( ) ( ) ( ) y x z y x A A curl A A x y x y ? A k Adding we get, ( ) ( ) ( ) x y z curl curl curl curl ? ? ? ? ? ? ? A i A j A k A ? ? ? ? ? ? ? A

A ? ? ? ? ? ? ? ? ? A
A ? ? 4.23 ? List of Vector Relations 4B.14 We list below some useful vector relations and vector equations which are frequently used in many areas of physics. Students should carefully go through these relations for their benefit. We have grouped the vector identities into two categories – one involving 'del' operator and the other not involving it. Similarly we grouped the vector equations in physics into two categories – one involving Laplacian and the other not. The symbols in equations have usual meaning.
148 ? NSOU ? CC-PH-04 4.23.1 : Vector relations not involving 'del' operator 1.

35%	MATCHING BLOCK 147/267	SA	mmp assignment 2.pdf (D118439176)

A × (B + C) = A × B + A × C 2. (B + C) × A = B × A + C × A 3. A × B.C = A.B × C 4. A × (B × C) = B(A.C) − C(A.B) 5. x y z x y z A A A B B B ? ? i j k A B 6. x y z x y z x y z A A A B B B C C C ? ? ? A B C 4.23.2. :

Vector relations involving 'del' operator 1. 3 ? ?

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r? 2. 0 ? ?r ? 3. 1 2 ˆ n n n r nr nr ? ? ? ?r r ? 4. 2 2 (1) n n r n n r ? ? ? ? 5. 2 1 0 r ? ? ? ? ? ? ? ? ? 6. ( ) m n m n ? ? ? ? ? ? ? ? ? ? 7. ( )?? ? ? ? ? ? ? ? ? ? ? 8. ( ) ( ) ( ) m n m n ? ? ? ? ? ?

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A B A B ? ? ? 9. ( ) ( ) ? ? ? ? ? ? ? ? A A A ? ? ? 10. ( ) ( ) ( ) m n m n ? ? ? ? ? ? A B A B ? ? ? 11. ( ) ( ) ( ) ? ? ? ? ? ? ? ? A A A ? ? ? 12. ( ) ( ) ( ) ? ? ? ? ? ? ? ? A B B A A B ? ? ? NSOU ? CC-PH-04 ? 149 13. ( ) ( ) ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? ? ? A B A B B A A B B A ? ? ? ? ? 14. 2 2 2 2 2 2 2 ( ) x y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 15. ( ) 0 ? ? ? ? ? 16. ( ) 0 ? ? ?A ? ? 17. 2 ( ) ( ) ? ? ? ? ? A A

A ? ? ? ? ? 4.23.3. :

Vector equation of variation branches of physics 1. Lorentz forces : F = q[E + v × B] 2. Maxwell's field equation (in vacuum) 0; t ? ? ? ? ? ? ? B D E ? ? 0; t ? ? ? ? ? ? ? D B H ? ? Where 0 0 and ? ? ? ? D E B H In vacuum 0 0 and ? ? ?J 3. Equation of continuity : ( ) 0 t ? ? ? ? ? ? ? ? v 4.23.4. : Vector equation involving Laplacian 1. Poisson's equation 2 0 ? ? ? ? ? ? 2. Laplace's equation : 2 0 ? ? ? 3. Wave equation : 2 2 2 2 1 c t ? ? ? ? ? ? 4. Diffusion (or heat conduction) equation : 2 1 h t ? ? ? ? ? ? Keywords : Gradient of a scalar function, divergence and curl of vector function, directional and normal derivatives, integral forms of divergence and curl.

150 ? NSOU ? CC-PH-04 4.24 ? Summary - II ? Constancy of direction and magnitude of vector function have been discussed with respect to derivatives. ? Derivatives of triple scalar and vector products have been discussed. ? Velocity, acceleration and relative velocity of a particle in terms of time derivatives have been exemplified. ? Gradient, divergence and curl have been defined, obtained their geometrical meanings. ? Physical meaning of directional derivative has been discussed. 4.25 ?? Vector Integration There are plenty of uses of integration in physics. Relevant integrals are set up to represent physical quantities such as volume, mass, moment of inertia etc. and then evaluated by suitable methods. The basic idea behind setting up of the evaluating is that an integral is the limit of a sum. Objective This unit deals with definition of multiple integrals with special reference to double and triple integrals with examples. Algebraic method suitable to find the elements of area, volume etc. in different co-ordinate systems is developed. Line, Surface and volume integrals of vector fields are discussed. Applications of Gauss's divergence theorem, Stoke's theorem and Green's theorem in a plane are discussed with examples. 4.26 ? Double and Triple Integral In case of single variable x, we define the definite integral ( ) b a f x dx ? as the limit of the sum of the areas of the rectangle as shown in the fig (4C.1) and use ( ) b a f x dx ? to calculate the area of the curve : 0 1 ( ) lim ( ), n b a h r b a f x dx h f a rh h n ? ? ? ? ? ? ? ? Fig (4C.1) NSOU ? CC-PH-04 ? 151 (4C.1) From the geometry of fig (4C.1) it is apparent that the sum of the areas of the rectangles will tend to the area under the curve in the limit n ? ?, Where 1 0 ( ) lim ( ) b n r r a n r f x dx f x r ? ? ? ? ?? ? ? ? ? , and ?r is the width of the rectangle. (4C.2) We define the double integral of f(x,y) over the area A is the (x, y) plane. [Fig 4C.2] as a limit of the sum and write it as, ? ? 1 0 (,) lim , r n r r r n r A A f x y dA f x y A ? ? ?? ? ? ? ? ? ?? (4C.3) Where the elementary area dA can be chosen accordingly. In Cartesian co-ordinate elementary area r r r A x y ? ? ? ? 0 0 lim x y dA x y dxdy ? ? ? ? ? ? ? ? ? . In polar co-ordinate elementary area . A r r r r ? ?? ? ??? ? ? 0 0 lim r dA r r rd dr ? ?? ??? ? ? ? ? ? ? ? In fig 4C.2 we have divided the (x, y) plane into little rectangles of area r r r A x y ? ? ? ? . Above each r r x y ? ? is a box reaching up to the surface. We can approximate the volume of this cylinder by a sum of those boxes as represented by the double integral (4C.3). Fig. (4C. 2) Fig. (4C.3)

152 ? NSOU ? CC-PH-04 Multiple integrals are usually evaluated by using interated (repeated) integrals. A triple integral of f(x, y, z) over a volume V, written (, ,) , V f x y z dxdydz ??? , is also defined as the limit of a sum and is evaluated by an iterated integral. We consider a function f(x, y, z) to be defined at every point in a region bounded by a volume V. If (x r , y r , z r) be any point in the r th element of volume r V? , then the limit of the sum ? ? 1 , , r r r r r f x y z V? ? ? ? If it exits as r ? ? and 0 r V? ? then, ? ? 1 0 lim , , r n r r r
r r r V

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f x y z V ? ? ?? ? ? ? (, ,) (, ,) V V f x y z dV f x y z

dx dy dz ? ? ??? ??? (4C.4) is known as the triple integral of f(x, y, z) over the volume V. Now volume element in Cartesian co-ordinate system is dV = dxdydz but in cylindrical and spherical co-ordinate systems are respectively dV = r drd?dz and 2 sin dV r d d dr ? ? ? ? , we summarize below expression for line element, surface element and volume element in different co-ordinate systems. Cylindrical co-ordinates (r, ?, z) : cos sin x r y r z z ? ? ? ? ? ? ? ? ? ? (transformation equations from rectangular to cyliderical co-ordinates) dV = r drd?dz} element of volume. ? 2 2 2 2 2 ( ) ( ) ( ) dS dr r d dz ? ? ? ? line element ? dA rd dz? ? surface element Spherical co-ordinate ? ? r,θ,? : sin cos sin sin cos x r y r z r ? ? ? ? ? ? ? ? ? ? ? ? ? ? transformation equations from rectangular to spherical co-ordinates X
NSOU ? CC-PH-04 ? 153 Fig. (4C.4) ? ? 2 sin dV r d d dr ? ? ? ? volume element ? 2 2 2 2 2 2 2 ( ) ( ) sin ( ) dS dr r d r d ? ? ? ? ? ? line element ? 2 sin dA r d d ? ? ? ? surface element. 4.26.1 : Examples of double integration Problem 1: Evaluated A xy dx dy ?? , where A

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is the domain bounded by X-axis, ordinate x = 2a and the curve x 2 = 4ay.

Solution 1: Point of intersection Q [fig 4C.4] is given by (2a,a). The domain of intersection is OPQ. 2 4 2 0 0 x a a A xydxdy xy dxdy ? ? ?? ? ? First method : Here first we integrate with respect to y, treating x as constant between the limits 0 to 2 4 x a . The limits of integrations are x = 0 to 2a and y = 0 to 2 4

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x a . 2 2 4 2 4 2 2 0 0 0 0 2 x a x a a a A y xy dxdy x ydy dx x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? 2 5 6 2 2 5 2 2 2 0 0 0 1 1 6 32 32 32 a a a x x dx x dx a a a ? ? ? ? ? ? ? ? ? ? ? ? ? 6 4 6 2 2 1 64 (2) 0 3 192 192 a a a a

a ? ? ? ? ? ? ? ?
154 ?
NSOU ? CC-PH-04 Second method : We now integrated w.r.t. x first, treating y as constant between the limits 4ay to 2a [fig 4C.5]. The limits of integration are y = 0 to a and 4 x ay ?
to 2

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a. 2 0 4 a a ay A xy dxdy y x dx dy ? ? ? ? ? ? ? ? ?? ? ? 2 2 0 4 2 a a ay x y dy ? ? ? ? ? ? ? ? ? ? ? ? 4 2 2 0 0 (2 2) 2 3 a a a y a ay dy a ay y

dy ? ? ? ? ? ? ? Problem 2 :
Evaluate 2 2 0 0 ( )
x

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x y x x y dx dy ? ? ? ? ? Solution 2 : 2 2 2 2 3 2 3 0 0 0 0 ( ) 2 x x x y x x y dx x x y dy dx x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 2 4 0 48 2 5 x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ?

Problem 3 : Find by double integration, the area inside the circle sin r a ? ? and the cardioid (1 cos) r a ? ? ? . Solution 3 : Points of intersection of the circle : sin r a ? ? and the cardioid : (1 cos) r a ? ? ? are (0, 0) and , 2 a ? ? ? ? ? ? ? ? since when , ; 0, 0 2 r a r ? ? ? ? ? ? ? ? ? required area within the circle and cardioid is : Fig. (4C.5)
NSOU ? CC-PH-04 ? 155 sin sin 2 2 0 (1 cos) 0 (1 cos) r a r

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a r a r a r d dr d r dr ? sin 2 2 0 (1 cos) 1 2 a a d r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 0 1 sin (1 cos) 2 d a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 0 sin 1 cos 2 a d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 0 sin 1 2cos cos 2 a d ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 0 1 2cos sin cos 2 a

d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 0 2cos 2cos 2 ? ? ? ? ? ? ? ? a d ? ? ? ? ? ? ? 2 2 2 0 cos cos ? ? ? a d ? ? ? ? ? ? ? 2 1 2 a ? ? ?
Problem 4 : Find the volume

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bounded by the cylinder x 2 + y 2 = 4 and the planes y+ z = 4 and z = 0 Solution 4 :

From fig (4C.7) it is evident that the required volume V, A V z dx dy ? ?? , where A is the circle x 2 + y 2 = 4 in XOY plane. (4) V y dx dy ? ? ? ?? Or Fig. (4C.6)
156 ? NSOU ? CC-PH-04 2 2 4 2 2 4
x

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y y y x y V dy dx ? ? ?? ?? ?? ? ? ? ? ? 2 2 4 2 0 2 (4) y x y y x dy y dx ?? ? ? ?? ? ? ? ? ? ? ? ? 2 2 4 0 2 2 4 y y y dy x yx ?? ? ?? ? ? ? 2 2 2 2 2 4 4 4 y y dy y y y ?? ?? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 4 4 2 4 y y y y y dy y y dy ?? ?? ?? ?? ? ? ? ? ? ? ? = 16? units Now to evaluate 2 2 2 2 4 4 y y y

dy ?? ?? ? ? we put 2 2cos 4cos y y ? ? ? ? ? and dy 2sin d? ? ? ? when 2,cos 1 0 y ? ? ? ? ? ? and when y 2,cos 1 ? ? ? ? ? and ? ?? ? the given integral 0 0 2 2 8sin (2sin) 32 sin d d ? ? ? ? ? ? ? ? ? ? ? ? 0 16 (1 cos 2) 16 d ? ? ? ? ? ? ? ? similarly it can be shown, 2 2 2 2 4 4 0 y y y dy ?? ?? ? ? ? 4.26.2 Change of order of integration In case of double integration we have two method of evaluting the double integration by using iterated integrals. It is often seen that one of the methods we use is more Fig. (4C.7)
NSOU ? CC-PH-04 ? 157 Fig. (4C.8) Fig. (4C.9) convenient; we choose the easier method. It is common experience that if we change the order, the corresponding limits of the variables are to be changed. Example 1 : Change the order of integration in the integral : 2 1 0 (,) x y x x f x y dy dx ? ? ? ? Solution 1 : In the given integral, integration with respect to y should be given first preference. So, the strip AB parallel to y- axis with thickness dx is considered. Finally the strip AB moves from x = 0 to x = 1 and the area is obtained. While changing the order, let us consider the strips CD with thickness dy, parallel to the x-axis is considered. Now C is on x = y 2 and D is on x y ? and y moves from 0 to 1, we write the above integral as : 2 1 0 (,) x y y y x y dy f x y dx ? ? ? ? ? ? Example 2 : Evaluated the following integral by changing the order of integration : 2 2 0 x a a a x xy dx dy ? ? ? Solution 2 : In the given order, first we integrate with respect to y, from 2 x y a ? to y = 2a – x along the vertical strip MN; then the strip MN moves parallel to x-axis from x = 0 to x = a as shown in fig (4C.9). When the order of integration is reversed, we have to consider the total area OPQ as a sum of two similar areas PQR and POR.

158 ? NSOU ? CC-PH-04 On the area OPR, we consider the strip ST parallel to x-axis and integrated with respect to x from x = 0 to x ay ? and ST slides from y = 0 to y = a. Again on the area RPQ, we consider the strips UV and integrated from x = 0 to x = 2a – y and finally the strips slides from
y =
a to y = 2
a. 2 2 0 0 0 0 a

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a ay a y x x y y I xy dx dy xy dx dy ? 2 2 2 2 0 0 0 2 2 a y ay y a a y y a x y x y dy dy ? 2 2 2 0 1 1 (2) 2 2 y a a y y a ay dy a y y dy ? ? ? ? ? ? ? ? ? 2 3 3 4 2 2 4 0 1 4 3 2 2 3 2 3 4 8 a a a a

y ay y a
y
a ? 4.26.3 Examples of triple integral 1. Volume as triple integral : The volume of a solid is obtained by evaluating the triple integral fig (4C.10), V V x y z ? ? ? ? ? ? ? ??? ??? If ? is the density of the solid, it's mass, M x y z ? ? ? ? ? ? ??? 2. Volume as volume of revolution : We consider an element of area x y ? ? on a plane area A. The revolution of the element about X-axis will generate a ring of volume (4C.11) : Fig. (4C.10)
NSOU ? CC-PH-04 ? 159 Fig. (4C.11) 2 2 ( ) V

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y y y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 y y y y y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2y y y x ? ? ? ? ? ? ? ? ? ? ? ? ?

Since y?
is small, ? ? 2 y? is smaller,, and we can neglect it in comparison to 2y y? . 2 V y y x ? ? ? ? ? ? therefore the whole volume generated by the entire area about the X-axis is 1 2 A V y dy dx ? ? ?? Similarly, V? about Y-axis is ? ? 2 2 V x x x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 , x x y ? ? ? ? neglecting ? ? 2 x? 2 2 A V x dx dy ? ? ?? 4.27 ? Change of Variables : Jacobian It is convenient to develop an algebraic method suitable to find the elements of area, volume etc. in different co-ordinate systems and also at the same time for any change of variables in a multiples integral. In two dimensions suppose x and y are given functions of two new variables u and v by the transformation equations. x = x(u, v) & y = y(u, v) The Jacobian of (x, y) with respect to (u, v) is the determinant in equation (4C.5)
160 ? NSOU ? CC-PH-04 , (,) , (,) x x x y x y u v J J y y u v u v u v ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (4C.5) Then the area element dy dx is replaced in (u, v) system by the area element dA = |J | du dv (4C.6) where |J | is the absolute value of the Jacobian given in equation (4C.5). In three dimensions, suppose =

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x(u, v, w); y = y(u, v, w) and z = z(u, v, w)

then the Jacobian of transformation from (x, y, z) to (

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u, v, w) is given by (, ,) (, ,) x x x u v w x y z y y y J u v w u v w z

z z u v w ? (4C.7) Then the volume element dx dy dz is replaced in (u, v, w) system by the volume element dV = | J | du dv dw (4C.8) Evaluation of double and triple integral becomes easier by change of variables. Two important formulae are listed below for the purpose. 1. Double integral : ? ? ( ) (,), (,) | | xy uv R R f xy dx dy f x u v y u v J du dv ? ?? ?? (4C.9) R xy and R uv are symbols of region in xy-plane and uv-plane respectively. Where (,) (,) x y J u v ? ? ? 2. Triple integral : (, ,) xyz R f x y z dx dy dz ???
NSOU ? CC-PH-04 ? 161 ? ? (, ,), (, ,), (, ,) uvw R f

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x u v w y u v w z u v w J du dv dw ? ??? (4C.10) where (, ,) (, ,) x y z J u v w ? ? ?			

Special cases : A< Cartesian to polar co-ordinate system : (,) (,) x y r ? ? Transformation equations are

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cos , sin x r y r ? ? ? ? ? 2 2 cos sin (,) cos sin sin cos (,) x x r x y r J r r y y r r r ? 2 2 cos sin r r ? ? ? ? ?		

Therefore, element of area dA dx dy J dr d r dr d ? ? ? ? ? And (,) (

76%	**MATCHING BLOCK 163/267**	**W**
cos , sin) xy r R R f x y dx dy f r r J dr d ? ? ? ? ? ? ?? ?? (		

cos , sin) r R f r r r dr d ? ? ? ? ? ? ?? B< Cartesian to cylindrical co-ordinate system : ? ? , , (, ,) x y z r z? ? Transformation equations are

46%	**MATCHING BLOCK 164/267**	**W**
cos , sin , x r y r z z ? ? ? ? ? cos sin 0 (, ,) sin cos 0 (, ,) 0 0 1 x x x r z r x y z y y y J r r z r		

z
z z z r
z ?
162 ?
NSOU ? CC-PH-04 Or, J = r Therefore (, ,) (cos , sin ,) xyz r z R R f x y z dx dy dz f r r z J dr d dz ? ? ? ? ? ??? ??? (cos , sin ,) r z R f r r z r dr d dz ? ? ? ? ? ? ??? C< Cartesian to spherical polar co-ordinates : ? ? , , (, ,)
x y z r ? ? ? Transformation equations are

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sin cos x r ? ? ? sin sin y r ? ? ? cos z r ? ? ? ? sin cos cos cos sin sin , , sin sin cos sin sin cos (, ,) cos sin 0 x x x r r r x y z y y y J r r r r r z z z r ? 2 2 2 2 2 2 2 sin sin (sin cos) cos sin cos r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 sin			

r ? ? ? Volume element 2 sin dV dxdydz J drd d r dr d d ? ? ? ? ? ? ? ? Exercise of 4.26 and 4.27 : 1) Change the order of the integration in 0 y x e dxdy y ? ? ? ? ? and hence find its value. 2) Evaluate the integral ? ? 2 2 2 V x y z dx dy dz ? ? ??? where V is the volume of the sphere with center at the origin and radius R.

NSOU ? CC-PH-04 ? 163 Figure : Solution (1) 3) Evaluate 2 ( ) R x y dx dy ? ?? , where R is the parallelogram in the xy – plane with vertices (1, 0), (3, 1), (2, 2) and (0,1) using the transformation, u = x + y, v = x – 2y. 4) Transform the integral 1 1 1 x x V dx dy dz ? ? ? ? by the substitution i.e. u = 1 + x and v = xy 5) Given the transformation x = u 2 – v 2 , y = 2uv(a) compute its Jacobian; (b) Evaluate

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y u x ? ? ? ? ? ? ? ? and y v x ? ? ? ? ? ? ? ? . Solution : Solution 1 : The region of integration is bounded by y = x, x = 0

and infinity boundary. We take a strip parallel to x – axis to change the order. The extremities of the strips lies on x = 0 and y = x. Therefore limits of x are from x = 0 to x = y and the limits of y are from

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y = 0 to y = ? 0 0 0 0 y y y y e e dx dy dy dx y y ? ? ? ? ? ? ? ?? ?? ? ? 0 0 0 0 1 y y y y

e x dy e dy e y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Solution 2 : Using

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spherical polar co-ordinate sin cos , sin sin , cos x r y r z r ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 4

sin sin x y z dx dy dz r r d d dr r d d dr ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 4 0 0 0 sin R V r x y z dx dy dz r d d dr ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ??? ? ? ?
164 ? NSOU ? CC-PH-04 2 2 5 4 0 0 0 0 0 sin

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sin 5 R R d d r dr d d ? 5 5 0 2 2 cos [cos cos0] 5 5 R R ? ? ? ? ? ? ? ? ? ? ? ? 5 5 5 2 2 4 [1 cos] [1 (1)] 5 5 5 R R R ? ? ? ? ? ? ? ? ? ? ?

Solution 3 : The region R, i.e. the parallelogram ABCD in xy – plane because they became the region R / , i.e. the rectangle A´B´C´D´ in the uv – plane. (Figure solution 3) Where u = x + y, v = x – 2y 1 1 (2), ( ) 3 3 x u v y u v ? ? ? ? ? Now Jacobian of transformation, (,) 1 (,) 3 x y x y u u

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x y u v v v ? ? ? ? ? ? ? ? ? ? ? ? ? ? the given integral is, 4 1 4 2 2 2 1 2 1 2 1 1 1 [ ] 3 3 u v u R u J du dv u du dv u du v ? ? ?? ? ? ? ? ?? ? ? ? v NSOU ? CC-PH-04 ? 165 4 4 2 2 1 1 1 .3 3 u u u du u du ? ? ? ? ? ? 4 3 3 3 1 4 1 1 63 (64 1) 21 3 3 3 3 3 u ? ? ? ? ? ? ? ? ? ? ? ? ? ? Solution 4 : u = 1 + x; v = xy 1, 1 v x u y u ? ? ? ? ? ? 2 1 (,) 1 (1) (,) 1 1 0 1 v x y x y u u u jacobian x y u v u v v u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

so, 1 1 dx dy J du dv du dv u ? ? ? Now, the limits of y are x to 1 x also v = xy, so v varies from x 2 to 1 i.e. (u – 1) 2 to 1. Limits of x varies from – 1 to + 1. So, limits is u varies from 0 to 2. Hence 1 2 1 2 1 1 0 (1) 1 1 x
u v x u v u V dx dy V du dv u ? ? ? ? ? ? ? ? ? ? ? ? ? ? where V´ is the function V changed in u and v. Solution 5 : a) The Jacobian J(u, v) of u, v with respect to x, y is, 2 2 2 2 (,) 4( ) 2 2 x x

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u v u v J u v u v y y v u u v ? ? ? ? ? ? ? ? ? ? ? ? ? 166 ? NSOU ? CC-PH-04 b) We have u 2 − v 2 = x and 2uv = y ? ? ? ? ? 2 2 2 2 2 2 2 2 2 2 4 u v u v u

v

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x y ? ? ? ? ? ? ? ? ? ? 2 2 2 1 2 u x x y ? ? ? ? Differentiating partially with respect to x 2 2 2 2 2 2 2 2 2 1 1 2 1 2 2 y x x y u x u u x x y x y x y ? 2 2 2 y u u x x y ? ? ? ? ? ? ? ? ? ? ? Again ? ? 2 2 2 1 2 v x y x ? ? ? 2 2 2 2 2 1 2 1 2 y u x v v x u v x y ? 2 2 2( ) y

v v x u v ? ? ? ? ? ? ? ? ? ? ? ? ? 4.28 ?
Ordinary integrals of vector Vectors which are functions of single variable, are integrated in the same way as scalar. Thus if V(t) = iV x (t) + jV y (t) + kV z (t), then ( ) ( ) ( ) ( ) x y z t dt V t dt V t dt V t dt ? ? ? ? ? ? ? V i j k (4C.11) (4C.11) is indefinite integral of V(t) However if ( ) ( ) d t t dt ? V R then,
NSOU ? CC-PH-04 ? 167 Fig. (4C.13) ( ) ( ) t dt t ? ? ? V R C , where C is a constant vector. The definite integral of V(t) between the limits

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t = T 0 to t = T, is given by, ? ? 0 0 0 0 ( ) ( ) ( ) ( ) ( ) T T T T T T d t dt t dt t t T

dt ? ? ? ? ? ? ? V R R C R R (4C.12) 4.28.1 Line integral of a vector field We consider a vector field with field vector V. We draw a continuous curve C in the field (Fig. 4C.12) where V is defined at every point on it. Let us choose (n−1) points C i , which divide the curve C into n segments ?C i and form the sum 1 0 . ? ? ? n i i i ? V C (4C.13), where V i is the value of V at C i and ?C i is the vector whose rectangular components are i?x i , j?y i , k?z i and which joins C i−1 and C i . Now taking the limit of the sum (4C.13), e.g. 1 0 0 lim i n i i i i C? ? ? ? ? ?? ? ? ? V C (4C.13) and if then the sum approaches a definite limit, then this limit is defined as the line integral . C d ? V r along the curve C. i.e. 1 0 0 lim . i n i i i i C C d ? ? ? ? ?? ? ? ? ? ? ? V C V r (4C.14) In general the value of the line integral depends upon V and the path C joining the end points C 0 and C. We note the followings : 1. If V(r) be the force acting on a particle which moves along the curve from C 0 to C, then the line integral (4C.14) represents the work done by the force. If this line integral along a closed path . C ?? V dr is zero, then V(r) is called a conservative force field and in that case V(r) is also called irrotational. 2. If V(r) is a conservative force filed, then the line integral (4C.14) does not depend on the choice of path connecting any two points on the curve i.e. it becomes
168 ? NSOU ? CC-PH-04 independent of path, between any two points on the curve and in that case V(r) can be expressed as a gradient of a scalar, called potential function (say ?) of the force field i.e. ( ) ? ? ? V r ? 3. If V(r) represents the velocity of fluid flow, then the integral is called the circulation of V along the closed curve C. When the circulation of V(r) along a closed curve is zero the V(r) is called irrotational. Essentially line integral is an integration along a curve and there is only one independent variable. Therefore to evaluate a line integral we have to transform the integrand in terms of a single variable using the equation of the curve along which integration is required to be evaluated. Example Art of 4.28.1 : Example 1 : Prove that if the closed line integral of a field vector A vanishes, then it is the gradient of a field scalar . Solution 1 : Let P 1 and P 2 be two points connected by any two curves P 1 Q P 2 and P 1 RP 2 , forming a closed curve P 1 QP 2 RP 1 . Now 2 1 . . 0 P P C d ? ? ? ? ? Adr A r Or, 1 2 2 1 . . . 0 C ? ? ? ? ? ? ? ? PQP P RP Adr Adr Adr Or, 1 2 2 1 1 2 . . . ? ? ? ? ? ? PQP P RP P RP Adr Adr Adr (i) From (i) we see that line integral of a vector A is independent of path connecting two points P 1 and P 2 and depends only on the co-ordinates of ends points P 1 and P 2 therefore 2 2 1 1 . p p p p ? ? ? ? ? Adr (ii) where ? is some scalar field. . d? ? ? Adr . If P 1 and P 2 are two very close points in the field. Then . ? ? ? ? Adr dr Fig. Example (1)

NSOU ? CC-PH-04 ? 169 Fig. Example (3) or, ? ? 0 A ? ?? ? ?dr which is true for any dr. Therefore ? ? ? A (iii) Example 2 : If F = (5xy – 6x 2)i + (2y – 4x)j, then calculate the line integral . C ?? F dr along the curve C in the xy plane given by y = x 3 from the point (1, 1) to (2,8) Solution 2 : F.dr = (5xy – 6x 2)dx + (2y – 4x)dy We convert F.dr in terms of x only by substituting
y =

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x 3 , dy = 3x 2 dx ? F.dr = (5x.x 3 – 6x 2)dx + (2x 3 – 4x)3x 2 dx = (5x 4 – 6x 2)dx + (6x 5 – 12x 3)dx ? ? 2 2 5 4 3 2 6 5 4 3 1 1 . 6 5 12 6 3 2 x C x x x x x dx x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?		

F
dr = (64 – 1) + (32 – 1) – 3(16 – 1) – 2(8 – 1) = 65 + 31 – 45 – 14 = 35 Example 3 : If ? ? ?A ? , where ? is some scalar functions of position, show that the line integral of A along a curve C linking two points A and B is independent of the choice of the curve C. Solution 3 : We consider a close line integral along APBQA of the vector A, i.e. . APBQA ?? Adr Therefore . . 0 APBQA APBQA APBQA d ? ? ? ? ? ? ? ? ? ? ? Adr dr ? (i) . . 0 B A A B P Q ? ? ? ? ? Adr Adr Or, . . . B A B A B A P Q Q ? ? ? ? ? ? Adr Adr Adr (ii) For equation (ii) we see that line integral is independent of path connecting A and B. 170 ? NSOU ? CC-PH-04 Example 4 : Calculate the work done when a force F = 3xyi – y 2 j moves a particle in the xy – plane from (0, 0) to (1,2) along the parabola y = 2x 2 Solution 4 : F = 3xyi – y 2 j = 3x(2x 2)i – (2x 2) 2 j = 6x 3 j – 4x 4 j now, r = xi + yj or, dr = dxi + dyj = dxi + 4xdxj 1 1 3 5 4 6 0 0 3 8 7 . (6 16) 2 2 6 C x dx x dx x x unit ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? F dr Example 5 : A particle of constant mass m is moving in a conservative force field ? ? ? F ? . If A and ? be two points in space, prove that 2 2 1 1 ( ) ( ) 2 2 A B A mv B mv ? ? ? ? ? , where v A and v B are the magnitudes of velocities of the particle at A and B respectively. Solution 5 : The work done by the force F is 2 2 . d r d dr dr dw m m dt dt dt dt dt ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? F dr dr 2 2 1 1 ( ) 2 2 d dr m dt m d v dt dt ? ? ? ? ? ? ? ? ? where dr dt ?v ? total work done, ? ? 2 2 2 1 1 ( ) 2 2 B B A A W m d v m v v ? ? ? ? (i) but ? ? ? F ? . . B B B A B A A A W dr d ? ? ? ? ? ? ? ? ? ? ? ? ? F dr ? (ii) from (i) and (ii) : 2 2 1 1 2 2 B A A B mv mv ? ? ? ? ?
NSOU ? CC-PH-04 ? 171 or, 2 2 1 1 2 2 B B A A mv mv ? ? ? ? ? where A ? and 2 1 2 A mv are respectively potential energy and kinetic energy of the particle at A. Thus total energy at A and B are equal (conservation of energy). This is known as work-energy theorem in mechanics. Exercise of Art 4.28.1 : 1) If F = (3x 2 + 6y)i – 14yzj + 20 xz 2 k. Evaluate .? F dr along the straight line from (0,0,0) to (1,1,1). 2) Find the work done in going around a unit circle in the xy plane, (i) counter clockwise from 0 to ? (ii) clockwise from 0 to – ? against a force field given by, 2 2 2 2 y x F x y x y ? ? ? ? ? ? i j Solution : Solution 1 : We take the parameter t such that x = t, y = t, z = t varies from 0 to 1. Now F.dr = (3x 2 + 6y)dx – 14yz dy + 20xz 2 dz = (3

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t 2 + 6t)dt – 14t 2 dt + 20t 3 dt = (3t 2 + 6t – 14t 2 + 20t 3)dt ? ? ? ? 1 1 2 2 3 2 3 0 0 13 . 3 6 14 20 11 6 20 3 C t t t t t t dt t t t			

dt ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? F dr Solution 2 : We have 2 2 y x x y ? ? ? ? i j F Now for unit circle x 2 + y 2 = 1 2 2 y x y x x y ? ? ? ? ? ? ? ? i j F i j
Now dr = idx + jdy
172 ? NSOU ? CC-PH-04 ? F.dr = – ydx + x dy, also cos , sin , sin , cos x y dx d dy d ? ? ? ? ? ? ? ? ? ? ? i) Let the counter clockwise path be C 1 (0 to ?). ? work done along the counter clockwise is ? ? ? ? 1 0 sin sin cos cos C y dx x dy d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 0 0 sin cos d d ? ? ? ? ? ? ? ? ? ? ? ? ? ii) The work done along the clockwise path C 2 is ? ? 2 2 2 0 0 (sin cos) C y dx x dy d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Therefore we see that work depends on path. Therefore F is not conservative. Now 2 2 0 0 . . . 2 2 d d ? F dr F dr F dr Or, using the results of (i) and (ii) 1 2 0 0 . . . . . ( ) 2 C C C ? F dr F dr F dr F dr F dr Fig. : Solution (2) Fig : Solution (2) for c 1 : ? = 0 to ? For c : ? = 0 to 2? for c 2 : ? = 0 to – ? Origin 0 is within circle Origin 0 is within circle. o o

NSOU ? CC-PH-04 ? 173 Fig (4C.13) ? Note : However if the origin o is outside the circle, work done . 0? ?? F dr as shown in fig. below. Fig. : Solution (2) Origin O is outside the circle. In this case . . 0 ? ? ? ? ? ? ? ? F dr F dr 4.28.2 Surface integral of a vector field We consider a surface defined by z = f(x, y) having continuous first order partial derivatives. Let ds be a small area of the surface and ˆ n the limit normal vector in the outward direction to this small area. Then the area vector corresponding to this small portion of the surface is ˆ d ds ? n ? . The normal surface integral of a continuous vector point function V(r) is defined as ˆ S S V d V ds ? ? ? ?? ?? n ? (4C.15) Now the projection of a vector area ˆ nds on the xy – plane [Fig 4C.13] whose unit normal is k is given by ˆ ˆ ds ds ? ? ? n k n k But the projection of ds on xy plane is dxdy. Hence ˆ( )ds dxdy ? ? n k , Therefore ˆ dxdy ds ? ?n k and equation (4C.15) can be written as

174 ? NSOU ? CC-PH-04 ˆ ˆ ˆ S s S dxdy d ds ?? ? ? ? ? ? ? ?? ?? ?? V V n V n n k (4C.16) Similarly, considering the projection of vector area on yz plane and zx plane, the surface integral can be expressed as : ˆ ˆ S dzdy ? ? ?? V n n i and ˆ ˆ S dxdz j ? ? ? ?? V n n respectively When a surface enclosed a cetain volume it is called a closed surface. The positive normal to the closed surface is drawn outward from the surface and that is the direction of elementary surface dS on the closed surface S. For open surface the direction of the positive normal is determined from sense of traversing its boundary. If it is right handed then the positive normal is outward. When left handed it is inward as shown in figures 4C.14 and 4C.15 respectively. Flux of a vector field : The quantity ˆ. S S d ds ? ?? ?? V σ V n is called the flux of the vector filed V. In most case flux cannot be equated to any physical concept. However in the following case we can relate flux to specific physical quantity. 1. When V = v, the velocity vector of flow of liquid, ˆ. S v ds ?? n gives the volume of he liquid crossing the surface S per second normally. 2. When ??V v , where ? is the density of the following liquid and v its velocity of flow, ˆ. S ds ? ?? v n represents the mass of liquid crossing per second normally through the surface. 3. In case of electric and magnetic field flux represents the total number of lines of force crossing the surface normally. We have also flux of particles and flux of heat, defined similarly. Fig (4C.15) Fig (4C.14)

NSOU ? CC-PH-04 ? 175 Exercise of 4.28.2 1. Evaluate ˆ. ds ??? An

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where S is the surface of unit cube bounded by x = 0, x = 1; y = 0, y = 1; z = 0, z = 1			

or

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bounded by co-ordinate planes and the planes x = 1, y = 1, z = 1			

when i) A = r; ii) A = 4xzi − y 2 j + yzk 2. Evaluate ˆ. S ds ?? An , over the entire surface S

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of the region bounded by the cylinder x 2 + z 2 = 9, x = 0, y = 0, z = 0 and y = 8, if A = 6zi + (2x + y)j − xk			

Solution 1 : (fig. solution 1) We have ? ? . . ? ? ? ? ? ? ? x y z S S d ds ds ds A S A i j k or, ? ? . . S S dy dz dx dz dx dy ? ? ? ? ? ? ? Ads A i j k . . . ABCD EFGH BFGC dy dz dy dz dx dz ? ? ? ? ?? ?? ?? Ai A i A j . . . AEHD DCGH ABFE dx dz dx dy dx dy ? ? ? ? ? ?? ?? ?? A j Ak A k i) When A = r = ix + jy + kz For the surface ABCD, x = 1 1 1 0 0 ( ). 1 ABCD x y z i dy dz dx dz ? ? ? ? ? ?? ? ? i j k

176 ? NSOU ? CC-PH-04 For the surface EFGH, x= 0 1 1 0 0 ( ). (0) 0 ? ? ? ? ? ? ? ?? ? ? EFGH x y z dy dz dy dz i j k i Similarly . 1and . 0 ? ? ? ? ?? ?? BFGC AEHD dx dz dx dz A j A j and . 1 and . 0 DCGH ABEF dx dy dx dy ? ? ? ? ?? ?? Ak A k (1 0) (1 0) (1 0) 3 ? ? ? ? ? ? ? ? ? ? ??? A ds ii) When A = 4xzi − y 2 j + yzk For the surface ABCD, x = 1 1 1 1 1 2 0 0 0 0 (4). 4 4 ABCD xz y yz dydz z dz dy z dz dy ? ? ? ? ? ? ?? ? ? ? ? i j k i ? ? 1 1 2 0 0 2 2.1.1 2 z y ? ? ? ? ? ? ? ? For the surface EFGH, x = 0 1 1 2 0 0 (4). (0) 0 EFGH xz y yz dy dz dy dz ? ? ? ? ? ? ? ?? ? ? i j k i For the surface BFGC, y = 1 1 1 1 1 2 2 0 0 0 0 (4). 1 BFGH xz y yz dxdz y dx dz dx dz ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? i j k j For the surface AEHD, y = 0 1 1 2 0 0 (4). (0) 0 AEHD xz y yz dx dz dx dz ? ? ? ? ? ? ? ?? ? ? i j k j

NSOU ? CC-PH-04 ? 177 For the surface DCGH, z = 1 ? ? 1 1 1 1 2 0 0 0 0 1 1 4 1 2 2 DCGH xz y yz dx dy ydx dy y dy dx ? ? ? ? ? ? ? ? ? ?? ? ? ? ? i j k k For the surface ABFE, z = 0 ? ? 1 1 2 0 0 4 (0) 0 AEHD xz y yz dxdy dx dy ? ? ? ?? ? ? ?? ? ? i j k k 1 3 (2 0) (1 0) 0 2 2 S ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? A ds Solution 2 : (fig. solution 2) Here the surface S is composed of the bottom circular surface S 1 top circular surface S 2 , curved surface S 3 . Therefore 1 2 3 ˆ ˆ ˆ ˆ . S S S S ds ds ds ds ? ? ? ? ? ? ?? ?? ?? ?? ? An A n A n A n But ? ? 1 1 1 ˆ 6 (2) .( ) (2) S S s ds z x y x ds x y ds ? ? ? ? ? ? ? ? ? ? ?? ?? ?? A n i j k j (i) Now ? ? ? ? 2 2 2 ˆ 6 2 (2) S S S ds z x y xk ds x y ds ? ? ? ? ? ? ? ? ? ?? ?? ?? A n i j j (ii)
178 ? NSOU ? CC-PH-04 Evaluation of integral (i) : 1 1 1 (2) 2 0 , S S
x y ds x dx dz y in s ds
dx dz ? ? ? ? ? ? ? ?? ?? ? ? ? 2 1 3 9 3 2 0 0 0 (2) 2 2 9
x S

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x z x x y ds x dx dz x xdx ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? 3 2 2 0 2 9 1. 9 x x dx x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3 3 2 2 1 1 0 0 9 9 9 9 2 sin 2 sin 2 2 3 2 2 3 x x x x x x x ? 9 9 2.3. . 2. . 9 2 2 2 2 2 ? ? ? ? ? ? ? ? ? ? ? ? Evaluation of integral (ii) 2 ˆ (2) S x y dx dz ds dx dz and ? ? ? ? ?? ? n j 2 2 3 9 3 9 2 0 0 0 0 2 8 8 x x x z x z x dx dz dx dz y in S ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3 3 3 2 2 2 1 0 0 0 9 9 2 9 8 9 9 8 sin 2 2 3 x x x x x x x dx x

dx ? 9 9 8 9 18 27 2 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
again 3 ˆ ˆ . ˆ. S R dy dz ds ? ? ? ? ?? ?? A n A n ni Now 2 2 (,) 9 x z x z ? ? ? ? ? constant
NSOU ? CC-PH-04 ? 179 ˆ ˆ. 3 3 x z x ? ? ? ? ? ? ? ? ? i k n ni ? ? 3 3 5 5 ˆ ˆ . . 5 3 3 x S dydz xz ds xz z dy dz ? ? ? ? ? ? ?? ?? ?? A n An 3 3 8 3 3 0 3 ˆ. 5 40 0 S z y ds z dy dz z dz ? ? ?? ? ? ? ? ? ? ? ? ?? ? ? ? ? An ˆ. 9 27 0 18 ds ? ? ? ? ? ? ? ? ? ? ??? An 4.28.3 Volume integral of a vector field We consider a closed surface in space enclosing a volume V. If V(r) be a continuous vector point function, then volume integral is defined as, ( ) V r dV ??? V In Cartesian co-ordinate it is written as, ? ? x y z V V V V dx dy dz ? ? ??? i j k ( ) ( ) ( ) x y z V V V V dx dy dz V dx dy dz V dx dy dz ? ? ? ??? ??? ??? i j k 4.29 ? Green's theorem in plane Statement : It states that

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if M and N are continuous functions of x and y having continuous derivative in

a region R of the xy-plane bounded by a closed curve C,

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then ( ) C R N M M dx N dy dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? (4C.17)

The line integral in counter clockwise around the boundary of area R
180 ? NSOU ? CC-PH-04 Proof : Let us assume that the region R, bounded by a closoed curved C be such that the vertical lines at x = p and at x = q as shown in Fig (4C.16) cuts the curve C in at most two points at P and Q. Similarly the horizontal lines at y = m and y = n cuts the curve C in at most two points at M and N. Let the equation to the curve PMQ be 1 ( ) y x?? and that to the curve QNP be 2 ( ) y x ?? which is denoted by C 1 and C 2 respectively. Now ? ? 2 2 1 1 ( ) ( ) ( ) ( ) (,)
x q
y

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x x q x x x p y x x p R M M dx dy dy dx M x y dx y y ?? ? ? ? ? ? ? ? ? 2 1 , ( ) , ( ) x q x p M x x M x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 1 ( ) ( ) , ( ) , ( ) ? ? ? ? ? ? ? ? ? x q x q x p x

p along C along C M x x dx M x x
dx ? ? ? ? ? ? ? 2 1 , ( ) , ( ) (,) p q q p C M x x dx M x x dx
M x y
dx ? ? ? ? ? ? ? ? ? ? ?? (4C.18) Again, let the equation to the curve NPM be 3 ( ) x y ?? and that to MQN be 4 ( )

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x y ?? ? ? 4 4 3 3 ( ) ( ) ( ) ( ) (,) n y n y y y m y y m R N N dx dy dx dy N x y dy x x ?? ? ? ? ? ? ? ? 4 3 ( ), ( ), ? ? ? ? ? ? ? ? ? y n y m N y y N y y dy ? ? ? ? ? ? 4 3 ( ), ( ), ? ? ? ? ? ? ? ? y n y m y m y n N y y dy N y y

dy ? ? (,)
c N x y
dy ? ?? (4C.18A) Fig (4C.16) C 1
NSOU ? CC-PH-04 ? 181 C 1 is the curve y = x 2 C 2 is the curve y = x Fig Example (1) Combining (4C.18) and (4C.18A) we get, ? ? (,) (,) R C N M dx dy M x y dx N x y dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? (4C.17) In vector form equation (4C.17) can be re-written as : . . C R d dx dy ? ? ? ?? ? V r V k ? Where V = Mi + Nj, r = xi + yj, dr = i dx + j dy ? V.dr = M dx + N dy and . N M x y ? ? ? ? ? ? ? ? ? ? ? ? ? V k ? Example of Art 4.29 :

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Example 1: Verify Green’s theorem for ? ? 2 2 C xy y dx x dy ? ? ? ? ? ? ? ?? , where C is bounded by y = x, and y = x 2 .

Solution 1 : The curve C 1 and the line C 2 intersect at (0, 0) and (1,1). The positive direction in traversing C is as shown in figure. Along the curve C 1 , the given integral become ? ? 1 2 4 2 1 0 2 I xx

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x dx x x dx ? ? ? ? ? ? ? ? ? ? 1 3 4 0 19 3 20 x x dx ? ? ? ? Along the curved C 2 , the integral become ? ? 0 0 2 2 2 2 2 1 1 3 1 I x x dx x dx x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 1 2 19 1 1 20 20

C xy y dx x dy I I ? ? ? ? ? ? ? ? ? ? ? ? ? ?? (i)
182 ? NSOU ? CC-PH-04 Now applying Green’s theorem, we set that 2 2 , . M xy y N x ? ? ? Now 2 , 2 N M

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x x y x y ? ? ? ? ? ? ? ? 2 2 2 N M x x y x y x y ? ? ? ? ? ? ? ? ? ? Now ? ? 2 2 C xy y dx x dy ? ? ? ? ? ? ? ?? 2 1 0 (2) (2) x x N M dx dy x y dx dy dx x y dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? [We have 1 2 C C C f f ? ? ?? . Along C 1 , y = x 2 , x varies from 0 to 1. Along C 2 , y = x and x varies from 1 to 0] ? ? ? ? 2 1 1 1 2 2 2 3 4 4 3 0 0 0 1 – 20 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? x x dx xy y dx x x x x dx x x (

ii) Since (i) and (ii) equal, Green’s theorem is verified. Example 2 : Apply Green’s theorem in the plane to evaluate the integral 3 [(2)] C x y dx xy dy ? ? ?? over

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the boundary of the region bounded by the circles x 2 + y 2 = 1 and x 2 + y 2 = 9.

Solution 2 : Here M = 2x – y 3 N = – xy 2 3 M N y and y y x ? ? ? ? ? ? ? ? ? ? ? ? ? 3 2 2 3 C x y dx xy dy y y dx dy ? ? ? ? ? ? ? ? ? ? ? ? ?? ? Now in plane

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polar co-ordinate cos , sin , x r y r dxdy r dr d ? ? ? ? ? ? . Here r varies from 1 to 3 and ? varies from 0 to 2 ? . ? ? ? ? 3 2 3 2 2 1 0 2 sin 3 sin C r x y dx xy dy r r r dr d ?

NSOU ? CC-PH-04 ? 183 2 3 2 2 2 0 1 sin 3 sin r r d dr r d dr ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 3 2 3 2 2 3 0 1 0 1 sin sin 3 d r dr d r dr ? ? ? ? ? ? ? ? ? ? ? ? ? 3 4 4 4 1 3 3 3 0 3 1 .80 60 4 4 4 r ? Since 2 2 2 0 0 sin 0 sin d and d ? ? ? ? ? ? ? ? ? ? ? Exercise of Art (4..29) : 1) If C be the boundary of the rectangle (in xy plane) defined by y = 0, x = a; y = b, x = 0; evaluate the integral . C ?? F dr , where F = (x 2 + y 2)i – 2xyi by applying Green's theorem. 2) Verify Green's theorem in the plane to evaluate the integral ? ? 2 2 C xy x dx

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x y dy ? ? ? ? ? ? ?? over the triangle bounded by the line y = 0, x = 1 and y = x. 3) Apply Green's theorem

to prove that the area enclosed by a plane curve is 1 ( ) 2 C x dy y dx ? ?? . Hence find the area of an ellipse whose semi-major and minor axes are of lengths a and b. Solution 1 : Now applying Green's theorem we have M = x 2 + y 2 ; N = –2xy 2 ; 2 N M y y x y ? ? ? ? ? ? ? ? ? ? ? 2 2 2 (2 2) C x y dx xy dy y y dx dy ? ? ? ? ? ? ? ? ? ? ? ?? ?
184 ? NSOU ? CC-PH-04 2 2 0 0 4 4 [ ] 2 2 a y b

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y y dx dy dx y dy a b ab ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? Solution 2 : Along OA : y = 0 dy = 0 along AB : x = 1, dx = 0 along BO : y = x, dy = dx Now ? ? 2 2 C xy x dx x y dy ? ? ? ? ? ? ? ?? 2 3 OA AB BO x dx y dy x dx ? ? ? ? ? ? ? 1 1 0 2 3 0 0 1 1 1 1 1 3 2 4 12 x dx y dy x dx ? ? ? ? ? ? ? ? ? ? ? ? ? (i) Now M = xy – x 2 N = x 2 y M x y ? ? ? 2 N xy x ? ? ? Therefore using Green's theorem ? ? 1 2 2 0 0 (2) (2) y x OABO y xy x dx x y dy xy x dx dy dx xy x dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? 1 1 1 4 3 2 3 2 0 0 0 0 1 1 1 4 3 4 3 12 x x x dx xy xy dx x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (

ii) From equation (i) and (ii) we see that Green's theorem is verified.

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Solution 3 : We have from Green's theorem, [ ] C R N M Mdx Ndy dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? (

i) Fig. Solution (2)
NSOU ? CC-PH-04 ? 185 Now we put N = x and M = – y 1, 1 N M x y ? ? ? ? ? ? ? ? [ ] (1 1) C R y dx x dy dx dy ? ? ? ? ? ? ?? ? or, [ ] 2 2 C R x dy y dx dx dy A ? ? ? ? ? ?? ? where A is the area of the plane curve. 1 [ ] 2 C A x dy y dx ? ? ? ? ?? (ii) Now equation of the ellipse : 2 2 2 2 1 x y a b ? ? Therefore cos x a ? ? and sin y b ? ? , is the parametric equation of the ellipse. ? ? 1 cos . cos sin . (sin) 2 C A a b d b a d ? ? ? ? ? ? ? ? ? ? ?? 2 2 1 cos sin 2 C ab d d ? ? ? ? ? ? ? ? ? ? ?? ? ? 2 2 2 0 1 1 cos sin 2 2 C ab d ab d ab ? ? ? ? ? ? ? ? ? ? ? ? Now for a circle a = b 2 A a? ? ? 4.30 ? Gauss's Divergence Theorem Statement : This theorem states that, the volume integral of the divergence of the vector V taken over any volume ? in its field is equal to the surface integral of V over the closed surface enclosing the volume ? . In vector notation, the theorem is written as ( ) ˆ . . surface enclosing d d ? ? ? ? ??? ?? ?V V n (4C.19) (ˆ n points out of the closed surface ?)

186 ? NSOU ? CC-PH-04 Proof : We consider a small volume element i d? of the total volume ? shown in Fig (4C.17) in a vector field V. From the definition of divergence of vector fields, we get net outflow from each i d? as . i d? ?V and then adding the total outflow from the entire volume ? as . i i d? ? ?V (4C.20) which is explained below. From Fig (4C.17) it is seen that an outflow from a to b is an inflow from b to a, so that such outflows across interior faces cancel. The total sum in (4C.20) then equals just the total outflow from the entire volume ? . When i d? ? 0, the sum (4C.20) is converted into a triple integral over the volume ? : 0 lim . . i i i i d d d ? ? ? ? ?? ? ? ? ??? ? ? V V (4C.21) Now consider the Fig (4C.18) below, outflow or flux of vector field V through d? is ˆ . d? V n and the total outflow from the volume enclosed by the surface is ˆ . d? ?? V n (4C.22) Where ˆ n is the unit normal to the surface element d? and pointing outwards. (? is the surface enclosing volume ?) Thus both the equations (4C.21) and (4C.22) give the total outflow from the total volume ? and hence they are equal to each other and we get equation (4C.19) Example 4.28 and 4.20 Example 1: Evaluate ˆ . S d? ??? r n Solution 1: Using divergence theorem, ˆ . . 3 3 S V V ds dV dv V ? ? ? ?? ? ? ? r n r? Fig (4C.17) Fig (4C.18)
NSOU ? CC-PH-04 ? 187 Example 2 : Using divergence theorem evaluate 2 . S ??? ?r ds Solution 2 : We have 2 2 . . S V dV ? ?? ? ? ? ? ?
r ds
r Now, ? ? ? ? ? ? 2 2 2 2

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r r r r x y z ? ? ? ? ? ? ? ? ? i j k ? where r 2 = x 2 + y 2 + z 2 . 2 2 2 2 r x y z ? ? ? ? i j k ? 2 . (2) (2) (2) 2 2 2 6 r x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 . . 6 6

S V V r r
dV dV V ? ? ? ? ? ?? ? ? ? ds ? ? ? Example 3 : Prove that ˆ ( ) ( ) S V ds dV ? ? ? ?? ??? ? n B B ? Solution 3: We have from Gauss's divergence theorem, ˆ . . S V ds dV ? ?? ??? ? A n A? Let

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A = C × B, where C is a constant vector. ˆ . . V S dV ds ? ? ? ? ? ??? C B C B n ? Now . . .( ) .( ) C ? ? ? ? ? ? ? ? ? ? ? ? C B B C C B B , since C is a constant vector,, 0 ? ?C ? ˆ ˆ . . V S S dV ds ds ?? ? ? ? ? ? ? ? ?? ?? ? ? C B C B n C n B ? ˆ

V S dV ds ? ? ? ? ? ? ??? B n B ? (1)
188 ? NSOU ? CC-PH-04 [now if the vector B is always normal to a given closed surface S, then B and ds is parallel and B × ds = 0 and hence from equation (1) ˆ V S dV ds ? ? ? ? ? ? ??? B B n ? 0] S ? ? ? ? ? ??? B ds Example 4 : Prove that ? ? . 0 S ? ? ? ? ??? ds ? ? Solution 4 : We put ? ? ? ? A ? ? then, ? ? . (.) . S V V dV dV ? ? ? ? ? ? ?? ? ? ? Ads A? ? ? ? Now, . . . 0 ? Since 0 ? ? ? ? ? ? and 0 ? ? ? ? ? ? ? ? . 0 S ? ? ? ? ? ? ??? ds ? ? Example 5 : ? ? ? ? ? 2 2 . V V dV ? ? ? ? ? ? ? ? ? ? ? ? ? ds ? ? ? ? Where V is the volume bounded by the surface S and ,? ? are scalar fields. Solution 5 : Let ? ? ?A ? Now by divergence theorem ˆ . . V S dV ds ? ? ?? ? A An or ˆ .( ) . . V S dV ds ? ? ? ? ? ? ? ?? ? ? ? n But 2 . . . . ? therefore, 2 . ( ). V S dV ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ds (i) Now interchanging ? and ? , we get 2 . . V S dV ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ds (ii)
NSOU ? CC-PH-04 ? 189 Subtracting equation (ii) from equation (i), we get 2 2 ( ). V S dV ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ds Example 6 : Applying Gauss's divergence theorem evaluate 3 ˆ . S n ds r ?? r , where S represents any closed surface enclosing volume V. When the origin is outside S. Solution 6 : Let 3 r ? r A . Now A is continuously differentiable throughout the volume enclosed by S. By Gauss's divergence theorem, ˆ . . V S ds dV ? ? ? ? ? A n
A Now 3 3 3 1 1 . . (.)

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r r r ? ? ? ? ? ? ? r A r r Now 4 3 5 1 3 ˆ 3r r r ? ? ? ? ? ? ? r r 3 5 3 3 3 3 . 3 3 . 0 r r r r ? ? ? ? ? ? ? ? r r A 3 3 ˆ(.) . 0 V S r ds dV r r ? ? ? ? ?? ? ? ? ? r

n

Exercise of Art 4.28 and 4.30 : 1)
Evaluate ˆ . dS ?

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F n , where F = 4xzi − y 2 j + yzk and S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 & z = 1			

by using divergence theorem. 2)
Evaluate 2 x y
dV ? ,

96%	**MATCHING BLOCK 198/267**	**SA**	MMP.pdf (D118446089)
where V is the closed region bounded by the planes 4x + 2y + z = 8, x = 0, y = 0 & z = 0. 190 ?			

NSOU ? CC-PH-04 Solution 1 : ˆ . . S V ds dV ? ? ? ? ? F n F now ? ? 2 . (4) ( ) xz y yz x y z ? ? ? ? ? ? ? ? ? ? ? ? F or, . 4 2 4 z y y z y ? ? ? ? ? ? ? F ? ? ? ? 1 1 1 1 1 2 0 0 0 0 0 . (4) 2 2 V dV z y dxdy dz z yz dx dy y dx dy ? ? ? ? ? ? ? ? ? ??? ?? ?? ? F 1 1 1 2 0 0 0 3 3 2 2 2 2 y y dx dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? Solution 2 : In the problem, z varies from 0 to 8 − 4x − 2y, y varies from 0 to 4 − 2x and x varies from 0 to 2. ? the
given integral 8 4 2 2 4 2 2 4 2 2 2 2 0 0 0 0 0 (8 4 2)

62%	**MATCHING BLOCK 197/267**	**W**	
x y x x V x y z x y x y dV x y dx dy dz x y x y dx dy ? ? ? ? ? ? ? ? ? ? ? ? ? ??? ? ? ? ? ? 4 2 2 4 2 2 2 2 3 2 2 3 2 0 0 0 0 (8 4 2) 8 4 2 2 2 3 x x x y x y y y x y x y dy dx x x x dx ? 4 2 2 2 2 3 2 2 3 0 0 2 4 2 3 x x x y x y x y dx ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 3 2 2 3 0 2 4 (4 2) 2 (4 2) (4 2) 3 x x x x x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 3 4 5 6 2 3 4 5 0 0 64 8 64 8 128 32 16 32 16 3 3 3 3 4 5 3 6 45 x x x x x x x			

x
x
dx ?
NSOU ? CC-PH-04 ? 191 4.31 ? Stoke's Theorem Statements : It

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states that if ? is an open two-sided surface bounded by a			

simple closed curve and if V be continuously differentiable point function, then ˆ . . curved bounding open surface ? ? ? ? ? ?? ? V dr V n dσ ? (4C.23) where the boundary is traversed in the counter clockwise direction. ˆ n is the outward drawn unit normal to the surface element d? . Proof : We consider an open surface which is two sided and whose bounding curve is simple (i.e. it must not cross itself) and closed (Fig 4C.19). We consider the surface to be divided into a large number of elementary surfaces, d? with a unit vector ˆ n normal to each area element and lying on the same side of the surface (Fig. 4C.20). Fig. (4C.19) Fig. 4C. 20) Now from the difinition of curl ? ? 0 1 ˆ. lim . d around d d ? ? ? ? ? ? ?? V n V dr ? or, ˆ . . around d d d ? ? ? ? ? ? ?? ? V dr V n ? (4C.24) For each area element d? . Adding now for all the area elements we get ˆ . . all d around d surface d d ? ? ? ? ? ? ? ? ?? ? V r V n ? (4C.25)

192 ? NSOU ? CC-PH-04 From Fig 4C.20 we see that all the interior line integral cancel because along a common curve between two d? 's, the two integral, are in opposite direction. Therefore left hand side of equation (4C.25) becomes simple the line integral around the outside curve bounding the surface. Therefore we can write, ˆ . . . all d around d curved bounding open surface d d d ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? V r V r V n ? (4C.26) which is Stoke's theorem. Example of Art 4.31 : Example 1 : Using Stoke's theorem prove that ? ? ˆ S S C d d dS ? ? ? ? ? ? ? ? ? ? ? ? r S n ? ? Where ? is a scalar function of r, S is a open surface bounded by a closed curve C. Solution 1 : Let A be a constant vector. Then by Stoke's theorem . . S

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C d ? ? ? ? ? ? ?? ? A r A dS ? or ? ? . ( ) . C S d? ? ? ? ? ? ? ? ? ? ?? ? A r A A dS ? ? since A is constant vector, 0 ? ?A ? . . . C S S d

d d ? ? ? ? ? ? ? ? ? ?? ?? ? A r A S A S ? ? or, . . ; C S d d ? ? ? ? ? ? ?? ? A r A S ? or, C S d d ? ? ? ? ? ? ?? ? r S ? Example 2 : Evaluate by Stoke's theorem the integral, ? ? 2 x C e dx y dy dz ? ? ?? where C is the curve x 2 + y 2 = 4, z = 2.
NSOU ? CC-PH-04 ? 193 Fig. Example (3) Solution 2 : ? ? 2 . x

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x y z C C C e dx y dy dz A dx A dy A dz ? ? ? ? ? ? ? ? ? ? ? ? ? Adr 2 1 x x y z A e A y A ? ? ? ? ? Now 0 2 1 x x y z

e y ? ? ? ? ? ? ? ? ? ? ? ? i j k A By Stoke's theorem : ˆ . . C S dS ? ? ? ? ?? ? Adr An ?

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where S is the surface bounded by the circle. x 2 + y 2 = 4 and z = 2.

Also ˆ ?n k . 2 0 0 x C C e dx y dy dz ? ? ? ? ? ? ? ? ? ? ? ? Adr A ? Example 3 : Verify Stoke's theorem for A = (2

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x − y)i − yz 2 j − y 2 zk, the region of integration being the upper half surface of the sphere x 2 + y 2 + z 2 = 1 and C its boundary. Solution 3 : The boundary is the circle of unit radius in xy − plane

in C is a circle of x 2 + y 2 = 1 = r 2 ? r = 1. Substituting x = r cos ? = cos ? and y = r sin ? = sin ? and z = 0, we get 2 2 . (2) C C A dr x y dx yz dy y z dz ? ? ? ? ? ? ? ? ? ? ? ? ? or ? ? ? ? 2 0 . 2cos sin sin 0 C d z ? ? ? ? ? ? ? ? ? ? ? ? ? ? Adr ? ? 2 2 2 2 2 0 0 0 sin 2sin cos sin sin2 d d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
194 ? NSOU ? CC-PH-04 or, ? ? 2 2 0 0 1 . 1 cos2 sin2 2 C d d ? ? ? ? ? ? ? ? ? ? ? ? ? Adr 2 2 2 0 0 0 1 1 cos2 sin2 2 2 d d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 0 0 1 .2 cos2 sin2 0 2 d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Now, 2 2 2 x

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y z x y yz y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k A k ? ? ˆ ˆ . . S S ds k ds ? ? ? ? ?? ?? ? A n n

Now the projection of vector area ˆ n dS on the xy plane whose unit normal is k, is given by ? ? ˆ ˆ . . dS dS ? n k n k . But the projection of dS on xy plane is dx dy. Hence ˆ(.)dS dx dy ? nk ˆ ˆ ( ). . S S S dS dS dx dy ? ? ? ? ? ?? ?? ?? A n k n ? Now 1 2 0 0 1 .2 2 dx dy r dr d r dr d ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? ? Stoke's theorem is verified.
NSOU ? CC-PH-04 ? 195 Fig. (Solution 1) Exercise of Art 4.31 : 1) Verify Stoke's theorem for A = (
y − z + 2)i + (yz + 4)j − zxk over

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the surface of the cube x = 0, y = 0, z = 0; x = 2, y = 2, z = 2; above the xy plane. 2) Verify Stoke's theorem		

for the function F = x 2 i − xyj integrated round the square in the line x = 0, y = 0; x = e, y = a Solution 1 : We have from Stoke's theorem ˆ . ( ) . C S dS ? ? ? ?? ? Adr A n ? (i) We first evalute R.H.S. of equation (1), we have (1) 2 4 y z x y z y z yz xz ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k A i j k F ? Now we consider the cube Now . ( ) ( ) x y z S S S dS dS dS dy dz dx dz dx dy ? ? ? ? ? ? ? ? ? ? ? ? F dS F i j k F. i j k . . . . ABCD EFGH BFGC AEHD dy dz dy dz dx dz dx dz ? ? ? ? ? ? ? ?? ?? ?? ?? F i F i F j F j . . DCGH ABFE dx dy dx dy ? ? ? ?? ?? F k F k therefore . . . S S ABFE dx dy ? ? ?? ? ?? ? F dS F dS F k
196 ? NSOU ? CC-PH-04 Now 2 2 2 2 2 0 0 0 0 . [ ] 4 2 ABCD y dy dz y dy dz y dy dz z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? F i 2 2 0 0 . 4 EFGH dy dz y dy dz y dy dz ? ? ? ? ? ? ? ?? ?? ? ? F i 2 2 2 0 0 2 . (1) (1) 2 2 0 2 BFGC BFGC dx dz z dx dz dx z dz ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? F j . (1) 0 AEHD AEHD dx dz z dx dz ? ? ? ? ? ? ?? ?? F j 2 2 0 0 . (1) 4 DCGH DCGH dx dy dx dy dx dy ? ? ? ? ? ? ? ?? ?? ? ? F k . 4 4 0 0 4 4 ? ? ? ? ? ? ? ? ? ? ?? F dS Now we evaluate L.H.S. of equation (1) ? ? . (2) (4) C C y z dx yz dy xz dz ? ? ? ? ? ? ? ? ? ? ? Adr . . . . EA ? ? ? ? ? ? ? ? ? AB BF FE Adr Adr Adr Adr Now along EA : y = 0, dy = 0, z = 0, dz = 0 2 0 . 2 4 EA dx ? ? ? ? ? Adr Along AB : x = a, dx = 0, z = 0, dz = 0 2 0 . 4 8 AB dy ? ? ? ? ? Adr
NSOU ? CC-PH-04 ? 197 Fig. Solution (2) Along BF : z = 0, dz = 0, y = 2, dy = 0 0 2 . 4 8 BF dx ? ? ? ? ? Adr Along FE : z = 0, dz = 0, x = 0, dx = 0 0 2 . 4 8 FE dy ? ? ? ? ? Adr . 4 8 8 8 4 C ? ? ? ? ? ? ? ? ?? Adr ? L.H.S of equation (1) is − 4 and R.H.S of equation (1) is also − 4. Therefore Stoke's theorem is verified. Solution 2 : F = x 2 i − xyj y ? ? ? ? F k ? Now, ˆ ˆ . . S S dS y dx dy ? ? ? ?? ?? F n k n ? . S y dx dy ? ? ?? k k 0 0 ˆ . a a S S ds y dx dy y dy dx ? ? ? ? ? ? ?? ?? ? ? F n ? 3 2 a ? ? Now, F.dr = (x 2 i − xyj) (i dx + jdy) = x 2 dx − xy dy Now . . . . . OA AB BC CO C ? ? ? ? ? ? ? ? ? ? F dr F dr F dr F dr F dr
198 ? NSOU ? CC-PH-04 Along OA : y = 0, dy = 0 3 2 0 . 3 a OA a x dx ? ? ? ? ? F dr Along AB : x = a, dx = 0 3 0 . 2 a AB a ay dy ? ? ? ? ? ? ? F dr Along BC : y = a, dy = 0 3 2 0 . 3 a BC a x dx ? ? ? ? ? ? F dr Along CO : x = 0, dx = 0 . 0 CO ? ? ? F dr 3 3 3 3 . 0 3 2 3 2 C a a a a ? ? ? ? ? ? ? ?? F dr ˆ . . C S ds ? ? ? ? ? ?? ? F dr F n ? ? Stoke's theorem is verified. Keywords Double and triple integral, Jacobian, line, surface, volume integral of a vector field, Gauss's divergence theorem, Stoke's theorem and Green's theorem in plane. 4.32 ? Summary ? Definitions of double and triple integration as a limit of sums are given. Change of order of integration is exemplified and example of triple integral are given.
NSOU ? CC-PH-04 ? 199 ? Change of variables of integrands with respective Jacobians are introduced. ? Line, surface, volume integrals of vector field are discussed with examples. ? Elementary proofs of Gauss's divergence theorem, Stoke's theorem and Green's theorem in a plane are given. Also verification of these theorems with examples are provided.
200 ? NSOU ? CC-PH-04 Unit 5 ??Orthogonal Curvilinear Co-ordinates Structure 5.1 Objectives 5.2 Introduction 5.3 Curvilinear Co-Ordinates 5.4 Orthogonal Curvilinear Co-Ordinates 5.4.1 Elements of Arc Length, Area and Volume 5.4.2 ? ? i r u and ? i u (i = 1, 2, 3) Forms A Reciprocal System of Triads : 5.5 Gradient in Orthogonal Curvilinear Co-Ordinates 5.5.1 Gradient in circular cylindrical co-ordinates 5.5.2 Gradient in spherical polar co-ordinates 5.6 Divergence in Orthogonal Curvilinear Co-Ordinates 5.6.1 Divergence in circular cylindrical co-ordinates 5.6.2 Divergence in spherical polar co-ordinate 5.7 Curl in Orthogonal Curvilinear Co-Ordinates 5.7.1 Curl in circular cylindrical co-ordinates 5.7.2 Curl in spherical polar co-ordinate 5.8 Laplacian in Orthogonal Curvilinear Co-Ordinates 5.8.1 Laplacian in circular cylindrical co-ordinate 5.8.2 Laplacian in spherical co-ordinate system 5.8.3 Exercise 5.9 Summary 5.1 ? Objectives Objective of this chapter is to set up an orthogonal curvilinear co-ordinate system and find its unit vectors. The line element, area and volume elements are expressed in terms of orthogonal curvilinear co-ordinate. Now we have derived expressions for gradient, divergence, curl and Laplacian in terms of orthogonal curvilinear co-ordinates and have

NSOU ? CC-PH-04 ? 201 shown this expressions in some special co-ordinate system like circular cylindrical and spherical polar co-ordinate system. 5.2 ? Introduction In rectangular co-ordinate system the co-ordinate surfaces are planes and they intersect at right angles to each other producing straight co-ordinate axes. In the previous chapters we have defined gradient, Divergence, curl and Laplacian in rectangular co-ordinates x, y, z. But in solving many physical problems, depending on the symmetry of the problems, we have to express those vector operators in other co-ordinate systems like, cylindrical, spherical etc. in which the surfaces are not all planes and the intersection of the surfaces are curve lines rather than straight lines. Therefore, it has now become necessary to define a co-ordinate system whose co-ordinate surfaces are curved surfaces and the intersections of these curved surfaces produce curved lines as axes of co-ordinate system. This preferred co-ordinate system is called curvilinear co-ordinate system. When the curved surfaces intersect at right angles, we have orthogonal curvilinear co-ordinate system and orthogonal curvilinear co-ordinates are convenient to study the physical problems. 5.3 ? Curvilinear co-ordinates Three curve surfaces u 1 = constant, u 2 = constant and u 3 = constant are taken such that any two surfaces always intersect to produce a curve and all the three surfaces intersect at a point. For example, (Fig 5.1), surfaces u 1 = constant and u 2 = constant intersect along a curve called u 3 − axis and similarly u 1 − axis and u 2 − axis are defined. We can take these curves of intersections as reference axes to construct a co-ordinate system, called curvilinear co-ordinate system. Let the Cartesian co-ordinates and the curvilinear co-ordinates of a same point P be (x, y, z) and (u 1 , u 2 , u 3) respectively. Since there need be point to point correspondence between the co-ordinate systems : ? ? ? ? ? ? 1 2 3 1 2 3 1 2 3 , , , , , , x
x

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u u u y y u u u z z u u u ? ? ? ? ? ? ? ? (5.1) Fig (5.1) P 202 ? NSOU ? CC-PH-04 And 1 1 2 2 3 3 (, ,) (, ,) (, ,) u u x y z u u x y z u u

x y z ? ? ? ? ? ? ? ? (5.2)
The functions defined by (5.2) ae continuous having first order continuous derivatives. If r be the position vector of the point P, then the vectors along tangents to u 1 , u 2 , u 3 axes will be 1 2 3 , and u u u ? ? ? ? ? ? r r r respectively and the unit vectors along these tangents are : 1 1 2 2 3 3 1 1 1 , , , ? ? ? ? ? ? ? ? ? 1 2 3 h u h u h u r r r e e e (5.3) respectively,, where ? ? ? i i h u r (5.4) ; i = 1, 2, 3 and called scale factors which may have dimensions. Now in order that the co-ordinate surfaces u 1 = constant, u 2 = constant and u 3 = constant uniquely defines a point of intersection, the vectors 1 2 3 , and u u u ? ? ? ? ? ? r r r should be non-coplaner and their triple scalar product do not vanish i.e. 1 2 3 . 0 u u u ? ? ? ? ? ? ? ? r r r where r = ix + jy + kz i.e. 1 1 1 2 2 2 3 3 3 0

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x y z u u u x y z u u u x y z u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (5.5) or, ? ? 1 2 3 (, ,) 0 , , x y z u u u ? ? ? (5.6)

NSOU ? CC-PH-04 ? 203 where (5.6) gives the Jacobian of transformation : (see article 4C.5). 5.4 ? Orthogonal Curvilinear Co-ordinates The curvilinear co-ordinate system would be orthogonal if the unit vectors given by equations (5.3) along the tangent to the axes are orthogonal. i.e. 0 . 1 ? ? ? ? ? ? ? ? ? i j for i j for i j

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e e (5.7) and e 1 × e 2 = e 3 ; e 2 × e 3 = e 1 ; e 3 × e 1 = e 2 5.4.1 :

Elements of Arc length, Area and volume Arc length = since

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r = r(u 1 , u 2, u 3), 1 2 3 1 2 3 u u u u u u ? ? ? ? ? ? ? ? ? ? ? ? r

r r dr (5.8a) = h 1 e 1 du 1 + h 2 e 2 du 2 + h 3 e 3 du 3 (5.8b) using equation (5.3) Now square of the differential arc length ds is given by : ? ? ? ? ? ? 2 2 2 2 2 2 2 1 1 2 2 3 3 . ds h du h du h du ? ? ? ? dr dr (5.9) Using the equation (5.7) or, 2 2 2 2 1 2 3 ds ds ds ds ? ? ? (5.10) where ds 1 = h 1 du 1 , ds 2 = h 2 du 2 and ds 3 = h 3 du 3 are the differential arc lengths along u 1 ,u 2 and u 3 respectively. Now plane area is a vector given by the cross-product of two vectors. Thus element of area in this case is, ds 1 × ds 2 = h 1 du 1 e 1 × h 2 du 2 e 2 = e 1 × e 2 h 1 h 2 du 1 du 2 Or, ds 1 × ds 2 = e 3 h 1 h 2 du 1 du 2 or, |ds 1 × ds 2 | = h 1 h 2 du 1 du 2 (5.11) Similarly, |ds 2 × ds 3 | = h 2 h 3 du 2 du 3 (5.12) And |ds 3 × ds 1 | = h 3 h 1 du 3 du 1 (5.13)
204 ? NSOU ? CC-PH-04 Again, volume element having sides ds 1 , ds 2 and ds 3 is given by triple scalar product dV = ds 1 .ds 2 × ds 3 = (e 1 .e 2 × e 3)h 1 h 2 h 3 du 1 du 2 du 3 Or, dV = h 1 h 2 h 3 du 1 du 2 du 3 (5.14) using equation (5.7). Equation (5.14) gives the volume element in orthogonal curvilinear co- ordinate system. 5.4.2 : ? ? i r u and i u? (i = 1,2,3) forms a reciprocal System of Triads [see article 4A.3.21] We have dr = idx + jdy + kdz, and 3 1 2 1 u u u u x y z ? ? ? ? ? ? ? ? ? i j k ? . i i u d du ? ?r ? (5.15) Again if we multiply both sides of equation (5.8a) scalarly by 1 u? , we get 1 1 1 1 2 1 3 1 2 3 . . . .

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u d u u u u u u u u u ? r r r r ? ? ? ? (5.16) Comparing equation (5.15) and (5.16), we get 1 1 1 2 . 1, . 0, u u u u ? ? ? ? ? ? r r ? ? 1 3 . 0 u u ? ? ?

r ? Similarly, by multiplying equation (5.8a) scalarly by 2

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u? and 3 u? , we get 2 2 2 1 2 3 . 1; . 1; . 0 u u u u u u ? ? ? ? ? ? ? ? ? r r r ? ? ? And 3 3 3 1 2 3 . 0; . 0; . 1 u u u u u u ? ? ? ? ? ? ? ? ?

r r r ? ? ? From these relations we can say that vectors i u ? ? r and i u? (i = 1, 2, 3) form a reciprocal system of triads. Therefore,
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u u h h h h u h h h h h h h

r r r
u u u

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e e e e e e e e ? Since e 1 .e 2 × e 3 = 1 & e 2 × e 3 = e 1 ,

Similarly 2 2 2 ?u h e ? and 3 3 3 ?u h
e ? Or, we get 1 1 1 2 2 2 3 3 3 h u h u h u ? ? ? ? ? ? ? ? ? ? ? e e e (5.17) 5.5 ? Gradient in orthogonal curvilinear co-ordinates We consider a scalar function ?(

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u 1 , u 2 , u 3) where u 1 , u 2 , u 3 are functions of x, y, z defined by equation 5.2. Then we have : 3 1 2 1 2 3 3 1 2 1 2 3 3 1 2 1 2 3 u u u x u x u x u x u u u y u y u y u y u u u z u z u z u

z ? (5.18)
Multiplying equation (5.18) by i, j, k respectively and adding we get : 1 2 3 1 2 3 u u u x y z u u u ? i j k or, 3 1 2 1 1 2 2 3 3 h u h u h u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
e e e (5.19) Using (5.17).
206 ? NSOU ? CC-PH-04 Fig. 5.2 5.5.1 : Gradient in circular cylindrical co-ordinates In this co-ordinate system, the three curvilinear co-ordinates are : u 1 = r, 0 r? ? ? ? 2 , u ?? 0 2 ? ? ? ? ? u 3 = z, z ?? ? ? ? ?? Transformation equations are : cos sin x r y r z z ? ? ? ? ? ? ? ? ? ? (5.20) Now r = ix + jy + kz = cos sin
r r z ? ? ? ? ? i
j k

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cos sin r ? ? ? ? ? ? ? ? r i j ? ? (sin) sin sin cos r r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? r i j i j z ? ? ? r k Therefore unit vectors are : cos sin sin cos ? r r r r r r r

z e i j e i j e k (5.21) Scale factors are, 1; ; 1 z h h h z ? ? ? ? ? ? ? ? ? ? ? ? ? ? r r r r r r
NSOU ? CC-PH-04 ? 207 Therefore 3 1 2 1 1 2 2 3 3 h u h u h u ? ? ? ? ? ? ? ? ? ? ? ? ? ? e e e 1 z ? ? ? ? ? ? ? ? ? ? ? ? ? ? r θ e e k r r (5.22) 5.5.2 : Gradient in spherical polar co-ordinates In this co-ordinate system, 1 2 3 ; ; u r u u ? ? ? ? ? ?
Transformation equations are sin cos sin sin cos
x r y r z r ? ? ? ? ? ? ? ? ? ? ? ? ? ? (5.23) We have r = ix + jy + kz or,
sin cos sin
sin cos

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r r r ? ? ? ? ? ? ? ? ? r i j k sin cos sin sin cos r ? ? ? ? ? ? ? ? ? ? ? r i j k 1 1 r h h r ? ? ? ? ? ? r cos cos cos sin sin r r r ? ? ? ? ? ? ? ? ? ? ? ? r i j k 2 h h r ? ? ? ? ? ? ? ? ? r sin (sin) sin cos r r ? ? ? ? ? ? ? ? ? ? ? r i j 3 sin h h r ? ? ? ? ? ? ? ? ? ? r Fig. 5.3 208 ? NSOU ? CC-PH-04 1 2 3 1 sin r h h h h r h h r ? (5.24) Now, 1 2 3 sin cos sin sin cos cos cos cos sin sin sin cos r r ? r r r r r r θ r

e e i j k e e i j k e e i j (5.24a) Therefore, replacing ? by ? ; in (5.19) 3 1 2 1 1 2 2 3 3 1 sin h u h u h u r r ? r r e e e e e e (5.25) 5.6 ? Divergence in orthogonal Curvilinear Co-ordinates We consider a vector point function A(u 1 ,u 2 ,u 3) having components A 1 ,A 2 ,A 3 along

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the unit vectors e 1 , e 2 , & e 3 respectively, such that A = A 1 e 1 + A 2 e 2 + A 3 e 3 = A 1 + A 2 + A 3 (5.26)

Since

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the unit vectors are orthogonal, e 1 = e 2 × e 3 ; e 2 = e 3 × e 1 ;

e 3 = e 1 × e 2 (5.9) Let
us consider the component A 1 of the vector point function. We have, 1 1 1 1 2 3 1 2 3 2 3 A A A h h u u ? ? ? ? ? ? ? ? A e e e , using equation (5.17) Therefore, 1 1 2 3 2 3 A h h

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u u ? ? ? ? ? ? ? A ? ? 1 2 3 2 3 1 2 3 2 3 A h h u u A h h u u ? ? ? ? ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 209 But 2 3 3 2 2 3 0 u u u u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (see article 4B.13) ? ? 1 1 2 3 2 3 A h h u u ? ? ? ? ? ? ? ? ? A (5.27) Now for any function f(u 1), we have 1 1 1 1 1 1 1 1 1 ( ) ( ) ( ) ( ) f u f u f u u u u f f f f u x y z u x u y u

z ? i j k i j k 1 1 f u u ? ? ? ? (5.28)
Using the identity (5.28) we get, from equation (5.27), ? ? ? ? 3 1 2 1 1 2 3 1 2 3 1 2 3 1 1 1 2 3 ? ? ? ? ? ? ? ? ? ? ? ? A

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h h u u u A h h u u h h h e e e A ? ? ? ? using (5.17) ? ? ? ? 1 2 3 1 2 3 1 2 3 1 1 2 3 1 1 A h h A h h h h h u h h h u ? ? ? ? ? ? ? ? 1 2 3 e e e 1 2 3 1 ? ? ? ?

e e e
Similarly, we can find, ? ? 2 2 3 1 1 2 3 2 1 A h h h h h u ? ? ? ? ? A ? ? 3 3 1 2 1 2 3 3 1 A h h h h h u ? ? ? ? ? A Thus, we have 1 2 3 ? ? ? ? ? ? ? ? ? ? ? ? A A A A ? ? ? ? ? ? 1 2 3 2 3 1 3 1 2 1 2 3 1 2 3 1

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A h h A h h A h h h h h u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (5.29)

Equation (5.29) gives the divergence of a vector point function in orthogonal curvilinear co-ordinates.
210 ? NSOU ? CC-PH-04 5.6.1. Divergence in circular cylindrical co-ordinates In this co-ordinates system u 1 = r, u 2 = ?, u 3 = z; h 1 = h r = 1; h 2 = h ? = r; h 3 = h z = 1 and e 1 = e r , e 2 = e ?? , e 3 = e z = k from equation (5.24), Therefore 1 1 ( ) z r A A r A r r r z ? ? ? ? ? ? ? ? ? ? ? ? ? A (5.30) where 1 2 3 , , r z A A A A A A ? ? ? ? . 5.6.2. Divergence in spherical polar co-ordinate In this co-ordinate system 1 2 3 , , ; u r u u? ? ? ? ? 1 2 3 1; , sin ; r h h h h r h h r ? ? ? ? ? ? ? ? ? 1 2 3 1 2 3 , , ; , , r A A A A A A ? ? ? ? ? ? ? ? ? r θ e e e e e e Therefore from equation (5.29), ? ? ? ? 2 2 1

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sin sin sin r A r A r A r r r r ?

A 5.7 ? Curl in Orthogonal Curvilinear Co-ordinates We have A = A 1 e 1 + A 2 e 2 + A 3 e 3 Now using equation (5.17), we get, 1 1 1 2 2 2 3 3 3
A
A

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h u A h u A h u ? ? ? ? ? ? ? ? ? ? ? ? 1 1 1 2 2 2 3 3 3 A h u A h u A h u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A (5.31) Now ? ? ? ? ? ? 1 1 1 1 1 1 1 1 1 1 1 A h u A h u A h u A h u ? 1 1 1 1 1 1 2 1 1 1 3 1 1 2 3 A h u u A h u u A h u u u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (5.32) NSOU ? CC-PH-04 ? 211 Now from equation (5.17) 3 2 1 2 1 1 2 1 2 u u h h h h ? ? ? ? ? ? ? e e e 3 1 2 3 1 1 3 1 3 u u h h h h ? ? ? ? ? ? ? e e

e Therefore equations (5.32) becomes ? ? ? ? ? ? 3 2 1 1 1 1 1 1 1 1 3 3 1 2 2 ? ? ? ? ? ? ? ?

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A h u A h A h h h u h h u e e ? ? Similarly we get, ? ? ? ? ? ? 3 1 2 2 2 2 2 2 2 1 2 1 3 2 3 A h u A h A h h h u h h u ? ? ? ? ? ? ? ? ? e e ? ? ? ? ? ? 1 2 3 3 3 3 3 3 3 1 3 2 1 3 3 A h u A h A h h h u h h u ? ? ? ? ? ? ? ? ? e e Thus ? ? ? ? ? ? ? ? 1 2 3 3 2 2 1 1 3 3 2 3 2 3 1 3 3 1 A h A h A h A h h h u u h h u u ? e e

A ? ? ? ? 3 2 2 1 1 1 2 1 2 A h A h
h h u u ? ? ? ? ? ? ? ? ? ? ? ? e (5.33)
Equation (5.33) can be written in a determine from 1 1 2 2 3 3 1 2 3 1 2 3 1 1 2 2 3 3 1 ? ? ? ? ? ? ? ?

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h h h h h h u u u A h A h A h

e e e A ? (5.34) 5.7.1. Curl in circular cylindrical co-ordinates In this co-ordinate system, h 1 = h r = 1; h 2 = h ? = r; h 3 = h z = 1; e 1 = e r , e 2 = e ? , e 3 = e z = k; A 1 = A r , A 2 = A ? , A 3 = A z From equation (5.34),
212 ? NSOU ? CC-PH-04 1 r z r r r z A rA A ? ? ? ? ? ? ? ? ? ? ? r θ e e k A (5.35) 5.7.2. Curl in spherical polar co-ordinates In this co-ordinate system : h 1 = h r = 1; h 2 = h ? = r; h 3 = h ? = r sin ??; u 1 = r, u 2 = ?, u 3 = ? ; A 1 = A r , A 2 = A ?? , A 3 = A ? Therefore from equation (5.34) 2 sin 1 sin sin ? ? ? ? ? ? ? ? r r r r r A rA r A ? ? ? ? ? ? ? ? ? r θ e e e A ? (5.36) 5.8 ? Laplacian in Orthogonal Curvilinear Co-ordinate From equation (5.19) : 3 1 2 1 1 2 2 3 3 h u h u h u ? ? ? ? ? ? ? ? ? ? ? ? ? ? e e e let 1 1 2 2 3 3 A A A ? ? ? ? ? ? A e e e (5.37), comparing (5.19) and (5.37) we get, 1 2 3 1 1 2 2 3 3 1 1 1 , , A A
A

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h u h u h u ? ? ? ? ? ? ? ? ? ? ? ? (5.38) From equation (5.29) : ? ? ? ? ? ? 1 2 3 2 3 1 3 1 2 1 2 3 1 2 3 1 , A h h A h h A h h h h h u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A substituting A 1 , A 2 , A 3 from equation (5.38), we get NSOU ? CC-PH-04 ? 213 2 3 3 1 1 2 1 2 3 1 1 1 2 2 2 3 3 3 1 h h h h h h h h h u h u u h u u h u ? A (5.39) 2 2 3 3 1 1 2 1 2 3 1 1 1 2 2 2 3 3 3 1 h h h h h h h h h u h u u h u u h u ? (5.40) 5.8.1.

Laplacian in
circular cylindrical co-ordinate In this co-ordinate system : h 1 = h r = 1; h 2 = h ? = r; h 3 = h z = 1; u 1 = r, u 2 = ?, u 3 = z ; A 1 = A r , A 2 = A ? , A 3 = A z ; e 1 = e r , e 2 = e ? , e 3 = k Therefore from
equation (5.40) 2 1 1

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r r r r r r z z ? 2 2 2 2 2 1 1 r r r r r z ? (5.41) 5.8.2.

Laplacian in spherical co-ordinate system In this co-ordinate system : h 1 = h r = 1; h 2 = h ? = r; h 3 = h ? = r sin ??; u 1 = r, u 2 = ?, u 3 = ? ; A 1 = A r , A 2 = A ? , A 3 = A ? ; e 1 = e r , e 2 = e ? , e 3 = e ? Therefore from equation (5.40) we get changing ??to ? 2 2 2 2 2 1 1 sin sin sin sin r r r r ? (5.42) 5.8.3. Exercise 1) Express the vector V = ix + j2y + kz in circular cylindrical co-ordinates. 2) Express the vector V = i2x – jy + 3kz in spherical co-ordinates. Solution : Solution 1 : Transformation equations in circular cylindrical co-ordinates are cos sin x r y r z z ? ? ? ? ? ? ? ? ? ?

214 ? NSOU ? CC-PH-04 using equation (5.20). cos 2 sin r r z ? ? ? ? ? ? ? V i j k Now i, j and k are given in terms of e r , e ? and e z by solving equation (5.21) and we get cos sin ? ? ? ? r θ i e e sin cos ? ? ? ? r θ j e e k = e z Therefore ? ? ? ? cos cos sin 2 sin sin cos r r z ? ? ? ? ? ? ? ? ? ? ? ? r θ r θ z V e e e e e ? ? ? ? 2 2 cos 2 sin sin cos 2 sin cos r r r r z ? ? ? ? ? ? ? ? ? ? ? ? ? r θ z e e e ? ? 2 1 sin sin cos r r z ? ? ? ? ? ? ? ? r θ z e e e Solution 2 : Transformation equations for spherical co-ordinates are, sin cos sin sin cos x r y r z r ? ? ? ? ? ? ? ? ? ? ? ? ? ? using equation (5.23). 2 sin cos sin sin 3 cos r r r ? ? ? ? ? ? ? ? ? V i j k Now i, j, k are given in terms of , , ? r θ e e e by solving equations (5.24a) and we get sin cos cos cos sin ? ? ? ? ? ? ? ? ? r θ i e e e sin sin cos sin sin ? ? ? ? ? ? ? ? ? ? r θ j e e e cos sin ? ? ? ? r θ k e e Therefore ? ? 2 sin cos sin cos cos cos sin r ? ? ? ? ? ? ? ? ? ? ? ? r θ V e e e
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sin sin sin cos cos sin sin r ? ? ? ? ? ? ? ? ? ? ? ? r θ e e e ? ? 3 cos cos sin r ? ? ? ? ? ? r θ e e ? ? 2 2 2 2 2 2sin cos sin sin 3cos r ? ? ? ? ? ? ? ? ? r e ? ? 2 2 2sin cos cos sin cos sin 3sin cos r ? ? ? ? ? ? ? ? ? ? ? ? θ e ? ? 2 2sin sin cos sin sin r ? ? ? ? ? ? ? ? ? e ? ? ? ? ? 2 2 2 2 2 3sin cos 4cos 1 sin cos 2cos sin 3 r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? r

θ e e sin sin (2cos sin)
r ? ? ? ? ? ? ? ? e Keywords Curvilinear co-ordinate system, orthogonal curvilinear co-ordinate system; gradient, divergence, curl and Laplacian. 5.9 ? Summary ? We have defined curvilinear and orthogonal curvilinear systems. ? Expressions for elements of arc length, area and volume have been obtained in orthogonal curvilinear co-ordinate system. ? Expression for gradient, divergence, curl and Laplacian have been obtained in orthogonal curvilinear co-ordinate systems and in circular cylindrical and spherical polar co-ordinate systems.
216 ? NSOU ? CC-PH-04 Unit 6 ??Dirac Delta Function Structure 6.1 Objective 6.2 Introduction 6.3 Definition 6.4 Step Up / Step Down Function : Unit Impulse Function 6.5 Different Representation of the Delta Function 6.5.1 Properties of Delta Function 6.5.2 Delta Function in Three Dimension 6.6 Summary 6.1 ? Objectives The objective of this chapter is to introduce Direc Delta function to the students. It's definition and properties are explained. Also various representations of delta function have been discussed. 6.2 ? Introduction Delta function appears in many physical problems. It was first used by P.A.M Dirac in quantum mechanics and thereafter it became popular among physicists and mathematicians and is popularly known now as Dirac Delta function ( )x? . The point to be remembered is that ( )x? is not a function at all in the usual sense. Since its value is not finite at x = 0 and it is only treated as if it were a function for certain clearly defined purpose in physics and mathematics. 6.3 ? Definition In one dimension, the Dirac Delta function ( )x? , can be thought of as a function on the real line which is zero everywhere except at the origin where it has such a large value that the integral of the function over an interval containing the point x = 0 is equal to unity. Thus
NSOU ? CC-PH-04 ? 217 ( ) 0,for 0 , for 0 and ( ) 1 x x x x dx ? ? ?? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (6.1) ? – function has the unit area under the curve. When the centre of the delta function is shifted to x = a from the origin equation 6.1 is rewritten as : ( ) 0,for 0 , for 0 and ( ) 1 x a x a x a x a dx ? ? ?? ?? (6.1a) Equation 6.1a is shown in figure (6.3) 6.4 ? Step up/step down function : Unit Impulse Function We consider a function, 1 2 ( ) , 0, y x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (6.2) As shown in fig (6.1) We can make an approximation to ? -function by making a step-up / step-down function shown in fig (6.1). Now let ( ) ( ) ( ) y x u x u x ? ? ? ? ? ? ? ? ? Where ( ) u x ?? the step is up function and ( ) u x ?? is the step down function. The width of the curve being ( ) 2 ? ? ? ? ? ? ? and the height is 1 2? so that area of the curve is 1 (2) 1 2 ? ? ? ? . Now when 0 ? ? , we get Fig. 6.1
218 ? NSOU ? CC-PH-04 0 ( ) lim ( ) x x ? ? ? ? ? ? ? (6.3) And equation (6.3) is represented by fig (6.2). Equation (6.2) also defines a unit impulse function of impulse F t (x) × t = 1 where F t (x) = 1 ' 2 2 0, 2 t t x t t x ? ? ? ? ? ? ? ? ? ? ? ? ? A rule for integration of its product with another function f(x) is given by ( ) (0) (0) f x x dx f ? ?? ?? ? ? ? (6.4) when ( )x? centred at origin. When ( )x? is centred at x = a, we get ( ) ( ) ( ) f x x a dx f a ? ?? ?? ? ? ? (6.5) Equation (6.5) is valid for any continuous function f(x), because ( ) 0 x a ? ? ? for x a? and we can replace the function f(x) by its value at x = a while integrating since, ( ) ( ) ( ) ( ) ( ) ( )

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f x x a dx f a x a dx f a x a

dx ? ? ? ?? ?? ?? ?? ?? ?? ? ? ? ? ? ? ? ? Now ( ) 1 x a dx ? ?? ?? ? ? ? by equation (6.1a), and equation (6.5) follows. The range of integration of equation 6.1 or 6.1a or 6.4 or 6.5 need not be from to ?? ??. It may be over any region containing the centre of the ? -function where it does not vanish. It is to be noted that if x has the dimensional length, ( ) x a ? ? would have the dimension of inverse length. Fig. 6.3 Fig. 6.2
NSOU ? CC-PH-04 ? 219 Similarly if x has the dimension of time, then ( ) x a ? ? would have the dimension of (time) –1 . 6.5 ? Different representation of the ?-function a. ?-function as a limiting form of rectangular function : We suppose : 1 , for ( ) 2 0, for 0 x a y x x a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (6.6) We see that as ? decreases, the rectangular distribution becomes narrower as sharper.. The integral 1 ( ) 1 2 a a y x dx dx ? ? ? ? ?? ? ?? ? ? ? ? ? This is true for any value of ? . Thus even in the limit 0 ? ? the structure becomes infinitely peaked, however still retaining the area under the curve as unity so, 0 lim ( ) ( ) y x x a ? ? ? ? ? ? ? Also 1 ( ) ( ) ( ) 2 a a f x y x dx f x dx ? ? ? ? ?? ? ?? ? ? ? ? Assuming f(x) to be continuous at x = a and when in the infinitesimal integral x a ? ? ? ? ? ? , f(x) may be assumed to be a constant, we get 0 ( ) ( ) lim ( ) ( ) a f x x a dx f x y a dx ? ? ?? ?? ? ?? ?? ? ? ? ? ? 0 lim ( ) ( ) a a a f a dx f a ? ? ?? ? ? ? ? ?? ? ? ? ? Fig. 6.6 ? 1 < ? 2 < ? 3
220 ? NSOU ? CC-PH-04 Therefore, the distribution ( ) y x ? in the limit 0 ? ? represent of ? -function. b. Gaussian representation of the ?-function. A Gaussian is denoted by 2 ( ) 2 2 2 1 ( ) ; 0 2 x a y x e ? ? ? ? ?? ? ? ? , again as ? decreases the Gaussian becomes sharper and in the limit 0 ? ? and will get a ? -function. Also the integral, ( ) 1 y x dx ?? ?? ? ? Further it has a width ? and at x = a it has a value 2 1 2?? . So, 2 ( ) 2 2 2 0 1 ( ) lim 2 x a x a e ? ? ? ?? ? ? ? ? 6.5.1 Properties or characteristics of delta function 1. ( ) ( ) x x ? ? ? ? (6.7) It states that the delta function is an even function of x. 2. ( ) 0 x x? ? (6.8) Since, if we take a continuous function f(x) and find that ( ) ( ) 0 f x x x dx ? ? ? . This shows that ( ) x x? as a factor in the integral is equivalent to zero. 3. 1 ( ) ( ), 0 x x a a ? ? ? ? (6.10) We consider ( ) ( ) (0) ( ) . f x ax dx f ax dx ? ? ?? ?? ?? ?? ? ? ? Now putting ( ) (0) , ( ) l f l ax f x l a a ? ? ?? ?? ? ? ? (6.11) Fig. 6.7
NSOU ? CC-PH-04 ? 221 But we have ( ) ( ) (0) f x ax dx f ? ?? ?? ? ? (6.4) Comparing equation (6.11) and (6.4) ( ) ( ) ( ) l ax ax

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a a ? ? ? ? ? 4. ? ? ? ? 2 2 1 ( ) ( ) , 0 2 x a x a x a a a ? ? ? ? ? ? ? ? ? (6.12) We have ? ? ? ? 2 2 ( )( ) x a x a x a ? ? ? ? ? ? ? 1 1 ( ) ( ) x a x a x a x

a ? ? ? ? ? ? ? ?
using (6.4) Hence,
considering an arbitrary continuous
function f(x), we can write, ? ? 2 2 ( ) ( ) ( ) ( ) ( )
f x f x f x

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x a dx x a dx x a dx x a x a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 ( ) ( ) 2 f a f a

a ? ? ? (6.13)
using equation (6.10). The right hand side of the equation (6.13) can be written as 1 ( ) ( ) ( ) ( ) 2
f x x a dx f
x

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x a dx a ? ? ? ? ? ? ? ? ? ? ? ? Hence we get ? ? ? ? 2 2 1 ( ) ( ) 2 x a x a x a a ? ? ? ? ? ? ? ? ? 5. ( ) ( ) ( ) ( ) f x x a f a x a ? ? ? ? ? (6.14) Since ( ) 0 x a ? ? ? at x = a

but is zero for all other value of x, the product ( ) ( ) f x x a ? ? will remain non-zero for x = a and will result in ( ) ( ) f a x a ? ? .

222 ? NSOU ? CC-PH-04 6.5.2 Delta function in Three Dimension The three dimensional delta functions are defined as : 3 ( ) 0, for 0 ( ) 1 r r and r d r ? ? ? ? ? ? ? ? ? ? ? (6.15) Equation (6.4) and (6.5) in three dimensional forms are 3 ( ) ( ) (0) f d f ? ? ? r r r (6.16) 3 ( ) ( ) ( ) f d f ? ? ? ? 0 0 r r r r r (6.17) Key Words Delta functions, unit impulse function. 6.6 ? Summary ? Dirac delta function is defined and explained. Shift of origin considered. ? Rectangular and Gaussian representation discussed. ? Listed the properties of Dirac delta functions.

NSOU ? CC-PH-04 ? 223 Unit 7 ? ? Matrices Structure 7.1 Objective 7.2 Introduction 7.3 Definition, Notation and Terminology 7.4 Complex Matrices 7.5 Matrix Algebra 7.6 Characteristic Equation of a Square Matrix : Eigenvalues and Eigen Vectors 7.6.1 Some Theoretical Aspects of Eigenvalues and Eigen Vectors of Matrices 7.7 Diagonalisation 7.8 Solutions of Systems of Linear Homogeneous and Non-Homogeneous Equations 7.8.1 We consider a set of m non-homogeneous linear equations in unknowns : (m > n) 7.8.2 Solutions of homogeneous equations : (m = n) 7.8.3 Solutions for non-homogeneous system of equations : (m = n) 7.9 Solutions of Coupled Linear Ordinary Differential Equations in Terms of Eigenvalue Problem 7.10 Functions of a Matrix 7.10.1 Functions of a diagonalizable matrix 7.10.2 Powers of a matrix 7.11 Cayley-Hamilton's Theorem 7.11.1 Evaluation of Functions of Any Matrix, Diagonalisable or not, Using Cayley-Hamiltonian Theorem 7.11.2 Inner Product 7.12 Summary

224 ? NSOU ? CC-PH-04 7.1 ? Objectives In this chapter we shall discuss various arithmetic operation with matrices covering various terminologies and notation. We shall define a number special matrices which frequently occur in physics and discuss methods of matrix algebra that are useful in solving a system of linear equations in some unknowns. 7.2 ? Introduction Historically study of matrices arose in connection with, successive linear transformtions in vector spaces. The simplest of such transformations are the linear transformtions of components of vectors under rotation of co-ordinate axes as discussed in chapter 4 : 3 1 i ij j a ? ? ? ? j A A (7.1) Where i ? A are the components of vector A in the new co-ordinate system and A j are those in old co-ordinate system. where 2 2 1(1,2,3) ? ? ? ? ? ij ji j j a a i (7.2) 0 ( ) ij ik ji ki i i a a a a j k ? ? ? ? ? (7.3) Now we consider a further linear transformation of the co-ordinate system in which the same vector has components , i ?? A which are linearly related to the components i ? A by 3 1 i ik k k b ? ? ?? ? ? ? A A (7.4) It is possible to eliminate the intermediate co-ordinate system and obtain a transformation directly from the components A i to i ?? A 3 3 1 1 i ik kj j k a a ? ? ?? ? ?? j A A (7.5)

NSOU ? CC-PH-04 ? 225 3 3 1 1 ik kj j k b a ? ? ? ? ?? j A 3 1 ij j c ? ? ? j A (7.6) Where 1 1 , n ij ik kj k c b a i j n ? ? ? ? ? In dealing with such transformation it is convenient to introduce the concept of matrices. Now using equation (7.1) and (7.4) we get (7.5), which is the result of two successive linear transformation in the vector space. In fact it is in the study of such successive linear transformations that the branch of matrix algebra historically developed. For a proper understanding of the basic concept of quantum mechanics, a sound foundation in matrix algebra is essential. Matrices occurs in physics mainly two ways: first in the solution of linear equation and second, in the solution of eigenvalue problems in classical and quantum mechanics. In this chapter we shall discuss various arithmetic operation with matrices covering various terminologies and notation. We shall define a number special matrices which frequently occur in physics and discuss methods of matrix algebra that are useful in solving a system of linear equations in some unknowns. 7.3 ? Definition, Notation and Terminology A rectangular array of numbers (real or complex) is called a matrix. The array consists of m rows and n colomns. The individual members of the array are called the elements. Sometime the elements may be functions like f 1 (x) etc. If a matrix has m rows and n columns, the matrix is of order m × n (called m by n). A general m by n matrix can be written

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as, 11 12 1 21 22 2 1 2 ... ... ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? n n m m mn a a a a a a a a a

A (7.1)
226 ?
NSOU ? CC-PH-04 Or ij m n a ? ? ? ? ? ? A (7.8), is shorthand notation. Terminologies : 1. Row matrix : If there be only one row of elements in the matrix, it is called a row matrix. Thus A = [a, b, c, d], is a row matrix of order 1 × 4. 2. Column matrix : A matrix having elements in one column only is called a column matrix. Thus ? ? , , , p q p q r s r s ? ? ? ? ? ? ? ? ? ? ? ? ? ? A , to save space, is a column matrix of order 4 × 1. 3. Null matrix : The matrix A of arbitrary order is said to be a null (or zero) matrix if, and only if, every element of A equals zero. We denote a null matrix by 0. Thus if A = 0, then ij m n a ? ? ? ? ? ? A = 0, thus 0 0 0 , 0 0 0 ? ? ? ? ? ? ? A is a null matrix of order 2 × 3. It is evident that for any arbitrary matrix A – (– A) = 0 4. Negative matrix : – A is the negative matrix of A, when sign of all the element of A is reversed. If a b c d ? ? ? ? ? ? ? ? A , then a b c d ? ? ? ? ? ? ? ? ? ? ? ? A 5. Transpose of matrix : If the rows and the columns of a matrix are interchanged, the resulting matrix is called the transpose of the former matrix e.g. if 2 4 6 3 5 7 1 3 5 ? ? ? ? ? ? ? ? ? ? ? ? A
,
NSOU ? CC-PH-04 ? 227 the transpose of A i.e. A T or ? A (called A-tilde) is given by, 2 3 1 4 5 3 6 7 5 ? ? ? ? ? ? ? ? ? ? ? ? A . In notation ij m n a ? ? ? ? ? ? A then ji a ? ? ? ? ? ? T n m A 6.

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Square matrix : If the number of rows and columns of a matrix are equal

i.e. m = n, the matrix is said to be a square matrix. If A is a square matrix of order n × n we say that A is of order n. Square matrices gives rise to various types of special matrices which frequently occur in physics. For example 2 3 4 2 5 6 0 9 0 ? ? ? ? ? ? ? ? ? ? ? ? ? A is a square matrix of 3 × 3. 7. Diagonal matrix : A square matrix having all its non-diagonal elements as zero is called a diagonal matrix. Let ij n a ? ? ? ? ? A be a square matrix of order n. The elements a 11 , a 22 , a 33 , ... a nn form the principle diagonal of the matrix. The elements a ii are called the diagonal elements of the equare matrix A. All the remaining elements a ij for i j? are called the off-diagonal elements. Thus in a diagonal matrix A, 0, 0, ii ij a a ? ? or in short ij ij ij n a a ? ? ? ? ? ? . For example : 3 0 0 0 5 0 0 0 4 ? ? ? ? ? ? ? ? ? ? ? ? A , is a diagonal matrix of 3 × 3. 8. Scalar matrix : If the elements of a diagonal matrix are all equal, then the matrix is called a scalar matrix. Thus a ii = x, a ij = 0, 0 0 0 0 0 x x x ? ? ? ? ? ? ? ? ? ? ? ? A , is a scalar matrix of order 3 × 3.
228 ? NSOU ? CC-PH-04 9. Unit matrix : If the elements of a diagonal matrix are all equal to unity, then the matrix is called a unit or identity matrix i.e. a ii = 1, a ij = 0; A = I = 1 0 0 0 1 0 0 0 1 ? ? ? ? ? ? ? ? ? ? , is a unit matrix or order 3 × 3. 10. Singular matrix : A square matrix A is a singular matrix, if det A = 0. Thus 2 5 19 1 2 4 3 2 0 ? ? ? ? ? ? ? ? ? ? ? ? ? ? A , is a singular matrix, since det A = 0, if det 0?A , then the matrix is called a regular or non-singular matrix. 11. Determinant of a matrix : The determinant whose elements are corresponding elements of

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a square matrix A is called the determinant of matrix A and denoted by det A or |A|.

Thus if 1 2 3 2 3 4 3 4 5 ? ? ? ? ? ? ? ? ? ? ? ? A , then det 1 2 3 2 3 4 3 4 5 ?A .
Now we see that det A = 0 i.e. A is a singular matrix. Again let 1 1 4 3 4 9 6 5 6 2 ? ? ? ? ? ? ? ? ? ? ? ? A and we see det 1 1 4 3 4 9 6 0 5 6 2 ? ? A , therefore A 1 is a non-singular matrix. 12. Triangular matrices : A square matrix in which all the elements below the principle or

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leading diagonal are zero is called an upper triangular matrix.

If however, all the elements above the principle diagonal of a square matrix are zero, then it is called a lower triangular matrix. For example. 1 1 2 2 3 0 4 5 2 0 0 0 4 0 0 0 7 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A u
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Is an upper triangular matrix and 1 2 0 0 5 3 0 2 0 2 ? ? ? B L , is an lower triangular matrix. 13. Symmetric matrix : If a square matrix A = [a ij] such that A T = A (7.9) i.e. [a ij] = [a ji] for all i, j. Then the matrix A is called a symmetric matrix.

For example : if a h g h b f g f c ? ? ? ? ? ? ? ? ? ? ? ? A , a h g h b f g f c ? ? ? ? ? ? ? ? ? ? ? ? ? T A i.e. ij ji a a ? ? ? ? ? ? ? ? ? ? for all i,j . 14.

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Skew-symmetric or anti-symmetric matrix : A square matrix A = [a ij] is called a skew-symmetric or an anti-symmetric matrix if– i) a ij = – a ji for all values

of i, j ii) a ii = 0 i.e. all the leading diagonal elements are zero. Above two properties are satisfied if A T = – A (7.10) for example the matrix : 0 0 0 h g h f g f ? ? ? ? ? ? ? ? ? ? ? ? ? ? , is a skew-symmetric matrix or anti-symmetric matrix. Any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix in the following manner : 1 1 2 2 ? ? ? ? ? ? ? ? ? ? ? ? ? T T A A A A A (7.11) Where first part in R.H.S is a symmetric matrix and the second part is skew-symmetric. 15. Constant matrix : If all the non-vanishing elements of a diagonal matrix happen to be equal to each other, it is said to be a constant matrix. The elements of a constant
230 ? NSOU ? CC-PH-04 matrix are thus given by ij ij n A a a? ? ? ? ? ? ? ? (7.12), where = a ij = a = constant for all i, j and a is a scalar when a = 1, we get unit matrix. Thus A = aI (7.13) This shows that a constant matrix is a constant multiple of the unit-matrix. 16.

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Equal matrices : Two matrices A and B are said to be equal if and only if a ij = b

ij for all values of j. This requires that i) they are of the same order and ii) they have their corresponding element identical. A = [a ij], B = [b ij] 7.4 ? Complex Matrices Study of complex matrices with complex elements is useful in quantum mechanics. 17. Conjugate matrix : A be a given matrix having complex elements, then the conjugate matrix of A, written A*, is the matrix whose corresponding elements are the complex conjugates of the elements of A. That is if ? ? ? ? ? ? ij a m n A , then * ? ? ? ? ? ? ij a m n A* . Also if c is any scalar, then, ( )* * * c c? A A (7.14) For matrix A whose elements are real numbers, the conjugate matrix A* = A. 18. Hermitian conjugate : When the two operations of complex conjugation and transposition are carried out one after another on a matrix, the resulting matrix is called the Hermitian conjugate of the original matrix and will be denoted by A + (called A-dagger). The order of the two operation is immaterial,
thus

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A + = (A*) T .... (7.15) For example : 2 3 2 3 2 3 ? ? ? ? ? ? ? ? ? ? ? ? i i A i ? ? ? ? * 2 3 2 2 3 2 * 2 3 3 2 3 3 T ? T i i i i A A A* i i 19. Hermitian matrix : A complex square matrix A = [a ij] is called Hermitian matrix if (A*) T = A or A + = A. Example : 0 0 * 0 0 A A ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i i i i NSOU ? CC-PH-04 ? 231 ? ? 0 0 T ? ? ? ? ? ? ? ? ? ? i A* A i Or, A + = A i.e. A is Hermitian.

Every diagonal element of a Hermitian matrix must be real ; since A + =
A,

or, ? ? ? ? ii ii a a ? ? Or, ? ? ? ? * T ii ii a a? or, ? ? * ii ii a a? ? a ii is real. 20. Symmetric and Hermitian matrix We consider equality of A, A T and A + . The equality of A and A T or of A and A + will be defined only if A is a square matrix with m = n. We then get the following four special matrices e.g. symmetric, anti-symmetric, Hermitian, anti-Hermitian. Symmetric : A = A T ; Hermitian : A = A + ; Anti-symmetric : A = − A T ; Anti-Hermitian (skew-Hermitian) : A = − A + . 21. Orthogonal matrix : A square matrix A is called orthgonal if AA T = I, where I is an identity or unit matrix. Since AA T = I, A T = + A −1 i.e. if transpose of A is equal to its inverse. The orthogonality condition of two square matrices is that the determinant. |AA T | = I Example : cos sin sin cos ? ? ? ? ? ? ? ? ? ? ? ? A A matrix A satisfying the relations AA T = I n (7.16a) A T A = I m (7.16b) Where I n and I m are two unit matrices, not necessarily of the same order, is called an orthogonal matrix. Again it can be shown that if A is a finite matrix satisfying both equations (7.16), then A must be square and AA T = I, ? AA T A = I. 232 ? NSOU ? CC-PH-04 Let det A = d. Taking the determinants of both sides of equation 7.16a, we have 2 1 1 d d ? ? ? ? . This shows that the determinant of an orthogonal matrix can only have values +1 or −1. At the same time this shows that A is non-singular, so that A −1 exists. Multiplying equation 7.16a by A −1 from the right, we have A −1 = A T . 22. Unitary matrix : A complex square matrix A is said to be unitary [not unit or identity matrix], if A −1 = A + . Therefore AA −1 = AA + = I So, if the product of the matrix and its Hermitian conjugate is an identity matrix, it is a unitary matrix. 7.5 ? Matrix Algebra Matrix algebra is different from ordinary algebra in as much as vector algebra is different from a scalar algebra. We ordinarily indicate a matrix by a bold face letter like A or B etc. but the later does not have a numerical values, it simply stands for the array. The various operation of addition, subtraction, multiplication etc. on matrices are called its algebra. 7.5.1 Addition
of Matrices :

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If A and B be two matrices of the same order then		

their sum A + B is defined as the matrix whose elements are obtained by adding the corresponding elements

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of A and B. If A = [a ij] and B = [b ij], then A + B = [a ij + b ij]. (7.17) Therefore the sum of two matrices,		

each of order m × n, is a matrix of the same order m × n with each element being the sum of the corresponding elements of the given matrices. Thus it is evident that matrices are useful in case which are added by components, for example, vectors. To explain, suppose, 1 1 1 2 2 2 1 1 1 2 2 2 and a b c a b c d e f d e f ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B and 1 2 1 2 1 2 1 2 1 2 1 2

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a a b b c c d d e e f f ? ? ? ? ? ? ? ? ? ? ? ? ? ?			

A B
NSOU ? CC-PH-04 ? 233 Suppose the column in A and B represent the displacement of three particles. Then first particle has displacement ia 1 + jd 1 in A, the first particle has displacement at a later time ia 2 + jd 2 B. Then the total displacement of the first particle is i(a 1 + a 2) + j(d 1 + d 2) is the first column of matrix A + B. Similarly the second and third columns represent displacement of the second and third particles. Properties of matrix addition : 1< Matrix addition is commutative :

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A + B = B + A 2< Matrix addition is associative : A + (B + C) = (A + B) + C			

Two matrices must be conformable for addition,; by conformity we mean both the matrices must

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have the same number of rows and the same number of columns i.e. they are of the

same order. 7.5.2 : Scalar multipliction of a matrix : If a matrix A is multiplied by a scalar quantity k, the each element of A gets multiplied by k. example : if A = [a ij], then kA = [ka ij]. In determinants however only one row or one column is multiplied and not every elements of the determinant. It is known that the components of a vector A = ai + bj may be conveniently written as elements of a matrix either a b ? ? ? ? ? ? ? A , called a column matrix or a column vector or A T = (a b) called a row matrix or row vector. The row matrix A T is

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the transpose of the matrix A. Now a vector twice the length of A is 2 2 2 2 & 2 (2 2) 2 a a a b a

b b b ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? T i j A 7.5.3 : Matrix Multiplication : Let a b e f and c d g h ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B Then a b e f ae bg af bh c d g h ce dg cf dh ? AB C
234 ? NSOU ? CC-PH-04 Thus all the elements of C may be obtained by using the following simple rule. The element in row m and column n of the product matrix AB is equal to row m of A times column n of B. In notation ? ? mn k ? ? mk kn AB A B (7.18) We consider the elements in a row or a column of a matrix as the components of a vector. Then row times column multiplication for the matrix product AB corresponds to finding the dot product of the row vector of A and a column vector of B e.g. (ai + bj). (ei + gj) = ae + bg The product AB (maintaining the order) can be found if and only if the number of elements in a row of A equals the number of elements in a column of B: the matrices A,B in that order are then called conformable. The number of rows in A and of column in B have nothing to do with the question of whether we can find AB or not. Matrix multiplication is not commutative i.e. in general, matrices do not commute under multiplication. We define the commutator of the matrices A and B by [A, B] = AB – BA = commutator of A and B (7.19) If [A, B] = 0, then A and B commutes (in some special matrices). Commutators are widely used in classical and quantum mechanics. Properties of matrix multiplication : 1. Non-commutative : ? AB BA in general 2. Associative : A(BC) = (AB)C = ABC 3. Distributive : A(B + C) = AB + AC and (A + B)C = AC + BC 4. AI = IA = A, where I is a unit matrix conformable with A. 5. AA –1 = A –1 A = I provided 0?A 6. [AB] T = B T A T 7. If A.B = 0, it does not mean necessarily that A or B is a null matrix. 8.

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If A and B are two square matrix of the same order then

det AB = detBA = (det A) (det B) even when A and B do not commute.
NSOU ? CC-PH-04 ? 235 7.5.4 : Adjoint and inverse of a matrix : The cofactor of an element in a square matrix A means exactly the same thing as the cofactor of that element
in det A since if 1 1 1 1 1 1 2 2 2 2 2 2 2 3 3 3 3 3 3 ,

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a b c a b c a b c a b c a b c a b c ? ? ? ? ? ? ? ? ? ? ? ? A A The matrix obtained from the cofactor of |A| is given by C, where 1 1 1 2 2 2 3 3 3 A B C A B C A B C ? ? ? ? ? ? ? ? ? ? ?

C
The transpose of the matrix of the co-factor is 1 2 3 1 2 3 1 2 3 ? ? ? ? ? ? ? ? ? ? A A A B B B C C C =

adjoint of matrix A = adj A = C T , (7.20) Now we define the inverse of a matrix A as the matrix A −1 such that AA −1 and A −1 A are both equal to unit matrix I. It is to be noted that only square matrices can have inverse and acatually some square matrices do not have inverse either. Now if A −1 A = I, then (det A −1) (det A) = det I = 1. If two numbers have product equal to one, then neither of them is zero. Thus det 0?A is a requirement for A to have an inverse. Thus the condition for a square matrix A to have an inverse is that A is non-singular i.e. 0?A . If a matrix has an inverse, it is called an invertible and if it does not have an inverse, it is called singular. If A be an invertible matrix, then the inverse of matrix A is given by, 1 adj ? ? ? T A C A A A (7.21)
236 ? NSOU ? CC-PH-04 7.5.5 : Properties of inverse of a matrix : 1. Inverse of matrix is unique. 2. Every matrix commutes with its inverse, i.e. AA −1 = A −1 A = I 3. Inverse of the product of a number of matrices (all square and of the same order), none of which is singular, equals the product of the inverses taken in the reverse order i.e. (ABC) −1 = C −1 B −1 A −1 4. If A be an invertible matrix, then A −1 is invertible and (A −1) −1 = A. 5. Inverse of the transpose of a square matrix is the transpose of its inverse, i.e. [A −1] T = [A T] −1 7.5.6 : Properties of orthogonal matrix : [see Art 7.2.2., item 21] 1. Every orthogonal matrix is non-singular i.e. if A is an orthogonal matrix, 0?A 2. Unit matrix is an orthogonal matrix. 3. If A is an orthogonal matrix, then its determinant, 1 ? ? A 4. The product of two orthogonal matrices is also orthogonal. 5. The transpose of a orthogonal matrix is also orthogonal. 6. The inverse of an orthogonal matrix is also orthogonal. 7.5.7 : Properties of unitary matrix : [see 7.4 : item no 22] A matrix U satisfying the relations, UU + = I n ? (7.22a) U + U = I m ? (7.22b) is called a unitary matrix. If U is a finite matrix satisfying both equations (7.22), then U must be a square matrix, and ? ? ? ? ? UU I U U I . The elements of a unitary matrix may be complex. In fact it is evident from equations (7.22) that a real unitary matrix is orthogonal. Let det U = d. Taking the determinants of both sides of equation (7.22a) and noting that U + = d*, we have * 1 1 ? ? ? dd d
NSOU ? CC-PH-04 ? 237 This shows that the determinant of a unitary matrix can be a complex number of unit magnitude, i.e. a number of the form e i? , where ?? is real. It also shows that a unitary matrix is non-singular and possess an inverse properties : 1) The inverse of a unitary matrix is unitary. 2) The Hermitian conjugate of a unitary matrix is its inverse i.e. U + = U −1 3) The product of two unitary matrices is also unitary. 4) A unitary matrix with elements as real numbers is orthogonal. 7.5.8 : Trace of matrix : The trace of a square matrix A = [a ij] is defined as the sum of its diagonal elements. It is also called spur or the diagonal sum and is denoted by T r A or S p A. Thus, 1 n r ii i T a ? ? ? A (7.23) Properties : 1) The trace of sum (or difference) of two matrices is the sum (or difference) of their traces. 2) The trace of the product of two matrices A and B is independent of the order multiplication i.e. T r (AB) = T r (BA). This property is true even when AB ? BA and the above equation implies that the trace of any commutator [A, B] is zero. 3) The above equation also gives, T r (ABC) = T r (BCA) = T r (CAB) i.e. the trace is the invariant under cyclic permutation of matrices in a product. It is important to note that trace of a number of matrices is not invariant under any permutation, but only under a cyclic permutation of

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the matrices. 7.5.9 : Rank of a matrix : An integral number r is said to be the Rank of a matrix A; if, i. There is at least one

square sub-matrix of A of order r whose determinant is non- zero. ii. All the square sub-matrices of A of order (r + 1), have determinants zero.
238 ? NSOU ? CC-PH-04 Generally speaking the rank r of a matrix is the largest order of any non-vanishing minor of the matrix. Example : Let 1 2 3 2 3 4 3 4 5 ? ? ? ? ? ? ? ? ? ? ? ? A be a matrix of 3 rd order and |A| = 0. Since |A| is zero, the rank of A is not 3. But, there is at least one 2 × 2 sub-matrix 3 4 4 5 ? ? ? ? ? ? whose determinant is not zero. In fact none of the minors is zero. So, the rank of A i.e. r(A) = 2. We observe the following : 1)

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The rank of a non-singular square matrix of order n is n and that of a singular square matrix of order n is less than n. 2) The rank of a null matrix is obviously zero. 3) The rank of the transpose of a matrix A is the same as the rank A. 4) The rank of product of two matrices never exceeds the rank of either matrix. 5) The rank of a matrix is not altered, (i) If interchange of rows is made, (ii) If the elements of any row are multiplied by a non-zero number, (iii) If ? times the elements of a row is added to corresponding elements of another row, ? being any number, (iv) If the matrix is pre or post multiplied by a non-singular matrix. Sub-matrices and rank : Let A be a matrix of order m × n. Any matrix obtained from A by omitting some of its rows or columns is called a sub-matrix A. When a matrix is partitioned into a number of blocks, each block is a sub-matrix of the original matrix. If a matrix A has at least one square non-singular sub-matrix of order r but every square sub-matrix of A of order greater than r is singular, then r

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is said to be the rank of the matrix A. The rank of the matrix A

given below is 3 because it has non-singular square sub-matrices of order 3 but not higher than 3. 3 5 9 1 2 0 1 3 3 6 1 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? A
NSOU ? CC-PH-04 ? 239 Example : Find the rank of the matrix : 1 0 2 4 3 1 1 2 5 1 5 6 4 1 3 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Solution : We note that the determinant of the given matrix is zero. So that its rank is less than 4. Next we consider all possible sub-matrices of order 3 (there are 16 of them) and observe that all of them are also singular, so that rank is less than 3. Finally, we note that the given matrix has non-singular sub-matrices of order 2, so that its rank is 2. A simple method to find the rank of a matrix is to find the maximum number of linearly independent row vectors or column vectors of the matrix. This maximum number is the rank of the matrix. 7.5.10 : Normal form : A matrix which commutes with its own Hermitian conjugate is said to be a normal matrix or in normal form. Thus if matrix A is in normal form if and only if [A, A +] = 0 (7.24) It can be easily seen that symmetric, antisymmetric, Hermitian and antihermitian matrices are also normal matrices. For this reason they often occurs in physics. Properties : 1) The inner product of the i th and j th rows of the normal matrix equals the inner product of the i th and j th columns. 2) The norm of the i th row of a normal matrix equals that of the i th column. 7.6 ? Characteristic Equation of a Square Matrix : Eigenvalues and Eigen vectors of Matries Let A be a square matrix of order n and X a non-zero column vector. If there exists a scalar ? such that AX = ?X (7.25)
240 ? NSOU ? CC-PH-04 Then the vector X is defined as an eigenvector and ? is defined as an eigenvalue corresponding to the eigenvector X. Equation (7.25) is called eigenvalue equation and may be written as ( ) 0 ? ? ? ? ? ? AX IX A I X (7.26) Characteristic matrix : For a given square matrix A, the matrix ( )??A I is called the Characteristic matrix of A, where ? is a scalar parameter and I the unit matrix of the same order. Characteristic polynomial : The determinant ??A I , an expansion will give rise to a polynomial and is known as the

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characteristic polynomial of matrix A. Characteristic equation : The equation ??A I = 0 is known as the characteristic equation of matrix A

determines the eigenvalues of the matrix A. Eigenvectors or characteristic vectors : For each eigenvalues ? , we have a non- zero column vector X that satisfies the equation ( ) 0 ?? ? A I X (7.27) The non-zero vector X is known as the eigenvector or the characteristic vector. Orthogonal vectors : Two vectors X 1 and X 2 are said to be orthogonal vectors if the condition 1 2 0 ? ?

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X X is satisfied. Let 1 4 1 2 2 5 3 6 x x x and x x x ? X X ? ? 4 1 2 1 2 3 5 1 4 2 5 3 6 6 0 x x x x x x x x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? X X

Normalised form of vectors : With the condition 1 1 1 ? ? X X we can find out normalised form of vector X 1 , similarly from the condition 2 2 1 ? ? X X , we can find out nromalised form of X 2 . 7.6.1 : Some theoretical aspects of Eigenvalues and Eigenvectors of matrix Theorem I : The determinant of matrix A is the products of its eigenvalues. Proof : We have 0 ?? ? A I (7.28)
NSOU ? CC-PH-04 ? 241 Equation (7.28) can be expanded as 11 12 1 21 22 2 1 2 ... ... 0 ... ...

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n n n n nn a a a a a a a a a ? ? ? ? ? ? ? (7.29) or, ? ? 1 1 (1) ... 0 n n n n ? ? ? ? ? ? ? ? ? ? (7.30)

Where 1 2 , ... n ? ? ? are constants expressed in terms of the coefficient a ij of the matrix A given by equation (7.7). (see Art 7.2.1) The identity ? ? 1 1 (1) ... n n n n ? ? ? ? ? ? ? ? ? ? ? ? A I (7.31) If 1 2 , ... n ? ? ? are the eigenvalues of A, the roots of the right hand side polynomial of (7.31) will be 1 2 , ... n ? ? ? Therefore ? ? ? ? ? ? 1 2 ... n ? ? ? ? ? ? ? ? ? ? ? ? A I (7.32) Putting 0 ? ? on both sides 1 2 ... ? n ? ? ? A (7.33) Hence theorem is proved. Also if any of the eigenvalues of A is zero, then |A| = 0 i.e. the matrix A is singular. Theorem II : If A be a square matrix, then its trace is the sum of its eigen values. Proof : If A be a square matrix of order n, then 1 ( ) n r ii i T a ? ? ? A Now if 1 2 , ... n ? ? ? be eigenvalues of A, then we have the identity ? ? 1 1 (1) ... n n n n ? ? ? ? ? ? ? ? ? ? ? ? A I (7.34)
242 ? NSOU ? CC-PH-04 The coefficient of 1 n ? ? in L.H.S is (1) (1) n n ii i i a ? ? ? ? (7.35) and the coefficient of 1 n ? ? in R.H.S. is (1) (1) n n i i i ? ? ? ? ? (7.36) Equating these quantities, we have (1) (1) (1) (1) n n n n ii i i i i i a ? ? ? ? ? ? ? ? ? ? Therefore 1 n n ii i i i i a ? ? ? ? ? ? Thus the theorem is proved. 7.7 ? Diagonalization We have a square matrix A of order n which we have to diagonalize. We now construct a matrix P of order n whose columns are the eigen vectors of the given matrix A. Since the eigenvectors are linearly independent, P is non-singular. Therefore P –1 exists. Now we state that the matrix P –1 AP is diagonal whose diagonal element are the eigenvalues of A. Example 1 : Consider the matrix 3 3 i i ? ? ? ? ? ? ? ? A . Find the eigenvalues and eigen vectors and hence construct the unitary matrix U that diagonalize A. Also compute U –1 AU. Solution : The characteristic equation of matrix A is 3 0 3 i i ? ? ? ? ? ? ? ? ? A I Or, ? ?? ? 2 4 0 ? ? ? ? ?
NSOU ? CC-PH-04 ? 243 ? the eigen values are 1 2 2, 4 ? ? ? ? for 1 2; 2 , ? ? ? ? ? ? ? ? ?

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x y ? AX X X so that 3 2 2 3 i x x x i y y y ? ?? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? I or, 3 2 0 0 3 0 2 i x i y ? or, 1 0 1 i x i y ? ?? ? ? ? ? ? ?? ? ? ? ? ?? ? or, 0 0 x

iy ix y ? ? ? ? ? ? ? ? Both the equations are equivalent. Choosing x = 1, the convenient eigenvector 1 1 i ? ? ? ? ? ? ? ? X . Similarly for 2 4 ? ? , the eigenvector is 2 1 i ? ? ? ? ? ? ? ? ? X Normalised eigenvectors are 1 1 2 i ? ? ? ? ? ? ? and 1 1 2 i ? ? ? ? ? ? ? Therefore 1 1 1 2 i i ? ? ? ? ? ? ? ? ? U 1 1 1 2 i i ? ? ? ? ? ? ? ? ? ? U 1 0 0 1 ? ? ? ? ? ? ? ? ? ? ? UU I thus U is unitary.. now 2 0 i ? ? ? U
244 ? NSOU ? CC-PH-04 1 1 1 1 1 1 1 2 2 i i i i i ? U 1 1 3 1 1 1 1 1 2 0 1 1 3 2 2 0 4 2 i i i i i i i i i i ? ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? D U AU Therefore

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D is a diagonal matrix whose diagonal elements are the eigenvalues of A.

Example 2 : A square matrix 0 0 i i ? ? ? ? ? ? ? ? ? A is given. Find the eigenvalues and eigenvectors of the matrix A. Construct an appropriate matrix which will diagonalize A and find the diagonal matrix. Solution : Characteristic equation : 0 0 i i ? ? ? ? ? ? ? ? ? ? ? A I Therefore eigenvalues are 1 ? ? ? Using the equation : ? ? 0 ?? ? A I X , we get the eigenvectors 1 2 x x ? ? ? ? ? ? ? ? X For 1 2 1 1, 0 1 x i x i ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? or, – x 1 – ix 2 = 0, ix 1 – x 2 = 0 These equations are not independent so that x 1 and x 2 are not unique and infinitely many solutions can be obtained. If we take x 1 = 1 we get x 2 = i Therefore, eigenvector corresponding to 1 ? ? is 1 1 i ? ? ? ? ? ? ? ? X and that corresponding to 1 ? ? ? is 2 1 i ? ? ? ? ? ? ? ? X Now we construct a 2 × 2 matrix P with the column vectors X 1 and X 2 Therefore 1 1 i i ? ? ? ? ? ? ? ? ? P

NSOU ? CC-PH-04 ? 245 Therefore 1 1 1 1 2 2 T Adj Adj i i ? ? ? ? ? ? ? P P P C P Where C is the cofactor matrix of A and is given by 11 12 21 22 c c c c ? ? ? ? ? ? ? C Where 11 12 21 22 ( ); ( ), (1), (1) c i c i c c ? ? ? ? ? ? ? ? ? 1 1 i i ? ? ? ? ? ? ? ? ? ? ? C Therefore 1 1 1 1 T T i i i i ? C Therefore 1 1 1 1 2 i i i ? ? ? ? ? ? ? ? ? ? ? ? ? P Now, 0 1 1 0 i i i i ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? AP 1 1 1 1 2 0 1 0 1 1 1 0 2 0 1 2 2 i i i i i i i i ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? P AP D where D is the diagonal matrix with diagonal elements as the

52%	**MATCHING BLOCK 256/267**	**W**

eigen values of A. Thus we can state the following theorem. If a matrix of order n has n linearly independent

eigenvectors, then it is related through a similarity transformation to a diagonal matrix whose diagonal elements are the eigenvalues of the matrix. However the matrix P is not a unique matrix, because we could arrange the eigenvectors X 1 , X 2 in any order in the construction of P i.e. we could form P in example 2 as 1 1 i i ? ? ? ? ? ? ? ? P
246 ? NSOU ? CC-PH-04 We can therefore state the general rule that in the process of Diagonalisation P −1 AP = D, the order of the eigenvalues in D corresponds to the order of the eigenvectors of A in constructing P. The matrices A and D are said to be related by a simiarity transformation. The inverse transformation A = PDP −1 is also similarity transformations. 7.8 ? Solutions of systems of linear homogenous and non- homogenous equations : An application of theory of matrices 7.8.1 : We consider a set of m non-homogeneous linear equations in n unknowns : (m >

61%	**MATCHING BLOCK 258/267**	**SA**	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

n) 11 1 12 2 1 1 ... n n a x a x a x b ? ? ? ? 21 1 22 2 2 2 ... n n a x a x a x b ? ? ? ? ... ... 1 1 2 2 ... m m mn n m a x a x a x b ? ? ? ?

Which can be represented as, 11 12 1 1 1 21 22 2 2 2 1 2 ... ...
n n m m mm
n m

44%	**MATCHING BLOCK 259/267**	**SA**	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

a a a x b a a a x b a a a x b ? or, AX = B (7.37) and A b = 11 12 1 1 21 22 2 2 1 2 ... ... n n m m mm m a a a b a a a b a a a b ? ? ? ? ? ? ? ? ? ? ? ? ? A b = [A, B] is

called the augmented matrix of order n × (m + 1) Now, i) The equations are consistent, if rank A = rank A b
NSOU ? CC-PH-04 ? 247 ii) The solutions are unique, if rank A = rank A b = n, where n= numbers of unknowns. iii) Infinite solutions, if rank A = rank A b = r, r > n iv) The equations are inconsistent, if rank A ? rank A b 7.8.2 : Solutions of homogeneous equations : (m = n) For homogeneous system, equatiosn (7.37) can be written as AX = 0 (7.38) If 0?A then A −1 exists Pre-multiplying both sides by A −1 , we get A −1 AX = A −1 0 or, IX = 0 Which shows that X = 0 i.e x 1 = x 2 = ... = x n = 0 This is called the trivial solution. If however the matrix A is singular i.e. |A| = 0, then the equations under considerations will have infinite solutions where some solutions may be non-zero (non-trivial solution). 7.8.3 : Solutions for non-homogeneous system of equations : (m = n) A system of non-homogeneous equations is represented by : AX = B (7.39) Now if 1 0, ? ?A A exists. Therefore A −1 AX = A −1 B or, IX = A −1 B ? X = A −1 B (7.40) Thus finding the value of A −1 , we can find out the solution using equation (7.40). This method of finding out the solution however fails if A is singular. However the solution given by equation (7.40) is unique. 7.9 ? Solutions of Coupled Linear Ordinary Differentical Equations in Terms of Eigenvalue Problems We want to reduce a system of coupled ordinary differential equation to an eigenvalue problems.

248 ? NSOU ? CC-PH-04 We exemplified the process by a specific problems of discussing the vibrations of the two coupled springs shown in fig (7.1) Fig. (7.1) In the figure y 1 , y 2 are the displacements of the two masses. The equations of motion for the coupled vibrations can be written as ? ? ? ? 1 1 1 1 2 2 1 2 2 2 2 1 m y k y k y y m y k y y ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? (7.41) where the dots denote time derivatives. ? ? 1 2 1 1 2 2 1 2 2 2 2 k k m y y k m and k k m m ? y A (7.42) Equation (7.41) can be written as ? ?? y Ay (7.43) With the trial solution, t? ?y xe ? (7.44) we get from equation (7.43) We get 2 t t e e ? ? ? ? x Ax 2 ?? Ax x (7.45) We now define

NSOU ? CC-PH-04 ? 249 Equation (7.45) is the eigenvalues equation 2 ? being the eigenvalues corresponding to the matrix A with the eigenvector x. Thus 2 ? gives the frequency of oscillation and eigenvector x gives the displacement equation of the system. Now suppose m 1 = 3, m 2 = 2, k 1 = 18, k 2 = 6 Therefore 8 2 3 3 ? ? ? A (7.46) The eigen values and eigenvectors of this matrix are found to be, 2 2 1 2 2 1 9, ; 2, 1 3 x x ? The most general solution of the system is therefore, 3 3 2 2 1 1 2 1 3 2 4 2 ( ) it it it it b e b e b e b e ? ? ? ? ? ? y t x x x x or, 1 1 2 1 3 2 4 2 ( ) cos3 sin3 cos 2 sin 2 a t a t a t a t ? ? ? ?

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y t x x x x (7.47) where a and b are the arbitrary constants. 7.10 ?

Functions of a Matrix There are two methods by which a function of a matrix can be evaluated 1) Functions of a diagonalizable matrix 2) Functions of any matrix based on the existence of a minimal polynomial. 7.10.1 : Functions of a diagonalizable matrix We have P −1 AP = D, A = PDP −1 (7.48) Where A be a diagonalizable square matrix, P be the diagonalising matrix for A and D is the diagonal matrix containing the eigenvalues of A as its diagonal elements. Now if f is any function of a matrix, then we have f(A) = P f (D)P −1 (7.49) Thus, if we can define f(D), we can define and evaluate f(A).

250 ? NSOU ? CC-PH-04 7.10.2 : Powers of a matrix We have from second equation of (7.48), we taking k th power A k = (PDP −1) (PDP −1) .... (k times) = PD k P −1 (7.50) Similarly, if m = − k is a negative integer and 0?A , then A m = PD m P −1 = P(D −1) k P −1 (7.51) Example : Find A k , where k is any integer, positive or negative, where 3 3 i i ? ? ? ? ? ? ? ? A Solution : The eigenvalues of the matrix A are 2, 4. Eigen vectors are 1 i ? ? ? ? ? ? and 1 i ? ? ? ? ? ? ? Therefore 1 1 1 2 0 , 0 4 i i ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? P P AP D Now 0?A . For any integral k, therefore we have : 1 1 1 1 2 2 1 1 2 2 1 1 2 0 1 2 (1 2) 2 0 4 (1 2) 1 2 ? k k k k k i k k k i i i i i k A PD P (7.52) 7.10.3 : Roots of a matrix We consider a diagonal matrix D, whose elements are given by i ij d ? ? ? ? ? ? D . It is evident that D k is again a diagonal matrix whose diagonal elements are d i k , i.e. k k i ij d ? ? ? ? ? ? D . Now let 1 p k ? and consider a diagonal matrix D 0 whose elements are given by 0 p i ij d ? ? ? ? ? ? D . Clearly the k th power of D 0 will equal D i.e. D 0 k = D. Then consider the matrix B = PD 0 P −1 = PD P P −1 . Taking the k th power of B, we find, B k = (PD P P −1) (PD P P −1) ... (PD P P −1)(k th times) = PDP −1 = A (7.53) Thus B = PD P P −1 is a k th root of A. The same result holds good for any fractional power. Thus if q is any fraction, we have A q = PD q P −1 (7.54)

NSOU ? CC-PH-04 ? 251 7.10.4 : Series Suppose u is a series in matrix A with scalar co-efficient a k as in equation (7.55) 0 k k u a ? ? ? ? k A (7.55) Now if and only if every element of the right hand side converges then the series converges. In each case, equation (7.55) can be written as : 0 ( ) k k f u a ? ? ? ? ? k A A (7.56) Where f(A) is a matrix of the same order as A and whose elements are given by ? ? ? ? 0 ( ) k ij ij k f a ? ? ? ? k A A (7.57) Now we may state that a series f(A) in a matrix A is convergent if and only if the coresponding algebraic series ( )f ? is convergent for every eigenvalue i ? of A. Thus, if 0 ( ) k k f a ? ? ? ? ? k A (7.58) exist for R ? ? , then, 0 ( ) k k f a ? ? ? ? k A A (7.59) exists if and only if every eigenvalue i ? of A satisfy i R ? ? . R is called the radius of convergence of the series. 7.11 ? Cayley-Hamilton's Theorem The theorem can be stated as follows, The square matrix satisfies its own characteristic equation.

252 ? NSOU ? CC-PH-04 Stated mathematically : If 2 0 1 2 ... 0
n n x

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a a x a x a x ? ? ? ? ? ? ? ? A I be the characteristic equation of a square matrix A, then 2 0 1 2 ... 0 n n a a a a ? ? ? ? ? I A A A (7.60) Where every x is replaced by A, and thus 0 0 0 0 0 0 a a x a a ? ? ? A

I Thus, for any square matrix of order n,
there is a matrix polynomial of degree not more than n which equals the null matrix. If A is a square matrix where characteristic polynomial is |A − xI| = D(x), then D(A) = 0. If i ? are the eigenvalues of A, equation (7.60) can be written as ? ? 1 2 ( )...( ) 0 n ? ? ? ? ? ? ? ? A I A I A I (7.61) Example : Consider the matrix 11 10 5 5 4 5 20 20 4 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A Show that this matrix satisfies the cayley-Hamilton theorem. Solution : The eigenvalues of A are 4, −6, −1, we have (A − 4I)(A + 6I)(A + I) 15 10 5 5 10 5 10 10 5 5 0 5 5 10 5 5 5 5 0 20 20 0 20 20 10 20 20 5 ? 7.3.16.1 : Minimal polynomial In case of some matrices having eigenvalues, with multiplicity greater than unity, there may exist polynomial of degree less than n which equal the zero matrix. The following example follows the statement. Example 1 : If q p p p q p p p q ? ? ? ? ? ? ? ? ? ? ? A , where p, q are scalar and 0p ? , show that,
NSOU ? CC-PH-04 ? 253 ? ?? ? (2) ( ) 0 q p q p ? ? ? ? ? A I A I Where q − p, q − p and q + 2p are the eigenvalues of A. Solution : The eigenvalues of the given matrix are found to be q + 2p with multiplicity unity and q − p with multiplicity two, greater than unity. From cayley-Hamilton theorem, we get ? ?? ? 2 (2) ( ) 0 q p q p ? ? ? ? ? A I A I (7.62) However, now we consider the given matrix polynomial ? ?? ? 2 (2) ( ) 2 2

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p p p p p q p q p p p p p p p p p p p p ?

A I A I 0 0 0 0 0 0 0 0 0 ? ? ? ? ? ? ? ? ? ? ? (7.63) This shows that for the matrix under consideration we have got a polynomial of degree 2 which is equal to zero. i.e. ? ? 2 2 2 (2) 2 0 q p q pq p ? ? ? ? ? ? ? A A I (7.64) From equation (7.61) we see that highest degree of polynomial of A is 3. But from equation (7.64) we see that highest degree under the given condition is 2. Polynomial is equation (7.63) can be termed as minimal polynomial and polynomial in equation (7.61) is called characteristic polynomical of the matrix. In the given problem the highest degree of the minimal polynomial is evidently 3 when it is equal to the characteristic polynomial. It is also evident that minimal polynomial divides the characteristic polynomial exactly.
254 ? NSOU ? CC-PH-04 7.3.16.2 : Matrix polynomial An expression of the form 1 1 1 0 ( ) ... k k f a a a a ? ? ? ? ? ? ? ? k k A A A A I (7.65) Where a i are scalar co-efficient, is called a matrix polynomial in A of degree K. The existence of a minimal polynomial [it can be shown] for every matrix provides a greater simplification in evaluating matrix polynomials. 7.11.1 Evaluation of functions of any matrix, Diagonalisable or not, using cayley-Hamiltonian theorem If the degree of the minimal polynomial of a matrix A is m, any functions f(A), which is sufficiently differentiable, can be expressed as a linear combination of the m linearly independent matrices I, A, A 2 .... A m − 1 , i.e. f(A) = s(A) (7.66), where 1 2 1 2 ( ) m m s ? ? ? ? ? ? ? ? m m A A A 1 0 ... ? ? ? ? ?A I (7.67), where i ? are scalars which can be suitably evaluated by a process. 7.11.2 : Inner product : In some practical situation in basic science we cannot do without some concept of measurement for distance and angle between two lines etc. The inner product in some way is a generalisation of these everyday concepts. We discussed in 7.1 the way matrices occurs in physics by virtue of transformation of vectors in vector space and the operation of linear operators on vector space. A linear transformation from one set to another can be written as 1 (1,2..) (1,2,3,...,) p i ik k k i n b i n ? ? ? ? ? ? ? A A (7.68) Or as A´ = BA where B = [b ik] is now an n × p matrix. If now the variables A´ i are further related to the m variables A´´ i by a linear transformation of the form 1 (1,2...) (1,2,...,) p i ik k k i n c i m ? ?? ? ? ? ? ? A A (7.69) which can be written in the matrix notation as A´´ = CA´ where
A´´ = {A´´ i }

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is a column vector of order m × 1 and C = [C ij] is a matrix of order m ×

n.

NSOU ? CC-PH-04 ? 255 Therefore the combined transformation can be written as : ?? ? ? ? ? A CA CBA DA (7.70) The product matrix D = CB will be of order m × p. Now we define inner product space in matrix notation in line with equation (7.70) which gives a reason of definition of matrix multiplication as we have given earlier equation (7.70). A vector space V defined over a field F, where F is the field of real or complex numbers, becomes an inner product space if with every pair of element. ? and ,V ?? there is associated a unique scalar belonging to the field F, denoted by (,? ?) and called the inner product or the scalar product of (,? ?); for which the following properties hold (,) (,)* (,) * (,) (,) *(,) *(,) a b a b a b a b ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ψ ψ ψ ψ ψ ψ (7.71) These three equations in equations (7.71) together form the definition of inner product of vectors. The vector space of n-tuplets of real or complex numbers can be made an inner product space if we define the inner product of two vectors by * 1 (,) n i i i ? ? ? ? ? ψ ψ (7.72) Now when we regard vectors as column matrix, their inner product defined in equation (7.72) can also be written in a concise way in the matrix notation. If ? ? 1 2 , ,..., n ? ? ? ?ψ and ? ? 1 2 , ,..., n ? ? ? ? ? are two column vectors, 1 2 * * * , ,..., n ? ? ? ? ? ? ? ? ? ψ and ? ? 1 2 * * * 1 2 , ,..., , ,..., n n ? ? ? ? ? ? ? ? ? ? ? ? ? ψ * 1 n i i i ? ? ? ? ψ (7.73)
256 ? NSOU ? CC-PH-04 Which is the same expression in equation (7.72). Hence, the inner product can be expressed as (,)? ? ? ? ψ ψ (7.74) The orthogonality condition is now 0 ? ? ? ψ (7.73) Examples : Example 1 : Show that the matrix 1 2 2 1 2 2 i i ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A is unitary.. Solution : 1 2 2 (*) 1 2 2 T i i ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? T

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A A or, 1 2 2 , 1 2 2 i i ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A Now 1 0 0 1 ? ? ? ? ? ? ? ? ? ? A A I ? A is unitary. Example 2 : Show that the matrix cos sin sin cos ? ? ? ? ? ? ? ? ? ? ? is orthogonal. Solution : Let cos sin cos sin sin cos sin cos ? T A A Now 2 2 2 2 cos sin cos sin sin cos sin cos cos sin sin cos ?

T AA 1 0 0 1 ? ? ? ? ? ? ? ? = I = unit matrix ? A is orthogonal.
NSOU ? CC-PH-04 ? 257 Example 3 : If cos sin sin cos i i ? ? ? ? ? ? ? ? ? ? ? A show that A is unitary.. Solution :

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cos sin cos sin (*) sin cos sin cos T i i i i ? T A A cos sin cos sin sin cos sin cos i i i i ? A A 2 2 2 2 cos sin cos sin sin cos 1 0 0 1 sin cos sin

sin sin cos
i i i i ? = unit matrix ??A is unitary matrix. Example 4 : Consider the following transformation in three dimension : cos sin x x y ? ? ? ? ? sin cos y x y ? ? ? ? ? ? z´ = z i) Write down the transformation matrix ( ) A θ . ii) Show that ? ? 1 2 1 2 ( ) ( ) ? ? ? ? ? ? A A A iii) Show that ( )?A is unitary.. Solution : i) The transformation in this case is

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cos sin 0 sin cos 0 0 0 1 x x y y z z ? 258 ?

NSOU ? CC-PH-04 The transformation matrix is

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cos sin 0 ( ) sin cos 0 0 0 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A ii) ? ? 1 1 2 2 1 2 1 1 2 2 cos sin 0 cos sin 0 ( ) sin cos 0 sin cos 0 0 0 1 0 0 1 ? A A ? ? ? ? ? ? ? ? ? ? ? 1 2 1 2 1 2 1 2 1 2 cos sin 0 sin cos 0 0 0 1 ? A

iii) ? ? * 1 ( ) ( ) ( ) T T ? ? ? ? ? ? ? ? ? ? ? A A A since ( )?A is real matrix. Now ? ? ? ? cos sin 0 cos sin 0 ( ) ( ) ( ) ( ) sin cos 0 sin cos 0 0 0 1 0 0 1 T ? A A A A 1 0 0 0 1 0 0 0 1 ? ? ? ? ? ? ? ? ? ? ? ? I Exercises : Pauli matrices : The following set of three 2 × 2 matrices is called Pauli spin matrices. 1 2 3 0 1 0 1 0 1 0 0 0 1 i i ? σ σ σ Prove that : i) 2 2 i j j i ij ? ? ? σ σ σ σ I where I 2 is a 2 × 2 unit matrix, 0, 1, ij i j i j ? ? ? ? ? ? ?
NSOU ? CC-PH-04 ? 259 ii) , , , i j k i i j k ? σ σ σ is a cyclic permutation of 1,2,3 iii) 2 2 i ? σ I Keywords Adjoint, inverse, Orthogonal, Hermitian, Trace, Rank, Normal form, matrix functions, Eigen value and Eigen functions. 7.12 ? Summary 1. Definitions, notation and terminology of real and complex matrices have been discussed with examples. 2. Properties of inverse, orthogonal, unitay matrices, have been stated. 3. Procedures for Diagonalisation and to find rank of matrix have been discussed. 4. Eigen value equations have been set up and procedure to obtain eigenfunction and eigen values have been indicated. Coupled linear ordinary differential equation have been discussed in terms of eigen value problem. 5. Evaluations of function of any matrix have been incorporated.
260 ? NSOU ? CC-PH-04 Unit 8 ??????????C and C++ Programming Fundamentals Structure 8.1 Objectives 8.2 Introduction to Programming 8.3 Constants, Variable and Data Types 8.3.1 C Tokens 8.3.2 C Constants 8.3.3 Variables 8.3.4 Data Types 8.4 Operators and Expressions 8.5 I/O Statements 8.6 Manipulators for Data Formatting 8.7 Control System 8.8 Loop Statements 8.8.1. Jumping out of loops 8.8.2 Goto statement transfer of control branching within a loop 8.9 Summary 8.10 Exercises 8.11 References 8.1 ????? Objectives This chapter is designed for software programmers with a need to understand the C programming language starting from scratch. This chapter will give you enough understanding on C programming language from where you can take yourself to higher level of expertise. 8.2 ????? Introduction to Programming In computing, a program is a specific set of ordered operations for a computer to perform. In the modern computer that John von Neumann outlined in 1945, the program
NSOU ? CC-PH-04 ? 261 contains a one-at-a-time sequence of instructions that the computer follows. Typically, the program is put into a storage area accessible to the computer. The computer gets one instruction and performs it and then gets the next instruction. The storage area or memory can also contain the data that the instruction operates on. Note that a program is also a special kind of "data" that tells how to operate on "application or user data". The processs of writing computer programs is known as Programming, and the language of writing programs is known as Programming Language. Example : C & C++. C is a general-purpose, high-level language that was originally developed by Dennis M. Ritchie to develop the UNIX operating system at Bell Labs. C was originally first implemented on the DEC PDP-11 computer in 1972. In 1978, Brian Kernighan and Dennis Ritchie produced the first publicly available description of C, now known as the K&R (Kernighan and Richie) standard. The UNIX operating system, the C compiler, and essentially all UNIX application programs have been written in C. C has now become a widely used professional language for various reasons : Easy to learn Structured language it produces efficient programs it can handle low-level activities. It can be compiled on a variety of computer platforms. Keywords : C, C++, auto, break, case, char, const, continue, default, do, double, else, enum, extrn, float, for, goto, if, int, long, register, return, short, signed, sizeof, static, struct, switch, typedef, union, unsigned, void, volatile, while. Facts about C : C was invented to write an operating system called UNIX. C is a successor of B language which was introduced around the early 1970s. The language was formalized in 1988 by the American National Standard Institute (ANSI). The UNIX OS was totally written in C. Today C is the most widely used and popular System Programming Language. Most of the state-of-the-art software have been implemented using C. Today's most popular Linux OS and RDBMS MySQL have been written in C. Why Use C : C was initially used for system development work, particularly the programs that make-up the operating system. C was adopted as a system development language because it produces code that runs nearly as fast as the code written in assembly language. Some examples of the use of C might be :

262 ? NSOU ? CC-PH-04 ? Operating Systems ? OVERVIEW C Programming Language ? Compilers ? Assemblers ? Text Editors ? Print Spoolers ? Network Drivers ? Modern Programs ? Databases ? Language Interpreters ? Utilities C Programs : A C program can vary from 3 lines to millions of lines and it should be written into one or more text files with extension ".c"; for example, hello.c. You can use "vi", "vim" or any other text editor to write your C program into a file. This tutorial assumes that you know how to edit a text file and how to write source code inside a program file. Sample : # include>stdio.h< //header file void main(){ //main Function //code } 8.3 ????? Constants, Variable and Data types 8.3.1 C Tokens : The smallest individual units and punctuation marks are known as C Tokens. C Programming Language has six types of Tokens. Keywords : Keywords are special words that are used to give a special meaning to the program and can't be used as variable and constant. They are basically a sequence of characters that have fixed to mean for example break, for, while, do-while, do, if, int, long, char.

NSOU ? CC-PH-04 ? 263 'C' tokens Keyword Constant String Operator void, int 65, 11.7 "jdfh", "year" +,-,* Identifier Special symbols main, amount &,#,$ Identifiers : Identifiers refer to the variable name, array name, function name. It is user defined and collection of letters and digits but first letter always character. Constants : The quantity which does not change during the execution of a program is known as constant. There are types of constant. Variables : Variables are used to give the name and allocate memory space. An entity that may vary data during execution. For example, sum, area, a, b, age, city. String : String is a collection of character. For example, "RAM", "Meerut", "Star" String is represented by a pair of double quotes. Operators : Operators act as connectors and they indicate what type of operation is being carried out. The values that can be operated by these operators are called operands. They are used to perform basic operations, comparison, manipulation of bits and so on. 8.3.2 C Constants C Constants Numeric constant Character constant Integer constant Real constant Character constant String constant

264 ? NSOU ? CC-PH-04 Integer Constants : An integer constant is a sequence of digits from 0 to 9 without decimal points or fractional part or any other symbols. There are 3 types of integers namely decimal integer, octal integers and hexadecimal integer. Decimal Integers consists of a set of digits 0 to 9 preceded by an optional + or - sign. Spaces, commas and non digit characters are not permitted between digits. Example for valid decimal integer constants are int y=123;//here 123 is a decimal integer constant Octal Integers constant consists of any combination of digits from 0 through 7 with a O at the beginning. Some examples of octal integers are Real Constants Real Constants consists of a fractional part in their representation. Integer constants are inadequate to represent quantities that vary continuously. These quantities are represented by numbers containing fractional parts like 26.082. Example of real constants are float x = 6.3; //here 6.3 is a double constant. float y = 6.3f; //here 6.3f is a float constant. float z = 6.3 e + 2; //here 6.3 e + 2 is a exponential constant. float s = 6.3L ; //here 6.3L is a long double constant Real numbers can also be represented by exponential notation. The general form for exponential notation is mantissa exponent. The mantissa is either a real number expressed in decimal notation or an integer. The exponent is an integer number with an optional plus or minus sign. Single Character Constants A single character constant represent a single character which is enclosed in a pair of quotation symbols. Example for character constants are char p = 'ok' ; // p will hold the value 'o' and k will be omitted char y = 'u'; // y will hold the value 'u' char k = '34' ; // k will hold the value '3', and '4' will be omitted char e = ' '; // e will hold the value ' ', a blank space chars = '\45'; // s will hold the value ' ', a blank space

NSOU ? CC-PH-04 ? 265 All character constants have an equivalent integer value which are called ASCII Values. String Constants A string constant is a set of characters enclosed in double quotation marks. The characters in a string constant sequence may be a alphabet, number, special character and blank space. Example of string constants are "VISHAL" "1234" "God Bless" "!.....?" Backslash Character Constants [Escape Sequences] Backslash character constants are special characters used in output functions. Although they contain two characters they represent only one character. Given below is the table of escape sequence and their meanings. 8.3.3 Variables C variable is an identifier that is used to store a data value and whose value may be changes during the program execution. C variable might be belonging to any of the data type like int, float, char etc. Variable Declaration in C : All variable which are used in the program should be declared before use. Declaration consists of one or more variable name (that are chosen by programmer) with data type and ending with semicolon. Example int sum; Data type variable name Condition for declaring Variable 1. They must begin with letter. 2. Length of a variable should not be more than 31 characters. 3. It should not be Keyword. 4. No white space is allowed. 8.3.4 Data Types Data types specify a particular kind of data item, as defined by the values variable can take. C language has some predefined set of data types to handle various kinds of data that we can use in our program. These data types have different storage capacities.

266 ? NSOU ? CC-PH-04 C language supports 2 different types of data types : 1. Primary data types : These are fundamental data types in C namely integer (int), floating point (float), character (char) and void. 2. Derived data types : Derived data types are nothing but primary data types but a little twisted or grouped together like array, structure, union and pointer. These are discussed in details later. Data type determines the type of data a variable will hold. If a variable x is declared an int. It means x can hold only integer values. Every variable which is used in the program must be declared as what data-type it is. Primary Data type Character Integer Float Void Char signed unsigned float Signed char int int double Unsigned short int short int char long double long int long int Summary : This has been a lengthy and perhaps disconcerting article. The alphabet of C although of relevance, is not normally a day to day consideration of practicing programmers, so it has been discussed but can now be largely ignore. Much the same can be said regarding keywords and identifiers, since the topic is not complicated and simply becomes committed to memory. The declaration of variables is rarely a problem, although it is worth re-emphasizing the distinction between a declaration and a definition. If that still remains unclear, you might find of benefit to go back and re- read the description. The standard has substantially affected parts of the language described in this chapter. In particular, the changes to the conversions and the change from 'unsignedness preserving' to 'value preserving' rules of arithmetic may cause some surprises to experienced C programmers. Even they have some real relearning to do.

NSOU ? CC-PH-04 ? 267 8.4 ????? Operators and Expressions An operator is a symbol that tells the compiler to perform specific mathematical or logical functions. C provides the following types of operators ? Arithmetic Operators ? Relational Operators ? Logical Operators ? Bitwise Operators ? Assignment Operators ? Misc Operators Arithmetic Operators Operator Operation of Operator Example + Adds two operands. A + B – Subtracts second operand from the first. A – B * Multiplies both operands. A*B / Divides numerator by de-numerator B/A % Modulus Operator and remainder of after an B % A integer division. ++ Increment operator increases the integer value by one A++ -- Decrement operator decreases the integer value by one. A-- Relational Operators Operator Operation of Operator Example = = Check if the values of two operands are equal or (A == B) not. If yes, then the condition becomes true. != Check if the values of two operands are equal or (A != B) not. If the values are not equal, then the condition becomes true < Check if the value of left operand is greater than (A < B) the value of right operand. If yes, then the condition becomes true.

268 ? NSOU ? CC-PH-04 > Check if the value of left operand is less than the value (A > B) of right operand. If yes, then the condition becomes true. <= Check if the value of left operand is greater than or (A <= B) equal to the value of right operand. If yes, then the condition becomes true. >= Check if the value of left operand is less than or equal (A >= B) to the value of right operand. If yes, then the condition becomes true. Logical Operators Bitwise Operators Operator Operation of Operator Example && Called Logical AND operator. If both the operands (A && B) are non-zero, then the condition becomes true || Called Logical OR Operator. If any of the two (A || B) operands is non-zero, then the condition becomes true ! Called Logical NOT Operator. It is used to reverse ! (A && B) the logical state of its operand. If a condition is true, then Logical NOT operator will make it false. Bitwise operator works on bits and perform bit-by-bit operation. The truth tables for &, I, and ^ is as follows – A B A&B A | B A^B 0 0 0 0 0 0 0 1 0 1 1 1 1 1 1 0 1 0 0 1 1 Assignment Operator This type of Operators are used to assisgn the result of an expression to an identifier. The most common assignment operator is " = ". Example : C = A + B will assign the value of A + B to C.

NSOU ? CC-PH-04 ? 269 Misc Operators Operator Operation of Operator Example sizeof() Returns the size of a variable. sizeof(a), where a is integer, will return 4. & Returns the address of a variable. & a; returns the actual address of the variable. * Pointer to a variable * a; ? : Conditional Expression. If condition is true? then value X : otherwise value Y C Programming Expresson : 1. In programming, an expression is any legal combination of symbols that represents a value. 2. C Programming provides its own rules of Expression, whether it is legal expression or illegal expession. For example, in the C language x +5 is a legal expression. 3. Every expression consists of at least one operand and can have one or more operators. 4. Operands are values and Operators are symbols that represent particular actions. Valid C Programming Expression : C Programming code gets compiled firstly before execution. In the different phases of compiler, C programming expression is checked for its validity. Expressions Validity a + b Expression is valid since it contain + operator which is binary operator ++ a + b Invalid Expression Priority and Expression : In order to solve any expression we should have knowledge of C Programming Operators and their priorities.

270 ? NSOU ? CC-PH-04 Types of Expression : In programming, different varities of expressions are given to the compiler. Expressions can be classified on the basis of Position of Operators in an expression – Type Explanation Example Infix Expression in which Operator is in between Operands a + b Prefix Expression in which Operator is written before + a b Operands Postfix Expression in which Operator is written after Operands ab + These expressions are solved using the stack. Example of Expression : Now we will be looking into some of the C Programming Expressions, Expression can be created by combining the operators and operands Each of the expression results into the some resultant output value. Consider few expressions in the table below. Expression Examples, Explanation n1 + n2, this is an expression which is going to add two numbers and we can assign the result of addition to another variable. x = y. This is an expression which assigns the value of right hand side operand to left side variable. v = u + a* t, We are multiplying two numbers and result is added to 'u' and total result is assigned to v x >= y, This expression will return Boolean value because comparison operator will give us output either true or false ++j, This is expression having pre increment operator, it is used to increment the value of j before using it in expression [/table]. Summary : This chapter has described the entire range of control of flow available in C. The only areas that cause even moderate surprise are the way in which cases in a switch statement are not mutually exclusive, and the fact that goto cannot transfer control to any function except the one that is currently active. None of this is intellectually deep and it has never been known to cause problems either to beginners or programmers experienced in other languages. The logical expressions all give integral results. This is perhaps slightly unusual, but once again take very little time to learn.

NSOU ? CC-PH-04 ? 271 8.5 ????? I/O Statements C programming has several in-built library functions to perform input and output tasks. Two commonly used functions for I/O (Input/Output) are printf() and scanf(). The scanf() function reads formatted input from standard input (keyboard) whereas the printf() function sends formatted output to the standard output (screen). Example 1 : C Output #include>stdio.h<// Including header file to run printf() function. int main() { printf("C Programming");// Displays the content inside quotation return O; } Output C Programming How this program works ? ? All valid C program must contain the main() function. The code execution begins from the start of main () function. ? The printf() is a library function to send formatted output to the screen. The printf() function is declared in "stdio.h" header file. ? Here, stdio.h is a header file (standard input output header file) and #include is a preprocessor directive to paste the code from the header file when necessary. When the compiler encounters printf() function and doesn't find stdio.h header file, compiler shows error. ? The return O; statement is the "Exit status" of the program. Example 2 : C Integer Output #include>stdio.h< int main() { inttestInteger=5; printf("Number = %d", testInteger);
272 ? NSOU ? CC-PH-04 return 0; } Output Number = 5 Inside the quotation of printf() function, there is a format string "%d" (for integer). If the format string matches the argument (testInteger in this case), it is displayed on the screen. Example 3 : C Integer Input/Output #include>stdio.h< int main() { inttestInteger; printf("Enter an integer:"); scanf("%d", &testInteger); printf("Number = %d", testInteger); return 0; } Output Enter an integer : 4 Number = 4 The scanf() function reads formatted input from the keyboard. When user enters an integer, it is stored in variable testInteger. Note the '&' sign before testInteger; & testInteger gets the address of testInteger and the value is stored in that address. Example 4 : C Floats Input/Output #include>stdio.h< int main() { float f; printf("Enter a number:");
NSOU ? CC-PH-04 ? 273 //%f format string is used in case of floats scanf("%f",&f); printf("Value = %f", f); return 0; } Output Enter a number : 23.45 Value = 23.450000 The format string "%f" is used to read and display formatted in case of floats. Example 5 : C Character I/O #include>stdio.h< int main() { char chr; printf("Enter a character: "); scanf("%c",&chr); printf("You entered %c.",chr); return 0; } Output Enter a character : g You entered g. Format string %c is used in case of character types. Little bit on ASCII code When a character is entered in the above program, the character itself is not stored. Instead, a numeric value (ASCII value) is stored. And when we displayed that value using "%c" text format, the entered character is displayed. Example 6 : ASCII Code #include>stdio.h<
274 ? NSOU ? CC-PH-04 int main() { char chr; printf("Enter a character: "); scanf("%c",&chr); //When %c text format is used, character is displayed in case of character types printf("You entered %c.\n", chr); //When %d text format is used, integer is displayed in case of character types printf("ASCII value of %c is %d.",chr,chr); return 0; } Output Enter a character : g You entered g. ASCII value of g is 103. The ASCII value of character 'g' is 103. When, 'g' is entered, 103 is stored in variable chr instead of g. You can display a character if you know ASCII code of that character. This is shown by following example. Example 7 : C ASCII Code #include>stdio.h< int main() { int chr=69; printf("Character having ASCII value 69 is %c.", chr); return 0; } Output Character having ASCII value 69 is E. More on Input/Output of floats and Integers Integer and floats can be displayed in different formats in C programming.

NSOU ? CC-PH-04 ? 275 Example #7 : I/O of Floats and Integers #include>stdio.h< int main() { int integer = 9876; float decimal=987.6543; // Prints the number right justified within 6 columns printf("4 digit integer right justified to 6 column : %6d\n", integer); //Tries to print number right justified to 3 digits but the number is not right adjusted because there are only 4 numbers printf("4 digit integer right justified to 3 column: %3d\n", integer); //Rounds to two digit places printf("Floating point number rounded to 2 digits: %.2f\n", decimal); //Round to 0 digit places printf("Floating point number rounded to 0 digits: %.f\n", 987.6543); //Prints the number in exponential notation (scientific notation) printf("Floating point number in exponential form: %e\n", 987.6543); Return 0; } Output 4 digit integer right justified to 6 column : 9876 4 digit integer right justified to 3 column : 9876 Floating point number rounded to 2 digits : 987.65 Floating point number rounded to 0 digits : 988 Floating point number in exponential form : 9.876543e+02 8.6 ????? Manipulators for data formatting Formatting output using manipulators Formatted output is very important in development field for easily read and understand.

276 ? NSOU ? CC-PH-04 C++ offers the several input/output manipulators for formatting, commonly used manipulators are given below . Manipulator Declaration in endl iostream.h setw iomanip.h setprecision iomanip.h setf iomanip.h endl endl manipulator is used to Terminate a line and flushes the buffer. Difference b/w '\n' and endl When writing output in C++, you can use either std::endl or '\n' to produce a newline, but each has a different effect. ? std::endl sends a newline character '\n' and flushes the output buffer. ? '\n' sends the newline character, but does not flush the output buffer. The distinction is very important if you're writing debugging messages that you really need to see immediately, you should always use std::endl rather than '\n' to force the flush to take place immediately. The following is an example of how to use both versions, although you cannot see the flushing occurring in this example. #include >iostream.h< int main() { cout>>"USING'\\n' ...\n"; cout>>"Line 1 \nLine 2 \nLine3 \n"; cout>>"USING end ....">>endl; cout>> "Line 1" >>endl>> "Line 2" >>endl >> "Line 3" >>endl; return 0; }

NSOU ? CC-PH-04 ? 277 Output USING '\n' ... Line 1 Line 2 Line 3 USING end ... Line 1 Line 2 Line 3 setw() and setfill() manipulators setw() manipulator sets the width of the filed assigned for the output. The field width determines the minimum number of characters to be written in some output representations. If the standard width of the representation is shorter than the field width, the representation is padded with fill characters (using setfill). setfill() character is used in output insertion operations to fill spaces when results have to be padded to the field width. Syntax setw([number_of_characters]); setfill([character]); Consider the example #include >iostream.h< #include >iomanip.h< int main() { cout>>"USING setw() ....\n"; cout>>setw(10) >>11>>"\n"; cout>>setw(10)>>2222>>"\n"; cout>>setw(10) >>33333>>"\n"; cout>>setw(10) >>4>>"\n"; cout>>"USING setw() &setfill() [type- I]..\n":

278 ? NSOU ? CC-PH-04 cout>>setfill('0'); cout>>setw(10) >>11>>"\n"; cout>>setw(10)>>2222>>"\n"; cout>>setw(10) >>33333>>"\n"; cout>>setw(10) >>4>>"\n"; cout>>"USING setw() &setfill() [type- II]..\n": cout>>setfill('-')>>setw(10) >>11>>"\n"; cout>>setfill('*')>>setw(10)>>2222>>"\n"; cout>>setfill('@')>>setw(10) >>33333>>"\n"; cout>>setfill('#')>>setw(10) >>4>>"\n"; return 0; } Output USING setw()........... 11 2222 33333 4 USING setw() &setfill() [type- I]... 0000000011 0000002222 0000033333 0000000004 USING setw() &setfill() [type-II]... ------- – 11 ******2222 @@@@@33333 #########4 setf() and setprecision() manipulator setprecision manipulator sets the total number of digits to be displayed, when floating point numbers are printed.

NSOU ? CC-PH-04 ? 279 Syntax setprecision([number_of_digits]); cout>>setprecision(5)>>1234.537; //output will be : 1234.5 On the default floating-point notation, the precision field specifies the maximum number of meaningful digits to display in total counting both those before and those after the decimal point. Notice that it is not a minimum and therefore it does not pad the displayed number with trailing zeros if the number can be displayed with less digits than the precision. In both the fixed and scientific notations, the precision field specifies exactly how many digits to display after the decimal point, even if this includes trailing decimal zeros. The number of digits before the decimal point does not matter in this case. Syntax setf([flag_value], [field bitmask]); field bitmask flag values adjustfield left, right or internal basefield dec, oct or hex floatfield scientific or fixed Consider the example #include >iostream.h< #include >iomanip.h< int main () { cout>>"USING fixed ....\n"; cout.setf(ios::floatfield,ios::fixed); cout>>setprecision(5)>>1234.537>>endl; cout>>"USING scientific ....\n"; cout.setf(ios::floatfield,ios::scientific); cout>>setprecision(5)>>1234.537>>endl; return 0; }

280 ? NSOU ? CC-PH-04 Output USING fixed .... 1234.53700 USING scientific .... 1234.5 Consider the example to illustrate base fields #include >iostream.h< #include >iomanip.h< int main() { intnum=10; cout>>"Decimal value is :">>num>>endl; cout.setf(ios::basefield,ios::oct); cout>>"Octal value is :">>num>>endl; cout.setf(ios::basefield,ios::hex); cout>>"Hex value is :">>num>>endl; return 0; } 8.7 ????? Control System In any programming language, there is a need to perform different tasks based on the condition and we can control the flow of program in such a way so that it executes certain statements based on the outcome of a condition. In C programming Language we have following decision control statements 1. if statement 2. if-else & else-if statement 3. switch-case statement If Statement It is basically a two way decision statement, used to decide whether a certain statement or block of statements will be executed or not.

NSOU ? CC-PH-04 ? 281 Syntax of if statement If (condition) { Body Of If Statement } Flow Diagram of if statement

282 ? NSOU ? CC-PH-04 Example of if statement Output If-Else Statement If-Else statement is an extension of if statement. If the condition of if statement is TRUE the If block is executed otherwise Else block is executed. Syntax of if-Else statement if (Condition) { Body Of If } else { Body of Else }

NSOU ? CC-PH-04 ? 283 Example of if-else statement Output The Else If Statement The else...if statement is useful when you need to check multiple conditions Syntax Of Else-If Statement if (condition 1) Body Of Else If } else if (Condition 2) { Body Of Else If } else { Body Of Else }

284 ? NSOU ? CC-PH-04 Example Of Else-If Statement Output Nested If ..Else statement When an if else statement is present inside the body of another "if" or "else" then this is called nested if else. "Enter the value of A" "C is big" "B is big" "A is big" "% d" "Enter the value of B" "Enter the value of C" "% d" "% d"

NSOU ? CC-PH-04 ? 285 Example Of Nest If Statement Output Switch Statement in C/C++ The switch statement is used when we have multiple options and we need to perform a different task for each option. switch (n) { case 1 : //code to be executed if n = 1; break ; case 2://code to be executed if n = 2; break; default://code to be executed if n doesn't match any cases } "\n enter the value of A" "C is big" "A is big" "% d" "\n enter the value of B" "\n enter the value of C" "% d" "% d" "C is big" "B is big"

286 ? NSOU ? CC-PH-04 Example of Switch Statement Output "Wrong choice" "% d" "January" "February" Entry your choice In Press for January In Press for February

NSOU ? CC-PH-04 ? 287 8.8 ????? Loop Statement Loop : Loop is used when we need to repeatedly execute a block of statements according to the condition given in the loop. C programming language provides the following types of loops to handle looping requirements. 1. for loop 2. while loop 3. do... while loop For Loop The syntax of for loop is : for (initialization Statement; testExpression; updateStatement) { // codes } How for loop works ? The initialization statement is executed only once. Then, the test expression is evaluated. If the test expression is false (0), for loop is terminated. But if the test expression is true (nonzero), codes inside the body of for loop is executed and the update expression is updated.

288 ? NSOU ? CC-PH-04 This process repeats until the test expression is false. The for loop is commonly used when the number of iterations is known. To learn more on test expression (when test expression is evaluated to nonzero (true) and 0 (false)), check out relational and logical operators. For Loop Flowchart Example for Loop // Program to calculate the sum of first n natural numbers // Positive integers 1,2,3...n are known as natural numbers #include>stdio.h< int main() { int num, count, sum =0;

NSOU ? CC-PH-04 ? 289 printf("Enter a positive integer: "); scanf("%d", &num); //for loop terminates when n is less than count for (count = 1; count >=num;++count) { sum += count; } printf("Sum = %d", sum); return 0; } Output Enter a positive integer : 10 Sum = 55 While loop The syntax of while loop is : while (testExpression) { //codes } where, testExpression checks the condition is true or false before each loop. How while loop works? The while loop evaluates the test expression. If the test expresion is true (nonzero), codes inside the body of while loop are exectued. The test expression is evaluated again. The process goes on until the test expression is false. When the test expression is false, the while loop is terminated.

290 ? NSOU ? CC-PH-04 Flowchart of while loop Example 1 : While Loop // Program to find factorial of a number // For a positive integer n, factorial = 1*2*3...n #include>stdio.h< int main() { int number; long long factorial; printf("Enter an integer: "); scanf("%d", &number); factorial = 1; //loop terminates when number is less than or equal to 0 while (number <0) { factorial *= number://factorial = factorial* number; --number; } printf("Factorial= %lld", factorial);

NSOU ? CC-PH-04 ? 291 return 0; } Output Enter an integer: 5 Factorial = 120 do...while loop The do..while loop is similar to the while loop with one important difference. The body of do... while loop is executed once, before checking the test expression. Hence, the do...while loop is executed at least once. do...while loop Syntax do { // codes } while (testExpression); How do...while loop works? The code block (loop body) inside the braces is executed once. Then, the test expression is evaluated. If the test expression is true, the loop body is executed again. This process goes on until the test expression is evaluated to 0 (false). When the test expression is false (nonzero), the do... while loop is terminated. Flowchart of do... while Loop

292 ? NSOU ? CC-PH-04 Example 2: do...while loop //Program to add numbers until user enters zero #include>stdio.h< int main() { double number, sum = 0; //loop body is executed at least once do { printf("Enter a number: "); scanf("%lf".&number); sum += number; } while(number !=0.0); printf("Sum = % %.2lf", sum); return 0; } Output Enter a number : 1.5 Enter a number : 2.4 Enter a number : −3.4 Enter a number : 4.2 Enter a number : 0 Sum = 4.70 Nested Loop : C programming allows to use one loop inside another loop. Loop inside the another loop is called Nested Loop.

NSOU ? CC-PH-04 ? 293 Syntax while (condition) { while (condition) { statement (s); } statement (s); } Loop Control Statements : 1. Break : When a break statement is encountered inside a loop, the loop is immediately terminated and the program control resumes at the next statement following the loop. 2. Continue : When a continue statement is encountered inside a loop, control jumps to the beginning of the loop for next iteration, skipping the execution of statements inside the body of loop for the current iteration.

294 ? NSOU ? CC-PH-04 Theory Questions 1. State whether the following statement are True or False (a) Every statement in C program should end with a semicolon. (b) Every C program ends with and END word. (c) A printf Statement can generate only one output line. (d) Like variable constant have a type./ (e) All static variable are automatically initialized to zero. (f) The statement a+ = 10 is valid. (g) The expression !(a<=b) is same as a<b. (h) = & '==' both are same. (i) While loop is an entry control loop. (j) An exit controlled loop is executed a minimum of one time. 2. Fill the blanks in the following statements. (a) The ___________ Function is used to display the output of the screen. (b) The ___________ header file contains mathematical functions. (c) A variable can be made constant by using the keyword ____________. (d) __________ is the increment operator. (e) ?: Operator known as __________ operator.

NSOU ? CC-PH-04 ? 295 (f) The __________ statement is used to immediately exit from the loop. (g) The first part of for loop declaration is __________ . (h) The do-while is known as __________ control loop. (i) A for loop within a for loop is known as __________ loop. (j) while (1) is known as __________ loop. 3. Programming problems (a) C "Hello, World!" Program. (b) C Program to Print an Integer (Entered by the User). (c) C Program to Add Two Integers. (d) C Program to Swap Two Numbers. (e) C Program to find Factorial of a Number. (f) C Program to Make a Simple Calculator Using switch...case. (g) C Program to Generate Multiplication Table. (h) C Program to Check Whether a Number is Prime or Not. (i) C Program to Check Armstrong Number. Answer 1. 2. (a)T (b)F (c)F (d)T (e)T (f)T (g)F (h)F (i)T (a)printf( ) (b)math.h (c)constant (d)++ (e)ternary (f)break (g)initialization (h)exit (i)nested (j)infinite

296 ? NSOU ? CC-PH-04 8.8.1 Jumping out of Loops While executing any loop, it becomes necessary to skip a part of the loop or to leave the loop as soon as certain condition becomes true, which is called jumping out of loop. C language allows jumping from one statement to another within a loop as well as jumping out of the loop. There are two keywords in C language to Jumping out or Break Loop. ? break statement ? continue statement Break Statement When break statement is encountered inside a loop, the loop is immediately excited and the program continues with the statement immediately following the loop. While (condition check) { Statement − 1; Statement − 2 ; if (some condition) { break; } Statement − 3 ; Statement − 4 ; } jumps out of the loop no matter how many cycles are left, loop is excited Continue Statement It causes the control to go directly to the test condition and then continue the loop process. On encountering continue, curson leave the current cycle of loop, and starts with the next cycle. →

NSOU ? CC-PH-04 ? 297 while (condition check) { Statement − 1 ; Statement − 2 ; If (some condition) { Continue ; jumps to the next cycle directly. Statement − 3 ; Statement − 4 ; } 8.8.2 Goto Statement transfer of control branching within a loop C supports a unique form of a statement that is the goto statement which is used to branch unconditionally within a program from one point to another. Although it is not a good habit to use the goto statement in C, there may be some situations where the use of the goto statement might be desirable. The goto statement is used by programmers to change the sequence of execution of a C program by shiffing the control to a different part of the same program. The general form of the goto statement is : goto level : Not executed for the cycle of loop in which continue is executed. ? ? ? →

298 ? NSOU ? CC-PH-04 A lebel is an identifier required for goto statement to a place where the branch is to be made. 8.9 ????? Summary This chapter has introduced many of the basics of the language although informally. Functions in particular, from the basic building block for c. Art 8.4 provides a full descriptions of this fundamental objects, but you should by now understand enough about them to follow their informal use in the intervening material. 8.10 ????? Exerceises 1. History of C Why we use C programming language ? 2. Procedure to create a Program in C Programming Language. 3. What is the importance of header files ? 8.1 ????? References 1. Mathematical Methods for physicists, 6th edition : Georg B Arfken, Hans J Weber 2. Mathematical methods in the physical sciences (3rd edition) : Marry L Boas 3. Matrices and Tensors in physics (third edition) : A. W Joshi

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Submitted text	As student entered the text in the submitted document.
Matching text	As the text appears in the source.

1/267 **SUBMITTED TEXT** 128 WORDS	**48%** **MATCHING TEXT** 128 WORDS
x y x y x y ? ? ? Solution : 3 3 2 2 (,) (0,0) (,) (0,0) lim lim 0, x y x y x y x xy y x y ? ? ? ? ? ? ? ? 0 x y ? ? Example 2: Find 2 (,) (0,0) lim x y x xy x y ? ? ? Solution : ? ? ? ? ? ? ? ? ? ? ? ? 2 (,) (0,0) (,) (0,0) lim ( ) lim x y x y x x y x y x xy x y x y x y x y ? ? ? ? ? ? ? ? ? ? ? ? (,) (0,0) lim 0 ? ? ? ? x y x x y Exercise Art 1.4.1 : 1) Find the limit : 0 2 2 1 lim : 2 2 x x ans x ? ? ? ? ? ? ? ? ? 2) Find 2 4 2 (,) (1,1) ( ) 1 lim : 2 x y x y ans x y ? ? ? ? ? ? ? ? 1.4 ?????	x 3 y 2 (1-x-y) = x 3 y 2 - x 4 y 2 – x 3 y 3 = 3x 2 y 2 – 4x 3 y 2 -3y 3 = 2x 3 y – 2x 4 y -3x 3 y 2 For maxima & minima = 0 x 2 y 2 – 4x 3 y 2 -3x 2 y 3 = 0 =< x 2 y 2 (3 – 4x -3x 3 y – 2x 4 y -3x 3 y 2 = 0 =< x 3 y(2 – 2x -3y) = 0 ----------------< (2) From (1) & (2) 4x + 3y – 3 = 0 2x + 3y - 2 = 0 2x = 1 =< x = ½ 4 (½) + 3y – 3 = 0 =< 3y = 3 -2 , y = (1/3) l = 2 2 x f ? ? = 6xy 2 -12x 2 y 2 -6xy 3 ? ? ? ? ? ? ? ? ? ? ? 2 2 x f (1/2,1/3) = 6(1/2)(1/3) 2 -12 (1/2) 2 (1/3) 2 -6(1/2)(1/3) 3 = 1/3 – 1/3 -1/9 = -1/9 m = y x f ? ? ? 2 = ? ? ? ? ? ? ? ? ? ? ? ? ? y f x = 6x 2 y -8 y – 9x 2 y 2 ? ? ? ? ? ? ? ? ? ? ?

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2/267 **SUBMITTED TEXT** 75 WORDS	**26%** **MATCHING TEXT** 75 WORDS
x x f x x ? ? ? ? ? ? Since f(x) = x when x < 0 And 0 0 lim ( ) lim ( ) 0 x x f x x ? ? ? ? ? ? ? Since f(x) = – x when x > 0 Again f (0) = 0 Therefore, 0 0 lim ( ) lim ( ) (0) x x f x f x f ? ? ? ? ? ? Therefore the function is continuous at x = 0 Now 0 ( ) ( ) ( ) lim h f x h f x	x 5 -3x 4 + 5 ('08 S-1) Sol: Given f(x) = x 5 -3x 4 + 5 f 1 (x) = 5x 4 – 12x 3 for maxima or minima f 1 (x) =0 5x 4 – 12 x 3 = 0 x =0, x= 12/5 f 11 (x) = 20 x 3 – 36 x 2 At x = 0 =< f 11 (x) = 0. So f is neither maximum minimum at x = 0 At x = (12/5) =< f 11 (x) =20 (12/5) 3 – 36(12/5) =144(48-36) /25 =1728/25 < 0 So f(x)

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3/267	SUBMITTED TEXT	68 WORDS	34%	MATCHING TEXT	68 WORDS

f(x) = 2x 2 + 3 at x = 1 Solution : 2 1 1 lim ( ) lim 2 3 5 x x f x x ? ? ? ? ? ? ? ? 2 1 1 lim ( ) lim 2 3 5 x x f x x ? ? ? ? ? ? ? ? And f(1) = 5 Therefore 1 1 lim ( ) lim ( ) (1) x x f x f x f ? ? ? ? ? ? And f(x) is continuous at x = 1

f 1 (x) =0 5x 4 – 12 x 3 = 0 x =0, x= 12/5 f 11 (x) = 20 x 3 – 36 x 2 At x = 0 =< f 11 (x) = 0. So f is neither maximum nor minimum x = 0 At x = (12/5) =< f 11 (x) =20 (12/5) 3 – 36(12/5) =144(48-36) /25 =1728/25 < 0 So f(x) is minimum at x = 12/5

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4/267	SUBMITTED TEXT	61 WORDS	22%	MATCHING TEXT	61 WORDS

h f h f f h f h h ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 11 Since f (0) = 0 Now 0 0 ( ) lim lim 1 h h f h h h h ? ? ? ? ? ? ? Since f(x) = x when x< 0 and h → 0 + means 0h ? And 0 0 ( ) lim lim 1 h h f h h h h ? ? ? ? ? ? ? ? ?

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

5/267	SUBMITTED TEXT	38 WORDS	50%	MATCHING TEXT	38 WORDS

f x h ? ? ? ? ? ∴ 0 (1) (1) (1) lim h f h f f h ? ? ? ? ? ? Or, 2 0 2(1) 3 5 (1) lim h h f h ? ? ? ? ? ? ? (1) 5 f ? ? 2 0 2 4 lim h h h h ? ? ? 12 ?

SA MMP.pdf (D118446089)

6/267	SUBMITTED TEXT	61 WORDS	39%	MATCHING TEXT	61 WORDS

h h h a b a h h h h h ? 2 1 a b ? ? ? 0 0 1 2 (2) lim lim 1 h h ah h h h ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Or, – 1 = 0 lim 2 (2) 4 h a h a ? ? ? ? ? 1 4 a? ? ?

SA MMP.pdf (D118446089)

7/267	SUBMITTED TEXT	7 WORDS	90%	MATCHING TEXT	7 WORDS

a b a f x dx f x dx f x

SA Mathematical Physics - I SLM full.pdf (D113782471)

8/267 **SUBMITTED TEXT** 57 WORDS **43%** **MATCHING TEXT** 57 WORDS

a function f(x) about x = a ; 2 ( ) 1 1 ( ) ( ) ( ) ( ) ( ) ( ) ... ( ) ( ) ... 2! ! n n f x f a x a f a x a f a x a f a n ? ?? ? ? ? ? ? ? ? ? ? ? (1.3) Where f (n) (

SA M P Vol 1.pdf (D134397037)

9/267 **SUBMITTED TEXT** 41 WORDS **57%** **MATCHING TEXT** 41 WORDS

Submitted text	Matching text
of the function ( ) 4 f x x ? ? at x = 0 Solution : 1 2 2 2 32 2 (0) (0) 1 1 1 ( ) (0) 2 2 1! 2! 4 2! 4 14 f f f x f x x x x x	of the function f(x) = x 5 -3x 4 + 5 ('08 S-1) Sol: Given f(x) = x 5 -3x 4 + 5 f 1 (x) = 5x 4 – 12x 3

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10/267 **SUBMITTED TEXT** 74 WORDS **34%** **MATCHING TEXT** 74 WORDS

f a f a f x f a x a x a ? ?? ? ? ? ? ? ? Now 3 4 2 6 ( ) ; ( ) ? ?? ? ? ? f x f x x x ; we write 4.41 = (2 + 0.1) 2 , implies a = 2 & x = 2.1 2 2 (2) (2) ( ) (2) (2.1 2) (2.1 2) 1! 2! f f f x f ? ?? ? ? ? ? ? ? 2 2 6 8 16 (2) (2.1 2) (2.1 2) 1! 2!

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11/267 **SUBMITTED TEXT** 25 WORDS **54%** **MATCHING TEXT** 25 WORDS

Submitted text	Matching text
P x P x y Q x dx ? ? (2.2) Where P 0 (x), P 1 (x), and P 2 (x) are called co-efficient functions and Q(x)	P 2 (x) + --------+ P n (x) .y = Q(x) Where P 1 (x), P 2 (x), P 3 (x)… ….. P n (x) and Q(x) (

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12/267 **SUBMITTED TEXT** 37 WORDS **33%** **MATCHING TEXT** 37 WORDS

Submitted text	Matching text
x x x e y y y x x x ?? ? ? ? ? ? ? ? Comparing with the standard form, we get 2 2 3 cos ( ) , ( ) 1 1 x x x x x x ? ? ? ? ? ? and 2 ( ) 1 x e Q x x ? ?	x x x y y d y ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 4 1 2 0 0 1 1 2 2 4 x x y y x x dx ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 2 2 4 1 0 0 1 . 2 2 2 4 x x y x y y x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 2 2 2 2 0 1 . 2 1 2 1 1 8 x x x x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 4 6 1 3 5 0 0 1 1 2 2 8 8 2 4 6 x x

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13/267 SUBMITTED TEXT 44 WORDS	100% MATCHING TEXT 44 WORDS
p p p p p p p p p p ? (1)(2)...(1) ! p p p p p	

SA Mathematical Physics - I SLM full.pdf (D113782471)

14/267 SUBMITTED TEXT 37 WORDS	45% MATCHING TEXT 37 WORDS
is a solution of (2.27), 2 1 1 2 ( ) ( ) 0 d y x y x y dx ? ? ? ? ? ? (2.28) similarly y 2 (x) is a solution of (2.27) 2 2 2 2 2 ( ) ( ) 0 d y x y	is a multiple of d, y 2: d, so that x - y :::: x-d. Since x - y is a multiple of d, x - y 2:

W https://link.springer.com/content/pdf/bbm:978-0-387-22602-6/1.pdf

15/267 SUBMITTED TEXT 24 WORDS	90% MATCHING TEXT 24 WORDS
y x y x y y x ? ? ?? ? ? ? ? = 0 0 0 , ( ) y y x y ? ? ?	y ? ? = 0 = 4x 3 -4x + 4y = 0 ? x 3 – x + y = 0 -------------------< (1) = 4y 3 +4x - 4y = 0 ? y 3 + x – y = 0 -------------------< (2)

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16/267 SUBMITTED TEXT 47 WORDS	53% MATCHING TEXT 47 WORDS
x y C y x C y x y ? ? ? ? ? ? ? (2.32) In matrix form, equation (2.32) can be written as 1 0 2 0 0 1 1 0 2 0 0 2 ( ) ( ) ( ) ( ) y x y x y C y x y x y	

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17/267 SUBMITTED TEXT 22 WORDS	86% MATCHING TEXT 22 WORDS
y x y x W y y y x y x y x y x y x y x ? ? ? ? ? ? ? (2.35)	y 2 (1-x-y) = x 3 y 2 - x 4 y 2 – x 3 y 3 = 3x 2 y 2 – 4x 3 y 2 -3x 2 y 3 = 2x 3 y – 2x 4 x 3

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18/267 SUBMITTED TEXT 48 WORDS	38% MATCHING TEXT 48 WORDS
e 2x + C 2 e –4x , now the wronkian of y 1 = e 2x and y 2 = e –4x 2 4 2 2 2 1 2 2 4 1 2 4 2 6 0 2 4 x x x x x x x x y y e e W e e e y y e	

SA MMP Assignment.pdf (D112572984)

19/267 SUBMITTED TEXT 84 WORDS	48% MATCHING TEXT 84 WORDS
y x y x W y y y x y x y x y x y x y x ? ? ? ? ? ? ? ? Again 2 2 ( ) ( ) y x Cy x ? ? ? NSOU ? CC-PH-04 ? 33 Therefore 1 2 1 1 1 1 (,) ( ) ( ) ( ) ( ) 0 W y y C y x y x C y x y x ? ? ? ? ? (2.37) Again we assume that y 1 (x) and y 2 (x)	y 2 (1-x-y). Sol: Given f(x) = x 3 y 2 (1-x-y) = x 3 y 2 - x 4 y 2 – x 3 y 3 = 3x 2 y 2 – 4x 3 y 2 -3x 2 y 3 = 2x 3 y – 2y -3x 3 y 2 For maxima & minima = 0 and = 0 ? 3x 2 y 2 – 4 x 3 y 2 -3x 2 y 3 = 0 =< x 2 x -3y) = 0 ---------------< (1) ? 2x 3 y – 2 x 4 y -3x 3 y 2 = 0 =< x 3

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20/267 SUBMITTED TEXT 269 WORDS	51% MATCHING TEXT 269 WORDS
x x y x ? ? ?? ? ? ? ? (2.38) And 2 2 2 ( ) ( ) ( ) ( ) ( ) 0 y x x y x x y x ? ? ?? ? ? ? ? (2.39) Now multiplying equation (2.38) by y 2 (x) and (2.39) by y 1 (x), we get respectively : 1 2 1 2 1 2 ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 y x y x x y x y x x y x y x ? ? ?? ? ? ? ? (2.40) And 2 1 2 1 2 1 ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 y x y x x y x y x x y x y x ? ? ?? ? ? ? ? (2.41) Subtracting equation (2.40) from (2.41), 2 1 1 2 2 1 1 2 ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 y x y x y x y x x y x y x y x y x ? ?? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.42 Now wronskian ? ? 1 2 2 1 1 2 ( ), ( ) ( ) ( ) ( ) ( ) W y x y x y x y x y x y x ? ? ? ? ? ? ? ? Therefore ? ? 1 2 2 1 1 2 ( ), ( ) ( ) ( ) ( ) ( ) W y x y x y x y x y	x) = x 3 y 2 (1-x- y). Sol: Given f(x) = x 3 y 2 (1-x- y) = x 3 y 2 - x 4 y 2 – y 3 = 3x 2 y 2 – 4x 3 y 2 -3x 2 y 3 = 2x 3 y – 2x 4 y -3x 3 y 2 maxima & minima = 0 x 2 y 2 – 4x 3 y 2 -3x 2 y 3 = 0 =< x 2 y 2 (3 – 4y) = 0 ---------------< (1) ? 2x 3 y – 2x 4 y -3 x 3 y 2 = 0 =< x 3 y(2 – 2x -3y) = 0 ----------------< (2) From (1) & (2) 4x + 3y – 3 = 0 2x + 3y - 2 = 0 2x = 1 =< x = ½ 4 (½) + 3y – 3 = 0 =< 3y = 3 -2 , y = (1/3) x f ? ? = 6xy 2 -12x 2 y 2 -6xy 3 ? ? ? ? ? ? ? ? ? ? 2 2 x f (1/2,1/3) = 6(1/2)(1/3) 2 -12 (1/2) 2 (1/3) 2 -6(1/2)(1/3) 3 = 1/3 – 1/3 -1/9 = -1/9 m = x f ? ? ? 2 = ? ? ? ? ? ? ? ? ? ? ? ? y x = 6y -8 x 3 y – 9x 2 y 2 ? ? ? ? ? ? ? ? ? ? ? y x f 2 (1/2 ,1/3) = 6(1/2) 2 (1/3) -8 (1/2) 3 (1/3) -9(1/2) 2 (1/3) 3 = = y f ? ? = 2x 4 -6x 3 y

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21/267 SUBMITTED TEXT 104 WORDS	43% MATCHING TEXT 104 WORDS
A A x A x ? ? ? (3) Substituting equation (3) in equation (1), we get ? ? 2 2 2 1 2 0 1 2 2 2 Aa A A x A A x A x x x ? ? ? ? ? ? ? Or, 2A 2 – 2A 2 x 2 + A 1 – 2A 1 x + 2A 2 x – 2A 0 = x 2 – x Or, – 2A 2 x 2 – 2A 1 x + 2A 2 x + 2A 2 + A 1 – 2A 0 = x 2 – x	a a x y a x a a ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 b a a x a b a y a a R x y dxdy x y dxdy ? ? ?? ? ? ? ? ? ? ? ?? ? ? ? ? ? 2 2 2 2 3 2 2 2 0 0 2 2 3 a x b a a a x a a x a y a y x y dxdy x y ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3 3 2 2 2 2 2 2 3 2 . 3 a a b b x a x a x dx a ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3 3 2 2 2 2 2 2 3 0 4 3 a b x x

W https://www.iare.ac.in/sites/default/files/CSE_LINEAR_ALGEBRA_AND_CALCULUS_LECTURE_NOTES.pdf

22/267 SUBMITTED TEXT 44 WORDS	100% MATCHING TEXT 44 WORDS
A 2 + A 1 – 2A 0 = 0 x 1 → 2A 2 – 2A 1 = – 1, A 1 = 0 x 1 → – 2A 2 = 1, 1 2 2 A ? ? 1 0 2 A ? ?	a a a x a a a x a a a

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23/267 **SUBMITTED TEXT** 25 WORDS	**75%** **MATCHING TEXT** 25 WORDS
Solve the equation (D – 1) (D + 5)y = 7e 2x (1) Solution : Characteristic equation is (D – 1)(D + 5) = 0 (2) ∴	Solve the Differential equation(D 2 +5D+6)y=e x Sol : Given equation is (D 2 +5D+6)

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24/267 **SUBMITTED TEXT** 29 WORDS	**47%** **MATCHING TEXT** 29 WORDS
y x y x ? constant and the general solution of equation (2.34) can be written as : y(x) = C 1 y 1 (x) + C 2 y 2 (x) (2.36)	

SA M P Vol 1.pdf (D134397037)

25/267 **SUBMITTED TEXT** 48 WORDS	**58%** **MATCHING TEXT** 48 WORDS
D F D a ? ? ? Now, F(D) = D 2 – 2D + 4 F(D + 1) = (D + 1) 2 – 2(D + 1) + 4 = D 2 + 2D + 1 – 2D – 2 + 4 = D 2 + 3 ∴ 2 2 1 1	D f) (1 .. cos) (1 ? ? Formulae 1. = (1 – D) -1 = 1 + D + D 2 + D 3 + ------------------ 2. = (1 + D) -1 = 1 - D + D 2 - D 3 + ------------------ 3. = (1 – D) -2 = 1 + 2D + 3D 2 + 4

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26/267 **SUBMITTED TEXT** 58 WORDS	**23%** **MATCHING TEXT** 58 WORDS
cos 2x + C 2 sin 2x (1) We assume particular integral y p = A sin x + B cos x (2) Substituting (1) in the original equation we get, (– A sin x – B cos x) + 4(A sin x + B cos x) = 2 sin x Comparing co-efficient of sin x and cos x	

SA Examnr_138343258_LinAlg_Vect.pdf (D139552471)

27/267 **SUBMITTED TEXT** 50 WORDS	**87%** **MATCHING TEXT** 50 WORDS
x x x x F D D D 2 2 1 2 2 (2 1) 3 9 27 D D x x ? ? ? ? ? ? ? ? ? ? ? 2 1 4 4 2 4 1 3 9 27 3 9 3 x x x ? ? ? ? ? ? ? 2 1 2 2 (2 1) (2 2) (2) 3 9 27 x	x ? =) 4 1 3 x ? = [1 + + + + ...+ ? ? 3 4 1 x ? ? ? ? ? ? ? ? ? 3 3 3 2 2 3 4 1 8 1 4 4 1 2 2 1 1 2 1 x D D D D D D ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = [1 - + - D] 1-4) = [(1-4) - + - (-12 = [-4x 3 +6x 2 -30x +16] = = [2x 3 -3x 2 +15x -8]

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28/267 **SUBMITTED TEXT** 73 WORDS	**39%** **MATCHING TEXT** 73 WORDS
cos2 2sin 2) (2 3) (12 32) sin 2 40 (12 32) (12 32) x x D D x x D D ? ? ? ? ? ? ? ? ? 2 (6cos 2 2sin 2) (2 3)(8)sin 2 40 4(9 64) x x D D x x D ? ? ? ? ? ? ? or 3cos 2 sin 2 (2 3)(6cos 2 8sin 2) 20 4(100) x x D x x PI x ? ? ? ? ? ? ? (3cos 2 sin 2) 48sin 2 14cos 2 20 400 x x x x	

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29/267 **SUBMITTED TEXT** 45 WORDS	**31%** **MATCHING TEXT** 45 WORDS
f x where 0 (,) (,) lim x x f x x y f x y f x ? ? ? ? ? ? ? ? (3.1) Similarly treating x as constant, we get ? ? 0 , (,) lim y y f x y y f x y f f y y ? ? ? ? ? ? ? ? ? ? ? (3.2)	f x x f x x x f x f x dx x h ?? ? ? ? ? ? ? 5 10 10 (0) 0 ( ) 100 0.0, 0.1, 0.2, ... , 1.0 i y y y x y y n x ?? ? ? ? ? ? ? ? ? • 22. 22 Finite Difference: Example 1 1 1 1 2 2 1 1 2 5 10 10 2 5 10 10 2 1 5 5 1 1 10 2 10 2 2 i i i i i i i i i i i y y

W https://www.slideshare.net/SyeilendraPramuditya/finite-difference-method-250591577

30/267 **SUBMITTED TEXT** 42 WORDS	**42%** **MATCHING TEXT** 42 WORDS
f f f f f f x y x y ? ? ? ? ? ? ? ? ? ? 48 ? NSOU ? CC-PH-04 3 3 3 3 2 2 , , xxx xxy xxz f f f f f f x x y x z ? ? ? ? ? ? ? ? ? ? ?	f f f f f f i j k y z x x y ? 3 3 2 1 2 1 .( ) f f f f f f div curl f f x y z y x z

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31/267 **SUBMITTED TEXT** 63 WORDS	**35%** **MATCHING TEXT** 63 WORDS
sin (cos) (cos) x x x e x x e x xdx e x x x dx D D D ? ? ? ? ? ? ? ? ? ? ? ? (cos sin) x e x x x dx ? ? ? ? ? ? sin sin cos x e x x x dx x ? ? ? ? ? ? ? ? ? = – e x (x sin x + 2 cos x) ∴	

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32/267 **SUBMITTED TEXT** 42 WORDS	**43%** **MATCHING TEXT** 42 WORDS
x = r cos ?, y = r sin ? 2 2 1 , tan y r x y x ? ? ? ? ? ? ? ? ? ? ? ? (i) Now 2 2 2 cos 2 sin r x x x r x y r y y r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (ii) 2 2 2 2 2 2 2 2 2 1 sin 1 1 1 cos 1 y x y x y y x r x x y	x 2r+2s*2b y 2b+1 ™ = sin y 1 X r=1 (2r + 1) ″ (x + y) 2r + (x * y) 2r * 2y 2r — * sin x 1 r=1 (1 * 2 *2r) (2x + y) 2r * (x * y) 2r — = x + y)+A(x * y)*2A(y) — * sin x ″ B(x + y)*B(x * y) — ; 984

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33/267 **SUBMITTED TEXT** 96 WORDS	**52%** **MATCHING TEXT** 96 WORDS
x r r r y r y y r r ? (iv) 2 2 sin sin cos cos x x r r r r x ? 2 2 2 2 2 2 2 2 2 2 2	X r=1 (2r + 1) ʺ (x + y) 2r + (x * y) 2r * 2 r — * sin x 1 X r=1 (1 * 2 *2r + 1) ʺ (x + y) 2r * (x *

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34/267 **SUBMITTED TEXT** 224 WORDS	**54%** **MATCHING TEXT** 224 WORDS
r x y z ? ? ? Show that 2 2 2 2 2 2 0 u u u x y z ? ? ? ? ? ? ? ? ? Solution : ? ? ? ? 1 2 1 2 2 2 2 2 2 2 1 u x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? 3 3 2 2 2 2 2 2 2 2 1 . 2 2 u x y z x x x y z x ? ? ? ? ? ? ? ? ? ? ? ? ? And ? ? ? ? 3 5 2 2 2 2 2 2 2 2 2 2 3 1. 2 2 u x y z x x y z x x ? 50 ? NSOU ? CC-PH-04 ? ? 5 2 2 2 2 2 2 2 2 3 x y z x y z x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 5 2 2 2 2 2 2 2 5 2 2 2 2 2 x y z x y z r x y z ? ? ? ? ? ? ? ? ? ? Similarly ? ? 2 5 2 2 2 2 2 u r y x z y ? ? ? ? ? ? and ? ? 2 5 2 2 2 2 2 u r z x y z ? ? ? ? ? ? ? ? ? ? 2 2 2 5 2 2 2 2 2 2 2 2 2 2 2 0 u u u r x y z x y z x y z ? 2)	

SA 07200140 (2).pdf (D118447009)

35/267 **SUBMITTED TEXT** 263 WORDS	**20%** **MATCHING TEXT** 263 WORDS
sin cos sin sin 2sin cos cos r r r r r ? (v) Similarly 2 2 2 2 2 2 2 2 2 2 2 2sin cos cos cos sin r r r y r r ? 2 2sin cos r ? ? ? ? ? ? (vi) Adding (v) and (vi) we get, 2 2 2 2 2 2 2 2 2 1 1 r r x y r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 1 1 0 u u u u u r r x y r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3) Prove that if f(x, y, z) = 0 I. ? ? 1 x y z y z x ? ? ? ? ? ? ? ? ? ? ? and two similar relation II. 1 x y z x y z y z x ? ?? Proof : We have, f(x, y, z) = 0 i.e. x = x(y, z) and z = z(x, y) y z x x dx dy dz y z ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (i) 52 ? NSOU ? CC-PH-04 and y x z z dz dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (ii) Eliminating dz from (i) and (ii) we get y y z x x x z z dx dy dx dy y z x y ? (	

SA 07200140 (2).pdf (D118447009)

36/267	SUBMITTED TEXT	77 WORDS	70%	MATCHING TEXT	77 WORDS

y z x x x z y z y ? or, 1 0 x y z y z x z x y ? or, 1 x y z y z x z x y ? (

SA Unit I.docx (D111988832)

37/267	SUBMITTED TEXT	15 WORDS	100%	MATCHING TEXT	15 WORDS

du u dx u dy dt x dt y dt ? ? ? ? ? ? ? ? ?

SA M P Vol 1.pdf (D134397037)

38/267	SUBMITTED TEXT	15 WORDS	100%	MATCHING TEXT	15 WORDS

du u dx u dy dt x dt y dt ? ? ? ? ? ? ? ? ? (

SA M P Vol 1.pdf (D134397037)

39/267	SUBMITTED TEXT	285 WORDS	39%	MATCHING TEXT	285 WORDS

x y z x y z xyz x y z xyz x y z xyz ? 2 2 2 2 2 2 3 3 ( ) x y z xy yz xz u u u x y z x y z x y z x y z xy yz xz ? Now 2 u u u u x y z x y z x y z ? ? ? ? ?? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? 3 x y z x y z ? NSOU ? CC-PH-04 ? 55 2 2 2 2 3 3 3 9 ( ) ( ) ( ) ( ) x y z x y z x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? Solution 2 : We have z x? and 1 z y ? . x z k y ? ? , where k is a constant. Now when x = 3, y = 5, 1 3 3 5 z k ? ? 5 9 k? ? 5 9 x z y ? ? ? ln z = ln 5x – ln 9y = ln 5 + ln x – ln 9 – ln y 0 0 z x y z x y ? ? ? ? ? ? ? ? max z x y z x y ? ? ? ? ? ? ? ? ? ? ? ? ? 100 100 100 z x y z x y ? ? ? ? ? ? ? ? ? = 1 + 0.5 = 1.5% ?

SA 07200146.pdf (D118456064)

40/267 **SUBMITTED TEXT** 99 WORDS	**34%** **MATCHING TEXT** 99 WORDS
x 2 y − x 3)dy = 0 Here M(x, y) = x 2 y − 2xy 2 2 4 M x y y ? ? ? ? ? N(x, y) = 3x 2 y − x 3 2 6 3 N xy x x ? ? ? ? ? Therefore, M N y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? and dz is not exact differential. Now 2 2 2 3 2 (,) 2 2 y y M x y x y xy x x x ? 3 y x x ? ? ? ? ? ? ? ? ? M(x, y)	x 3 y 2 (1-x- y) = x 3 x 4 y 2 − x 3 y 3 = 3x 2 y 2 − 4x 3 y 2 -3x 2 y 3 = 2x 3 x 4 y -3x 3 y 2 For maxima & minima = 0 and = 0 ? 3x 2 y 2 − 4x 3 x 2 y 3 = 0 =< x 2 y 2 (3 − 4x -3y) = 0 ---------------< (1) ? 2x 3 y − 2x 4 y -3x 3 y 2 = 0 =< x 3 y(2 − 2 x -3y) = 0 ----------------< (2)

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41/267 **SUBMITTED TEXT** 70 WORDS	**25%** **MATCHING TEXT** 70 WORDS
x y x dx dy x y x y ? ? ? ? Or, 2 1 2 3 0 x dx dy y x y y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Or, 2 1 2 3 0 x dx dy dx dy y x y y ? ? ? ? Or, 2 2 3 0 ydx xdy dx dy x y y ? ? ? ? NSOU ? CC-PH-04 ? 59 Or, 2 (log) 3 (log) 0 x d d x d y y ? ? ? ? ? ? ? ? ? Therefore 2 2 2log 3log dz x d x y y x	x y xyz x x y x y z xyz dx dy dz ? ? ? ? ? ? ? ? ? ? 2 2 2 1 1 1 0 0 0 x x y x y z dx dy xyz dz ? ? ? ? ? ? ? ? ? ? 2 2 2 1 2 1 1 0 0 0 2 x y x y z z dx xy dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 1 2 2 0 0 1 1 2 x x y dx xy x y dy ? ? ? ? ? ? ? ? ? ? 2 1 1 2 3 0 0 1 1 2 x x y d x x x y y d y ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 4 1 2 0 0 1 1 2 2 4 x x y y x

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42/267 **SUBMITTED TEXT** 74 WORDS	**34%** **MATCHING TEXT** 74 WORDS
x 2 + y 2 − R 2) 2 2 U y x x ? ? ? ? ? ? ? 2 2 U x y y ? ? ? ? ? For 0, 2 2 0 U y x x ? ? ? ? ? ? (i) And 0,2 2 0 U x y y ? ? ? ? ? ? (ii) From equation (i) and (ii), we get, y x x y ? ? ? ? ? or, x 2 = y 2 i.e. x = y (iii) 2 2 2 2 2 2 R R x y x y ? ? ? ? ? ? 2	x 3 y 2 (1-x-y) = x 3 y 2 - x 4 y 2 − x 3 y 3 = 3x 2 y 2 − 4x 3 y 2 -3x 2 y 3 = 2x 3 y − 2x 4 y -3x 3 y 2 minima = 0 and = 0 ? 3x 2 y 2 − 4 x 3 y 2 -3x 2 y 3 = 0 =< x 2 y 2 (3 − 4 x -3y) = 0 ---------------< (1) ? 2x 3 y − 2x 4 y -3

W https://www.iare.ac.in/sites/default/files/AERO_LINEAR_ALGEBRA_AND_CALCULUS_LECTURE_NOTES.pdf

43/267 **SUBMITTED TEXT** 133 WORDS	**21%** **MATCHING TEXT** 133 WORDS
x y z x y ? ? ? ? ? 1 0.5 1.5 100 100 100 x y z z z z x y ? error in z is 1.5% of z. Solution (3) : We have ( ) ( ) z f x ct x ct ? ? ? ? ? 56 ? NSOU ? CC-PH-04 ( ) ( ) ( ) ( ) z f x ct x ct x ct x ct x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ( ).1 ( ).1 f x ct x ct ? ? ? ? ? ? ? ? 2 2 ( ) ( ) z f x ct x ct x ? ? ?? ?? ? ? ? ? ? ? (i) Again ( ) ( ) ( ) ( ) ( ) ( ) z f x ct x	

SA 07200146.pdf (D118456064)

44/267	SUBMITTED TEXT	96 WORDS	35%	MATCHING TEXT	96 WORDS
	x 2 + y 2 + z 2 – a 2 = 0 66 ? NSOU ? CC-PH-04 Solution : Let 2 2 2 2 (, ,) x y z x y z a ? ? ? ? ? (i) We consider (, ,) (, ,) 4 (, ,) U F x y z x y z xyz x y z ?? ?? ? ? ? ? (ii) 4 2 0 U yz x x ? ? ? ? ? ? ? (iii) 4 2 0 U yz y y ? ? ? ? ? ? (iv) 4 2 0 U yz z z ? ? ? ? ? ? (			x + y + z =3a Sol: x 2 +y 2 +z 2 = x + y + z - 3a = 0 Using Lagrange's function F(x , y , z) = u(x , y , z) + (x , y , z) For or minima ? ? F x = + = 2x + = 0 ------------ (1) ? ? F y = + = 2y + = 0 ------------ (2) ? ? F z = + = 2z + = 0 ------------ (3) (1) , (2) & (3) = -2	

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45/267	SUBMITTED TEXT	124 WORDS	23%	MATCHING TEXT	124 WORDS
	x y z x y z r ? ? ? ? ? ? (ii) U V ?? ? ? ? (iii) Now, 0; U V x x x ? ? ? ? ? ? ? ? ? ? ? or, 8 (2) 0 yz x ?? ? (iv) 0; U V y y y ? ? ? ? ? ? ? ? ? ? ? or, 8 (2) 0 xz y ?? ? (v) 0; U V z z z ? ? ? ? ? ? ? ? ? ? ? or, 8 (2) 0 xy z ?? ? (vi) From (iv), 2 2 8 x xyz ? ? ? From (v), 2 2 8 y xyz ? ? ? From (vi), 2 2 8 z xyz ? ? ? 2 2 2 2 2 2 2 2 2 ; . . x y z ie x y z ? ? ? ? ? ? ? ? ? x = y = z				

SA Unit I.docx (D111988832)

46/267	SUBMITTED TEXT	25 WORDS	80%	MATCHING TEXT	25 WORDS
	U x y z V x y z ?? ? ? (i) or, 2 2 2 2 2 2 (, ,) 8 x y z U x y				

SA M P Vol 1.pdf (D134397037)

47/267	SUBMITTED TEXT	45 WORDS	39%	MATCHING TEXT	45 WORDS
	y z b c ? 2 2 2 2 2 2 x y z a b c ? ? ? Again we have 2 2 2 2 2 2 2 2 1 1; or, 3 x y z x a b c a ? ? ? ? , , and 3 3 3 a b c x y z ? ? ? ?				

SA mmp assignment 2.pdf (D118439176)

48/267	SUBMITTED TEXT	41 WORDS	75%	MATCHING TEXT	41 WORDS
	x y z f x y z x y z ?? ? ? (i) Where f(x, y, z) = x 2 + y 2 + z 2 and (, ,) x y z				

SA M P Vol 1.pdf (D134397037)

49/267	**SUBMITTED TEXT**	136 WORDS	**40%**	**MATCHING TEXT**	136 WORDS

a b c a b c p ? Or, ? ? 2 2 2 2 2 2 2 2 2 ; or, p a b c p a b c ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 , , ap bp cp x y z a b c a b c a b c ? ? ? ? ? ? ? ? ? ? minimum values of ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 a p b p c p f a b c a b c a b c ? ? ? ? ? ? ? ? ? or, ? ? ? ? 2 2 2 2 2 2 2 2 p a b c f a b c ? ? ? ? ? 70 ?

SA M P Vol 1.pdf (D134397037)

50/267	**SUBMITTED TEXT**	16 WORDS	**76%**	**MATCHING TEXT**	16 WORDS

Line Integral of a Vector Field 4.28.2 Surface Integral of a Vector Field 4.28.3 Volume Integral of

SA M P Vol 1.pdf (D134397037)

51/267	**SUBMITTED TEXT**	12 WORDS	**71%**	**MATCHING TEXT**	12 WORDS

A A a is a unit vector in the direction of the vector A.

SA M P Vol 1.pdf (D134397037)

52/267	**SUBMITTED TEXT**	29 WORDS	**71%**	**MATCHING TEXT**	29 WORDS

cos x y z A A A A A A ? ? ? ? ? ? 2 2 2 2 2 x y z A A A A ? ? ? ? A (4A.8) 4.3.8

SA M P Vol 1.pdf (D134397037)

53/267	**SUBMITTED TEXT**	36 WORDS	**34%**	**MATCHING TEXT**	36 WORDS

of vectors [Graphical representation] The sum of vectors A and B is a vector C by placing the origin of B on the terminus of A and joining the initial point of A to the terminus of B.

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

54/267	**SUBMITTED TEXT**	11 WORDS	**100%**	**MATCHING TEXT**	11 WORDS

that the sum of two vectors A and B is

SA M P Vol 1.pdf (D134397037)

55/267	**SUBMITTED TEXT**	107 WORDS	**34%**	**MATCHING TEXT**	107 WORDS

z A A A ? ? ? A i j k where 2 2 2 ; x y z A A A A ? ? ? ? A and , x y z B B B ? ? ? B i j k where 2 2 2 x y z B B B B ? ? ? ? B Therefore ? ? ? ? ? ? x x y y z z A B A B A B ? ? ? ? ? ? ? ? A B i j k and ? ? ? ? ? ? 1 2 2 2 2 x x y y z z A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B

SA 07200146.pdf (D118456064)

56/267	**SUBMITTED TEXT**	30 WORDS	**63%**	**MATCHING TEXT**	30 WORDS

A A A A (4A.9) Then the system of the vectors A 1 , A 2 , A 3 ... is said to be

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

57/267	**SUBMITTED TEXT**	62 WORDS	**70%**	**MATCHING TEXT**	62 WORDS

j k i j k i j k ???? 2 2 2 (1) (2) (6) 41 PQ PQ ? ? ? ? ? ? ? ? ? ???? ? ? 3 4 4 (3 5) 2 QR ? ? ? ? ? ? ? ? ? ? ? i j k i j k i j k ???? 2 2 2 (2) (1) (1) 6

SA MMP.pdf (D118446089)

58/267	**SUBMITTED TEXT**	22 WORDS	**83%**	**MATCHING TEXT**	22 WORDS

A = 2i + 3j − k and B = 3i − j + 5k. Find the value of A ±

SA VECTOR ANALYSIS.docx (D118765685)

59/267	**SUBMITTED TEXT**	98 WORDS	**83%**	**MATCHING TEXT**	98 WORDS

A = A x i + A y j + A z k = 2i + 3j − k B = B x i + B y j + B z k = 3i − j + 5k ? A + B = (A x + B x)i + (A y + B y)j + (A z + B z)k = (2 + 3)i + (3 − 1)j + (−1 + 5)k = 5

SA 07200146.pdf (D118456064)

60/267	**SUBMITTED TEXT**	23 WORDS	**47%**	**MATCHING TEXT**	23 WORDS

the unit vectors i, j, k along the rectangular co-ordinate system, i.i = j.j = k.k = 1 (4A.13) and i.j = j.k = k.

SA 07190529mmp.pdf (D118455416)

61/267	**SUBMITTED TEXT**	55 WORDS	**47%**	**MATCHING TEXT**	55 WORDS

x + jA y + kA z (4A.15) B = iB x + jB y + kB z (4A.16) will be written as A.B = A x B x + A y B y + A z B z (4A.17)

SA MMP.pdf (D118446089)

62/267	**SUBMITTED TEXT**	13 WORDS	**75%**	**MATCHING TEXT**	13 WORDS

A, B & C we can write A.(B + C) = A.B + A.C (4A.18)

SA M P Vol 1.pdf (D134397037)

63/267	**SUBMITTED TEXT**	56 WORDS	**87%**	**MATCHING TEXT**	56 WORDS

A + B).(A − B) = A.A + B.A − A.B. − B.B. = A 2 + B 2 2. (A + B) 2 = (A + B).(A + B) = A.A. + A.B. + B.A. + B.B. = A 2 + B 2 + 2A.B 4.5 ?

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

64/267	**SUBMITTED TEXT**	17 WORDS	**70%**	**MATCHING TEXT**	17 WORDS

the cross product of two vectors represents the area of the parallelogram having the two vectors as

SA Yeshy.pdf (D112546741)

65/267	**SUBMITTED TEXT**	66 WORDS	**39%**	**MATCHING TEXT**	66 WORDS

A × [B + C + D + ...] = A × B + A × C + A × D + ... 3. A × A = 0 4. i × j = k; j × k = i; k × i = j 5. i × i = j × j = k × k = 0 6. x y z x y z A A A B B B ? ?

SA MMP Assignment.pdf (D112572984)

66/267	**SUBMITTED TEXT**	152 WORDS	**42%**	**MATCHING TEXT**	152 WORDS

j k i ? ? ? B C B C We have B × C = i(B y C z − B z C y) + j(B z C x − B x C z) + k(B x C y − B y C x) We replace x, y, z by 1, 2, 3 Now the first component of 1 1 ( ) ? ? jk j k B C ? B C Now if j, k = 2, 3 or 3, 2 1 123 2 3 132 3 2 ( ) ? ? ? B C B C ? ? B C Now 123 1 ? ? ? and 123 1 ? ? ? ? (B × C) 1 = B 2 C 3 − B 3 C 2

SA MMP.pdf (D118446089)

67/267	**SUBMITTED TEXT**	48 WORDS	**62%**	**MATCHING TEXT**	48 WORDS

j k B C ? B C With j, k = 3, 1 or 1, 3 2 231 3 1 213 1 3 ( ) ? ? ? B C B C ? ? B C = B 3 C 1 − B 1 C 3

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

68/267	**SUBMITTED TEXT**	11 WORDS	**100%**	**MATCHING TEXT**	11 WORDS

A.B × C = B.C × A = C.A × B (4A.20)

SA MMP.pdf (D118446089)

69/267	**SUBMITTED TEXT**	14 WORDS	**100%**	**MATCHING TEXT**	14 WORDS

x y z x y z x y z A A A B B B C C C ?

SA M P Vol 1.pdf (D134397037)

70/267	**SUBMITTED TEXT**	17 WORDS	**78%**	**MATCHING TEXT**	17 WORDS

A × B.C = B × C.A = C × A.B 3. If any of the two

SA M P Vol 1.pdf (D134397037)

71/267	**SUBMITTED TEXT**	15 WORDS	**87%**	**MATCHING TEXT**	15 WORDS

A.B × C = B.C × A = C.A × B = volume of the

SA M P Vol 1.pdf (D134397037)

72/267	**SUBMITTED TEXT**	43 WORDS	**35%**	**MATCHING TEXT**	43 WORDS

A × (B × C) or (A × B) × C, parentheses is essential, since A × B × C is meaningless. We have A × (B × C) = (A.C)B − (A.B)C (4A.21) The value of a triple vector product is a linear combination of

SA M P Vol 1.pdf (D134397037)

73/267	**SUBMITTED TEXT**	12 WORDS	**100%**	**MATCHING TEXT**	12 WORDS

A × (B × C) lies in the plane of B and C.

SA M P Vol 1.pdf (D134397037)

74/267	**SUBMITTED TEXT**	53 WORDS	**41%**	**MATCHING TEXT**	53 WORDS

A × B) × C = (A.C)B − (B.C)A (4A.22) Now (B × C) × A = (B.A)C − (A.C)B = − [(A.C)B − (A.B)C] = − A × (B × C) Proof of equation (4A.21) : B × C is a vector perpendicular to the plane of B and C. thus A × (B × C)

SA M P Vol 1.pdf (D134397037)

75/267	**SUBMITTED TEXT**	47 WORDS	**43%**	**MATCHING TEXT**	47 WORDS

B, C and D is defined as (A × B). (C × D) Now let's suppose C × D = N Then (A × B).(C × D) = (A × B).N = A.B × N = A.B × (C × D) = A. [(B.D)C − (B.C)D] = (A.C)(B.D) − (A.D) (B.C)

SA M P Vol 1.pdf (D134397037)

76/267	**SUBMITTED TEXT**	76 WORDS	**23%**	**MATCHING TEXT**	76 WORDS

A × B) × (C × D) = N × (C × D) = (N.D)C − (N.C)D = (A × B.D)C − (A × B.C)D = lC − mD (4A.24B) where l and m are scalar. Therefore (A × B) × (C × D) lies in the plane of C and D. Now let C × D = N ? (A × B) × (C × D) = (A × B) × N = (A.N)B − (B.N)A = (A.C × D)B − (B.C ×

SA M P Vol 1.pdf (D134397037)

77/267	**SUBMITTED TEXT**	27 WORDS	**68%**	**MATCHING TEXT**	27 WORDS

vectors A = 4i + 3j + k and B = 2i − j + 2k. Also find a unit vector perpendicular to both A and B.

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78/267	**SUBMITTED TEXT**	50 WORDS	**43%**	**MATCHING TEXT**	50 WORDS

j + k and C = j − k. Find a vector B such that, A × B = C and A.B = 3 Solution : Suppose ? ? ? ? ? ? B i j k Now A × B = C gives, 1 1 1 ? ? ? ? ? i j k j k

SA 07200140 (2).pdf (D118447009)

79/267	**SUBMITTED TEXT**	111 WORDS	**34%**	**MATCHING TEXT**	111 WORDS

a = 4i + 3j + k, b = 2i − j + 2k, find a unit vector ˆ n perpendicular to vector a and b such that a, b, ˆ n form a right handed system. Find the angle between the vectors a and b. Solution : We have, 4 3 1 7 6 10 2 1 2 a b? ? ? ? ? ? i j k i j k And 2 2 2 (7) (6) (10) 185 ? ? ? ? ? ? ? ? a b Therefore 7 6 10 ˆ 185 ? ? ? ? ? ? ? a b i j k n a b Also 2 2 2 4 3 1 126 ? ? ? ? a 2 2 2 2 1 2 3 ? ? ? ? b

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

80/267	**SUBMITTED TEXT**	34 WORDS	**66%**	**MATCHING TEXT**	34 WORDS

If ? be the angle between a and b, then sin ,? ? ? a b a b then 0 185 sin sin 62 42; 3 26 ? ? ? ? ? ? ? ? a b a b 0 62 42 ? ? ? ? 94 ?

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

81/267	**SUBMITTED TEXT**	28 WORDS	**70%**	**MATCHING TEXT**	28 WORDS

A = 2i − j + k B = i + 2j − 3k C = 3i + ?j + 5k, are coplanar. Solution : There vectors A, B, C

SA VECTOR ANALYSIS.docx (D118765685)

82/267	SUBMITTED TEXT	166 WORDS	19%	MATCHING TEXT	166 WORDS

B) × C = A × (B × C) only when A and C are collinear or (A × C) × B = 0 Solution : Given (A × B) × C = A × (B × C) This is possible if, B(A.C) + C(A.B) – B(A.C) – A(B.C) = 0 or, if C(A.B) – A(B.C) = 0 or, if (A × C) × B = 0 This show that either B = 0 or A × C = 0, but 0?B , hence A × C = 0, hence A and C are collinear. Example 11 : If A, B and C satisfy the condition (A × B) + (B × C) + (C × A) = 0, show that the vector are coplanar. Solution : We have (A × B) + (B × C) + (C × A) = 0 ? [(A × B) + (B × C) + (C × A)].A = 0 or, A × B.A + B × C.A + C × A.A = 0 or, B × C.A = 0 ;

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

83/267	SUBMITTED TEXT	51 WORDS	30%	MATCHING TEXT	51 WORDS

three vectors A, B, C defined by the equation , , [ ] [ ] [ ] ? ? ? ? ? ? b c c a a b A B C abc abc abc (4A.26) Are called reciprocal vectors triads to the vectors a, b and c. The vector triads a, b and c and its reciprocal triads A, B,

SA VECTOR ANALYSIS.docx (D118765685)

84/267	SUBMITTED TEXT	34 WORDS	38%	MATCHING TEXT	34 WORDS

a, b, c) and (A, B, C) are mutually reciprocal. i.e. , , [ ] [ ] [ ] ? ? ? ? ? ? B C C A A B a b c ABC ABC ABC Where A, B, C

SA M P Vol 1.pdf (D134397037)

85/267	SUBMITTED TEXT	96 WORDS	32%	MATCHING TEXT	96 WORDS

b.B = c.C = 1 (4A.27) Proof : We have [ ] ? ? b c A abc [ ] . 1 [ ] [ ] . ? ? ? ? ? a b c abc a A abc abc Similarly b.B = c.C = 1 ; Then a.A + b.B + c.C = 3 And 1 1 1 , , ? ? ? A B C a b c 2. If a, b, c and A, B, C are reciprocal triad of vectors, then a.B = a.C = 0 b.A = b.C = 0 (4A.28) 96 ? NSOU ? CC-PH-04 c.A = c.B = 0 Proof : [ ] . 0 [ ] [ ] . ? ? ? ? c a acb a B a

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

86/267	SUBMITTED TEXT	64 WORDS	34%	MATCHING TEXT	64 WORDS
A, B, C. Proof : ? ? 3 ( ) ( ) [ ] . ( ). [ ] c a a b abc ? ? ? ? ? ? ? ABC AB C b c , using equation (4A.26) Now (c × a) × (a × b) = (c × a) × N when N = a × b = a(c.N) − c(a.N) = a(c.a × b) − c(a.a × b) = a[abc] since a.a ×					

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

87/267	SUBMITTED TEXT	35 WORDS	70%	MATCHING TEXT	35 WORDS
j − k) − (i + 2j + 3k) = 4i + 2j − 4k Resultant forces F = (4i + j − 3k) + (3i + j − k) = 7i + 2j − 4k ?					

SA MMP.pdf (D118446089)

88/267	SUBMITTED TEXT	49 WORDS	55%	MATCHING TEXT	49 WORDS
z x y z A A A A A A ? ? ? ? ? ? ? ? ? ? ? ? A i j k i j k (4A.34) and 2 2 2 2 2 2 x y z x y z A A A A A A ? ? ? ? ? ? ? ? ? (4			z x a i j a i x a i x a i ? = 2 2 2 2 . . . . a a a a a a i i j i k i i x x y y z x x x x ? 2 2 2 2 2 2 2 2 ( ) . (.) a a a a a i a i a		

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89/267	SUBMITTED TEXT	79 WORDS	37%	MATCHING TEXT	79 WORDS
x y x y x y ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 cos sin 2 sin cos sin cos 2 sin cos x y xy x y xy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 cos sin cos sin x y x y ? ? ? ? ? ? ? ? ? ? (4					

SA M P Vol 1.pdf (D134397037)

90/267	SUBMITTED TEXT	61 WORDS	46%	MATCHING TEXT	61 WORDS
x + jA y + kA z and B = iB x + jB y + kB z and their scalar products A.B = A x B x + A y B y + A z B z					

SA MMP.pdf (D118446089)

91/267	**SUBMITTED TEXT**	12 WORDS	**66%**	**MATCHING TEXT**	12 WORDS

cos sin cos sin x x y y x y z z A A A A A A

SA M P Vol 1.pdf (D134397037)

92/267	**SUBMITTED TEXT**	185 WORDS	**27%**	**MATCHING TEXT**	185 WORDS

A B A B A B ? ? ? ? ? ? ? ? ? ? ? A B (4A.38) Now substituting equation (4A.36) and (4A.37) in equation (4A.38), we get ? ?? ? . cos sin cos sin x y x y A A B B ? ? ? ? ? ? ? ? ? A B ? ? ? ? sin cos sin cos x y x y z z A A B B A B ? ? ? ? ? ? ? ? ? ? 2 2 2 cos cos sin sin cos sin sin x x x y y x y y x x A B A B A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? 2 sin cos cos sin cos x y y x y y z z A B A B A B A B ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 cos sin cos sin x x y y z z A B A B A B ? ? ? ? ? ? ? ? ? . x x y y z z A B A B A B ? ? ? ? A B (4A.39) equation (4A.39)

SA 07200146.pdf (D118456064)

93/267	**SUBMITTED TEXT**	131 WORDS	**39%**	**MATCHING TEXT**	131 WORDS

y ? ? ? ? ? ? ? ? ? ? A B A B A B (4A.40) ( ) ( ) sin ( ) cos y x y ? ? ? ? ? ? ? ? ? ? ? A B A B A B (4A.41) ( ) ( ) z z ? ? ? ? ? ? A B A B (4A.42) NSOU ? CC-PH-04 ? 101 Now squaring equations (4A.40) and (4A.41) both sides and adding, we get 2 2 2 2 2 2 ( ) ( ) ( ) ( ) ( ) ( ) x y z x y z ? A B A B A B A B A B or, 2 2 ? ? ? ? ? ? A B A B ? ? ? ? ? A B A B (4A.42)

SA 07200146.pdf (D118456064)

94/267	**SUBMITTED TEXT**	16 WORDS	**95%**	**MATCHING TEXT**	16 WORDS

Find a unit vector parallel to the sum of vectors A 1 = 2

SA Yeshy.pdf (D112546741)

95/267	SUBMITTED TEXT	49 WORDS	26%	MATCHING TEXT	49 WORDS

A, B and C are a, b, c. 3) If |A + B| = | A − B|, then show the A and B are perpendicular. 4) If A.B = A.C, does it necessarily follow that B and C are equal. 5) If |A| = |B|, prove that A + B

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96/267	SUBMITTED TEXT	47 WORDS	58%	MATCHING TEXT	47 WORDS

A × B).(C × D) = (A.C)(B.D) − (A.D)(B.C) b) (A × B).(C × D) + (B.C).(A × D) + (C × A).(B × D) = 0 10) If A,B,C,D are such that A × C = B × D and A × B = C ×

SA M P Vol 1.pdf (D134397037)

97/267	SUBMITTED TEXT	80 WORDS	35%	MATCHING TEXT	80 WORDS

A + B| = |A − B| Squaring both sides, |A + B| 2 = |A − B| 2 Or A 2 + B 2 + 2A.B = A 2 + B 2 − 2A.B Or, 4A.B = 0 A.B = 0 i.e. A is perpendicular to B and vice-versa. Solution (4) : If A.B = A.C, then A.(B − C) = 0, i.e. either A is perpendicular to B− C or B− C = 0

SA 07200146.pdf (D118456064)

98/267	SUBMITTED TEXT	36 WORDS	67%	MATCHING TEXT	36 WORDS

A| 2 = |B| 2 ; or A.A = B.B Now, (A + B).(A − B) = A.A − A.B+ B.A − B.B = 0 Since A.A = B.B ? A + B is

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

99/267	SUBMITTED TEXT	32 WORDS	68%	MATCHING TEXT	32 WORDS

cos y x z A A A ? ? ? ? ? ? A A A ? ? 2 2 2 2 2 2 2 2 2 cos cos cos 1 x y z A A A ? ? ? ? ? ? ? ? ? ? ? ? A A A (

SA M P Vol 1.pdf (D134397037)

100/267 **SUBMITTED TEXT** 119 WORDS **42%** **MATCHING TEXT** 119 WORDS

A + B = − C, now B × (A + B) = − B × C Or, B × A + B × B = − B × C, or, − A × B = − B × C Or, A × B = B × C (i) Similarly, B × C = C × A (ii) Therefore, A × B = B × C = C × A b) We have A × B = B × C ? − A × B = − B × C ? B × A = C × B or, B × A + B × B = C × B = − B × C or, B × A + B × B + B ×

a 2 -b 2 -c 2 =0 We get c = b 2 +2b 2 =3b 2 ? a = From the diagonal elements of I 4b 2 +c 2 = 1 ? 4 b 2 +2 b 2 =1 (c 2 =2b 2) ? b = a= = b= c = = ? ? ? ? ? ? ? ? ? ? ? ? c b a c b a c b2 0 ? ? ? ? ? ? ? ? ? ? ? ? c b a c b a c b2 0 I c c c b b b a a ? ? ? ? ? ? ? ? ? ? ? ? ? 2 0 ? 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 4 c b a c a c b c b a c b a b c b c b c b ? ? ? ? ? ? ? ? ? ? 1 0 0 0 1 0 0 0 1 b2 ? b3 ? 6 1 ? b3 ? 2 1 ? 6 1 ?

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101/267 **SUBMITTED TEXT** 13 WORDS **90%** **MATCHING TEXT** 13 WORDS

j k A i j k A A A A A A 104 ?

SA 07200140 (2).pdf (D118447009)

102/267 **SUBMITTED TEXT** 129 WORDS **24%** **MATCHING TEXT** 129 WORDS

C × D) = (C × D).P = C.D × P = C.D × (A × B) = C.[(D.B)A − (D.A)B] = (C.A)(D.B) − (C.B)(D.A) = (A.C)(B.D) − (A.D)(B.C) b) Now, (A × B).(C × D) = (A.C)(B.D) − (A.D)(B.C) Similarly (B × C).(A × D) = (B.A) (C.D) − (B.D) (A.C) (C × A).(B × D) = (C.B) (A.D) − (C.D) (A.B) ? (A × B).(C × D) + (B × C).(A × D) + (C × A).(B × D) = 0 Solution (10) : We have (A − D) × (B − C) = (A − D) × B − (A × D) × C = A × B − D × B − A × C + D × C = A × B + B × D − A × C − C ×

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

103/267 **SUBMITTED TEXT** 11 WORDS **84%** **MATCHING TEXT** 11 WORDS

the position vector of the point P(x,y,z) with respect to

SA M P Vol 1.pdf (D134397037)

104/267 **SUBMITTED TEXT** 21 WORDS **45%** **MATCHING TEXT** 21 WORDS

r(u) = ix(u) + jy(u) + kz(u). And 0 lim ? ? ? ? ? u d du u r r 0 ( ) ( ) lim u u u u u ? ? ? ? ? ? ?

SA M P Vol 1.pdf (D134397037)

105/267 **SUBMITTED TEXT** 31 WORDS	**70%** **MATCHING TEXT** 31 WORDS
u ( ) ( ) 0 u u u ? ? ? ? A A 0 ( ) ( ) lim 0 ( ) u u u u d u du ? ? ? ? ? ? ? ? A	

SA M P Vol 1.pdf (D134397037)

106/267 **SUBMITTED TEXT** 38 WORDS	**38%** **MATCHING TEXT** 38 WORDS
A a a ˆ ˆ d dA d A du du du ? ? ? ? ? ? ? ? ? ? ? A a A A a ˆ ˆ ˆ dA d A A du du ? ? ? ? ? ? ? ? ? a a a 2 ˆ ˆ ˆ ˆ 0 dA d A A du du ? ? ? ? ? a a a a	a a a a a a a a a a a .... 2 1 2 1 2 22 21 1 12 11 n×n ? ? ? ? ? ? ? ? ? ? 33 32 31 23 22 21 13 12 11 a a a a a a a a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 4 2 3 3 1 3 1 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 4 2 3 3 1 3 1 1 4 2 3 1 3 4 2 3 1 1 4 4 3 3 ? ? ? ? ? ? ? ? ? ? ?

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107/267 **SUBMITTED TEXT** 39 WORDS	**41%** **MATCHING TEXT** 39 WORDS
A A Proof : Consider ˆ( )a u to be a unit vector in the direction of A(u), the A(u) = ˆ ˆ ˆ ( ) ( ) ( ) ( ) u u A u u A ? ? A a	

SA M P Vol 1.pdf (D134397037)

108/267 **SUBMITTED TEXT** 9 WORDS	**100%** **MATCHING TEXT** 9 WORDS
A B C B C A C A B The	

SA M P Vol 1.pdf (D134397037)

109/267 **SUBMITTED TEXT** 36 WORDS	**88%** **MATCHING TEXT** 36 WORDS
The gradient of a scalar field (, ,) x y z ? at a point (x 0 , y 0 , z 0) is a vector,	The gradient of a scalar function ?(x,y,z) at a point P(x,y,z) is a vector

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110/267 **SUBMITTED TEXT** 69 WORDS	**21%** **MATCHING TEXT** 69 WORDS
dt dt ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? v r θ a r θ r θ θ ? ? ?? ? ? ? ?? ? ? where 2 2 d r r dt ??? and 2 2 d dt ? ? ? ?? . Now substituting the values of ˆd dt r and ˆ d dt θ , we get Fig. Solution 1 NSOU ? CC-PH-04 ? 115 ? ? ? ? 2 ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ 2 r d d a r r r r r r r r	

SA MMP.pdf (D118446089)

111/267 **SUBMITTED TEXT** 29 WORDS	**80%** **MATCHING TEXT** 29 WORDS
Find directional derivative of ??= x 2 − 2y 2 + 4z 2 at the point (1, 1, − 1) in the direction 2	Find directional derivative of the function f = x 2 -y 2 +2z 2 at the point P =(1,2,3) in the direction

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112/267 **SUBMITTED TEXT** 38 WORDS	**89%** **MATCHING TEXT** 38 WORDS
x y z x y z x y z x y z ? i j k (4	

SA Yeshy.pdf (D112546741)

113/267 **SUBMITTED TEXT** 48 WORDS	**80%** **MATCHING TEXT** 48 WORDS
Unit vector along the direction 2i + j − k is ? ? 1 2 6 ? ? i j k ? Required directional derivative is, ? ? ? ? 1 4 4 8 8 2 4 8 . 2 6 6 6 ? ? ? ? ? ? ? ? ? i j k i j k	

SA VECTOR ANALYSIS.docx (D118765685)

114/267 **SUBMITTED TEXT** 38 WORDS	**89%** **MATCHING TEXT** 38 WORDS
Find the angle between the surface x 2 + y 2 + z 2 = 4 and z = x 2 + y 2 − 5 at	Find the angle between the surfaces x 2 +y 2 +z 2 =9, and z = x 2 +y 2 - 3 at

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115/267 **SUBMITTED TEXT** 49 WORDS	**50%** **MATCHING TEXT** 49 WORDS
x y z x y z ? ? ? ? ? ? 2 2 2 2 2 x y z ? ?? ? ? ? ? ? ? i j k i k at the point (1, 0, −1). Now unit normal to (, ,) x y z ? ?	x 2 +y 2 +z 2 =29 and x 2 +y 2 +z 2 +4x-6y-8z-47 =0 at the point (4,- 3,2). Sol:- Let f = x 2 +y 2 +z 2 -29

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116/267	**SUBMITTED TEXT**	60 WORDS	**48%**	**MATCHING TEXT**	60 WORDS
x y z ? ? ? ? ? ? ? ? ? i j k i j k or, 2 1 5 4 2 1 x y z ? ? ? ? ? ? ? Exercise of Art 4.14, 4.15 and 4.18 : 1) Find unit vector normal to the surface x 2 + y − z = 1 at the point (1, 0, 0). 2)					

SA 2021asgn.PG.pdf (D112583081)

117/267	**SUBMITTED TEXT**	81 WORDS	**71%**	**MATCHING TEXT**	81 WORDS
x x y x y ? ? ? 2 2 x x y z x y ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k 2 2 2 2 .0 x x x y x y x y ? i j k ? ? ? ? 2 2 2 2 2 2 2 2 1 (2) (2) x x x y x y x y x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?					

SA mmp assignment 2.pdf (D118439176)

118/267	**SUBMITTED TEXT**	36 WORDS	**80%**	**MATCHING TEXT**	36 WORDS
x y z x y z ? ? ? ? ? ? and 2 (, ,) x y z ? 2 2 2 5 0 x y z ? ? ? ? ? Now, the					

SA Mathematical Physics - I SLM full.pdf (D113782471)

119/267	**SUBMITTED TEXT**	76 WORDS	**35%**	**MATCHING TEXT**	76 WORDS
y x z A A A x y z ? ? ? ? ? ? ? ? ? ? ? A (4B.9) And curl A or rot A or ? × A by ? ? ? ? ? ? ? ? x y z x y z A A A i j k A ? y y x x z z A A A A A A y z z x x y ?			y x z ? 2 2 2 2 a a i i i i k x x y x z ? Page | 154]0 . .,1 .[. . . 2 2 2 2 2 2 ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ki ji ii k zx a i j yx a i x a i x a i ? = 2 2 2 2 . . . . a a a a a a i i j i k i i x x y y z x x x x ? 2 2 2 2 2 2 2 2 ( ) . (.) a a a a a i a i a x x x y		

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120/267	SUBMITTED TEXT	259 WORDS	36%	MATCHING TEXT	259 WORDS

r x y z ? ? ? ? ? r ? ? 2 2 2 2 2 2 2 1 ln ln 2 x y z x y z ?? ? ? ? ? ? ? ? ? 2 2 2 1 ln 2 x y z x y z ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? i j k Fig. Solution (4) 122 ? NSOU ? CC-PH-04 2 2 2 2 2 2 2 2 2 2 2 2 1 2 2 2 2 x y z x y z x y z x y z x y z x y z ? i j k i j k 2 ˆ r r ? ? r r ii) We have ? ? 2 2 2 2 n n r x y z ? ? ? ? ? 2 2 2 2 n n r x y z x y z ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k ? ? ? ? ? ? 2 2 2 2 2 2 2 2 2 2 2 2 n n n x y z x y z x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k Or ? ? ? ? 1 1 2 2 2 2 2 2 2 2 2 .2 .2 2 2 n n n n n r x y z x x y z y ? i j ? ? 1 2 2 2 2 .2 2 n n x y z z ? ? ? ? ? ? ? ? ? ? ? ? k ? ? 1 2 2 2 2 ( ) n n x y z x y z ? ? ? ? ? ? i j k ? ? 1 2 2 1 2 ˆ

SA 07200140 (2).pdf (D118447009)

121/267	SUBMITTED TEXT	162 WORDS	15%	MATCHING TEXT	162 WORDS

div r n r ? ? r Solution 1 : Let ? ? ? ? 1 2 2 2 2 1 2 3 , n n r r x y z r x y z ? ? ? ? ? ? ? ? ? ? ? ? r i j k i j k 3 1 2 n div r x y z ? ? ? ? ? ? ? ? ? ? ? ? ? r Fig. (4B.9) d? ˆn NSOU ? CC-PH-04 ? 129 now 1 n xr ? ? 1 1 2 2 1 n n n n n n r x r nxr r nxr r nr x x x r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? similar expressions for 3 2 , y z ? ? ? ? ? ? can be obtained and are given by 2 2 2 n n r nr y y ? ? ? ? ? ? 2 2 3 n n r nr z z ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 3 3 (3) n n n n n div r r nr x y z r nr r n ? ? ? ? ? ? ? ? ? ? r Note : When n = – 3, 3 0. n r div r

SA MMP Assignment.pdf (D112572984)

122/267 **SUBMITTED TEXT** 513 WORDS **19%** **MATCHING TEXT** 513 WORDS

n r n n r ? ? ? ? Solution 2 : ? ? 2 2 2 n n n n r r nr n r ? ? ? ? ? ? ? ? ? ? ? ? ? r r Let 2n r ? ? ? . Now using formula (9) in Art 4B.14.2, we can write 2 n r n n n ? ? ? ? ? ? ? ? ? ? ? ? ? r r r Now x y z x y z ? ? ? ? ? ? ? ? ? ? ? r = 1 + 1 + 1 = 3, where r = ix+ jy + kz and 2 3 ˆ (2) n n r n r ? ? ? ? ? ? ? ? ? r = (n − 2) r n−4 r 2 2 4 2 2 3 (2) . 3 (2) n n n n n r nr n n r nr n n r ? ? ? ? ? ? ? ? ? ? ? ? ? r r ? ? 2 2 2 2 3 (2) (1) n n n n n n r n n r n n r ? ? ? ? ? ? ? ? ? ? ? ? ? ? 130 ? NSOU ? CC-PH-04 Example 3 : Prove that 2 2 2 2 ( ) d f df f r r dr dr ? ? ? Solution 3 : 2 ( ) ( ) f r f r ? ? ? ? ? Now, ( ) ( ) ( ) ( ) ( ) ( ) ( ) f r f r f r f r r f r r f r r f r x y z r x r y r z ? i j k i j k ( ) ( ) ( ) 1 ( ) ( ) ( ) f r x f r y f r z f r f r x y z r r r r r r r r r ? r i j k i j k Now 2 1 ( ) ( ) df r f r r dr ? ? ? ? r ( ) ( ) ( ) x df r y df r z df r x r dr y r dr z r dr ? ( ) ( ) ( ) x y z f r f r f r x r y r z r ? Now, 1 1 ( ) ( ) ( ) ( ) x x f r f r x f r f r x r r x r r x ? ( ) 1 ( ) ( ) f r r x r x f r f r r r r x r r x ? 2 2 3 3 ( ) ( ) ( ) ( ) ( ) ( ) f r

SA 07170523.pdf (D54981551)

123/267 **SUBMITTED TEXT** 76 WORDS **37%** **MATCHING TEXT** 76 WORDS

x x x f r x x x f r f r f r f r r r r r r r r r ? ? ? ?? ? ? ?? ? ? ? ? ? ? ? ? 2 2 3 2 ( ) ( ) ( ) f r x x f r f r r r r r ? ? ? ?? ? ? ? ? (i) Similarly 2 2 3 2 ( ) ( ) ( ) ( ) ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? y f r y y f r f r f r y r r r

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124/267 **SUBMITTED TEXT** 51 WORDS **51%** **MATCHING TEXT** 51 WORDS

x y z x y z f r f r f r f r r r r ? ? ? ? ? ? ? ?? ? ? ? ? ? 3 ( ) 2 ( ) ( ) ( ) ( ) f r f r f r f r f r r r r ? ? ? ?? ?? ? ? ? ? ? ? ?

SA 07200140 (2).pdf (D118447009)

125/267	SUBMITTED TEXT	115 WORDS	34%	MATCHING TEXT	115 WORDS

y)i + (y − 2z)j + (x + az)k is solenoidal. Solution : Solution (1) : x y z ? ? ? ? ? ? ? i j k ? = constant vector Now linear velocity x y z x y z ? ? ? ? ? ? ? i j k v r ? or, ? ? ? ? ? ? ? y z z x x y z y x z y x ? ? ? ? ? ? ? ? ? ? ? ? ? v i j k Now, ? ? ? ? ? ? ? 0 0 0 0 y z z x x y z y x z y x x y z ?

SA 07200146.pdf (D118456064)

126/267	SUBMITTED TEXT	324 WORDS	27%	MATCHING TEXT	324 WORDS

r ? ? ? ? ? ? ? ? ? Solution (3) : ? ? 3 3 3 3 x y z xyz ? ? ? ? ? F or, ? ? ? ? 3 3 3 3 3 3 3 3 x y z xyz x y z xyz x y ? ? ? ? ? ? ? ? ? ? ? ? ? F i j ? ? 3 3 3 3 x y z xyz z ? ? ? ? ? ? ? k ? ? ? ? ? ? ? 2 2 2 3 3 3 3 3 3 x yz y xz z xy ? ? ? ? ? ? ? i j k Now, ? ? ? ? ? ? ? 2 2 2 3 3 3 3 3 3 x yz y xz z xy x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? F = 6x + 6y + 6z = 6(x + y + z) Solution (4) : 2 2 2 2 r x y z r r r r ? ? ? ? ? ? F i j k where r 2 = x 2 + y 2 + z 2 Now, 2 2 2 x y z x y z r r r ? F Now, ? ? 2 2 2 2 2 2 2 2 2 2 2 2 1 2 x x x y z x x x r x y z x y z ? 2 2 4 1 2x r r ? ? Similarly, 2 2 2 4 1 2 y y y r r r ? ? ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 133 Fig (4B.10) and 2 3 2 4 1 2 z z z r r r ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 4 2 2 2 2 3 3 2 1 x y z r r r r r ? ? ? ? ? ? ? ? ? ? F Solution 5 : A = (x+ 3y)i + (y − 2z)j + (x + az)k Now, (3) (2) ( ) 1 1 2 x y y z

SA 07200140 (2).pdf (D118447009)

127/267	SUBMITTED TEXT	29 WORDS	47%	MATCHING TEXT	29 WORDS

is y z z y z y A A A dz A dz dy A dy dz A dy z y ? [

SA M P Vol 1.pdf (D134397037)

128/267	SUBMITTED TEXT	42 WORDS	88%	MATCHING TEXT	42 WORDS

y y x x z z A A A A A A z y z x x y ?

SA M P Vol 1.pdf (D134397037)

129/267 **SUBMITTED TEXT** 72 WORDS **68%** **MATCHING TEXT** 72 WORDS

x y z x y z x y z x y z ? r r r v r ? ? ? ? ? ? ? 1 3 3 2 2 x y z ? ? ? ? ? ? ? ? ? ? ? ? ? i j k

SA 07200140 (2).pdf (D118447009)

130/267 **SUBMITTED TEXT** 100 WORDS **26%** **MATCHING TEXT** 100 WORDS

Find the constant a, b, c so that the vector, A = (x + 2y + az)i + (bx – 3y – z)j + (4x + cy + 2z)k, is irrotational. Solution 2 : Since A is irrotational, 0 ? ?A ? Or, 0 2 3 4 2 x y z x y az bx y z x cy z ? ? ? ? ? ? ? ? ? ? ? ? ? i j k Or, (c + 1)i + j(a – 4) + k(b – 2) = 0 Therefore, 1 0 1 c c ? ? ? ? ? 4 0 4 a a ? ? ? ? 2 0 2 b

SA 2021asgn.PG.pdf (D112583081)

131/267 **SUBMITTED TEXT** 95 WORDS **28%** **MATCHING TEXT** 95 WORDS

y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A i j k ? Or, if A x , A y and A z are the components of the vector A, we can write, x y z A A A x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A i j k i j k (2 4) x A x y z x ?? ? ? ? ? ? ? ? ? (i) (2 3) y A x y z y ?? ? ? ? ? ? ? ? (ii) (4 2) z A x y

SA 07170523.pdf (D54981551)

132/267 **SUBMITTED TEXT** 66 WORDS **46%** **MATCHING TEXT** 66 WORDS

f 1 (y, z) = – zy 2 2 3 2 ? ? y z 2 2 2 2 2 3 3 (,) 4 2 , (,) 2 2 2 x y f x z zx x z f x y yz ? ? ? ? ? ? 2 2 2 3 2 4 2 2 x y z yx yz xz c ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? where c is a constant

f 1 (y,z) ? ? ? y ? 2y-z ??= 2xy-3y 2 /2-yz+f 2 (z,x) ? ? ? z ? 4x-y+2z ??= 4xz-yz+z 2 +f 3 (x,y) Hence ?= x 2 /2 -3y 2 /2+ z 2 +2xy+4zx-yz+c 11: If ? is a constant

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z f x y ? ? ? ? ? ? (vi) 138 ? NSOU ? CC-PH-04 f 1 (y, z), f 2 (x, z) and f 3 (x, y)

SA M P Vol 1.pdf (D134397037)

134/267 SUBMITTED TEXT 54 WORDS	**66%** MATCHING TEXT 54 WORDS
x y z ? ? ? ? ? ? ? ? ? ? ? ? 1 2 3 ; ; F F F x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 139 1 2 3 F F F x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?	x+y 3 1 2 1, 1, 0 F F F x y z ? ? ? ? ? ? ? ? ? ? and 3 1 2 1 1 0 2 F F F x y z ? ? ? ? ? ? ? ? ? ? ? ?

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135/267 SUBMITTED TEXT 120 WORDS	**51%** MATCHING TEXT 120 WORDS
F ? or, 1 2 3 0 x y z F F F ? ? ? ? ? ? ? ? i j k or, 3 3 2 1 2 1 0 F F F F F F y z z x x y ? i j k or, 3 3 2 1 2 1 0 F F F F F F y z z x x y ? = ? F 1 dx + F 2 dy + F 3	f x y z f f f ? ? ? ? ??? ? ? ? ? 3 3 2 1 2 1 f f f f f f i j k y z x z x y ? 3 3 2 1 2 1 .( ) f f f f f f div curl f f x y z y x z z x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? 0 1 2 2 2 1 2 3 2 2 2 3 2 ? ?? ? ? ?? ? ? ?? ? ? ?? ? ? ?? ? ? ?? ? ? f xz f zy f

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136/267 SUBMITTED TEXT 14 WORDS	**76%** MATCHING TEXT 14 WORDS
x, y, z); Q = Q(x, y, z); R = R(x, y, z),	

SA MMP.pdf (D118446089)

137/267 SUBMITTED TEXT 13 WORDS	**87%** MATCHING TEXT 13 WORDS
If A and B are irrotational, show that Ax B is solenoidal. 3)	

SA VECTOR ANALYSIS.docx (D118765685)

138/267 SUBMITTED TEXT 41 WORDS	**61%** MATCHING TEXT 41 WORDS
We have 0, 0 ? ? ? ? A B ? ? ( ) 0 ? ? ? ? ? ? ? ? ? A B B A A B ? ? ? (See Art 4B.14.2 item no. 12) ? A × B	

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

139/267 SUBMITTED TEXT 72 WORDS	**56%** MATCHING TEXT 72 WORDS
r ? ? ? Now x y z x y z ? ? ? ? ? ? ? ? i j k r ? .0 .0 .0 0 z y x z y x y z z x x y ? i j k i j k Since x, y, z	

SA mmp ass 2.pdf (D118459575)

140/267	SUBMITTED TEXT	79 WORDS	77%	MATCHING TEXT	79 WORDS

x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k 2 2 2 2 2 2 y z z y z x x z x y y x ?

x y z l grad x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = 2 2 2 2 2 2 0 i j y z z y x z z x y y x ?

W https://www.iare.ac.in/sites/default/files/CSE_LINEAR_ALGEBRA_AND_CALCULUS_LECTURE_NOTES.pdf

141/267	SUBMITTED TEXT	314 WORDS	24%	MATCHING TEXT	314 WORDS

x y z A A A x y z ? A r r A A r A r ? ? ? 3 x y z A A A x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? r r r A 3 3 2 x y z A A A ? ? ? ? ? ? ? ? ? ? ? ? A i j k A A A where A = iA x + jA y + kA z ii) We have x y z x y z A A A ? ? ? ? ? ? ? ? i j k A ? y z z x x y A A A A A A z y x z y x ? i j k Now, ? ? y z z x x y i j k A A A A A A z y x z y x x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A r ? z x x y x y A A z A A y A A x x z y x y x ? i i j y z y z z x A A z A A y A A x z y z y x z ? j k k = – iA x – iA x – jA y – jA y – kA z – kA z = – 2[

SA 07170523.pdf (D54981551)

142/267	SUBMITTED TEXT	135 WORDS	16%	MATCHING TEXT	135 WORDS

A) ( ) ? ? ? A ? ? 2 ( ) ? ? ?A A ? ? ? Proof : 2 ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? A A A A A ? ? ? ? ? ? ? ? ? Using the formula for A × (B × C), treating ? as a normal vector and also an operator which differentiate A. Laplacian of a vector, 2 A ? , simply means a vector whose components are 2 2 2 , , x y z A A A ? ? ? 144 ? NSOU ? CC-PH-04 4< , ? ? ? ? ? ? ? ? ? ? A A A where ? is a scalar.. Proof : ? ? ? ? A ? ? ? ? ? ? ? ? ? ? ? ? A A A

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

143/267	SUBMITTED TEXT	45 WORDS	80%	MATCHING TEXT	45 WORDS

A A 5< grad (.) (.) (.) (.) ( ) ( ) ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B A B A B B A A B B A

SA mmp ass 2.pdf (D118459575)

144/267 **SUBMITTED TEXT** 975 WORDS **15%** **MATCHING TEXT** 975 WORDS

x y y z z grad A B A B A B x ? ? ? ? ? A B y y x x z z x x y y z z B A B A B A A B A B A B x x x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? x x x x x x x x x y z y z x y z B B B B B A A A A A A A A B B B x y z y z x y z ? y y x x z z y z y y z z B A A A B A B B A B A B y z x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? . y y x x x z x x y y z B A B A B B B A A B A x y x y z x ? A B x z z A A B z x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? . x x x x B A curl curl ? ? ? ? ? ? ? ? ? A B A B B A Hence, considering y and z components of L.H.S and adding them all, we get NSOU ? CC-PH-04 ? 145 (.) (.) (.) ( ) ( ) ? ? ? ? ? ? ? ? ? ? A B A B B A A curl B B curl A Now we find out ( ) ( ) ( ) x y z z y A A ? ? ? A curl B curl B curl B y x x z y z B B B B A A x y z x ? Similarly, (B × curl A) x = B y (curl A) z – B z (curl A) y y x x z y z A A A A B B x y z x ? 6< ( ) div ? ? ? ? ? ? ? A B B A A B ? ? Proof : ( ) ( ) div x j z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A B i j k A B ? ? ? ? ? ? y z z y z x x z x y y x A B A B A B A B A B A B x y z ? i j k i j k ? ? ? ? ? ? y z z y z x x z x y y x A B A B A B A B A B x y z ? ? ? ? ? ? ? ? ? ? ? ? y y x x z z x y z A A A A A A B B B y z z x x y ? y y x x z z x y z B B B B B B A A A y z z x x y ? B A A B ? ? 7< ( ) ( ) ( ) curl ? ? ? ? ? ? ? ? ? A B A B B A B A A B ? ? ? ? = A div B – B div A + B grad A – A grad B 146 ? NSOU ? CC-PH-04 Proof : ( ) y z z y z x x z x y y x curl x y z A B A B A B A B A B A B ? ? ? ? ? ? ? ? ? ? ? i j k A B Considering the x-component only ? ? ? ? ( ) x x y y x z x x z curl A B A B A B A B y z ? ? ? ? ? ? ? ? ? A B Adding and subtracting ? ? x x A B x ? ? , we get ( ) x curl ?A B ? ? ? ? ? ? x x x y x z A B A B A B x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? x x y x z x A B A B A B x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? y y x x z z x x y z B A B A B A A B B B x y z x y z ? y y x x z z x x y z A B A B A B B A A A x y z x y z ? . . . . x x x x A A B B ? ? ? ? ? B B A A ? ? ? ? ? Considering components in y and z direction and adding. We get curl(A × B) ? ? ? ? ? ? ? ? . . x y z x y z A A A B B B ? ? ? ? ? ? B i j k A i j k ? ? ? ? ? ? ? ? ? ? . . x y z x y z A

SA mmp ass 2.pdf (D118459575)

145/267	**SUBMITTED TEXT**	50 WORDS	**57%**	**MATCHING TEXT**	50 WORDS

A A B B B ? ? ? ? ? ? i j k B i j k A ? ? ? ? ? ? ? ? ? ? ( ) . . . . ? ? ? ? ? ? ? A B B A A B A B

SA Mathematical Physics - I SLM full.pdf (D113782471)

146/267	**SUBMITTED TEXT**	245 WORDS	**19%**	**MATCHING TEXT**	245 WORDS

A A A is a scalar Proof : ( ) curl ? ? ? ? ? ? ? ? ? ? A A A A ? ? ? ( ) ( ) ( ) x y z A A A x y z ? i j k i j k ( ) ( ) ( ) ( ) ( ) ( ) y y x x z z A A A A A A y z z x x y ? i j k Considering only x-components ( ) ( ) ( ) y z x z y A A curl A A y z y z ? A i Similarly ( ) ( ) ( ) x z y x z A A curl A A z x z x ? A j And ( ) ( ) ( ) y x z y x A A curl A A x y x y ? A k Adding we get, ( ) ( ) ( ) x y z curl curl curl curl ? ? ? ? ? ? ? A i A j A k A ? ? ? ? ? ? ? A

SA mmp assignment 2.pdf (D118439176)

147/267	**SUBMITTED TEXT**	73 WORDS	**35%**	**MATCHING TEXT**	73 WORDS

A × (B + C) = A × B + A × C 2. (B + C) × A = B × A + C × A 3. A × B.C = A.B × C 4. A × (B × C) = B(A.C) − C(A.B) 5. x y z x y z A A A B B B ? ? i j k A B 6. x y z x y z x y z A A A B B B C C C ? ? ? A B C 4.23.2. :

SA mmp assignment 2.pdf (D118439176)

148/267	**SUBMITTED TEXT**	68 WORDS	**48%**	**MATCHING TEXT**	68 WORDS

r? 2. 0 ? ? ?r ? 3. 1 2 ˆ n n n r nr nr ? ? ? ? ?r r ? 4. 2 2 (1) n n r n n r ? ? ? ? 5. 2 1 0 r ? ? ? ? ? ? ? ? ? 6. ( ) m n m n ? ? ? ? ? ? ? ? ? ? ? 7. ( )?? ? ? ? ? ? ? ? ? ? ? 8. ( ) ( ) ( ) m n m n ? ? ? ? ? ?

SA MMP.pdf (D118446089)

149/267	**SUBMITTED TEXT**	22 WORDS	**62%**	**MATCHING TEXT**	22 WORDS
f x y z V ? ? ? ?? ? ? ? (, ,) (, ,) V V f x y z dV f x y z				f(x , y , z) = 0 ------------(1) (x , y , z) = 0 ------------- (2) F(x , y , z) = f(x , y , z) + (	

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150/267 SUBMITTED TEXT 18 WORDS	**80%** MATCHING TEXT 18 WORDS
is the domain bounded by X-axis, ordinate x = 2a and the curve x 2 = 4ay.	is the region bounded by x-axis and x=2a and the curve x 2 =4ay.

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151/267 SUBMITTED TEXT 54 WORDS	**48%** MATCHING TEXT 54 WORDS
x a . 2 2 4 2 4 2 2 0 0 0 0 2 x a x a a a A y xy dxdy x ydy dx x dx ?? ? ? ? 2 5 6 2 2 5 2 2 2 0 0 0 1 1 6 32 32 32 a a a x x dx x dx a a a ? ? ? ? ? ? ? ? ? ? ? ? ? 6 4 6 2 2 1 64 (2) 0 3 192 192 a a a a	x b a a a x a a x a y a y x y dx dy x y ? 3 3 2 2 2 2 2 2 3 2 . 3 a a b b x a x a x dx a a b b x a x a x dx a a ? ? ? ? ? ? ? ? ? ? ?

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152/267 SUBMITTED TEXT 33 WORDS	**40%** MATCHING TEXT 33 WORDS
a. 2 0 4 a a ay A xy dxdy y x dx dy ? ? ? ? ? ? ? ? ?? ? ? 2 2 0 4 2 a a ay x y dy ? ? ? ? ? ? ? ? ? ? ? ? 4 2 2 0 0 (2 2) 2 3 a a a y a ay dy a ay y	a x a a x a y x ay y x ay xy dx dy xdx ydy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 0 2 2 a a y x a y x yd y ? ? ? ? ? ? ? ? ? ? 2 0 2 2 a y a ay y dy ? ? ? ? ? ? ? ? 2 2 3 0 2 2 2 3 a y

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153/267 SUBMITTED TEXT 46 WORDS	**68%** MATCHING TEXT 46 WORDS
x y x x y dx dy ? ? ? ? ? Solution 2 : 2 2 2 2 3 2 3 0 0 0 0 ( ) 2 x x x y x x y dx x x y dy dx x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 2 4 0 48 2 5 x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ?	x y x x y z z dx xy dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 1 2 2 0 0 1 1 2 x x y dx xy x y dy ? ? ? ? ? ? ? ? ? ? 2 1 1 2 3 0 0 1 1 2 x x y dx x x y y x x y x dx ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 2 2 4 1 0 0 1 . 2 2 2 4

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154/267 SUBMITTED TEXT 74 WORDS	**35%** MATCHING TEXT 74 WORDS
a r a r a r d dr d r dr ? sin 2 2 0 (1 cos) 1 2 a a d r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 0 1 sin (1 cos) 2 d a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 0 sin 1 cos 2 a d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 0 sin 1 2cos cos 2 a d ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 0 1 2cos sin cos 2 a	a r a r a r ? sin 4 4 2 2 2 2 2 2 0 0 0 1 2 1 2 sin 0 2 a a r d a a a d ? 4 4 0 0 cos 1 sin a d a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? sin 0 0 4 4 a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 1 2 4 4 2 2

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155/267

SUBMITTED TEXT 179 WORDS	51% MATCHING TEXT 179 WORDS
A B A B ? ? ? 9. ( ) ( ) ? ? ? ? ? ? ? ? ? A A A ? ? ? 10. ( ) ( ) ( ) m n m n ? ? ? ? ? ? A B A B ? ? ? 11. ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? A A A ? ? ? 12. ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? A B B A A B ? ? ? NSOU ? CC-PH-04 ? 149 13. ( ) ( ) ( ) ( ) ( ) ? ? ? ? ? ? ? ? ? ? ? ? A B A B B A A B B A ? ? ? ? ? 14. 2 2 2 2 2 2 2 ( ) x y x ? 15. ( ) 0 ? ? ? ? ? 16. ( ) 0 ? ? ? ?A ? ? 17. 2 ( ) ( ) ? ? ? ? ? A A	

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

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SUBMITTED TEXT 66 WORDS	50% MATCHING TEXT 66 WORDS
y y y x y V dy dx ? ? ?? ?? ?? ? ? ? ? ? 2 2 4 2 0 2 (4) y x y y x dy y dx ?? ? ? ? ?? ? ? ? ? ? ? ? ? ? 2 2 4 0 2 2 4 y y y dy x yx ?? ? ?? ? ? ? ? 2 2 2 2 2 4 4 4 y y dy y y y ?? ?? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 2 4 4 2 4 y y y y y dy y y dy ?? ?? ?? ?? ? ? ? ? ? ? = 16? units Now to evaluate 2 2 2 2 4 4 y y y	y y y y x xdx y dy x d y dy ? 2 2 2 1 2 0 1 0 0 2 2 y y y y x x x x y dy y y y y y y dy dy ? ? ? ? ? ? ? ? ? 1 2 2 2 3 0 1 1 1 . 4 4 2 2 y y dy y y dy ? ? ? ? ? ? ? ? 1 2 3 2 3 4 0 1 1 1 4 4 . . 2 3 2 2 3 4 y y y

W https://www.iare.ac.in/sites/default/files/AERO_LINEAR_ALGEBRA_AND_CALCULUS_LECTURE_NOTES.pdf

157/267

SUBMITTED TEXT 79 WORDS	38% MATCHING TEXT 79 WORDS
a ay a y x x y y I xy dx dy xy dx dy ? 2 2 2 2 0 0 0 2 2 a y ay y a a y y a x y x y dy dy ? 2 2 2 0 1 1 (2) 2 2 y a a y y a ay dy a y y dy ? ? ? ? ? ? ? ? ? 2 3 3 4 2 2 4 0 1 4 3 2 2 3 2 3 4 8 a a a a	a ay a ay y y y x y x a a dxdy dx dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 4 4 2 0 0 4 2 4 a a ay y y y x a y x dy ay dy a ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 3 3 2 0 1 2 . . 3 4 3 2 a y y a a ? ? ? ? ? ? ? ? ? ? Page | 60 3 4 1 . .4 4 .64 3 12 a a a a

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158/267

SUBMITTED TEXT 46 WORDS	78% MATCHING TEXT 46 WORDS
y y y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 y y y y y x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2y y y x ? ? ? ? ? ? ? ? ? ? ? ?	y y y x y y y y y y x h h h h y y y x

W https://www.slideshare.net/SyeilendraPramuditya/finite-difference-method-250591577

159/267

SUBMITTED TEXT 28 WORDS	71% MATCHING TEXT 28 WORDS
bounded by the cylinder x 2 + y 2 = 4 and the planes y+ z = 4 and z = 0 Solution 4 :	

SA manjucal.pdf (D104415716)

160/267 **SUBMITTED TEXT** 15 WORDS	**66%** **MATCHING TEXT** 15 WORDS
x(u, v, w); y = y(u, v, w) and z = z(u, v, w)	

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

161/267 **SUBMITTED TEXT** 27 WORDS	**48%** **MATCHING TEXT** 27 WORDS
u, v, w) is given by (, ,) (, ,) x x x u v w x y z y y y J u v w u v w z	

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

162/267 **SUBMITTED TEXT** 50 WORDS	**75%** **MATCHING TEXT** 50 WORDS
cos , sin x r y r ? ? ? ? ? 2 2 cos sin (,) cos sin sin cos (,) x x r x y r J r r y y r r r ? 2 2 cos sin r r ? ? ? ? ? ?	cos , sin x r y r ? ? ? ? ? ? ? ? ? cos sin , sin cos , x x r x y r y y r r r ? 2 2 cos sin r r ? ? ? ? ? ? ? ? ? ? 1 ,

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163/267 **SUBMITTED TEXT** 21 WORDS	**76%** **MATCHING TEXT** 21 WORDS
cos , sin) xy r R R f x y dx dy f r r J dr d ? ? ? ? ? ?? ?? (	cos sin r r ? ? ? ? ? ? ? ? ? ? 1 , cos , sin R R f x y dx dy f r r r dr d ? ? ? ? ? ?? ??

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164/267 **SUBMITTED TEXT** 39 WORDS	**46%** **MATCHING TEXT** 39 WORDS
cos , sin , x r y r z z ? ? ? ? ? cos sin 0 (, ,) sin cos 0 (, ,) 0 0 1 x x x r z r x y z y y y J r r z r	cos , sin x r y r ? ? ? ? ? ? ? ? ? cos sin , sin cos , x x r x y r y y r r r ? 2 2

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165/267 **SUBMITTED TEXT** 28 WORDS	**50%** **MATCHING TEXT** 28 WORDS
x u v w y u v w z u v w J du dv dw ? ??? (4C.10) where (, ,) (, ,) x y z J u v w ? ? ?	

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

166/267	**SUBMITTED TEXT**	26 WORDS	**52%**	**MATCHING TEXT**	26 WORDS

y u x ? ? ? ? ? ? ? ? and y v x ? ? ? ? ? ? ? ? . Solution : Solution 1 : The region of integration is bounded by y = x, x = 0

y d x d y a x y ? ? ? ? ? ? ? ? ? ? ? ? ? Sol. The region of integration is given by 2 , 4 y x x

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167/267	**SUBMITTED TEXT**	26 WORDS	**55%**	**MATCHING TEXT**	26 WORDS

y = 0 to y = ? 0 0 0 0 y y y y e e dx dy dy dx y y ? ? ? ? ? ? ?? ?? ? ? 0 0 0 0 1 y y y y

y dy ? 2 1 2 0 1 2 . 2 2 y y y y y dy y dy ? ? ? ? ? ? ? ? ? 1 2 2 2 3 0 1 1 1 . 4 4 2 2 y y y dy y y y

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168/267	**SUBMITTED TEXT**	29 WORDS	**66%**	**MATCHING TEXT**	29 WORDS

spherical polar co-ordinate sin cos , sin sin , cos x r y r z r ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 4

spherical polar coordinates sin cos , sin sin , x r y r ? ? ? ? ? ? cos z r ? ?

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169/267	**SUBMITTED TEXT**	61 WORDS	**63%**	**MATCHING TEXT**	61 WORDS

sin 5 R R d d r dr d d ? 5 5 0 2 2 cos [cos cos0] 5 5 R R ? ? ? ? ? ? ? ? ? ? ? 5 5 5 2 2 4 [1 cos] [1 (1)] 5 5 5 R R R ? ? ? ? ? ? ? ? ? ?

sin r r d d r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 0 0 cos r r d r ? ? ? ? ? ? ? ? ? ? 1 0 cos cos 0 2 r r d r ? ? ? ? ? ? ? ? ? 1 2 1 1 0 0 0 1 1 0 1 0 2 2 2 r r

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170/267	**SUBMITTED TEXT**	112 WORDS	**26%**	**MATCHING TEXT**	112 WORDS

sin cos x r ? ? ? sin sin y r ? ? ? cos z r ? ? ? ? sin cos cos cos sin sin , , sin sin cos sin sin cos (, ,) cos sin 0 x x x r r r x y z y y y J r r r r r z z z r ? 2 2 2 2 2 2 2 sin sin (sin cos) cos sin cos r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 sin

SA mmp ass 2.pdf (D118459575)

171/267 **SUBMITTED TEXT** 122 WORDS	**26%** **MATCHING TEXT** 122 WORDS
x y u v v v ? ? ? ? ? ? ? ? ? ? ? ? ? ? the given integral is, 4 1 4 2 2 2 1 2 1 2 1 1 1 [ ] 3 3 u v u R u J du dv u du dv u du v ? ? ?? ? ? ? ? ?? ? ? ? v NSOU ? CC-PH-04 ? 165 4 4 2 2 1 1 1 .3 3 u u u du u du ? ? ? ? ? ? 4 3 3 3 1 4 1 1 63 (64 1) 21 3 3 3 3 3 u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Solution 4 : u = 1 + x; v = xy 1, 1 v x u y u ? ? ? ? ? 2 1 (,) 1 (1) (,) 1 1 0 1 v x y x y u u u jacobian x y u v u v v u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?	

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

172/267 **SUBMITTED TEXT** 104 WORDS	**36%** **MATCHING TEXT** 104 WORDS
x y ? ? ? ? ? ? ? ? ? ? 2 2 2 1 2 u x x y ? ? ? ? Differentiating partially with respect to x 2 2 2 2 2 2 2 2 2 1 1 2 1 2 2 y x x y u x u u x x y x y x y ? 2 2 2 y u u x x y ? ? ? ? ? ? ? ? ? ? ? Again ? ? 2 2 2 1 2 v x y x ? ? ? 2 2 2 2 2 1 2 1 2 y u x v v x u v x y ? 2 2 2( ) y	x 2 y 2 – 4x 3 x 2 y 3 = 2x 3 y – 2x 4 y -3x 3 y 2 For maxima & and = 0 ? 3x 2 y 2 – 4 x 3 y 2 -3x 2 y 3 = 0 =< x 2 y 2 (3 – 4x -3y) = 0 ---------------< (1) ? 2x 3 y – 2 x 4 y -3x 3 y 2 = 0 =< x 3 y(2 – 2x -3y) = 0 ----------------< (2) From (1) & (2) 4x + 3 y – 3 = 0 2x + 3y - 2 = 0 2y = 3 -2 ,

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173/267 **SUBMITTED TEXT** 57 WORDS	**32%** **MATCHING TEXT** 57 WORDS
u v u v J u v u v y y v u u v ? ? ? ? ? ? ? ? ? ? ? ? ? 166 ? NSOU ? CC-PH-04 b) We have u 2 – v 2 = x and 2uv = y ? ? ? ? 2 2 2 2 2 2 2 2 2 2 2 4 u v u v u	

SA M P Vol 1.pdf (D134397037)

174/267 **SUBMITTED TEXT** 90 WORDS	**60%** **MATCHING TEXT** 90 WORDS
x 3 , dy = 3x 2 dx ? F.dr = (5x.x 3 – 6x 2)dx + (2x 3 – 4x)3x 2 dx = (5x 4 – 6x 2)dx + (6x 5 – 12x 3)dx ? ? 2 2 5 4 3 2 6 5 4 3 1 1 . 6 5 12 6 3 2 x C x x x x x dx x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?	x dy ? ? ? ? ? ? ? ? ? ? 2 1 1 2 3 0 0 1 1 2 x y dx x x y dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 4 1 2 0 0 1 1 2 2 4 x x y x x dx ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 2 2 4 1 0 0 1 . 2 2 2 4 x x y x y y x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 2 2 2 2 0 1 . 2 1 2 1 1 8 x x x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 4 6 1 3 5 0 0 1 1 2 2 8 8 2 4 6 x x x x

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175/267 **SUBMITTED TEXT** 32 WORDS **52%** **MATCHING TEXT** 32 WORDS

t = T 0 to t = T, is given by, ? ? 0 0 0 0 ( ) ( ) ( ) ( ) ( ) T T T T T T d t dt t dt t t T

SA VECTOR ANALYSIS.docx (D118765685)

176/267 **SUBMITTED TEXT** 61 WORDS **60%** **MATCHING TEXT** 61 WORDS

t 2 + 6t)dt − 14t 2 dt + 20t 3 dt = (3t 2 + 6t − 14t 2 + 20t 3)dt ? ? ? ? 1 1 2 2 3 2 3 0 0 13 . 3 6 14 20 11 6 20 3 C t t t t t t dt t t t

SA VECTOR ANALYSIS.docx (D118765685)

177/267 **SUBMITTED TEXT** 28 WORDS **90%** **MATCHING TEXT** 28 WORDS

where S is the surface of unit cube bounded by x = 0, x = 1; y = 0, y = 1; z = 0, z = 1

SA Yeshy.pdf (D112546741)

178/267 **SUBMITTED TEXT** 16 WORDS **90%** **MATCHING TEXT** 16 WORDS

bounded by co-ordinate planes and the planes x = 1, y = 1, z = 1

SA 17691A0431-2.docx (D33209716)

179/267 **SUBMITTED TEXT** 175 WORDS **25%** **MATCHING TEXT** 175 WORDS

x z x x y ds x dx dz x xdx ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? 3 2 2 0 2 9 1. 9 x x dx x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3 3 2 2 1 1 0 0 9 9 9 9 2 sin 2 sin 2 2 3 2 2 3 x x x x x x x ? 9 9 2.3. . 2. . 9 2 2 2 2 ? ? ? ? ? ? ? ? ? ? Evaluation of integral (ii) 2 ˆ (2) S x y dx dz ds dx dz and ? ? ? ?? ? n j 2 2 3 9 3 9 2 0 0 0 0 2 8 8 x x x z x z x dx dz dx dz y in S ? ? ? ? ? ? ? ? ? ? ? ? ? 3 3 3 2 2 2 1 0 0 0 9 9 2 9 8 9 9 8 sin 2 2 3 x x x x x x dx x

x x x y x dx x dx x dx ? 3. Evaluate ? ? 2 5 2 2 0 0 x x x y dxdy ? ?? x x x x xy x x y dy dx x y dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 5 6 5 5 2 3 7 6 8 8 3 2 5 0 0 0 ( ) 1 5 5 . . 3 3 6 3 8 6 24 x x x x x x x x x dx x Evaluate 2 1 1 2 2 0 0 1 x dydx x y ? ? ? ? ? Sol: ? ? 2 2 1 1 1 1 2 2 2 2 0 0 0 0 1 1 1 x x x y dydx dy dx x

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180/267 **SUBMITTED TEXT** 15 WORDS	**90%** **MATCHING TEXT** 15 WORDS
if M and N are continuous functions of x and y having continuous derivative in	if M and N are continuous functions of x and y having continuous derivatives in

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181/267 **SUBMITTED TEXT** 20 WORDS	**78%** **MATCHING TEXT** 20 WORDS
then ( ) C R N M M dx N dy dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? (4C.17)	Then c R N M M dx N dy dxdy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 [(2) ( )] (2 2) c

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182/267 **SUBMITTED TEXT** 78 WORDS	**34%** **MATCHING TEXT** 78 WORDS
x x q x x x p y x x p R M M dx dy dy dx M x y dx y y ?? ? ? ? ? ? ? ? ? 2 1 , ( ) , ( ) x q x p M x x M x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 1 ( ) ( ) , ( ) , ( ) ? ? ? ? ? ? ? ? x q x q x p x	x=0 to x=1 ? ? ? ? 2 2 1 1 2 2 0 0 x x x y x x y x xy x y dy dx x y xy dy dx ? 2 3 2 1 2 0 2 3 x x y x y xy dx ? ? ? ? ? ? ? ? ? ? ? ? 4 4 6 7 1 0 2 3 2 3 x x x x x dx ? ? ? ? ? ? ? ? ? ? ? ? 4 6 7 1 0 5 6 2 3 x x x x

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183/267 **SUBMITTED TEXT** 83 WORDS	**27%** **MATCHING TEXT** 83 WORDS
x y ?? ? ? 4 4 3 3 ( ) ( ) ( ) ( ) (,) n y n y y y m y y m R N N dx dy dx dy N x y dy x x ?? ? ? ? ? ? ? ? 4 3 ( ), ( ), ? ? ? ? ? ? ? ? ? y n y m N y y N y y dy ? ? ? ? ? ? 4 3 ( ), ( ), ? ? ? ? ? ? ? ? y n y m y m y n N y y dy N y y	x d y ? ? ? ? ? ? ?? ?? 1 2 2 0 0 1 0 y y y x y x xdx y dy x d x y dy ? 2 2 2 1 2 0 1 0 0 2 2 y y y y x x x y dy y dy ? 2 1 2 0 1 2 . 2 2 y y y y y dy y dy ? ? ? ? ? ? ? ? ? ? 1 2 2 2 3 0 1 1 1 . 4 4 2 2 y y y dy y y y dy ? ? ? ? ? ? ? ? 1 2 3 2 3 4 0 1 1 1 4 4 . . 2 3 2 2 3 4 y y

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184/267 **SUBMITTED TEXT** 39 WORDS	**92%** **MATCHING TEXT** 39 WORDS
of the region bounded by the cylinder x 2 + z 2 = 9, x = 0, y = 0, z = 0 and y = 8, if A = 6zi + (2x + y)j − xk	

SA VECTOR ANALYSIS.docx (D118765685)

185/267 **SUBMITTED TEXT** 31 WORDS	**72%** **MATCHING TEXT** 31 WORDS
Example 1: Verify Green's theorem for ? ? 2 2 C xy y dx x dy ? ? ? ? ? ? ? ?? , where C is bounded by y = x, and y = x 2 .	

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186/267 **SUBMITTED TEXT** 137 WORDS	**25%** **MATCHING TEXT** 137 WORDS
x x y x y ? ? ? ? ? ? ? 2 2 2 N M x x y x y x y ? ? ? ? ? ? ? ? ? ? Now ? ? 2 2 C xy y dx x dy ? ? ? ? ? ? ? ?? 2 1 0 (2) (2) x x N M dx dy x y dx dy dx x y dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? [We have 1 2 C C C f f ? ? ?? . Along C 1 , y = x 2 , x varies from 0 to 1. Along C 2 , y = x and x varies from 1 to 0] ? ? ? ? 2 1 1 1 2 2 2 3 4 4 3 0 0 0 1 – 20 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? x x dx xy y dx x x x x dx x x (	x x y x y z xyz dxdydz ? ? ? ? ? ? ? ? ? 2 2 2 1 1 1 0 0 0 x x y x y z dx dy xyz dz ? ? ? ? ? ? ? ? ? ? 2 2 2 1 2 1 1 0 0 0 2 x y x x y z z dx xy dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 1 2 2 0 0 1 1 2 x x y dx xy x y dy ? ? ? ? ? ? ? ? ? ? 2 1 1 2 3 0 0 1 1 2 x x y dx x x y y dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 4 1 2 0 0 1 1 2 2 4 x x y y x x dx ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 2 2 4 1 0 0 1 . 2 2 2 4 x x y x y y x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 2 2 2 2 0 1 . 2 1 2 1 1 8 x x x x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 4 6 1 3 5 0 0 1 1 2 2 8 8 2 4 6 x x

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187/267 **SUBMITTED TEXT** 54 WORDS	**56%** **MATCHING TEXT** 54 WORDS
x dx x x dx ? ? ? ? ? ? ? ? ? ? 1 3 4 0 19 3 20 x x dx ? ? ? ? Along the curved C 2 , the integral become ? ? 0 0 2 2 2 2 2 1 1 3 1 I x x dx x dx x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 1 2 19 1 1 20 20	

SA MMP.pdf (D118446089)

188/267 **SUBMITTED TEXT** 71 WORDS	**61%** **MATCHING TEXT** 71 WORDS
polar co-ordinate cos , sin , x r y r dxdy r dr d ? ? ? ? ? ? . Here r varies from 1 to 3 and ? varies from 0 to 2 ? . ? ? ? ? 3 2 3 2 2 1 0 2 sin 3 sin C r x y dx xy dy r r r dr d ?	polar co-ordinates cos , sin , x r y r dx dy r dr d ? ? ? ? ? ? Here 'r' varies from 0 to a and ' ' ? varies from 0 to 2 ? ? ? 2 2 2 2 2 2 0 0 0 0 a a y a r x y dx dy r rdrd ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 4 2 2 0 0 0 4 4 a r a d ? ? ? ? ? ? ? ? ? ? ? ? ? 4 8

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189/267 SUBMITTED TEXT 30 WORDS	**78%** MATCHING TEXT 30 WORDS
the boundary of the region bounded by the circles x 2 + y 2 = 1 and x 2 + y 2 = 9.	

SA Tutorial 5.pdf (D58525697)

190/267 SUBMITTED TEXT 183 WORDS	**19%** MATCHING TEXT 183 WORDS
y y dx dy dx y dy a b ab ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? Solution 2 : Along OA : y = 0 dy = 0 along AB : x = 1, dx = 0 along BO : y = x, dy = dx Now ? ? 2 2 C xy x dx x y dy ? ? ? ? ? ? ?? 2 3 OA AB BO x dx y dy x dx ? ? ? ? ? ? ? ? 1 1 0 2 3 0 0 1 1 1 1 1 3 2 4 12 x dx y dy x dx ? ? ? ? ? ? ? ? ? ? ? ? ? (i) Now M = xy − x 2 N = x 2 y M x y ? ? ? 2 N xy x ? ? ? Therefore using Green's theorem ? ? 1 2 2 0 0 (2) (2) y x OABO y xy x dx x y dy xy x dx dy dx xy x dy ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? 1 1 1 4 3 2 3 2 0 0 0 0 1 1 1 4 3 4 3 12 x x x dx xy xy dx x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (	y y dy dy ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 1 1 8 4. 2 y dy y dy ? ? ? ? ? ? 2 3 1 4 4.7 28 4. 8 1 3 3 3 3 y? ? ? ? ? ? ? ? ? ? ? ? 2. Evaluate 2 0 0 x y dy dx ?? Sol. 2 2 0 0 0 0 x x y x y y dy dx y dy dx ? 2 2 2 2 2 3 2 2 0 0 0 0 0 1 1 1 1 8 4 0 8 0 2 2 2 2 3 6 6 3 x x x x y x dx x dx x dx ? 3. x x x y dxdy ? ?? Sol. ? ? 2 2 5 5 3 2 2 3 0 0 0 0 3 x x x y x y xy x x y dy dx x y dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 5 6 5 5 2 3 7 6 8 8 3 2 5 0 0 0 ( ) 1 5 5 . . 3 3 6 3 8 6 24 x x x x x x x x dx x dx ? 4. Evaluate 2 1 1 2 2 0 0 1 x dydx x

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191/267 SUBMITTED TEXT 26 WORDS	**73%** MATCHING TEXT 26 WORDS
Solution 3 : We have from Green's theorem, [ ] C R N M Mdx Ndy dx dy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? (	Solution: We have by Green's theorem C R N M Mdx Ndy dxdy x y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?

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192/267 SUBMITTED TEXT 26 WORDS	**55%** MATCHING TEXT 26 WORDS
x y dy ? ? ? ? ? ? ? ?? over the triangle bounded by the line y = 0, x = 1 and y = x. 3) Apply Green's theorem	

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193/267 SUBMITTED TEXT 86 WORDS	**22%** MATCHING TEXT 86 WORDS
A = C x B, where C is a constant vector. ˆ . . V S dV ds ? ? ? ? ? ??? C B C B n ? Now . . .( ) .( ) C ? ? ? ? ? ? ? ? ? ? ? ? ? C B B C C B B , since C is a constant vector,, 0 ? ? ?C ? ˆ ˆ . . V S S dV ds ds ?? ? ? ? ? ? ? ? ? ?? ?? ? ? C B C B n C n B ? ˆ	a c b2 0 ? ? ? ? ? ? ? ? ? ? ? ? c b a c b a c b2 0 l c c c b b b a a ? ? ? ? ? ? ? ? ? ? ? ? ? 2 0 ? 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 4 c b a c b a c b c b a c b a c b c b c b

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194/267 **SUBMITTED TEXT** 81 WORDS **64%** **MATCHING TEXT** 81 WORDS

r r r r x y z ? ? ? ? ? ? ? ? ? i j k ? where r 2 = x 2 + y 2 + z 2 . 2 2 2 2 r x y z ? ? ? ? i j k ? 2 . (2) (2) (2) 2 2 2 6 r x y z x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 . . 6 6

SA 07200146.pdf (D118456064)

195/267 **SUBMITTED TEXT** 49 WORDS **62%** **MATCHING TEXT** 49 WORDS

r r r ? ? ? ? ? ? ? ? r A r r Now 4 3 5 1 3 ˆ 3r r r ? ? ? ? ? ? ? r r 3 5 3 3 3 3 . 3 3 . 0 r r r r ? ? ? ? ? ? ? ? r r A 3 3 ˆ (.) . 0 V S r ds dV r r ? ? ? ? ?? ? ? ? ? r

SA 07200140 (2).pdf (D118447009)

196/267 **SUBMITTED TEXT** 43 WORDS **93%** **MATCHING TEXT** 43 WORDS

F n , where F = 4xzi − y 2 j + yzk and S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 & z = 1

SA VECTOR ANALYSIS.docx (D118765685)

197/267 **SUBMITTED TEXT** 157 WORDS **62%** **MATCHING TEXT** 157 WORDS

SUBMITTED TEXT	MATCHING TEXT
x y x x V x y z x y x y dV x y dx dy dz x y x y dx dy ? ? ? ? ? ? ? ? ? ? ? ? ? ??? ? ? ? ? ? 4 2 2 4 2 2 2 2 3 2 2 3 2 0 0 0 0 (8 4 2) 8 4 2 2 2 3 x x x y x y y y x y x y dy dx x x x dx ? 4 2 2 2 2 3 2 2 3 0 0 2 4 2 3 x x x y x y x y dx ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 3 2 2 3 0 2 4 (4 2) 2 (4 2) (4 2) 3 x x x x x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 3 4 5 6 2 3 4 5 0 0 64 8 64 8 128 32 16 32 16 3 3 3 3 4 5 3 6 45 x x x x x x x	x y xyz dxdydz ? ? ? ? ? ? Sol. 2 2 2 1 1 1 0 0 0 x x y x y z xyz dxdydz ? ? ? ? ? ? ? ? ? 2 2 2 1 1 1 0 0 0 x x y x y z dx dy xyz dz ? ? ? ? ? ? ? ? ? ? 2 2 2 1 2 1 1 0 0 0 2 x y x y z z dx xy dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 1 2 2 0 0 1 1 2 x x y xy x y dy ? ? ? ? ? ? ? ? ? ? 2 1 1 2 3 0 0 1 1 2 x x y dx x x y y dy ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 4 1 2 0 0 1 1 2 2 4 x x y y x dx ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 2 2 4 1 0 0 1 . 2 2 2 4 x x y y y x x x x x dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 4 6 1 3 5 0 0 1 1 2 2 8 8 2 4 6 x x x x x x

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198/267 **SUBMITTED TEXT** 29 WORDS **96%** **MATCHING TEXT** 29 WORDS

where V is the closed region bounded by the planes 4x + 2y + z = 8, x = 0, y = 0 & z = 0. 190 ?

SA MMP.pdf (D118446089)

199/267 SUBMITTED TEXT 14 WORDS	**95%** MATCHING TEXT 14 WORDS
states that if ? is an open two-sided surface bounded by a	

SA VECTOR ANALYSIS.docx (D118765685)

200/267 SUBMITTED TEXT 45 WORDS	**46%** MATCHING TEXT 45 WORDS
C d ? ? ? ? ? ?? ? A r A dS ? or ? ? . ( ) . C S d? ? ? ? ? ? ? ? ?? ? A r A A dS ? ? since A is constant vector, 0 ? ?A ? . . . C S S d	

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

201/267 SUBMITTED TEXT 24 WORDS	**80%** MATCHING TEXT 24 WORDS
where S is the surface bounded by the circle. x 2 + y 2 = 4 and z = 2.	where and S is the surface bounded by the region x 2 +y 2 =4, z=0 and z=3.

W https://www.iare.ac.in/sites/default/files/AERO_LINEAR_ALGEBRA_AND_CALCULUS_LECTURE_NOTES.pdf

202/267 SUBMITTED TEXT 43 WORDS	**31%** MATCHING TEXT 43 WORDS
x y z C C C e dx y dy dz A dx A dy A dz ? ? ? ? ? ? ? ? ? ? ? ? Adr 2 1 x x y z A e A y A ? ? ? ? ? Now 0 2 1 x x y z	

SA MMPass2.pdf (D118457090)

203/267 SUBMITTED TEXT 60 WORDS	**51%** MATCHING TEXT 60 WORDS
x − y)i − yz 2 j − y 2 zk, the region of integration being the upper half surface of the sphere x 2 + y 2 + z 2 = 1 and C its boundary. Solution 3 : The boundary is the circle of unit radius in xy − plane	

SA VECTOR ANALYSIS.docx (D118765685)

204/267 SUBMITTED TEXT 29 WORDS	**92%** MATCHING TEXT 29 WORDS
the surface of the cube x = 0, y = 0, z = 0; x = 2, y = 2, z = 2; above the xy plane. 2) Verify Stoke's theorem	the surface of the cube. x=0, y=0, z=0, x=2, y=2, z=2 above the xy plane. By Stoke's theorem,

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205/267 **SUBMITTED TEXT** 31 WORDS **55%** **MATCHING TEXT** 31 WORDS

y z x y yz y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? i j k A k ? ? ˆ ˆ . . S S ds
k ds ? ? ? ?? ?? ? A n n

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

206/267 **SUBMITTED TEXT** 40 WORDS **41%** **MATCHING TEXT** 40 WORDS

u u u y y u u u z z u u u ? ? ? ? ? ? ? ? (5.1) Fig (5.1) P 202 ?
NSOU ? CC-PH-04 And 1 1 2 2 3 3 (, ,) (, ,) (, ,) u u x y z
u u x y z u u

SA M P Vol 1.pdf (D134397037)

207/267 **SUBMITTED TEXT** 47 WORDS **87%** **MATCHING TEXT** 47 WORDS

e e (5.7) and e 1 × e 2 = e 3 ; e 2 × e 3 = e 1 ; e 3 × e 1 = e
2 5.4.1 :

e e e e e e e e e e e

W https://www.iare.ac.in/sites/default/files/AERO_LINEAR_ALGEBRA_AND_CALCULUS_LECTURE_NOTES.pdf

208/267 **SUBMITTED TEXT** 37 WORDS **86%** **MATCHING TEXT** 37 WORDS

x y z u u u x y z u u u x y z u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
? ? ? (5.5) or, ? ? 1 2 3 (, ,) 0 , , x y z u u u ? ? ? (5.6)

SA M P Vol 1.pdf (D134397037)

209/267 **SUBMITTED TEXT** 34 WORDS **95%** **MATCHING TEXT** 34 WORDS

r = r(u 1 , u 2, u 3), 1 2 3 1 2 3 u u u u u u ? ? ? ? ? ? ? ? ? ?
? ? r

SA M P Vol 1.pdf (D134397037)

210/267 **SUBMITTED TEXT** 62 WORDS **53%** **MATCHING TEXT** 62 WORDS

u d u u u u u u u u u ?
? ? ? ? ? ? ? r r r r ? ? ? ? (5.16) Comparing equation (5.15)
and (5.16), we get 1 1 1 2 . 1, . 0, u u u u ? ? ? ? ? ? ? r r ? ? 1 3
. 0 u u ? ? ?

SA M P Vol 1.pdf (D134397037)

211/267	SUBMITTED TEXT	51 WORDS	50%	MATCHING TEXT	51 WORDS

u? and 3 u? , we get 2 2 2 1 2 3 . 1; . 1; . 0 u u u u u u ? ? ? ? ? ? ? ? ? r r r ? ? ? And 3 3 3 1 2 3 . 0; . 0; . 1 u u u u u u ? ? ? ? ? ? ? ? ?

SA M P Vol 1.pdf (D134397037)

212/267	SUBMITTED TEXT	12 WORDS	85%	MATCHING TEXT	12 WORDS

u u h h h h u h h h h h h h

SA mmp ass 2.pdf (D118459575)

213/267	SUBMITTED TEXT	37 WORDS	90%	MATCHING TEXT	37 WORDS

e e e e e e e e ? Since e 1 .e 2 × e 3 = 1 & e 2 × e 3 = e 1 ,

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

214/267	SUBMITTED TEXT	78 WORDS	36%	MATCHING TEXT	78 WORDS

u 1 , u 2 , u 3) where u 1 , u 2 , u 3 are functions of x, y, z defined by equation 5.2. Then we have : 3 1 2 1 2 3 3 1 2 1 2 3 3 1 2 1 2 3 u u u x u x u x u x u u u y u y u y u y u u u z u z u z u

SA M P Vol 1.pdf (D134397037)

215/267	SUBMITTED TEXT	40 WORDS	37%	MATCHING TEXT	40 WORDS

cos sin r ? ? ? ? ? ? ? r i j ? ? (sin) sin sin cos r r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? r i j i j z ? ? ? r k Therefore unit vectors are : cos sin sin cos ? r r r r r r

SA mmp ass 2.pdf (D118459575)

216/267	**SUBMITTED TEXT**	121 WORDS	**22%**	**MATCHING TEXT**	121 WORDS

r r r ? ? ? ? ? ? ? ? ? r i j k sin cos sin sin cos r ? ? ? ? ? ? ? ? ? ? r i j k 1 1 r h h r ? ? ? ? ? ? r cos cos cos sin sin r r r ? ? ? ? ? ? ? ? ? ? ? ? r i j k 2 h h r ? ? ? ? ? ? ? ? ? r sin (sin) sin cos r r ? ? ? ? ? ? ? ? ? ? r i j 3 sin h h r ? ? ? ? ? ? ? ? ? ? r Fig. 5.3 208 ? NSOU ? CC-PH-04 1 2 3 1 sin r h h h h r h h r ? (5.24) Now, 1 2 3 sin cos sin sin cos cos cos cos sin sin sin cos r r ? r r r r r r θ r

SA mmp ass 2.pdf (D118459575)

217/267	**SUBMITTED TEXT**	64 WORDS	**71%**	**MATCHING TEXT**	64 WORDS

the unit vectors e 1 , e 2 , & e 3 respectively, such that A = A 1 e 1 + A 2 e 2 + A 3 e 3 = A 1 + A 2 + A 3 (5.26)

SA Math_Debalina.docx (D24237902)

218/267	**SUBMITTED TEXT**	36 WORDS	**95%**	**MATCHING TEXT**	36 WORDS

the unit vectors are orthogonal, e 1 = e 2 × e 3 ; e 2 = e 3 × e 1 ;

SA VECTOR ANALYSIS.docx (D118765685)

219/267	**SUBMITTED TEXT**	143 WORDS	**18%**	**MATCHING TEXT**	143 WORDS

u u ? ? ? ? ? ? ? A ? ? 1 2 3 2 3 1 2 3 2 3 A h h u u A h h u u ? ? ? ? ? ? ? ? ? ? ? ? NSOU ? CC-PH-04 ? 209 But 2 3 3 2 2 3 0 u u u u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (see article 4B.13) ? ? 1 1 2 3 2 3 A h h u u ? ? ? ? ? ? ? ? ? A (5.27) Now for any function f(u 1), we have 1 1 1 1 1 1 1 1 1 ( ) ( ) ( ) ( ) f u f u f u u u u f f f u x y z u x u y u

SA M P Vol 1.pdf (D134397037)

220/267	**SUBMITTED TEXT**	62 WORDS	**54%**	**MATCHING TEXT**	62 WORDS

h h u u u A h h u u h h h e e e **A** ? ? ? ? using (5.17) ? ? ? ? 1 2 3 1 2 3 1 2 3 1 1 2 3 1 1 A h h A h h h h u h h h u ? ? ? ? ? ? ? ? 1 2 3 e e e 1 2 3 1 ? ? ? ?

SA mmp ass 2.pdf (D118459575)

221/267	SUBMITTED TEXT	23 WORDS	76%	MATCHING TEXT	23 WORDS

A h h A h h A h h h h h u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? (5.29)

SA MMP.pdf (D118446089)

222/267	SUBMITTED TEXT	18 WORDS	80%	MATCHING TEXT	18 WORDS

sin sin sin r A r A r A r r r r ?

SA M P Vol 1.pdf (D134397037)

223/267	SUBMITTED TEXT	180 WORDS	28%	MATCHING TEXT	180 WORDS

h u A h u A h u ? ? ? ? ? ? ? ? ? ? ? ? 1 1 1 2 2 2 3 3 3 A h u A h u A h u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A (5.31) Now ? ? ? ? ? ? 1 1 1 1 1 1 1 1 1 1 1 1 A h u A h u A h u A h u ? 1 1 1 1 1 1 2 1 1 1 3 1 1 2 3 A h u u A h u u A h u u u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (5.32) NSOU ? CC-PH-04 ? 211 Now from equation (5.17) 3 2 1 2 1 1 2 1 2 u u h h h h ? ? ? ? ? ? ? ? e e e 3 1 2 3 1 1 3 1 3 u u h h h h ? ? ? ? ? ? e e

SA mmp ass 2.pdf (D118459575)

224/267	SUBMITTED TEXT	140 WORDS	36%	MATCHING TEXT	140 WORDS

A h u A h A h h h u h h u e e ? ? Similarly we get, ? ? ? ? ? ? 3 1 2 2 2 2 2 2 2 1 2 1 3 2 3 A h u A h A h h h u h h u ? ? ? ? ? ? ? ? ? e e ? ? ? ? ? ? 1 2 3 3 3 3 3 3 3 1 3 2 1 3 3 A h u A h A h h h u h h u ? ? ? ? ? ? ? ? ? e e Thus ? ? ? ? ? ? ? ? 1 2 3 3 2 2 1 1 3 3 2 3 2 3 1 3 3 1 A h A h A h A h h h u u h h u u ? e e

SA mmp ass 2.pdf (D118459575)

225/267	SUBMITTED TEXT	13 WORDS	90%	MATCHING TEXT	13 WORDS

h h h h h h u u u A h A h A h

SA mmp ass 2.pdf (D118459575)

226/267	**SUBMITTED TEXT**	227 WORDS	**39%**	**MATCHING TEXT**	227 WORDS

h u h u h u ? ? ? ? ? ? ? ? ? ? ? ? (5.38) From equation (5.29) : ? ? ? ? ? ? 1 2 3 2 3 1 3 1 2 1 2 3 1 2 3 1 , A h h A h h A h h h h h u u u ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A substituting A 1 , A 2 , A 3 from equation (5.38), we get NSOU ? CC-PH-04 ? 213 2 3 3 1 1 2 1 2 3 1 1 1 2 2 2 3 3 3 1 h h h h h h h h h u h u u h u u h u ? A (5.39) 2 2 3 3 1 1 2 1 2 3 1 1 1 2 2 2 3 3 3 1 h h h h h h h h h u h u u h u u h u ? (5.40) 5.8.1.

SA mmp ass 2.pdf (D118459575)

227/267	**SUBMITTED TEXT**	64 WORDS	**85%**	**MATCHING TEXT**	64 WORDS

r r r r r z z ? 2 2 2 2 2 1 1 r r r r r z ? (5.41) 5.8.2.

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228/267	**SUBMITTED TEXT**	117 WORDS	**32%**	**MATCHING TEXT**	117 WORDS

sin sin sin cos cos sin sin r ? ? ? ? ? ? ? ? ? ? ? ? r θ e e e ? ? 3 cos cos sin r ? ? ? ? ? ? r θ e e ? ? 2 2 2 2 2 2sin cos sin sin 3cos r ? ? ? ? ? ? ? ? ? r e ? ? ? 2 2 2sin cos cos sin cos sin 3sin cos r ? ? ? ? ? ? ? ? ? ? ? ? θ e ? ? ? 2 2sin sin cos sin sin r ? ? ? ? ? ? ? ? ? e ? ? ? ? ? 2 2 2 2 2 3sin cos 4cos 1 sin cos 2cos sin 3 r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? r

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229/267	**SUBMITTED TEXT**	15 WORDS	**75%**	**MATCHING TEXT**	15 WORDS

f x x a dx f a x a dx f a x a

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230/267	**SUBMITTED TEXT**	59 WORDS	**70%**	**MATCHING TEXT**	59 WORDS

a a ? ? ? ? ? 4. ? ? ? ? 2 2 1 ( ) ( ) , 0 2 x a x a x a a a ? ? ? ? ? ? ? ? ? (6.12) We have ? ? ? ? 2 2 ( )( ) x a x a x a ? ? ? ? ? ? 1 1 ( ) ( ) x a x a x a x

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231/267	**SUBMITTED TEXT**	65 WORDS	**48%**	**MATCHING TEXT**	65 WORDS

x a dx a ? ? ? ? ? ? ? ? ? ? ? Hence we get ? ? ? ? 2 2 1 ( ) ( ) 2 x a x a x a a ? ? ? ? ? ? ? ? 5. ( ) ( ) ( ) ( ) f x x a f a x a ? ? ? ? ? (6.14) Since ( ) 0 x a ? ? ? at x = a	x a +? curl a Proof : curl (? x(? a)= ? ? ? ?) (a x i ? = ? ? ? ? ? ? ? ? ? ? ? ? ? x a a x i ? ? = ? x a i a i = ??x a +(?x grad ?)x a +?

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232/267	**SUBMITTED TEXT**	29 WORDS	**61%**	**MATCHING TEXT**	29 WORDS

x a dx x a dx x a dx x a x a ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 ( ) ( ) 2 f a f a

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233/267	**SUBMITTED TEXT**	26 WORDS	**100%**	**MATCHING TEXT**	26 WORDS

as, 11 12 1 21 22 2 1 2 ... ... ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? n n m m mn a a a a a a a a a

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234/267	**SUBMITTED TEXT**	16 WORDS	**78%**	**MATCHING TEXT**	16 WORDS

Square matrix : If the number of rows and columns of a matrix are equal

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235/267	**SUBMITTED TEXT**	11 WORDS	**100%**	**MATCHING TEXT**	11 WORDS

leading diagonal are zero is called an upper triangular matrix.	leading diagonal are zero is called an Upper triangular matrix.

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236/267	**SUBMITTED TEXT**	71 WORDS	**35%**	**MATCHING TEXT**	71 WORDS

Is an upper triangular matrix and 1 2 0 0 5 3 0 2 0 2 ? ? ? B L , is an lower triangular matrix. 13. Symmetric matrix : If a square matrix A = [a ij] such that A T = A (7.9) i.e. [a ij] = [a ji] for all i, j. Then the matrix A is called a symmetric matrix.

is either an upper triangular matrix or lower triangular matrix 15. Symmetric matrix: A square matrix A =[a ij] is said to be symmetric if a ij = a ji for every i and j a symmetric matrix if A T = A Eg: is a symmetric matrix 16.

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237/267	**SUBMITTED TEXT**	17 WORDS	**73%**	**MATCHING TEXT**	17 WORDS

a square matrix A is called the determinant of matrix A and denoted by det A or |A|.

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238/267	**SUBMITTED TEXT**	23 WORDS	**81%**	**MATCHING TEXT**	23 WORDS

Equal matrices : Two matrices A and B are said to be equal if and only if a ij = b

Equal matrices : Two matrices A = [a ij] and b= [b ij] are said to be equal if and only if (i) A and B

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239/267	**SUBMITTED TEXT**	129 WORDS	**28%**	**MATCHING TEXT**	129 WORDS

A + = (A*) T .... (7.15) For example : 2 3 2 3 2 3 ? ? ? ? ? ? ? ? ? ? ? i i A i ? ? ? ? * 2 3 2 2 3 2 * 2 3 3 2 3 3 T ? T i i i i A A A* i i 19. Hermitian matrix : A complex square matrix A = [a ij] is called Hermitian matrix if (A*) T = A or A + = A. Example : 0 0 * 0 0 A A ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i i i i NSOU ? CC-PH-04 ? 231 ? ? 0 0 T ? ? ? ? ? ? ? ? ? ? ? i A* A i Or, A + = A i.e. A is Hermitian.

A A A A A A ? ? ? ? ? ? ? ? ? ? is a Hermitian matrix. ? ? ? A A ? ? 2 1 is also a Hermitian matrix Now ? ? ? ? A A A A ? ? ? ? ? ? ? ? A A ? ? ? ? ? ? A A ? ? ? ? Hence A A ? ? a skew-Hermitian

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240/267	**SUBMITTED TEXT**	13 WORDS	**95%**	**MATCHING TEXT**	13 WORDS

If A and B be two matrices of the same order then

If A and B be two square matrices of the same order, then 29.

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241/267 SUBMITTED TEXT 41 WORDS	**52%** MATCHING TEXT 41 WORDS
of A and B. If A = [a ij] and B = [b ij], then A + B = [a ij + b ij]. (7.17) Therefore the sum of two matrices,	of A and B is by A+B. Thus [a ij] + [b ij] = [a ij +b ij] and [a ij +b ij] = [a ij] + [ij] 19. The difference of two matrices:

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242/267 SUBMITTED TEXT 37 WORDS	**63%** MATCHING TEXT 37 WORDS
Skew-symmetric or anti-symmetric matrix : A square matrix A = [a ij] is called a skew-symmetric or an anti-symmetric matrix if– i) a ij = – a ji for all values	

SA Mathematical Physics - I SLM full.pdf (D113782471)

243/267 SUBMITTED TEXT 19 WORDS	**87%** MATCHING TEXT 19 WORDS
a a b b c c d d e e f f ? ? ? ? ? ? ? ? ? ? ? ? ? ?	

SA Yeshy.pdf (D112546741)

244/267 SUBMITTED TEXT 23 WORDS	**89%** MATCHING TEXT 23 WORDS
A + B = B + A 2< Matrix addition is associative : A + (B + C) = (A + B) + C	

SA Yeshy.pdf (D112546741)

245/267 SUBMITTED TEXT 26 WORDS	**50%** MATCHING TEXT 26 WORDS
the transpose of the matrix A. Now a vector twice the length of A is 2 2 2 2 & 2 (2 2) 2 a a a b a	the transpose of the matrix A is called the conjugate transpose of A and is denoted by A θ ? ? ? ? ? ? ? ? ? ? ? 2 0 0 0 1 0 0 0 3 ? ? ? ? ? ? ? ? ? ? ? 2 0 0 0 2 0 0 0 2 A ? A ? B ? ? A 32 32 3 4 0 5 2 3 2 3 4 0 5 2 3 2

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246/267 SUBMITTED TEXT 14 WORDS	**76%** MATCHING TEXT 14 WORDS
If A and B are two square matrix of the same order then	If A and B be two square matrices of the same order, then 29.

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247/267 SUBMITTED TEXT 17 WORDS	73% MATCHING TEXT 17 WORDS
have the same number of rows and the same number of columns i.e. they are of the	

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248/267 SUBMITTED TEXT 28 WORDS	40% MATCHING TEXT 28 WORDS
the matrices. 7.5.9 : Rank of a matrix : An integral number r is said to be the Rank of a matrix A; if, i. There is at least one	The rank of a matrix is $\leq$r if all minors of (r+1) th order are zero. 2. The rank of a matrix is $\geq$r, if there is at least one

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249/267 SUBMITTED TEXT 52 WORDS	47% MATCHING TEXT 52 WORDS
a b c a b c a b c a b c a b c a b c ? ? ? ? ? ? ? ? ? ? ? ? A A The matrix obtained from the cofactor of |A| is given by C, where 1 1 1 2 2 2 3 3 3 A B C A B C A B C ? ? ? ? ? ? ? ? ? ? ?	

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250/267 SUBMITTED TEXT 58 WORDS	55% MATCHING TEXT 58 WORDS
The rank of a non-singular square matrix of order n is n and that of a singular square matrix of order n is less than n. 2) The rank of a null matrix is obviously zero. 3) The rank of the transpose of a matrix A is the same as the rank A. 4) The rank of product of two matrices	

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251/267 SUBMITTED TEXT 16 WORDS	62% MATCHING TEXT 16 WORDS
is said to be the rank of the matrix A. The rank of the matrix A	

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252/267 **SUBMITTED TEXT** 64 WORDS	**75%** **MATCHING TEXT** 64 WORDS
X X is satisfied. Let 1 4 1 2 2 5 3 6 x x x and x x x ? X X ? ? 4 1 2 1 2 3 5 1 4 2 5 3 6 6 0 x x x x x x x x x x x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? X X	X 9 * 25 X 7 + 42X 5 * 25X 3 + 4s 16 (x) = 36X 13 * 245X 11 + 539X 9 * 660X 7 + 539X 5 * 245X 3 + 36X; x) = 24X 15 * 154X 13 + 273X 9 * 143X 7 + 273X 5 * 154X 3 + 24X; a 12 (X 8 * 3

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253/267 **SUBMITTED TEXT** 22 WORDS	**73%** **MATCHING TEXT** 22 WORDS
characteristic polynomial of matrix A. Characteristic equation : The equation ??A I = 0 is known as the characteristic equation of matrix A	

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254/267 **SUBMITTED TEXT** 23 WORDS	**86%** **MATCHING TEXT** 23 WORDS
n n n n nn a a a a a a a a a ? ? ? ? ? ? ? (7.29) or, ? ? 1 1 (1) ... 0 n n n n ? ? ? ? ? ? ? ? ? ? (7.30)	

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255/267 **SUBMITTED TEXT** 67 WORDS	**50%** **MATCHING TEXT** 67 WORDS
x y ? AX X X so that 3 2 2 3 i x x x i y y y ? ?? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ?? ? ? ? ? ? I or, 3 2 0 0 3 0 2 i x i y ? or, 1 0 1 i x i y ? ?? ? ? ? ?? ? ? ? ?? ? or, 0 0 x	

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256/267 **SUBMITTED TEXT** 20 WORDS	**52%** **MATCHING TEXT** 20 WORDS
eigen values of A. Thus we can state the following theorem. If a matrix of order n has n linearly independent	Eigen values of A. Diagonalization of a Theorem: If a square matrix A of order n has n linearly independent

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257/267	SUBMITTED TEXT	13 WORDS	84%	MATCHING TEXT	13 WORDS

D is a diagonal matrix whose diagonal elements are the eigenvalues of A.

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258/267	SUBMITTED TEXT	60 WORDS	61%	MATCHING TEXT	60 WORDS

n) 11 1 12 2 1 1 ... n n a x a x a x b ? ? ? ? 21 1 22 2 2 2 ... n n a x a x a x b ? ? ? ? ... ... 1 1 2 2 ... m m mn n m a x a x a x b ? ? ? ?

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259/267	SUBMITTED TEXT	90 WORDS	44%	MATCHING TEXT	90 WORDS

a a a x b a a a x b a a a x b ? or, AX = B (7.37) and A b = 11 12 1 1 21 22 2 2 1 2 ... ... n n m m mm m a a a b a a a b a a a b ? ? ? ? ? ? ? ? ? ? ? ? ? A b = [A, B] is

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260/267	SUBMITTED TEXT	16 WORDS	75%	MATCHING TEXT	16 WORDS

y t x x x x (7.47) where a and b are the arbitrary constants. 7.10 ?

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261/267	SUBMITTED TEXT	61 WORDS	69%	MATCHING TEXT	61 WORDS

a a x a x a x ? ? ? ? ? ? ? A I be the characteristic equation of a square matrix A, then 2 0 1 2 ... 0 n n a a a a ? ? ? ? ? I A A A (7.60) Where every x is replaced by A, and thus 0 0 0 0 0 0 a a x a a ? ? ? A

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262/267 **SUBMITTED TEXT** 40 WORDS	**86%** **MATCHING TEXT** 40 WORDS
p p p p p p q p q p p p p p p p p p p p p p p ?	

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263/267 **SUBMITTED TEXT** 103 WORDS	**30%** **MATCHING TEXT** 103 WORDS
A A or, 1 2 2 , 1 2 2 i i ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A Now 1 0 0 1 ? ? ? ? ? ? ? ? ? ? A A I ? A is unitary. Example 2 : Show that the matrix cos sin sin cos ? ? ? ? ? ? ? ? ? ? ? is orthogonal. Solution : Let cos sin cos sin sin cos sin cos ? T A A Now 2 2 2 2 cos sin cos sin sin cos sin cos cos sin sin cos ?	A = A T = Consider A. A T = = A is orthogonal matrix. 2. Prove that the matrix is orthogonal. Sol: Given A = Then A T = Consider cos sin sin cos ? ? ? ? ? ? ? ? ? ? ? cos sin sin cos ? ? ? ? ? ? ? ? ? ? ? cos sin sin cos ? ? ? ? ? ? ? ? ? ? ? cos sin sin cos ? ? ? ? ? ? ? ? ? ? ? cos sin sin cos ? 2 2 2 2 sin cos sin cos cos sin cos

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264/267 **SUBMITTED TEXT** 24 WORDS	**61%** **MATCHING TEXT** 24 WORDS
is a column vector of order m × 1 and C = [C ij] is a matrix of order m ×	

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265/267 **SUBMITTED TEXT** 68 WORDS	**51%** **MATCHING TEXT** 68 WORDS
cos sin cos sin (*) sin cos sin cos T i i i i ? T A A cos sin cos sin sin cos sin cos i i i i ? A A 2 2 2 2 cos sin cos sin sin cos 1 0 0 1 sin cos sin	

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266/267 **SUBMITTED TEXT** 41 WORDS	**100%** **MATCHING TEXT** 41 WORDS
cos sin 0 sin cos 0 0 0 1 x x y y z z ? 258 ?	

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267/267 **SUBMITTED TEXT** 112 WORDS **52%** **MATCHING TEXT** 112 WORDS

cos sin 0 ( ) sin cos 0 0 0 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? A ii) ? ? 1 1 2 2 1 2 1 1 2 2 cos sin 0 cos sin 0 ( ) sin cos 0 sin cos 0 0 0 1 0 0 1 ? A A ? ? ? ? ? ? ? ? ? ? 1 2 1 2 1 2 1 2 1 2 cos sin 0 sin cos 0 0 0 1 ? A

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PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, discipline specific, generic elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success.
Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor
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Course Code : GE-PH-11

Unit 1 ????? Vectors Structure 1.1 Objectives 1.2 Introductions 1.3 Vector Algebra 1.4 Vector Product or Cross Product 1.5 Exercises 1.1 Objectives This unit helps you to develop on idea about vector and its properties. Also you will learn vector algebra & application in daily life. 1.2 Introductions The Physical quantities we often come acrors are of two types. Those which have magnitudes only such as mass, volume etc are known as scalar, whereas those which have magnitude and direction are vectors. Force, velocity, acceleration etc are some very common vectors. We write a vector A ? having a magnitude | | A ? or simply A and geometrically represented by a line segment of length proportional to A and whose direction is along A ? . In onthogonal rectangular cartesian system the vector A ? is expressed as ˆ ˆ ˆ x y z A i A j A k A ? ? ? ? where ˆ ˆ ˆ , , i j k are unit vectors along the axes and

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A x , A y and A z are components of A ? along the

axes. The magnitude of the vector in terms of its components is 2 2 2 x y z A A A A ? ? ?, where

8 ? NSOU ? GE-PH-11 A x = A cos α, A y = A cos β and A z = A cos γ ; where α, β and γ are the angles between the vector and the three co-ordinate axes. cos α, cos β and cos γ are the direction cosines of the vector and 2 2 2 cos cos cos 1 ? ? ? ? ? ? 1.3 Vector Algebra Sum of two vectors : Sum of two vectors A ? and B ? is written as ? ? ? ? ? A B C and is geometrically represented as following. We draw a line OP along the direction of A ? and with magnitude OP proportional to A. OP ? represents A ? . From the end point P of A ? we draw a line PQ along the direction B ? and of magnitude proportional to B. ? P? represents the vector B ? . OQ ? , the line joining. The start point of A ? to the end point of B ? is the vector C ? represents the Sum of A ? and B ? in direction and magnitude. From simple geometry the magnitude of the vector sum is 2 2 2 cos C A B AB? ? ? ? where θ is angle made by B ? with A ? . The vector C ? will make angle α with A ? given by sin tan cos ? ? B A B ? ? ? In case of difference ? ? A B A B ? ? ? ? ? ? ? ? ? We draw OP A? ? ? . From P we draw PR proportional to B but opposite to the direction of B ? . PR ? represents − B ? . The line joining O to R is the vector A B ? ? ? in direction and magnitude. D 2 = A 2 + B 2 − 2AB cosθ

NSOU ? GE-PH-11 ? 9 Scalar or dot product The Scalar or a dot

58%	MATCHING BLOCK 2/68	SA	Physics_Vol-1 EM.pdf (D40552326)

product of two vectors is scalar quantity equal to the product of magnitudes of the two vectors and the cosine of angle between them.

Thus ? ? ? ? ˆ ˆ ˆ ˆ ˆ ˆ . cos ? ? ? ? ? ? ? ? ? x z x z AB AB i A j Ay k A i A j Ay k A ? ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ; . . . 1; . . . ? ? ? ? ? ? ? ?

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x x y y z z A B A B A B i i j

j k k i j j k k j The work down W by a force F ? to make a displacement S ? is given by . cos W F S FS ? ? ? ?? is a scalar quantity equal to the product of force (F) and the displacement in the direction of the force S cos θ . 1.4 Vector product or Cross product A vector product or a cross product of two linearly independent vectors expressed as A B C ? ? ? ? ? is a vector perpendicular to both A ? and B ? and therefore normal to the plane containing A ? & B ? , such that when a right hand screw

58% **MATCHING BLOCK 4/68** **SA** Physics_Vol-1 EM.pdf (D40552326)

is rotated from A ? to B ? the direction of motion of the screw head gives the direction of

the product and the magnitude is

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equal to the product of the magnitudes of two vectors and the sine of the angle between them.

We write ˆ sin C A B A B n ? ? ? ? ? ?? where ˆn is the unit vector along C ? and θ is the angle between the vectors A ? and B ? . The cross product C ? of A ? and B ? gives the area of the parallelogram with vectors A ? and B ? as sides. In terms of components– ? ? ? ? ˆ ˆ ˆ ˆ ˆ ˆ

59% **MATCHING BLOCK 6/68** **SA** Physics_Vol-1 EM.pdf (D40552326)

x y z x y z A B i A j A k A i B j B k B ? ? ? ? ? ? ? ? ? ? ? ? ? ? ˆ ˆ ˆ y z y z z x z x x y x y i A B B A j A B B A k A B B

A ? ? ? ? ? ?
10 ? NSOU ? GE-PH-11 ˆ ˆ ˆ ˆ i i j j k k ? ? ? ? ? ? = 0 ˆ ˆ ˆ

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x y z x y z i j k A A A B B B ?

and ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ ˆ i j k j k i k i j ? ? ? ? ? ? Obviously ˆ ˆˆ B A A B ? ? ? ? ? The torque N ? i.e. the moment of force F ? and r ? is the position vector of point of application of force then, N r F ? ? ? ? ? Derivative of a vector with respect to a parameter : We consider a vector ˆ ˆ ˆ x y z A i A j A k A ? ? ? ? The derivative of the vector A ? with respect to a parameter α say is defined as. ? ? 0 ( ) A A dA Lt d ?? ? ?? ? ? ?? ? ? ? ? ? ? ? 0 ( ) ( ) ( ) ( ) ( ) ( ) ˆ ˆ ˆ y y x x z z A A A A A A Lt i j k ?? ? ?? ? ? ?? ? ? ?? ? ?? ?? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ˆ ˆ ˆ y x z dA d A dA i j k d d d ? ? ? ? ? ? The time derivative of velocity v ? w.r.t t is ˆ ˆ ˆ

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y x z dv dv dv dv i j k dt dt dt dt ? ? ? ?

which is the acceleration ˆ ˆ ˆ .

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x y z a a i a j a k ? ? ? ? ; ; x x z

x x z dv dv dv a a a dt dt dt ? ? ?
NSOU ? GE-PH-11 ? 11

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v x , v y and v z are components of

velocity and

84%	**MATCHING BLOCK 11/68**	**SA**	BSc_Physics_1st sem_Block B.pdf (D129738380)

a x , a y and a z are components of acceleration along the

co-ordinate areas. Solved problems (1) Find the angle between the vectors ˆ ˆ ˆ 4 2 6 A i j k ? ? ? ? and ? ? ˆ ˆ ˆ 9 6 3 B i j k ? ? ? ? . Solution : From the definition of dot products, if θ be the angle between the vectors, ? ? . cos

96%	**MATCHING BLOCK 12/68**	**SA**	Physics_Vol-1 EM.pdf (D40552326)

x x y y z z A B A B A B A B

A B ? ? ? ? ? ? ?
or 2 2 2 2 2 2 4 9 2 (6) 6 3 cos 4 2 6 9 (6) 3 ? ? ? ? ? ? ? ? ? ? ? ? ? 42 42 56 126 2 4 7 7 2 3 3 ? ? ? ? ? ? ? ? 42 42 1 7 2 2 3 84 2 ? ? ? ? ? ? = cos 60 o ∴ θ = 60 o 2. Find the area of a parallelogram whose sides are determined by the vectors ˆ ˆ ˆ 3 A i j k ? ? ? ? and ˆ ˆ ˆ 2 7 i j k ? ? The area of the parallelogram A B ? ? ? ? ? ˆ ˆ 1 1 3 2 7 1 i j h A B ? ? ? ? ? ? ˆ ˆ ˆ (1 21) (6 1) (7 2) i j k ? ? ? ? ? ? ? ? ˆ ˆ ˆ 20 5 5 i j k ? ? ? 2 2 2 20 5 5 450 15 2 A B ? ? ? ? ? ? ? ? ? ?
12 ? NSOU ? GE-PH-11 (3) Find the unit vector along ˆ ˆ ˆ 5 3 7 P i j k ? ? ? ? The unit vector 2 2 2 ˆ ˆ ˆ 5 3 7 5 (3) 7 P i j k P ? ? ? ? ? ? ? ? ? ? ? ˆ ˆ ˆ 5 3 7 1 ˆ ˆ ˆ 5 3 7 25 9 49 83 i j k i j k ? ? ? ? ? ? ? ? (4) Find the dot product of ˆ ˆ ˆ 4 3 7 a i j k ? ? ? ? and ˆ ˆ ˆ 2 5 4 b i j k ? ? ? ? The dot product is . ? ? ? ? ?

96%	**MATCHING BLOCK 13/68**	**SA**	Physics_Vol-1 EM.pdf (D40552326)

x x y y z z a b a b a b a b = 4 × 2 + 3 × 5 + 7 × 4 = 8 + 15 + 28 = 51 1.5

Exercises (1) Find the vector product of ˆ ˆ ˆ 4 3 7 P i j k ? ? ? ? and ˆ ˆ ˆ 2 5 4 Q i j k ? ? ? ? . (2) Find the component of ˆ ˆ ˆ 3 4 A i j k ? ? ? ? in the direction of ˆ ˆ ˆ B i j k ? ? ? ? . (3) Show the vector ˆ ˆ ˆ 3 2 A i j k ? ? ? ? and ˆ ˆ ˆ 9 7 15 B i j k ? ? ? ? ? are perpendicular to each other. (4) Vector ˆ ˆ ˆ 2 6 27 A i j k ? ? ? ? and vector ˆ ˆ ˆ B i j k ? ? ? ? ? ? are parallel. Find the values of λ and μ . (5) If ˆ ˆ ˆ 5 3 P i j k ? ? ? ? and ˆ ˆ ˆ 3 5 Q i j k ? ? ? ? Show that P Q? ? ? and P Q? ? ? are mutually perpendicular. (6) Using vector prove that area of a triangle is equal to 1 sin 2 ab ? , where a and b are two adjacent sides and θ is the angle between them.

NSOU ? GE-PH-11 ? 13 Unit 2 ????? Ordinary Differential Equation Structure 2.1 Objectives 2.2 Introductions 2.3 Separable Equation 2.4 Exercises 2.1 Objectives By reedy this unit you will learn about different kind of differential equation & its technique of solving such equations. 2.2 Introductions An equation involving unknown function of an independent variable and the derivatives of the function is known as differential equation. It is a very useful device to solve problems in physics. In applications, the function or the functions usually represents physical quantities, the derivatives represent their rate of change and the differential equation defines a relation between them. If y(t) represents a function of an independent variable 't' and dy dt is its derivative, then , , 0 dy F y t dt ? ? ? ? ? ? ? is a ordinary differential equation. The differential equation is ordinary if it does not involve partial derivatives. The order of a differential equation is determined by the term with the highest derivative. 5 dy y x dx ? ? is a first order linear differential equation. First order since only dy dx not 2 2 d y dx or 3 3 d y dx appears in the equation.

14 ? NSOU ? GE-PH-11 Linear differential equation is a differential equation that is defined by a linear polynomial in the unknown function and its derivatives. 2 2 0 d y kx dx ? ? (2) k = constant is a second order linear differential equation. A differential equation is homogeneous if once all the terms involving unknown functions are collected together on one side of the equation, the other side is identically zero, Equation (2) is a second order linear homogeneous equation. A first order differential equation is said to be homogeeous if it may be written as : f(x, y) dy = g (x, y) dx where f and g are homogeneous function of the same degree of x and y. We introduce an variable y v x ? . f(x dv + v dx) = g dx or (g – vf) dx = x dv f ( ) dx f dv h v dv x g vf ? ? ? which can be solved by integration. Solving a first order linear differential equation using integrating factor. We take a linear differential equation in the form ( ) ( ) ? ? dy P x y Q n dx (1) multiply both sides of equation (1) by a + ve function μ (x), that transforms the LHS into the derivative of the product μy dy Py Q dx ? ? ? ? ? (2) As per the choice of μ LHS = ( ) dy d d y y dx dx dx ? ? ? ? ?

NSOU ? GE-PH-11 ? 15 equation (2) becomes ( ) d y Q dx ? ? ? y Q dx ? ? ? ? or 1 y Q dx ? ? ? ? (3) Eqa (3) is the solution of eqn (1) in terms of μ(x) and Q(x). μ(x) is called the integrating factor of eqn. (1). From the condition imposed on μ. ( )? ? d dy y P y dx dx ? ? ? or, ? ? ? dy d dy y P y dx dx dx ? ? ? ? or, , ? ? ? pdx d P e dx ? ? ? μ is called integrating factor of eqn (1) because its presence makes the equation integrable, Solve the equation : 2 3 dy x x dx ? ? , (1) x<0, We put the eqn. in standard form 3 dy y x dx x ? ? ; 3 ( ) , ( ) P x Q x x x ? ? ? The integrating factor is 3 ( ) dx Pdx x x e e ? ?? ? ? ? 3ln ( )x e ? ? 3ln 0 x e x ? ? ?

16 ? NSOU ? GE-PH-11 3 1 x ? The solution is 1 ( ) ( ) ( ) ( ) y x x Q x dx x ? ? ? ? 3 3 3 2 1 x x dx x dx x x ? ? ? ? 3 1 ; x c x ? ? ? ? ? ? ? ? ? c = constant or y = – x 2 + cx 3 . 2.3 Separable Equation A differential equation is said to be separable if the variables can be separated and can be written as F(y) dy = G(x) dx once this is done, all that is needed to solve that equation is to integrate both sides, Let us solve the equation x dx + see x sin y dy = 0 (1) We rewrite eqn (1) as See x sin y dy = – x dx or sin y dy = – x cos dx Integrating both sides. cos sin sin y x x x dx ? ? ? ? ? ? ? ? ? = – (x sin x + cos x + c) ∴ the required Soln. is cos y = x sin x + cos x + c ; c = constant

NSOU ? GE-PH-11 ? 17 Exact differential equation A first order differential equation (of one variable) is called exact or exact differential if it is the result of a simple differentiation. The equation P(x, y) dx + Q (x, y) dy = 0 is exact if Q P x y ? ? ? ? ? . In this case there will be function R(x, y) such that and R R P Q x y ? ? ? ? ? ? . R(x, y) is the solution of the problem. Solve the differential equation by exact differential. e y dx + (2y + xe y) dy = 0. (1) Comparing with the standard equation P(x, y) dx + Q(x, y) dy = 0 (2) we get P(x, y) = e y and Q(x, y) = 2y + xe y ; y y Q P e e x y ? ? ? ? ? ? . i.e. Q P x y ? ? ? ? ? , hence the given equation is exact. To find the soln R(x, y) we write y R e x ? ? ? (3) and 2 y R y x e y ? ? ? ? (4)

18 ? NSOU ? GE-PH-11 from 3. (,) ( ) y y R x y e dx x e y ? ? ? ? ? (5) from (5) ( ) y R xe y y y ? ? ? ? ? ? ? ? ? ? ? ( ) y x e y ? ? ? ? (6) equating the rhs of (6) and (4) we have. ( ) 2 y y x e y y xe ? ?? ? ? or ( ) 2 y y ? ? ? 2 ( )y y ? ? ? From (5) R(x, y) = xe y + y 2 The general solution is xe y + y 2 = c. Second order linear homogeneous equation with constant coefficients are of great value in physical problems. As an illustration we discuss damped harmonic vibration. Suppose a mass m is subjected to a restoring force proportional to displacement (– x?) and a resistive force proportional to instantaneous velocity (– kx?), the equation of motion can be written as. mx x kx ? ? ? ? ?? ? or 0 k x x x m m ? ? ? ? ?? ? or 2 2 0 x bx w x ? ? ? ?? ? (1) where 2 k b m ? and 2 w m ? ?

NSOU ? GE-PH-11 ? 19 Let x = e pt be a trial solution, 2 and x px x p x ? ? ? ?? putting this in eqn (1) we get. p 2 + 2bp + w 2 = 0 or 2 2 p b b w ? ? ? ? has two values say 2 2 b b w ? ? ? ? ? and 2 2 b b w ? ? ? ? ? and the solution of the problem can be witten as. , x x x Ae Be ? ? ? ? A and B are constants. We discuss three different causes. Case 1 b 2 < w 2 i.e large resistive force. ? ? 2 2 2 2 b w t bt bt b w x Ae e Be e t ? ? ? ? ? ? both the terms gradually decreases to zero with time. There is no oscillation. The motion of the mass is shown in the figure. This motion is known as over damped motion. Case 2. b 2 > w 2 , Small resistive face. We write the soln as ? ? 2 2 ; bt i t i t x e Ae Be b ? ? ? ? ? ? ? ? ? ? ? 1 2 cos sin bt e c t c t ? ? ? ? ? cos ( ) bt e D t ? ? ? ? ? [c 1 , c 2 and δ are constants]
20 ? NSOU ? GE-PH-11 The motion is an oscilatory one. Its amplitude De –bt is exponentially decreasing with time. This is known as damped vibration. Case 3. b 2 = w 2 , ? ? ? In this case the solution of the differential equation is ? ? bt x c Dt e ? ? ? This motion is known as critical damped motion. It is critical in the sense that if b<w the motion is non-oscillatory and if b > w the motion is oscillatory. (1) Solve the differential equation 2 6 dy y x dx ? given that 1 (1) 25 y ? 2 6 dy y x dx ? or y –2 dy = 6x dx Integrating 2 6 ? ? ? ? y dy x dx or 2 1 3 , constant. x c c y ? ? ? ? putting 1 25 y ? ? and x = 1, we get 2 1 3.1 1 25 c ? ? ? or C + 3 = – 25 C = – 28 ∴ The required solution is 2 1 3 28 x y ? ? ?
NSOU ? GE-PH-11 ? 21 or 2 1 28 3 y x ? ? (2) Show that 2 2 2 d y dy dx dx ? has a solution 2 1 2 x y c c e ? ? , where c 1 and c 2 are constant. we have y = c 1 + c 2 e 2x 2 2 2 x dy c e dx ? 2 2 2 2 4 2 x d y dy c e dx dx ? ? Thus, y = c 1 + c 2 e 2x is a solution of 2 2 2 d y dy dx dx ? . (3) Find the solution of 2 3 4 4 2 4 dy x x dx y ? ? ? ? given that y(1) = 3. We rewrite the differential equation as dy(2y – 4) = (3x 2 + 4x – 4) dx. or 2ydy – 4dy = 3x 2 dx + 4x dz – 4dx Integrating both sides separately we have 2 3 2 4 4 4 , 2 y y x x x c ? ? ? ? ? or with y = 3 at x = 1, we get. 3 2 – 4 × 3 = 1 2 + 2.1 2 – 4.1 + c
22 ? NSOU ? GE-PH-11 or 9 – 12 = 1 + 2 – 4 + c or, – 3 = – 1 + c, or, c = – 2 ∴ y 2 – 4y = x 3 + 2x 2 – 4x – 2 is the required solution. 2.4 Exercises (1) State the order and degree of the following differential equations. i) 3 2 2 3 2 8; d y dy x y dx dx ? ? ? ? ? ? ? ? ? ? (2, 1) ii) 5 2 3sin –sin ; dy x x y dx ? ? ? ? ? ? ? ? (1,5) iii) 4 7 2 2 2 57 3 d y dy dx dx ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2,4) (2) Find the general solution of the differential equation dy + 7x dx = 0. Obtain the particular solution with y(o) = 2. (3) Establish the differential equation of a simple harmonic motion and obtain its general solution.
NSOU ? GE-PH-11 ? 23 Unit 3 ????? Laws of Motion Structure 3.1 Objectives 3.2 Introduction 3.3. Frame of Reference 3.3.1 Interial frame pf Referemce 3.3.2 Non-interial fram of Reference 3.4 Dynamics of System of Particle Centre of Mass 3.5 Total angular Mamentum of System 3.6 Total Kainetic Energy of a System of Particles 3.7 Racket Motion 3.8 Solved Problems 3.9 Exercises 3.1 Objectives By this unit you will gather knowledge about motion of objects & its relevant lows. An Idiea of centre of mass & its motion will also develop. Finally you will learn about Rocket motion in detail. 3.2 Introductions Newton's Laws of motion describe

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the relationship between a body and the forces acting upon it and its motion in response to those forces.

Three laws known as Newton's laws of motion together laid the foundation of classical mechanics. First Law :–

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The state of motion of a body does not change until and external force is applied on it. Second Law :– The rate of change of linear momentum of a body is proportional to the

external force applied on
it.
24 ?

NSOU ? GE-PH-11 Third Law :– Every action has an equal and opposite reaction. First law was actually present in topics of scientific discussion as Galelio's Law of Inertia before the statement by Newton was made. The law introduced the concept of force as an agent responsible for the change of state of motion of a body. While the 1st Law gives a qualitative definition of force, the second Law gives a quantitative statement. The linear momentum p ? of a body is the product of its mass 'm' and velocity v ? p mv ? ? ? The second law mathematically may be written as ( ) d mv F dt ? ? ? , F ? is total external force applied on the body. If we confine to the cases where m = constant, then dv F k m dt ? ? ? k = a constant of proportionality. or F k ma ? ? ? dv a dt ? ? ? = rate of change of volocity or acceleration. If the force that produce unit acceleration on unit mass is taken as unit of force k becomes unity and we have F ma ? ? ? (1) The force that produce 1m s –2 of acceleration on 1 kg of mass is 1 Newton (IN). From eqn. (1) or F F m a a m ? ? i.e. greater is the value of m more it is difficult to produce acceleration i.e. more it is difficult to make the change of state of motion. Mass is therefore defined as inertia of the body. Again if F = 0, a = 0 i.e. no force means no acceleration. No acceleration means no change of state of motion. This statement is nothing but the statement of first law of motion. According to the third law of motion no force in nature is alone but always accompanied with reaction. This law plays a very important role in our daily lie. As an illustration we explain how a horse draw a cart.

NSOU ? GE-PH-11 ? 25 The horse with his hooves applies an oblique force F on the ground. The ground in reply, applies a force of reaction F´ (F´ = F). F´ has an upward vertical component F´ sin θ which balances the weight of the horse. Tension T is created in the rope. The frictional force f of the ground is applied on the cart in backward direction. If T < f the cart will move forward. If F´ cos θ < T the horse will move forward. The equation of motion of the horse is F / cos θ – T = Ma (1) ; M = Mass of the horse The equation of motion of the cart is T – f = ma, (2) ; a = acceleration of the system m = mass of the cart Adding equation (1) and equation (2) we get the equation of motion of the horse and cart composite system as F / cos θ – f = (M + m) a The equal and opposite force of action and reaction can never cancel each other since they act on different bodies. 3.3 Frame of Reference A framework that is used for observation and description of physical phenomenon and formulation of Physical laws usually consists of an observer, a coordinate system and a clock to assign time at a position with respect to the coordinate system is a frame of reference. Proper choice of frame of reference makes the study of motion of the body easy and solution of the problem less complicated. To an observer in a running train all co-passangers are at rest but the objects outside train are in motion. On the other hand to an observer standing in the station, all passenger in the train are moving while the trees and land are at rest.

26 ? NSOU ? GE-PH-11 3.3.1 Inertial frame of Reference We consider two orthogonal Cartesian coordinate systems (S) and (S /) as two frames of references. The coordinate axes are parallel. The origin O of s is at rest but o / of s / is moving with uniform velocity v with respect to 0. r ? and r ? ? are position of a point particle P at a time t and OO vt ? ? ? . (At t = 0, O and O / are taken to be coincident) obviously r r vt ?? ? ? ? r r v ?? ? ? ? ? ? ? ? and 0 r r ?? ? ? ? ? ?? ?? (1) Newton's second law of motion for the point particle P of mass m (say) in s frame is mr F? ? ? ? ?? (2) From equation (1) and (2) we write mr F ? ? ? ? ? ?? (3) Equation (2) and (3) are of exactly same form. The reference from s at rest, is known as inertial frame and Newton's Laws are valid in this frame. Equation (3) states that Newton's Laws are also valid in s / frame. In fact all frames moving with uniform velocity with respect to an inertial frame are also inertial. Two other facts are very relevent from this discussion that uniform velocity and rest are not distinguishable and the state of motion of an inertial frame can not be determined by an observer in the frame. 3.3.2 Non-inertial frame of Reference Let the S / frame is moving with uniform acceleration a ? and at t = 0. O and O / are coincident and v = 0. At time t. r r OO ? ? ? ? ? ? 2 1 2 r at ?? ? ? ?

NSOU ? GE-PH-11 ? 27 r r at ?? ? ? ? ? ? ? ? and r r a ?? ? ? ? ? ? ?? ?? (4) The Newton's second law of motion looks in s frame like mr F? ? ? ? ?? (5) with the help of eqn. (4) and (5) the equation of motion of the particle m S / frame becomes. mr ma F ? ? ? ? ? ? ? ?? or mr F ma ? ? ? ? ? ? ? ?? (6) Though the left hand side of equation (6) contains the product of mass and acceleration right hand side contains over and above the applied force, another force term equal to negative of the product of mass of the particle and acceleration of the frame of reference. Equation (6) is different from Newton's law. Thus Newton's laws are not valid in such accelerating frames. Such frames are known as non-inertial frames. Along with the externally applied free F ? another force term ma ? ? appears in the equation of motion. This force is due only for the acceleration of the frame. This force is not originated due to interaction of some physical quantities and only originated due to acceleration of frame. Such force of reference frame vanishes when viewed from an intertial frame, is known as pseudo force or ficticious force'. We realise this force in our daily life when we move in a lift. When the lift is moving uniformly the man in it experiences only his own weight. When the lift is going upward with an acceleration 'a' his apparent weight become mg + ma he feels heavier and when the lift is going down-ward with an acceleration 'a' his apparent weight become ms − ma and he feels lighter. In case the suspension cord breaks down and he suflers free fall, a = g, the man feels weightless. A rotation is an acceleration. A body at rest in a rotating frame of reference feels a pseudo force known as centrefugal force. When a car moving with uniform motion changes its direction of motion a man sitting in it feels a force opposite to the direction of turning. A boy riding a marry go-round feels a force drawing him radially outward. These are contrefugal forces whose value is mr ω 2
where m

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is the mass of the body, r is the distance of the body from the centre of rotation			

and ω is the magnitude angular velocity
of the rotating frame.
28 ? NSOU ? GE-PH-11 If a body in a rotating frame is not at rest but has a velocity, over and above the centrefugal force it feels another pseudo force known as coreolis force perpendicular to the velocily of the body and the axis of rotation of the frame. (1) Two

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blocks of mass 50 kg and 30 kg are in contact on a smooth surface. A force of 100 N is applied on the 50			

kg block as shown. 100N 50 kg 30kg Find the acceleration of the system and the contact force between the blocks. Solution : If f is the acceleration, then 100 = (50 + 30) × f. or 100 80 f ? = 12.5 ms −2 If P be the contact force, the force diagram for the 50 kg mass is 100 50kg P 100 − P = 50 × 1.25 = 62.5 ∴ P = 100 − 62.5 = 37.5 N The force diagram for 30 kg mass is P = 30 × 1.25 = 37.5 N. (2) A 75 kg man stands on a platform scale in an elevator. Standing from rest the elevator ascendes, attains its maximum speed of 1.20 m/s´ in ls. It travels with this constant speed for next 10s. The elevator then undergoes a retardation for 1.7s and comes to rest. What does the scale register. i) before the elevator starts to move ? ii) during the 1st 1 sec ? iii) While elevator is travelling at constant speed? iv) during the time it is slowing down ? (g = 10 ms −2) → → ← 30 kg P →
NSOU ? GE-PH-11 ? 29 Solution : mg R i) There is no acceleration ∴ R = mg = 75 × 10 = 750 N ii) during the 1st s. the acceleration 1.2 1 f ? = 1.2 ms −2 R − mg = mf R = mg + mf = 75 × 10 + 75 × 1.2 = 750 + 90 = 840 N. iii) Since acceleration is zero R − ms = 0, R = mg = 750 N iv) If the retardation is a 1.2 = a × 1.7 ; 1.2 0.7 1.7 a ? ? mg − R = ma R = mg − ma = 750 − 75 × .7 = 750 − 52.5 = 697.5 N. (3) A system of two masses m 1 and m 2 capable of moving through a frictionless pully P connected with a string is known as Atwood Machine. Compute the acceleration of the masses and tension of the string. Let us suppose that m 2 is moving down with acceleration f and m 1 is going up with same acceleration and T is the tension of the string. For the mass m 1 we write T − m 1 g = m 1 f .... (1) For the mass m 2 we write
30 ? NSOU ? GE-PH-11 m 2 g − T = m 2 f .... (2) Adding equation (1) and (2)

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m 2 g − m 1 g = m 1 f + m 2 f or 2 1 1 2 ( ) m m g f m m ? ? ? From (1) 2 1 1 1 1 2 1 1 m m T m g m f m g m m ? ? ? ? ? ? ? ? ? ? ? ? 1 2 1 2 2m m g m m ? ? (4)

A 20 kg object has a velocity 1 ˆ 4i ms ? at t = 0. A constant force ? ? ˆ ˆ 2 4 i j ? N Acts on the object for 3s. What is the magnitude of velocity at the end of 3s ? Solution : For x– motion u x = 4ms −1 , F x = 2N. acceleration 2 2 0.1ms 20 ? ? ? x f 2 1 1 4 3 0.1 9 2 2 ? ? ? ? ? ? ? ? x x x v u t f t = 12 + 0.45 = 12.45 ms −1 For Y - motion, u y = 0, F y = 4 N. acceleration 2 4 0.2 ms 20 ? ? ? y f 1 1 0 0.2 9 0.9 ms 2 ? ? ? ? ? ? ? y v ∴ The magnitude of velocity at the end of 3s. is ? ? 2 2 12.45 (0.9) v ? ?
NSOU ? GE-PH-11 ? 31 155.0025 .81 155.8125 ? ? ? = 12.48 ms −1 3.4 Dynamics of System of Particles Centre of Mass The centre of mass of a distribution of mass in space is the unique point where the weighted relative position of the distributed mass sums to zero. This is the point to which a force may be applied to cause linear acceleration without any angular acceleration. Calculations in mechanics are often simplified when formulated with respect to the centre of mass. We consider a system of n particles P i (i = 1, 2, 3, ... n) having mass m i are located in space with co-ordinates i r ? (i = 1, 2, 3, ... n). The co-ordinate R ? of the centre of mass satisfies the condition ? ? 1 0 n i i i m r R ? ? ? ? ? ? ? .... (1) or 1 1 n n i i i i i

58% **MATCHING BLOCK 18/68** **SA** PARMESHWAR (M.Phil., Physics).docx (D57914933)

m R m r ? ? ? ? ? ? ? ? or i i i i m r R m ? ? ? ? ? ? or i i m r R M ? ? ? ? .... (2) Where 1 n i i M m ? ? ? =

mass of the system If i r ? ? is the co-ordinate of the i- th particle with respect to the centre of mass from (1) 1 0 n i i i m r ? ? ? ? ? (3) Equation (2) or (3) may be taken as the definition of centre of mass.
32 ? NSOU ? GE-PH-11 Dynamics of a System of particles We consider a system on n particles of mass m i (i = 1, 2, ... n) at position vectors i r ? (i = 1, 2, ... n) with respect to an arbitrary origin O. If i F ? denotes the total force on the i th particle, the equation of motion of the i th particle may be written as 2 2 i i i d r m F dt ? ? ? ? (1) The total force i F ? on the i th particle consists of the external force on the i th particle i F ? e and forces of interaction of all other particles 1 2 ,.... i i ni F F F ? ? ? ; on the i th particle. ∴ We rewrite equation (1) as, 2 2 ? ? ? ? ? ? ? ? n e i i i ji j i d r m F F dt Summing over all the particles we get. 2 2 1 1 ? ? ? ? ? ? ? ? ? ? ?? ? ? ? n n n n e i i i ji i j i i j i d r m F F dt (2)

66% **MATCHING BLOCK 20/68** **SA** MPDSC 1.1 Classical Mechanics.pdf (D133919389)

The first term on the right hand side of eqn (2) is the total external force 1? ? ? ?

n e e i n F F and the second terms contains each suffix twice and since ji ij F F ? ? ? ? from third law of motion. We get 1 0 n n ji i j i F ? ? ? ? ? ? ? We write eqn (2) as 2 2 1 n e i i i d r m F dt ? ? ? ? ? (3)
NSOU ? GE-PH-11 ? 33 If

61% **MATCHING BLOCK 21/68** **SA** ES3C3 Assignment.pdf (D25382439)

R ? is the position vector of the centre of mass and i r ? ? is the position vector of the i th particle with respect to the centre of mass

i i r R r ? ? ? ? ? ? ? 2 2 2 2 2 2 i i d r d r d R dt dt dt ? ? ? ? ? ? putting this in eqn (3) we get 2 2 2 2 1? ? ? ? ? ? ? ? ? n e i i i i d r d R m m F dt dt or 2 2 ? ? ? e d R M F dt (4) Since 2 2 1 n i i i d r m dt ? ? ? ? 2 2 1 0 n i i i d m r dt ? ? ? ? ? ? . From the defn. of centre of mass Equation (4) states that the centre of mass moves as if the total external force were acting on the entire mass of the system concentrated at the centre of mass. Purely internal forces obeying third law of nation have no effect on the
motion of the centre of mass. The linear momentum of a system of n particles

is 1 n i i p p ? ? ? ? ? ? i p ? is the linear momentum of the i th particle. ? ? 1 1 n n i i i i i i d m r m R r dt ? ? ? ? ? ? ? ? ? ? ? ? ? 1 n n i i i i i dR d m m r dt dt ? ? ? ? ? ? ? ?
34 ? NSOU ? GE-PH-11 or ? ? ? ? p M R (5) Since 1 0 n i i i m r ? ? ? ? ? from the defn. of c.m. The total linear momentum

83%	**MATCHING BLOCK 22/68**	**SA**	PHY17R121-Mechanics and Properties of Matter_A ... (D110170498)

of a system of particles is the total mass of the system

times the velocity of centre of mass. ? ? ? ? e p F

58%	**MATCHING BLOCK 23/68**	**SA**	ELMP-1 - Mechanics.pdf (D137599141)

Rate of change of total momentum of a system is equal to total external force on the

system. Therefore if 0 e F ? ? , p ? = constant vector. Total linear momentum of a system of particles is consurved if there is no external force

61%	**MATCHING BLOCK 24/68**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

on the system. 3.5 Total angular momentum of system The total angular momentum of a system of n particles

is given by. 1 n i i i L r p ? ? ? ? ? ? ? ? Since the angular momentum of the i th paricle is given by i i Li r p ? ? ??? ? ? or 1 n i i i L

32%	**MATCHING BLOCK 25/68**	**SA**	Physics_Vol-1 EM.pdf (D40552326)

r m r ? ? ? ? ? ? ? ? ? ? ? ? 1? ? ? ? ? ? ? ? ? ? ? ? ? n i i i i R r m R r 1 1 1 ? n n n i i i i i i i i i i R m R R m r r m R r m r ? ? 1 1 1 n n n i i i i i i i i i i i d R m R R m r m r R r

p dt ? 1 n i i R MR L ? ? ? ? ? ? ? ? ? ? ?
NSOU ? GE-PH-11 ? 35 Since the second and third terms contain i i m r ? ? ? which is zero from the defn. of c.m. or 0 cm L L L ? ? ? ? ? where 0 L R MR ? ? ? ? ? ? , is the angular momentum of entire system concentrated at cm about the origin R P ? ? ? ? i Lcm L? ? ? ? ? , is
the angular momentum of system about the c.m. Thus
the angular momentum
of a system of particles
about
an arbitrary origin
is equal to

59%	**MATCHING BLOCK 26/68**	**SA**	MPHS - 11 full draft.pdf (D114327742)

the sum of the angular momentum of the entire mass concentrated at the centre of mass about the origin and the angular momentum of the system about the centre of mass. 3.6

Total Kinetic energy

62%	**MATCHING BLOCK 27/68**	**SA**	Mechanics Properties of Matter-PHY17R121.docx (D109220287)

of a System of Particles The total kinetic energy of a system of n particles is 2 1 1 2

n i i i i T T m v ? ? ? ? ? , where
the kinetic energy of the i the particle is 2 1 2 i i T m v ? or, T ? ? ? ? 1 1 1 1 . . 2 2 n n i i i i i i i i

61%	**MATCHING BLOCK 28/68**	**SA**	Mechanics Properties of Matter-PHY17R121.docx (D109220287)

m v v m v v v v ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 1 1 . . 2 n i i i i i i i i m v m v v m v v m v ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 1 1 1 1 . 2 2 ? n n i i i i i i i m v v m v m v 2 1 1 . 2 ? ? ? ? ? ? ? ? ? ? ? ? ? n i i i i i mv T m v m

r 0 i i d m r dt ? ? ? ? ? i v ? = vel of the i th particle w.r.t. the orgin v ? = vel. of cm. w.r.t the origin i v ? ? = vel of i- th particle wrt cm.
36 ? NSOU ? GE-PH-11 and 2 1 2 i i i T m v ? ? ? or T = T 0 + T cm ; 2 0 1 and 2 ? ? ? ? cm i i T mv T T The

52%	**MATCHING BLOCK 29/68**	**SA**	M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

kinetic energy of the system consists of two parts, the kinetic energy obtained if the entire mass is concentrated at the centre of mass (

T 0) and the kinetic energy of the motion of the system about the centre of mass (T cm) Conservation of energy

71%	**MATCHING BLOCK 30/68**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

The work done by all the forces in moving the system from an initial configuration (1) to a final configuration (2)

is 2 12 1 1 1 . . ?? ? ? ? ? n n n i i i ji i i i j i W F ds Fe F ds 1 1 . . ? ? ? ? ? ? ?? ? ? ? ? ? ? n n n e i i ji i i i j i F ds F ds The Second term for say (ij) pair is . . ? ? ? ? ? ji i ij j F ds F ds ? ? . ji i j F ds ds ? ? ? ? ? ij ji F F ? ? ? ? ? . 0, ji ij F ds ? ? ? ? Since || ji ji F ds ? ? Hence we are left with 2 2 12 1 1 . . ? ? ? ? ? ? ? ? ? ? ? e i i i i i W F ds m v v dt 2 2 2 2 1 2 1 1 1 1 1 2 2 2 ? n i i i i i i n i i d m v m v m v
NSOU ? GE-PH-11 ? 37 T 2 – T 1 (1) Again if the external force is derivable in terms of gradient of a potential (conservative force) ? ? 2 2 12 1 1 1 . . n i i i i i i i W Fe ds V ds V ? ? ? ? ? ? ? ? ? ? ? ? ? ? or W 12 = V 1 – V 2 (2) From (1) and (2) T 1 + V 1 = T 2 + V 2 Total initial energy is equal to total final energy in case of conservative force field. 3.7 Rocket Motion The rocket motion is based on the principle of conservation of momentum. Rocket is a device that can apply acceleration to itself using thrust by expelling a part of its mass with high velocity and can therefore move due to conservation of momentum. We establish the formula for rocket motion with the assumption that the motion is in vacuum, with no gravity and no air resistance. We consider two stages of motion of the rocket at two very close
38 ? NSOU ? GE-PH-11 consecutive time 't' and '(t + dt)'. m is the mass of rocket and the propellant in it at the instant t and (m – dm e) is that at time t+ dt. m e and dm e are the masses of rocket exhaust exited before t and during dt. v and v + dv velocity of the rocket at t and t + dt, J e is the linear momentum of rocket exhaust at t and also at t + dt and v e is the velocity of exhaust. All velocities are with respect to ground (inertial reference frame). From principle of conservation of linear momentum we write J e + mv = J e – dm e V e + (m – dm e) (v + dv) or O = m dv – (v + v e) dm e [neglecting the second order term dm dv] or ( ) . e e dm dv v v m dt dt ? ? (1) The left hand side of above equation must represent the thrush acting on the rocket resulting the acceleration dv dt . We write the thrush as - ( ) e e dm T v v dt ? ? . or . e dm T u dt ? (2) u = v + v e is the velocity of exhaust relative to the rocket. This is approximetely constant in rockets. The term e dm dt is the burn rate of rocket proplelant. We rewrite equation (1) as . e dm dv u m ? (3) The mass of rocket exhausa dm e is during the time interval dt is equal to the negative of the mass change of the rocket. Equation (3) becomes. dm u dv m ? ? Integrating i i v m v m dm dv u m ? ? ? ?

NSOU ? GE-PH-11 ? 39 or ln , i i m v v u m ? ? m i is the mass of the rocket with its contents and v i is the velocity rocket at t = 0. It we take v i = 0 ln i m v u m ? (4) Equation (4) is an important equation in rocket motion known as Tsiolkovsky equation. When rocket starts from ground it is necessary to include the force of gravity and the effect of air resistance. We then modify equation (1) as ? ? ? e D dm dv u mg F m dt dt ; F D is the atmospheric ohag. or T − mg − F D = ma, a acceleration of rocket = dv dt . Problem : (1) Calculate the mass ratio needed to escape earth's gravity starting from rest. It is given that escape velocity is 11.2 × 10 3 ms −1 and the exhaust velocity is 2.5 × 10 3 ms −1 . Solution : We know that ln i m v u m ? or 3 3 11.2 10 ln 4.48 2.5 10 i m v m u ? ? ? ? ? 4.48 88 i m e m ? ? i.e. 87 88 part of final and 1 88 part of rocket, Problem : (2) A rocket engine ejects at the rate of 40 kg s −1 with an exhaust velocity 4000 ms −1 with an exhaust velocity 4000 ms −1 . Rocket's initial mass is 24000 kg. What is the change of relocity of the rocket in 24 s?

40 ? NSOU ? GE-PH-11 Solution : 40 / dme kg s dt ? fuel spent in 24s is 40 × 24 = 960 kg. using the rocket formula. ln i m v u m ? 24000 4000ln 4000 ln1.04 24000 960 ? ? ? ? = 4000 × .0408 = 163.2 ms −1 . Problem : (3) In a rocket launching the initial mass was 2.8 × 10 6 kg. The fuel was found at the rate of 1.4 × 10 +4 kg s −1 to burn. The exhaust velocity was 2.4 × 10 3 ms −1 calculate the initial acceleration. The thurst is 3 4 2.4 10 1.4 10 e dm T u dt ? ? ? ? ? kgm s −2 = 3.36 × 10 7 kg ms −2 acceleration produced 7 6 3.36 10 12 2.8 10 e T m ? ? ? ? ? ms −2 considering gravitation effect the net acceleration is 12 − 9.8 = 2.2 ms −2 3.8 Solved Problem (1) A boy of mass 40 kg jumps from rest into a trolley of mass 80 kg, moving with velocity 10 ms −1 . What will be the velocity of the trolley after the boy jumped in ? Solution : Total linear momentum before jump's 40 × 0 + 80 × 10 = 800 kgms −1 If v is the velocity of the trolley and boy after the jump, the total linear momentum is (80 + 40) v = 120 v.

NSOU ? GE-PH-11 ? 41 From the priniciple of conservation of linear momentum. 120 v = 800 v = 800 6.67 120 ? ms −1 (2) A 5 kg gun fires a bullet of 15 gm at a velocity 1000 ms −1 . What will be the velocity of recoil of the gun. Total linear momentum of the system of gun and bullet before fire is 15 50 0 0 0 1000 ? ? ? ? . If v is the velocity of recoil, the total linear momentum of the system is 15 50 1000 5 15 1000 v v ? ? ? ? ? ∴ 5v + 15 = 0, v = − 3 ms −1 . The recoil velocity is 3 ms −1 oppositive to the direction of motion of the bullet. (3) Two masses m 1 and m 2 move along a line, m 1 following m 2 with velocity v 1 while m 2 moves with a velocity v 2 (v 1 < v 2). They collide and stick with each other and continue moving along the same line with velocity (v 1 + v 2)/2. Find the relation between the masses. Solution : Total linear momentum along the path before collission is m 1 v 1 + m 2 v 2 Total linear momentum in the same direction after the collission is ? ? ? ? 1 2 1 2 2 v v m m ? ? from convervation of linear momentum ? ? ? ? 2 1 1 1 2 2 1 2 2

75%	MATCHING BLOCK 31/68	SA	Mechanics Properties of Matter-PHY17R121.docx (D109220287)

v v m v m v m m ? ? ? ? or 2m 1 v 1 + 2m 2 v 2 = m 1 v 2 + m 1 v 1 + m 2 v 2 + m 2 v 1 or m 1 (v 1 − v 2) − m 2 (v 1 − v 2) = 0 (m 1 − m 2) (v 1 − v 2) = 0 Since 1 2 v v ? , m 1 = m 2 42 ?

NSOU ? GE-PH-11 3.9 Exercises (1) A 1.5 kg mass has an acceleration ? ? ˆ ˆ 4 3 i j ? ms −2 . Two forces act on the mass. one is . What is the other force ? (2) Two masses 4 kg and 8 kg at the ends of a rope are hung and pass through a frictionless pulley (At wood machine) calculate the acceleration of the masses and the tension of the rope. (3) A 100 m long iron chain of mass 10 kg m −1 is being dragged by a force of 200 N. What will be the tension of the chain at its mid point ? (4) A mass of 1.5 kg is hung as shown in the figure below. Compute the value of the tensions T 1 and T 2 . (5) How do Newton's Laws define and measure force ? (6) How we derive Newton's first law and the measure of inertia from Newton's Second law ? (7) Two different masses m 1 and m 2 moving directly towards each other with velocities v 1 and v 2 (1 2 v v ?). After collission they stick together and stops. Find the condition. (8) On a smooth surface a ball A of mass 100 g at a velocity 10 ms −1 elastically collide with another ball B of mass 200 g moving with velocity 5 ms −1 in the same direction. They collide and keep on moving in the same direction. Compute their velocities after collission. (9) A train car of mass 42000 kg moving with velocity 10 m/s yowardss another train car. After the two car collide, they couple together and move with velocity 6 ms −1 . What is the mass of the second train car ?

NSOU ? GE-PH-11 ? 43 Unit 4 ????? Rotational Motion Structure 4.1 Objectives 4.2 Introductions 4.3 Angular Momentum 4.5 Kinetic Energy 4.6 Solved Problem 4.7 Exercises 4.1 Objectives A knowledge of circular motion & its application will develop by going through this unit. 4.2 Introductions A point mass m moving along line AB as shown, seems to rotate about an arbitrary origin O. In small time Δt it moves from A to B with linear speed v. AB subtends an angle Δθ at O. The angular velocity of the point mass about O is defined as 0 0 ˆ ? ? ? ? ???? ? t t Lt Lt n t t ? ? ?? ?? ? ? ? ?? ???? is a vector quantity given by ˆn ?? ?? ? ???? where ˆn is a unit vector given by the direction of motion of a right hand screw head when rotation from A to B. s v t r r ? ? ?? ? ?

44 ? NSOU ? GE-PH-11 0 ˆ ˆ ? ? ? ? t v n Lt n t r ? ?? ? ? ? v r ? is the magnitude of the angular velocity 4.3 Angular Momentum Angular momentum is the moment of linear momentum of the point mass about the origin is given by L r p ? ? ? ? ? p mv ? ? ? , is the linear momentum Angular momentum L ? is a vector quantity r ? both to the radius vector r ? and linear momentum p ? . In magnitude (when r & v are perpendicular) 2 v L rmv mr r ? ? = mr 2 ω = I ω 2 I = mr 2 is known as moment of inertia of the point mass about the origin. 4.4 Kinetic Energy The kinetic energy of point mass is 2 2 2 2 1 1 2 2 ? ? v T mv mr r 2 1 2 ? I? In linear motion the linear momentum and kinetic energy are given by p = mv and 1 2 2 mv T ? whereas corresponding physical quantities in rotational motion are angular momentum L = Iω and kinetic energy 1 2 2 ?T I ? . These similarity of expressions may be used to give a meaning to the new physical quantity moment of inertia.

NSOU ? GE-PH-11 ? 45

67%	**MATCHING BLOCK 32/68**	**SA**	Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

Moment of inertia plays the same role in rotational motion as played by mass in linear motion.

Mass is the inertia in linear motion and moment of inertia is the inertia in rotational motion. The moment of inertia of extended bodies with different shapes having different axes of rotation are of inmense importance in Physics and Engineering and can be calculated with the use of simple mathematics. A few are given below. 1. Thin rod of length 'L' and mass 'm'. Axis perpendicular to the rod and through the centre. 2 1 12 I mL ? 2. Thin rod of length L and mass 'm'. Axis perpendicular to the rod and through and end. 2 1 3 I mL ? 3. Thin solid disc of radius 'r' and mass 'm'. Axis ⊥ 2 to the disc through the centre and any axis in the plane of disc and through the centre (Through any diameter of the disc. 2 1 4 x y I I m ? ? ? 2 1 2 z I m? ?

46 ? NSOU ? GE-PH-11 4. Solid cylinder of mass m and raius 'r' about the axis of the cylinder. 2 1 2 I m? ? 5. Solid sphere of mass 'm' and radius 'r', about any diameter. 2 2 5 ?I mr 6. Thin rectangular plate or rod. Axis ⊥ r to the plane through the centre. ? ? 2 2 1 . 12 I m a b ? ? Torque and conservation of angular momentum. We write Newton's Second law of motion as, ? ? ? ? F p (1) Taking the Cross product by ? r on both sides of equation (1) from left are have ? ? ? ? ? ? ? ? r F r p (2) The angular momentum is defined as L r p ? ? ? ? ? The time rate of change of angular momentum is L r p r p ? r p r mr = 0 ? ? ? ? ? ? r p The lefthand side of eqn (2) is the moment of force also known as torque (N ?)

NSOU ? GE-PH-11 ? 47 and right hand side is equal to the rate of change of angular momentum. We write eqn. (2) as N L ? ? ? ? in words

57%	**MATCHING BLOCK 33/68**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

the rate of change of angular momentum is equal to the torque applied on the point mass. If the applied torque in zero the angular momentum

will remain conserved. In case of a system of (n) particles the total angular momentum is 1 n i i L L ? ? ? ? ? i L ? is the angular momentum of the i th particle. 1 n i i i r p ? ? ? ? ? ? The time rate of change of total angular momentum is 1 1 n n i i i i i i L r p r p ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? i i r p ? ? ? ? 1 n e i i ji j r F F ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 0for each i ? or 1 1 n n n e i i j ji i i j i L r F r F ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? (3) The last term of eqn (3) can be considered as sum of pairs ? ? i ji j ij i j ji r F r F r r F ? ? ? ? ? ? ? ? ? ? ? ? ? Since the mutual force

95%	**MATCHING BLOCK 34/68**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

between i th and j th particle is along the line joining

them ? ? || i j ji r r F ? ? ? ? hence each pain on the right hand side of eqn (3) is zero and equation (3) looks like i i i r m r ? ? ? ? ? ?
48 ? NSOU ? GE-PH-11 1 n e i i i L r F ? ? ? ? ? ? ? ? e e i i i r F N ? ? ? ? ? is the external torque on the ith particle 1 n e i i N ? ? ? ? or (4); e e e i L N N N ? ? ? ? ? ? ? ? total external torque on the system of particles. In case 0, e N L ? ? ? ? const. Equation (4) states that the time

61%	**MATCHING BLOCK 35/68**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

rate of change of angular momentum of a system of particles is equal to the total external torque on the system.

In absence of external torque the angular momentum of the system will remain conserved. 4.5 Solved problem 1) If the earth had its radus suddenly decreased by half when spinning about its axis, what would be the length of the day ? If L 1 and L 2 are the angular momentum of earth before and after decrease of the radius and I 1 and I 2 and ω 1 and ω 2 are respectively the moment of inertia and angular velocity in two cases ; from the conservation of angular momentum. L 1 = L 2 or I 1 ω 1 = I 2 ω 2 or 2 2 1 2 1 2 2 2 2 2 ; 5 5 MR MR T T ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? or 2 2 1 2 1 2 R R T T ? or 2 2 2 1 1 R T T R ? ? ? ? ? ? ? M = mass of earth R 1 and R 2 and radii and T 1 & T 2 are the period of revolution before & after. NSOU ? GE-PH-11 ? 49 = 2 1 24 2 ? ? ? ? ? ? ? = 6 hrs. 4.6 Exercises (1) A ballet dancer spins about a vertical axis at 90 rpm with her arms outstretched. With her arms folded, the moment of inertia about the axis of rotation changes to 75%. Calculate the new rate of rotation. (2) The mass and radius of earth are respectively 5.972 × 10 24 kg and 6.378 × 10 6 m. Compute the angular momentum of the earth. (3)

76%	**MATCHING BLOCK 36/68**	**SA**	Physics_Vol-1 EM.pdf (D40552326)

A disc of mass M and radius R is rotating about it axis

with angular velocity ω, If a bob of clay of mass m is dropped on the rotating disc at distance 2 R from centre, calculate the resulting angular velocity.
50 ? NSOU ? GE-PH-11 Unit 5 ????? Gravitation Structure 5.1 Objectives 5.2 Introduction 5.3 Application of Artificial Satellites 5.4 Global Positioning System (GPS) 5.5 Astronaut's Health Hazards 5.6 Solved Problems 5.7 Exercises 5.1 Objectives You will learn basic of Gravition & it application. Also you will gather knowledge abotu artificial satellite, GPS system etc. 5.2 Introduction In the universe any two bodies attract each other with a force depending on their masses and distance between them. Newton discovered a law known as Newton's law of gravitation which may be stated as a point particle of mass M attracts another point particle of mass m at a distance r ? from it

92%	**MATCHING BLOCK 37/68**	**SA**	PHY17R121-Mechanics and Properties of Matter_A ... (D110170498)

with a force proportional to the product of the masses and inversely proportional to the square of the distance between them

and the
force is directed along the line joining
them. Mathematically. 2 3 ( ) GMm r k F r r r r r ? ? ? ? ? ? ? ? ? ( ) F r ? ? is the force by M on m, r ? is the radius vector of m with rspect to the point mass M. The -ve sign ensures the force is attractive. k = GMm is a constant. G is a constant known as universal gravitational constant, universal in nature and is equal to 6.674 × 10 −11 Nm +2 kg −2 . Such force which is a function of radius vector and directed towards a fixed point is known as central force.

NSOU ? GE-PH-11 ? 51 The motion of planets about the star or those of satellites about the planets are examples of motion under central force, and the force between them are central forces. Central motion is confined in a plane Let L ? is angular momentum of body under central motion. It is given by L r p ? ? ? ? ? L r p r p ? ? ? ? ? ? ? ? ? ? ? ? ? ; Since p m r ? ? ? ? 0 r p ? ? ? ? ? or ( ) L r F r ? ? ? ? ? ? ? ? Since central for ( ) F r ? ? is parallel or antiparallel to 0 r r F ? ? ? ? ? . ∴ In central motion 0 L ? ? ? i.e. the angular momentum vector is constant. It is constant in magnitude as well in direction in space. From definition of angular momentum L r p ? ? ? ? ? the angular momentum vector is always ⊥ r to the plame containing r ? and i.e ? ? ? p r and since direction of L ? is fixed r ? and p ? are always in a constant plane i.e. the motion is confined in a plane. Let the body under central force has a mass m and instantaneous velocity v ? . In time Δt it moves from A to B. The radius vector r ? sweeps an area AOB in time Δt. The area AOB 1 2 r s? ? or 1 2 A r r ? ?? ? ? The area swept by the radius vector per second is known as areal velocity and is given by 1 2 2 2 0 0 1 2 ? ? ? ? t t t r A Lt Lt r Lt t t t ? ? ? ?? ? ?? ? ? ?
52 ? NSOU ? GE-PH-11 2 0 1 , 2 ? ? ? ? ? ? ? t r Lt t ? ?? ? ? ? ? angular velocity 2 1 2 v r r ? 1 2 2 mrv rv m ? ? , 2 L m ? L = mrv = magnitude of the angular momentum = Constant. This is a contant vector since the motion is confined in a plane. Kepler's Laws 1) A

80%	**MATCHING BLOCK 38/68**	**SA**	BSc_Physics_1st sem_Block B.pdf (D129738380)

planet moves in an elliptial orbit with sun at one of its foci. 2)

A line segment joining a planet and the sun sweeps equal ara during equal interval

87%	**MATCHING BLOCK 39/68**	**SA**	BSc_Physics_1st sem_Block B.pdf (D129738380)

of time. 3) The square of time period of revolution of a planet about the sun is proportional to the cube of the semi major axis of the elliptical orbit.

Artificial Satellites Earth has a natural satellite, which move in an elliptic orbit. Man has placed artificial satellites in circular orbits around earth. If

57%	**MATCHING BLOCK 40/68**	**SA**	BSc_Physics_1st sem_Block B.pdf (D129738380)

a satellite of mass 'm' orbits round the earth (mass M) in a circular orbit of radius r (the

distance of the Satellite from earth's centre) with velocity v, the centrifugal force on the satellite is 2 mv r . The gravitational force of earth on the satellite is 2 GmM r . The satellite will remain in the orbit if 2 2 ; GmM mv GM v r r r ? ? Geostationary Satellite A circular orbit in earth's equitorial plane of radius 4216 km and earth's centre as the centre is known as geostationary orbit. An object, an artificial satellite, in such orbit, known
NSOU ? GE-PH-11 ? 53 as geostationary satellite has an orbital period equal earth's rotation period of one sidereal day (23 hrs 56 min 4s). Such satellite looks motionless with respect to an observer on the surface of earth. Geosyrchronous Satellite Geosynchronous satellite is a satellite in geosynchronus orbit, with an orbital period same as earth's rotation priod (23 hrs 56 min 4 sec). Such a Satellite returns to the same position in the sky after each Sidereal day. 5.3 Application of Artificial Satellites 1. Weather forecasting : Various climate factors such as air pressure and temperature, humidity are monitored by using special cameras and instruments stationed in the artificial satellites. The records obtained are of immense importance for forcasting present and future weather. 2. Communication : Geostationary satellites are used for communication purpose such as long distance telephone, telex, radio, TV etc. 3. Spying – Satellites are used to keep an eye the enemy troops, their position, movement etc. 4. Study of the outer space. 5. Information about natural resources on earth such as underground water, minerals, oil wells, natural gas, coal deposits etc. Artificial satellites are therefore of immense importance in research, defence, remote sensing, movement of fish in oceans, global positioning and navigation. 5.4

91%	**MATCHING BLOCK 41/68**	**SA** Unit II Notes.docx (D113412638)
Global Positioning System (GPS) Global positioning system is a Satellite navigation system		

that allows land sea and airbornes users to determine their location, velocity and time 24 hrs a day in all weather condition, anywhere in the world. It is a system of 31 satellites designed to help navigation. To GPS receivers are included in many commercial product such as automobiles, smart phones etc.
54 ? NSOU ? GE-PH-11 5.5 Astronaut's Health hazards Human are well adopted to the physical conditions at the surface of earth. Journey into the space brings several changes in their body and mind. Some are temporary and some other are of critical concern. Common problems felt by astronauts in the initial condition of weightlessness is known as Space Adaptation Syndrone. It includes nausea, vomiting, vertigo, headache, lethergy etc. Long term weightlessness causes muscle atrophy, detercoration of skeleton known as space flight osteopenia, cardio vascular system functions are slowed down, decrease in the production of red blood cell, balance disorder of immune system is effected, loss of bodymass, nasal congestion, sleeplessness, flautulence, change of position and structure of brains, damage of gastrointestinal tissues and aging effect etc. 5.7 Solved Problem (1) A 1500 kg satellite orbits earth at an altitude of 2.5 × 10 6 m. (i) What is the orbital speed of the satellite ? (ii) What is the period of rotation ? (iii) What is the kinetic energy of rotation ? We know 2 2 GMm mv R R ? or 11 24 2 6 6 6.67 10 5.96 10 6.4 10 2.5 10 GM v R ? ? ? ? ? ? ? ? ? ? 13 7 6 6.67 5.96 10 4.466 10 8.9 10 ? ? ? ? ? ? 3 1 44.66 10 6682 v ms ? ? ? ? 2 10 2 1 5274 4.32 10 2 ? ? ? ? ? ? R T S KE mv J T ?
NSOU ? GE-PH-11 ? 55 (2) Two bodies of masses 5 kg and 6 × 10 24 kg are placed with their centres 6.4 × 10 6 m apart. Calculate the force of attraction between two masses and initial accelerations. Solution : Force ? ? 24 11 2 2 6 5 6 10 6.67 10 6.4 10 GmM F R ? ? ? ? ? ? ? ? = 48.85 N Acceleration of 5 kg mass 48.85 9.77 5 ? ? ms −2 Acceleration of 6 × 10 24 kg mass 24 48.85 8.142 10 6 24 ? ? ? ? ? ms −2 (3) An object, dropped on the surface of a planet falls 27 m in 3 s. The radius of the planet is 8 × 10 6 m. What is the mass of the planet ? Solution : The acceleration of the falling body is 2 2 2 27 6 3 3 h g t ? ? ? ? ? ms −1 Now from Newton's law of gravitation 2 GmM mg R ? ∴ 2 6 6 11 6 8 10 8 10 6.674 10 gR M G ? ? ? ? ? ? ? ? = 5.75 × 10 24 kg. (4) What is the velocity of a low altitude satellite of earth ?
56 ? NSOU ? GE-PH-11 Solution : From Law of gravitational force 2 2 (0) (0) mv GmM R R ? ? ? 11 24 2 6 6.67 10 5.97 10 6.378 10 Gm v R ? ? ? ? ? ? ? = 62.21 × 10 6

80%	**MATCHING BLOCK 42/68**	**SA** Mechanics - Block I -5 SLM.pdf (D116041085)
m = mass of the Satellite, M = Mass of earth R = Radius of earth		

G = Gravitational Constant. 3 62.21 10 7887 v ? ? ? ms −1 5.7 Exercises (1) A 1000 kg Satellite is in synchronous orbit around earth. Calculate the i) orbital radius, ii) altitude of the satellite and iii) Kinetic energy of the satellite. (2) A sphere of mass 100 kg is attracted by another sphere of mass 11.75 kg by a force of 19.6 × 10 −7 N, when the distance between them is 0.2 m. Find the value of G. (3) (i) If suddenly the gravitational attraction between earth and its satellite becomes zero what will happen to the satellite ? (ii) Why is the weight of a body at the pole is more than that at the equater ? (iii) The time period of a satellite is 5 hrs. If the separation between the earth and the satellite is increased to four times what will be the time period of the satellite ? (4) The gravitational protential energy of a 500 kg satellite around a planet of mass 4.2 × 10 23 is – 4.8 × 10 9 J. Calculate the (i) orbital velocity of the satellite (ii) the kinetic energy and (iii) total energy of the satellite. [Ans. i) 3.098 × 10 3 m, (ii) 2.4 × 10 9 J (iii) – 2.4 × 10 9 J]
Unit 6 ????? Fluids : Surface Tension Structure 6.1 Objectives 6.2 Introductions 6.3 Surface Energy 6.4 Angle of contact 6.5 Jaeger's method 6.6 Solved problems-I 6.7 Viscosity 6.8 Poiseulle's law 6.9 Lubrication 6.10 Reynold's number 6.11 Solved Problem - II 6.12 Exercises 6.1 Objectives A knowledge of fluid dynamis will be gatherd by you thogh this unit. The basics of surface tention & viscuity in discussed here. 6.2 Introductions The free surface of a liquid behaves like a streched membrane and tries to minimise its surface area. The property of liquid surface by virtue of which it tries to minimise its surface area may be taken as qualitative definition of surface tension. A quantitative definition of surface tension can be obtained as follows. We imagine a line on the surface of a liquid. Both sides of line is pulled by a force due to contractile property of the liquid surface. The force of pull perpendicular to the line per unit length tangential to the surface is the measure of the surface tension. The unit of surface tension is Nm −1 .

58 ? NSOU ? GE-PH-11 Molecular Theory of Surface Tension Let us consider some liquid in a container. The medium above the liquid is air say. A is a liquid molecule well inside the liquid. Liquid molecules symmetrically distributed about A pull A by short range molecular force. This force between molecules of same liquid is known as cohesive force. The net force on A is zero. B is a molecules on the liquid surface. From below B is pulled by liquid molecules and from above by air molecular. Molecules force between liquid molecules & air molecules is known as adhesive force. In this case cohesive force is stronger than the adhesive force. As a result the molecule B experiences a net downward force. Thus all molecules on the surface are pulled downwards. This unbalanced force results in a potential energy in the surface. Nature wants to minimise this potential energy by minimising the surface area and therefore the surface behaves as a stretched membrane. 6.3 Surface Energy Due to unbalanced force on the surface molecules of a liquid the surface becomes a sheet of potential energy. If we take a soap film in a loop and pierce it with a pin we will see the liquids of the film to be blown around. This is a direct evidence of existence of surface enery. To estimate the amount of energy on the liquid surface we take a rectangular liquid film formed in a rectangular frame ABCD. Whose CD can be moved. A force F = 2 × CD × T must be applied on CD to held the film. (The film has two surface and T is surface tension.) If the side CD is pulled further by dx the work by F is Fdx = 2. CD. Tdx. This work is stored in film as potential energy of the inereased area. If E stands for potential energy per unit area, the increase in surface energy will be 2E.CD.dx. Thus
NSOU ? GE-PH-11 ? 59 2E.CD.dx = 2F.CD.dx ∴ E = T The process is taken to be adiabatic. The surface energy density is equal numerically to the surface tension having unit Jm −2 . Excess pressure

95% **MATCHING BLOCK 43/68** **SA** PHY17R121-Mechanics and Properties of Matter_A ... (D110170498)

Due to the property of surface tension a drop or bubble tries to contract and so compresses the matter enclosed. This in term increases the internal pressure which prevents further contraction and equilibrium is achieved. So in equilibrium pressure inside a bubble or

a

54% **MATCHING BLOCK 44/68** **SA** PHY17R121-Mechanics and Properties of Matter_A ... (D110170498)

drop is greater than that outside and the difference of pressure is called excess pressure. Expression of excess pressure ABCD is

an elementary portion of a curved liquid surface. AB = CD = R 1 θ 1 and BC = AD = R 2 θ 2 where R 1 and R 2 are respective radii of curvatures and θ 1 and θ 2 are the angles in radian subtended by AB and BC at the centre of their curvatures. Component of surface tension at A perpendicular to AB is T sin 2 2 ? and that at B perpendicular to BC is T sin 1 2 ? . The net inward force on AB is 2 2 1 1 2 sin 2 2 2 T AB T R ? ? ? ? ? ? [2 ? is small] 1 1 2 TR ? ? ? The net inward force on BC is 2 1 2 TR ? ? The excess force 1 1 2 2 1 2 TR TR ? ? ? ? ? ? SOLID T sin 2 2 ?
60 ? NSOU ? GE-PH-11 ? ? 1 2 1 2 T R R ? ? ? ? If P 1 and P 2 are the pressures inside and outside the curved surface ? ? ? ? 1 2 1 2 1 2 . P P AB BC T R R ? ? ? ? ? or, the excess pressure ? ? 1 2 1 2 1 2 1 1 2 2 T R R P P P R R ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 1 2 1 2 1 1 T R R T R R R R ? ? ? ? ? ? ? ? ? ? ? For spherical drop R 1 = R 2 = R say 2T P R ? ? For sphenical bubble having two liquid surfaces 4T P R ? ? For cylindrical drop 1 2 , R R R ? ? ? T P R ? ? For cylindrical bubble 2T P R ? ? 6.4 Angle of contact

85% **MATCHING BLOCK 45/68** **SA** PHY17R121-Mechanics and Properties of Matter_A ... (D110170498)

The angle conventionally measured through the liquid where liquid - vapour interface meets solid surface. It quantifies the wettability of a solid surface by a liquid. A given system of solid, liquid and vapour at a given temperature and pressure has a unique contact angle.

NSOU ? GE-PH-11 ? 61 A liquid drop on a solid surface in a gaseous medium is shown in the figure. T lg is the surface tension of the liquid in the gas medium acts tangentially on the liquid surface at the meeting point of solid liquid and gas. T sg and T sl are similar terms like surface tension for solid – gas and solid liquid pain. T sg , better to be called solid surface free energy and T sl solid-liquid interfacial free energy act as shown in the figure. The young -Laplace equation for equilibrium is lg cos ? ? sg sl T T T ? or lg cos ? ? sg sl T T T ? If T sg < T sl ; θ is acute, the liquid is said to wet the solid surface as in the case of water on glass surface. In case T sg > T sl ; θ is obtuse liquid does not wet the surface as in the case of mercury on glass surface. Capillary rise A Capillary tube is vertically immersed in a liquid in a container. If the liquid wets the surface, liquid will rise in the tube. The free surface within the tube is spherical. The liquid touches tube along a circle of radius r equal to the radius of the capillary tube. At the point of contact the surface tension T will act making an angle θ with the tube surface down-wards, where θ is the angle of contact. The reaction of the surface tension has a vertically upward component T cos θ, everywhere on the circle of contact. Hence the liquid column in the capillary tube will be pulled up by a force cos 2 T r ? ? ? . At equilibrium this force will be
62 ? NSOU ? GE-PH-11 equal to the weight of the liquid column of height h. The net volume of liquid column is 2 3 2 1 4 ( ) 2 3 3 r r h r r r h ? ? ? ? ? ? ? ? ? ? ? ? ? ? . If d is the density of the liquid 2 cos 2 3 r T r r h dg ? ? ? ? ? ? ? ? ? ? ? ? or 1 cos 2 3 r T h rdg ? ? ? ? ? ? ? ? ? or 2 cos . T h dgr ? ? (1) Since r >> h or 2 . T h dgr ? (2) for small θ . from equation (1) or (2) for a particular liquid tube pair hr = constant This is known as Jurin's law. By measuring the height of the liquid column and radius of the capillary tube surface tension of liquid may estimated using equation (2). 6.5 Jaeger's method In Jaeger's method

67%	MATCHING BLOCK 46/68	SA	PHY17R121-Mechanics and Properties of Matter_A ... (D110170498)

surface tension of a liquid is determined by measuring the pressure required to cause air to flow from a capillary tube immersed in the liquid. One end of a

capillary tube is vertically immersed in the experimental liquid kept in a container C. From other end air is pushed into the tube. The pressure inside is measured with the help of a momometer M. Bubbles are formed at the tip of the capillary tube. It is assumed that the radius of bubble formed is equal to the radius of the capillary tube.
NSOU ? GE-PH-11 ? 63 The excess pressure inside the bubble is 2T

68%	MATCHING BLOCK 47/68	SA	Mechanics Properties of Matter-PHY17R121.docx (D109220287)

r , where T is the surface tension of the liquid to be measured and r is the radius of

the capillary tube, which can be measured using a travelling microscope. If h 2 be maximum difference in liquid heights in the manometer, then the air pressure inside the bubble when it is just on the point of being detached from the orifice is h 2 ρ 2 g + p. Where p is the atmospheric pressure and ρ 2 is the density of manomatric liquid. The pressure outside the bubble is h 1 ρ 1 g + p, where ρ 1 is the density of the experimental liquid. Thus the pressure difference is ? ? ? ? 2 2 1 1 2T P h g P h g r ? ? ? ? ? ? or ? ? 2 2 1 1 2 rg T h h ? ? ? ? Jaeger's method is very useful for the study of variation of surface tension with temperature, comparison of surface tension of different liquids, variation of surface tension of solution with concentration of solute, determination of surface tension of molten metals etc. Synclastic Surface : The curved surfaces having centres of curvatures on the same side of the surface such as dome shaped surface. Anticlastic Surface : The curved surfaces having centres of curvatures on the opposite sides of the surface such as saddle shaped surface.

64 ? NSOU ? GE-PH-11 Variation of surface tension with temperature : – For small ranges of temperatures the surface tension varies linearly with temperature, as T t = T o (1 – at) (T t and T 0 are surface tension at temperature t and at 0 o c, a is known as temperature coefficient of surface tension) However the most comprehensive relation given by Eotuos may be written as ? ? 2 3 c TV k ? ? ? ?; V is the molar volume c ? = Critical temperature where T = 0 ? = Temperature in Kelvin. For water surface tension varies from 58.8 dyn cm –1 to 75.6 dyn cm –1 for temperature between 100 o c to 0 o c. Surface tension & condensation of vapour : We consider a narrow verticle glass tube, dipped into a liquid which does not wet the glass. The entire assembly is enclosed in air tight system as shown in the figure. If P is the saturated vapom pressure on the plane surface that on the convex surface is P h g ? ? . where ? = density of saturated vapour and h is the depression liquid column in the tube. The pressure just inside the meniscus is P h g ? ? , where ? is the density of the liquid. Thus the excess pressure is ? ? ? ? 2T P h g P h g r ? ? ? ? ? ? or ? ? 2 0 T hg r ? ? ? ? ? Thus saturation vapour pressure on convex surface is greater than that on the plane surface. Similarly it can be shown that saturation vapour pressure on concave surface is less than that on plane surface. If we place a drop of water in a space where vapour pressure is at saturation value for a plane surface, the drop will begin to evaporate for the vapour pressure is less than the saturation vapour pressure for the drop and it will therefore

NSOU ? GE-PH-11 ? 65 be converted into vapour in order to increase the vapour pressure to its own saturation value. This will result in further decrease in radius of the drop and consequent rise in the saturation value of its own vapour pressure and it will therefore evaporate more and more rapidly. That is why saturated vepour does condense into drop, for as soon as a tiny drop is formed it begins to evaporate. Thus condensation may not take place even when the vapour becomes super saturated. If however dust particles or charged ions be introduced into the saturted vapour they offer flatter surface to it and condensation starts. Phenomena associated with surface tension : 1) An immersed glass rod is taken out of the water, drops of water are seen sticking at the end of the rod. 2) Spider walks on the surface of water without rapturing it. 3) Blade or neddle left gently on water surface remains floating. 4) Paint brushes when taken out of paint the bristles are drawn closer. 5) Rain drops are spherical. 6) Irregular shaped camphor piece dances on water surface. Water surface in contact with camphor has lower surface tension, camphor moves to other side of higher surface tension. 7) Hot water having less surface tension is better than cold water for washing purpose 8) Detergent works on the principle of lower surface tension of solution. Problem : One thousand drops of water each of diameter 0.2 mm combine to form a single drop. Calculate the loss of energy. T = 72 dyn. cm –2 Volume of 1000 drops = volume of single combined drop of radius r say ∴ 3 3 4 4 (0.01) 1000 3 3 r ? ? ? ? r 3 = (0.01) 3 × (10) 3 , ∴ r = 0.01 × 10 = 0.1 cm. Total energy of 1000 small drops. = 1000 × 4π (0.01) 2 × 72

66 ? NSOU ? GE-PH-11 Total energy of the large drop is = 4 π (0.1) 2 × 72 Loss of energy = ? ? 2 2 4 72 1000 (0.01) (0.1) ? ? ? ? erg ? ? 4 0.1 0.01 72 82.79 ?? ? ? ? erg 6.6 Solved problems - I (1) n droplets of equal size

87%	**MATCHING BLOCK 48/68**	**SA**	PHY17R121-Mechanics and Properties of Matter_A ... (D110170498)
of radius ‘r’ coalesce to form a bigger drop of radius ‘R’.			

If T is the surface tension of the liquid, then show that the energy liberated is 2 2 3 4 r T n n ? ? ? ? ? ? ? ? . Solution The volume remains constant, therefore 3 3 4 4 ; 3 3

50%	**MATCHING BLOCK 49/68**	**SA**	Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)
n r r ? ? ? ? R = 2 3 r n R 2 = 2 2 3 r n decrease in surface area 2 2 2 2 3 4 4 4 n r R r n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∴ the			

energy liberated 2 2 3 4 r T n n ? ? ? ? ? ? ? ? ? (2) Surface tension of water of 0 o c is 70 dyn cm –1 . Find the surface tension of water at 25 0 c . [α = 0.027] Solution 0 (1) T T t ? ? ? = 70 (1 – 0.0027 × 25) = 65.28 dyn cm –1 (3) A glass tube of internal diameter 3.5 cm and thickness 0.5 cm held vertically with

NSOU ? GE-PH-11 ? 67 its lower end immensed in water. Find the downward pull on the tube due to surface tension. (Surface tension of water = 0.074 N/m) Solution : Internal diamen = 3.5 cm ; internal radius = 1.75 cm External radius = 1.75 + 0.5 = 2.25 cm Length along which the water and glass surface meet = 2π (1.75 + 2.25) × 10 −2 m = 8π × 10 −2 m ∴ Downward pull = 8π × 10 −2 × 0.074 N. = 0.0186 N (4) A niddle 10 cm long can just rest on the surface of water. Find the weight of the needle. (S.T. = 0.07 Nm −1) Solution : Water wets the heedle from two sides along the length, hence effective length is 10 × 2 = 20 cm. The force of surface tension on the needle is equal to the weight of the needle. Hence the weight of the needle 3 1 20 0.07 14 10 0.014 100 N ? ? ? ? ? ? ? ? N. (5) A reetangular plate 16 cm × 10 cm rests with its face on surface of water of surface tension 0.07 Nm −1 . Calculate the force required to pull the plate out of water. Solution : The perimeter 1 2 (16 10) 100 ? ? ? m = 52 × 10 −2 m ∴ The required force 16 16 10 10
68 ? NSOU ? GE-PH-11 = 52 × 10 −2 × 0.07 = 364 × 10 −4 N = 0.0364 N 6.7 Viscosity Viscosity is the property by virtue of which there is a resisting force against relative motion between two layes of a fluid. This property is reciprocal to fluidity and may be taken as the qualitative definition of viscosity. Quantitatively it is the force per unit area between two layers of fluid in contact per unit velocity gradient. Mathematically we write F A? F = resisting force dv dx ? A = area of layer in contact. or dv F A dx ? dv dx = velocity gradient or dv F A dx ?? ? = coefficient of viscosity 1 1 dv FA dx ? ? ? ? ? ? ? ? ? ? The unit of coefficient of viscosity is Nm −2 s. This is also known as Pascal second or Pas. Though Pas is the S1 unit of coefficient of viscosity it is not very popular in science and technology. Instead dynes per square centemeter dyns cm −2 , known as poise [P] is in use. 1 Pa s = 10 P 6.8 Poiseulle's law Poseuille's law is a physical law that gives the pressure drop in an incompressible and Newtonian fluid in Laminar fluid through a long cylindrical tube of constant crossction. Laminar flow in the cylindrical tube prescribes that bunch of circular of layers fluid each having velocity determined by their radial distance from the axis of the tube – centre moving fastest while liquid in contact of the tube wall is stationary. NSOU ? GE-PH-11 ? 69 Let

100%	**MATCHING BLOCK 50/68**	**SA**	Mechanics Properties of Matter-PHY17R121.docx (D109220287)
L and R are the length and radius of the capillary tube			

through which the fluid is passing. A concentric solid cylinder of the fluid of radius r is considered. If ΔP is the pressure difference at the two ends of the tube the force on the liquid cylinder pushing the liquid out is 2 P r ? ? . The viscous force against the motion of the cylindrical fluid is 2 dv rL dr ? ? , where dv dr is the velocity gradient. For no acceleration 2 2 dv rL P r dr ? ? ? ? ? or . 2 P dv rdr L ? ? ? (1) Value of fluid velocity at axis is maximum, at radius r is v and at radius R is zero. Integration equation (1) we get 0 2 v r R P dv r dr L ? ? ? ? ? ? or ? ? 2 2 ( ) 4 P v r R r L ? ? ? ? (2) Next we consider a cylindrical cell of the fluid between radii r and r + dr coming out of the capillary tube with fluid of volume dQ per see is given by ? ? 2 2 2 2 . 4 P dQ r dr v r R r dr L ? ? ? ? ? ? ? ? or ? ? 2 2 2 ? ? P dQ R r rdr L ?? ? Total fluid coming out of tube per sec is ? ? 2 2 0 . 2 R P Q R r rdr L ?? ? ? ? ? ? r
70 ? NSOU ? GE-PH-11 4 4 2 2 4 P R R L ?? ? ? ? ? ? ? ? ? ? ? ? ? or 4 8 P R Q L ?? ? ? (3) Poiscuilles method of 'experimental determination of viscosity of liquid'. In the diagram the experimental arrangement for the determination of viscosity of liquid is shown. AB is a long capillary tube. Experimental liquid from a constant level tank T enters the capillary tube from the end A. From the end B the outcoming fluid is collected in the container C. In the manometric arrangement two vertical tubes T 1 and T 2 in connection with the end A and B respectively show different heights in manometric liquid. The height difference is gives the pressure difference Δp at the two ends A and B. ΔP = hdg, d = density of manometric liquid The volume V of liquid collected in the container C in time t gives the rate of volume collected Q per unit time. ? V Q t .

NSOU ? GE-PH-11 ? 71 Using the Poiscuille's formula 4 8 ? P Q R L ? ? ? The coefficient of viscosity (?) is estimated from 4 8 ? PR QL ? ? ? R is measured by the help of travelling microscope. Variation of viscosity with temperature. In liquid due to strong cohesive forces between the molecules in any layer in moving liquid tries to drag the adjacent layer and produce the effect of viscosity. Increase in temperature breaks the bonding between the atoms and cohesive forces decreases. As a result the viscosity decreases. In gases where cohesion is less, the source of viscosity is the collision between the molecules. Higher the temperature more frequent the collision greater is the value of coefficient of viscosity. In these cases the cofficient of viscosity is proportional to square root of absolute temperature. 6.9 Lubrication Lubrication is the action of applying a substance such as oil or grease to an engine or component as to minimise friction and allow smooth movement. Lubricant is a substance introduced to reduce friction between surfaces in mutual contact. Lubricity, the property of reducing friction must have (1) High boiling point and low freezing point. (2) High viscosity index (3) Thermal Stability (4) Hydrolic stability (5) Ability of corrosion prevention and η temp → →
72 ? NSOU ? GE-PH-11 (6) High resistance to oxidation. If the lubicant is too thick it will require large amount of energy to move and if it is too thin the surfaces will come in contact and friction will increase. 6.10Reynold's number If fluid flows through a tube with small velocity the flow is steady. As velocity is gradually increased at one stage the flow becomes turbulent. The largest velocity that allows a steady flow is called critical velocity. Reynold showed that the critical velocity v c of a fluid in a tube is i) 1 c v ? ? , ? desnsity of the fluid iii) c v ?? , ? = coefficient of viscosity of the fluid. iii) 1 c v r ? , r = radius of the tube. Thus c v k r ? ? ? ; k = constant of proportionality known as Reynold's number. If k > 2000, the flow is steady. Example : The velocity of water in a river is 20 km h −1 near the surface. If the river is 10 m deep, find the shear between the horizontal layers of the water of the river ? = 10 −2 poise. We know F dv A dx ?? is the shear. In this case 3 20 10 0.56 60 60 10 dv dx ? ? ? ? ? s −1 2 4 10 0.1 .56 5.6 10 F A ? ? ? ? ? ? ? N m −2 6.11 Solved problems - II (i) A tank 100 cm × 100 cm × 100 cm is full of water. Water is coming out through a vertical tube of length 40 cm and radius 1 mm from the bottom of the tank. If the
NSOU ? GE-PH-11 ? 73 coefficient of viscosity of water be 0.01 poise how much time will be required to loose half the tank of water ? Solution : From the Poiseuille's law the rate of flow is ? ? 4 2 1 , 8 ? ? P P r Q l ? ? 100 100 100 100 ? ? ? ? dx Q dt , when the height of water is x in the tank. ? ? dx Q dt . The pressure difference P 2 − P 1 = xg 4 . 8 dx x g r dt l ? ? ? ? ? or 0.50 4 1.00 0 8 t dx g r dt x l ? ? ? ? ? ? or ? ? 4 3 9.8 3.14 10 1 ln 0.01 40 0.5 8 10 100 t ? ? ? ? ? ? or 12 4 9.8 3.14 10 0.693 . 8 4 10 t ? ? ? ? ? ? ? ? 8 7 8 4 0.693 10 7.2 10 . 9.8 3.14 t s ? ? ? ? ? ? ? (2) The flow rate of blood in a coronary artery of a man is reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced ? n x l
74 ? NSOU ? GE-PH-11 Solution From Poiseuille's law ? ? 4 2 1 8 ? ? P P r Q l ? ? We write 1 2 4 4 1 2 ? Q Q r r or 4 1 1 2 2 Q r Q r ? ? ? ? ? ? ? or 4 1 1 1 2 2 1/2 2 ? ? ? ? ? ? ? ? ? ? r Q Q r Q Q 1 1 4 2 (2) 1.189 r r ? ? 1 2 1 0.84 1.189 r r r ? ? Thus the decrease in the artery radius is 16%. (3) Two horizontal plates 250 mm apart have oil of viscosity 20 poise in between. Calculate the shear stress in oil if the upper plate is moved with velocity 1250 mm s −1 . Stress = viscosity × velocity gradient 3 3 20 1250 .0 10 250 10 ? ? ? ? ? ? = 19 NM −2 6.12 Exercises (1) A water film is formed between two straight parallel wires of length 10 cm each with a separation of 0.1 cm. If the distance between the wires is increased by 0.1 cm, how much work is to be done ? (S. T. of water = 0.072 N m −1) (2) The radii of the two vertical arms of a u-tube are r 1 and r 2 . When a liquid of density ρ and angle of contact zero is taken in this U-tube the difference of liquid in two arms is h. Show that the surface tension of the liquid is
NSOU ? GE-PH-11 ? 75 ? ? 1 2 2 1 2 h g r r r r ? ? . (3) Find the workdone in blowing

87%	**MATCHING BLOCK 51/68**	**SA**	BSc_Physics_1st sem_Block C.pdf (D129738406)
a soap bubble of radius 10 cm. The surface tension of soap solution			

is 30 dyn cm −1 . (4) A thin wire is bent in the from of a rectangle of dimension 5 cm × 4 cm. What force due to surface tension does the sides experience when soap film is formed in the frame ? (Surface tension of soap solution is 0.03 Nm −1) (5) Mercury has an angle of contact 120 o with glass. A narrow tube of radius 2 mm made of this glass is dipped in a trough containing mercury. Calculate the capillary descent. S. T. of mercury = 0.456 Nm −1 . Density of mercury = 13.6 × 10 3 Kgm −3 . (6) A plate 10 cm × 10 cm is pulled with a velocity 0.05 ms −1 in a liquid of viscosity 1.0 poise above a fixed plate at a distance 0.25 mm. Find the force to maintain the velocity. (7) A plate in water 0.05 mm above a fixed plate is moving with velocity 100 cm s −1 . It requires a force at 2 Nm −2 to maintain the speed. Determine the viscosity.

76 ? NSOU ? GE-PH-11 Unit 7 ????? Elasticity Structure 7.1 Objectives 7.2 Introductions 7.3 Poison Ratio 7.4 Load-Elongation / Stress-Stain Relation 7.5 Determination Young's modules by Searle's method 7.6 Work done in streaching a wire 7.7 Strain energy 7.8 Twist of a wire or a cylinder 7.9 Rigidity modules by static torsion 7.10 Elastic constants by Searles method 7.11 Rigidity modules by dynamic method 7.12 Moment of inertia by torsion balance 7.13 Exercises 7.1 Objectives The main objective is to give you an idea about Elasticity of solid material and its different application in daily life. 7.2 Introduction Elasticity is a property of matter by virtue of which it resists any external force trying to deform its shape and size (and the body regains its original shape and size when the external force is withdrawn) Iron is much more difficult to be stretched than rubber hence iron has greater elasticity than rubber. Strain – The fractional deformation of a body is known as strain.

NSOU ? GE-PH-11 ? 77 Longitudinal strain – suppose a rod of length L is linerly deformed by an amount ΔL (i.e. it is alongated to L + ΔL or contracted to L – ΔL) then the fractional deformation i.e. ? ? L L ? is the longitudinal strain. It is a dimensionless quantity. Volume strain – Suppose a body of volume V is deformed by ΔV the fractional volume change ? ? V V ? is known as the volume strain. Shearing Strain or Shear :– We consider a cube of each side equal to L. on the upperside a tangential force F is applied. Due to friction an equal and opposite force F will be applied on the lower surface tangentially. The shape of the cube will be deformed. Let the upper side moves by an amount ΔL along the force. The fractional deformation L L ? ?? is known as shearing strain or shear. Whenever a body is strained a resisting force is generated within the body to oppose the deformation. This resisting force generated measured per unit area of application is called stress. Corresponding to above three type of strains we have longitudinal stress, volume stress and shearing stess. The unit of stress is Nm –2 . Strain is the cause and stress is the result. Hookes Law Within elastic limit the stress is proportional to strain. i.e. Stress constant Strain ? This constant is in general called modules of elasticity. In case of longitudinal (tensile or contractile) Longitudinalstress Y Longitudinalstrain ? ; the young's modulus. In case of volume Volumestress K Volumestrain ? ; Bulk modulus

78 ? NSOU ? GE-PH-11 In case of Shear Shearing strain Shearing strain ?? ; rigidity modulus. A cube ABCD of each side L is given a shear θ as shown in the figure. The diagonal DB changes to DB / and the elongation is DB / – DB = OB / OB BB BB ? ? ? ? ? ? 1 . 2 ? ? ? BB BC BC 0 45 ? ? ? BB O 2 ? L ? Shear BB AA BC AD ? ? ? ? ? ? ∴ Longitudinal tensile strain along DB is 1 2 2 ? ? ? OB L DB L ? 1 2 ? ? BC DB 2 ? ? Similarly longitudinal contractile strain along AC is also 2 ? . AC and DB are perpendicular to each other. Thus a shear is equizalent to two mutually perpendicular strains one tensile and other contractile each of value half the strain and vice versa. 7.3 Poisson Ratio When ever a body is elongated due to some external force, there is a countraction in the perpendicular direction of the body and similarly counteraction in a direction causes elongation in perpendicular direction. Therefore longitudinal tensile strain is accompanied by

NSOU ? GE-PH-11 ? 79 lateral contractile strain and vice versa.

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The ratio of lateral strain to longitudinal strain is known as Poisson ratio.

In L and l are the longitudinal and lateral dimension of a body and ΔL and Δl are the changes in the dimensions, then Longitudinal strain = L L ? and Lateral strain = l l ? and Poisson ratio is L dl l L l L l dL ? ? ? ? ? Tensile deformation is considered + ve where as contractile deformation is considered – ve. In order that materials have + ve Poisson ratio the definition of Possion ratio contains a minus sign. 7.4 Load-Elongation / Stress-Stain Relation One end of a long vertical cylinder of the material under study is fixed at a point in the ceiling. A mass M is hung at the lower end of the cylinder. Slowly but steadily the cylinder elongates, then stops and becomes steady. The external downward force Mg increases the length of the cylinder. If 'L' and 'l' be the original length and increase in length of the cylinder, l/L is the longitudinal strain in the material. The strain produces longitudinal stress in the material of the cylinder. At the steady state the external deforming force per unit area of the cylinder i.e. Mg/A is the measure of the stress,

where A is the area of cross section of the

cylinder. Loads are gradually increased in small steps and each time the corresponding increase in length is noted. A graph of stress against strain is obtained, which gives an insight of the elastic propertics of the material of the cylinder. At the beginning the graph is a straight line (OA). The stress is proportional to the strain, showing the validity of Hooke's law. The

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slope of the graph gives the young's modulus of the material of the			

cylinder. After that the graph bends down a bit shown by the portion ABC. After A there is always a limiting value of load before which strain totally disappears on removal of the load. If AB is such a portion, B is known as elastic limit. After 80 ? NSOU ? GE-PH-11 B on removal of the load the body does not regain its original size. (Shown by B / O /), instead assumes a new set. After C the body becomes plastic, even when the stress is zero strain is not zero 'C' is called yield point. The point 'D' is the ultimate tensile strength of the material. Hence if any additional strain is produced beyond this point fracture can occur (E). Between the points D and E at a weak point the cylinder becomes narrow known as 'neck' and then the material ruptures. If 'D' and 'E' are close to each other the material is 'brillle' and if they are far apart the materials is 'ductile'. Relations between elastic constants : We consider a cube with sides parallel to co-ordinate axes. On the two opposite faces perpendicular to y-axis tensile force F is applied. If L is the length of the side and ΔL is increase of the length due to application of the force. The longitudinal stress along Y-axis is F/L 2 and the longitudinal strain is ΔL/L. The Young's modulus is 2 F L F Y L L L L ? ? ? ? F L YL ? ? (1) X- and Z- dimension will decrease by Δl, (say). Then lateral strain is Δl/L and the poisson ratio is l L l L L L ? ? ? ? ? ? ? (2) or F l L Y L ? ? ? ? ? ? (3) If (F – F) forces are simultaneously applied along X–, Y– and Z– directions, each side will increase by (ΔL – 2Δl). The volume V = L 3 of the cube will increase by ? ? 3 3 2 V L L l L ? ? ? ? ? ? ? or 3 3 2 1 1 V L L L L ? ? ? ? ? ? ? ? ? ? ? ? ? X o Y Z ↑ →
NSOU ? GE-PH-11 ? 81 or 2 1 3 ... ...–1 V L L V L ? ? ? ? ? ? ? ? ? ? 3 2 L L L ? ? ? ? ? Volume Strain According to definition, the bulk modulus is ? ? 2 3 2 2 3 1 F L F K l L l L L L L ? ? ? ? ? ? ? ? ? ? ? ? ? ? . 3 (1 2) F YL L F ? ? ? from (1) and (2) 3(1 2) Y ? ? ? or 3 (1 2) Y K ? ? ? (4) Application of (F – F) force along Y-direction causes Y direction elongation L F YL ? ? and X– and Z – direction contraction given by F L YL ? ? ? as shown above. Application of (F – F) contractile force along X – direction cause contraction L F YL ? ? along X and elongation by along Y and Z direction given by F l YL ? ? ? . If the tensile force (F – F) along Y and contractile force (F – F) along X are applied simultaneously X dimension will decrease by (ΔL + Δl) and the strain along this direction is 2 (1) L l F YL F YL F L L YL ? ? ? ? ? ? ? ? ? This strain is contractile. The Y-direction will increase by same amount and the tensile strain along Y- is 2 (1) F YL ?? .
82 ? NSOU ? GE-PH-11 We know a pair of equal longitudinal strains one tensile and other contractile along mutually perpendicular direction is equivalent to a shear of value twice the either longitudinal strain. Thus in our case the shear is given by 2 2 (1) F YL ? ? ? ? and rigidity modulus ? is given by 2 2 ShearingStress ShemingStrain 2 (1) F L F YL ? ? ? ? ? or 2 (1) Y ? ? ? ? (5) Equation (4) and (5) can be solved for other two relations as 2 (1) 3(1 2) K ? ? ? ? ? ? (6) and 9 3 K Y K ? ? ? ? (7) from relations (5) and (6) the Poisson ratio may be expressed in terms of other elastic constants as 1 2 Y ? ? ? ? (8) an 3 2 6 2 K K ? ? ? ? ? ? (9) Limiting values of Poisson ratio. Relation (4) suggests that .5 ? ? and relation (5) suggests that –1 ? ? . Thus theoretically – 1 > σ > .5. But the material to have only + ve value of σ we write 0 > σ > .5. 7.5 Determination Young's modules by Searle's method Two wires, a control wire AB and a test wise CD. Of equal length are attached to a rigid support at the upper ends. The lower ends of the wires are attached to a horizontal bar supporting a spirit level. The bar is hinged to the control wire so that when the test wire
NSOU ? GE-PH-11 ? 83 is extended on increasing weight, the spirit level gets tilted by small amount. By turning the micrometer attached to the test wire the spirit level is brought back to its original position. By measuring the amount of turning of the micrometer the increase in length of the test wire due to increase of the weight is obtained. The weight on the test wire side is increased in steps and each time the extension 'l' is measured by adjusting the micrometer. A graph of load against extension is drown from the reading. The youngs modulus is given by Stress Strain Y ? or F A Y l L ? 2 FL r l ? ? 2 . gL m l r? ? or 2 1 tan gL Y r ? ? ? L is measured with the help of a meter scale. r is measured with a screw gauge. Measuring the slope of the curve young's modulus is obtained from the above expression. 7.6

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Work done in streaching a wire We consider a vertical wire of length L and			

area of cross section A fixed at upper end. A downward force F is applied at the lower end. The wire is elongated by l (say). The θ Extn l F = mg, the force A = π r 2 = area of crossection of the wire. r = radius of the wire L = length of the experiment wire.
84 ? NSOU ? GE-PH-11 process of elongation is slow and takes a time. Let x is the instantaneous elongation (x > l) and the restoring force is f at an intermediate time. F A f L Y x L Ax ? ? or YAx f L ? In streaching further by dx the work done against the restoring force is YA fdx xdx L ? The work done to perform the complete elongation l is 0 0 ? ? ? ? l L YA W f dx x dx L 2 1 2 2 ? ? ? ? YA l YAl Fl F L L 7.7 Strain energy The work done by the external force

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in streching the wire is stored in the wire as potential energy

and is known as strain energy of the wire. Thus the strain energy say 'U' is given by 1 1 . . 2 2 F l U F l AL A L ? ? ? ? ? 1 2 ? × stress × strain × volume. Thus the strain energy density of a stretched wire is equal to 1 2 U V ? (stress × strain) 7.8 Twist of a wire or a cylinder We consider a wire / cylinder of length 'L' and radius 'r'; whose upper end is fixed and a twist θ is applied at the lower end. In the figure a co-axial cylindrical cell of redii L F ↑ ? ↓ ↑↑↑↑↑ x dx ? ↓ ↑
NSOU ? GE-PH-11 ? 85 between x and x + dx (0 > x > r) is considered within the wire. AB is a vertical line on the surface of the cylindrical cell before application of the twist. On application of the twist θ at the bottom B moves to B' (say). If φ is the shear BB / = Lφ = x θ or x L ? ? ? The shearing stress on the annular region at the bottom of the wire is given by stress = shearing strain × rigidity modulus. x L ? ?? ? ? ? The restoring force on the annular region opposing the twist is equal to the shearing stress multiplied by the area. ∴ The restoring force 2 n x xdx L ? ? ? ? 2 2

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x dx L ??? ? The moment of this force about the axis 00 / of the

wire is 3 2
x dx L ??? The moment of total restoring force also known as twisting coupe is 4 3 0 0 2 2 4 r r x x dx L L ??? ??? ? ? ? ? ? or 4 2 r L ? ? ? ? ? or C ? ? ? where 4 2 r C L ? ? ? , is known as moment of restoring couple per unit twist. This also known as tortional rigidity of the wire. φ
86 ? NSOU ? GE-PH-11

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Work done in twisting a wire. We consider a wire of length 'L' and radius 'r' fixed at one end and

a torque Γ is applied to the other end to produce a twist θ. An equal and opposite restoring torque will be produced in the wire at equilibrium. 4 2 r L ? ? ? ? ? or 4 , 2 r C C L ? ? ? ? ? ? ? a constant for the wire known as restoring torque per unit twist. The twist takes a time to allend the final value θ. Let α is an instantaneous value of twist at an intermediate time and τ is the corresponding restoring torque c ? ? ? For a further twist dα, the work done is dW d? ? ? The total work done by the external torque against the restoring torque to produce twist θ is 2 0 0 1 1 2 2 W d C d c ? ? ? ? ? ? ? ?? ? ? ? ? ? ?
7.9 Rigidity modulus by static torsion A cylinder of diameter d is hung from a torsion head T with the help of a wire 'E' of the material whose rigidity modulus is to be determined. Let 'L' and 'r' be the length & radius of the wire. Two ends of a long thread after spiraling on the cylinder hold two pans C – C via two small pulleys P – P. If weights are placed on the pans the thread unwinds and gives rotation to the cylinder and twist to the specimen wire. An indicator I is fixed on the top of the cylinder which can move over a circular scale 'S' and reads the rotation of the cylinder and the twist 'θ' of the wire. Equal weights are placed on the pans and the twist is noted. The weights are increased in steps and each time the reading of the indicator T

NSOU ? GE-PH-11 ? 87 is taken. A graph of the twist 'θ' against the mass 'm' placed on the pans is drawn. Average value of m/θ is obtained from the graph and the value of rigidity modulus is calculated from the relation below. The external torque twisting the wire is mg × d and the internal restoring torque produced is 4 2 r L ? ? ? . At equilibrium 4 2 r mgd L ? ? ? ? or 4 2g dL m r ? ? ? ? 'L' is measured by a meter scale 'd' is measured by a slide calipers. 'r' is measured by a screw gauge. 7.10 Elastic constants by Searles method The following experiment enables us to obtain all the elastic constants. Two identical bars A and B of square or circular cross-section are connected together at their middle points by a wire E of the material whose elastic constants are to be measured. The bars are suspended by two silk fibres C & D from a rigid support such that the bars A and B and the wire E are in same horizontal plane. Mass 'M' of each bar is measured with the help of a physical balance. The length 'L' of the bars is measured by a meter scale. For bars of square cross section breadth – 'b' or for the bars of circular cross- section the radius 'r' is measured by a slide calipers.
88 ? NSOU ? GE-PH-11 The length 'l' and radius 'r' of the speciman E are measured by a metre scale and screw gange respectively. The nearer ends of the bars A and B are brought closer with the help of a small thread. The thread is burned. The bars will move to and fro about the silk suspension. The time period of oscillation is measured. It T 1 is the time period and I is the moment of inertia of the bars about the suspension, the Young's modulus of the material of wire E is 2 4 1 8 ? Il Y T r ? ? ? 2 2 2 M I L b ? ? : for square cross section 2 2 12 4 L R M ? ? ? ? ? ? ? ? ? ? : for circular cross section Next the silk fibres C and D are removed. One of the bars say A is attached to the rigid support, such that the specimen wires is vertical and bar B can be made to oscillate about the specimen wire. If T 2 be the time period of oscillation of B, the rigidity modulus of the material of specimen is 2 2 8 4 ? Il T r ? ? The poisson ratio is 2 2 2 1 1 2 T T ? ? ? and the bulk modulus is 3(1 2) Y K ? ? ? 7.11 Rigidity modulus by dynamic method A wire 'E' of the material, whose rigidity modus is to be determined is hung from the torsion head T in a rigid support. At the lower end of the wire a cylinder C is attached.
NSOU ? GE-PH-11 ? 89 If the cylinder is given a small rotation by an external torque, the wire gets twisted. The moment of the restoring couple is equal and opposite to moment of the twisting couple. The cylinder's simple harmonic motion for small θ is governed by the equation of motion ? ? ?? I c ? ? where c is the moment of restoring couple per unit twist of the wire. or 2 2 0, ? ? ? ?? c I ? ? ? ? or 2 2 ? ? ? ? ? ? ? ? c T I ? or 2 2 4 , ? I T c ? T = time period of Oscillation We know 4 2 ? r c L ? ? 'r', the radius of the wire and is measured with the help of a screwgauge, 'L', the length of the wire, measured with the help of a meter scale. From the above relation we get 2 2 4 ? ? c I T ? 2 2 2 4 1 2 MR T ? ? ?

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M is the mass of the cylinder and R is the radius of the			

cylinder. or 4 2 2 2 2 2 ? r MR L T ? ? ? or 2 2 4 4 ? MR L T r ? ?
90 ? NSOU ? GE-PH-11 with the help of a stopwatch and the minor M the time period of oscillation can be measured. 7.12 Moment of inertia by torsion balance The experimental arrangement consists of a circular table known as inertia table D fitted with a pair of small vertical pillars P–P and a horizonal bar B. The whole assembly is suspended by a thin wire W from the torsion head T inside a rigid frame F. The frame is mounted on a heavy iron base. A mirror M is used for the measurement of time period of oscillation. The entire apparatus is enclosed in a glass cover. The table D is set into torsional vibration and the time period (t 0) is precisely measured, which is related to the moment of inertia I 0 of inertia table an restoring couple per unit twist c of the wire W by 0 0 2 I t C ? ? (1) An object whose moment of inertia I is to be measured is placed on the inertia table. Again the inertia table is set into torsional vibration. If 't' by the time period of oscilation, 0 2 I I t C ? ? ? (2) The experimental body is replaced by another body of known moment of inertia say I´, the time period of oscillation will be given by 0 2 I I t C ? ? ? ? ? (3) Solving for I, from equation (1), (2) and (3) we get 2 2 0 2 0 t t I I t t ? ? ? ? ? Perfectly rigid body : A body will be called perfectly rigid body if it does not F

NSOU ? GE-PH-11 ? 91 suffer any deformation whatever be the external deforming force. No body is perfectly rigid. Perfectly elastic body : Whatever be the magnitude of deforming force, if a body completely regains its shape and size the body is said to be perfectly elastic. No body is perfectly elastic. Perfectly plastic : A body is said to be perfectly plastic if it retains its changed configuration even after removal of external force. Problem 1 : – A piece of copper wire has twice the radius of a steel wire. One end of the copper wire is joined to the one end of the steel wire so that both can be subjected to the same longitudinal deforming force. By what fraction of its length will the steel wire be stretched when the length of the copper wire has increased by 0.5 %. Y for steel is twice that of the copper. We know that 2 F L Y L r ? ? ? For steel 2 s s s s L F Y L r ? ? ? ? (1) For Copper 2 c c c c L F Y L r ? ? ? ? (2) Dividing eqn (1) by eqn (2) we have 2 2 s c s c c s c s Y r L L Y L L r ? ? ? ? ? or 2 = 0.5 4 100 s s L L? ? ? or 4 0.5 1 2 100 100 s s L L ? ? ? ?

92 ? NSOU ? GE-PH-11 Problem 2 : A load of 24 kg is suspended from one end of a wire whose radius is 1 mm. What will be the change of temperature of the wire if the wire suddenly snaps ? For the material of the wire Y = 12 × 10 11 dyn.cm −2 , density = 9 gm cc −1 sp.heat = 0.1 and J = 4.2 × 10 7 ergs cal −1 . Solution : We know mg L Y L ? ? ? or ? mgL L Y ? ? 3 11 24 10 980 0.01 12 10 L ? ? ? ? ? ? ? ? ? 5 2 980 196 106 10 L L ? ? ? ? ? ? ? ? Work done 3 1 1 196 . 24 10 980 2 2 105 L W L F ? ? ? ? ? ? ? ? ? ? 196 12 98 10 L ? ? ? ? ? ? ? Heat H = msθ θ = rise of temperature = π × (0.1) 2 L × 9 × 0.1 × θ = 9πLθ × 10 −3 Again W = JH or, 7 3 196 12 98 4.2 10 9 10 10 L L? ? ? ? ? ? ? ? ? ? ? ?

NSOU ? GE-PH-11 ? 93 or 2 7 3 196 12 98 10 4.2 10 9 10 ? ? ? ? ? ? ? ? ? ? ? = 0.0061 o C Problem 3 : A metal wire of length 3 m and diameter 2 mm is stretched by 10 kg- wt. Find the contraction of the diameter. Y = 20 × 10 11 dyn.cm −2 and σ = 0.26 Solution : F L Y L ? ? ? , or L F L Y ? ? ? D L D L ? ? ? ? or L D F L D Y ? ? ? ? ? ? ? 11 0.26 0.2 10000 980 0.01 20 10 DF D Y ? ? ? ? ? ? ? ? ? ? ? ? ? ? = 8 × 10 −6 cm. 9.13 Exercises 1) A wire 2m long and 2.0 mm in diameter stretched by 8 kg. The length increases by 0.24 mm. Find stress, strain and Y. 2) A wire is stretched by the application of a force of 50 kg-wt per square cm. What is the percentage increase in length Y = 7 × 10 10 Nm −2 3) A tangential force of 0.245 N is applied on the upper surface of a block (60 mm × 60 mm × 60 mm). The upper surface is displaced by 5 mm with respect to the lower surface. Find shear stress, shear strain and modulus of rigidity. 4) A cylindrical mass 5 kg of radius 4 cm is hung with a metal wire of length 100 cm and radius 2 mm. When the cylindrical mass is made to oscillate it makes 25 oscillations in 20s. Compute the rigidily modulus of the material of the wire.

94 ? NSOU ? GE-PH-11 Unit 8 ????? Special theory of relativity Structure 8.1 Objectives 8.2 Introductions 8.3 Lorenz transformation 8.4 Length Contraction 8.5 Time dialation 8.6 Relativisitic velocity addition 8.7 Exercises 8.8 Suggested Readings 8.1 Objectives .... 8.2 Introductions Einstein in 1905 at the age of 26 yrs. through a paper "On the electrodynamics of moving bodies" introduced special theory of relativity. The inconsistancy of Newtonian mechanies with Maxwell's equations of electromagnetism and the failure of experimental proof of existance of a hypothesided luminiferous ether led Einstein to propose the theory based

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on two postulates. 1. The physical laws are invariant in all inertial system. 2. The speed of light in vacuum is the same for all observers regardless of motion of the light source. The

propagation of waves could not be entertained without a medium at the time light was initially thought to be wave in nature. An omnipresent hypothetical medium called

NSOU ? GE-PH-11 ? 95 eather was required the light to creep through. As time passed the enrichment of scientific knowledge took place and ether became more and more self contradictory. Though none was in favour of ether hypothesis everybody had to swallo it for absence of suitable alternative physical theory. Michelson – Morley's Experiment (1887) failed to find any measurable property of ether and the null result of the above experiment was only explained with the help of Einstein's postulates, which laid the foundation of special theory of relativily. 8.3 Lorenz transformation Einsteins second postulate of constancey of velocity of light was a blow to the Galilean transformation laws in Newtonian mechanics. Let S and S / are two inertial frames, S is at rest and S / is moving along common X-axis with uniform velocity v. If P is a point whose space and time coordinates are (x, y, z, t) and (x / , y / , z / , t /) inS and S / frames respectively, then

Galilean transformation equations are

x = x / + vt ; y = y / , z = z / and t =

t / .

We suppose an observer in s and light is moving in the frame s / with velocity c along x-axis (x & x / axis coincident) Differentiating galilean transformation equation with respect to time we get dx dx v dt dt ? ? ? = c + v < c The velocity of light appears to be c + v to the observer in s frame, which is not possible according to the second postulate of Einstein. Thus Galilean transformations need modification. Lorentz derived the new set of transformation equations as 2 2 1 x vt x v c ? ? ? ? D

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y y ? ? z z ? ? 2 2 2 1 v t x c t v c ? ? ? ?

or Inverse Lorentz transformatin as 2 2 1 ? ? ? ? ?

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x vt x v c y y ? ? z z ? ? 2 2 2 1 ? ? ? ? ? v t x c t v c

While deriving the above

relations Lorenlz kept the following facts in his thinking. 1) Constancy of velocity of light 2) New relations must boil down to Galilean relations in the limit v >> c. 3) The relations must be linear in space and time. 8.4 Length Contraction We imagine a rod lying at rest along x / axis in S / frame. Its end points are measured to be at 2 x ? and 1 x ?. So that the rest length is 2 1 0 x x L ? ? ? ? say. In order to measure the length of the rod from S-frame we must read the end positions X 2 and X 1 simultaneously say at a time t. Using Lorentz transformation

NSOU ? GE-PH-11 ? 97 2 1 2 1 2 2 2 2 , 1 1 x vt x vt x x v c v c ? ? ? ? ? ? ? ? 2 1 2 1 2 2 1 x x x x v c ? ? ? ? ? ? Thus the length measured from s frame is ? ? 2 2 2 1 2 1 x x x x v c ? ? ? ? ? ? or 2 2 0 1 L L v c ? ? Thus a body's length is measured to be greatest when the body is at rest relative to the observer. When it moves with a velocity v with respect to the observer its measured length is contracted in the direction of its motion by the factor 2 2 1 v c ? , where as its dimensions perpendicular to the direction of motion are unaffected. The frame in which the observed body is at rest is known as proper frame and the length of the rod in such a frame is known as proper length. 8.5 Time dialation We consider a clock to be at rest

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at the position x / in the S / -frame. Let t / 1 and t / 2 are times recorded by the

clock at x / in s / . The times will be recorded as t 1 and t 2 say from s-frame. Using Lorentz inverse transformation we get. 1 2 1 2 2 1 v t x c

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t v c ? ? ? ? ? and 2 2 2 2 2 1 v t x c t v c ? ? ? ? ? 98 ? NSOU ? GE-PH-11 1 2 2 1 2 2 1 t t t t v c ? ? ? ? ? ? ? 2 1 2 1 t t t t ? ? ? ? ?

Therefore from the point of view of observer S, the moving S / -clock appears slowed down by a factor 2 2 1 v c ? . The proper time interval 2 1 t t d? ? ? ? ? (say) is the time interval recorded by a clock attached to the observed body. A non proper time interval t 2 – t / 1 = dt (say) would be a time interval measured by two different clocks at two different places, thus 2 2 1 d dt v c ? ? ? 8.6 Relativistic velocity addition We write down the Lorentz transformation equations as 2 2 1

91%	MATCHING BLOCK 64/68	SA	Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

x vt x v c ? ? ? ? ? y y ? ? z z ? ? and 2 2 2 1 v t x c t v c ? ? ? ? ?

Taking the
differentials we get. 2 2 1 dx
v dt dx v c ? ? ? ? ? (1) dy dy ? ? (2) dz dz ? ? (3) and 2 2 2 1 v dt dx c dt v c ? ? ? ? ? (4)
NSOU ? GE-PH-11 ? 99 Dividing equation (1) by equation (4) we get 2 2 1 dx v dx dx v dt dt v dx dt dt v c dx dt c ? ? ? ? ? ? ? ? ? ? ? ? ? ? or 2 1

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x x x u v u v u c ? ? ? ? ? (5) where u x and u / x

are velocities of a body as observed from s and s / frames respectively, along common x-axis. Dividing equation (2) by equation (4) 2 2 2 2 2 2 1 1 1 dy v c dy v c dy dt v v dx dt dt dx c dt c ? ? ? ? ? ? ? ? ? ? ? ? or 2 2 2 1 1
y

50%	MATCHING BLOCK 66/68	SA	Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Pan ... (D76782351)

x u y v c u v u c ? ? ? ? ? (6) and similarly 2 2 2 1 1 z x u z v c u v u c ? ? ? ? ? (7)

u y
and u z are the y and z component of velocities as observed from s-frame and u / y and u / z are the respective velocities as observed from s / frame of the said body. Problem : The velocity of light with respect s / framme moving with velocity 0.9 c is. c. If s he the rest frame, what will the velocity of light with respect to s frame. Assume all velocities are along x-asis.
100 ? NSOU ? GE-PH-11 We know 2 1 x x u v u vu x c ? ? ? ? ? 2 0.9 0.9 1 c c c c c ? ? ? ? = c. Problem : At an attitude of 10000 m μ-mesons are created with half-life 1.56 μs. None of them are therefore expected to be present on earth surface. But a large number of μ-mesons are present on the earth surface. If the velocity of μ-mesons are taken to be 0.98C, the time taken by the mesons to reach earth 10000 34 0.98 3 10 T s ? ? ? ? ? 34 21.8 1.56 ? ? halflives ∴ Survival rate = 2 – 21.8 = 0.27 × 10 –6 mesons are not expected to reach earth. If relativic effect is taken from earth's fame, 8 10000 34 0.98 3 10 T s ? ? ? ? ? The half life is 2 1.56 1 (0.98) s? ? = 7.8 μs 34 4.36 7.8 T? ? ? halflives The survival rate = 2 – 4.36 = 0.049
NSOU ? GE-PH-11 ? 101 i.e. 49000 out of one million is expected on the earth surface. If relativistic effects is considered from meson's frame. 2 8 6 10000 1 (0.98) 2000 0.98 3 10 98 3 10 T ? ? ? ? ? ? ? = 6.8 μs = 6.8 4.36 1.56 ? half life Survival rate = 2 –4.36 = 0.049 same as when observed from earth's frame. This explains the presence of muon on earth. 8.7 Exercises (1) A 2 m tall and 50 cm wide cosmonant is approaching earth in a spaceship moving with velocity 0.97c. The length of the cosmonant is parallel to the direction of motion. What are the height and width of the cosmonant according to an observer on earth ? (2)

65%	MATCHING BLOCK 67/68	SA	ELMP-1 - Mechanics.pdf (D137599141)

The average life time of a π meson in its own frame is 26 ns. The meson moves with speed 0.95c with respect to earth. What is

the life time with respect to

100%	**MATCHING BLOCK 68/68**	**SA** ELMP-1 - Mechanics.pdf (D137599141)
an observer at rest on earth. What is the average distance it travels before decaying as measured by		

the observer of earth ? (3)
Two spaceships approach each other, each moving with same speed as measured by a stationary observer on earth. Their relative speed is 0.7c. Determine the
velocity of each space from earth. 8.8 Suggested Reading 1. Mathematics of Physics and Chemistry. By : H. Margenu and G. M. Murphy 2. Mathematical methods for Physicists By : George B. Arfken. 3. Vector Analysis By : Murray R. Speigel 102 ? NSOU ? GE-PH-11 4. General properties of Matter By : D. S. Mathur 5. General properties of Matter By : F. H. Newman and V. H. L. Searle 6. Classical Mechanics By : Herbert Goldstein 7. Classical Mechanics By : R. G. Takwale and P. S. Puranik 8. Physics By : D. Halli day and R. Resnick

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1/68	**SUBMITTED TEXT**	16 WORDS	**96%**	**MATCHING TEXT**	16 WORDS
A x , A y and A z are components of A ? along the **SA** BSc_Physics_1st sem_Block B.pdf (D129738380)					

2/68	**SUBMITTED TEXT**	23 WORDS	**58%**	**MATCHING TEXT**	23 WORDS
product of two vectors is scalar quantity equal to the product of magnitudes of the two vectors and the cosine of angle between them. **SA** Physics_Vol-1 EM.pdf (D40552326)					

3/68	**SUBMITTED TEXT**	15 WORDS	**100%**	**MATCHING TEXT**	15 WORDS
x x y y z z A B A B A B i i j **SA** Physics_Vol-1 EM.pdf (D40552326)					

4/68	**SUBMITTED TEXT**	18 WORDS	**58%**	**MATCHING TEXT**	18 WORDS
is rotated from A ? to B ? the direction of motion of the screw head gives the direction of **SA** Physics_Vol-1 EM.pdf (D40552326)					

5/68	SUBMITTED TEXT	18 WORDS	100%	MATCHING TEXT	18 WORDS

equal to the product of the magnitudes of two vectors
and the sine of the angle between them.

SA Physics_Vol-1 EM.pdf (D40552326)

6/68	SUBMITTED TEXT	57 WORDS	59%	MATCHING TEXT	57 WORDS

x y z x y z A B i A j A k A i B j B k B ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
ˆ ˆ ˆ y z y z z x z x x y x y i A B B A j A B B A k A B B

SA Physics_Vol-1 EM.pdf (D40552326)

7/68	SUBMITTED TEXT	11 WORDS	92%	MATCHING TEXT	11 WORDS

x y z x y z i j k A A A B B B ?

SA Physics_Vol-1 EM.pdf (D40552326)

8/68	SUBMITTED TEXT	11 WORDS	84%	MATCHING TEXT	11 WORDS

y x z dv dv dv dv i j k dt dt dt dt ? ? ? ?

SA Physics_Vol-1 EM.pdf (D40552326)

9/68	SUBMITTED TEXT	15 WORDS	87%	MATCHING TEXT	15 WORDS

x y z a a i a j a k ? ? ? ? ; ; x x z

SA Physics_Vol-1 EM.pdf (D40552326)

10/68	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

v x , v y and v z are components of

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

11/68	SUBMITTED TEXT	16 WORDS	84%	MATCHING TEXT	16 WORDS

a x , a y and a z are components of acceleration along the

SA BSc_Physics_1st sem_Block B.pdf (D129738380)

12/68 **SUBMITTED TEXT** 14 WORDS **96%** **MATCHING TEXT** 14 WORDS

x x y y z z A B A B A B A B

SA Physics_Vol-1 EM.pdf (D40552326)

13/68 **SUBMITTED TEXT** 32 WORDS **96%** **MATCHING TEXT** 32 WORDS

x x y y z z a b a b a b a b = 4 × 2 + 3 × 5 + 7 × 4 = 8 + 15 + 28 = 51 1.5

SA Physics_Vol-1 EM.pdf (D40552326)

14/68 **SUBMITTED TEXT** 33 WORDS **57%** **MATCHING TEXT** 33 WORDS

The state of motion of a body does not change until and external force is applied on it. Second Law :– The rate of change of linear momentum of a body is proportional to the

Unit–1 ? Laws of Motion 7-46 Unit–2 ? Rotational Dynamies 47-88 Unit–3 ? Gravitation 89-124 Unit-4 ? Central Force Motion 125-152 Unit-5 ? Elasticity 153-181 Unit-6 ? and fluid dynamics 182-209 Special Theory of Relativity 210-237 UG : Physics (HPH) Course : Mechanics

SA PARMESHWAR (M.Phil., Physics).docx (D57914933)

15/68 **SUBMITTED TEXT** 19 WORDS **100%** **MATCHING TEXT** 19 WORDS

the relationship between a body and the forces acting upon it and its motion in response to those forces.

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is the mass of the body, r is the distance of the body from the centre of rotation

SA BSc_Physics_1st sem_Block C.pdf (D129738406)

17/68 **SUBMITTED TEXT** 47 WORDS **42%** **MATCHING TEXT** 47 WORDS

m 2 g – m 1 g = m 1 f + m 2 f or 2 1 1 2 ( ) m m g f m m ? ? ? From (1) 2 1 1 1 1 2 1 1 m m T m g m f m g m m ? ? ? ? ? ? ? ? ? ? ? ? 1 2 1 2 2m m g m m ? ? (4)

SA PARMESHWAR (M.Phil., Physics).docx (D57914933)

18/68 **SUBMITTED TEXT** 16 WORDS **58%** **MATCHING TEXT** 16 WORDS

m R m r ? ? ? ? ? ? ? ? or i i i i m r R m ? ? ? ? ? ? or i i m r R M ? ? ? ? .... (2) Where 1 n i i M m ? ? ? =	Angular Momentum of a System of Particles

SA PARMESHWAR (M.Phil., Physics).docx (D57914933)

19/68 **SUBMITTED TEXT** 25 WORDS **66%** **MATCHING TEXT** 25 WORDS

blocks of mass 50 kg and 30 kg are in contact on a smooth surface. A force of 100 N is applied on the 50

SA Physics_Vol-1 EM.pdf (D40552326)

20/68 **SUBMITTED TEXT** 18 WORDS **66%** **MATCHING TEXT** 18 WORDS

The first term on the right hand side of eqn (2) is the total external force 1? ? ? ?

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

21/68 **SUBMITTED TEXT** 28 WORDS **61%** **MATCHING TEXT** 28 WORDS

R ? is the position vector of the centre of mass and i r ? ? is the position vector of the i th particle with respect to the centre of mass

SA ES3C3 Assignment.pdf (D25382439)

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of a system of particles is the total mass of the system

SA PHY17R121-Mechanics and Properties of Matter_All Unit-1.pptx (D110170498)

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Rate of change of total momentum of a system is equal to total external force on the

SA ELMP-1 - Mechanics.pdf (D137599141)

24/68	**SUBMITTED TEXT**	18 WORDS	**61%**	**MATCHING TEXT**	18 WORDS

on the system. 3.5 Total angular momentum of system The total angular momentum of a system of n particles

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

25/68	**SUBMITTED TEXT**	88 WORDS	**32%**	**MATCHING TEXT**	88 WORDS

r m r ? ? ? ? ? ? ? ? ? ? ? 1? ? ? ? ? ? ? ? ? ? ? ? n i i i i R r m R r 1 1 1 ? n n n i i i i i i i i i i i R m R R m r r m R r m r ? ? 1 1 1 n n n i i i i i i i i i i d R m R R m r m r R r

SA Physics_Vol-1 EM.pdf (D40552326)

26/68	**SUBMITTED TEXT**	30 WORDS	**59%**	**MATCHING TEXT**	30 WORDS

the sum of the angular momentum of the entire mass concentrated at the centre of mass about the origin and the angular momentum of the system about the centre of mass. 3.6

SA MPHS - 11 full draft.pdf (D114327742)

27/68	**SUBMITTED TEXT**	15 WORDS	**62%**	**MATCHING TEXT**	15 WORDS

of a System of Particles The total kinetic energy of a system of n particles is 2 1 1 2

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

28/68	**SUBMITTED TEXT**	91 WORDS	**61%**	**MATCHING TEXT**	91 WORDS

m v v m v v v v ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 1 1 . . 2 n i i i i i i i i m v m v v m v v m v ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 1 1 1 1 . 2 2 ? n n i i i i i i i i m v v m v m v 2 1 1 . 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? n i i i i i mv T m v m

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

29/68	**SUBMITTED TEXT**	24 WORDS	**52%**	**MATCHING TEXT**	24 WORDS

kinetic energy of the system consists of two parts, the kinetic energy obtained if the entire mass is concentrated at the centre of mass (

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

30/68	**SUBMITTED TEXT**	21 WORDS	**71%**	**MATCHING TEXT**	21 WORDS

The work done by all the forces in moving the system from an initial configuration (1) to a final configuration (2)

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

31/68	**SUBMITTED TEXT**	92 WORDS	**75%**	**MATCHING TEXT**	92 WORDS

v v m v m v m m ? ? ? ? or 2m 1 v 1 + 2m 2 v 2 = m 1 v 2 + m 1 v 1 + m 2 v 2 + m 2 v 1 or m 1 (v 1 – v 2) – m 2 (v 1 – v 2) = 0 (m 1 – m 2) (v 1 – v 2) = 0 Since 1 2 v v ? , m 1 = m 2 42 ?

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

32/68	**SUBMITTED TEXT**	17 WORDS	**67%**	**MATCHING TEXT**	17 WORDS

Moment of inertia plays the same role in rotational motion as played by mass in linear motion.

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

33/68	**SUBMITTED TEXT**	25 WORDS	**57%**	**MATCHING TEXT**	25 WORDS

the rate of change of angular momentum is equal to the torque applied on the point mass. If the applied torque in zero the angular momentum

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

34/68	**SUBMITTED TEXT**	13 WORDS	**95%**	**MATCHING TEXT**	13 WORDS

between i th and j th particle is along the line joining

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

35/68	SUBMITTED TEXT	21 WORDS	61%	MATCHING TEXT	21 WORDS

rate of change of angular momentum of a system of particles is equal to the total external torque on the system.

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

36/68	SUBMITTED TEXT	14 WORDS	76%	MATCHING TEXT	14 WORDS

A disc of mass M and radius R is rotating about it axis

SA Physics_Vol-1 EM.pdf (D40552326)

37/68	SUBMITTED TEXT	21 WORDS	92%	MATCHING TEXT	21 WORDS

with a force proportional to the product of the masses and inversely proportional to the square of the distance between them

SA PHY17R121-Mechanics and Properties of Matter_All Unit-1.pptx (D110170498)

38/68	SUBMITTED TEXT	14 WORDS	80%	MATCHING TEXT	14 WORDS

planet moves in an elliptial orbit with sun at one of its foci. 2)

SA BSc_Physics_1st sem_Block B.pdf (D129738380)

39/68	SUBMITTED TEXT	28 WORDS	87%	MATCHING TEXT	28 WORDS

of time. 3) The square of time period of revolution of a planet about the sun is proportional to the cube of the semi major axis of the elliptical orbit.

SA BSc_Physics_1st sem_Block B.pdf (D129738380)

40/68	SUBMITTED TEXT	19 WORDS	57%	MATCHING TEXT	19 WORDS

a satellite of mass ‘m’ orbits round the earth (mass M) in a circular orbit of radius r (the

SA BSc_Physics_1st sem_Block B.pdf (D129738380)

41/68 **SUBMITTED TEXT** 12 WORDS **91%** **MATCHING TEXT** 12 WORDS

Global Positioning System (GPS) Global positioning system is a Satellite navigation system

SA Unit II Notes.docx (D113412638)

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m = mass of the Satellite, M = Mass of earth R = Radius of earth

SA Mechanics - Block I -5 SLM.pdf (D116041085)

43/68 **SUBMITTED TEXT** 41 WORDS **95%** **MATCHING TEXT** 41 WORDS

Due to the property of surface tension a drop or bubble tries to contract and so compresses the matter enclosed. This in term increases the internal pressure which prevents further contraction and equilibrium is achieved. So in equilibrium pressure inside a bubble or

SA PHY17R121-Mechanics and Properties of Matter_All Unit-1.pptx (D110170498)

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drop is greater than that outside and the difference of pressure is called excess pressure. Expression of excess pressure ABCD is

SA PHY17R121-Mechanics and Properties of Matter_All Unit-1.pptx (D110170498)

45/68 **SUBMITTED TEXT** 44 WORDS **85%** **MATCHING TEXT** 44 WORDS

The angle conventionally measured through the liquid where liquid - vapour interface meets solid surface. It quantifies the wettability of a solid surface by a liquid. A given system of solid, liquid and vapour at a given temperature and pressure has a unique contact angle.

SA PHY17R121-Mechanics and Properties of Matter_All Unit-1.pptx (D110170498)

46/68	**SUBMITTED TEXT**	28 WORDS	**67%**	**MATCHING TEXT**	28 WORDS

surface tension of a liquid is determined by measuring the pressure required to cause air to flow from a capillary tube immersed in the liquid. One end of a

SA PHY17R121-Mechanics and Properties of Matter_All Unit-1.pptx (D110170498)

47/68	**SUBMITTED TEXT**	19 WORDS	**68%**	**MATCHING TEXT**	19 WORDS

r , where T is the surface tension of the liquid to be measured and r is the radius of

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

48/68	**SUBMITTED TEXT**	12 WORDS	**87%**	**MATCHING TEXT**	12 WORDS

of radius ‘r’ coalesce to form a bigger drop of radius ‘R’.

SA PHY17R121-Mechanics and Properties of Matter_All Unit-1.pptx (D110170498)

49/68	**SUBMITTED TEXT**	41 WORDS	**50%**	**MATCHING TEXT**	41 WORDS

n r r ? ? ? ? R = 2 3 r n R 2 = 2 2 3 r n decrease in surface area 2 2 2 2 3 4 4 4 n r R r n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∴ the

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

50/68	**SUBMITTED TEXT**	13 WORDS	**100%**	**MATCHING TEXT**	13 WORDS

L and R are the length and radius of the capillary tube

SA Mechanics Properties of Matter-PHY17R121.docx (D109220287)

51/68	**SUBMITTED TEXT**	13 WORDS	**87%**	**MATCHING TEXT**	13 WORDS

a soap bubble of radius 10 cm. The surface tension of soap solution

SA BSc_Physics_1st sem_Block C.pdf (D129738406)

52/68 **SUBMITTED TEXT** 13 WORDS **76%** **MATCHING TEXT** 13 WORDS

The ratio of lateral strain to longitudinal strain is known as Poisson ratio.

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53/68 **SUBMITTED TEXT** 13 WORDS **75%** **MATCHING TEXT** 13 WORDS

slope of the graph gives the young's modulus of the material of the

SA ELMP-1 - Mechanics.pdf (D137599141)

54/68 **SUBMITTED TEXT** 15 WORDS **70%** **MATCHING TEXT** 15 WORDS

Work done in streaching a wire We consider a vertical wire of length L and

SA Properties of Matter Complete Book (2).docx (D111988805)

55/68 **SUBMITTED TEXT** 12 WORDS **83%** **MATCHING TEXT** 12 WORDS

in streching the wire is stored in the wire as potential energy

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x dx L ??? ? The moment of this force about the axis 00 / of the

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57/68 **SUBMITTED TEXT** 21 WORDS **73%** **MATCHING TEXT** 21 WORDS

Work done in twisting a wire. We consider a wire of length 'L' and radius 'r' fixed at one end and

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58/68	SUBMITTED TEXT	15 WORDS	89%	MATCHING TEXT	15 WORDS

M is the mass of the cylinder and R is the radius of the

SA BSc_Physics_1st sem_Block B.pdf (D129738380)

59/68	SUBMITTED TEXT	31 WORDS	65%	MATCHING TEXT	31 WORDS

on two postulates. 1. The physical laws are invariant in all inertial system. 2. The speed of light in vacuum is the same for all observers regardless of motion of the light source. The

SA M_Sc_ Physics - 345 11 - Classical Mechanics.pdf (D101798669)

60/68	SUBMITTED TEXT	4 WORDS	100%	MATCHING TEXT	4 WORDS

y y ? ? z z ? ? 2 2 2 1 v t x c t v c ? ? ? ?

SA Dr. Kusam_Book Mechanic-B.Sc.I-Semester-II-Panjab Uni..pdf (D76782351)

61/68	SUBMITTED TEXT	9 WORDS	100%	MATCHING TEXT	9 WORDS

x vt x v c y y ? ? z z ? ? 2 2 2 1 ? ? ? ? ? v t x c t v c

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62/68	SUBMITTED TEXT	24 WORDS	64%	MATCHING TEXT	24 WORDS

at the position x / in the S / -frame. Let t / 1 and t / 2 are times recorded by the

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

63/68	SUBMITTED TEXT	46 WORDS	41%	MATCHING TEXT	46 WORDS

t v c ? ? ? ? ? and 2 2 2 2 2 1 v t x c t v c ? ? ? ? ? 98 ? NSOU ? GE-PH-11 1 2 2 1 2 2 1 t t t t v c ? ? ? ? ? ? ? ? 2 1 2 1 t t t t ? ? ? ? ?

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64/68	**SUBMITTED TEXT**	12 WORDS	**91%**	**MATCHING TEXT**	12 WORDS

x vt x v c ? ? ? ? ? ? y y ? ? z z ? ? and 2 2 2 1 v t x c t v c ? ? ?
? ?

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65/68	**SUBMITTED TEXT**	12 WORDS	**70%**	**MATCHING TEXT**	12 WORDS

x x x u v u v u c ? ? ? ? ? ? (5) where u x and u / x

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66/68	**SUBMITTED TEXT**	13 WORDS	**50%**	**MATCHING TEXT**	13 WORDS

x u y v c u v u c ? ? ? ? ? ? (6) and similarly 2 2 2 1 1 z x u z v
c u v u c ? ? ? ? ? ? (7)

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67/68	**SUBMITTED TEXT**	27 WORDS	**65%**	**MATCHING TEXT**	27 WORDS

The average life time of a π meson in its own frame is 26 ns. The meson moves with speed 0.95c with respect to earth. What is

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68/68	**SUBMITTED TEXT**	17 WORDS	**100%**	**MATCHING TEXT**	17 WORDS

an observer at rest on earth. What is the average distance it travels before decaying as measured by

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NSOU ? CC-PH-06 7 Unit 1 ? To find the number of lines per centimeter of a transmission grating and to measure the wavelength of an unknow spectral line Structure 1.0 Objectives 1.1 Introduction 1.2 Apparatus required 1.3 Theory and formula used 1.4 Experimental procedure 1.5 Observation 1.6 Result 1.7 Precautions 1.8 Oral Questions 1.0 Objective To find the number of lines per centimetre of a transmission grating and to measure the wave length of an unknown spectral line. 1.1 Introduction A grating is a diffractive optical element which contains a large number of equidistant slits of same width. Gratings are fabricated by several techniques: (i) mechanically by ruling equidistant parallel lines on a glass plate by a diamond point, (ii) replica grating produced by depositing a thin film of cellulose acetate on mechanically ruled grating, (iii) optically by interfering two coherent parallel beams on a photosensitive plate. In case of mechanically ruled grating the region where a line is drawn act as an opaque and spaces in between the lines acts as slits. Number of lines on a transmission grating typically is of the order of 15000 per inch. 1.2 Apparatus required Spectrometer, sodium vapour lamp, grating, spirit level, magnifying glass. 7

NSOU ? CC-PH-06 8 1.3 Theory and formula used When a parallel beam of monochromatic light produced by the collimator, incident normally on a grating, the beam of light will be diffracted through the grating. On both the sides of central maximum, principal maxima of different orders will be formed. $(a + b) \sin \theta = n\lambda$ .... (1) where θ is the angle of diffraction corresponding to the nth order principal maximum The width of transparent portion of the slit is a and width of opaque portion is b, the distance a + b is grating element. Fig. 1. Diffraction angle measurement.

NSOU ? CC-PH-06 9 Grating element is related to is number of rulings per cm of the grating (N) by the relation 1 (a + b) = — N equation (1) becomes sin θ = nλ or N n = sinθ λ .... (2) and λ θ = sin Nn .... (3) By measuring θ experimentally, we can find out number of rulings per cm of the grating using equation (2), and wavelength of any unknown spectral line using equation (3). 1.4 Experimental procedure 1. Adjust the telescope and collimator for parallel rays. 2. Level the spectrometer including grating table by a spirit level. 3. Make telescope and collimator in line with each other. Now rotate the telescope through 90°, so that collimator is perpendicular to the axis of the telescope. Keeping grating on the table, rotate the table until telescope receives the image of the slit, and image coincides with intersection of the cross wires. For making grating surface normal to the incident light, rotate the table containing the grating from this position through 45°. 4. Rotate the telescope towards left side of the direct beam to receive the first order spectrum (shown in Fig. 1). Take the reading of the scale of both the verniers. Now rotate the telescope towards right side of the direct beam to observe the first order spectrum on other side of direct beam. Take the readings both the verniers. 5. Similarly set the position of the telescope for observing second order diffracted image on either side of direct beam. Take the reading of both the verniers. 6. Repeat the above procedure.

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NSOU ? CC-PH-06 12 1.5 Observation Find the vernier constant of the spectrometer Vernier constant = 1 M.S.D. – 1 V.S.D. 1.6 Result Number oflines per centimetre of a transmission grating (N) = lines/cm Wave length CA.) of an unknown spectral line = ... nm 1.7 Precautions 1. The grating surface must be normal to the incident rays. 2. The ruled surface should face the telescope. 3. Do not touch the surface of the grating. 1.8 Oral Questions 1. What is the difference between interference and diffraction? 2. Give an example of diffractive optical element. 3. How grating can be fabricated? 4. How do you set grating for normal incidence? 5. What is grating element?

NSOU ? CC-PH-06 13 Unit 2 ? To study photo current versus intensity and wanelength of light; maximum photo electrons versus frequency of light

Structure 2.1 Objectives 2.2 Introduction 2.3 Theory 2.4 Apparatus 2.5 Experimental Procedure 2.6 Experimental Results 2.7 Discussion 2.8 Summary 2.9 Exercise

and Answer 2.1 Objective To show light behaves like a particle when a partiele it interacts with matter, such as electrons on a metal serface. And also estimate to the value of Planck's constant h. 2.2 Introduction Photo electgric effect, phenomenon in which electrically charged particles are released from or within a materid when it absorbs electromagnetic radiation. The effect is often defined as the ejection of electrons from a metal plate when light falls on it. The phenomenon was fundamentally significant in the development of modern phy6sics because of the puzzling questions it raised about the nature of light-particle vs wave like behaviour and finally resulved by Albert Einstein in 1905. The effect remains important for research in areas from material science to astrophysics, as well as forming the basis for a variety of useful devices. 13

NSOU ? CC-PH-06 14 2.3 Theory We known that some every is required to remove the least tightly bound electorus from the mental surface. This minimum energy required to release one electron from the surface is called work function ω 0 . The work function is measure of efficiency of the mental to serve as electron emitter. Metals with low ω 0 value are good electron emitters. e.g Cs, Na, etc. When an electron at the mental surface absords the energy hv of the incident rediation a certain part is used by the electron to do work equal to ω 0 . So as to overcome the attractive forces of the positive ions of the mental. The remaining energy (hv − ω 0) is— 1 2 2 mv max = hv − ω 0 .... (1) where m is mass of the electron & V max is the maximum speed of mitted electron. This is known as Einstein's Photoelectric equation. The threshold frequencies v 0 for a mental surface is the frequency of photon which is just sufficient to selease an electron from the surface with zero K.E. so, 0 = hv 0 − ω 0 or, hv 0 = ω 0 With this equation (1) takes the form 1 2 mv max 2 = h(v − v 0) At stopping potential (v s), there will be h 0 electron at the anode surface. The nagative potential repels all electrons to search the anode surface. All electrons will stop their motion i.e. 1 2 mv max 2 = eV s Therefore, Equation (B) becomes eV s = h(v − v 0)

NSOU ? CC-PH-06 16 Again photo electric current increases linearly with increase of intensity of incident light at a fixed frequency and fixed accelerating potential. This aspect can also be verified by blowly varying the intersity in steps and noting the corresponding photoelectric current at a constant frequency and fixed accelerating potential. Fig. 2.2 2.4 Apparatus Light source, light filters of different colours. Photoelectric cell, micro/milli- ammeter, voltmeter luxmeter variable voltage source. v μA Photosensitive Plate Electron collecting Plate Incident light

NSOU ? CC-PH-06 18 2.6 Experimental Results Table 1. Recording of stopping potential. Sl. No. Colours Retarding Potential Photoelectric in Volt Current in μμμμμA Red Blue Yellow Green O V O O V O O V O O O

NSOU ? CC-PH-06 19 Table 2. Data for stopping potentials vs. Frequency graph (extracted from Table 1) Sl. Colours Wavelength (λλλλλ) Frequency Stopping Potential No. v c = λ (v s) in volt Table 3. Recording of Photoelectric current for varying incident intensities for an accelerating potential = ... volt. Sl. No. Colours of Incident Intensity in Lux Photoelectric light filter (or, LDR current in μμμμμA) Current (μ(μ(μ(μ(μA) Red Blue Yellow

NSOU ? CC-PH-06 20 Green Alternative Table 3(a). If Lux meter or LDR arrangement is not available Recording of Photoelectric current for varying distance of the constant intensity source for an Accelerating potential = ... volt Sl. Colours of Distance of the source Value of Photoelectric No. light filter from the photo cell 1 2 d cm −2 current (μμμμμA) (d) in cm Red Blue Yellow 5 10 20 30 40 0 10 20 30 40 5 10 20 30 40

NSOU ? CC-PH-06 21 Draw photoelectric current (i) at particular colour (frequency or war length) againsth against incident intensity. Conclude about your observation from the graph about the variation of photo electric current at particular frequency or wavelength against incident intensity. Alternatively, draw photoelectric current (i) vs 1 2 d graph. Draw V s versus v graph, and calculate h. Value of h = electric charge (e) × slope of V s versus v graph. Green 5 10 20 30 40 Fig. 2.3 : Variation of photoelectric current (i) with the intensity of photocell incident light Radiation intensity (i) Photoelectgric current (i)

NSOU ? CC-PH-06 22 2.7 Discussion 1. The alignment of source, filter and cathode surface of the photocell should be made carefully. 2. Recording of the photoelectric current with increasing retarding potential should be made systematically to get stopping potential, otherwise exact value of retarding potential for which photoelectric current just reduces to zero may be overlooked. 2.8 Summary ? Photoelectric effect experiment showing light is also a particls Energy comes in particle like chums-basis of quantum physics. ? Based on the wave model of light, physicists predicted that increasing light amplitude would increase the kinetic energy of emitted photoelectrons, while increasing the frequency would increase measured current. ? Experiments showed that increasing the light frequency increased the kinetic energy of the photoelectrons, and increasing the light amplitude increased the current. 2.9 Exercise and Answer 1. Write down Einstein's postulates on photoelectric effect. Ans. — See theory. 2. Write down Einstein's equation of photoelectric effect. Ans. — See theory. 3. What is threshold frequency for photoelectric effect? Ans. — There is a certain minimum frequency v 0 of incident radiation below which there is no emission of photoelectrons. This frequency is called the threshold frequency which depends on the material and the nature of the emitting surface.

NSOU ? CC-PH-06 23 4. What do you meant by photoelectrons? Ans. — The metals that exhibit the photoelectric effect are called photosen- sitive materials and the emitted electrons are called photoelectrons. 5. What is the stopping potential or cut-off voltage? Ans. — The negative anode voltage which just stops the most energetic photoelectrons and reduces photocurrent to zero is called stopping potential or stopping voltage or cut-off voltage. 6. Define work function for photoelectric effect? Ans. — Work function for photoelectric effect is defined as the energy needed to remove the least fightly bound electrons from the surface of a metal. 7. Draw the graph intensity of radiation vs frequency. Ans. — See the theory. 8. What is the significant of photoelectric effect? Ans. — The photoelectric effect is significant because it demonstrates that light has particle like qualities. It established that we can consider light as quanta of (packets) of energy (photon) where one photon interacts with one electron and each photons must have sufficient energy to remove each electron.

NSOU ? CC-PH-06 24 Unit 3 ? Determination of slit width by studying the single slit diffraction pattern Structure 3.1 Objectives 3.2 Introduction 3.3 Apparatus required 3.4 Theory and Formula used 3.5 Experimental Procedure 3.6 Observation 3.7 Calculation 3.8 Precautions 3.9 Oral Questions 3.1 Objective Determination of slit width by studying the single slit diffraction pattern. 3.2 Introduction When light waves face an obstacle (size is comparable to the wavelength) in its path, the waves bend round the edges. This bending of light waves is known as diffraction. There are two types of diffraction (i) Fresnel diffraction and (ii) Fraunhofer diffraction. In Fresnel diffraction, source and screen are at finite distances from the obstacle. In Fraunhofer diffraction, source and screen are at infinite distances from the obstacle. 3.3 Apparatus required Apparatus required: Laser source, single slit, screen, photo detector, multimeter. 24

NSOU ? CC-PH-06 25 3.4 Theory and Formula used A plane wavefront is incident normally on a slit of width d as shown in Fig. l. According to Huygens's principle, each point on the plane wavefront become sources of secondary wavelets. The secondary wavelets which travel in a direction parallel to the direction of incident beam make a central maximum. The secondary wavelets which travel in a direction making an angle θ with central axis will have maximum and minimum intensity, depending upon their path difference. Apart from central maximum, secondary maxima will form in between secondary minima on both sides of the central maximum. Fig. 1. Diffraction through a single slit. If the distance of the first secondary minimum from the centre of the central maximum is y, then width of the central maximum is twice i.e. 2y The directions of secondary minima are given by equation $d \sin \theta = m\lambda$ .... (1) where d is the width of the slit, λ is the wavelength of the light, $m = \pm l, \pm 2, \pm 3$ ....

NSOU ? CC-PH-06 26 For first minimum, m = 1 $d \sin \theta = \lambda$ .... (2) For a large value of D, we can approximate sine by y/D. Thus, our equation becomes $d(y/D) = \lambda$ .... (3) λD y = — .... (4) d 2λD Width of the central maximum = —— .... (5) d 2λD Slit width d = ———————————— .... (6) width of central maximum 3.5 Experimental Procedure 1. For emission of constant light intensity from the laser source, switch ON the laser before starting the experiment. 2. Setup the laser and make incident the beam on to the slit, keep the screen/ photo detector far behind the slit. 3. A diffraction pattern as shown in Fig. 2 will be formed behind the slit, try to observe it on a screen first and then allow to fall on the photo detector. 4. For plotting intensity profile of the diffraction pattern, a photodetector connected with a multimeter is being used. 5. Set the position of photo detector on second minima of the diffraction pattern and note down the reading of current. Now shift the photo detector with the help of translation stage towards the central maximum and take the recordings current vs position of the photo detector. Continue the shifting of the photodetector inthe same direction until second minima will come again in opposite direction. 6. Plot a graph between position of photo detector vs current. From the graph, measure the distance between two first minima on either side of central maxima, which is width of the central maxima.

NSOU ? CC-PH-06 27 7. Measure the slit and photodetector's separation and calculate the width of the slit using equation (6). 8. Repeat the above procedure. Note: If photo detector is not available, then one can proceed by an alternative method. Using a graph paper in place of photo detector (mentioned in point number 3) where diffraction pattern will form, and marking the pattern (maxima and minima including central maximum) by a pencil, and then measuring the width of the central maximum by a scale, one can calculate slit width from the relation 2Dλ d = ——————————— , where 0 is distance between slit and graph paper. width of central maximum 3.6 Observation Fig. 2. (a-b) Photographs of single slit diffraction patterns of two different slit widths.

NSOU ? CC-PH-06 28 Table 1. Intensity profile data Sl. Photodetector's Position Current value No. 1. 2. 3. 4. 5. 3.7 Calculation (i) Plot a graph between photodetector's position vs current. (ii) From the graph, measure width of the central maximum = distance between two first minimum on either side of central maxima = ... mm Table 2. Slit width measurement Sl. Waveleng Distance Width of the Slit width 2?D No. th of laser between slit central maximum d = ———————————— source (?) and (from graph) width of central maximum photodetector (D) 1. 2. 3. 4. 5. Mean slit width = ... /µm 3.8 Precautions 1. Never keep your eyes in the path of the laser beam. 2. Laser should be switched ON before starting the experiment.

NSOU ? CC-PH-06 29 3. Photo detector should be far away from the slit. 4. Translation stage should be move gradually. 3.9 Oral Questions 1. What is diffraction? 2. What is the basic difference between interference and diffraction phenomena? 3. What are two types of diffraction, mention the differences between them? 4. Give an example which can be explained by diffraction phenomenon.

NSOU ? CC-PH-06 30 Unit 4 ? Use of an OPAMP as adder, subtractor, invering and non-inverting amplifier Structure 4.1 Objectives 4.2 Introduction 4.3 Apparatus required 4.4 Adder or Summing Amplifier 4.5 Differential Amplifier or Subtractor 4.6 Inverting Amplifier 4.7 Non-inverting Amplifier 4.8 Precautions 4.9 Oral Questions 4.1 Objective Use of an OPAMP as adder, subtractor, inverting and non-inverting amplifier. 4.2 Introduction Operational amplifier (OPAMP) is a direct coupled high gain differential amplifier. It is basically used to compute mathematical functions like addition, subtraction, integration, differentiation etc. An ideal OPAMP has some important characteristics like infinite voltage gain, infinite input impedance, zero output impedance, infinite bandwidth, characteristics is independent of temperature and perfect balance. Pin diagram of IC 741 is shown in Fig. 1. It has two input terminals and one output terminal. Terminal marked with (+) is non-inverting input terminal and marked with (–) is inverting input terminal. 30

NSOU ? CC-PH-06 31 Fig. 1. Pin diagram of IC 741. 4.3 Apparatus required IC OPAMP 741, DC power supply, Function generator, Digital storage oscilloscope, Resistances, Connecting wires. 4.4 Adder or Summing Amplifier Op-amp is used to design a adder circuit or summing amplifier whose output is the sum of input signals. Schematic of the circuit diagram of adder or summing amplifier is shown in Fig. 2. Fig. 2. Circuit diagram of Adder or summing amplifier.

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V R V R V R f 1 1 2 2 0 + = – (1) V R R V R R V f f 0 1 1 2 2 = – + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ If R 1 = R 2 = R V R R V V f 0 1 2 = – + ( ) For R f = R V 0 = – (V 1 +

V 2) (2) Output voltage is equal to sum of input voltages V 1 and V 2 . Procedure 1. Setup the circuit as per diagram shown in Fig. 2. 2. Connect the +ve and -ve supply voltages to pin number 7 and 4. 3. Apply voltages V 1 and V 2 to the input terminals. 4. Vary the input voltages and note down the corresponding output voltages in table 1. Table 1 Input Voltage Output Voltage V 1 V 2 V 0

NSOU ? CC-PH-06 33 4.5 Differential Amplifier or Subtractor Schematic of the circuit diagram of Differential Amplifier or Subtractor is shown in Fig. 3. Fig. 3. Circuit diagram of Differential Amplifier or Subtractor circuit. Potential at point a and b is V x V V R V V R x x 1 1 0 2 – = – (3) V V R V R x x 2 1 2 – = (4) Subtracting equation (3) and (4) V R R V V 0 2 1 2 1 = – ( ) (5) Output voltage is amplified version of the difference of input voltages V 2 and V 1 . Procedure 1. Setup the circuit as per diagram shown in Fig. 3. 2. Connect the +ve and –ve supply voltages to pin number 7 and 4.

NSOU ? CC-PH-06 34 3. Apply the voltages V 1 and V 2 to the input terminals. 4. Vary the input voltages and note down the corresponding output voltages in table 2. Table 2 Input Voltage Output Voltage V 1 V 2 V 0 4.6 Inverting Amplifier Schematic of circuit diagram of inverting amplifier using OPAMP is shown in Fig. 4. Fig. 4. Circuit diagram ofInverting Amplifier.

NSOU ? CC-PH-06 35 Applying Kirchhoffs current law at the point G gives V V R V V R f 1 1 0 – = – (6) As the point G is a virtual ground, V ≈ 0 V R V R f 1 1 0 = – V V R R f 0 1 1 = – (7) A V V R R f f = = – 0 1 1 (8) Voltage gain Ar is called the closed loop gain, the negative sign shows that output voltage is 180° out of phase with respect to the input voltage. PROCEDURE 1. Setup the circuit as per diagram shown in Fig. 4. 2. Connect the +ve and –ve supply voltages to pin number 7 and 4. 3. Apply input voltage to the inverting terminal from a function generator. 4. Measure corresponding output voltages and hence calculate the gain. Table 3 Input Input Output Gain = (V o /V i) Gain = –(R f /R 1) Frequency Voltage (V i) Voltage (V o) (Practical) (Theoretical) (Hz)

NSOU ? CC-PH-06 36 4.7 Non-inverting Amplifier Schematic of circuit diagram of non-inverting amplifier using OPAMP is shown in Figure 5. Fig. 5. Circuit diagram of Non-inverting Amplifier. V V R R f 0 1 1 1 = + ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ (9) A V V R R f f = = + 0 1 1 1 (10) A f is called voltage gain of the amplifier, which is greater than 1 by a factor R R f 1 ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ . There is no phase difference between input voltage and output voltage. PROCEDURE 1. Setup the circuit as per diagram shown in Fig. 5. 2. Connect the +ve and –ve supply voltages to pin number 7 and 4. 3. Apply input voltage to the non-inverting terminal from a function generator. 4. Measure corresponding output voltages and hence calculate the gain.

NSOU ? CC-PH-06 37 Table 4 Input Input Output Gain = (V o /V i) Gain = 1+(R f /R 1) Frequency Voltage (V i) Voltage (V o) (Practical) (Theoretical) (Hz) 4.8 Precautions 1. Connections should be verified before switch ON the instrument. 2. Input voltage should remain constant during the period of observation. 4.9 Oral Questions 1. What is an operational amplifier? 2. What are the applications of OPAMP? 3. How many output terminals are there in OPAMP? 4. What is the function of inverting terminal?

NSOU ? CC-PH-06 38 Unit 5 ? To test a transistor using multimeter. To design a switch (NOT gate) using a transistor & study its performance Structure 5.1 Objectives 5.2 Introduction 5.3 Apparatus required 5.4 Experimental Procedure 5.5 Precautions 5.6 Oral Questions 5.1 Objective To test a transistor using a multi meter. To design a switch (NOT gate) using a transistor and study its performance. 5.2 Introduction A bipolar junction transistor (BJT) consists of two pn junctions. Transistors have three terminals-emitter, base and collector. The emitter is highly doped, base is lightly doped and collector is moderately doped. BJTs are of two types npn and pnp. Fig. 1. (a) npn transistor (b) pnp transistor EMITTER EMITTER COLLECTOR COLLECTOR BASE BASE 38

NSOU ? CC-PH-06 39 5.3 Apparatus required Transistor, multimeter, resistors. A. Transistor testing Transistor testing is carried out by applying the concept of pn-junction biasing. When a forward bias is applied to the pn-junction the junction allows current to pass through it and when reverse bias, it behaves as an open circuit. we can identify a transistor whether it is NoPN or PNP by testing the junction's continuity in the forward biasing mode and reverse biasing mode. Experimental Procedure First of all, set the knob of digital multimeter to Diode/Continuity position. Testing of NPN Transistor (i) Connect the positive probe of multimeter to the base terminal of the transistor and negative or common probe to the either emitter terminal or collector terminal (as shown in Fig. 2). In both the cases forward biasing condition will satisfy and multimeter will show some reading. Fig. 2 (ii) Connect the negative probe of multimeter to the base terminal of the transistor and positive probe to either emitter terminal or collector terminal (as shown in Fig. 3). In both the cases reverse biasing condition will satisfy and multi meter will show open circuit condition.

NSOU ? CC-PH-06 40 Fig. 3 (iii) Connect the negative probe of multi meter to the emitter and the positive probe to the collector. The multimeter will show open circuit condition. After interchanging, the positive probe to emitter and the negative probe to the collector (as shown in Fig. 4), still multi meter will show open circuit condition. Multimeter will show an open circuit condition between emitter and collector for both the directions. Fig. 4 Testing of PNP Transistor (i) Connect the negative probe of multimeter to the base terminal of the transistor and positive probe to either emitter terminal or collector terminal (as shown in Fig. 5). In both the cases forward biasing condition will satisfy and multimeter will show some reading.

NSOU ? CC-PH-06 41 Fig. 5 (ii) Connect the positive probe of multimeter to the base terminal of the transistor and negative probe of multi meter to either emitter terminal or collector terminal (as shown in Fig. 6). In both the cases reverse biasing condition will satisfy and multimeter will show open circuit condition. Fig. 6 (iii) Connect the negative probe of multi meter to the emitter and the positive probe to the collector. The multimeter will show open circuit condition. After interchanging, the positive probe to emitter and the negative probe to the collector (as shown in Fig. 7), multimeter will show open circuit condition. Multimeter will show an open circuit condition between emitter and collector for both the directions.

NSOU ? CC-PH-06 42 Fig. 7 B. Design of switch (NOT gate) using transistor A NOT gate is called inverter because it inverts the input. For high input, output will be low and for low input, output will be high. A switch or NOT gate can be realized using a transistor. Fig. 8. NOT Gate using a Transistor 5.4 Experimental Procedure (i) At first connect the circuit as per diagram shown in Fig. 8. (ii) When high voltage is applied as input to terminal A of the transistor. Transistor move to ON state. When the transistor is in ON state, transistor

NSOU ? CC-PH-06 43 behaves like short-circuited, so the voltage V cc gets dropped at R 2 and no voltage appears at the output terminal which is in logic low. (iii) When low voltage is applied as input to terminal A of the transistor. Transistor moves to OFF state. When the transistor is in OFF state, supply voltage has no path to move, entire voltage Vcc will appear at the output terminal Y which means that the output is at logic high. Table 1. Truth table for NOT Gate Input Output High Low Low High 5.5 Precautions 1. Choose the leads of the transistor carefully. 2. Connect the circuit components properly. 5.6 Oral Questions 1. Draw the logic symbol of NOT gate. 2. Give the truth table of A NOT gate. 3. Why the emitter of transistor doped heavily compared to others?
NSOU ? CC-PH-06 44 Unit 6 ?

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To verify and design AND, OR, NOT and XOR gate using NAND gates

Structure 6.1 Objectives 6.2 Introduction 6.3 Apparatus required 6.4 Result 6.5 Precautions 6.6 Oral Questions 6.1 Objective

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To verify and design AND, OR, NOT and XOR gate using NAND gates. 6.2

Introduction The NAND and NOR gates are said to be universal gates, because realization of other logic gates with the help of NAND and NOR gates are possible. le 7400 consist of four NAND gates. Each NAND gate utilizes two input pins and one output pin. This le has fourteen pins including one power supply and one ground pin. Schematic of the pin diagram is shown in Fig. 1. Fig. 1. le 7400 pin diagram. 44
NSOU ? CC-PH-06 45 6.3 Apparatus required IC 7400, trainer kit, wires, probes, etc. Experimental Procedure 1. Insert IC 7400 to the IC base of trainer kit and connect positive supply and ground to the pin 14 and 7 respectively. 2. Make connections of the circuit as per diagram shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5. 3. Provide various combinations of inputs to the input terminals and note down the corresponding output from the output LEDs in 4. Verify the Truth Table (Table 1, Table 2, Table 3 and Table 4). (i) AND gate from NAND gate: If the output of one NAND gate is inverted by another NAND gate, the final output will be an AND gate. Fig. 2. Realization of AND gate using NAND gate. Y =A·B Observation Table 1. Truth Table of NAND Gate INPUT OUTPUT A B Y 0 0 0 0 1 0 1 0 0 1 1 1
NSOU ? CC-PH-06 46 (ii) OR gate from NAND gate: If the output of two NOT gates realized by NAND gates are connected to another NAND gate, the final output will be an OR gate. Fig. 3. Realization of OR

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gate using NAND gate. Y A B Y A B Y A B = · = + = +

Observation Table 2. Truth Table of OR Gate INPUT OUTPUT A B Y 0 0 0 0 1 1 1 0 1 1 1 1
NSOU ? CC-PH-06 47 (iii) NOT gate from NAND gate: If all the inputs of a NAND gates are connected together, the output will be NOT gate. Fig. 4. Realization of NOT gate using NAND gate. Observation Table 3. Truth Table of NOT Gate INPUT OUTPUT A Y 0 1 1 0 (iv) XOR gate from NAND gate: Fig. 5. Realization of XOR gate using NAND gate.
NSOU ? CC-PH-06 48 Observation Table 4. Truth Table of XOR Gate INPUT OUTPUT A B Y AB AB = + 0 0 0 1 0 1 0 1 1 1 1 0 6.4 Result AND, OR, NOT and XOR gates are realized using NAND gates. Truth table for these gates are verified. 6.5 Precautions 1. IC should be tested before performing the experiment. 2. Connections should be tight. 6.6 Oral Questions 1. Why NAND and NOR gates are called universal gates? 2. Realize NOR gate using minimum number of AND gates. 3. Give the truth table of XNOR Gate.
NSOU ? CC-PH-06 49 Unit 7 ? To design a combinational logic system for a specified Truth Table
Structure 7.1 Objectives 7.2 Introduction 7.3 Theory 7.4 Apparatus 7.5 Experimental Procedure 7.6 Experimental Results 7.7 Discussion 7.8 Summary 7.9 Exercise

and Answer 7.1 Objective Students can solve any logical Boolean expression by making combinational logic system, also can simplify a logical expression. 7.2 Introduction The truth table displays the logical operations on input signals in a table format. Every Boolean expression can be viewed as a truth table. The truth table identifies all possible inputs combinations and the outputs for each. It is common to create the table so that the input combinations produce an unsigned binary up count. A combinational logic gate can specified a truth table. For a given Boolen expression, we can make a combinational logic circuit. 7.3 Theory Problem 1 : Verify the logic identify : AB + AC + BC – = AC + BC – 49 NSOU ? CC-PH-06 50 7.3(1) 0 Theory : The logic circuit required for the verification of the logic identity AB + AC + BC – = AC + BC – is shown in Fig. 7.4-1. The outputs y 1 and y 2 are to be measured for all possible values ('0' and '1') of A, B and C. It is found that y 1 – y 2 for all possible combination of A, B and C then the identify will be verified. 7.4(1) 0 Procedure : i) At first construct the logic circuit of Fig. 7.4-1 on a bread board by wing ICs-7432, 7408/7409, 7404, a 5V regulated power suppy and logic switch units. Indicate the pin number used. One such possible pin number are indicated in Fig. 7.4-1. For conveniences insert the ICs with the Pins 1-7 on one side and Pins 8-14 on the other side of the central groove on the bread board connect the +ve and –ve terminals of the 5V supplying to two side lines of the bread board, which may be called '+5V live' and 'ground live'. Connect pins 14 of all ICs to this +5V live and pins 7 to ground line. ii) Now, measure the voltage at the points A, B, C, y 1 and y 2 for all possible comibination of input voltages at A, B and C by using a dc voltmeter (0-5V). Record all these voltages in a table. iii) Detive suitable voltage ranges '0' and '1' and obtain the truth table show that y 1 – y 2 for all possible input combinations. Fig. 7.4-1
NSOU ? CC-PH-06 51 ? Experimental data : Types of IC Used : IC – 7432 (Quad 2 input AND) IC – 740817409 (Quad 2-input AND) IC – 7404 (Hex inverter) (A) Data for input and output voltages to verify the identify Table I Input Voltages in V at Output Voltage in V at A B C y 1 y 2 0.0 0.0 0.0 ... ... 5.0 0.0 0.0 ... ... 0.0 5.0 0.0 ... ... 0.0 0.0 5.0 ... ... 5.0 5.0 0.0 ... ... 0.0 5.0 5.0 ... ... 5.0 0.0 5.0 ... ... 5.0 5.0 5.0 ... ... (B) Truth table as obtained from TABLE I : Detive 0.0 – 0.8 V as '0' and 3.0 – 5.0 V as '1' Table II Row Input Output A B Cy 1 y 2 1 0 0 0 0 0 2 1 0 0 0 0 3 0 1 0 1 1 4 0 0 1 0 0 5 1 1 0 1 1 6 0 1 1 0 0 7 1 0 1 1 1 8 1 1 1 1 1 Note : Following the above procedure you can verify any other logic identity set in the examination. Further examples for practice.
NSOU ? CC-PH-06 52 (a) AB +
A –

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C = (A + C) (A – + B) (b) (A + B) (A + C) = A + BC (c) (A + B) (B + C) (C + A) =		

AB + BC + CA ? Problem (2) : A truth table is given. Write down the Boolean expression and construct a logic circuit to realize the truth table. ? Theory : Suppose the following is the given truth table : A B y 1 y 2 0 0 0 0 0 1 1 0 1 0 1 0 1 1 0 1 Corresponding Boolean expression can be obtained by 'the sum of the fundamental products' that produce 1 outputs. To get the output 1 for the input A = 0 and B = 1, we require the fundemental product A – B. Thus. y 1 = A – B + AB – y 2 = AB The logic circuit implementing the Boolean expression (7.4-1) and (7.4-2) is shown in Fig. 7.4-2 Fig. 7.4-2
NSOU ? CC-PH-06 53 ? Procidure : (i) Construct the logic circuit of Fig. 7.4-2 on a bread board in operation (i) of problem (1). (ii) Measure the voltages at A, B, y 1 and y 2 for all possible input combinations. Record the voltages in a table. (iii) Define suitable voltage ranges as '0' and '1' and obtain the given truth table. ? Experimental data : Types of IC used : IC – 7432 IC – 7408 IC – 7404 (A) Data for input and output voltages : Table I Input voltages in V at Output voltages in V at A B y 1 y 2 0.0 0.0 ... ... 5.0 0.0 ... ... 0.0 5.0 ... ... 5.0 5.0 ... ... (B) Truth table as obtained from Table I : Define .... to .... as '0' and .... to .... as '1' Table II Input voltages in V at Output voltages in V at A B y 1 y 2 0.0 0.0 ... ... 5.0 0.0 ... ... 0.0 5.0 ... ... 5.0 5.0 ... ... Note : Following the procedure as describe above you can obtain the Boolean expression corresponding to any given truth table. Then you can construct the logic circuit and finally by measuring input the output voltages the given truth table can be verified. However, the following points are note worthy for consideration.
NSOU ? CC-PH-06 54 (a) The 'sum-of-products' from may be more complen than necessary. In that case, simplify the Boolean expression by wing postulates before implementing the logic circuit. (b) Sometimes an alternative 'products-of-sums' form may come out to be convenient. For example, consider the truth table of OR gate : A B y 0 0 0 1 0 1 0 1 1 1 1 1 In the 'sum-of-products' form the logic function is, Y = AB – + A – B + AB Here one considers those rows for which y = 1; it a variable has the value 0 it is complemented and it the variable has the value 1 it is left unchanged.

Y =

69% **MATCHING BLOCK 6/11** W

A(B – + B) + A – B = A + A – B = A(1 + B) + A – B = A + (A + A –)B = A + B

But in the 'product-of-sums' form the logic function is obtained at once as y = A + B. Here one considers only those rows for which y = 0; if a variable has the value 0 it is left unchanged and it the variable has the value 1 it is complemented. ? Problem (2) : Implement the logic circuit of an equality detector which gives an output 1 if A and B are both 1 or both 0. 7.3(ii) ? Theory : The truth table corresponding to the given problem is the following : A B y 0 0 1 1 0 0 0 1 0 1 1 1

NSOU ? CC-PH-06 55 The 'sum-of-product' form of the Boolean expression is Y = A – . B – + AB The logic circuit implementing this Boolean expression is shown in Fig. 7.4-3. 7.4 Apparatus 7.4(ii) ? Procedure and Experimental data : [Similar to that in problem (2)] Fig. 7.4-3 7.8 Summary A truth table is a mathematical table used in logic specially in connection with Boolean Algebra, boolean functions and propositional calculus which sets out the functional values of logical expressions on each of their functional arguments i.e. for each combination of values taken by their logical variables. In particular, truth tables can be used to show wether a propositiona expression is true for all legititime input values, i.e. logically valid.

NSOU ? CC-PH-06 56 7.9 Exercise and Answer ? What is a truth table? ? Instead of describing in words the input-output relationship of a logic gate for all possible input combinations can be shown in a table. This table is known as truth table. ? What is a LED? ? LED stands for light emitling diode. It is a special type of semiconductor diode made of direct gap semiconductors like GaAs. It emits light in forward biased condition due to recombination of holes and electron. In Si and Ge recombination occured via traps so, the energy liberated is converted into heat. But in GoAs, direct recombination leval place without the aid of traps and hence the energy released appears in the form of light. ? What are the practical used of the basic logic gated? ? They are the basic 'building blocks' of more complex digital circuits including a digital computer. ? What do you use a resistor in series with the LED? ? To limit the current through the LED to the safety limit (typical current rating is at the order of 50 mA). ? What is an inverter? ? Since a NOT circuit inverts the sense of the output with respect to the input, the circuit is also called an inverter. ? In a NOT circuit what condition of a transistor indicate a logical 1 and a logical 0 state? ? When the transistar is in cutoff the output is high i.e., a logical 1 state. When the transistor is in sathration, the output is low i.e. a logical 0 state. ? Can you identify the material of the transistar from the given data? ? Yes, it V BE = 0.7 V then it is Si-made, it V BE ≈ 0.3 V then it is Ge-made.

NSOU ? CC-PH-06 57 Unit 8 ? To design Half Adder. Full Adder using ICs

Structure 8.1 Objectives 8.2 Introduction 8.3 Theory 8.4 Apparatus 8.5 Experimental Procedure 8.6 Experimental Results 8.7 Discussion 8.8 Summary 8.9 Exercise

and Answer 8.1 Objective To design Half Adder and Full Adder using ICs. 8.2 Introduction In the combinational circuits, different logic gates are used to design encoder, multiplexer, decoder and de-multiplexer. These circuits have some characteristics, like output of this circuit, mainly depends on the levels which are there at input terminals at any time. This circuit does not have any influence on the current state. The inputs and outputs of a combinational circuit are 'n' no. of inputs and 'm' no. of circuits are half adder and full adder, substractor, encoder, decoder, and multiplexer. 8.3 Theory A half-adder is a logic circuit which can add two binary digits and provides a sum (S) and a carry (C). The truth table of a half-adder is shown in table A. From the truth table we obtain the Boolean expressions for S and C as 57

NSOU ? CC-PH-06 58 S = A – B + AB – and C = AB Table A Input Output A B S C 0 0 0 0 0 1 1 0 1 0 1 0 1 1 0 1 The circuit realization of a half-adder using basic gates is shown in Fig. 8(i). A full-adder is a logic circuit which can add three binary digits at a time giving a sum and a carry. The truth table of a full-adder is shown in table B, where A n and B n are the n-th order bits of two binary numbers to be added and C n–1 is the carry generated from the of the lower (n – 1)th order bits. From the table we get the Boolean expressions S n and C n as Fig. 8(i)

NSOU ? CC-PH-06 59 S n =

A

n B n C

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n−1 + A n B n C n−1 + A n B n C n−1 + A n B n C n−1 C n = A n B n C n−1 + A n B n C n−1 + A n

B n C n−1 +
A n B n
C n−1 Fig. 8(ii) Table B Input Output A n B n C n−1 S n C n 0 0 0 0 0 0 0 1 1 0 0 1 0 1 0 0 1 1 0 1 1 0 0 1 0 1 0 1 0 1 1 1 0 0 1 1 1 1 1 1
NSOU ? CC-PH-06 60 Using Boolean postulates these expressions can be simplified to S n = (A n + B n + C n−1) C n + A n B n C n−1 and C n = A n B n + B n C n−1 + C n−1 A n The logic circuit implementing these Bodean expressions using basic gates is shown in Fig. 7.5-2. 8.4 Apparatus (i) IC-7432, IC-74087409, IC-7404, IC-7400, IC-7402, (ii) a regulated dc power supply (5V, 1A), (iii) a dc voltmeter (0-5V) or a digital multimeter, (iv) a bread board, some single strand wires and logic switch (SPDT) units. 8.5 Experimental Procedure (i) At first construct the logic circuit of Fig. 7.5-1 on a breed board by using ICs 7404, 7408 and 7432, a 5v regulated power supply and logic switch units. Inseart the ICs with the pins 1-7 on one side and pins 8-14 on the other side of the central groove on the bread board. Connect the tve and − ve terminals of the 5v power supply to the two side lines of the bread board which may be called '+5v line' and 'ground line'. Connect the pin 14 of all ICs to this +5v line and pins 7 to ground line. (ii) Now measure the voltage at the points A, B, C and S for all possible combinations of input voltage at A and B by using a dc voltmeter '0-5v' or a digital multimeter. Record all these voltages in a table. (iii) Define suitable voltage ranges as 0 and 1 and oftain the truth of the half- adder. (iv) Construct the full-adder circuit of Fig. 7.5-2 on the bread board as before. Measure voltage at points A n , B n , C n − 1 , S n and C n for all possible combinations of the input voltages. Define suitable range as 0 and 1 and verify the truth of the full-adder. 8.6 Experimental Results IC-7404 (Hex inverter) IC-7408/7409 (Quad 2-input AND)
NSOU ? CC-PH-06 61 IC-7432 (Quad 2-input OR) (A) Data for input and output voltage for half-adder : Table I Input voltage in v at Output voltage in v at A B S C 0.0 0.0 ... ... 0.0 5.0 ... ... 5.0 0.0 ... ... 5.0 5.0 ... ... (B) Truth table as mentioned from Table I : Define 0.0−0.8v as 0 and 3.0−5.0v as 1 Table II Input Output A B S C 0 0 0 0 0 1 1 0 1 0 1 0 1 1 0 1 (C) Data for the input and output voltage for full-adder : Table III Input Voltages in V at Output Voltage in V at A B Cy 1 = S n y 2 = C n 0.0 0.0 0.0 ... ... 5.0 0.0 0.0 ... ... 0.0 5.0 0.0 ... ... 0.0 0.0 5.0 ... ... 5.0 5.0 0.0 ... ... 0.0 5.0 5.0 ... ... 5.0 0.0 5.0 ... ... 5.0 5.0 5.0 ... ...
NSOU ? CC-PH-06 62 (D) Truth table as mertioned from Table III Define 0.0−0.8v as 0 and 3.0−5.0v as 1. Table IV Input Output A n B n C n−1 S n C n 0 0 0 0 0 0 0 1 1 0 0 1 0 1 0 0 1 1 0 1 1 0 0 1 0 1 0 1 0 1 1 1 0 0 1 1 1 1 1 1 8.7 Discussions (i) The bread board makes circuit connections highly flexible. There is no need of soldering or using binding screws. (ii) Unlive analog circuits, here absolute values of the input and output voltages are not important the only requirement is that these voltages must be high or low and lie within certain specified ranges. (iii) Three LEDs with proper current limiting resistances can be used at the points A, B and Y for easy, quick and visual identification of '0' and '1' states. If a LED glows then it is a '1' state and if it does not glow then it is a '0' state. (iv) Implementation of digital gates by using IC is more popular than their implementation by using discrete circuit components. This is due to low cost, small size, low power requirement and improved performance of ICs.
NSOU ? CC-PH-06 63 (v) While connecting the +5v dc supply to the ICs, care should be taken. Connection to any wrong pin may damage the IC. (vi) Here the circuits have been designed by using basic gates. The circuits can be conveniently designed by using Ex-OR ICs (7486). 8.8 Summary With the help of half adder, we can disign

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circuits that are capable of performing simple addition with the help of logic gates.

Though

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the implementation of larger logic diagrams is possible with the full adder logic a simpler symbol is mostly used to represent the operation. 8.9

Exercise and Answer ? What is clock? ? Clock is usually multivibrator to generate pulse at an exact frequency to synchronize the logic circuits. ? What are set and reset inputs? ? In a flip-flop circuit a set input flips the output to the opposite condition from its initial state. A reset input flops the output back to its initial stage. ? What is half-adder? Why is it so-called? ? See, theory of Expt. 7.5 A half adder cannot handle the carry coming from the lower order bits. Thus a half-adder is only a step towards the binary addition and to complite the process are require a full-adder. That is why it is called a half- adder. ? What is full-adder? ? See, theory of Expt. 7.5. ? Construct a half adden using NAND/NOR only. ? See the suggested work at the end of Expt. 7.5.

NSOU ? CC-PH-06 65 Unit 9 ? To design a Half Subtractor, Full Subtractor, Adder Subtractor using Full Adder IC Structure 9.1 Objectives 9.2 Introduction 9.3 Theory 9.4 Apparatus 9.5 Experimental Procedure 9.6 Experimental Results 9.7 Discussion 9.8 Summary 9.9 Exercise

and Answer 9.1 Objective To design and verify Half Subtractor and Full Subtractor using Universal logic gates. 9.2 Introduction Quite similar to the half adder, a half subtractor subtracts two 1-bit binary numbers to give two outputs, difference and borrow. Since it neglects any borrow inputs and essentially performs half the function of a subtractor, it is known as the half subtractor. Let's write the truth table based on this information and general binary subtraction rules. 0 – 0 = 1 0 – 1 = 1, borrow 1 1 – 0 = 1 1 – 1 = 0 A full subtractor accounts for the borrow that a half subtractor neglects. Hence it has three inputs and two outputs. 65

NSOU ? CC-PH-06 66 9.3 Theory Half Subtractor : Half Subtractor is a combinational circuit which consists of two binary input variables called minued and subtrahend, and two binary output variables called difference and borrow. In the two bit subtraction result, the lower significant bit is called as difference and higher significant bit is called as borrow. The truth table of the subtractor is given below in that the difference becomes logic '1' when both inputs are different each other and it is equal to logic '0' when both inputs are equal. And borrow is equal to logic '1' when minuend is smaller than subtrahend. Full Subtractor : Full subtractor is a combinationd circuit which consists of three binary inputs variables called minuend and subtrahend and two binary outputs variables called difference and borrow out. In this subtraction result, the lower significant bit is called as difference and the higher significant bit is called as borrow out. The truth table of the full subtractor descirbes all the eight possible input variations. The full subtractor results the output are equal to logic '0' when all the applied inputs are equal to logic '0' or most significant bit and any one of the least significant bit is equal to logic '1' and the outputs are equal to logic '1' when all the inputs are equal to logic '1' or any of the subtrahend is equal to logic '1'. The difference is equal to 1 when odd numbers of inputs are equal to 1 from the applied three inputs. The borrow out is equal to 1 if any one of the subtrahend or all the applied inputs are equal to logic '1'. 9.4 Apparatus Component Specification Quantity AND Gate IC 7408 1 EX-OR IC 7486 1 OR Gate IC 7432 1 IC Trainer Kit — 1 Patch Chord — Adequate

NSOU ? CC-PH-06 67 9.5 Experimental Produre (i) Verify the truth table of the given logic Gates. (ii) Connection to be made as per the circuit diagram. (iii) All the possible input variations are to be given. (iv) Observe the output and verify the truth table. Full Subtractor K-Map for Difference K-Map for Borrow (Bout) : 00 01 11 10 00 01 11 10 0 0 1 0 1 0 0 1 1 1 1 1 0 1 0 1 0 0 1 0 Sum (S) = + + + ABB ABB ABB ABC in in in Borrow (Bout) = + + AB AB BB in in = + ( ) + + ( ) AB AB B AB AB B in in = + + + + ( ) AB A B B B A A BB in in ( ) = ⊕ ( ) + ⊕ ( ) A B B A B B in in = + + + AB ABB ABB ABB in in in Sum (S) = ⊕ ⊕ A B B in = + + + ( ) AB ABB AB AB B in in = A – B + (A O . B) B in (Bout) = + ⊕ ( ) AB A B B in A BB in A BB in

NSOU ? CC-PH-06 68 9.6 Experimental Procedure Half Subtractor ⇒ Logic Diagram Truth Table A B BORROW DIFFERENCE 0 0 0 0 0 1 1 0 1 0 1 0 1 1 0 0 Half Subtractor K-Map for Difference K-Map for Borrow (Bout) : 00 01 00 01 00 1 00 1 01 1 01 DIFFERENCE = A – B + AB – BORROW = A – B A B A B

NSOU ? CC-PH-06 69 Full Subtractor ⇒ Full Subractor using two Half Subtractor. Truth Table Input Output A B B in B out D 0 0 0 0 0 0 0 1 1 1 0 1 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 0 0 0 1 1 0 0 0 1 1 1 1 1 9.7 Discussion You have already learned different binary adder circuits like Half Adder, Full Adder, Parallel Adder and different binary Subtractor, Full Subtractor and also a combination Parallel Adder & Subtractor Circuits.

NSOU ? CC-PH-06 70 9.8 Summary It is possible to convert the full adder circuit into full subtractor by simply complementing the input A before it is applied to the gates to produce the final borrow bit Output (Bout). 9.9 Exercise and Answer ? What is clock? ? Clock is usually multivibrator to generate pulse at an exact frequency to synchronize the logic circuits. ? What are set and reset inputs? ? In a slip-flop circuit a set input flips the output to the opposite condition from its initial state. A reset input flops the output back to its initial stage. ? What is half-adder? Why is it so-called? ? See, theory of Expt. 7.5 A half adder cannot handle the carry coming from the lower order bits. Thus a half-adder is only a step towards the binary addition and to complite the process are require a full-adder. That is why it is called a half- adder. ? What is full-adder? ? See, theory of Expt. 7.5. ? Construct a half adden using NAND/NOR only. ? See the suggested work at the end of Expt. 7.5. ? What is a latch? ? It is another name for a SR flip-flop which can store 1 bit of information. It is called a latch as the information remains locked or latched in the circuit until driggered into alternate state. ? What do you mean by set and reset? ? In a flip-flop the state Q = 1 is referred to as the 'set' and the 'set' and the state Q = 0 as 'reset'. ? What is the need of a clocked flip-flop? ? Very often it is required to set or reset the flip-flops in synohronism with a train of pulses known as clock. Thus the need arises of clocked flip-flops?

NSOU ? CC-PH-06 71 Unit 10 ? To study the diffraction pattern of a crossed grating with the help of a LASER source Structure 10.1 Objectives 10.2 Introduction 10.3 Theory 10.4 Apparatus Required 10.5 Experimental Procedure 10.6 Observation 10.7 Precautions 10.8 Oral Questions 10.1 Objective To study the diffraction pattern of a crossed grating with the help of a LASER source. 10.2 Introduction When n number of slits of same width placed side by side separated by opaque spaces, an optical element is formed produced known as grating. Diffraction pattern produced by a grating is known as grating spectrum. In crossed grating arrangement two gratings are placed at right angles to each other. 10.3 Theory When an unexpanded laser beam incident normally on the grating the beam of light will be diffracted through the grating and a number of principal maxima of different orders on both sides of central maximum will be formed. If we rotate the grating by 90° from its previous position pattern of the diffracted beam will also rotate by 90°. 71

NSOU ? CC-PH-06 72 When the diffracted order beam produced from first grating is allowed to enter into another grating positioned at right angles to the first one, each diffracted order beam will again be diffracted by second grating and it will produce a diffraction pattern with reticular structure. 10.4 Apparatus Required Laser source, diffraction grating - two, screen, mount etc. 10.5 Experimental Procedure (i) Switch ON the laser. (ii) Place the first grating in the path of laser beam as shown in Fig. 1 and try to observe the diffraction pattern (pattern is shown in Fig. 4). Now rotate the grating by 90°. from its previous position and again try to observe the diffraction pattern (pattern is shown in Fig. 5). (iii) Place the crossed grating arrangement in the path of laser beam as shown in Fig. 3 and try to observe the diffraction pattern (pattern is shown in Fig. 6). (iv) Observe the diffraction pattern Fig. 1. Experimental setup for observing diffraction pattern from a grating probed by a laser beam.

NSOU ? CC-PH-06 73 Fig. 2. Experimental setup for observing diffraction pattern from a grating rotated by 90° from its previous position (shown in Fig. l) and probed by a laser beam. Fig. 3. Experimental setup for observing diffraction pattern from a crossed grating probed by a laser beam. 10.6 Observation Number of lines per centimetre of I st grating = ... lines/cm Number of lines per centimetre of 2nd grating = ... lines/cm Fig. 4. Diffraction pattern formed by a grating when probed with a laser beam (shown in Fig. l).

NSOU ? CC-PH-06 74 Fig. 5. Diffraction pattern formed by a grating (rotated by 90°) when probed with a laser beam (shown in Fig. 2). Fig. 6. Diffraction pattern formed by a crossed grating when probed with a laser beam (shown in Fig. 3). If both the gratings have same grating constant then the pattern will be a square shape otherwise it will be a rectangular shape.

NSOU ? CC-PH-06 75 10.7 Precautions 1. Grating surfaces should be normal to the incident rays. 2. Both the gratings should be in crossed position. 3. Never keep your eyes in the path of the laser beam. 4. Do not touch the surface of the grating. 10.8 Oral Questions 1. What is a diffraction grating and how does it work? 2. Define grating constant? 3. What are the uses of diffraction grating?

NSOU ? CC-PH-06 76 Unit 11 ? To draw the characteristics of a JFET and hence to determine relevant parameters Structure 11.1 Objectives 11.2 Introduction 11.3 Apparatus Required 11.4 Experimental Procedure 11.5 Precautions 11.6 Oral Questions 11.1 Objective To draw the characteristics of a JFET and hence to determine relevant parameters. 11.2 Introduction The difference between bipolar junction transistor (BJT) and filed effect transistor (

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FET) is : BJT is a current controlled device whereas FET is a voltage controlled device.

In case of BJT, input current controls the output characteristics whereas for FET input voltage controls the output characteristics. Current conduction in FET takes place either by electrons or holes, hence is known as a unipolar device. Schematic diagram of n-channel JFET and p-channel JFET is shown in Fig. l(a-b) 76
NSOU ? CC-PH-06 77 Fig.1. Basic structure of (a) n-channel JFET (b) p-channel JFET. 11.3 Apparatus Required FET, bread board, resistor, connecting wires, D.C. power supply, two voltmeters, a milli-ammeter. JFET parameters (i) Drain resistance change in drain source voltage r D = ——————————————— at constant V GS change in drain current (ii) Transconductance change in drain current g fs = ——————————————— at constant V DS change in gate-source voltage (iii) Amplification factor change in drain source voltage μ = ——————————————— at constant I D change in gate-source voltage
NSOU ? CC-PH-06 78 Relationship among JFET parameters μ μ = = × Δ Δ Δ Δ Δ Δ V V V V I I DS GS DS GS D D ; μ μ = × = × Δ Δ Δ Δ V I I V r g DS D D GS d fs ; 11.4 Experimental Procedure Output Characteristics (i) At first connect the circuit as shown in Fig. 2. (ii) Keep V GS = 0V and vary Drain-Source voltage V DS in step of 0.5 volt and note the values of Drain-Source voltage V DS and corresponding values of Drain current I D till the Drain current becomes constant. (iii) Keep the Gate-Source bias at – (...)V and note the values of Drain-Source voltage V DS and corresponding values of Drain current I D till constant readings of I D are obtained. (iv) Repeat with a Gate- Source bias of – (...)V. Fig 2. Circuit for determining characteristics of JFET.
NSOU ? CC-PH-06 79 Transfer Characteristics (i) Set the Drain-Source voltage V DS constant at + (...)V (as per the data of FET used) (ii) Vary the Gate-Source voltage V GS from zero towards negative values in steps of 0.5V. Note the values of V GS and corresponding values of Drain current I D . As the negative Gate Source voltage increases the Drain current I D decreases, take the reading till the current I D becomes zero. (iii) Similarly repeat with Drain Source voltage of + (...)V.

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Observation TABLE 1. Output characteristics Sl. No. V GS = 0V V GS = – (...)V V GS = – (...)V V DS I D V DS I D V DS I D TABLE 2. Transfer characteristics Sl. No. V DS = (...)V V DS = (...)V V GS I D V GS

I D
NSOU ? CC-PH-06 80 Graph (i) For output characteristics, plot graphs between Drain current I D (along Y- axis) and Drain-Source voltage V DS (along X-axis) for various values of Gate-Source voltage V GS . (ii) For transfer characteristics, plot graphs between Drain current I D (along Y- axis) and Gate Source bias V GS (along X-axis) for various values of Drain Source voltage Y DS . Calculation (i) from drain characteristics Drain resistance can be calculated. r D = Δ Δ V I DS D at constant V GS (ii) from transfer characteristics Transconductance can be calculated. g fs = Δ Δ I V D GS at constant V DS (iii) Amplification factor can be calculated by Drain resistance and Transconductance value. μ = r d × g fs Result 1. r D = ... 2. g fs = ... 3. μ = ... 11.5 Precautions 1. Check maximum rating of FET up to which it can be used. 2. According to n-channel or p-channel FET apply the Gate-Source voltage and Drain Source voltage. 3. Connections should be tight.
NSOU ? CC-PH-06 81 11.6 Oral Questions 1. What is the difference between BJT and FET? 2. What is Transconductance? 3. What is pinch off voltage? 4. What is Shorted gate drain current? 5. What is Gate to source cut off voltage?
NSOU ? CC-PH-06 82 Unit 12 ? Determination of thickness of a thin film by using Fresnel's bi-prism
Structure 12.1 Objectives 12.2 Introduction 12.3 Theory 12.4 Apparatus 12.5 Experimental Procedure 12.6 Experimental Results 12.7 Discussion 12.8 Summary 12.9 Exercise

and Answer 12.1 Objective To determine the thickness of a thin sheet of mica with the help of Fresnel's biprism 12.2 Introduction The Results of Young's double slit experiment quite clearly indicate interference and the wave nature of light, when the experiment was first done objections were raised that the results were not conclusive since there could have been difference effect from the edge of the slits. To counter this, Augustin Fresnel proposed a series of interference experiments that would have no diffracting edge. The most notable of these is the Fresnel's Biprism, where two virtual source are created by refraction through a biprism. 82

NSOU ? CC-PH-06 83 12.3 Theory It, in Fig. 12.3-(i), a thin sheet of mica of thickness t is introduced in the path of one of the interfering beams by covering one-helf of the biprism with mica sheet, the optical path length of the beam increases by t( ) μ λ –1 when μ is the refractive index of the sheet for the wave length ? of the light used. As a result, the frige patttern on the serean AB is displaced towards the beam in the path of which the mica sheet is placed. If s is the shift of the central fringe on the screen and x is the fringe width, the increase in the optical path is also given by s/x wave lengths. So, t s x ( ) μ λ – = 1 .... (i) But λ = xd D .... (ii) where d is the distance between the virtual sources S 1 and S 2 and D is the distance between the slit and the screen. Fig. 12(i) Noting that D = D 1 + ?, where D 1 is the aparent distance between the slit and the focal plane of the eye-piece, and ? is the index error, we obtain from (i) and (ii)

NSOU ? CC-PH-06 84 t sd D = + – ( )( ) 1 1 β μ .... (iii) The index correction can be avoided by observing the shifts s 1 and s 2 for two apparent distance D 1 and D 2 between the slit and focal plane of the eye-piece respectively. Then eq. (iii) gives D s d t 1 1 1 + = – β μ( ) & D s d t 2 2 1 + = – β μ( ) ∴ – = – – D D d s s t 2 1 2 1 1 ( ) ( ) μ or, t d s s D D = – = – – μ 1 2 1 2 1 .... (iv) To determine d, a convex lens whose focal length f is such that D is greater than if, is placed between the biprism and the eye piece. For two positions of the lens, real images of the virtual sources s 1 & s 2 are obtained on the focal plane of the eye-piece. If d 1 and d 2 are the distance between the real images of s 1 and s 2 for the two positions of the lens, then d d d = 1 2 .... (v) 12.4 Apparatus (i) A Feresnel biprism (ii) an source of menochromatic light (iii) a source of white high (iv) a slit (v) micrometer eye-piece (vi) a convex lens of suitable focal length (vii) an optical bench with four stands (viii) a plump line

NSOU ? CC-PH-06 85 (ix) an index rod (if required) (x) a thin mica sheet and (xi) a clamped stand (if required) 12.5 Experimental Procedure The procedure consists of two parts : (a) adjustment, and (b) measurements (a) Adjustments : 1. Mount the upright carrying the slit near the zero on the scale on the optical bench. Set up a phemp line near the slit and make the datter vertical by the tangent screw attached to it. Look along the axis of the bench from the other end and bring the slit in the middle of the bench by moving the slit stand perpendicular to the bend. Make the slit very narrow by the screw for adjusting the slit width. 2. Mount the biprism on its stand with its plane face dioected towards the slit and at right angles to the optical bench. Move the stand towards the slit until it is in contact with the slitstand. Adjust the height of the biprism on that the centre of the biprism and that of the slit are in the same height. 3. Illeminate the slit and sodium light and adjust the position of the flame so that its brightest part of in the front of the slit. Place a screen with a linear apperture between the slit and the source to cut off any stray light. On looking at the slit through the biprism along the middle line of the optical path, turn the transverse screw of the biprism till the common base of the biprism is found to move across the slit. Then places the biprism in the middle of the bench. If there is an appreciable angle between the slit and the base of the biprism, adjust the tangent screw of the biprism till they are parallel. 4. Place the upright carrying the eye piece on the optical bench and close to the biprism. Adjust the height of the eye piece so that its axis may be at the same height at the centre of teh biprism. Focus the eye piece on the cross

NSOU ? CC-PH-06 86 wires and move it perpendicular to the length of the bench until its axes passes through the centres of the biprism and the slit. Now intistinct fringes appear in the field of view. If on looking through the eye piece you nither find a fringe system nor a bright patch of light in the field view, turn the transverse screw of piprism till it is found. Then make fringes as distinct as possible by rotating the prism in own plane by tangent screw. 5. After getting well defind fringes when the bases of slit stand and the biprism stand are in contact proceed to cheeck whether d 1 and d 2 will be observable. For this, take canvex lens for d 1 and make a rough estimate for focal length. Shift eyepiece slowly and place it at 4.5 times the focal length from the slit. While moving eye piece, take care that the fringes never go out of sight. To keep the fringes in the field view, adjust the transverse screw of biprism continuously. Now mount the lens on the optical bench with an upright between prism and eyepiece. Adjust height and position so that centre is an the line joining everything move the lens along bench to get real image of virtual source s 1 and s 2 in the focal plane of the eye piece for two position of the lens. 6. Remove the lens from the bench. Slowly shift the upright carrying eyepiece away from biprism and place it near the end of optical bench. While moving eyepiece keep fringes in field view by adjusting transverse screw. If intensity decreases with distance, adjust slit width, position and intensity of sodium flame. If fringes are too broad, move the biprism slowly away from slit till fringe width lie within 1 to 2 milimiters. Note that the biprism must not be moved to a distance greater than that corresponding to the position of the lens nearer to the biprism in the previous operation. 7. Place and switch on a high power electric lamp behind the sodium flame. Adjust in height so that the incandescent filament is at the same height as the axis of the eye piece. Coloured fringes with a white one at centre will be observed in field view. Bring the central fringe on the cross wires by adjusting transverse screw of eye piece. Move the eye piece close to prism,

NSOU ? CC-PH-06 87 keeping central fringe on the cross wire by adjusting micrometer screw. Next move eyepiece away from biprism and bring it near end of optical bench. Keep the central fringe on the cross wires by continuously adjusting the transverse screw. This are repeated till the central fringe is an the cross wires for all position of eye piece. (b) Measurments : 1. Remove the white light. Determine the vernier constant for bench stands and the least count for the micrometer of eye piece. Fix the eye piece at a distance of about 4.5 times the focal length from the slit. Place the stand carrying the lens of the optical bench between biprion and eye piece, make it coanial with the latter. Now on moving the lens stand along the bench, magnified real image of virtual sources s 1 and s 2 will be seen in the field of eye piece. Bring the images at equal distances from the cross wires by rotating lens about vertical axis. This ensures that images of both s 1 and s 2 appear simultaneously in focus at each of the two positions of the lens. Measure the distance d 1 between two frings by shifting the eye price perpendicular to the bench and setting the cross wires on the same eye (left or right) of the image. Make this measurement four times, once shifting the eye piece frame left to right again fram right to left. The mean of four values gives d 1 . Re-adjust the lens in this position repeat readings for the more independent settings of lens. Take mean value for d 1 . Next move the lens towards the eyepiece keeping the latter fixed in previous position till sharp images of virtual sources s 1 and s 2 are seen in field view. Determine the distance between of two images d 2 in a same manner. From this value of d 1 and d 2 , d is calculated from Eq. (vi) [If time permit determine d for two other position of eyepiece on bench, close to previous position. Use mean value to calculate t. Note that in each case the position of eye piece should be such that its distance from slit is greather than 4 times the focal length of lens'. 2. Replace the sodium light by an incandescent lamp (white light) so that a central white fringe flanked by a few coloured fringe is obtained. So the

NSOU ? CC-PH-06 88 cross wires of the eye piece on the white fringe and take the reading of the micrometer screw. The stand carrying the mica sheet is now so placed that the mica sheet covers only one half of the prism. Using a lateral motion screw and looking towards the slit and start edge of the mica film is made parallel to the prism edge. This answers that the sheet is introduced in the path of only one of the intefering beams. The fringe pattern is displaced consequently. Shift the eye piece by furning the micrometer screw and set the cross wires of the eye piece on the white frings again. Take the reading of the screw. The difference between the two readings given the shifts of the pattern on the insertion of the mica sheet. Measures once again by removing the mica sheet and shifting the eye piece in the reverse direction. Take the mean of these two values. Repeat the readings for two more impedent settings of the mica sheet. Find the grand means. 3. Read D 1 . 4. If the index correction is to be avoided, repeat the step 2 for another distance D 2 of the eye piece without changing the positions of the slit and the Biprism. 5. If the index correction is required find the error ? as in Exp. No. 01. Taking value of ? for mica for the D-line of sodium from Physical tables, Calculate the thickness t from Eq. (iii) and Eq. (iv). 12.6 Experimental Results Table 1 Measurement of the slights of the fringe pattern. (a) Position of the slit on the bench = ——— + ——— = ——— cm (a) Position of the biprism on the bench = ——— + ——— = ——— cm (a) Position of eye piece on the bench = ——— + ——— = ——— cm Apparent distance between the slit and the eye piece. D 1 = (c – a) cm

NSOU ? CC-PH-06 89 No. Position Direction Reading (mm) of the micrometer screw for the Pattern Means Grand of of mica of eye eye piece set at the central white fringe shift (cm) mean obs sheet piece without the mica sheet with mica sheet (x 2 ~ x 1) (cm) on the move- Linear Circular Total Linear Circular Total (mm) bench ment (x 1) (x 2) 1. forward .... backward .... 2. forward .... backward .... 3. forward .... backward .... Table 2 Today error between the slit and the eye piece Position of Position of the Apparent length Actual length Index error the slit eyepiece when of the rod of the rod β = (l 1 – l 2) a 1 (cm) the end of the l 1 = (a 1 ~ a 2) l 2 cm cm rod is in clear cm focus a 2 (cm) .... Table 3 Determinatia of teh thickness of the mica sheet (meaning the index correction) Distance Apparent distance Index error D 1 + β Pattern Frefractive Thickness (cm) D 1 (cm) β (cm) (cm) shift index of t (cm) (cm) mica μ [The table for determining t avoiding the index correction is left for the Hudeep to male]

NSOU ? CC-PH-06 90 12.7 Discussion 1. In the measurement of the distance (d) between the two virtual sources S 1 and S 2 , the distance between the slit and the screen should be nearly equal to 4.5 times the focal length of the Convex Lens so that d 1 and d 2 do not differ largely. This will reduce the error is measuring d. 2. While using the micrometer screw, core should be taken to avoid back lash error arising from the misfit between the micrometer screw and the nut in an old instrument. To do this, the eye-piece should be moved beyond the image cocerned before reversing its direction of movement from left to right, or vice versa. One can also find the distance from the initial and final readings of the screw and counting the number of complete turns without depending on the linear scale. 3. While measuring d 1 and d 2 . the images may be distored due to spherical aberration. To avoid this distortion, a stop with a passage of light through the central portion of the lens may be used. N.B. : If possible, repeat the experiment for another value of D 2 , say D 3 . Calculate three a values of λ from Eq. (v) for D 2 ~ D 1 , D 3 ~ D 2 and D 3 ~ D 1 , and find mean λ. If, however, any of the three differences, D 2 ~ D 1 , D 3 ~ D 2 and D 3 ~ D 1 is quite small, the percentage error in λ will be large. In that case calculate three values of λ from Eq. (iv) for D 1 , D 2 and D 3 making index correction and obtain the mean 'λ'. 12.8 Summary Fresnel's biprism can be used to determine the wavelength of a light source (monochromatic light), refractive index of medium etc. Fresnel's biprism retracts and produces two virtual images which acts like cohorent sources. An interference pattern is produced which can be seen with an eye piece.

NSOU ? CC-PH-06 91 12.9 Exercise and Answer 1. In the Fresnel biprism experiment, how are two coherent sources realisted? Ans. The two virtual sources produced by refraction of light from a single source through the two halves of the biprism, serve as the two coherent sources in the Fresnel biprism experiment. 2. If, instead of a monochromatic light white light is used in Fresnel biprism experiment, what will you see in the fringe system? Ans. While light is a combination of seven colours the path difference between two interfering rays is zero for all wavelengths. Hence the central fringe will be while. Again the spacing between two consecutive bright or dark fringes decreases with the decrease of wavelength [Eq. (i)]. As a result, the bright fringe nearer to the central fringe will have strong violet tinge, followed by other bright fringes having a strong fringe of colours in the spectral order. After some fringes the path difference between two interfering light waves becomes so large that the conditions for constructive and destructive inference may be satisfied at a given point simultaneously for a number of wavelengths. So, the dark fringes of some wavelengths are completely marked by the bright fringes of other wavelengths. Moreover, at some points, so many colours may superpose that the resultant illumination essentially appears white. 3. Who do you use a narrow slit? What will be the effect on the fringes if you broaden the slit? Ans. A narrow slit is used because it produces distinct fringes. When the slit is gradually broadened, the fringe system first becomes blurred, because a broad source will serve as many narrow sources will produce two coherent virtual sources and the fringe system due to one pair of virtual sources will not be formed exactly in the same position as that due to other pairs, resulting in a blurred fringe system.

NSOU ? CC-PH-06 92 When the slit is sufficiently broadened, the dark fringes due to one pair of virtual source will be completely marked by the bright fringes due to another pair. As a result no fringe system will be observed. 4. What is the order of angle of the biprism, φφφφφ? What will happen if φ φ φ φ φ is increased? Ans. About 30'. When φ is increased, d increased and the fringes become narrower. 5. What will happen if one half of the bipirism is covered with lamp block? Ans. The fringes will disappear because one of the two coherent sources will be removed. 6. Can the covex lens in the experiment have any focal length? Ans. No. the focal length f should be such that D < 4f. Only then the real images of the virtual sources can be focused at the focal plane of the eye piece. 7. Why do you take a thin mica sheet? Ans. Because, if the mica sheet is thick the fringe pattern will be displaced significantly. The central fringe may then go beyond the range of movement of the micrometer screw of the eye-piece. 8. Will the fringe width change open the insertion of the mica sheet? Ans. No. 9. If the mica sheet is introduced in the path of both the interfering beams, what will happen? Ans. There will be no shift of the fringe pattern, because the optical paths of the two interfering beams change equally. 10. What is 'optical path'? Ans. It is the product of the geometrical path length traversed by light and the refractive index of the medium.

NSOU ? CC-PH-06 93 11. Why do you use white light to measure 's'? Ans. With white light, the central fringe is white, and so it can be readily located. This facilitates the measurement of S. If a monochromatic source is used, the shift of the central fringe can not be measured because all the fringes 100 K identical. 12. Can you measure the refractive index of mica in your experiment? Ans. Yes; if the thickness t of the mica sheet is known, its refractive index μ can be found from Eq. (iii), the other quantities being measured in this experiment. 13. Why do you choose mica here? Ans. Because mica sheets can be made very thin.

NSOU ? CC-PH-06 94 Unit 13 ? To Calibrate a thermocouple to measuere temperature in a Specified Range using (i) Null Method (ii) Direct measurement using Op-Amp difference amplifier and to determine Neutral temperature (i) Null Method Structure 13.1(i) Objectives 13.2(i) Introduction 13.3(i) Theory 13.4(i) Apparatus 13.5(i) Experimental Procedure 13.6(i) Experimental Results 13.7(

i) Discussion 13.1(i) Objective To draw the thermo emf-temperature curve of a given thermocouple and to caliprate the thermocouple to measure temperature in a specified range. 13.2(i) Introduction Ordinarily a copper-constantan couple is employed for this experiment. The couple gives about 40 micro-volt per 0°C. To prepare this couple, three pieces of wires, each of one meter long are taken, of which one is of constantan while the other two are of copper. The two ends of constantan wire are cleaned and are joined by twisting with one end of each of the copper wires. The junctions are then soldered with minimum amount of solder covering a length of about 2 or 3 mm of each 94

NSOU ? CC-PH-06 95 junction. Two glass are introduced in the copper wires to be sure that the metals touch at the junction only. 13.3(i) Theory When one junction of a thermo-couple is kept at 0°C while its other junction is maintained at a higher temperature, thermo-emf e will be developed in the couple. If this emf e be balance against the potential difference existing at the ends of a length of potentiometer wire of total length L having the potential drop P per Unit length then. e = Pl If E be the emf of the storage bettery B, R be the resistance of the potentiometer wire of length L and R 1 be the resistance applied in the resistance box kept in the potentiometer circuit then P ER R R L = +( ) 1 From we get e ER R R L = + l ( ) 1 By measuring thermo-e.m.f. e with the help of Eq. for different temperatures of the hot junction, a curve may be drawn by plotting temperature (t°C) of the hot junction along the x-axis, while the corresponding thermo-emf e along y-axis. Within a small range of temperature (which is far away from neutral temp) the curve would be a straight line as is shown in Fig. This curve is called the calibration curve of the given thermo couple. To find the thermo-electric power P de dt = at a given temperature 0°C of the hot junction a tangent is drawn to curve at the point corresponding to ?°C of the hot junction. By measuring to slope (BC/AC in Fig.) of this tangent line P de dt = can be determined at ?°C. If e is measured in ?V and t in °C then P will be given in units of ?V/°C.

NSOU ? CC-PH-06 96 To find the unknown temperature one junction of the thermo-couple is kept at 0°C while its other junction is kept within a bath of unknown temp (7°C). Then by measuring the thermo-emf e from the relation the unknown temperature can be found out from the calibration curve (as the temperature corresponding to the emf e). 13.4(i) Apparatus (a) A potentiometer (b) a Galvanometer (c) a Battery (usually Alkali cell) (d) a Plug key (e) two Resistance box (f) Copper and Constant wire (g) a tunnel (h) two thermometers (i) Crushed ice. 13.5(i) Experimental Procedure (i) The resistance R of the potentiometer wire is measured by a P.O. box. the emf E of the storage battery B is measured by an occurate voltmeter reading 0.0/volts. The emf of the battery B, should be measured also after the experiment to see whether if remained constant throughout the experiment or not. (ii) A thermo couple is constructed from the supplied wires as described earlier. (iii) To make the process of null point (l) determination easier and time saving and at the same time kuping the order of accuracy same as in the measurment of the quantities, it is usual to choose a proper value of R 1 and to keep it constant throughout the experiment.

NSOU ? CC-PH-06 97 If we take the usual copper constantan couple & draw the calibration curve for a maximum temperature difference of 100°C then a p.d. of 5 ?V/cm along the potentiometer wire will give a null point at the middle of the 10th wire with the hot junction at 100°C. The required value of R 1 is calculated from the relation by putting P = 5 × 10 −6 V/cm. The applied value of R 1 is a hound number near about the calculated value of R 1 . (iv) After making connections of circuit in the manner as shown in Fig, the hot junction J 2 is introduced into water at room temperature (t 1 °C). [If t 1 ~ 30°C, the null point may vbe obtained near about the 3rd wire]. After finding a rough null point with a high resistance S, the value of S should be made zero to get the accurate null point. This null point is noted several times and their mean value (l 1) is obtained.

NSOU ? CC-PH-06 98 From this mean value of null point, the thermo-emf (e 1) at room temperature (t 1 °C) is calculated by the relation. (v) Temperature of water (in which the junction J 2 is kept immersed) is now raised by steps of 10°C and at each setp, several rull points are noted by constantly stirring the water and maintaining its temperatrue constant for at least 3 or 4 minutes. The mean value of several null points at each temperature by using the relation proceding in this way, the final mean null point is determined at a temperature (t 2 °C) of about 90°C and the corresponding thermo-e.m.f. is also calculated from the relation. (vi) A graph is then drawn by plotting the temperature (t°C) of the hot junction along x-axis and its corresponding thermo-emf e (in mV) along y-axis. This curve is called the calibration curve of the thermo-couple. As the neutral temperatrue of the couple is far away from the highest temperature employed for the hot junction and as the range of temperature employed is small, the curve would be a straight line as in shown in the Fig. (vii) To find the thermo-electric power P de dt = ⎛ ⎝ ⎜ ⎞ ⎠ ⎟ at a given temperatrue ?°C a tangent is drawn to the calibration curve at a point corresponding to ?°C. Form the stop of this tangent we get P de dt BC AC = = in mV/°C. (viii) The hot function J 2 is now immersed in an unknown temperature bath. Generally as an unknown temperature the students are asked find the melting point of a solid. The powdered solid (whose milting point is required) is taken in sufficient amount in a test tube the hot junction J 2 of the couple is introduced at the middle region of this powder. The test tube is then introduced in the water taken in a large beaker. The temperature of this water can be varied. The resistance R 1 is kept unaltered in the potentiometer circuit and the null points on the potentiometer wire are noted after an interval of half minute from the beginning of multing of solid till the whole of half minute is adopted from the beginning of feeling of liquid till the

NSOU ? CC-PH-06 99 whole of it melts. The same procedure of noting the null points, after an interval of half minute after an interval of half minute is adopted from the beginning of freezing of liquid till the whole of it is solidified. (ix) Two curves are drawn

by plotting time along x-axis and corresponding null points along y-axis. The nature of the curves will be like those shown in Fig. (during the

melting of solid] and in other fig. [during the fruzing of liquid]. Both the curves will blow a horizontal part when the melting or freezing in going on. The ordinate l' of this horizontal part will be the actual null point. The mean of these two values of l' (one during multing and another during fruzing) in employed to calculate thermo-emf l' corresponding to the melting point T°C of solid, by using the Eg. (x) The melting point of the solid is now found out the calibration curve as the temperature corresponding to the thermo-e.m.f. l'. 13.6(i) Experimental Result (A) Determination of the resistance (R) of Pot. wire :

NSOU ? CC-PH-06 100 Table I Value of Q Value of P Ratio Resistance in Galvanometer Nature of resistance in the Unknown (1st arm) (2nd arm) Q/P the third arm deflection R-arm, in comparison with resistance in ? in ? in ?(R) the unknown resistance (x) (X) in ? 5000 right too high 0 left too low 100 right too high X lies 10 10 1 between 50 left too low 60 & 61? 60 slightly left slightly low 61 slightly right slightly high 607 right high X lies 10 100 1 10 between 606 slightly right slightly high 60.5 & 605 slightly left slightly small 60.6? 6055 slightly left slightly small 10 1000 1 100 X = 6056 no deflection equal 60.56 ? 6057 slightly right slightly high (B) Nothing of the EMF(E) of the cell B Table II Stage of Expt EMF of cell Mean EMF Remark B in V in V Before Expt ..... The emt of ..... battery is After Expt ..... remaining constant (C) Calculation of R 1 : From Eq. R ER PL R 1 = –

NSOU ? CC-PH-06 101 Putting P = 5 × 10 –6 V/cm (for Cu constantant couple) L = 1000 cm and the measured values of E & R we can calculate R 1 (D) Temperature – Null point record : Temp. of cold junction = 0°C EMF of battery B = E = ... V Res of Pot wire = R = ... ? Length of Pot. wire = L = 1000 cm. Table III No Temperature Resistance in Null Points Total length in Thermo emf of of hot the pot circuit On At the scale Mean scale cm required l l = x + EM R R L 10 3 1 ( ) in mV Obs Junction in ? (R 1) wire readings readings for balance (l) in °C (t) no. in cm in cm Room 19.9 1. temp = ... 3rd 19.8 19.8 219.8 ..... t 1 °C 19.7 51.9 2. t 1 + 10 ..... 4th 51.8 51.8 348.2 ..... 51.7 etc etc etc etc etc etc etc .... 3 .... (E) Drawing of (e – t) curve, the temperature (t) of the hot junction in °C is plotted along x-axis while the corresponding thermo-emf (e) in milli volts is plotted abiog y-axis. As the range of temperature is small and the highest temperatrue applied at the hot junction far below the neutral temperature, the curve would be a straight line (straight portion of a parabola). The nature of the curve is shown in the fig. (F) To find thermo-electric power from the graph at a specified temperature

NSOU ? CC-PH-06 102 Table IV Temp ?e = Bc ?e ?t = Ac Value of ??in °C in mV in ?V in °C Δ Δ e t in ?V/°C .... (G) Time-Null point record during melting & freezing of the solid : Temperature of fold Junction = 0°C, Resistance R 1 = ...? Table V Null point during melting Null point during freezing Time wire Scale Total Wire Scale Total in no. read length no. read length min in cm in cm in cm in cm 0 1 2 1 1 1 2 ii) The water taken in the beaker (in which hot junction J 2 is introduced) should be large and it should be heated slowly so that the temperature may remain constant for an appreciable time. iii) The function (J 1 and J 2) should be kept at the middle region of the bath so that the temperatures of the functions may not change due to a small variation of the temperature of the surroundings. iv) The experiment should be performed within a small range of temperature so that the (e – t) curve writhin that range may be approximately a straight line.

NSOU ? CC-PH-06 103 (ii) Direct measurement using Op-Amp Structure 13.1(ii) Objectives 13.2(ii) Introduction 13.3(ii) Theory 13.4(ii) Apparatus 13.5(ii) Experimental Procedure 13.6(ii) Experimental Results 13.7(ii) Discussion 13.8 Summary 13.9 Exercise and Answer 13.1(ii) Objective To draw the thermo emf-temperature of a given thermo-couple and to calibrate the thermocouple to measure in a specified rangs. 13.2(ii) Introduction Let us now consider the most commonly used op-amp circuit. This circuit will be used in measuring thermo emf. It is called inverting amplifier. It is so called because the non-inverting input is grounded and the input voltage V in is comucted through resistor R i to the inverting input. The resistance R t is called the feedback resistor, because it feeds part of the output back to the input. If voltage at the input end is V i and the output voltage is V 0 . We have V 0 = – AV i .

NSOU ? CC-PH-06 104 Now voltage gain is –A, negative because the amplifier is in an inverting mode. We notic V in ? V i , there is a resistance between them. A is OP amps inhert gain when there is no feed-back loop, it is often called open-loop gain of the OP-amp. From Ohm's law we can write I V V R and I V V R in i F i P 1 1 0 = – = – Applying Kirchaoff's law at the inverting input we have I1 = I f + I – where I – is the current entering at the inverting input. Now we assume that the OP-amp is ideal. The characteristics of an ideas OP- amp which are mecessary for out present purpose are : (1) Input resistance, R i = ?, (2) Output resistance R 0 = 0, (3) Open loop voltage gain A = ?, (4) Perfect balance, ie. V 0 = Owhum V 1 = V 2 As R i = ?, I – = 0 ? I 1 = I f V V R V V R in i i i o f – = – ..... (1) Again for an ideal OP-amp amplifier A = ?, That implies V V A i = – = 0 0, , Substituting this in eqn. (i) we get V R V R V R R R in f in f 1 0 0 1 = – = The ratio V 0 /V in is the voltage gain of the inverting amplifier circuit. It is called close loop gain of the amplifier. That means, it is the gain when operational ampklifier is used with feedback. The gain is negative, as it should be for an investing amplifier. But more interesting thing is that magnitude of the voltage gain depends only on the ratio of the two resistances R 1 and R f and not on the amplifier itself So, we can choose the two resistances to produce the required voltage gain.

NSOU ? CC-PH-06 105 Thermo emf is measured by operational amplifier. We need not know the inner mechanism of OP-amp. If is enough for our purpose to know that it is a voltage amplifying device. Which can produce an output voltage in the range of mV from the input of thermo emf, which is of the order of ?V. The output voltage can be directly read by the voltmeter. One wire of the thermocouple is connected to inverting input terminal and the other wire is connected to the non-inverting terminal which is kept at zero potential by connecting to the ground. For example, for a copper constant thermocouple, the hot juncton is connected to the inverting terminal and the cold one to the non-inverting terminal. It is voltage gain is A input voltage is V in , Output is V out = AV in . Measuring V out . We can know V in (thermo emf). 13.3(ii) Theory A thermo couple consists of two dissimialr metallic wires joined at their two ends to form a closed circuit. If one junction is at 0°C and the temperature (t°C) of the other junction is increased from zero. thermo emf (e), of teh order of micro-volt. Increases from zero according to the equation given by e = at + bt 2 volt .... (i) a & b are constants for a particular thermocouple thermo emf (e) at different temperatures (t) of the hot junction are measurd by an op-amp & e is plotted against t. The e–t curve so obtain is the calibration curve. As the temperature t is far away fro the neutral temperature, the calibration curve is part of the straight portion of parabolic (e–t) curve. From the calibration curve we can find the thermoelectric power. Thermoelectric power at temperature t is p de dt = = slope of the calibration curve at the poin corresponding to the temperature. The temperature of the hot function at which e is maximum si called neutral temperature (t o) when t t de dt a bt t a b n n n = = + = = – , , 2 0 2 .... (ii)

NSOU ? CC-PH-06 106 To find neutral temperature, we require two experimental values of thermo emfs and corresponding temperatures : (e 1 , t 1) and (e 2 , t 2) we have e 1 = at 1 + bt 1 2 & e 2 = at 2 + bt 2 2 from these two relation we get a e t e t t t t t b e t e t t t t t = – – = – – 1 2 2 2 1 2 1 2 2 1 2 1 1 2 1 2 2 1 ( ) & ( ) .... (ii) We can find a and b uysing eqn. (iii) and neutral temperature for the couple can be calculated using eqn (ii). To measure e by OP-amp, the required circuit is shown in Fig. For constant couple. Copper wire at the cold junction (–ve) is joined to inverting pin 2 through resistance R 1 . The feedback resistance R f is joined between pin 2 & pin 6. The hot junction of the couple is joined to the non-investing pin 3, which is eathed. The output voltage (V 0) is measured by the voltameter V 1 . Pins 7 & 4 are Jaind to two power supplies +V –V respectively. To remove the off set voltage a potentiometer p is joined betwen pin 1 and 5 & its wiper is connected tot 4eh negative supply V. 13.4(ii) Apperatus Thermocouple, power supply (+12V or +15V), IC OP AMP (IC 741), two resistance boxes, two keys ice bath containg melting ice & beaker containing water jobced over a tripod.

NSOU ? CC-PH-06 107 13.5(ii) Experiment Procedure (1) Preparation of the thermocopule : Take two pieces of copper wire (Cu) and one piece of constants wire (Cn), each about one meter long. Clean the ends of the wires. Join one end of the constantan wire to one end of one copper wire and the other end of wire and the other end of the constantan wire to end of the other copper wires of twisting and then soldering the junctions with minimum of solder. Insert the copper wires into two thin glass tuber of lengths of about 25 cm, So that the two junctions rest at the lower ends of the glass tubes. Thus, we make it sure that copper and constantan wires do not touch each other at any points execpt at the two junctions. (2) The circuit is completed as shown in Fig. Take resistance R 1 = 1 K? and R f = 10 K? to get voltage gain 10. Exact values of the two resistances are determined by a multimeter. As the junction are now at the same (room) temperature, the output voltage, read by voltameter V 1 should be zero. If it is not zero, there is offset voltage./ To remove the offset voltage the wiper of the potentiometer P is rotated in the right direction to make the output voltage zero. (3) Now the junction, which is comected to the inverting pin, is placed in the ice bath, containing metting ice. The other junction, whihc is connected to the non inverted pin is placed in the water kept in a beaker placed over a tripod. From a stand a thermometer T, reading 1 / 10 th of a degree, is placed inside the water, with its bulb close to the junction. With the help of separtate clamps, the two junctions are kept field in their positions, equally surrounded by melting ice and water respectively. (4) After waiting for some time, note the temperature (t) of the water in the beaker, which should steady for at least 2 min. Note the output voltage (V 0) read by the voltameter V 1 . Now start hearing the Water and raise the temperature by 8 to 10°C. Hold the temperature steady for 2 mins and note the temperatrue (t) and the corresponding output voltage (V 0). Continue such observations by increasing the temperatrues by steps of 8 – 10°C until it rises upto about 95°C, Calculate the input voltage (V in) in lach case.

NSOU ? CC-PH-06 108 Which is the therm emf (e). ? V in = e = R R i f V 0 . (5) Plot e along y-axis and t along x-axis in a graph paper. Keeping the origin at (0, 0) as origin is a sure point of the curve. choose the scales along the two axes such that the whole of the given graph paper is utilised. Draw the mean the through the origin about which the experimental points are enenly distributed. This is the calibration curve. Thermodectric power P = slop of ther curve AB/BC. (6) To calculate neutral temperatrue choose any pair of the measured values (e 1 , t 1) (e 2 , t 2) and Find the values a & b of the thermocouple by eqn. (iii) From these values find the nutral temperature t n from eqn (ii) Repeat ther same calculation, if time permits, for other pairs, calculated the mean value. 13.6(ii) Experiment Results A. Resistance put in the resistance boxes R 1 & R f : R 1 = 1 K? & R p = 10 K? Measured values : R 1 = ... K? & R p = ... K? l t A B C

NSOU ? CC-PH-06 109 ? Voltage gain = R R f 1 = ... B. Off set voltage is removed by rotating the wiper of the potentiometer. C. Data for (e – t) graph & duauring of the calibration curve Table 1 Temperature of the cold junction = 0°C No of Temp. of the Amplified output Thermo emf obs hot junction t°C voltage V 0 mV l = R R f 1 V 0 mV 1 2 3 D. Deter,omation of thermoelectric power (P) From Fig. P AB BC = = — = ...V/°C E. Determination fo nutral point : For a pair of measured emf ? (e 1 = mV, t 1 = ... °C), (e 2 = ... mV, t 2 = ... °C a e t e t t t t = – – 1 2 2 2 1 2 1 2 2 1 ( ) mV/°C and b e t e t t t t t = – – 1 1 2 2 1 2 2 1 ( ) mV/°C Neutral temp t a b x n = – = = 2 2 .... °C No of Temp of the Thermo emf e Neutral temp Mean Obs hot junction t°C from Table-1 mV t n °C t n °C 1 ..... [for obs 1 & 2] 2 ..... 3 ..... [for obs 2 & 3]

NSOU ? CC-PH-06 110 13.7(ii) Discussion 1) Identification of the different pin numbers of op-amp is very important. Take help of the teacher and suritch on the circuit only after it is inspected by teachers. 2) Resistaces R 1 and R 2 should be measured very accurately by a multimeter. 3) Cold junction should be cheaked from time to time by pressing the powered ice with a glass rod and adding more ice as and when nesersary. 4) Tempratrue of hot water should be kept steady for at least 2 mins, before reading of voltameter is taken. 5) Temperatrue of the hot function should not exceed the boiling point of water to get a straight calibration curve. 13.8(ii) Summary From the induystrial view point, temperature measurement is one of the important measurements. The resistance temperature detector (RTD) and thermocouple (T c) are the major device of the temperature measuring instrument. Temperature measurment is important for monitoring and control purposes. 13.9(ii) Exercise and Answer 1. What will happen when two wires of two different metals are joined at their ends and a difference of temperature is maintained between the two function? Ans. A current will flow in the circuit. which is known as the thermo-current and this phenomenous is called seebeck effect. 2. What will happen if a cell is inserted in the thermo-couple circuit whose two junctions are kept at the same temperature? Ans. One of the junctions will be hot while the other junction will be cold. This phenomenon is known as peitier effect.

NSOU ? CC-PH-06 111 3. What will happen when a temperatrue difference is maintained between the two points of one conductor of a thermo couple? Ans. A potential difference will be set up between the two points of the same conductor having a difference of temperature. This phenomenon is known as Thomson effect. 4. What is the difference betwen peltier effect and Joule effect? Ans. Heating in peltier effect is proportional to the current while heating in Joule effect if proportional to the square of the current. 5. What is neuitral temperature of a couple? Ans. If is the temperature of the hot function of which the thormo emf generated in the couple is maximum. 6. What is the relationship between the thermo emf (e) and tem (t) of the hot junction when that of cold junction is 0°C? Ans. Within a small range of temperature the relation between e and t can be represented by e = at + bt 2 or a = at 6 . 7. What do you mean by thermo-electric power? Ans. Thermo electgric power of a couple at a particular temperature is the increase of the thermo emf when the temperature of the hot junction is increased by 1°C. If dF be the increase of thermo emf for a rise of temp. dT° at a temp. T of ther hot junction then thermo electric power at T° is dE dT . 8. What is the nature of the curve connecting thermo electric power P = (dE dT) & temperature? Ans. It is a straight line. The slope of this curve may be +Ve or –Ve. 9. What is pyro-electricity? Ans. It is an electrical effect in which certain crystals, especially tourmaline electrical sharges when heated or coded. 10. What is an inversion temperature? Ans. It is the temperature of the hot junction at which thermo emf is zero & is about to be reversed.

NSOU ? CC-PH-06 112 11. Can you measure the thermo emf by using a microvoltmeter? Ans. No, by using avoltmeter we get p.d and not emf. 12. What type of galvanometer is suitable for this experiment? Ans. A suspended coil dead beat and voltage sensitive type. 13. Why do you use a potentiometer and not a voltameter to measure the thermo emf? Ans. In potentiometer we take reading at null condition, when no current flows through the thermo couple. Hence the measurement become accurate. A voltmeter measures p.d. and not the emf. 14. What is the value of thermo electric power at neutral temperature? Ans. Zero.

NSOU ? CC-PH-06 113 Unit 14 ? To design Fourier spectrum of (i) square (ii) triangular and (iii) half sinusoidal wave form by CRO

Structure 14.1 Objectives 14.2 Introduction 14.3 Theory 14.4 Apparatus 14.5 Experimental Procedure 14.6 Experimental Results 14.7 Discussion 14.8 Summary 14.9 Exercise

and Answer 14.1 Objective To understand Fourier series representation of periodic signals. 14.2 Introduction Fourier's theorem states that a periodic function can be expressed by an infinite series of sine or cosine functions of hermonically related frequencies. The frequency of the sine or the cosine term in the infinite series is a harmonic of the frequency of the periodic function. 14.3 Theory Figure (i), (ii), (iii) and (iv) respectively show form periodic waves namely (i) a square wave of period T and amplitude V 1 (ii) a triangular wave of period T and peau 113

NSOU ? CC-PH-06 114 value V 1 (iii) a half finersoidal wave (which is the output of a half wave rectifier driven from a sincesoidal source) of period T and amplitue V and (iv) a half sinusoidal (which is the output of a half wave rectifies driven from a sinusoidal source) of period T and amplitade V. A A A A 0 1 2 4 1 2 2 3 2 15 : : : : : : = π The corresponding amplitude spectrum is shown in fig. 14 (vii). Considerily the AC components. A A A 1 2 4 1 4 3 4 15 : : : : = π π For the half sinusoidal waveform of Fig. 14 (iv), the dc compact is A V 0 2 = π , the amplitude are 2f 0 , 4f 0 , 6f 0 .

NSOU ? CC-PH-06 115 A V A V A V 2 4 6 4 3 4 15 4 35 = = = π π π , , A A A A 0 2 4 6 1 2 3 2 15 2 35 : : : : : : = The amplitude spectrum of the half simutiodal waveform is shown in Fig. hence considering the ac component only A A A 2 4 6 1 1 5 3 35 : : : : = As the amplitude doop off repidly with increasing frequency, the first few frequency components are significant. 14.4 Apparatus (i) Function Generator giving square and triangular waveforms. (ii) Half wave and full wave rectifiers giving half sinersoidals. (iii) Several active based pass filters (A detailed fiscussion of these filters and their design can be found in the book "Fundamental of Electric cicuit Theory by D. Chattopadhyay and P.C. Rakshit (S chand and Co New Delhi). (iv) A CRO (v) Connecting wires etc.

NSOU ? CC-PH-06 116 14.5 Experimental Procedure 1. Obtain a square wave form the function generator and display it on a CRO. Choose a suitable frequency, say 1 kH2 of the wave form. The Fourier series expansions of the four periodic waves are given by (i) V t V n n t n ( ) sin , , , ... = = ∝ ∑ 4 1 0 1 2 3 5 π ω (ii) V t V t t t ( ) sin sin sin ..... = − + ⌈ ⌊ ⌉ ⌉ ⌋ ⌉ 8 1 9 3 1 25 5 2 0 0 0 π ω ω ω (iii) V t V V t t t ( ) sin cos cos ..... = + − − ⌈ ⌊ ⌉ ⌋ ⌉ π ω π ω π ω 1 2 2 3 2 2 5 4 0 0 0 (iv) V t V V n t n where T n ( ) cos , , , ... = + − = = ∝ ∑ 2 4 1 2 0 2 2 4 6 0 π π ω ω π is the fundamental angular frequency v. The plot of teh amplitude of the harmonic components of teh Fourier series with frequency is known as teh amplitude spectrum of the waveform. For the square wave of Fig. 4 (i), the amplitudes of the frequency components f 0 = ? o /2?. 3f 0 and 5f 0 are respectively. A V A V and A V so that A A A 1 3 5 1 3 5 4 4 3 4 5 1 1 3 1 5 = = = = π π π , : : : : The amplitude spectrum of teh square wave is depicted in Fig. 3.EL32(V) For the triangular wave the amplitudes of the frequency components f 0 , 3f 0 and 5f 0 respectively. A V A V A V 1 2 3 2 5 2 8 8 9 8 25 = = = π π π , , A A A 1 3 5 1 1 9 1 25 : : : : =

NSOU ? CC-PH-06 117 The amplitude spectrum of the triangular wave is displayed. For the half sinusoidal of Fig. the dc compenent is A V 0 = π . The amplitude of the frequency components f 0 , 2f 0 , 4f 0 are A V A V A V 1 2 4 2 2 3 2 15 = = = , , π π . 2. Take a band-pass filter of centre frequency 1 KHz and of narrow bandwith and set up the circuit as in Fig. 3.EL32(ix). The band-pass filter allows only the frequency component f 0 (= 1.KHz) to pass through and rejects all other frequency from appearing at its output. 3. Display the output singal of the band-pass filter on the CRO and measure its amplitude A 1 by the CRO. 4. Replace the band-pass filter successfully by band. pass filters of centre frequencies 2f 0 , 3f 0 , 4f 0 and 5f 0 and measure by CRO the corresponding amplitudes A 2 , A 3 , A 4 and A 5 . Correct the amplitudes by the gains of the active band-pass filters at the centre frequencies. Function generator or Full wave rectifier Active band-pass filter To CRO

NSOU ? CC-PH-06 118 5. Plot the amplitude spectrum as in Fig. 3.EL32(V) and find A 1 : A 3 : A 5 . Compare it with the tgheoretical value. 6. Repeat the experiment for a triangular wave form of frequency f 0 = 1 KHz, say. 7. Replace the function generator by a half-wave rectifier giving a half sinusoidal wave form of frequency f 0 = 1 KHz. and repeat the experiment. The dc component of Fourier expansion of half sinusoidal waveform is not considered here only A 1 , A 3 and A 4 are measured by the CRO. 8. Repeat the experiment by replacing the half wave rectifier with a full-wave rectifier. Measure A 2 , A 4 and A 6 . 14.6 Experimental results Table 1 Results for a square wave f 0 = ... Hz. Frequency Amplitude A 1 : A 3 : A 5 Remarkes components (Hz) (volt) Experimental Theoritical ... (= f 0) ... (= A 1) ... (= 2f 0) ... (= A 2) etc etc Make simular tables for triangular and half sinusoidal wave forms. 14.7 Discussions 1. The CRO must be operated in the triggered mode so the pattern on the screen is steady. 2. If the gains of the active band-pass filters at the centre frequencess are equal, the amplitudes need not be corrected for these gains, since the gains cancel out the taking the amplitude ratios.

NSOU ? CC-PH-06 119 14.8 Summary In this method, the frequency of the input wave form is changed keeping the resonant frequency of the tuned circuit constant. The frequency must be varied slowly and carefully to get a maximum amaplitue of the output sine wave across the tuned circuit. The input waveform should be free from appreciable distortion. Otherwise, the agreement between the theory and the experiment will not be satisfactory 14.9 Exercise and Answer 1. State Fourier's theorem. Ans. See 'Theory'. 2. What are Dirichlet's conditions? Ans. The sufficient conditions required for the periodic function f(t) to express it as a convergent Fourier series are called Divichlet's conditions. The conditions are : (i) f(t) is single valued and finite, (ii) f(t) has a finite number of finite discontinuities in the period T s , (iii) f(t) has a finite number of maxima and minima in the interval T 1 and | ( )| f t dt t t T 0 0 + ∫ exists. 3. What are the amplitude and phase spectra? Ans. The Fourier seried of a periodic function can be expressed in the following form. f t A A nw t O o n o n n ( ) cos( ) = + − = • ∑ 1 where A o is constant, and A n and O n are respectively the amplitude and the phase of the nth hermonic. The variation of A n with n(or nw o) is called the amplitude spectrum, and the variation of O n with n(or nw o) is known as the phase spectrum of teh signal f(t). 4. What do you mean by even-function symmetry, odd-function symmetry, and have wave symmetry? Ans. The periodic function f(t) is said to process even-function symmetry if f(t) = f(−t). The function f(t) has odd-function symmetry if f(t) = −f(−t). The periodic function is said to have helf-wave symmetry if f(t) = −f(t − T / 2).

NSOU ? CC-PH-06 120 5. What is the effective (or nms) value of the periodic function f(t)? Ans. It is given by F T f t dt eff T = { } ∫ 1 2 0 ( ) 6. What is parallel resonance? Ans. Resonance accuring in a circuit containing an inductor in parallel with a capacitor, is called parallel resonance and the circuit is referred to as a parallel resonant circuit. parallel resonance is also called anti-resonance. 7. What is the quality factor Q of a parallel resonant circuit? Ans. Since the capacitor losses are very small, their Q-values are very high. So the Q of a parallel resonant circuit is given by Q of the inductor coil at resonance, w o L i.e. Q = ——, where R is the resistance of the coil. R 8. What is the capacitor current at resonance? Ans. It is Q times the current delivered by the generator. Thus the circuit gives current magnification. 9. What is the resistance and the reactance offered by the parallel resonant circuit at resonance? Ans. The resistance is the dinamic resistance r d and the reactance is zero. 10. When do you say that a circuit is resonent? Ans. The circuit is resonant when the current and the voltage are in phase, i.e., the power factor is unity. 11. Why is the parallel resonent circuit called a rejector circuit? Ans. As the inductor resistance R ? 0, the dinamic resistance r d ? •. So, the source current is a minimum at the parallel resonant frequency f p . The source current increases as the frequency changes from f p on either side. So, if the source contains a large number of frequency components, the circuit rejects the frequency equal to its resonant frequency f p by drawing minimum current from the source from the source. Hence the parallel resonant circuit is called a rejector circuit.

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n–1 + A n B n C n–1 + A n B n C n–1 + A n B n C n–1 C n = A n B n C n–1 + A n B n C n–1 + A n	n=[[a^n, n a^(n-1)],[0,a^n]] `for every positive integer n. play 19013 29.5 K+ 142.6 K+ 3:20 If `A=[(1,a),(0, 1)]` , then `A^n` (where `n in N)` equals `[(1,n a),(0, 1)]` ( b) `[(1,n^2a),(0, 1)]` (c) `[(1,n a),(0 ,0)]` (d) `[(n,n a),(0,n)]`

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circuits that are capable of performing simple addition with the help of logic gates.	

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the implementation of larger logic diagrams is possible with the full adder logic a simpler symbol is mostly used to represent the operation. 8.9

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FET) is : BJT is a current controlled device whereas FET is a voltage controlled device.

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1 PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, discipline specefic general elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the
university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC Open and Distance Learning (ODL) Regulations, 2017 have mandated compliance with CBCS for UGC 2020 programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Bachelors Degree Programme (BDP) level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success.
Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor
2 Printed in accordance with the regulations of the Distance Education Bureau of the University Grants Commission. First Print : June, 2022 Netaji Subhas Open University Under Graduate Degree Programme Choice Based Credit System ((CBCS) Subject : Honours in Physics (HPH) Course Code : CC -
PH - 07 Course : Mathematical Methods in Physics - II
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5 Netaji Subhas Open University Unit 1 q Fourier Series 7-40 Unit 2 q Frobenius Method and Special Function 41-81 Unit 3 q Some Special Integrals 82-107 Unit 4 q Theory of Errors 108-128 Unit 5 q Partial Differentiations 129-147 Unit 6 q Advance Mechanics 148-195 UG : Physics (HPH) Course : Mathematical Methods in Physics - II Course Code : CC-PH-07
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NSOU l CC-PH-07 7 Unit−1 q Fourier Series Structure 1.0 Objectives 1.1 Introduction 1.2 Periodic function 1.3 Harmonic Analysis 1.4 Orthogonal function 1.5 Fourier's theorem 1.6 Complex forms of Fourier series 1.7 Fourier co-efficients 1.8 Dirichlet's conditions 1.9 Even and Odd functions 1.10 Applications of Fourier series 1.11 Change of interval in Fourier expansion 1.12 Half-range series 1.13 Summary 1.14 Review Questions and Answer 1.0 Objectives After reading this unit you will learn 1. Definitions of periodic and Orthogonal Functions. 2. Harmonic analysis, Fourier's theorem, Fourier Series, Fourier Co-efficients, Dirichlet's conditions 3. Some examples of applications of Fourier series in different branches of physics. 1.1 Introduction Jean—Baptiste Joseph Fourier (21 March, 1768-16 May, 1830) was a French mathematician and Physcist, known as the investigator of Fourier series. The idea of Fourier series encountered to develop in mathematical science and engineering. The applications of Fourier series include in the general heat equation, vibrational modes of structural elements in buildings, quantum harmonic oscillators etc.
8 NSOU l CC-PH-07 1.2 Periodic function : Any funciton which repeats itself regularly over a given interval, is called periodic function. f x( ) → P → → → X Fig. 1.1 An illustration of a periodic function over an interval p. Here function f(x) is periodic i.e f(x + P) = f(x) 1.3 Harmonic Analysis : If a note has frequency n, then integer multiples of the frequency 2n, 3n and so on, are known as harmonics. The mathematical study of overlapping waves is called, harmonic analysis. 1.4 Orthogonal function : The functions f and g are orthogonal when this integral of the product of functions over the interval ( ) ( ) 0 f x g x dx ∗ = ∫ 1.5 Fourier's theorem : Fourier theorem

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is an expansion of a periodic function in terms of an infinite sum

NSOU l CC-PH-07 9 of sines and cosines. Mathematically, a periodic function f(x) can be represented by the sum of harmonic terms as 0 1 2 1 ( )
cos cos2 ...... cos 2 n f

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x a c x a x a n x = + + + + + b 1 sin x + b 1 sin 2x + ...+ b n sin nx = ( ) 0 1 1 cos sin 2 α = + + ∑ n n n a a nx b nx ......(1.1) It is a convergent series and a 0 , a n and b n

are called Fourier co-efficients. 1.6 Complex forms of Fourier series : Substituting a n = c n cos θ n and b n = c n sin θ n in equation (i) we get, ( ) 0 1 1 ( ) cos cos sin sin 2 n n n n n f x a c nx c nx α = = + θ + θ ∑ ( ) 0 1 1 cos 2 α = = + -θ ∑ n n n a c nx ... (1.2) where, 2 2 n n n C a b = + and 1 tan n n n b a - θ = Similarly, substituting cos nx 2 - + = inx in x e e 2 inx in x e e sinnx i - - = we get from equation (1.1)
10 NSOU l CC-PH-07 0 1 1 ( ) 2 2 2 in x in x in x in x n n n e e e e f

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x a a b i α - - = ? ? + - = + + ? ? ? ? ∑ = 0 1 1 2 2 2 in x in x in x in x n n e e e e a a ibn ∝ - - = ? ? + - + - ? ? ? ? ∑ = 0 1 1 2 2 2 in x in x n n n n a ibn a ib a e e ∝ - = - + ? ? + + ? ? ? ? ∑ = 0 1 1 1 2 2 2 in x in x n n n n n n

a ib a ib a e e ∝ -∝ = = - ? ? - + ? ? + ? ? ? ? ∑ ∑ ...(1.3) Now, put 0 0 1 2 a C= ( ) ( ) 1 2 1 2 - ? = - ? ? ? = - ? n n n n n n C a ib C a ib in equation (1.3) We get, 0 1 1 ( ) in x inx n n n n f x C C e C e ∝ -∝ - = =- = + + ∑ ∑ = in x n n C e ∝ =-∝ ∑ ... (1.4) Equation (1.4) represents the complex form of Fourier series and the co-efficient C n is known as complex Fourier co-efficient. Sine and Cosine integral for Fourier series in the integral −π to π.
NSOU l CC-PH-07 11 1. sin 0 nxdx for all n

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π -π = ∫ 2. π -π ∫ cos nx dx = 0 for 0 2 for 0 n n ≠ ? ? π = ? 3. 2 sin nx dx π -π = π ∫ 4. π -π ∫ cos 2 nx dx = for 0 2 for 0 n n π ≠ ? ? π = ? 5. π -π ∫ sin nx sin mx dx = 0 for n m≠ . 6. π -π ∫

cos nx cos mx dx = 0 for n m≠ . 7. π -π ∫ sin nx cos mx dx = 0 for all n and m. The integral formulas of number (5), (6) and (7) for sines and cosines, are known as orthogonality relations. 1.7 Fourier co-efficients : By multiplying equation (1.1) by cos mx dx and integrating both sides in the interval x = – π to π, we obtain,
12 NSOU l CC-PH-07 ( ) 0 1 cos cos 2 f x mxdx a mxdx π π -π -π = ∫ ∫ + 1 cos cos ∝ π -π = ? ? ? ? ? ? ? ? ? ? ∑ ∫ n n mx a mx dx + 1 cos sin n n mx b nx dx ∝ π -π = ? ? ? ? ? ? ? ? ? ? ∑ ∫ For m = 0, as stated in the integral formulas (1-7), we can write, ( ) 0 1 ( ) 2 2 f x dx a π -π = π ∫ or, ( ) 1 π -π = π ∫ n a f x dx For m = n and m, n < 0, ( ) 1 · cos π -π = π ∫ n a f x nx dx By multiplying equation (1.1) by sin mx. dx and integrating bothsides in the interal x = -π to π. we obtain, 1 ( )sin , n b f x nxdx

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π -π = π ∫ where, m = n and m,n < 0 0 1 ( ) 1 ( ) cos 1 ( ) sin n n a f x dx a f x nx dx b f x nx dx π -π π -π π -π ? ? = π ? ? ? = ? π ? ? ? = ? π ? ∫ ∫ ∫

NSOU l CC-PH-07 13 1.8 Dirichlet's conditions : (i) The function f(x) is periodic, single valued and finite. f x( ) −π 0 π 2π → x fig. (1.1a) (ii) f(x) is piece wise continuous that means there are finite number of discontinuities in any one period. Ψ x( ) x a = x b = x fig. (1.1b) (iii) f(x) has only a finite number of maxima and minima. The function ( ) 1 ( ) cos f x x = has infinite numbers of maxima and minima. So, Fourier expansion is not possible. 1.9 Even and Odd functions : Cosine terms are even functions i.e., cos x = cos (– x) and sine terms are odd functions i.e., sin x = −sin (−x) Now, f(x) = ( ) ( ) 1 2 f x f x ? + - ? ? ? + ( ) 1 ( ) 2 f x f x ? - - ? ? ? Even part Odd part
14 NSOU l CC-PH-07 In Fourier series cosine part represents even part and sine part represents odd part. Thus if f(x) is even functions, then Fourier series having only cosine terms and if f(x) is odd function, then only sine term will be their in Fourier series. f(x) = 0 1 1 cos 2 n n a a nx ∝ = + ∑ for f(x) is even function and f(x) = 0 1 1 2 n n a b ∝ = + ∑ sin nx for f(x) is odd function. 0 → x f x( ) 0 → x f x( ) → fig. 1.2 Even function fig. 1.3 Odd function For example, x 2 , cos x .... are even function, where as sin x, x, tan x .... are odd function, Graphically, an even function is symmetrical about y-axis and an odd function is symmetrical about the origin. 1.10 Applications of Fourier series : Fourier series help us to slove the problems in the field of Fourier Analysis. By expanding the functions in terms of sines and cosines, one can easily manipulate functions that are discontinuous or difficult to represent analytically. The fields of electronics, quantum mechenics and eletro dynamics are enriched with the use of Fourier series : 1. Sawtooth wave : Mathematically it can be expressed as f(x) = x for x -π ≤ ≤ π Graphically it can be represented in the interval [– π, π], shown in fig. 1.4.
NSOU l CC-PH-07 15 3π − 2π − π 0 π 2π 3π x f x( ) fig. 1.4 The Fourier series expansion for f(x) is given by ( ) 0 1 ( ) cos sin 2 n n n a f x a nx b nx ∝ = = + + ∑ f(x) = x is an odd function Now, 0 1 1 ( ) 0 2 a f x dx x dx π π -π -π = = = π π ∫ ∫ Also, ( ) 1 cos 0 n a f x nxdx π -π = = π ∫ since f(x) cos nx is odd function and b n = 1 ( ) sin . f x nx dx π π - π ∫ 2 sin x nx dx π π - = π ∫

16 NSOU l CC-PH-07 = ( ) 0 0 cos 2 cos [ ] x nx nx dx n

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n π π ? ? - - ? ? - π ? ? ? ? ∫ = [ ] 0 2 sin cos nx n n n π ? ? ? ? ? ? -π π + ? ? ? ? π ? ? ? ? ? ? = ( ) 2 1 n n - - = ( ) 1 2 1 n n + - The Fourier expansion is given by f(x) = x = ( ) 1 1 1 sin 2 n n nx n + ∝ = - ∑ = 2[sin x – 1 2 sin 2x + 1 3 sin 3x – 1 4 sin 4x +.........] 2.

Square Wave ; 0,for 0 ( ) ,for 0 - π ≤ ≤ ? = ? - ≤ ≤ π ? x f x h x The graphical representation is shown in fig. 1.5 –3π – 2π – π 0 π 2π 3π x f x( ) h fig. 1.5

NSOU l CC-PH-07 17 The Fourier series expansion is ( ) 0 1 ( ) cos sin 2 x n n a f x a nx b nx ∝ = = + + ∑ Now, 0 a = 1 ( ) f x dx π -π π ∫ = 0 0 1 ( ) ( ) f x dx f x dx

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π -π ? ? ? ? + π ? ? ? ? ∫ ∫ = 0 1 0 h dx h π ? ? ? ? + = π ? ? ? ? ∫ Also, 0 1 1 ( )cos ( )cos n a f x nxdx f x nx dx π π -π = = π π ∫ ∫ 0 1 cos 0 h nxdx π = = π ∫ and b n ( ) 1 sin 1 cos h h nxdx n n π -π = = - π π π ∫ = 0, for even 2h for odd n ? ? ? ? π ? n n f (x) = ( ) 1 sin 2 is odd only 2 n nx h h n n ∝ = + π ∑ 2

sin3 sin5 sin .... 2 3 5 h h x x x π ? ? = + + + + ? ? ? ?

18 NSOU l CC-PH-07 The square wave in Fourier series representation containing only odd terms in the sine series. This means square wave contains high frequency. 3. Rectangular wave : ,for 0 ( ) ,for 2 h x f x h x ≤ ≤ π ? = ? - π ≤ ≤ π ? The graphical representation is shown in fig. 1.6. – 2π – π 0 π 2π f x( ) X h – h fig. 1.6 We have ( ) 0 1 ( ) cos sin 2 n n n a f x a nx b nx ∝ = = + + ∑ Now, 2 0 0 1 ( ) a f x dx π = π ∫ = 2 0 1 0 h dx h dx π π π ? ? ? ? - = π ? ? ? ? ∫ ∫ 2 1 ( )cos n a f x nxdx π π = π ∫ 2 0 cos cos 0 h nxdx nxdx π π π ? ? ? ? = - = π ? ? ? ? ∫ ∫

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sin n b f x nxdx π = π ∫ = 2 0 sin sin h nxdx nxdx π π π ? ? ? ? - π ? ? ? ? ∫ ∫ = [ ] [ ] { } 2 0 cos cos h nx nx n π π π π - + - π = ( ) ( ) { } 1 cos 1 cos - π + - π π h n n n = ( ) 2 1 cos h n n - π π = 0, 4 , n iseven h n isodd n ? ? ? ? π ? Hence, 1 4 sin ( ) n h nx f x n ∝ = = π ∑ (n is odd only) = 4 sin3 sin5 sin .... 3 5 ? ? + + + ? ? π ? ? h x x x 4. Half-wave rectifier : f(x) = sin x, 0 x≤ > π = 0, 2 x π ≤ ≤ π 20

NSOU l CC-PH-07 The graphical representation of the function f(x) is shown in fig. 1.7 f x( ) π 2π 0 3π X fig. 1.7 We have, ( ) 0 1 ( ) cos sin 2 ∝ = = + + ∑ n n n a f x a nx b nx Now, 2 0 0 1 sin 0. a x dx dx π π π = + π ∫ ∫ 0 1 2 sin xdx π = = π π ∫ 0 1 sin cos n a x nx dx π = π ∫ 2 0, for odd 2 1 for iseven 1 n n n ? ? = ? - ? π - ? and 0 0 for all ,but 1 1 sin sin 1 for 1 2 n n n b x nx dx n π ≠ ? ? = = ? π = ? ? ∫

NSOU l CC-PH-07 21 The Fourier expansion of the function f(x) is given by 2 1 1 1 2 cos ( ) sin 2 1 n nx f x x n ∝ = = + - π π - ∑ (n is even only) 5. Full-wave rectifier : f(x) = sin x, 0 x≤ > π = – sin x, 0 x -π ≤ ≤ The graphical representation of the function is shown in fig. 1.8. – 3π – 2

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π – π 0 π 2π 3π x f x( ) fig. 1.8 Here, a 0 = 0 0 1 1 ( ) ( ) f x dx f x dx π -π + π π ∫ ∫ = 0 0 1 1 sin sin x dx xdx π -π - + π π ∫ ∫ = 0 2 4 sin x dx π = π π ∫ 0 0 1 1 sin cos sin cos n a x nxdx x nxdx π -π = - + π π ∫ ∫ 22 NSOU l CC-PH-07 0 2 sin cos x nx dx π = π ∫ = 2 0 for -odd 4 1 ,for -even 1 ? ? ? - ? π - ? n n n and 0 2 n b π = π ∫

sin x sin nx dx = 0 Hence, Fourier expansion of the function f(x) is given by 2 1 2 4 cos ( ) 1 ∝ = = - π π - ∑ n nx f x n (n is only even) 6. Riemann zeta function : The function is represented by, 1 1 1 1 ( ) .... 1 2 ∝ = φ = = + + ∝ ∑ t t t n t to n Let, f(x) = x 2 ,– x π ≤ ≤ π It can be graphically represented as in fig. 1.9. – π 0 π x f x( ) fig. 1.9 f(x) = x 2 is even function, so b n = 0, Hence

NSOU l CC-PH-07 23 2 0 1 cos 2 n n a x a nx ∝ = = + ∑ Now, 2 1 cos n a x nxdx π -π = π ∫ = 2 0 2 cos x nxdx

37%	**MATCHING BLOCK 10/115**	**SA**	019E1130_Mathematical Physics.pdf (D165097245)

π π ∫ = 2 0 0 2 2 2 sin sin x x nx nxdx n n π π ? ? - ? ? π π ? ? ∫ = 0 4 sin x nxdx n π = - π ∫ = 0 0 4 4 1 cos cos π π ? ? - - - ? ? π π ? ? ∫ x x nxdx n n n n = 2 0 4 4 1 1 cos · sin π ? ? π - ? ? π ? ? n nx dx n n n n = ( ) 2 4 1 n n - and 3 2 2 0 0 1 2 2 3 3 π π -π ? ? π = = = ? ?

π π ? ? ∫ x a x dx

24 NSOU l CC-PH-07 f(x) = x 2 = ( ) 2 2 1 4 1 cos 3 n n nx n α = π + - ∑ Now put x = π, then ( ) 2 2 2 2 1 1 4 1 3 n n n α = π π = + - ∑ or, 2 2 1 1 2 4 3 n n ∝ = π = ∑ or, 2 2 1 1 6 n n ∝ = π = ∑ If we put t = 2, then ( ) 2 2 1 1 2 6 n n ∝ = π φ = = ∑ By following the same process, If we put t = 4. ( ) 4 4 4 4 1 1 1 1 1 4 .... 1 2 3 n n ∝ = φ = = + + + ∝ ∑ With the help of Fourier series expansion we get by putting f(x) = x 4 , -π ≤ ≤ π x ( ) 4 4 90 π φ = Hence, 4 4 1 1 90 n n ∝ = π = ∑

NSOU l CC-PH-07 25 1.11 Change of interval in Fourier expansion : So far, the expansion was done in the interval [– π, π]. Now we discuss about the wider range. say [–l, l]. ( ) 0 1 ( ) cos sin 2 n n n a f x a x b x ∝ = = + φ + φ ∑ To calculate φ, Let us take f(x) = f(x + 2l), This is only true when π φ = n l Hence, 0 1 ( ) cos sin 2 n n n a n x n x f x a b l l ∝ = π ? ? π = + + ? ? ? ? ∑ If Dirichlet's condition is satisfied in the internal l x l - ≤ ≤ , then co-efficient are 1 ( )cos l n l n x a f x dx l l - π = ∫ 1 ( )sin l n l n x b f x dx l l π = ∫ and 0 1 ( ) l l a f x dx l - = ∫

26 NSOU l CC-PH-07 1.12 Half-range series : For a function f(x), ranging from 0 > x > l and 0 l x - > > is either odd or even. Then f(x) will contain either cosine or sine series. If f(x) is to be expanded in a half-range cosine series in the interval 0 > x > l, then we take f(x) = f(–x) in the range (–l > x > 0) i.e., f(x) is even over the entire period [–l, l]. Then 0 1 1 ( ) cos , 2 n n n x f x a a l ∝ = π = + ∑ where 0 0 2 ( ) l a f x dx l = ∫ , and 0 2 ( )cos l n n x a f x dx l l π = ∫ Now, if we want to expand in a half-range sine series in the internal 0 > x > l, then we have to choose f(x) = – f(–x) in the interval – l > x > 0. i.e, f(x) is odd over the entire period [–l, l]. Then 1 ( ) sin n n n x f x b l ∝ = π = ∑ , where 0 2 ( )sin π = ∫ l n m x b f x dx l l Example 1. Exapand the Fourier series of the function f(x) = x in the interval –1 > x > 1. Solution : For the interval – 1 > x > 1

NSOU l CC-PH-07 27 f(

x) = x = ( ) 0 1 cos sin 2

n n n a a n x b n x ∝ = + π + π ∑ Now, 1 1 1 2 0 1 1 1 ( ) 0 2 x a f x dx x dx - - - ? ? = = = = ? ? ? ? ∫ ∫ 1 1 cos 0 - = π = ∫ n a x n xdx and ( ) 1 1 2

sin 1 n n b x n x dx n - = π = - - π ∫ Hence ( ) 1 sin 2 ( ) 1 n n n

x

f x n ∝ = π = - - π ∑

Example 2. Express the function f(x) = x as a half-range sine series in the interval 0 > x > 2. Solution :

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We have, 1 ( ) sin 2 n n n x f x x b ∝ = π = = ∑ , where ( ) 2 0 2 sin 2 2 n n x b f x dx π = ∫ = 2 0 sin 2 n x x dx

π ∫

28 NSOU l CC-PH-07 = ( ) 2 2 0

42%	**MATCHING BLOCK 12/115**	**SA**	019E1130_Mathematical Physics.pdf (D165097245)

cos sin 2 2 2 2 ? ? π π - - ? ? ? ? - π ? ? π ? ? ? ? n x n x x n n = ( ) 4 4 cos 1 - π = - - π π n n n = 4 for is odd 4 for is even n n n n ? ? π ? ? - π ? Hence ( ) 1 1 4 ( ) sin 2 n n n x f x x n ∝ = - π = = - π ∑

Example 3. Expand the function ( ) | | f x x for l x l = - > > = - > > = - > > = - > > and prove that 2 2 2 2 1 1 1 .... 8 1 3 5 πππππ + + + = + + + = + + + = + + + = Solution : We have, 0 1 ( ) cos sin 2 n n n a n x n x f x a b l l ∝ = π ? ? π = + + ? ? ? ? ∑ Since f(x) = | x | is even, so b n = 0 Hence, 0 1 ( ) cos 2 n n a n x f x a l ∝ = π = + ∫ Now, 0 1 ( ) l l a f x dx l - = ∫ = 0 2 l x dx l l = ∫
NSOU l CC-PH-07 29 and 1 ( )cos - π = ∫ l

20%	**MATCHING BLOCK 13/115**	**SA**	MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

n l n x a f x dx l l = o 2 cos l n x x dx l l π ∫ = ( ) ( ) 2 sin cos 2 l o n x n x l l x l n n l l ? ? π π ? ? ? ? - ? ? ? ? ? ? ? ? ? ? ? ? - π ? ? π ? ? ? ? = ( ) 2 2 2 cos 1 l n n π- π = ( ) 2 2 2 1 1 n l n ? ? - - ? ? ? ? π = 2 2 0, is even 4 , is odd n l n n ? ? ? - ? π ? f(x) = |x| 2 2 1 4 1 cos 2 n n x l l l n ∝ =

π = - π ∑ (n is odd only) 2 2 2 4 1 3 1 cos5 cos cos .... 2 3 5 l l x x x l l l ? ? π π π = - + + + ? ? π ? ?
30 NSOU l CC-PH-07 Put, x = 0, then 2 2 2 2 4 1 1 1 0 .... 2 1 3 5 l l ? ? = - + + + ? ? π ? ? Hence 2 2 2 2 1 1 1 .... 8 1 3 5 π + + + = Example 4. Expand f(x) = x 2 as Fourier series in the internal – π > x > π and hence evaluate (i) 2 1 1 ∝ = ∑ n n and (ii) ( ) 2 1 1 2 1 ∝ = - ∑ n n Solution : The function f(x) = x 2 is even in the interval –π to π, hence b n = 0 0 1 ( ) cos ∝ = ∴ = + ∑ n n f x a a nx Now, 2 2 0 1 2 3 a x dx
π -π π = = π ∫ ( ) 2 2 0 2 4 cos 1 π = = - π ∫ n n a x nx dx n ∴ 2 2 2 2 2 1 1 1 ( ) 4 cos cos2 cos3 .... 3 1 2 3 f x x x x x x ? ? π = = - - + ? ? ? ? (1) Putting x = 0 we get, 2 2 2 2 1 1 1 0 4 .... 3 1 2 3 ? ? π = - - + ? ? ? ?
NSOU l CC-PH-07 31 = 2 2 2 2 2 2 1 1 1 1 2 2 4 .... 3 1 2 3 2 4 ? ? ? ? π - + + + - - ? ? ? ? ? ? ? ? = 2 2 2 2 1 1 1 2 1 4 4 3 2

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n n n n ∝ ∝ = = ? ? π - + ? ? ? ? ∑ ∑ = 2 2 1 1 2 3 n n ∝ = π - ∑ or, 2 2 1 1 6 n n ∝ = π = ∑ (ii) ( ) 2 2 2 2 1 1 1 1 .... 1 3 5 2 1 n n ∝ = = + + + - ∑ = 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 .... 1 3 5 2 4 2 n n ∝ = + + + + + - ∑ = 2 2 1 1 1 1 1 4 n n n n ∝ ∝ = = - ∑ ∑ = 2 1 3 1 4 n n ∝ = ∑ = 2 3 · 4 6

π = 2 8 π
32
NSOU l CC-
PH-07 Example : 5. An alternating current after passing through a rectification has the form i = i 0 sin x for 0 x≤ ≤ π = 0 for 2 x π ≤ ≤ π Where i 0 is the maximum current and the period in 2π. Express i as a Fourier series. Solution : 0 1 1 ( ) cos sin 2 n n n n a f x a nx b nx ∝ ∝ = = = + + ∑ ∑ Now, 2 0 0 0 0 0 2 1 1 ( ) sin i a f x dx i xdx

17%	**MATCHING BLOCK 15/115**	**SA**	019E1130_Mathematical Physics.pdf (D165097245)

π π = = = π π π ∫ ∫ 0 0 sin cos n i a x nxdx π = π ∫ ( ) ( ) 0 0 sin 1 sin 1 2 i n x n x dx π = ? + - - ? ? ? π ∫ ( ) ( ) ( ) ( ) 0 cos 1 cos 1 2 1 1 o n x n x i n n π ? + - ? = - + ? ? π + - ? ? ( ) ( ) 0 1 cos 1 cos 1 1 2 1 1 n n i n n ? - + π - π- ? = - + ? ? π + - ? ? ∴ a 1 = 0, 0 2 2 3 i a = - π , a 3 = 0, o 4 2 15 = - π i a , a 5 = 0, 0 0 2 35 i a = - π , ... 0 0 sin sin n i b x nxdx π = π ∫
NSOU l CC-PH-07 33 = ( ) ( ) 0 0 cos 1 cos 1 2 i n x n x dx π ? - - + ? ? ? π ∫ = ( ) ( ) 0 sin 1 sin 1 2 1 1 o n x n x i n n π ? - + ? - ? ? π - + ? ? ( ) 2 0 0 1 0 0 sin 1 cos2 2 2 o i i i b xdx x dx π π = = - = π π ∫ ∫ ∴ 0 0 0 0 0 2 2 2 ( ) sin cos2 cos4 cos6 .... 2 3 15 35 i i i i i f x x x x x = + - - - π π π π . = 0 0 0 2 1 2

cos2 sin 2 4 1 n i i i nx x n ∝ = + - π π - ∑ 1.13 Summary 1. Complex forms of Fourier series and even and odd functions and change of interval in Fourier expansion have been discussed. 2. Various types of wave form are discussed with reference to the example of electronics. 1.14 Review Questions and Answer : 1. Explain the periodic functon and state the Fourier's theorem. Ans. See articles no. (1.2) and (1.5) 2. Describe the complex form of Fourier's theorem. Ans. See article no. (1.6) 3. What is odd and even function? Explain with example. Ans. See article no. (1.9) 4. (a) What are the Dirichlet's conditions in Fourier's series? (b) What is the limitations of Fourier's theorem Ans. (a) See article no. (1.8).

34 NSOU l CC-PH-07 (b) We already discuss the Dirichlet's condition in Fourier's series. Therefore these conditions impose the limitations in the application of Fourier's theorem. 5. If F(x) have a Fourier Series expansion F(x) = 0 1 1

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cos sin , 2 n n n n a a nx b nx ∝ ∝ = = + + ∑ ∑ then prove that [ ] 2 1 ( ) f x dx

π -π π ∫ = ( ) 2 2 2 0 1 1 n n n a a b ∝ = + + π ∑ Ans. We have, [ ] 2 2 0 1 1 1 1 ( ) cos sin 2 π

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π ∝ ∝ = = -π -π ? ? ? ? = + + π π ? ? ? ? ∑ ∑ ∫ ∫ n n n n a f x dx a nx b nx dx = 2 2 2 2 2 0 1 1 1 1 1 1 cos . sin 4 n n n n a dx a nx dx b nx dx π π π ∝ ∝ = = -π -π -π + + π π π ∑ ∑ ∫ ∫ ∫ 0 0 1 1 1 1 cos sin π π ∝ ∝ = = -π -π + π π ∑ ∑ ∫ ∫ n n n n a a nx dx a b nx dx + 1 1 2 cos sin π ∝ ∝ = = -π π ∑ ∑ ∫ n n n n a nx b nxdx = 2 2 2 0 1 1 1 1 2 0 0 0 4 ∝ ∝ = = ? ? ? ? π+ π + π + + + π π ? ? ? ? ∑ ∑ n n n n

a a b = ( ) 2 2 2 0 1 1 2 n n n a a b ∝ = + + ∑ Hence, ( ) ( ) 2 2 2 2 0 1 1 2 n n n
a
F x
dx a b π ∝ = -π ? ? = + + ? ? π ∑ ∫ The above relation is called Parseval relation.
NSOU l CC-PH-07 35 6. Show that, if a real function f(x) be expanded into complex Fourier series, that is ( ) in x n f x a e ∝∝∝∝ -∝-∝-∝-∝ ==== ∑∑∑∑ , then show that C –n is the complex conjugate of c n. Ans. We have, 1 ( ) 2 in x n a f x e dx n π -π = ∫ ∴ 1 ( ) 2 π - - -π = π ∫ in x n C f x e dx = n c ∗ where * n c is the complex conjugate of c n . 7. Expand the function f(x) = e bx in the interval [–ππππππ, ππππππ] in Fourier series where b is non-zero constant, and show that (i) cos 2sin hbx hb π π = ( ) 2 2 1 1 1 cos 2 ∝ = + - + ∑ n n b nx b b n ( ) x -π ≤ ≤ π (ii) ( ) 1 2 2 1 cos 1 sin 2sin n n h bx n nx h b b n ∝ - = π = - π + ∑ ( ) x -π ≤ ≤ π Ans. We have the function ( ) = bx f x e Now, co-efficients of expansion are 0 2sin 1 b b bx hb e e a e dx b b π π -

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π -π π - ∴ = = = π π π ∫ ∴ 2 2 1 1 cos sin cos . bx n b nx n nx a e nx dx b n π π -π -π ? ? + = = ? ? π π + ? ? ∫ 36 NSOU l CC-PH-07 = ( ) 2 2 1 2 · sin h. , n b b b n - π π + and b n = 2 2 1 1 sin cos sin x b b bx n nx e nx dx b n π π -π -π ? ? - = ? ? π π + ? ? ∫ = ( ) 1 2 2 1 2 sin - - π π + n n

h b b n Hence, we can write ( ) bx f x e= in Fourier expansion form sin ( ) bx h b f x e b π = = π + ( ) 2 2 1 1 2 n n b b n ∝ = - π + ∑ sin h bπ cos nx + ( ) 1 2 2 1 1 2 n n n b n - ∝ = - π + ∑ sin h bπ sin nx = ( ) ( ) 2 2 1 1 2sin 1 cos sin 2 n n h b b nx n nx b b n ∝ = ? ? - π ? ? + - π ? ? + ? ? ∑ ... (1) Replacing x by (–x) in equation (ii) we get, ( ) ( ) 2 2 1 1 2sinh 1 cos sin 2 n bx n b e b nx n nx b b n ∝ - = ? ? - π ? ? = + + π ? ? + ? ? ∑ ... (2)
NSOU l CC-PH-07 37 Now, adding equation (1) and equation (2) we get, ( ) 2 2 1 2 1 2sin 1 cos ∝ - = ? ? - π ? ? + = + π ? ? + ? ? ∑ n bx bx n b h b e e nx b b n or, 2 cos 2sin π π h bx h b = ( ) 2 2 1 2 1 1 cos ∝ = - + + ∑ n n b nx b b n ∴ cos 2sin hbx hb π π = ( ) 2 2 1 1 1 cos 2 n n b nx b b n ∝ = + - + ∑ (Proved) Now subtracting equation (1) and (2) we get, ( ) 2 2 1 2sin 2 sin 1 n bx bx n h b n nx e e b n ∝ - = ? ? π ? ? - = - - π ? ? + ? ? ∑ or, ( ) 1 2 2 1 2 sin 2 sin 1 2sin n n h bx n nx h b b n ∝ - = π = - π + ∑ ∴ ( ) 1 2 2 1 sinh 1 sin 2sinh α - = π = - π + ∑ n n bx n nx b b n (Proved) 8. If ( ) ( ) 0 /2 = ≤ ≤ π f x x x = ( ) 2 x x π π - ≤ ≤ π , the express the function as a sine and cosine series and show that 2 2 2 2 1 1 1 1 .... 8 3 5 7 π - + - + = Ans. The graphical representation of the function is shown below :
38 NSOU l CC-PH-07 –π – /2π 0 π x f x( ) – /2π fig. : 1.10 The function is defined only over half range (0 to π), hence it can be represented by either cosine or sine series. When we represented as cosine series the function repeated in the range (–π to 0) as shown in dotted line i.e., as an even function and is given by f(x) = f(–x). The function given in the range (0 to π) is odd i.e., f(x) = – f(–x) and can be represented by sine series. For cosine series, the expansion over the half range (0 ro

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π) is given by 0 1 ( ) cos , 2 ∝ = = + ∑ n n a f x a nx where 0 0 1 ( ) 2 a f x dx π = π ∫ ∴ ( ) /2 2 0 0 0 2 2 · 4 2 a x dx x dx π π ? ? π π ? ? = + π - = = π π ? ? ? ? ∫ ∫ and 0 1 ( ) cos 2 π = π ∫

n a f x nx dx
NSOU l CC-PH-07 41 Unit–2 q Frobenius Method and Special Function Structure : 2.0 Objectives 2.1 Introduction 2.2 Frobenius Method 2.2.1 Ordinary and singular point and analytic function 2.2.2 Existence of power series method 2.2.3. Steps for solving series solution 2.3 Legendre differential equation 2.4 Rodrigue's formula for Legendre Polynomial 2.5 Generating function of P n (x) 2.6 Orthogonality of Legendre Polynomials 2.7 Recurrence formulae for P n (x) 2.8 Bessel's equation 2.9 Recurrence formula for Bessel functions 2.10 Generating function for J n (x) 2.11 Zeros of Bessel's Function 2.12 Orthogonalities of Bessel functions 2.13 Hermite's equation 2.14 Hermite polynomial H n (x) 2.15 Generating function of H n (x) 2.16 Rodrigue's formula of H n (x) 2.17 Recurrence formula of H n (x) 2.18 Orthogonality of Hemite Polynomials 2.19 Summary 2.20 Review Questions and Answer 2.0 Objectives 1. To know what is Frobenius method. 2. To know Legendre differential equation, Bessel's equation, Hermite's equation and their generating functions, recurrence formula and orthogonalities. 2.1 Introduction As we have already remarked, in some simple cases it is possible to find the general solution of the homogeneous equation y'' + Py' + Qy = 0 Equation (A) in terms of familiar elementary functions. For the most part, however, the equations
42 NSOU l CC-PH-07 of this type having the greatest significance in theoretical physics are beyond the reach or elementary methods, and one has no choice but to resort to the method of series solution. The central fact about the homogeneous equation (A) is that the behaviour of its solutions near a point x 0 depends on the behaviour of the coefficients P and Q near this point. A point x 0 is called an ordinary point of Eqn. (A) if both P and Q are analytic at x 0 , in the sense that P(z) and Qtz), looked upon as functions of the complex variable z, are analytic at z = x 0 . It is well known to us that in this case both P and Q will have Taylor series expansions in some neighbourhoods of the point x 0 . Any point that is not an ordinary point of Eqn. (A) is called a singular point of Eqn. (A). A singular point x 0 of Eqn. (A) is said to be regular if both (x - x 0) P and (x - x 0) 2 Q are analytic at x 0 , and irregular otherwise. A singular point x 0 of Eqn. (A) is said to be regular if both (x - x 0)P and (x - x 0) 2 Q are analytic at x 0 , and irregular otherwise. The reason behind such a classification of points in relation to a given homogeneous linear differential equation will become clear as we proceed. kn 2.2 Frobenius Method : This method is developed by the German mathematician Ferdinand Georg Frobenius (1849-1917) to find out infinite series solution for a second-order ordinary differential equation of the form ( ) ( ) ( ) 2 2 0 d u du P x Q x R x u dx dx + + = ... (2.2.1) ∴ ( ) 2 2 ( ) d u du A x B x u a dx dx + + = ... (2.2.2), where ( ) ( ) ( ) Q x A x P x = and ( ) ( ) ( ) R x B x P x = are the functions of x. The solutions of the differential equation 2 2 0 - = d u u dx is u = Ae x + Be -x and 2 2 0 + = d u u dx is u = A sin x + B cos x
NSOU l CC-PH-07 43 The above two equations are valid for power series solution Let us take the differential equation 2 2 4 1 0 d u u dx x + = , having the solutions u = a 0 + a 1 x + a 2 x 2 + a 3 x 3 + ... On solving the equation, we obtain a 0 = a 1 = a 2 = ... 0. So there is no series satisfying the above differential equation. Before going to the discussion of existance or non-existance of series solution of the form of equation. (2.2.1) we must have to know about the ordinary or singular points of the differential equation. 2.2.1 Ordinary and singular point and analytic function : If ( ) 0 P a ≠ , then A(x) and B(x) have finite values and x = a is called an ordinary point of the equation. For ordinary point power series method exists. If ( ) 0= P a , then x = a is called singular point and at that point A(x) and B(x) have the infinite values, At x = a,if (x – a) A (x) and (x – b) 2 B(x) have finite values, then solution can be developed by power series method. A function is said to be analytic or regular at a point x = a, if the function is single valued, continues derivatives and expand as Taylor expansion at x = a i. e. ( ) ( ) ! n n f a x a n - ∑ exists. 2.2.2 Existence of power series method : The theorem by which we can examine whether the power series solution exists or not is called Frobenius-Fuch's theorem. The theorem is discussed below : (i) If x = a is ordinary point of eqation (2.2.1), then solution can be expressed as ( ) ( ) ( ) 2 0 1 3 u x a a x a a x a = + - + - .... (2.2.3) (ii) If x = a is a regular or non-essential singular point, then solution can be expressed by ( ) ( ) ( ) ( ) 2 0 1 2 ..... m u x x a a a x a a x a ? ? = - + - + - + ? ? ? ? ... (2.2.4) The equations (2.2.3) and (2.2.4) are convergent at every print within the circle, drawn by taking the radius of the circle x = a.

44 NSOU l CC-PH-07 Some examples are illustrated below : Example 1. 2 2 2 0 d u du x x u dx dx + + = By compairing with equation (2.2.1) we obtain ( ) 0 0 P ≠ , is an ordinary point. Hence series solution method is eligible. Example 2. 2 2 2 0 d u du x x u dx dx + + = By compaining with equation (2.2.1) we obtain P(0) = 0 i.e., x = 0 is singular point. The equation can be written as 2 2 2 1 1 0 d u du u x dx dx x + + = , Hence, ( ) 1 0 · x x - and ( ) 2 2 1 0 · x x - have the finite values at x = 0, so the singular point is non-essential or regular and solution can be expressed by power series method. Example 3. Laguerre equation is given by ( ) 2 2 1 0 d u du x x nu dx dx + - + = or, ( ) 2 2 1 1 0 d u du n u x dx x dx + - + = Hence ( ) ( ) 1 0 x x x - - and ( ) 2 0 · n x x - hvae the finite values at x = 0 and x = 0 is non-essential or regular point and can be expressed by power series method. Example 4. 2 2 4 1 0 d u u dx x + =
NSOU l CC-PH-07 45 at x = 0, ( ) 2 4 1 0 · x x - →∝ . Hence the singular point is essential and series solution method is not applicable Example 5. 2 4 3 2 0 d u du x x xu dx dx + + = or, 2 2 3 1 1 0 d u du u x dx dx x + + = For above equation ( ) 0 1 0 | 1 x x x = - = but, ( ) 2 0 3 1 0 · | x x x = - →∝ . Hence the singular point is essential and series solution method is not applicatble. 2.2.3. Steps for solving series solution : (i) First we have to check the ordinary and singular point. (ii) Then check equation (2.2.3) or eqation (2.2.4), which one be the solution of the equation. (iii) Put , du u dx and 2 2 d u dx in said equation. (iv) Find the co-efficients a 0 , a 1 , a 2 ,.......... (v) Finally put the values in equation (2.2.3) or equation (2.2.4) and get the final result.
46 NSOU l CC-PH-07 2.3. Legendre differential equation : Legendre differential equation is given by ( ) ( ) 2 2 2 1 2 1 0 d u du x x n n u dx dx - - + + = ... (2.3.1) Here n is real constant or, ( ) 2 2 2 2 1 2 · 0 1 1 n n d u x du u dx dx x x + - + = - - ... (2.3.2) Comparing equation (2.2.1) we obtain at 1, x = ± ( ) 1 0 P ± = . Hence 1
x = ± is called singular point and ( ) ( ) 1 2 2 | 1 1

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x x A x x x = - = - - is finite and ( ) ( ) ( ) 1 2 2 1 1 =- - = + - x x A x x x is also finite Again, ( ) ( ) ( ) ( ) 1 2 2 1 1 1 = + = - - x n n B x x x is finite and ( ) ( ) ( ) ( ) 1 2 2 1 1 1 =- + = + - x n n B x x x

is also finite. By the above argument, we must say 1 x = ± are non-essential or regular singular points. Now, at x = 0, ( ) 2 0 0 1 | 1 x P x = = - = i.e., ( ) 0 0 P ≠ . Hence x = 0 is ordinary point and the trial solution can be expressed as equation (2.2.3) i.e.
NSOU l CC-PH-07 47 0 m m m u c x ∝ = = ∑ ... (2.3.3) Putting the value of equation (2.3.3) in equation (2.3.1) we get ( ) ( ) ( ) 2 2 1 0 0 0 1 1 2 1 0
m m m m m m m m m x m m c x x c mx n n c x ∝ ∝ ∝ - - = = = - - - + + = ∑ ∑ ∑ or, ( ) ( ) ( ) 2 1 0 0 0 0 1 1 2 1 0 ∝ ∝ ∝ ∝ - + = = = = - - - - + + = ∑ ∑ ∑ ∑
m m m m m m m m m m m m m m
c x m m c x c mx n n c x
To get the same power of x, we can write the above equation in the form To get the same power 5 x , we put 2 m

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s= + Hence, ( )( ) ( ) 5 5 2 0 2 2 1 1 s s s s s s c x s s c x ∝ ∝ + = = + + - - ∑ ∑ ( ) 5 5 1 0 2 1 0 ∝ ∝ = = - + + = ∑ ∑ s s s s sc x n n c x ...→ (2.3.4) From equation (2.3.4) we get, ( )( ) ( )( ) 2 1 2 1 s s n s n s C C s s + - + + = - + + ...→ (2.3.5)

This is called recurrence relation or recurssion formula. ( ) 2 0 1 2!

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n n C C + = - ( )( ) 3 1 1 2 3! - + = - n n C C 48 NSOU l CC-PH-07 ( )( ) 4 2 2 3 4.3 n n C C - - + = - ( )( ) 5 3 3 4 5.4 n n C C - + = ( )( )( )( ) 0 2 1 3 4! n n n n C - + + = ( )( )( )( ) 1 3 1 2 4 5! n n n n

C - - + + = C 6 = ... so on C 7 = ... so on Putting the values in equation (2.3.3) we get, u(x) = c 0 u 1 (x) + c 1 u 2 (
x) ....→ (2.3.6) Where, ( ) ( ) ( ) ( )( ) 2 4 1 1 2 1 3 1 2! 4! + - + + = - +

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n n n n n n u x x x + ...→ (2.3.7) and u 2 (x) ( )( ) ( )( )( )( ) 3 5 1 2 3 1 2 4 3! 5! n n n n n n x x x - + - - + + = - + + ... → (2.3.8)

Equation (2.3.7) and equation (2.3.8) contain even power of x and odd power of x, respectively. Legendre Polynomials P n (x) : The polynomials u 1 (x) and u 2 (x), multiplied by some constants, are called Legendre polynomials and are denoted by P n (x).
The coefficient of c n of the highest power x n is ( ) ( ) ( ) 2 2 ! 1.3.5................ 2 1 ! 2 ! n n n n c n n - = = ...→ (2.3.9)
From equation (2.3.5) we get, ( )( ) ( )( ) 2 2 1 1 s s s s c c n s n s + + + = - - + + ... → (2.3.10)
NSOU l CC-PH-07 49 Put s + 2 = n, in equation (2.3.10), then ( ) ( ) 2 1 2 2 1

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n n n n C C n - - = - - = ( ) ( ) ( ) ( ) 2 1 2 ! 2 2 1 2 ! n n n n n n - - [from equation (2.3.9)] = ( ) ( )( ) ( ) ( ) ( )( ) 1 2 2 1 2 2 ! 2 2 1 2 1 ! 1 2 ! n n n n n n n n n n n n - - - - - - - - = ( ) ( ) ( ) 2 2 ! 2 1 ! 2 ! n n n n - - - Similarly, ( )( ) ( ) 4 2 2 3 4 2 3 n n n n C C n - - - - - = - - - = ( ) ( ) ( ) 2 4 ! 2 2! 2 ! 4 ! n n n n - - -

and so on In general,
n – 2m < 0, ( ) ( ) ( ) ( ) 2 2 2 ! 1 2 ! ! 2 ! m n m n n m C m n m n m - - = - - -
Putting the value of C n – 2m in equation (2.3.3) we obtain the power series solution ( ) ( ) ( ) ( ) ( ) 2 0 2 2 ! 1 2 ! ! 2 !
m n m n m n m u x x m n m n
m ∝ - = - = - - - ∑ ..... → (2.3.11)
50 NSOU l CC-PH-07 The resulting solution of
equation (2.3.11) is called the Lengendre polynomial of degree n and is denoted by ( ) ( ) ( ) ( ) ( ) ( ) 2 2 2 ! 2 2 ! 2 1 ! 2 ! 2 ! - - x = - - -

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n n n n n n n x P x x n n n + ... → (2.3.12) The first few of these function ( ) ( ) ( ) ( ) ( ) 0 2 2 4 2 4 1 1 3 1 2 1 35 30 3 6 P x P x x P x x x ? = ? ? ? = - ? ? ? = - + ? ? ( ) ( ) ( ) ( ) ( ) 1 3 3 5 3 5 1 5 3 2 1 63 70 15 8 P x x P x x x P x x x x ? = ? ? ? = - ? ? ? = - + ? ? .... (2.3.13)

A
few of these function are shown in Fig. 2.1 Fig. 2.1 : Legendre function of the first kind P n (x)
NSOU l CC-PH-07 51 2.4 Rodrigue's formula for Legendre Polynomial : Rodrigue's formula is given for P
n (x) by ( ) ( ) 2 1 · 1 2 !
n n n n n d P x x n dx = - ....→ (2.4.1)
Proof : Let, t = (x 2 – 1) n ( ) 1 2 1 2 n dt n x x
dx - ⇒ = - ....→ (2.4.2) or, ( ) 2 1 2 dt x ntx dx - = ...→ (2.4.3) Differentiating (2.4.2) by Leibnitz theorem, (n + 1) times ( ) d D dx ≡ , ( ) ( ) 2 2 1 1 1 1 2 1 2 2
n

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n n n n x D t C x D t C D t + + + + - + + { } 1 1 1 2 + + = + n n n n x D t C D t or, ( ) 2 2 1 1 1 1 1 2 1 1 2 2 0 n n n n n n x D t x C n D t C n C D t + + + + + ? ? ? ? - + - + - = ? ? ? ? or, ( ) ( ) 2 2 1 1 2 1 0 n n n x D t x D t n n

D
t + + - + - + = or, ( ) ( ) 2 2 2 1 2 · 1 0

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d y dy x x n n y dx dx - + - + = , [

Put D n t = y]
52 NSOU l CC-PH-07 The above equation is Legendre equation. Hence y = P n (x) is also its solution. Now from equation (2.3.1) we can write ( ) ( ) ( ) 2 0 1 1 1, 1 2
d
P

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x P x x x dx = = - = , ( ) ( ) 2 2 2 2 2 2 1 1 2 2! = - d P x x dx = ( ) ( ) 2 2 2 1 1 · 2 1 2 3 1) 2 2 .2 ? ? - = - ? ? ? ? d x x x dx ( ) ( ) ( ) 3 3 2 3 3 3 3 1 1 1 5 3 2 2 3! = - = - d P x x x x dx , ( ) ( ) ( ) 4 4 2 4 2 4 4 4 1 1 1 35 30 3 6 2 ·4! = - = - + d P x x x x

dx
and
so on....... All the above solutions are matched with the equation (2.3.13) 2.5 Generating function of P n (x) : ( ) ( ) 1 2 2 0 1 2 ∝ - = - + = ∑

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n n n xy y y P x .... (2.5.1) Proof : ( ) ( ) 1 1 2 2 2 1 2 1 2 - - - + = ? - - ? ? ? xy y y x y NSOU l CC-PH-07 53 = ( ) ( ) ( ) 2 3 2 3 1.3 1.3.5 1 2 2 2 2 2.4 2.4.6 y x y y x y y x y + - + - + - + ... + ( ) ( ) 1.3.5. 2 1 2 2.4.6..........2 - + -

n n n y x
y
n The co-efficients of

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y n are ( ) ( ) ( ) ( ) 1 2 1 1.3.5.......... 2 1 1.3.5.......... 2 3 2 2 2.4.6.........2 2.4.6...........(2 2) n n n n n x x C n n - - - - - - + ... = ( ) ( ) ( ) 2 1.3.5.......... 2 1 1 .... ! 2 2 1 - - ? - ? - + ? ? - ? ? n n n n n x x n n = P n (x) ( ) ( ) ( ) ( ) ( ) 1 2 2 2 0 1 2 0 1 2 0 1 2 .... ∝ - = - + = = + + + ∑ n n n

xy y y P x P x yP x y P x ( ) 1 2 2 1 2 - - +xy y
is thus the generating function for Legendre polynomial P n (x). 2.6 Orthogonality of Legendre Polynomials : Legendre's polynomials are a set of orthogonal functions in the interval [– 1,1]. P n (

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x) is a solution of ( ) ( ) 2 2 2 1 2 1 0 - - + + = d y dy x x n n y dx dx ... (2.6.1) P m (x) is a solution of 54

NSOU l CC-PH-07 ( ) ( ) 2 2 2 1 2 1 0 - - + + = d t dt x x m m t dx dx ... (2.6.2) Multiplying (2.6.1) by t and (2.6.2) by y and substracting, We get, ( ) ( ) ( ) 2 2 2 2 2 1 2 1 1 0 ? ? ? ? - - - - + ? + - + ? = ? ? ? ? ? ? ? ? ? ? ? ? ? d y dy d t dt x t y x t y n n m m yt dx dx dx dx By integrating w. r. t. x from – 1 to + 1, we get, ( ) ( )( ) 1 1 2 1 1 1 1 0 + - - ? ? - - + - + + = ? ? ? ? ∫ dy dt x t y n m n m y tdx dx dx or, ( )( ) ( ) ( ) 1 1 1 0 + - - + + = ∫ n m n m n m P x P x dx [Since, y and t are the solutions of Legendre's differential equation] Hence, ( ) ( ) 1 1 0 + - = ∫ n m P x P x dx ... (2.6.3) The above equation is the required condition of orthogenality 2.7 Recurrence formulae for P n (x) : From equation (2.5.1) we get ( ) ( ) 1 2 2 0 1 2 ∝ - = - + = ∑ n n x xy y y P x

NSOU l CC-PH-07 55 Let, ( ) 1 2 2 1 2 - - + = xy y t ... (2.7.1) or, ( ) 2 2 1 2 1 - + = t xy y Differentiating w. r. t y we get, ( ) ( ) 2 2 2 1 2 2 2 0 - + + - + = dt t xy y t x y dy or, ( ) ( ) 2 1 2 0 - + + - = dt xy y t y x dy

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or, ( ) ( ) ( ) ( ) 2 1 0 0 1 2 0 n n n n n n xy y ny P x y x y P x ∝ ∝ - = = - + + - = ∑ ∑ ( ) 0 Put ∝ = ? ? ? ? = ? ? ? ? ∑ n n n t y P x The co-efficients of y n – 1 from both sides given ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 2 2 1 2 1 2 0 n n n n n nP x x n P x n P x P x xP x - - - - - - + - + - = or, ( ) ( ) ( ) ( ) ( ) 1 2 2 1 1 n n n nP x n xP x n P

x - - = - - - ......(2.7.2)

The above equation is the recurrence formulae for P n (x). 2.8 Bessel's equation : Bessel's equation of order n is expresed as ( ) + + - = 2 2 2 2 2 0 d u du x x x n u dx dx ... (2.8.1) x = 0 is a regular singular point. Therefore power series solution can be written as equation (2.2.4) i.e.

56 NSOU l CC-PH-07 ∝ + = = ∑ 0 m r r r u C x ... (2.8.2) Put the value of (2.8.2)

in equation (2.8.1) we get, ( )( ) 2 2 0 1

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m r r r x C m r m r x ∝ + - = + + - ∑ + ( ) ∝ + - = + ∑ 1 0 m r r r x C m r x + ( ) 2 2 0 0 m r r r x n C x ∝ + = - = ∑ or, ( )( ) ( ) 2 0 1 m r r r C m r m r m r n x ∝ + = ? ? + + - + + - ? ? ∑ + 2 0 0 m r r r C x ∝ + + = = ∑ or, ( ) 2 2 0 m r r r C m r n x ∝ + = ? ? + - ? ? ∑ + 2 0 0 m r r r C x ∝ + + = = ∑ Putting, r = 0,1

and Equation the co-efficient of x m and x m + 1 we get, 2 2 0 0 C m n ? ? - = ? ? and ( ) 2 2 1 1 0 C m n ? ? + - = ? ? ⇒ ( ) 2 2 1 0 m n ? ? + - ≠ ? ? , So C 1 = 0 , m n ⇒ = ± 0 0 C ≠ (indicial equation) Equating the co-efficient of the general term 2 m r x + + to zero, we get, ( ) 2 2 2 2 0 r r C m r n C + ? ? + + - + = ? ? ( ) 2 2 2 1 2 r r C C m r n + - ⇒ = + + - ... (2.8.3) NSOU l CC-PH-07 57 The above equation is recurrence relation. From (2.8.3) we get C 1 = C 3 = C 5 .... 0 and ( ) - = + - 2 0 2 2 1 C C 2 m n ( ) ( ) = ? ?? ? + - + - ? ?? ? 4 0 2 2 2 2 1 2 4 C C m n m n and so on If we say u = Au 1 (x) + Bu 2 (x), then for m = n, we can write, ( ) ( ) ( )( ) ? ? = - + ? ? + + + ? ? 2 4 1 0 1 .... 2 2 2 2.4 2 2 2 4

53%	MATCHING BLOCK 34/115	SA	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

n x x u x C x n n n ( ) ( )( ) ( ) 2 0 0 1 2 !..........2 1 2 .... r r n r r r x C x r n n n r ∝ = - = + + + ∑ ... (2.8.4) The solution (2.8.4) is called

the

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Bessel function of the first kind of order n and is denoted by ( ) ( ) ( ) ( ) 2 0 1 2 ! 1 r n r n r x J x r n r ∝ + = - =

Γ + + ∑ ... (2.8.5) Replacing n by −n in equation (2.8.5) we get, ( ) ( ) ( ) ∝ - + - = - - = Γ - + + ∑ 2 0 1 ( ) 2 ! 1 n r n r r x J x r n r ... (2.8.6)

The

solution (2.8.6)

is

called the Bessel function of the first kind of order (−n).

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NSOU l CC-PH-07 Hence the complete solution of equation (2.8.1) is ( ) ( ) . n n U AJ x B J x - - = + 2.9 Recurrence formula for Bessel functions : Formula : 1. + = - 1 J J J

n n n x

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n x ' Proof : We have, ( ) ( ) ( ) ∝ + = - ' = Γ + + ∑ 2 0 1 J 2 ! 1 r n r n r x r n r Differentiating w. r. t. x, we get, ( ) ( ) ( ) ( ) ∝ + - = - + = x Γ + + ∑ 2 1 0 1 2 1 J 2 2 ! 1 r n r n r n r x r n r ' ( ) ( ) ( ) ∝ + = - ∴ = Γ + + ∑ 2 0 1 J 2 ! 1 r n r n r x x n r n r ' + ( ) ( ) ( ) ∝ + - = - Γ + + ∑ 2 1 0 1 2 2 2 . ! 1 r n r r r x x r n r = ( ) ( ) ( ) ( ) ∝ + - = - + - Γ + + ∑ 2 1 1 1 2 1 ! 1 r n r n r x nJ x r n r = ( ) ( ) ( ) ∝ + + + = - +

Γ + + ∑ 1 2 1 1 1 2 ! 2 t n t n t x nJ x
t n t [Put, r – 1 = t] = ( ) ( ) ( ) ( ) ∝ + + = - - + Γ + + + ? ? ? ? ∑ 1 2
n 0 1 n J 2 ! 1 1 t n
t t x x t n t Here ' = - n n+1
xJ nJ J n x ... (2.9.1)
NSOU l CC-PH-07 59 Formulae : 2. - ' =- + 1 J J
J
n n n x n x Proof : We have, ( ) ( ) ( ) ( ) 2 1 0 1 2 1 J · 2 2 ! 1
r
n r n r n r x r n r ∝ + - = - + ' = Γ + + ∑ Hence, ( ) ( ) ( ) ( ) 2 0 1 2 J 2 ! 1
r n r n r n r
x x r n r ∝ + = - + ' = Γ + + ∑ ( ) ( ) ( ) ( ) ∝ + = - + - = Γ + + ∑ 2 0 1 2 2 2 ! 1 r n
r r n r n x r n r = ( ) ( ) ( ) ( ) ( ) ( ) 2 0 0 1 2 2 1 2 ! 1 ! 1 r r n r r r n r x n
r n r r n
r ∝ ∝ + = = - + - -
Γ + + Γ + + ∑ ∑ ( ) 2 2 +n r x = ( ) ( ) ( ) ( ) ( ) ∝ + = - + - + Γ + ∑ 2 0 2 1 2 ! r n r n r n r x nJ r n r n r = ( ) ( ) ( ) ( ) ∝ + - = - - Γ + ∑ 2 1 0 2 1 · J 2 2 ! r n r n r
x x n r
n
r = ( ) ( ) ( ) ( ) ∝ - - ? ? ? ? = - -
Γ - + + ? ? ? ? ∑ 1 2 0 1 J 2 ! 1 1 r n r n r x x n r n r - = - 1
n
n n
x
J
xJ nJ ' ... (2.9.2)
60 NSOU l CC-PH-07 Formula 3. From equation (2.9.1) and (2.9.2) we get, 1 1 2 - + = - + - n n n n n xJ xJ nJ nJ xJ '
Hence 1 1 2J J J n n n - + = - ' ... (2.9.3) Formula 4. Equating (2.9.1) and (2.9.2) we obtain - + - = - 1 1 J J J
J
n n n
n x n
n x ( ) - + = + 1 1 2 J J J n n n n x ... (2.9.4) Formula 5. Multiplying both sides by 1 n x - - of
equation (2.9.1) We get, - - - - + = - 1 n 1 J J J n n
n n n x nx x ' or, - - - - 1 J n n n x nx ' 1 J J n n n x - + = -
Hence, ( ) 1 J J n
n n n d x x dx - - + = - ... (2.9.5) Formula 6. Multiplying bothsides by 1n x - of
equation (2.9.2) We get, - - = - + 1 1 J J J n n n n n n x nx x ' or, 1 1 n n n n n x
J nx J x J - - + = ' Hence, ( ) 1 J J n n n n d x x
dx - = ...(2.9.6)
NSOU l CC-PH-07 61 2.10 Generating function for J n (x) : The generating function of Bessel's functions is given by ( ) ( )
1 2 J
x t

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n t n n e t x ∝ - =-∝ = ∑ ... (2.10.1) Proof : We have, ( ) - - = 1 2 2 2 · x x xt t t t e e e ( ) 2 1 1 .... 2 2! 2 xt xt ? ? = + + + × ? ? ? ? ( ) ? ? - + + ? ? ? ? 2 1 1 .... 2 2! 2 x x t t ... (2.10.2)

The co-efficients
of t n and t –n are given by ( ) ( ) ( ) ( ) - + - + = + 2 1 1 ....
J ! 2 2 1 !
n x n x x x n n [from (2.8.5) and ( ) ( ) ( ) ( ) ( ) 2 1 1 .... 2 2 ! 1 ! - - + - - + = - - + n n n x x J x n n [
From (2.8.6)] Hence, ( ) ( ) ( ) - - = + + + 1 1 2 0 1 -1 J J .... J ( ) x t t e x t x t x + ... or, ( ) 1 2 J ( ) x t n t n
n
e t x ∝ - =-∝ = ∑ (
Proved) Thus Bessel's functions can be derived from the co-efficients of different power of t of equation (2.10.1) 2.11 Zeros of Bessel's Function : From equation (2.8.5) we obtain ( ) ( ) ( ) ( ) 2 0 1 J 2 ! 1 r n r n r x x r n r ∝ + = - - = Γ + + ∑ 62 NSOU l CC-PH-07 All zero of ( ) J n x ,except x = 0, are simple. Specifically it states that for any integers 0 n ≥ and ≥ 1 m , the functions ( ) J n x and ( ) J n m x + have no common zeroes other than the one at x = 0. For n = 0, and m = 1

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J 0 (x) = ( ) ( ) ( ) 2 0 1 2 ! 1 r r r x r r ∝ = - Γ + ∑ ... (2.11.1) and J 1 (x) = ( ) ( ) ( ) 2 1 0 1 2 ! 2 r r r x r r ∝ + = - Γ + ∑ ... (2.11.2)

from equation (2.11.1) we get, J 0 (x) = ( ) ( ) ( ) - + 2 4 2 1 1 1 1! 2 2 2! x x + ... From equation (2.11.2) we get, J 1 (x) = ( ) ( ) - + 3 5 1 1 2 1! 2! 2 2! 3! 2 x x x – .... The graphs of the two functions are illustrated in Fig. 2.2. It shows that J 0 (x) and J 1 (x) have no common zeros. 2.12 Orthogonalities of Bessel functions : The Bessels equation can be written as ( ) Ψ Ψ + + α - Ψ = 2 2 2 2 2 2 0 d d x x x n dx dx ... (2.12.1) and ( ) φ φ + + β - φ = 2 2 2 2 2 2 0 d d x x x n dx dx ... (2.12.2) Fig. 2.2
NSOU l CC-PH-07 63 The solution of equation (2.12.1) is ψ = J n (αx) The solution of equation (2.12.2) is φ = J n (βx) Multiply (2.12.1) by x φ and (2.12.2) by x Ψ and subtracting we get, ? ? ? ? φ ψ ϕ Ψ φ - Ψ + φ - Ψ ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 2 2 2 d d d d x dx dx dx dx + ( ) 2 2 0 x α -β φΨ = or, ( ) ( ) φ Ψ ? ? φ - Ψ + α -β φΨ = ? ? ? ? ? 2 2 0 d d d x x dx dx dx Integrating both sides w. r. t. x from 0 to 1. ( ) φ Ψ ? ? φ - Ψ + α -β φψ = ? ? ? ? ∫ 1 2 2 0 1 0 0 d d x x dx dx dx or, ( ) ψ φ ? ? α -β φΨ = φ - ψ ? ? ? ? ∫ 1 1 2 2 0 0 d d x dx dx dx or, ( ) φ Ψ φ - Ψ = φΨ = α -β ∫ 1 2 2 0 1 d d x dx dx x dx or, ( ) ( ) = ψ φ φ - ψ φΨ = α -β ∫ 1 1 2 2 0 x d d dx dx x dx = ( ) ( ) ( ) ( ) ' ' α α β -β β α α -β 2 2 n n n n J J J J ∴ ( ) J n x ψ = α ∴ ( ) n d J x dx ψ ' = α α and ( ) . n J x φ = β ∴ ( ) n d J x dx φ ' = β β As α, β are distinct roots of
J n (x) = 0, then J n (α) = J n (β) = 0, Hence ( ) ( ) α β = ∫ 1 0 0 , n n x J x J x
dx This is known as orthogonlity relation of Bessel functions.
64 NSOU l CC-PH-07 2.13 Hermite's equation The Hermiti's equation is give by 2 2 2 2 0 d u du x nu dx dx - + = ... (2.13.1) Here n is positive constant. Comparing with equation (2.2.1) we obtain ( ) 0 | 0 x P x = ≠ i.e., x = 0 is ordinary point and solution can be expressed in the form of equation (2.2.3) i.e. ∝ = = ∑ 0 m m m u C x ... (2.13.2) Putting the values of equation (2.13.2) in equation (2.13.1) We get, ( ) 2 0 1 m m m C m m x ∝ - = - ∑ – 0 2 m m m C mx ∝ = ∑ + ∝ = = ∑ 0 2 0 m m m nC x ... (2.13.3) To obtain the same power of x, we can express the equation (2.13.3) in the form ( )( ) 5 2 0 2 2

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s s C s s x ∝ + = + + ∑ – 0 2 2 2 0 s s s s s s C sx nC x ∝ ∝ = = + = ∑ ∑ ... (2.13.4) From equation (2.13.4) we get, ( ) ( )( ) 2 2 2 1 S s s n C C s s + - = + + ... (2.13.5)

NSOU l CC-PH-07 65 The equation (2.13.5) is called the recursion or recurrence relation between the co-efficients. Equation (2.13.5) gives us ( ) 2 0 2 2.1 n C C - = ; ( ) 4 2 2 4.3

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n C - = , ( ) 2 0 2 2 2 4.3.2.1 n n C C - = - or, ( ) ( ) ( ) ( )( ) - - - = - = - 2 3 4 0 6 0 2 2 4 2 , 2 4! 6! n n n n n C C C C Similarly, ( ) ( ) ( ) - - - = = - = - 1 3 1 1 1 2 1 1 2 2 2 3.2 3! 3! n n n C C C C ( )( ) 5 3 1 2 1 6 2 6 2 5.4 5.4.3! n n n C C C - - - = = - = ( ) ( )( ) 2 1 1 3 2 5! n n C - - - ( ) ( )( )( ) 3 7 1 5 3 1 2.5 2 2 7.6 7.6.5! n n n n C C - - - - = = - = ( ) ( )( )( ) - - - - 3 1 5 3 1 2 7! n n n C = ( ) ( )( )( ) 3 1 1 3 5 2 7! n n n

C - - - - For even integer, the general from of the
co-efficients are ginen by ( ) ( ) ( ) ( ) 2 0 2 .... 2 2 2 2 ! m m n n n m C C m - - + = - ... (2.13.6) and for odd integer ( ) ( )( ) ( ) ( ) 2 1 1 1 3 .... 2 1 2 2 1 ! m m n n n m C C m + - - - + = - + ... (2.13.7)
66 NSOU l CC-PH-07 The solutions (2.13.2) becomes 0 m m m u C
x ∝ = = ∑ = + + + + 1 2 3 0 1 2 3 .... c c x c x c x = 2 3 0 2 1 3 .... ? ? ? ? + + + + + ? ? ? ? c c
x c x c
x = ( ) ( ) ( ) 2 2 4 0 2 1 2 2 .... 2! 4! n
n
n x x c - ? ? + - + - + ? ? ? ? ( ) ( ) ( )( ) - - ? ? - + + - + - + ? ? ? ? 2 3 5 1 1 3 1 2 2 .... 3! 5! n n n x x x c ...(2.13.8) = ( ) ( ) 0 1 1 2 c u x c u x + ( ) ( ) ( ) ( ) 2 2 4 1 2 1 2 2 .... 2! 4! n n n u x x x - = + - + - + ... (2.13.9) and ( ) ( ) ( ) ( )( ) 2 3 5 2 1 3 1 2 2 .... 3! 5! n n n u x x
x
x - - - = + - + - + (2.13.10)
The equation (2.13.8) is the power series solution of the Hermite's differential equation. 2.14 Hermite polynomial H n (x) : When n is even, then equation (2.13.8) becomes u = c 0 u 1 (
x) = ( ) ( ) ( ) - ? ? + - + + - + ? ? ? ? /2 2 0 2 ....2 1 2 .... 2 .... 2! !

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n n n n n x x C n ... (2.14.1) NSOU l CC-PH-07 67 Let us put c 0 = ( ) ( ) /2 ! 1 ! 2 n n n - , then co-efficient of x n of equation (2.14.1) given ( ) ( ) ( ) ( ) /2 /2 2 ......2 ! 1 · 2 ! ! 2 n n n n n n n n - - - = ( ) ( ) ( ) ( ) - 2 /2 /2 2 2 ....2 2 ! 2 2 n n n n n n = ( ) ( ) ? ? - ? ? ? ? = · 1 ....2 2 2 2 2 ! 2 n n n n n Similarly, the co-efficients of 2n x - is given by ( ) 2 1 2 1! n n n - - - ∴ ( ) ( ) ( ) ( ) ( )( )( ) ( ) 2 4 1 1 2 3 2 2 2 .... 1! 2! n n n n n n n n n n n

u x
x
x
x - - - - - - - = - + + (2.14.2) The equation (2.14.2) is known as Hermite polynomial of degree n and is denoted by H

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n (x). In general, ( ) ( ) ( ) ( ) /2 2 0 ! 1 2 ! 2 ! n s n s n s n H x x s n s - = = - - ∑ , n is even and ( ) ( ) ( ) ( ) /2 1 2 0 ! 1 2 ! 2 ! n s n s n s n H x x s n s - - = = - - ∑ , n

is odd ... (2.14.3)
68 NSOU l CC-PH-07 2.15.1 Generating function of H n (x) : The generating function of H

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n (x) is given by 2 2 0 ( ) ! n tx t n n t e H x n ∝ - = = ∑ ... (2.15.1)

Proof : Here, ( ) ( ) 2 3 2 2 2 2 2 2 1 1 1 (2) 2 .... 1! 2! 3! tx t tx t e tx t tx
t - - = + - + - + = 1 + 2

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t x – t 2 + - + + 2 2 3 4 4 4 .... 2 t x t x t = 1+2tx – t 2 + 2t 2 x 2 –2t 3 x + 4 2 t + ... = 1 + 2xt + (2x 2 – 1)t 2 + ... Now, ( ) 0 ! n n n t H x n ∝ = ∑ = ( ) 0 1 ( ) 1! t H x H x + + ( ) 2 2 2! t H x + ... = ( ) 2 2 1 (2) 4 2 .... 2! t t

x

x + + - + [from equation 2.14.3] = 1 + 2xt + (2x 2 – 1)t 2 + ... Hence, ( ) 2 2 0 ! n tx t n n t e H x n ∝ - = = ∑ (Proved). 2.16 Rodrigue's formula of H n (x) : The Rodrigue's formula of H n (x) is given by ( ) ( ) 2 2 1

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n n x x n n d H x e e dx - ? ? = - ? ? ? ? ... (2.16.1) NSOU l CC-PH-07 69 Proof : From equation (2.14.3) we obtain, H 0 (x) = 1 H 1 (x) = 2x H 2 (x) = (2x) 2 – 2! (2x) 2 – 2 [equation (2.14.3) is valid for s = 0 and 1] = 4x 2 – 2 H 3 (x) = (2x) 3 – 3! (2x) 1 = 8x 3 – 12x [

equation. (2.14.3) is valid for s = 0, 1] The left hand side of equation (2.16.1)
is for
n = 0, (–1) 0 - ? ? ° = = ? ? ° ? ? 2 2 0 1 ( )

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x x d e e H x dx for n = 1, ( ) - ? ? - ? ? ? ? 2 2 1 1 x x d e e dx = 2x = H 1 (x) for n = 2, (–1) 2 2 2 2 2 2 2 2 4 2 ( ) x x d e e x H x dx - ? ? = - = ? ? ? ? for n = 3, ( ) 2 2 3 3 3 1 x x d e e dx - ? ? - ? ? ? ? = 2 2 2 2 2 x x d e x e dx - ? ? - - ? ? ? ? = 2 2 2 2 4 2 x x x d e x e e dx - - ? ? - - ? ? ? ? = 2 2 2 2 3 8 8 4 x x x x e x e xe xe - - - ? ? - - + + ? ? ? ? 70 NSOU l CC-PH-07 = 8x 3 – 12x = H 3 (x)

Hence, the
equation (2.16.1) is proved. 2.17 Recurrence formula of H
n (x) : (1) 2nH n – 1 (x) = H'n(x) We have the equation (2.14.1) ( ) 2 2 0 ! n tx t

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n n t e H x n ∝ - = = ∑ Differentiating both sides w. r. t x we obtain ( ) 2 2 0 2 ! n tx t n n t te H x n ∝ - = = ∑ ' ( ) ( ) ∝ ∝ = = ⇒ = ∑ ∑ 0 0 2 ! ! n n n n n n t t t H n H x n n ' ( ) ( ) ∝ + = ⇒ = ∑ ∑ 1 0 2 ! ! n n n n n t t t H x H x n n ' The co-efficients of t n gives ( ) ( ) - = - 1 2 ( ) ! 1 ! n n H x H x n n ' or, ( ) ( ) 1 ! 2 ( ) 1 ! n n n H x H x n - = - ' ( ) ( ) 1 2 n n nH x H x - ∴ = ' ... (2.17.1) NSOU l CC-PH-07 71 (2) ( ) ( ) ( ) 1 1 2 2 0 n n n H x xH x nH x + - - + = Proof : From equation (2.16.1) we obtain, ( ) ( ) 2 2 1 n n x x n n d H x e e dx - ? ? = - ? ? ? ? ∴ ( ) ( ) 2 2 1 2 n n x x n n d H x x e e dx - ? ? = - ? ? ? ? ' + ( ) 2 2 1 1 1 n n x x n d e e dx + - + ? ? - ? ? ? ? or, ( ) ( ) ( )( ) 2 2 2 2 1 1 1 2 1 1 1 n n n x x x x x x n n n d d H x x e e e e dx dx + + - + ? ? = - + - - ? ? ? ? ' or, ( ) ( ) ( ) 1 1 2 2 n n n n H x x H x H x - + = - ∴ ( ) ( ) ( ) 1 1 2 2 0 n n n H x x H x n H x + - - + = ... (2.17.2) 3. ( ) ( ) ( ) 2 2 0

n

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n n H x x H x nH x - + = " ' Proof : We have equation (2.17.1), ( ) - = 1 ( ) 2 n n H x n H x ' ∴ ( ) ( ) ( ) - + ? ? = = - ? ? 1 1 ( ) 2 2 n n n n d H x n H x x H x H x dx " ' [From 2.17.2] or, ( ) ( ) ( ) 1 ( ) 2 2 n n n n H x H x xH x H x + = + - " ' ' = ( ) ( ) ( ) 2 ( ) 2 2 1 n n n H x x H x n H x + - + ' [From 2.17.1] 72 NSOU l CC-PH-07 = ( ) ( ) 2 2 n n xH x xH x - ' ∴ ( ) ( ) ( ) 2 2 0 n n n H x x H x n H x - + = ' ' [

From 2.17.3] (Proved) 2.18. Orthogonality of Hemite Polynomials: If H n (x) is the solution of equation (2.13.1), then we can write, ( ) ( ) ( ) 2 2 2 2 0 n n n
d H x dH x x nH x dx dx - + = ... (2.18.1) Multiplying equation (2.18.1) by 2 x e - we get, ( ) - - ? ? = - ? ? ? ? 2 2 2 ( )

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x x n n dH x d e n e H x dx dx ... (2.18.2) and ( ) 2 2 2 ( ) x x x m d Hm d e m e H x dx dx - - ? ? ? ? = - ? ? ? ? ...(2.18.3)

Multiplying equation (2.18.2) by H m (x) and equation (2.18.3) by H n (x) and subtracting both equation we get, ( ) ( ) ( ) ( ) 2 2 x

21%	MATCHING BLOCK 50/115	SA	PG_M.Sc._Physics_345 12_Mathematical physics-I ... (D111988815)

x m m n dHn x dH x d d H x e H x e dx dx dz dx - - ? ? ? ? - ? ? ? ? ? ? ? ? = ( ) ( ) ( ) 2 2 - - x m n m n e H x H x .... (2.18.4) Integrating both sides w. r. t. x from – ∝ to + ∝ of eqnation (2.18.4) we obtain, 2(m – n) ( ) ( ) 2 ∝ - -∝ ∫ x m n e H x H x dx = ( ) ( ) 2 ∝ - -∝ ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∫ n x m dH x d H x e dx dx dx – ( ) ( ) 2 ∝ - -∝ ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∫ m x m dH x d H x e dx dx dx = 0 – 0 = 0 Since, m ≠ n, then ( ) ( ) 2 0 ∝ - -∝ = ∫ x m n

e H x H x ... (2.18.5)
NSOU l CC-PH-07 73 Equation (2.18.5) slows Hermite polynomials are otthogonal w.r.t. 2 -x e 2.19 Summary 1. Frobenius method to find out infinite series solutions for a second order differential equation has been discussed. 2. Frobenius —Fuch's theorem to examine whether the power series solution exists or not has been discussed. 3. Steps for solving series solution has been discussed. 4. Legendre differential equation, Bessel's equation, Hermite equation and their generating functions, recurrence formula and orthogonalities have been discussed. 2.20 Review Questions and Answer : 1. What is Frobenius method ? Ans. The Frobenius means is a method by which one can expand a power series solution to much a diffrential equations of the form 2 2 2 ( ) ( ) 0 + + = d u du z ZP z q z u dz dz , where p(z) and q(z) are analytic at z = 0 or being analytic eleswhere. For more details see the text book. 2. What is singular point in a differential equation? Ans.
Let us consider a second-order differential equation ( ) ( ) 2 2 0 + + = d y dy P
x Q x y dx dx At any point x = a, if P(x) and Q(x) is finite, then x = a is called ordinary point and if P(x) or Q(x) diverges as x → a, then x = a is called singular point.
74 NSOU l CC-PH-07 Suppose P(x) and Q(x) diverges as

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x → a, but (x – a)P(x) and (x – a) 2 Q(x) remain finite as x → a, then x = a is called regular or nonessential singular point. If (x – a) P(x) and (x – a) 2 Q(x) remain infinite as x → a, then x = a

is called essential singular point or irregular singular point. See also the article No (2.2) also. 3. What is the indicial equation ? Ans. An indicial equation is also called a characteristic equation, when we solve a second-order ordinary differtial equation by Frobenius method, a recurrence relation we obtain, which is known as indicial equation Equation (2.3.5), Equation (2.8.3) are the indicial equations of Legendre and Bessel's equation respectively. 4. Find the series solution of the differencial equation (1 + x 2) u″″″″ + xu″″ – u = 0. Ans. The given equation is (1 + x 2)u″ + xu′ – u = 0 ... (1) At x = 0, ( ) 0 | 0 = ≠ x P x i.e., x = 0 is ordinary point of the equtation and series solution have the form 0 ∝ = = ∑ m m m u C x ... (2) Putting the value of (x) in equation (1) we get, ( ) ( ) 2 2 1 0 0 0 1 1 0 ∝ ∝ ∝ - - = = = + - + - = ∑ ∑ ∑ m m m m m
m m m m x m m C x x C mx x C x
NSOU l CC-PH-07 75 or, ( ) ( ) 2 0 0 0 0 1 1 0 ∝ ∝ ∝ ∝ - = = = = - + - + - = ∑ ∑ ∑ ∑
m m m m m m m m m m m m m m
C x m m C x m
C x C x .... (3) To obtain the same power of x, we can write eqation (3) in the form, ( )( ) 2 0 2 1 ∝ + = + + ∑

44%	MATCHING BLOCK 52/115	SA	019E1130_Mathematical Physics.pdf (D165097245)

s s s s s C x + ( ) 0 1 ∝ = - ∑ s s s s s C x + 0 0 0 ∝ ∝ = = - = ∑ ∑ s s s s s s sC x C x The co-efficient of x s is (s + 2)(s+1) C s + 2 + (s 2 – s + s –1) C s = 0 ∴ ( )( ) 2 2 1 1 2 2 1 + - - = = - + + + s s s s s C C C s s s ... (4) From equation (4) we get

the values 2 0 1 2 = +

68%	MATCHING BLOCK 53/115	W

C C 4 2 0 1 1 4 8 = = - C C C ( ) 6 4 0 0 3 1 1 1 · 6 2 8 16 = - = + = + C C C C C 8 = ... and so on. C 3 = 0, C 5 = C 7 = ...0

Putting the values of co-efficients in equation (2) we get the series solution u(x) = C 0 + C 1 x + C 2 x 2 + C 4 x 4 + C 6 x 6 + ...
76 NSOU l CC-PH-07 = 2 4 6 0 1 1 1 1 1 .... 2 8 16 ? ? + - + + ? ? ? ? C
x x x C x 5. Express 5x 3 + x in terms of Legendre Polynomials. Ans. We have, ( ) ( ) 3 3 1 5 3 2 = -
P

36%	MATCHING BLOCK 54/115	W

x x x or, 5x 3 = 2P 3 (x) + 5x ∴ f(x) = 5x 3 + x = 2P 3 (x) + 4x = 2P 3 (x) + 4P 1 (x) [. . . P 1 (x) = x] 6. Express f(x) = x 3 + 2x 2 – x – 3 interms of Legendre Polynomials. Ans. We have, ( ) ( ) 3 3 1 5 3 2 = - P x x x or, ( ) 3 3 2 3 5 5 = + x P x x ( ) ( ) 2 2 1 3 1 2 = - P x x or, ( ) 2 2 2 1 3 3 = + x P x P 1 (x) = x and P 0 (x) = 1 ∴ f(x) = x 3 + 2x 2 – x – 3 = ( ) ( ) 3 2 2 3 4 2 3 5 5 3 3 + + - + - P x x P x x = ( ) ( ) ( ) ( ) 3 2 1 0 2 4 2 7 5 3 5 3 + - - P x P x P x P x 7.

Prove that
P n (1) = 1 Ans. We have ( ) ( ) 1/2 2 0 1 2 α - = ? - - ? = ? ? ∑
n n t x t P x t
NSOU l CC-PH-07 77 Putting x = 1, we get, ( ) 1/2 2 0 1 2 1 α - = ? ? - + = ? ? ∑ n

24%	MATCHING BLOCK 55/115	SA	MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

n n t t P t or, ( ) ( ) 1/2 2 2 0 1 1 α - = ? ? - = ? ? ? ? ∑ x n t P t or, ( ) ( ) 1 2 0 1 1 α - = ? ? - = ? ? ? ? ∑ n n n t P t or, 1 + t + t 2 +......... + t n + ... = 1 + P(1).t + P 2 (t)t 2 + ... + ...P n (t)t n + ... Equating the co-efficients of t n an either side we get, P n (1) = 1 8. Show that P n (– x) = (– 1)

n P n (x) Ans. We have ( ) ( ) 1 2 2 2 0 1 2 ∝ - = - + = ∑

23%	MATCHING BLOCK 56/115	SA	019E1130_Mathematical Physics.pdf (D165097245)

n n xt t t P n Putting x = – x, we get ( ) ( ) ∝ - = + + = - ∑ 1 2 2 0 1 2 n n n xt t t P x ... (1) Also, putting t = – t we get, ( ) ( ) ( ) ∝ - = + + = - ∑ 1 2 2 2 0 1 2 1 n n n xt t t P x ... (2) 78 NSOU l CC-PH-07 From equation (1) & (2) we get, ( ) ( ) ( ) ∝ ∝ = = - = - ∑ ∑ 0 0 1 n n n n n n n n t P x t

P x Compairing the co-effcients of t n we get

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P n (– x) = (– 1) n P n (x) 9. If P n (x) is a Legendre Polynomial of degree n and ααααα is is such that P n (ααααα) 0, then show that P n – 1 (ααααα) and P n + 1 (ααααα) are of opposite signs. Ans. From

the recurrence relation we get, (2
n + 1)x P

52% **MATCHING BLOCK 58/115** **SA** MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

n (x) = (n + 1)P n + 1 (x) + nP n – 1 (x) ... (1) Putting, x = α is equation (1) we get, (2n + 1) α P n (α) = (n + 1)P n + 1 (α) + nP n – 1 (α) or, 0 = (n + 1)P n + 1 (α) + nP n – 1 (ααααα),. P n (α) = 0 or, ( ) ( ) 1 1 1 + - α = + α n n P n n P As n is positive integer so R. M. S is negative. Hence P n + 1 (α) and P n – 1 (α) are of opposite signs. 10.

Show that ( ) ( ) 0 1 = d J x J x dx ––––– Ans.
We have, ( ) ( ) ( ) ( ) ∝ + = = - Γ + + ∑ 2 0 1 1 2 ! 1

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n r r n r x J x r n r

Putting n = 0
NSOU l CC-PH-07 79 ( ) ( ) ( ) ( ) ∝ = = - Γ + ∑ 2 0 0 1 1 2 ! 1
r r r x J x r r ∴ ( ) ( ) ( ) ( ) ∝ - = = - Γ + ∑ 2 1 0 0 1 2 ! 1 r r r
d x r J x dx r r ( ) ( ) ( ) 2 1 0 1 1 2 ! ∝ - = = - Γ ∑ r r r x r r ( ) ( ) 1 1 - = = -
J x J x 11. Show that ( ) ( ) ( ) ? ? ? ? = - + - ? ? ? ? ? ? ? ? 4 1 0 3 2 48 8 24 1
J x J x J x
x x x Ans. We have, ( ) ( ) ( ) - + ? ? = + ? ? 1 1 2 n n
n x J
x J x J
x n or, ( ) ( ) ( ) + - = - 1 1 2 · n n n n J
x J
x J x x Putting, n = 1,
J 2 (x) 1 0 2 ( ) ( ) = - J
x
J
x x n = 2, ( ) ( ) ( ) 3 1 1 4 = - J
x J x J
x
x n = 3. ( ) ( ) 4 3 2 ( ) 6 = - J x
J
x
J x
x ∴ ( ) ( ) ( ) = - 4 3 2 6 J x
J x J x
x ( ) ( ) ( ) ( ) 2 1 1 0 6 4 2 ? ? = - - + ? ? ? ? J x J x J x J
x
x
x
x

80
NSOU l CC-PH-07 = ( ) ( ) - - + 2 1 1 0 2 24 6 2 ( ) ( ) J
x
J
x J x J x x x x = ( ) ( ) ? ? - - + ? ? ? ? 1 0 1 0 2 24 2 8 ( ) ( ) J x J
x
J x J x x x x = ( ) ( ) 1 0 3 2 48 8 24 1 ? ? ? ? - + - ? ? ? ? ? ? ? ? J x
J x x
x
x 12.
Prove that ( ) ( ) ∫ 0 1 J
x
J
n dx = (((( ))))
J x ? ? ? ? ? ? ? ? ---- ? ? ? ? ? ? ? ? 2 0 1 2 Ans. LHS = ( ) ( ) 0 1 ∫
J x J x dx = ( ) ( ) 0 0
J J ? ? - ? ? ? ? ∫
d x x dx dx [( ) ( ) 1 0 J J = - ∵ d x x dx] = ( ) 2 0 1 J 2 - ? ? ? ? ∫ d x dx dx = ( ) 2 0 1 J 2 - ? ? ? ? x (Proved) 13. Prove that ( ) ( ) ( ) - ? ? = + ? ? ? ? ∫ 2 2 2 2 1 0 0 1 1 2
x J x dx x J x J x
Ans. ( ) 2 0 ∫ xJ x dx = ( ) ( ) ( ) 2 2 2 0 0 0 2 2 2 - ∫ x x
J x
J x J x
dx ′ [integrating by parts] = ( ) ( ) ( ) 0 2 2 2 0 1 2 + ∫ x J x
x J x
J x dx [( ) ( ) 0 1 = - ∵ J x J x ′] = ( ) ( ) 2 2 0 1 1 2 + ? ? ? ? ∫
x
d J x
xJ xJ x dx dx [( ) ( ) 1 0 ? ? = ? ? ∵ d xJ x xJ x
dx]
NSOU l CC-PH-07 107 17. Compute erf (0.3). Solution. We have ( ) 3 5 7 2 ... 3 10 42 x x x erf x x ? ? = − + − + ? ? π ? ? Putting x = 0.3 ( ) ( ) ( ) ( ) 3 5 7 0.3 0.3 0.3 2 0.3 0.3 ... 3 10 42 erf ? ? ? ? = − + − + π ? ? ? ? = 1.128 [0.3 − 0.009 + 0.000243 + ...] = 0.3286.

108 NSOU l CC-PH-07 Unit−4 q Theory of Errors Structure 4.0 Objectives 4.1 Introduction 4.2 Definition of Errors 4.3 Propagation of errors 4.4 Normal law of Errors 4.5 Statistical methods in error analysis 4.5.1 Standard error and probable error 4.5.2 Conditions to find probable error 4.5.3 Advantages of standard error 4.6 Percentage Error 4.7 Summary 4.8 Review Questions and Answer 4.0 Objectives When you go through this unit, you may be able to learn 1. Definition, propagation and normal law of errors. 2. Statistical methods in error analysis. 4.1 Introduction The process of evaluating uncertainty regarding the measurement of any experimental results is called error analysis. The uncertainty of a single measurement of any kind of experiment is limited by the precision and accuracy of the measurement. Precision is a measurement without the reference of true value. It is based on the degree of consistency and agreement of independent measurements of the same quantity. Precision determines the reproducibility of the measurements. Where as accuracy is the closeness of agreement between the measured value and the true value. Hence we can say Error is nothing but the inaccuracy of the results which we want to measure. 4.2 Definition of Errors : The difference between the value, which we want to obtain and what we already measured, is called error. Due to wrong calibration of the measuring instrument, a 108

NSOU l CC-PH-07 109 constant error can take place during measurement. This type of error is called instrumental error, which can be adjusted only by the reference data. Except this instrumental error, there are different types of errors can arise in a measurement. Generally the errors are classified by two types : 1. Random errors and 2. Systematic errors (includes instrumental errors). 1. Random Errors : The errors occuring due to unknown reasons are called random errors. It can arise due to alteration of experimental conditions that are beyond the control ; examples are vibrations in the experiment, fluctuation of temperature and humidity, pressure etc. The detection of random errors are very difficult, because their effect on the experimental values is different for every repetition of the experiment. Instrumental resolution also be the cause of random erros, because all instruments have finite precision that limits the ability to resolve the small differences of the measurement. The alignment of eye with the pointer is one of the key reasons of random errors. Hence the statistical methods are used to obtain the random errors. 2. Systematic erros : The errors which are governed by some systematic rule is called systematic errors. For example, measuring a distance using the wrong end of a meter stick, incorrectly neglecting the effect of viscosity, air resistance, friction etc., that can provide a systematic shift of the measuring data. Some measuring devices require time to reach its equilibrium conditions. The most common example is taking temperature readings with a thermometer that is not thermally equilibrium with its environment. 4.3 Propagation of errors : As discussed earlier, there are uncertainties in measurement which we call them errors (systematic or Random). Since all measurements have uncertainties associated with them, then question arrises, how can we determine the values of uncertainties? If y is a function of x i.e.y = y(x), then uncertainty associated with x is x ± Δ x and in this way y also. As y = y(x) then y y x x ∂ Δ = Δ ∂ ... (4.3.1)
110 NSOU l CC-PH-07 Let us consider f is a function of independent variables x, y, z ...... i. e. f = f (x, y, z ...) ...(4.3.2) The expectation values and uncertainties associated with the independent variables are > x <, > y <, > z < .... and Δx, Δy, Δz ..... respectively. Hence the expectation value of f can be written as > f < = f (> x <, > y <, > z < .....) ...(4.3.3) and uncertainty of f is given by ...... ∂ ∂ ∂ Δ = Δ + Δ + +Δ + ∂ ∂ ∂ f f f f x y z x y z or, 2 2 2 2 ..... ∂ ∂ ? ? ? ? Δ = Δ + Δ + ? ? ? ? ∂ ∂ ? ? ? ? f f f x y x y ...(4.3.4) Hence the calculated quantity f is given by f = > f < ± Δ f ...(4.3.5) There are different ways by which we can calculate the values of f. The ways are discussed below : 1. Addition or Subtraction : Suppose two quantities x and y are added i.e., f (x, y) = x + y From equation (4.3.3)

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we get > f < = > x < + > y < and from equation (4.3.4) we get 2 2 Δ = Δ +Δ f x y Now for susbtraction f (x, y) = x − y ⇒ > f < = > x < − > y < and 2 2 Δ = Δ +Δ f x y [

From equation (4.3.4)]
NSOU l CC-PH-07 111 Hence for any combination of addition and subtraction the uncertainty for measurements of any values gives the same value i.e., 2 2 2 ... Δ = Δ +Δ +Δ + f x y z ...(4.3.6) 2. Multiplication or Division : Suppose two quantities are multiplied i.e., f = xy ⇒ >
f < = >
x < > y < where as 2 2 2 2 Δ = Δ + Δ f

43% **MATCHING BLOCK 61/115** **SA** MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

y x x y [From equation (4.3.4)] or, ( ) 2 2 2 2 2 2 ? ? Δ Δ Δ = + ? ? ? ? ? ? y x f x y x y = 2 2 2 y x f x y Δ Δ + For division say x f y = ⇒ x f y > < > <= > < and 2 2 2 2 4 Δ Δ = + Δ x x f y y y [From equation (4.3.4)] = 2 2 2 2 2 2 y x x y x y ? ? Δ Δ + ? ? ? ? = 2 2 2 y x f x y Δ Δ + 112

NSOU l CC-PH-07 Hence for division or multiplication, the uncertainty of any measurement gives the same result and the values are 2 2 2 2 2 2 ..... Δ Δ Δ Δ = + + + y x z f f x y z ...(4.3.7) 3. Raising to a power : Suppose the function f is defined as f = x n y m ⇒ log f = n log x + m log y or, > log(f) < = n > log (x) < + m > log(y) < and by differentiating both sides we get, f y x n m f x y Δ Δ Δ = + or, 2 2 2 2 Δ Δ Δ ? ? ? ? = + ? ? ? ? ? ? ? f y x n m f x y From equation (4.3.4) we obtain ( ) ( ) 2 2 1 2 1 2 − − Δ = Δ + Δ n m m n f nx
y x my x y = ( ) ( ) 2 2 2 2 2 2 2 2 Δ Δ + n m n
m y x n x y m x y x y = 2 2 2 2 2 2 Δ Δ + y x f n m x y

NSOU l CC-PH-07 113 In general, for multivariables 2 2 2 2 2 2 ..... Δ Δ Δ = + + y x f f n m x y ...(4.3.8) For all these equations Δx, Δy, Δz ..... represents the absolute error and , , y x z x y z Δ Δ Δ ..... represents relative error. So the percentage error is Error% 100 f f Δ = × . 4.4 Normal law of Errors : A series of errors are obeying the Normal law. The accidental errors associated with an extended series of observations, called normal law of errors. The exponential law for the distribution of acccidental errors of observation, discovered by Gauss has been a mathematical classic for over a century. If a measurement is subject to many small sources of random error and negligible systematic error, the limiting distribution will have the form of the smooth bell-shaped curved as shown in Fig. 4.1. P x( ) x x Fig. 4.1 This distribution curve is very often, called Gaussian distribution curve. This curve will be centered on the true value of the measured quantity. In general, this limiting distribution defines a function P(x). From the symmetry of the bell-shaped curve, we 114 NSOU l CC-PH-07 can say P(x) is centered on the average value of x. We know the mean or expectation value for finite number of measurements are given by 1 2 1 .... n n x x x x N ∝ = + + + + > <= = μ ∑ (say) ∴ 1 1 →∝ = μ = ∑ N i N i Lt x N ...(4.4.1) Here μ is the true value in the absence of systematic errors. The values of μ is confined with μ ± σ, where σ is the standard deviation, defined by ( ) 2 1 1 N i N i Lt x N →∝ = σ = −μ ∑ ...(4.4.2) The standard deviation, σ characterizes the average uncertainty in each of the measurements x i . Hence P(x) is directly related with μ and σ. Gauss showed that, for randomly distributed errors, the distribution function is given by ( ) 1 2 P x = σ π exp 2 1 2 x ? ? ? ? −μ ? ? − ? ? ? ? ? ? σ ? ? ? ? ...(4.4.3) The measurements whose limiting distribution N given by the Gauss function are said to be normally distributed. The significance of this function is shown in Fig 4.2. The fraction of measurements that fall in any small interval x to x + dx is equal to the area P(x) dx of the strip as shown in Fig 4.2. The total probability of our measurement falling anywhere between − ∝ to + ∝ must be unity i.e. x x x + dx x Fig. 4.2

NSOU l CC-PH-07 115 ( ) 1 P x dx ∝ −∝ = ∫ . ...(4.4.4) Now for finite number of measurements of x i with the results : x 1 , x 2 ..... x N , the probability of finding a value of x within the interval x 1 , x 1 + dx 1 ; x 2 , x 2 + dx 2 , ... x N , x N + dx N are given by ( ) 2 1 1 1 1 exp. 2 2 x P x ? ? −μ ? ? ? ? = − ? ? ? ? σ ? ? σ π ? ? ? ? ( ) 2 2 2 1 1 exp. 2 2 x P x ? ? −μ ? ? ? ? = − ? ? ? ? σ ? ? σ π ? ? ? ? .... ( ) 2 1 1 exp. 2 2 N N x P x ? ? −μ ? ? ? ? = − ? ? ? ? σ ? ? σ π ? ? ? ? ∴ P (x 1 , x 2 , ... x N) = P (x 1) P(x 2) ..... P(x N) ( ) 1 1 2 exp. 2 2 = ? ? ? ? = − −μ ? ? σ σ π ? ? ? ? ∑ N i i x ...(4.4.5) In equation (4.3.5), the numbers μ and σ are unknown, and we want to find the best estimates for μ and σ based on the given observations x 1 , x 2 ..... x N . From equation (4.4.5), we can say that probability is maximum when ( ) ( ) 1 2 2 = −μ σ ∑ N i i x is minimum. i.e., ( ) 1 0 N i i x = −μ = ∑

116 NSOU l CC-PH-07 or, 1 0 N i i x N = − μ = ∑ ∴ 1 1 N i i x N = μ = ∑ Proceeding in a same manner, the best estimation of ( ) 2 1 1 N i i x N = σ = −μ ∑ = ( ) 2 1 1 1 N i i x N = −μ − ∑ ...(4.4.6). Hence N is replaced by (N − 1), because the calculatlon of μ has used up one independent piece of information. One of the most popular method by which one can measure the near accurate value of any experiment is least square fitting. Most common types experiment involves the measurement of several values of two different variables to investigate the mathematical relation between two variables. Let us consider the general case of two variables x and y are related with a linear relation y = A + Bx. ...(4.4.7)., where A and B are constants. Now if we want to measure N different values {y 1 y 2 , ... y N } corresponding to N values of {x 1 , x 2 , ... x N }, then each points (x i , y i) would lie exactly on the line given in equation (4.4.7). Now we want to find out the values A and B, that give the best straight line fit to the measured data. If we know the constants A and B, then for any gieven values of x i , we can calculate the true value of the corresponding y i : y i = A + B x i ...(4.4.8)

NSOU l CC-PH-07 117 From our assumptions, the measurement of y i is governed by a normal distribution centered on this true value, with width parameter σ y (say). Therefore, the probability of obtaining the observed value y i is P(y i) = P(y 1) ..... P(y N) or, ( ) ( ) ( ) 2 2 1 2 2 1 1 1 exp. ..... 2 2 2 ? ? ? ? ? = − − − − ? ? σ π σ σ ? ? ? ? i
N

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y y y P y y y y y = 2 2 1 1 1 exp. 2 2 = ? ? ? ? ? ? − − ? ? ? ? σ π ? ? σ ? ? ? ? ∑ N i i y y y N y = 2 2 1 1 1 exp. 2 2 N i i i y y y

A Bx = ? ? ? ? ? ? − − − ? ? ? ? σ π ? ? σ ? ? ? ? ∑ = 2 2 1 exp. 2 2 y X y − σ π σ ...(4.4.9), where ( ) 2 2 1 N i i i X y A Bx = = − − ∑ The probability is maximum when X 2 is minimum in equation (4.4.9). This is why this method is called least square fitting. Now, ( ) ( ) 2 1 2 0 N i i i d X y A Bx dA = = − − − − = ∑ ...(4.4.10) and ( ) ( ) 2 1 2 0 N i i i i d X x y A Bx dB = = − − − − = ∑ ...(4.4.11) Equation (4.3.10) gives 1 1 N N i i i i AN B x y = = + = ∑ ∑ ...(4.4.12) and equation (4.3.11) gives 2 1 1 1 N N N i i i i i i A x B x x y = = = + = ∑ ∑ ∑ ...(4.4.13)

118 NSOU l CC-PH-07 Equation (4.4.12) and (4.4.13) are called normal equations. Equating these two equations we obtain 2 1 1 1 1

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N N N N i i i i i i i i i x y x x y = = = = − ∑ ∑ ∑ ∑ A = −−−−−−−−−−−−−−−−−−−− 1 1 1 = = = ? ? − ? ? ? ∑ ∑ ∑ N N N i i i i i i i N x y x y and B = −−−−−−−−−−−−−−−−−−− ...(4.4.14) 2 2 1 1 = = ? ? − ? ? ? ? ∑ ∑ N N i i i i N x x

Putting the values of equation (4.4.14) in equation (4.4.8), one can easily solve for the best least squre estimates of A and B. 4.5 Statistical methods in error analysis : The statistical methods help us to reduce the random errors. For a set of data containing N elements or measurements, given by {P 1 , P 2 , P 3 ..... P N }, the average or expectation value 1 2 3 1 ... 1 N N i i P P P P P P N N = + + + + 〈 〉 = = ∑ ...(4.5.1) The values of P〈 〉 is sometimes referred to as the best estimate of the actual value. The data P i are dispersed arround the mean or average. The measurement of this dispersion is called the standard deviation and is given by 2 2 1 1 = = ? ? − ? ? ? ? ∑ ∑ N N i i i i N x x
NSOU l CC-PH-07 119 ( ) 2 2 2 1 2 2 ... + + + ∆ = −〈 〉
N P P P P

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P N = ( ) 2 2 2 2 1 2 .... N P P P N P N + + + − 〈 〉 = 2 2 1 1 N i i P N P N = ? ? − 〈 〉 ? ? ? ? ∑ ...(4.5.2) Here ∆ P

is defined with a factor (N − 1) rather than N. When N is very large number, then the values of ∆ P is not diferred as N − 1 ≈ N, but when N is a small number, then deviation occurs. The question arises why we are taking (N − 1), rather than N? This is simply because, one information we are taking independent or reference from the set of values. Hence equation (4.2.2) becomes 2 2 1 1 1 N i i P P N P N = ? ? ∆ = − 〈 〉 ? ? − ? ? ∑ ...(4.5.3) When the standard deviation is small, then it is close to the mean value and they fall within the interval between (P − ∆ P) and (P + ∆ P). If the systematic errors are reduced, the random errors will dominate and it is impossible to get the accurate result. There is a way to reduce the random errors. Equation (4.5.3) shows that if N is large, then ∆ P is small. If we take the more and more data points as much as posssible in an experiment, then it is possible to reduce the random erros. This type of survey or data analysis is called statistical approach. 4.5.1 Standard error and probable error : 1. The standard error is a measure of the variability of a statistic. It is an estimate of the standard deviation of a sample distribution. The procedure of standard error calculation is given below. (a) First you have to calculate the mean. (b) Calculate the deviation of all samples from the mean i.e., mean minus the individual measurement.
120 NSOU l CC-PH-07 (c) Then square each deviation from mean, so that positive or negative values will give you only positive value. (d) Sum the standard deviation. (e) Divided that sum of standard deviation by the number of measurements minus one. (f) To get the standard deviation, square root the values of (e). (g) Divide the standard deviation by the square root of the sample size(n). In this way we obtain the standard error. (h) Add and subtract the standard error from the mean and record the number. 2. Probable error : Probable error defines the half-range of an interval about a central point for the distribution, such that half of the values from the distribution will lie, within the interval and half outside. The probable error is expressed as equal to 0.6745 times the standard deviation. The probable error is defined as Probable error 2 0.6745 l r N − = × (4.5.4), where N is the total number of observations and r is the correlation co-efficient of pairs of observations for any random sample. Correlation co-efficients is the covariance of the two variables divided by the product of their standard deviations. If there is a pair of random variables (X, Y), then ( ) Cov , x y X Y r = σ σ , [where covariance or variance is denoted by Cov (X,Y), and σ X , σ Y are dicussed earlier] If X = X (x 1 , x 2 ...... x N) and Y = Y (y 1 , y 2 , ... y N) ( ) ( ) ( ) 2 2 1 , N i i i Cov X Y x y = = > −μ <> −μ < ∑ and ( ) 2 1 1 1 N x i i x N = σ = −μ − ∑ and ( ) 2 1 1 1 N y i i y N = σ = −μ − ∑ [From equation...4.5.3] }

NSOU l CC-PH-07 121 The probable error is also defined as Probable error = 2 1 0.6745 N −ρ (1+1.086ρ 2 + 0.13ρ 4 + 0.002ρ 6), where ρ is the correlation of a population. This is also known as limit of the correlation of co-efficient. 4.5.2 Conditions to find probable error : 1. The distribution of the data must have the bell-shaped curve i.e., normal distribution curve. 2. It is important to calculate probable error measuring the statistics from the sample only. 3. The samples are taken in such a manner so that they must remain independent of each other's value. 4.5.3 Advantages of standard error : 1. Standard errors help us to reduce the sample errors as well as the measurement errors. 2. The standard error of any mean tells about the accuracy of the estimate clearly. 4.6 Percentage Error : The deviation is the measure of the precision of an experiment i.e., smaller the value of deviation means estimation tends to the accurate value. Generally in experimental physics it is need to account the accuracy i.e., how the difference is between the experimental values and established values (Theoritical values). The percentage error is represented by Experimentalvalue Theoriticalvalue Error%= ×100% Theoriticalvalue − ...(4.6.1) If we say experimental value = S 1 and theoritical value = S 2 , then deviation is 1 2 S= S S ∆ − ...(4.6.2) Most of the case the true experimental value is unknown. In this case, it is often useful to take the results from two different methods, so that a difference can

122 NSOU l CC-PH-07 be obtain and for our assumption we can take the mean of this two values to get near to the accurate value, then the percentage error may be represented by Errors% = 1 2 1 2 S 100% S 2 S S − × + ...(4.6.3) 4.7 Summary 1. Different types of errors, propagation of errors and normal law of errors have been discussed. 2. Statistical methods in error analysis such as standard error and probable error have been discussed. 3. Conditions to find probable error and advantages of standard error and percentage error calculation have been discussed. 4.8 Review Questions and Answer : 1. A thermometer reads 190ºC, when the actual temperature is 195ºC. Find the percentage error in the reading. Ans. Percentage error is given by Error% = 195 190 100% 195 − × = 2.564% 2. Distinguish between Random error and systematic error. Write down the two possible sources of random error and systematic error. Ans. See article no. (4.2). 3. The most probable value of a set of dispersed data is arithematic mean. Justify the statement. Ans. Let us take a set of experimental values [20, 20.5, 23, 23.5, 23.7, 24, 24.5, 25] for a single experiment. All the data are in the range of 20 to 25.The experiment was performed for eight (8) times and most probable value is around 23.

NSOU l CC-PH-07 123 The mean value is given by 1 1 N i i x N = μ = ∑ = 1 8 [20 + 20.5 + 23 + 23.5 + 23.7 + 24 + 24.5 + 25] = 23.025 In this simple manner we can easily justify the above statement. 4. What are the different sources of errors? Ans. (a) Systematic error. (see article no. (4.2)) (b) Random error. (see article no. (4.2)) (c) Least count error : Least count error is associated with the resolution of the instrument. (d) Constant error (e) Errors due to external factors : Discussed in systematic error. 5. The resistance value at a temperature t of a wire R t is given by the relation R t = R 0 (1 + α t), where R 0 is the resistance at 0ºC and α is the temperature co-efficient of resistance. The resistance values of the metal wire at different temperature is tabulated below. Obtain the values of R 0 and α using least square straight line fitting. Temperature (ºC) 20 40 60 80 100 (t) Resistance (Ω) 107.5 117 129 138 145.5 (R t)

124 NSOU l CC-PH-07 Ans. The given equation is R t = R 0 αt + R 0 ....(1) [y = βx + A] From equations (4.4.14),

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we obtain 2 1 1 1 1 0 2 2 1 1 N N N N i i i i i i i i i N N i i i i x y x x y A R N x x = = = = = = − = = ? ? − ? ? ? ? ∑ ∑ ∑ ∑ ∑ ∑ 1 1 1 0 2 2 1 1 N N N i i i i i i i N N i i i i N x y x y B R N x x = = = = = − − = α = ? ? − ? ? ? ? ∑ ∑ ∑ ∑ ∑ Here, 5 2 1 i i x = ∑ = 22000 ⇒ 5 2 1 5 i i x = ∑ = 110000 2 5 5 1 1 300 90000 i i i i x x = = ? ? = ⇒ = ? ? ? ? ∑ ∑ 5 1 637 i i y = = ∑ 5 1 40160 i i i x

y = = ∑ ∴ 0 637 22000 300 40160 98·3 110000 90000 R × − × = = −
J
NSOU l CC-PH-07 125 and 0 5 40160 300 637 0·485 110000 90000 R × − × α = = − J ∴ 0.485 0.0049 98.3 α = = 6. In an experiment, the time period of an oscillating object in five successive measurements is found to be 0.52 S, 0.56 S, 0.57 S, 0.54 S and 0.59 S. The least count of the watch used for the measurement of time period is 0.01S. What is the percentage error in measurement of time period T. Ans. The mean of the measured value is [ ] 1 1 1 0.52 0.56 0.57 0.54 0.59 5 N i i x S N = μ = = + + + + ∑ = 0.56 S From equation (4.3.2) we obtain ( ) 2 1 1 1 N i i x N = σ = −μ − ∑ = ( ) ( ) ( ) ( ) 2 2 2 3 2 1 0.04 0 0.01 0.02 0.03 4 ? ? − + + + + ? ? = 0.027S Standard error = 0.027 5 N σ = = 0.012S The percentage error = Standarderror ×100% mean = 0.012 100% 0.56 × = 2.14%

126 NSOU l CC-PH-07 7. How can we minimize errors? Ans. (i) Using instruments of higher precision, improving experimental techniques, etc. we can reduce least count error. (ii) Repeating the observations, several times and taking the arithmetic mean of all the observation, the mean value would be very close to the true value of the measured quantity. (iii) Gross error can be minimized only if the observer is very careful and sincere in his approach. 8. What are the different ways of expressing an error? Ans. (a) Absolute error (b) Relative error (c) Percentage error 9. What is called accuracy? Ans. The accuracy of an instrument is a measure of how close the output reading of the instrument with the correct value. 10. A resistor is market with 470ΩΩΩΩΩ, 10%. What will be the true value of the resistor? Ans. A resistor is marked with 470Ω, 10% means the actual (true value) value of the resistor lies within (470 + 470 × 10%)Ω to (470 – 470 × 10%)Ω or 517Ω to 423Ω. 11. What is absolute error? Ans. The absolute error of a measurement is the magnitude of the difference between the actual value (true value) and the value of the individual measurement. The actual or true value is taken by doing the arithmetic mean, because we are not sure about the actual value. If there are n number of readings [A 1 , A 2 , A 3 ,... A n]. Arithnetic mean, [ ] 1 2 1 ..... n A A A A n = + + + Hence for each measurement, corresponding errors can be represented as 1 1

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A A A Δ = – 2 2 A A A Δ = – .... n n A A A Δ = – So, absolute error [ ] 1 2 1 ..... n A A A A n Δ = Δ +Δ + +Δ

NSOU l CC-PH-07 127 12. The refractive index of water is found to have the values 1.29, 1.33, 1.34, 1.35, 1.32. Calculate the mean value, absolute error and percentage error. Ans. Mean value ( ) 1.29 1.33 1.34 1.35 1.32 1.326 5 + + + + μ = = Absolute error = 1 1.326 1.29 1.326 1.33 1.326 1.34 1.326 1.35 5 ? – + – + – + – ? 1.326 1.32? + – ? = 0.0168 Percentage error = Absolute error ×100% Mean = 0·0168 100% 1·326 × = 1.27% 13. How error are propagated or combined? Ans. See propagation error. 14. Find the probable error for the given correlation co-efficient 0.6 and the pairs of samples are 24. Ans. The equation (4.5.4) gives probable error= 2 1 0.6745 r N – × = ( ) 2 1 0.6 0.6745 24 – × = 0.088 Hence the error percentage of probable error = 0.088 × 100% = 8.8%.

128 NSOU l CC-PH-07 15. Find out the standard error for a measurement of height distribution (152 cm, 155 cm, 160 cm and 162 cm). Ans. Height distribution x i = [152, 155, 160, 162] Mean value [ ] 1 152 155 160 162 4 μ = + + + cm = 157.25 cm Standard deviation ( ) 2 1 1 1 N i i x N = ? ? σ = –μ ? ? – ? ? ∑ [From equation (4.4.2)] = ( ) ( ) ( ) 2 2 2 1 152 157.25 155 157.25 160 157.25 3 ? – + – + – ? ? ( ) 1 2 2 162 157.25 ? + – ? cm = 4.57 cm Standard error 4.57 2.29 4 N σ = = = cm Hence the true value = μ + standard error = (157.25 + 2.29) cm = 159.54 or 154.96 cm

NSOU l CC-PH-07 129 Unit–5 q Partial Differentiations Structure : 5.0 Objectives 5.1 Introduction 5.2 Differentials 5.3 Solution of partial differential Equations 5.4 Laplace's equation 5.5 Wave equation 5.5.1 Solution for vibrational modes of a stretched string 5.5.2 Two dimensional equation 5.6 Summary 5.7 Review Questions and Answer 5.0 Objectives 1. To solve a partial differential equation using method of separation of variables. 2. To illustrate the method to solve Laplace's equation and wave equation. 5.1 Introduction Partial differential equation contain the rate of change of variables that are independent to each other. A function can define with multivariables e.g., u = f (x, y, z, t .....) ...(5.1.1) A partial differential equation for the function u is an equation of the form , , , ...; , , , ... u u u f x y z t u x y t ∂ ∂ ∂ ? ? ? ? ∂ ∂ ∂ ? ? ... (5.1.2) 5.2 Differentials : If u = f(x), then for small change of Δx, a small change Δ u can be written as Δ = Δ du u x dx or, du du dx dx = ...(5.2.3)

130 NSOU l CC-PH-07 In equation (5.2.3) dx and du are called the differentials of x and u respectively. Now, if u = f(x, y), then for small change of Δx and Δy (two indepedent variables), we can write u u du dx dy x y ∂ ∂ = + ∂ ∂ ...(5.2.4) Here, , u u x y ∂ ∂ ∂ ∂ are called partial derivatives. For multivariables, equation (5.2.4) can expressed as 1 1 ..... n n u u du dx dx x x ∂ ∂ = + + ∂ ∂ = 1 n i i i u dx x = ∂ ∂ ∑ ...(5.2.5) Where u = f (x 1 , x 2 ....., x n) 5.3 Solution of partial differential Equations : The most widely used method to solve a partial differential equation is, separation of variables. In this method, it is assured that, u (x, y) = φ (x) ψ (y), i.e., the function of many variables is a product of function, each of which is a function of single variables. Limitation of this method is, it is applicable only for homogeneous function that means the function f(x, y) which

can be expressed in the form x n φ (x, y). In general, a function f (x, y, z, t ...) is a homogeneous function of degree n, it is possible to express it in the form , , ... n y z t x x x x ? ? φ ? ? ? ? Moreover it sometimes happens that coordinate system in which the separation is possible, is not suitable for applying the boundary conditions. We shall illustrate this method to solve the

NSOU l CC-PH-07 131 (1) Laplace's equaton. (2) Wave equation. 5.4 Laplace's equation : I. The Laplace's equation in certesian coordinates is given by 2 2 2 2 2 2 2 0 x y z ∂ φ ∂ φ ∂ φ + + = ∂ ∂ ∂ ...(5.4.1) Let us consider, φ (x, y, z) = X (x) Y (y) Z (z) ...(5.4.2) Now, substituting (5.4.2) in equation (5.4.1) we get, 2 2 2 2 2 2 1 1 1 0 d X d Y d Z X Y Z dx dy dz + + = Since, x, y, z are independent. Let us write, ( ) ...... 5.4.3 2 2 1 2 2 2 2 2 1 d X = k X dx 1 d Y = k Y dy ? − ? ? ? ? − ? ? Hence ( ) 2 2 2 2 2 1 3 2 1 d Z k k k z dz = + = From (5.4.3) we get, 1 1 k x k x X Ae Be − = + 2 2 − = + k y k y Y Ce De ...(5.4.4) and Z = E cos k 3 z + F sin k 3 z

132 NSOU l CC-PH-07 = ( ) ( ) 2 2 2 2 1 2 1 2 cos sin E k k z F k k z + + + ...(5.4.4) Putting the values of (5.4.4) in equation (5.4.2) we get, φ = XYZ = ( )( ) 1 1 2 2 k x k x k y k y Ae Be Ce De − − + + ( ) ( ) 2 2 2 2 1 2 1 2 cos sin E k k z F k k z ? ? + + + ? ? ? ? ...(5.4.5) Now, if we take, 2 2 1 2 1 d X k X dx = 2 2 2 2 1 d Y k Y dy = and ( ) 2 2 2 2 1 2 3 2 1 d Z k k k z dz = − + = − Then solution of equation (5.4.2) becomes φ = X Y Z = (A cos k 1 x + B sin k 1 x) (C cos k 2 y + D sin k 2 y) 2 2 2 2 1 2 1 2 k k z k k z Ee Fe − + − + ? ? + ? ? ? ? II. In cylindrical co-ordinate system (r, θ, z) x = r cos θ y = r sin θ and z = z

NSOU l CC-PH-07 133 Therefore, Laplace's equation in cylindrical form is 2 2 2 2 2 2 2 1 1 0 r r r r z φ ∂φ ∂ φ ∂ φ ∂ + + + = ∂ ∂ ∂θ ∂ ...(5.4.6) Let the solution is φ (r, θ, z) = R (r) Q (θ) Z(z) ...(5.4.7) Substituting equation (5.4.7) in (5.4.6) we get, 2 2 2 2 2 2 2 1 1 1 1 0 ? ? + + + = ? ? ? ? θ d Q d R dR d Z R r dr Z dr r Q d dz ...(5.4.8) Let, 2 2 2 1 = d Z k Z dz and d Q n Q d = - θ 2 2 2 1 ( ) and Then, or, 2 2 2 2 2 2 2 2 2 2 2 2 2 2 d Q 1 = n Q dθ 1 d R 1 dR n + +k =0 R r dr dr r d R dR r +r + k r n R=0 dr dr ? - ? ? ? ? ? ? ? - ? ? ? ? ? ? ? ? ? ? - ? ? ...(5.4.9) From, equation (5.4.9), we obtain the solution Z = (E e kz + F e −kz) Q = (C cos nθ + D sin nθ) and R = A J n (kr) + B J −n (kr), where J n (kr) and J −n (kr) are Bessel's function and complete solution of ( ) 2 2 2 2 2 2 2 1 0 d R rd R k r n R dr r dr + + − =

134 NSOU l CC-PH-07 Should be same as equation (2.6.1) Hence, equation (5.3.7) becomes φ (r, θ, z) = [A J n (k r) + B J −n (k r)] (C cos nθ + D sin nθ) (E e kz + F e −kz) ..... (5.4.10) This solution is known as the cylindrical harmonics. III. In spherical polar coordinate system (r, θ, φ) x = r sin θ cos φ y = r sin θ sin φ z = r cos θ So, Laplace's equation in spherical polar coordinate system becomes 2 2 2 2 2 2 2 2 2 2 2 1 cot 1 0 sin r r r r r r ∂ φ ∂φ ∂ φ ∂φ ∂ φ θ + + + + = ∂ ∂θ ∂ ∂θ θ ∂ϕ ...(5.4.11) Let, the solution be φ (r, θ, Q) = R (r) Q(θ) F(ϕ) ...(5.4.12) Substituting equation (5.4.12) in (5.4.11) we get, 2 2 2 2 2 2 2 2 1 1 1 2 cot 0 sin d Q dQ d R dR d F r r R dr Q d dr d F d ? ? ? ? + + + θ + = ? ? ? ? θ ? ? θ θ ϕ ? ? ...(5.4.13) Let us consider, ( ) 2 2 2 1 2 1) ? ? + = + ? ? ? ? d R dR r r n n R dr dr ...(5.4.14) and 2 2 2 1 d F m F d = − ϕ ...(5.4.15) Where − m 2 and n (n + 1) are the two seperation constants.

NSOU l CC-PH-07 135 Hence, ( ) 2 2 2 2 cos 1 0 sin d Q dQ m n n Q d d ? ? + θ + + − = ? ? θ θ θ ? ? This equation is called the associated Legender's equation. ... (5.4.16) From equation (5.4.15), we get, F = C cos mϕ + D sin mϕ ...(5.4.17) and from equation (5.4.14) we get, k (k − 1) + 2k = n (n − l) [By putting R = r k] or, k = n or, k = − (n + l) Hence, R = E r n + F r − (n + l) ...(5.4.18) The solution of equation (5.4.16) can be written as Q = A P n m cos θ + B Q n m sin θ Hence, equation (5.4.12) becomes ( ) 1 0 0 cos sin ∝ ∝ + = = ? ? φ = + + ? ? ? ? ∑ ∑ n n n m F Er C mq D mq r (A P n m cos θ + B Q n m sin θ) ...(5.4.19) This solution is known as spherical harmonics. 5.4.1 Illustrated examples : (1) Find the solution of the given equation 2 2 2 2 0 x y ∂ φ ∂ φ + = ∂ ∂ ; inside the annular regions bounded by the circles x 2 + y 2 = r 1 2 and x 2 + y 2 = r 2 2 that satisfies the conditions

136 NSOU l CC-PH-07 φ = φ 1 at r = r 1 and φ = φ 2 at r = r 2 Solution. The said equation is 2 2 2 2 0 x y ∂ φ ∂ φ + = ∂ ∂ This is two-dimentional Laplace's equation in certision form. For two-dimensional polar coordinate, the equation can be written as 2 2 1 0 r r r ∂ φ ∂φ + = ∂ ∂ or, 0 r r r ∂φ ∂ ? ? = ? ? ? ? ∂ ∂ or, , r A r ∂φ = ∂ constant or, r A r ∂ ∂φ = Integrating bothsides we get φ = A ln r + B (constant). Now put the boundary value conditions in the above equationn and we get, φ 1 = A ln r 1 + B φ 2 = A ln r 2 + B Equating the equations we obtain, 1 1 2 2 ln r A r ? ? = φ −φ ? ? ? ? ∴ 1 2 1 2 ln φ −φ = ? ? ? ? ? ? A r r

NSOU l CC-PH-07 137 and B = 1 1 2 1 1 1 2 ln ln ln ln r r r r φ −φ φ − − = 2 1 1 2 1 2 ln ln ln r r r r φ −φ ? ? ? ? ? ? Hence we get, the solution of the given equation is 1 2 2 1 1 2 1 1 2 2 ln ln ln ln ln φ −φ φ −φ φ = + ? ? ? ? ? ? ? ? ? ? ? ? r r r r r r 2. Suppose the following equation refers to a problem of two-dimensional steady flow of heat : 2 2 2 2 0 x y ∂ ψ ∂ ψ + = ∂ ∂ Boundary conditions are ψ (0, y) = 0, ψ (x, ∝) = 0 ψ (a, y) = 0, ψ (x, 0) = sin x a π Solution. Using the method of separation of variables, the solution can be written as ψ = ψ 1 (x) ψ 2 (y) ...(1) Putting the equation (1) in said equation we obtain, or, ψ 2 ψ 1 ″ + ψ 1 ψ 2 ″ = 0 or, ( ) 2 1 2 2 1 say k ″ ″ ψ ψ = − = − ψ ′ ψ ∴ 2 1 1 k ″ ψ = − ψ ...(2) and 2 2 2 k ″ ψ = ψ ...(3)

138 NSOU l CC-PH-07 From equation we get, ψ 1 = A sin kx + B cos kx and from equation (3) we get, ψ 2 = C e ky + D e −ky Hence, equation (1) becomes ψ (x, y) = (A sin k x + B cos k x) (C e ky + D e −ky) = e ky (A 1 sin k x + B 1 cos k x) + e −ky (A 2 sin k x + B 2 cos k x) ...(4) Using boundary condition ψ (0, y) = 0, in equation (4); we get, B 1 = B 2 = 0 and equation (4) becomes ψ (x, y) = A 1 e ky sin kx + A 2 e −ky sin kx ...(5) Using boundary condition ψ (x, ∝) = 0, in equation (5) we get, A 1 = 0 and equation (5) becomes ψ (x, y) = A 2 e −ky sin kx ...(6) Using boundary condition ψ (a, y) = 0, in equation (6) we get sina k = 0 ⇒ ka = nπ, [n = 0, 1, 2, 3...] and equation (6) becomes ψ (x, y) = A 2 e −ky sin n x a π ...(7) Using boundary condition ψ (x, 0) = sin x a π in equation we get, 2 sin sin 1 n x x A a a π π = ⇒ = and n = 1, Hence the required solution is ( ), sin ky x x y e a − π ψ = ... 5.5 Wave equation : The general form of wave equation is 2 2 2 2 1 ∂ φ ∇ φ = ∂c t ...(5.5.1)

NSOU l CC-PH-07 139 For one-dimensional case equation (5.5.1) becomes 2 2 2 2 2 1 x c t ∂ φ ∂ φ = ∂ ∂ ...(5.5.2) Here [ϕ (x, t)] has two independent variables x and t. Hence seperation of variables method gives the solution. φ = X (x) T (t) ...(5.5.3) Putting equation (5.4.3) in equation (5.4.2) we obtain 2 2 2 2 2 1 d X d T T dx c dt = x or, 2 2 2 2 2 2 = = − d X d T dx dt k X Tc (say) So, 2 2 2 0 + = d X k X dx and 2 2 2 2 0 d T c k T dt + = ...(5.5.4) The solution. of equation (5.4.4) are X = A cos kx + B sin kx and T = C cos ckt + D sin ckt Hence equation (5.4.3) becomes φ = (A cos kx + B sin kx) (C cos ckt + D sin ckt) ...(5.5.5) For 3-dimentional case, φ = X(x) Y(y) Z(z) T(t), so equation (5.4.5) is giving by the product of X, Y, Z, T and

140 NSOU l CC-PH-07 φ = (A 1 cos k 1 x + B 1 sin k 1 x) (A 2 cos k 2 y + B 2 sin k 2 y) (A 3 cos k 3 z + B 3 sin k 3 z) (C cos ckt + D sin ckt) ...(5.5.6) The equation (5.5.6) can be written in a different form ( ) 1 2 3 i ckt k x k y k z Ae ± ± ± ± φ = ...(5.5.7) For real k 1 , k 2 , k 3 , the above solution represents a plane progressive wave in the direction of the wave vector 1 2 3 ˆ ˆ ˆ k k i k j k k = + + and c the velocity with which the wave travels. 5.5.1 Solution for vibrational modes of a stretched string : For stretched string, two nodes should be produced at the end point, that means if the string has length l, then we can apply the following boundary conditions: (i) φ = 0 at x = 0 (ii) φ = 0 at x = l So for one-dimensional case, equation (5.4.5) gives A (C cos ckt + D sin ckt) = 0 ⇒ A = 0 and (C cos ckt + D sin ckt) (A cos kl + B sin kl) = 0 ⇒ sin kl = 0 or, kl = nπ (n is integer) [∵B ≠ 0] Hence equation (5.5.5) becomes 1 sin cos sin ∝ = π π π ? ? φ = + ? ? ? ? ∑ n n n n x n ct n ct C D l l l ...(5.4.8) Let us apply the initial cndition i.e., (i) at t = 0, ( ) 1 sin n n n x f x C l ∝ = π φ = = ∑ ...(5.4.9) (ii) at t = 0, 0 t ∂φ = ∂

NSOU l CC-PH-07 141 So, 1 sin 0 n n n c n x D l l ∝ = π π = ∑ ⇒ D n = 0 for all values of n. Hence, equation (5.5.8) becomes 1 sin cos ∝ = π π φ = ∑ n n n x n ct C l l ...(5.5.10) Where, ( ) 0 2 sin l n n x C f x dx l l π = ∫ [From Fourier expansion of the function f(x)] 5.5.2 Two dimensional equation From equation (5.5.1) we obtain the wave equation 2 2 2 2 1 ∂ φ ∇ φ = c dt For two-dimension case, Let us assume the solution. φ (x, y, t) = X (x) Y(y) T(t) ...(5.5.11) Putting the value of equation (5.4.11) in equation (5.5.11) we get, 2 2 2 2 2 2 2 1 1 1 d T d X d Y X Y c dt dx dy = + ...(5.5.12) Since x, y and t are independent. Hence we can write the equation (5.5.12) by choosing proper constants, 2 2 2 1 = − x d X k X dx

142 NSOU l CC-PH-07 or, 2 2 0 x d X k X dx + = and 2 2 2 1 = − d Y ky Y dy ...(5.5.13) or, 2 2 2 0 d Y k Y y dy + = Now, say k x 2 + k y 2 = k 2 , then 2 2 2 2 1 d T k C dt = − or, 2 2 2 2 0 d T k c T dt + = ...(5.5.14) From equation (5.5.13) and (5.5.14) we get, φ (x, y, t) = (A cos k x x + B sin k x x) (C cos k y y + D sin k y y) (E cos ckt + F sin ckt) ...(5.5.15) For rectangular membrane, boundary conditions are (i) φ = 0 when x = 0 (ii) φ = 0 when x = a (ii) φ = 0 when y = 0, (iv) φ = 0 when y = b Substituting condition (i) in equation (5.5.15) we get, A (C cos k y y + D sin k y y) (E cos ckt + F sin ckt) = 0 ⇒ A = 0 Now, equation (5.5.15) becomes φ (x, y, t) = B sin k x x (C cos k y y + D sin k y y) (E cos ckt + F sin ckt) ...(5.5.16) Substituting condition (ii) in equation (5.5.16) we get, sin k x a = 0, As B ≠ 0 ⇒ , x n k a π = n = 1, 2, 3 .....

NSOU l CC-PH-07 143 Now, equation (5.5.16) becomes φ (x, y, t) = B sin πn a x (C cos k y y + D sin k y y) (E cos ckt + F sin ckt) ...(5.5.17) Substituting the boundary condition (iii) and (iv) in equation (5.5.17), we obtain C = 0 and , m ky b π = m = 1, 2, 3 ..... Hence, φ (x, y, t) = BD sin sin π π n m x y a b (E cos ckt + F sin ckt) = sin sin n m BD x y a b π π (E cos ωt + F sin ωt) ...(5.5.18) Where, ω = ck = c 2 2 x y k k+ = 2 2 2 2 n m c a b π + Replacing the arbitrary constants in equation (5.5.18) we get φ (x, y, t) = 1 1 sin sin n m n m x y a b ∝ ∝ = = π π ∑ ∑ (E mn cos ωt + F mn sin ωt) (5.5.19) Again, from the initial condition φ = f(x, y) at t = 0 and 0 ∂φ = ∂t at t = 0, we obtain, F mn = 0 and

144 NSOU l CC-PH-07 ( ) 1 1 , sin sin mn n m n m f x y E x y a b ∝ ∝ = = π π = ∑ ∑ ...(5.5.20) The above expression is a double Fourier series. Where, ( ) 0 2 sin a mn n E f x xdx a a ? ? π = ? ? ? ? ? ? ∫ ( ) 0 2 sin b m f y ydy b b ? ? π ? ? ? ? ? ? ∫ = ( ) ( ) 0 0 4 sin sin a b n m f x f y x ydxdy ab a b π π ∫ ∫ = ( ) 0 0 4 , sin sin a b n m f x y x ab a b π π ∫ ∫ y [From double Fourier sine series] Another boundary condition can be taken into account, when the rectangular membrance defined within {0 > x > a, 0 > y > b} and has initial displacement f'g f (x, y) and initial velocity f' (x, y). In this case, the velocity function f' (x, y) = 1 1 sin sin mn m n n m B x y a a ∝ ∝ = = π π ω ∑ ∑ ...(5.5.21) Where, ( ) 0 0 4 , sin sin b a mn n m B f x y x ydxdy ab a b π π = ' ω ∫ ∫

NSOU l CC-PH-07 145 5.6 Summary : Method of separation of variables to solve partial differential equations such as Laplace's equation and Wave equation has been discussed. 5.7 Review Questions and Answer : (1) Solve the boundary value problem 4 , u u x y ∂ ∂ = ∂ ∂ given u (0, y) = 8e −3y by the method of seperation of variables. Ans. Let the solution of the given equation is u = ψ (x) φ (y) ...(1) Putting equation (1) in the given equation we obtain, 4 x y ∂ψ ∂φ φ = ψ ∂ ∂ or, 1 1 4 d d k dx dy ψ φ = = ψ φ (say) ∴ 4 d kdx ψ = ψ and d kdy φ = φ Integrating both equations we get, ln ψ = 4kx + lnA and ln φ = k y + lnB ∴ ψ = Ae −4kx ...(2) and φ = Be −k y ...(3) Putting (2) and (3) in equation (1) we get, u (x, y) = C e −4kx e −ky ...(4) Using boundary condition in equation (4) we get, C e −ky = 8 e −3y ⇒ C = 8 and k = 3 and equation (4) becomes u(x, y) = 8 e −12x e −3y

146 NSOU l CC-PH-07 or, ( ) ( ) 12 3 , 8 x y u x y e − + = This is the required solution. 2. Write down the three-dimentional equation of heat flow through a medium of thermal conductivity K. Show that under steady conditions this equation reduces to Laplace's equation ∇ 2 φ = 0, w were φ (r) denotes the temperature and can be written as ( ) 4 Q r B kr φ = + π Ans. Three-dimensional equation of heat flow through a medium is, given by 2 2 , ∂φ = α ∇ φ ∂t where K s α = ρ is the diffussivity of the medium of density ρ, Specific leat s. Under steady condition, 0 t ∂φ = ∂ ⇒ ∇ 2 φ = 0, well known by Laplace's equation. In spherical polar co-ordinates (r, θ, φ), the above equation can be written as 2 2 2 2 2 2 2 2 2 2 2 2 1 cot 1 0 sin r r r r r r ∂ φ ∂φ ∂ φ ∂φ ∂ φ θ + + + + = ∂ ∂θ ∂ ∂θ θ ∂ϕ [From equation (5.4.11)] Now, heat only flows radially in outward direction. Hence the above equation must be independent of θ and ϕ. ∴ 2 2 2 0 r r r ∂ φ ∂φ + = ∂ ∂ or, 2 0 d d r dr dr φ ? ? = ? ? ? ?

NSOU l CC-PH-07 147 or, 2 d r A dr φ = (constant) or, 2 dr d A r φ = Integrating bothsides we get, A B r φ = − + (constant) From the heat conduction eqnation we get Q = − K 4πr 2 d dr φ or, Q = − K 4πr 2 2 A r ? ? ? ? ? ? ? ∴ 4 Q A K = − π Thus we obtain, ( ) 4 Q r B kr φ = + π 3. A rectangular stretched membrane of sides a and b having edges parallel to the x-axis and y-axis, and bounded rigidly at the edges, is given a slight deformation in 2-direction perpendicular to its own plane. The differential equation for z is 2 2 2 2 2 2 2 2 1 , c t x y ∂ φ ∂ φ ∂ φ = + ∂ ∂ ∂ where c is constant. Solve the equation by the method of separation of variables assuming the initial condition ( ), f x y φ = and 0 t ∂φ = ∂ at t = 0. Ans. See the solution for the two-dimentional equation discussed in the article no. (5.5.2).

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Conservation of Linear momentum 6.6.5 Conservation of angular momentum 6.6.6 Conservation of energy 6.7			

Hamiltonian formulation of Mechanics 6.7.1 Hamiltonian 6.7.2 Variational principle 6.7.3 Hamilton's principle 6.7.4 Derivation of Lagrange's equation 6.7.5 Hamilton's equations of motion 6.7.6 Advantage of Hamiltonian Approach 6.7.7 Applications of Hamiltonian formulation 6.8 Review Questions and Answer 148

NSOU l CC-PH-07 149 6.0 Objectives 1. Classification of constraints with some examples, degrees of freedom, generalized coordinates, generalised displacement, virtual work. 2. D'Alembert's participle, Euler-Lagrange's equations with application. 3. Concept of symmetry, conservation of linear and angular momentum and conservation of energy. 4. Hamiltonian formulation of mechanics and its advantage and applications. 6.1 Introduction : Mechanics deals with the motion of physical objects, whether large or small. Advance Mechanics is the alternative way, by which we can express the Newton's laws of motion. Advance Mechanics donate that part of mechanics where the objects are neither too big nor too small interacting objects. Classical (advance) mechanics address a huge range of problems ranging from molecular dynamics to the motion of celestial bodies. Classical mechanics is very useful for analysing problems in which quantum and relativistic effects are negligible. Classical mechanics has many applications in the areas of Astronomy,dynamics of molecular collissions, propagation of seismic waves, generated by earthquakes etc. 6.2 Constraints : Constant is defined as a restriction to the fredom in the motion of an object. A motion under constraints is known as constrained motion.

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Motion along a specified path is the simplest example of a constrained motion. Imposing constraints

on a mechanical system is done to simplify the mathematical description of the system. The number of coordinates needed to specify the

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dynamical system becomes smaller when constraints are present in the system. Hence the degree of freedom of a dynamical system is defined as the minimum number of independent coordinates required to simplify the system completely along with the constraints. Thus if

k is the number of constraints and N is the number of particles in the system possessing motion in 3-dimensions then the number of degree of freedom are give by n = 3N – k ... (6.1), where n is the number of degrees of freedom.
150 NSOU l CC-PH-07 6.2.1 Classification of Constraints : There are many types of constraintts. A constraints is said to be 1. Scleronomic : If the constraint relations does not depend on time explicitly. 2. Rheonomic : If the constraint relation depends on time explicitly. 3. Holonomic : Constraints expressed in the form of equation f(r 1 , r 2 ....r n ; t) = 0 are called holonomic... (6.2) 4. Non-holonomic : Constraints that can not be expressed in the form of equation (6.2) are called non-holonomic. 5. Conservative : If the total energy of the system is conserved during the constrained motion, the constraints are called conservative. 6. Dissipative : If the total energy of the system is not conserved, the constraints are dissipative. 7. Bilateral and Unilateral : If constraint relations are in the form of equations, they are bilateral, but if relations are expressed in the form inequalites then constraints are unilateral. 6.2.2 Example of constraints : 1. Rigid body : In a rigid body the distances any pairs of particles are constant i.e. → → - i j r r = constant i r → and → j r J are the position vectors of i-th and j-th particles of the object. The constraint is conservative, scleronomic, holonomic, and bilateral. 2. Deformable bodies : For a deformable bodies, the distance between any pair of particular change in time, so 1 → → - j r r = f(t) for all i and j.
NSOU l CC-PH-07 151 The constraint is holonomic, bilateral, dissipative and Rheonomic. 3. Simple pendulum with rigid support : The position of the bob satisfy the relation → r = l where, l is the constant length of the pendulum. The constraint is scleronomic, holonomic, bilateral and conservative. 4. Pendulum with variable Length : Equation of constraint is ( ) ( ) 2 2 | | r t l t → = Hence, constraint is Rheonomic, holonomic, bilateral and dissipative. 5. Rolling without sliding : Frictional force without sliding do not work and total mechanical energy is conserved. Hence constraint is conservative and non- holonomic. 6. Gas filled hollow sphere : Gas molecules are constrained by the walls and can only move inside the sphere of radius R(say). So the constrains relation are i r R → ≤ . The constraint is scleronomic, holonomic, unilateral and conservative. 7. Expanding or contracting gas filled container : The dimension of container is changing with time, so, ( ) → ≤ i r R t The constraint is rheonomic, holonomic, unilateral and dissipative. 6.3 Degrees of freedom : Degrees of freedom is the minimum number of independent variables (q 1 , q 2 , ... q n , say) that is necessary to fix uniquely the position and the configuration of the given system, compatible with the given constraints. For a free particle, in three dimension space, the degrees of freedom can be specified by its three position coordinates, so number of degrees of freeding is 3.
152 NSOU l CC-PH-07 Henec, for N number of particles, degrees of freedom is 3N. Now if K number of constraint relations are their in the system, the degrees of freedom reduces to 3N – K. Example 1. Triatomic molecule linearly arranged : Here N = 3 and no. of constraints K = 2 [fixed distances between]. So, degrees of freedom is 3.3 – 2 = 7. Example 2. The bob of conical pendulum : Here the constraint is the fixed Legth l. Hence number of degrees of freedom is 3.1 – 1 = 2. Example 3. A dumbbell : A dumbbell consists of two heavy point particles connected to each other by a mass less rigid rod of length l. So, degrees x of freedom is 3.2–1 = 5 6.4 Generalised coordinates : To describe the configuration of a system, we consider the smallest possible number of variables, which are called the generalised coordinates of the system. A set of generalized co-ordinates is any set of co-ordinates which describe the configuration. How

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to choose a suitable set of generalised co-ordinates in a given situation? By three principles, we can solve the problem : (i) Their values determine the configuration of the system. (ii) They may be varied arbitrarily and independently of each other, without violating the constraints on the system. (iii)

There is no uniqueness in the choice of generalized coordinates. Then our choice should fall on a set of co-ordinates that will give us a reasonable mathematical simplification of the problem. For a system of N particles with K independent constaints the number of independent variables to specify the configuration is n = 3N – K. Notation for generalised co-ordinates : Generalised co-ordinates are denoted by q with numerical subscripts : q 1 , q 2 , ... q n represent a set of n generalised co-ordinates.
NSOU l CC-PH-07 153 In general we can express generalised co-ordinates as function of certain co-ordinates and time i.e., q 1 = q 1 (
x 1 , y 1 , z 1 ; .... ; t) q 2 = q 2 (x 1 , y 1 , z 1 , x 2 , y 2 , z 2 , ; .... , t) q n = q n (

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x 1 , y 1 , z 1 ; ....x n , y n , z n , t)

When a particle moves in a plane, it may be described by cartesian co-ordinates x, y and we write 1 2 = ? ? = ? q x q y For spherical symmetry, 1 2 3 = ? ? = θ ? ? = φ ? q r q q 6.4.1 Generalised Displacement : Let us consider a system of N particular have n degrees of freedom. The position vector in terms of generalised co-ordinate is defined as ( ) 1 2 , ,............ , i i n r r q q q t → → = ... (6.4.1) Hence, the small displacement, where n = 3N – K 1 n i i i j j j r r r q t r t → → → = ∂ ∂ ∂ = ∂ + ∂ ∂ ∂ ∑ as ∂t = 0, so 1 n i i j j j r r q q → → = ∂ ∂ = ∂ ∂ ∑ ... (6.4.2) ∂q j are called the generalised displacements or virtual displacement. This change in the system is not associated with a change in time i.e, there is no actual displacement and hence the displacement is termed as virtual displacement.
154 NSOU l CC-PH-07 6.4.2 Virtual Work : We first have to define the generalised force, associated with generalised displacement. Let us consider the total force ∑ i F acts on a system during a small displacement → ∂ i r .Then total work done. 1 · → → = ∂ = ∂ ∑ N i i i W F r = 1 1 · → = = ∂ ∂ ∂ ∑ ∑ N n i i j j i j r F q q = 1 n j j j Q q = ∂ ∑ ... (6.4.3) where Q j = 1 · → → = ∂ ∂ ∑ N i j i r F q ... (6.4.4) is the generalised force associated with co-ordinate q j . The product of Q j ; with the generalised displacement dq j is equal to the work done corresponding to the displacement. From equation (6.4.3), it follows that the product of dimension of generalised displacement and dimension of generalised force must have the dimension of work. As the displacement is virtual so, the work done for virtual displacement is called virtual work. 6.5 D'Alembert's principle :

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This method is based on the principle of virtual work.

Let us consider a system with equilibrium condition i.e., total force i F ∑ on each particles are zero, then work done by the force in a small displacement → δ i r will also zero. Hence for whole system of N particles
NSOU l CC-PH-07 155 1 · 0 → = δ = ∑ N i i i F r ... (6.5.1) Let the total force → i F is the sum of applied force ( ) → a i F and forces of constraints ( ) → c i F , then equation (6.5.1) becomes ( ) ( ) 1 1 · · 0 → → → → = = δ + δ = ∑ ∑ a c i i N N i i i i F r F r Virtual work done by the constraints is zero. so ( ) 1 · 0 a i N i i F r → → = δ = ∑ .... (6.5.2) The above equation is called principle of virtual work. To specify the equilibrium system. D' Alembert taken an idea of reversed force. When the applied force ( ) a i F → and the reversed force → Ɉ i P are equal then the system showd be in equilibrium condition i.e., ( ) → → = Ɉ a i i F P or, ( ) 0 → → - = Ɉ a i i F P and equation (6.5.2) becomes ( ) 1 · 0 a i N i i i F P r → → → = ? ? - δ = ? ? ? ? ∑ Ɉ ... (6.5.3) The equation (6.5.3) is called D'Alembert's principle. D'Alemmbert's principle is just one equation of motion. Since the constraint forces do not appear, it is sufficient if only applied forces are specified and also reverse effective force → ? ? ? ? ? ? i P reduces the problems of dynamics to that of statics.

156 NSOU l CC-PH-07 6.5.1. Derivation of Euler-Lagrange's equations : From equation (6.4.1) we get, ( ) 1 2 , ... , → → = i i n r r q q q t The velocity of the i-th particle is given by → → → → ∂ ∂ ∂ = = + ∂ ∂ ∂ ∑ j i i i i j j q dr r r v dt q t t ...(6.5.4) Further infinitesimal diplacement → ∂ i r can be written as 1 → → → = ∂ ∂ δ = δ + δ ∂ ∂ ∑ n i i i j j i r r q t q t For virtual displacement, δt = 0, hence 1 → → = ∂ δ = δ ∂ ∑ n i i j j j r r q q ... (6.5.5) Putting equation (6.5.5) in equation (6.5.3) we obtain, ( ) 1 1 · 0 → → → = = ? ? ∂ ? ? - δ = ? ? ∂ ? ? ? ? ∑ ∑ ⅃ N n a i i j i j i j r F P q q or, 1 1 · 0 → → → = = ? ? ∂ - δ = ? ? ∂ ? ? ∑ ∑ ⅃ N n i i i j j i j r F P q q [Just dropping the superscript a]
NSOU l CC-PH-07 157 or, · · 0 → → → → ∂ ∂

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δ - δ = ∂ ∂ ∑ ∑ j j i i i j i j j j i i r r F q P q q q or, · 0 → → ∂ ∂ - δ = ∂ ∑ ∑ j i j j i j j j i j r Q q P q q [

From equation (6.4.4)] .... (6.5.6) The 2nd term can be written as · · · → → → → ? ? ∂ ∂ ? ? δ = δ ? ? ∂ ∂ ? ? ? ? ∑ ∑ ⅃ ⅃⅃ j j i i i j i i

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j j j i i r r P q m r q q q = · · · → → → → ? ? ? ? ∂ ? ? ∂ ? ? δ - δ ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ? ? ∑ ∑ ⅃ ⅃ j j j i i i j i j j j i i r r d d m r q m r q dt q dt q ... (6.5.7) Now, · → → → ? ? ? ? ? ? ∂ ∂ ∂ ? ? ? ? ? ? δ = + ? ? ? ? ? ? ∂ δ ∂ ∂ ? ? ? ? ? ? ? ? ? ? ? ? ∑ i i i K j K j j K r r dq d d dt q q q dt dt q = 2 → → ? ? ∂ ∂ ∂ ? ? + ? ? ∂ ∂ ∂ δ ? ? ? ? ∑ i k i k j j K r q r d q q t dt q = → → ? ? ∂ ∂ ∂ ∂ ? ? + ? ? ∂ ∂ ∂ ? ? ? ? ∑ i k i j k K r q r q q t t 158 NSOU l CC-PH-07 = → ∂ ∂ i j r q ⅃ [from equation (6.5.4)] or, → → ? ? ∂ ∂ ? ? = ? ? ∂ ∂ ? ? ? ? i i j j r v d dt q q From equation (6.5.4) we get, → → → ∂ ∂ = + ∂ ∂ ∑ i i i j j j v r v q q t ⅃ and, → → ∂ ∂ = ∂ ∂ ⅃ i i j j v r q q Hence, equation (6.5.7) becomes · · · · → → → → → → ? ? ? ? ∂ ∂ ? ? ? ? δ = - δ ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ? ? ? ? ? ∑ ∑ ⅃ ⅃ ⅃ j j j i i i i j i i i j j j j i i v v v d P q mv m v q q dt q q = 2 2 1 1 2 2 ? ? ? ? ? ? ? ? ? ? ? ? ∂ ? ? - δ ? ? ? ? ? ? ? ? ? ? ? ∂ ? ? ? ? ? ? ? ? ? ? ? ? ∑ ∑ ∑ ⅃ j i i i i j j j j i i i d d m v m v q dt dq q = ? ? ? ? ∂ ∂ - δ ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ∑ ⅃ j j j j i d T T q dt q q where 2 1 · 2 = ∑ i i T m v represents the kinetic energy of i the system.

Now, from equation (6.5.6) we get, . 0 ? ? ? ? ∂ ∂ δ - - δ = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ∑

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j j j j i j d T T Q q q dt q q NSOU l CC-PH-07 159 or, 0 ? ? ? ? ∂ ∂ - - δ = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ j j j j d T T q Q dt q q

Since, q j are independent to each other hence the co-efficient of δq j should be zero i.e., 0 ? ? ? ? ∂ ∂ - - = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ j

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j j j d T T Q dt q q or, ? ? ? ? ∂ ∂ - = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ∑ j j j j d T T Q dt q q ... (6.5.8)

Case I. Conservative system : For conservative system, forces F i are derivable from potential function V = V(q j) i.e., only dependent on co-ordinates. So, we can write, → → ∂ = -∇ = - i i j V F V dr or, · · → → → → ∂ ∂ = = - ∇ δ ∂ ∑ ∑ i i i

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j i j j i i r r Q F V q q = · · → → ∂ ∂ ∂ - = - ∂ ∂ ∂ ∑ i j j i i r V V q q r Equation (6.5.8) becomes ? ? ? ?∂ ∂ ∂ - = - ? ? ? ? ∂ ∂ ∂ ? ? ? ? ∑ ∑ ⅃ j j i j d T T V dt q q

q

160 NSOU l CC-PH-07 or, ( ) ( ) · 0 ? ? ∂ - ∂ - - = ? ? ∂ ∂ ? ? ∑ ɺ ɺ j j i T V T V d dt q q [since V is not the function of ɺ j q] or, 0 ? ? ? ? ∂ ∂ - = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ɺ j j j f f d dt q q ... (6.5.9) (6.5.9) is known as Euler Lagrange equation of motion for conservative system, where = - f T V is function of q j , ɺ j q and t i.e. ( ) , , = ɺ j j f f q q t Case II. Non-conservative system : For non-conservative system, the potentials are velocity depndent, i. e., ( ) , = ɺ j j V V q q [q j is generalised coordinate and ɺ j q is generated velocity] so, ? ? ∂ ∂ = - + ? ? ∂ ∂ ? ? ɺ

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j j j V d V Q q dt q ....(6.5.10) Putting the equation (6.5.10) in equation (6.5.8) we obtain, ? ? ? ? ? ? ∂ ∂ ∂ ∂ - = - + ? ? ? ? ? ? ∂ ∂ ∂ ∂ ? ? ? ? ? ? ? ? ∑ ∑ ∑ ɺ ɺ ɺ j j j j j j j d T T V d V dt q q q dt q

or, ( ) ( ) 0 ∂ - ∂ - - = ∂ ∂ ∑ ∑ ɺ j j j j T V T V d dt q q or, 0, j j j f f d dt q q ? ? ? ? ∂ ∂ - = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ɺ [as L = T – V] The above equation is exactly same as equation (6.5.9). Hence we can say the Euler-Language equation is in the same form for conservative and non-conservative system.
NSOU l CC-PH-07 161 when = - = f T V L, then equation (6.5.8) and (6.5.9) becomes 0 ? ? ? ? δ δ - = ? ? ? ? δ δ ? ? ? ? ? ? ∑ j

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j j d L L dt q q ... (6.5.10) where L

recognised as Lagrangian The lagrangian function is useful to fix the equation of motion. If we want to know the information about the path adopted by the system during the motion, it is necessary to provide six initial values for a particle-three for position co-ordinates and three for velocity co-ordinates. But in configuration space, specification of the position of a single point provides only three initial values for each particle and hence specification of path is not possible. Thus if we difine the equation of motion, there should be infinite number of possible paths through any print in configuration space. Configuration space is introduced to represent the motion of a system in Lagrangion approach. 6.5.2 Application of Lagrange's equation of motion : 1. One dimensional simple harmonic oscillator : For simple harmonic oscillator, the kinetic energy of this system is 2 1 2 = ɺ T m x and potential energy V F dx = - ∫ = ( ) k x dx - - ∫ [here k is constant] = 2 1 2 kx c + + Hence, Lagrangian L can be expressed in the form 2 2 1 1 2 2 = - = - - ɺ L T V mx kx c From equations (6.5.9) we get ( ) 0 ∂ ∂ - = ∂ ∂ ɺ d L L dt x x [Here we put q j = x]
162 NSOU l CC-PH-07 or, 0 mx kx+ = ɺɺ or, 2 0 + ω = ɺɺx x where ? ? = ? ? ? ? k w m It is an equation of simple harmonic oscillator, where w is the frequency of oscillation given by ω = k m . 2. Falling body in uniform gravity : When an object falls from rest, its gravitational potential energy is converted to kinetic energy. Hence, kinetic energy 2 1 2 = ɺ T mx [at any distance x] and potential energy V = mg (h – x) So, Lagrangian L = T – V = ( ) 2 1 2 mx mg h x - - ( ) 0 d L L dt x x ∂ ∂ - = ∂ ∂ ɺ or, mx .. – mg = 0 or, mx mg =ɺɺ For free falling body the above equation represents the equation of motion and acceleration is g i.e., acceleration due to gravity. 3. Simple pendulum :

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The angle θ between rest position and deflected position is chosen as

generalised co-ordinate. If the length of the pendulum of mass m is l, then kinetic energy 2 1 2 = ɺ T mx 2 2 1 2 = θ ɺ ml h x
NSOU l CC-PH-07 163 as potential energy V = mg (l – l cos θ) Thus Lagrangian is L = T – V = ( ) 2 2 1 1 cos 2 θ - - θ ɺ ml mgl Lagrange's equation is 0 ∂ ∂ ? ? - = ? ? ∂θ ∂θ ? ? ɺ d L L dt or, 2 sin 0 ml mgl θ+ θ = ɺɺ sin 0 θ+ θ = m l ɺɺ For small θ, sin θ = θ So, 0 θ+ θ = m l ɺɺ The above equation represents the eqnation of motion of a simple pendulum with frequency ω = m l and time period 2 l T g = π . 4. Compound Pendulum : Let the point θ be fixed and taken it as origin. The point G is the centre of gravity in equilibrium position and G′, the centre of gravity in displaced position. Kinetic energy 2 1 2 = θ T I ɺɺ I is the moment of inertia of the body about the axis of rotation. o θ l C B x m o θ l G x G′ P. E=O

164 NSOU l CC-PH-07 Potential energy, V = – mgl cos θ ∴ Lagrangian L = T – V = 2 1 cos 2 θ + θ Ɉ I mgl ∴ ∂ = θ ∂θ L I ɈɈ sin L mgl mgl ∂ = - θ= - θ ∂θ We have, 0 ∂ ∂ ∂ ? ? - = ? ? ∂ ∂θ ∂θ ? ? Ɉ L L t or, 0 I mgl θ+ θ = ɈɈ 0 mgl I θ+ θ = ɈɈ or, 2 0 θ+ ω θ = ɈɈ , where ω = mgl I Hence, time period 2 I T mgl = π = 2 2 2 k l gl + π . 6.6 Concept of symmetry : If the property of a system does not charge under some defined operations, the system is said to have a symmetry with respect to the given operation. A system which does not interact with particles or field outside the system may exhibit space homogeneity and isotropy. The conservation theorems to be discussed are first integrals of equation of motion. These are the relations of the type f(q j , Ɉ j q , t) = constant. 6.6.1 : Cyclic or Ignorable co-ordinates : We know that Lagrangian is a function of generalised co-ordinate q j , generalised velocity Ɉ j q , and time t, i.e., ( ) , , = Ɉ j j L L q q t ... (6.6.1)
NSOU l CC-PH-07 165 If Lagrangian does not contain any coordinate q k from any set of co-ordinates that means L is independent of or 0 ∂ = ∂ k k L q q , then such co-ordinate is reffered to as an ignorable or cyclic co-ordinate. 6.6.2 Homogeneity of space : Homogencity of space implies that the physical laws are invariant under space translation. If we change the position of each particles by some vector dr → the Lagrangian of our system will be remain same, it implies the position vector → → → → → = + j j j r r r dr ' and change of Lagrangian of our system · → → ∂ δ = δ ∂ ∑ j j j L r L r = · → → ∂ δ ∂ ∑ j j L r r Symmetry under orbitrary tanslation implies 0 0 → ∂ δ = ⇒ = ∂ ∑ j j L L r Using Euler Language equaton, we can say 0 → ? ? ? ? ∂ = ? ? ? ? ∂ ? ? ∑ i j d L dt r Put, 2 1 ( ) 2 = - Ɉ j j i L m r v r then, ( ) 2 1 0 2 → ? ? ? ? ∂ = ? ? ? ? ∂ ? ? Ɉ j j j d m r dt r
166 NSOU l CC-PH-07 or, · 0 j j d m r dt → ? ? = ? ? ? ? Ɉ 0 → = Ɉ j p i.e., total momentum of the system is constant. Thus homogeneity of space leads to conservation of Linear momentum of the system. 6.6.3 Isotropy of space : Isotropy means rotation invariance in free space. If Lagrangian L is independent of the orientation of the system, it results the conservation of angular momentum of the system. Let us consider the rotation about the z-axis by an angle θ so that OA → OA'. It follows that , OA OA AA → → → = + ' ' , where the magnitute AA → ' is given by j r → ? ? δ ? ? ? ? → ? ? ' = ? ? ? ? j AA r sin θ dφ Thus sin j j j j j r r r r d → → → → ' → = + δ = θ θ = j d r → → φ × then j j j j v v v v → → → → → = + δ where j j v d v → → → δ = φ ×
NSOU l CC-PH-07 167 If the Lagrangian is invariant under rotation then we have, · · → → → → ∂ ∂ δ δ = δ + ∂ ∂ ∑ ∑ Ɉ Ɉ

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j j j j j L L r L r r r = · · → → → → ? ? ? ? ∂ ∂ ∂ δ δ + ? ? ? ? ∂ ∂ ? ? ∑ ∑ Ɉ Ɉ j j j j j j L L r r dt r r = · → → → → → → ? ? ? ? ? ? ? ? ∂ ∂ ∂ φ× φ× + ? ? ? ? ? ? δ ? ? ? ? ? ? ? ∂ ? ? ∑ ∑ Ɉ j j j j j j

L L r v d d dt r r Now, 2 1 2 j j j r L m V r → → ? ? = - ? ? ? ? ∑ Ɉ Hence, · → → → → → → ? ? ? ? δ = φ × + φ × ? ? ? ? ? ? ? ? ∑ ∑ j j j j j j mr L d d mr d r Ɉ Ɉ ɈɈ = · · → → → → → → ? ? ? ? φ × + φ × ? ? ? ? ? ? ? ? ? ∑ ∑ Ɉ Ɉ j j j j j j P d r P d r = · → → → → ? ? φ × ? ? ? ? ∑ j j i d d r P dt If ∂L = 0, then · 0 → → → → ? ? × = ⇒ × ? ? ? ? ∑ j j j j d r P r P dt = constant so the angular momentum of the system is constant.
168 NSOU l CC-PH-07 6.6.4 Conservation of Linear momentum : If a co-ordinate corresponding to a displacement is cyclic translation of the system has no effect i.e, description of system motion remians invariant under such a translation and linear momentum is conserved. Let, q j is the generalised co-ordinate and corresponding generalised displacement is dq j. From equation (6.5.9) we obtain. 0 j j j d dL L dt dq q ? ? ∂ - = ? ? ∂ ? ? ∑ Ɉ We know, L = T – V, where T is independent of q j i.e., 0 j j j d dL L dt dq q ? ? ∂ - = ? ? ∂ ? ? ∑ Ɉ ...(6.6.2) Further Kinetic energy 2 1 2 = Ɉ i T mr or, · → → ∂ ∂ = ∂ ∂ Ɉ Ɉ Ɉ Ɉ i i j j r T mr q q = · j j j r mr q → → ∂ ∂ Ɉ Ɉ Ɉ = · → → ∂ ∂ Ɉ Ɉ i i j r P q ... (6.6.3) If

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n͡ is the unit vector along the direction of translation,

then → ∂ i r = ˆ n ∂q j or, ˆ → ∂ = ∂ i j r n q

NSOU l CC-PH-07 169 Hence, ˆ · i T P n → ? ? = ? ? ? ? where, i P → represents the component of total linear momentum along the direction of translation. So from eqation (6.6.2) we can say ˆ · i d P n dt → ? ? ? ? ? ? = j L q ∂ ∂ If q j is cyclic, then 0 j L q ∂ = ∂ i.e., L is indepdendent of q j . Hence ˆ . → ? ? ? ? ? ? i P n = constant... (6.6.4) Linear momentum is conserved if a co-ordinate corresponding to a displacement is cyclic. 6.6.5 Conservation of angular momentum : If a co-ordinate corresponding to a rotation is cycdic, then system remain invariant under such a coordinate rotation and angular momentum is conserved. The figure shows the change of position vector under rotation of the system. Here chang in q j must correspond to an infinitesimal rotation of the vector i r → .
170 NSOU l CC-PH-07 ( ) ( ) i j i j j r q r q dq → → → + after rotation of i r → i.e. ( ) ( ) i i j j i j d r r q dq r q → → → = + → The magnetitude of i dr → is 1 sin i j dr r dq → ? ? = θ ? ? ? ? ˆ → → ∂ = × ∂ i i j r n r q From

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equation (6.6.3) we get, · → → ∂ ∂ = ∂ ∂ ∑ ∑ ɺ i i j j r T P q q = ˆ · i i P n r → → ? ? × ? ? ? ? ∑ = · i i n r P → → → ? ? × ? ? ? ? ∑ = ˆ ·n L → Hence j T q ∂ ∂ ∑ ɺ represent the total angular momentum along the

axis of rotation. Putting the value of j T q ∂ ∂ ∑ ɺ

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in equation (6.6.2) we get, ˆ · j j d L n L dt q → ? ? ∂ = ? ? ∂ ? ? ∑ If q j

is cyclic, then j L q ∂ ∂ =0, So, ˆ · 0 d n L dt → ? ? = ? ? ? ? or, ˆ ·n L → = constant. ... (6.6.5)
NSOU l CC-PH-07 171 If the rotation coordiante is cyclic, then angular momentum along the rotation is conserved. 6.6.6 Conservation of energy : For a conservative system, potential energy is a functin of coordinate not the velocity and constraints do not change with time. So, time will not involve explicity and hence L can be written as ( ) , j j L q qɺ . Thus total time derivative can be written as ∂ ∂ = + ∂ ∂ ∑ ∑ ɺ ɺɺ ɺ j

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j j j j j dL L L q q dt q q = j j j j j j d L L q q dt q q ? ? ∂ ∂ + ? ? ∂ ∂ ? ? ∑ ∑ ɺ ɺɺ ɺ ɺ ? ? ? ? ∂ ∂ = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∵ ɺ j j d L L dt q q = j j j d L q dt q ? ? ∂ ? ? ∂ ? ? ∑ ɺ ɺ = 0 j j j d T q dt q ? ? ∂ = ? ? ∂ ? ? ∑

ɺ ɺ [as V is independ of j qɺ] = ( ) j j j d q P dt ∑ ɺ Thus ( ) 0 j j j dL d q P dt dt - = ∑ ɺ or, ( ) 0 - = ∑ ɺ i i d q P L dt
172 NSOU l CC-PH-07 or, j j q P L- ∑ ɺ = constant... (6.6.6) We know, j j P mq = ɺ Hence, for simple consideration we can write ( ) 2 2 1 2. 2 2 j j j j j j q P m q m q T = = = ɺ ɺ ɺ Now, equation (6.6.6) becomes 2T – T + V = constant or, T + V = constant ... (6.6.7) The above equation shows that

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sum of kinetic energy and potential energy i.e., total energy of a

system is invariant if time t does not occur in L explicitly j means t is cyclic. 6.7 Hamiltonian formulation of Mechanics In Lagrangian formulation independent variables are generalised co-ordinates and time. As generated velocity is simply the time derivative of generalised coordinate, so generalised velocity is not the independent variable. In Hamiltonian formulation just we introduce a new independent variable, called generalised momentum P j . Like Lagrangian, a new function of this formalism is Hamiltonism H, which

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is function of generalised coordinates, generalised momenta and time i.e., ( ) , , = j j H H q P t .

Generalised momenta already have been defined by j j L P q ∂ = ∂ ⅃ , Which shows that, for each generalised coordinate, there is one component of generalised momentum. After providing the equal status of generalised momenta, the path adopted by the system during its motion must now be represented by a space of 6N dimensions (instead of 3N dimension) in which 3N dimensions for generalised co-ordinate and 3N dimensions for generalised momenta. This new space is called
NSOU l CC-PH-07 173 phase space. In configuration space there are infinite number of possible paths during motion, which we already discussed. But in this new formulation (i.e., Hamiltonian), specification of initial values of q j and P j at any instant on the path will fix the whole path of the moving system. Hence path in phase space refers almost the actual dynamical path, which is almost impossible to get in configuration space. 6.7.1 Hamiltonian : From equation (6.6.6) we obtain, constant j j j q P L- = ∑ ⅃ Now this constant is designated by a letter H. Thus we can write j j j q P L H - = ∑ ⅃ We recognise H as Hamiltonian and assign to it a basis of (q
j , P j) set, i.e., ( ) ( ) , ,

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j i j j j j H H q P q P L

q q = = - ∑
⅃ ⅃ ... (6.7.1) If H does not involve time, it is said to be a constant of motion, and represents the total energy of the system as discussed earlier. It is possible that

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H may be a constant of motion but not the total energy of the

system. 6.7.2 Variational principle : The principle of least action or more accurately,the principle of stationary action— is a variational principle that, when applied to the action of a mechanical system, can be used to obtain the equation of motion for that system. The principle is called "least" because its solution requires finding the path that has the least value. The principle can be used to derive Newtonian, Lagrangian and Hamiltonian equations of motion. Let f be a function of many independent variables q j and their derivatives j q⅃ . Then integral I, representing a path between two points 1 and 2 can be written as ( ) ( ) ( ) 2 1 , , = ∫ ⅃ j j I f q x q x x dx
174 NSOU l CC-PH-07 Mathematicaly the principle states that δI = 0, In words the path taken by the system between 1 and 2 and configurations from points 1 to 2 is the one for which the action is stationary (no change) to first order. 6.6.3 Hamilton's principle : Hamilton's variational principle or Hamilton's principle inovel with motion of the system. This principle states that the integral ( ) 2 1 t t T V dt - ∫ have a stationary value (extremum)

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t 1 where T is the kinetic energy and V is the potential energy of the system.

T is a function of cordinates and their derivatives and V is a function of
co-ordinates only. Hence Hamilton's principle is only for conservative system and state

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that the motion of the system from time t 1 to time t 2 is such that lime integral 2 1

t t
I L dt = ∫ , where L= T – V ....(6.7.2) is

the extremum for the path of motion, is defined as Lagrangian (already discussed earlier). 6.7.4 Derivation of Lagrange's equation : Let us consider a conservative system of particles. In the form of generalised coordinates equation (6.7.2) can be written as ( ) 2 1 , t

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j j t I L q q dt = ∫ ⅃ = ( ) ( ) 2 1 , t j j j t T q q V q

dt ? ? - ? ? ∫ ⅃
NSOU l CC-PH-07 175 According to Hamilton's principles, we have δI = 0 (Entremum condition) or, ( ) ( ) 2 1 , 0 t j j j t T q q V q dt ? ? δ - = ? ? ∫ ⅃ or, 2 1 0 ? ? ? ? ∂ ∂ ∂ δ + δ - δ = ? ? ? ? ∂ ∂ ∂ ? ? ? ? ? ? ∑ ∫ ⅃ ⅃ t j j j

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j j j j t T T V q q q dt q q q or, 2 2 1 1 ( ) 0 ∂ - ∂ δ + δ = ∂ ∂ ∑ ∑ ∫ ∫ ⅃ ⅃ t t j j j j j j t t T V T q dt q dt q q or, 2 2 1 1 ( ) t t j j j j j j t t T V d T q dt q dt q dt q ? ? ∂ - ∂ ? ? δ + δ ∂ ∂ ? ? ? ? ∑ ∑ ∫ ∫ ⅃ – 2 1 0 ? ? ∂ ? ? ∂ = ∂ ? ? ? ? ∑ ∫ ⅃ t j j j t d t q dt dt q or, ( ) 2 1 t j j j t T V q dt q ∂ - ∂ ∂ ∑ ∫ + 2 1 2 1 0 | t j j j j j j t t T d T q q dt q dt q

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t ? ? ∂ ∂ δ - δ = ? ? ∂ ∂ ? ? ∑ ∑ ∫ ⅃ ⅃ Since there is no co-ordiante variation at end points, so 2 0 1 | t t q j = δ and above equation reduces to 2 2 1 1 ( ) . 0 ? ? - ∂ δ - δ = ? ? ? ? ∑ ∑ ∫ ∫ j t t j j j j j t t d T

V d T q dt q dt dq dt dq or, 2 1 ( ) 0

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t j j j j t d T V d T q dt dq dt dq ? ? ? ? - ∂ - δ = ? ? ? ? ? ? ? ? ? ? ∑ ∫

⅃
176 NSOU l CC-PH-07 Since δq j is independent to each other, the coefficent of every δq j , is zero i.e., ( ) 0 j j T V d dT q dt dq ? ? ∂ - - = ? ? ∂ ? ? ⅃ For conservative system, we can write ( ) ( ) 0 j j T V T V d dt q q ? ? ∂ - ∂ - - = ? ? ∂ ∂ ? ? ⅃ ⅃ [As V is independent of j q⅃] or, 0 j j d L L dt q q ? ? ∂ ∂ - = ? ? ∂ ∂ ? ? ⅃ ⅃ [As L = T – V] For set of coordinates in a configuration system the equation can be written as 0 j j j d L L dt q q ? ? ? ? ∂ ∂ - = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ⅃ ... (6.7.3) This set of equations are called Lagrange's equation of motion. This equations follow directly from Hamilton's principle. 6.7.5 Hamilton's equations of motion : If we consider the Hamiltonian as defined in equation (6.7.1) is also the function of time, then Hamiltonian (H) can be represented by H = H (q j , P j , t) ... (6.7.4) and equation (6.7.1) becomes ( ) , , j j j j H q P L q q t = - ∑ ⅃ ⅃ [as ( ) , , j j L L q q t = ⅃] ... (6.7.5) The differential of eqn. (6.7.4) gives ∂ ∂ ∂ = + + ∂ ∂ ∂ ∑ ∑

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j j j j j j H H H dH dq dP dt q P t ....(6.7.6)

NSOU l CC-PH-07 177 and the differential of equation (6.7.5) gives ∂ ∂ = + - - ∂ ∂ ∑ ∑ ∑ ∑ ⅃ ⅃ ⅃

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j j j jj j j j j j j j L L dH q dp P dq dq dq q t ∂ - ∂

L dt t ... (6.7.7) we have, j j L P q ∂ = ∂ɺ and j j L P q ∂ = ∂ Putting this values in equation (6.7.7) we get, ∂ = + - - - ∂ ∑ ∑ ∑ ∑ ɺ ɺ ɺ ɺ

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j j j j j j j j j j j j L dH q dP P dq P dq P dq dt

t = ∂ - - ∂ ∑ ∑ ɺ ɺ

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j j j j j j L q dP P dq dt t ... (6.7.8) Compairing the coefficients of equations (6.7.6) and (6.7.8) we get, and ∂ ? = ? ∂ ? ? ∂ ? = - ∂ ? ? ɺ ɺ j j j j H q P H P q ... (6.7.9) L H t t ∂ ∂ - = ∂ ∂ ... (6.7.10)

Equation (6.7.9) are known as Hamiltons canonical equations of motion. Generalised momenta and generalised coordinates are dynamically equivalent sets of variables, because their role can be interchanged just by making a change of sign. Hence we must say that Hamilton's equation of motion are symmetric in q j and P j except for a change in sign. 6.7.6 Advantage of Hamiltonian Approach : In Lagrangian approach, two variables q j and j qɺ are not in equal status where as in Hamiltonian approach, co-ordinates and momenta are placed at equal footing,

178 NSOU l CC-PH-07 that provides the freedom of choosing sets of coordinates and momenta. The knowledge of Hamiltonian of a system is extremely important particualrly if we are interested in quantising a dynamical system, because equality of status of coordinates and momenta provides a convenient basis for the development of quantum mechanics. 6.7.7 Applications of Hamiltonian formulation : 1. Simple Harmonic oscillator : For simple Harmonic oscillator, Kinetic energy 2 1 2 = ɺ T mx and potential energy 2 1 2 V kx = Hence, Lagrangian 2 2 1 1 2 2 = - = - ɺ L T V mx kx x L P mx x ∂ = = ∂ ɺ ɺ ∴ Hamiltonian 2 2 2 1 1 2 2 x H P x L mx mx kx = - = - + ɺ ɺ ɺ = 2 2 1 1 2 2 mx kx + ɺ = 2 2 1 2 2 x P kx m + Canonical equations are x x P H x P m ∂ = = ∂ ɺ and x x H P kx P ∂ = - = - ∂ ɺ or, 0 mx kx+ = ɺɺ , This is equation of motion of simple Harmonic oscillator.

NSOU l CC-PH-07 179 In two dimensions ( ) 2 2 1 2 T m x y = + ɺ ɺ , 2 2 1 2 1 1 2 2 V k x k y = + ,: x y L L P mx P my x y ∂ ∂ = = = = ∂ ∂ ɺ ɺ ɺ ɺ ∴ x y H P x P y L = + - ɺ ɺ = ( ) 2 2 2 2 2 2 1 2 1 1 2 2 2 y x

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x y P P m P P k x k y m m + - + + + = 2 2 2 2 1 2 1 1 2 2 2 2 y x P P k x k y m m + + + Hence, Hamilton's Canonical equations are ; x x P H x P m ∂ = = ∂ ɺ ; y y P H y P m ∂ = = ∂ ɺ 1 ; x x H p k x P ∂ = - = - ∂ ɺ 2 y H p k y y ∂ = - = - ∂ ɺ or, 1 = - = - ɺ ɺɺ x p k x mx ; 2 = - = y p k y

my ɺ ɺɺ 1 2 0 and 0 mx k x my k y ∴ + = ? ? + = ? ɺɺ ɺɺ Proceeding similarly, for 3-dimensional relations we can write, 1 0 mx k x + = ɺɺ ; 2 0, my k y + = ɺɺ 3 0 mz k z + = ɺɺ 2. Simple Pendulum : We have 2 2 1 2 T ml = θ ɺ , v = mgl (1 – cos θ) ( ) 2 2 1 1 cos 2 L ml mgl = θ - - θ ɺ

180 NSOU l CC-PH-07 Hence, 2

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L P ml θ ∂ = = θ ∂θ ɺ ɺ Now, ( ) 2 2 2 2 1 1 cos 2 H P L ml ml mgl θ = θ- = θ - θ + - θ ɺ ɺ ɺ = ( ) 2 2 1 cos 2 ml mgl 1 θ + - θ ɺ = ( ) 2 2 1 cos 2 θ + - θ P mgl ml Hamilton's Canonical equation are 2 θ θ ∂ θ = = ∂ Ɉ P H P ml and θ ∂ = - ∂θ Ɉ H P = − mgl sin θ = −mglθ [ ] sin θ ≈ θ ∵ 2 2 θ θ θ = = - Ɉ ɈɈ P mgl ml ml or, g l θ = - θ ɈɈ 0 g l ∴ θ+ θ = ɈɈ 3. Particle in a central field of force : For central field, force is always directed towards the centre and potential energy is only the function of coordinates i.e., system is conservative. As the motion always confined in a plane, we consider it in (r, θ) coordinates. Now, Kinetic energy for the system is ( ) 2 2 2 1 2 T m r r = + θ Ɉ Ɉ and ( ) V V r = 2 m r θ Ɉ

NSOU l CC-PH-07 181 ∴ Lagrangian, L = T − V = ( ) 2 2 2 1 2 m r r+ θ Ɉ Ɉ − V(r) ... (1) Momenta r L P mr r ∂ = = ∂ Ɉ Ɉ ... (2) and 2 L P mr θ ∂ = = θ ∂θ Ɉ Ɉ ... (3) For central force, ( ) 2 1 F r r ∝ - or, ( ) 2 ˆ k F r r r → = - [Gravitational force and Coulomb force are two examples] We put, ( ) 2 ( ) - = = - dV r k F r dr r or, ( ) 2 = + ∫ k V r dr c r = k c r - + for ( ) , 0 0 r v r c →∝ → ∴ = So, ( ) k V r r = - ... (4) Lagrange's equation of motion is 0 d L L dt ∂ ∂ ? ? - = ? ? ∂θ ∂θ ? ? Ɉ or, ( ) 2 0 d mr dt θ = Ɉ [Putting ( ) ( ) 2 2 2 1 2 = + θ - L m r r V r Ɉ Ɉ or, 2 mr θ Ɉ = constant = l (say) Hence, 2 P mr l θ = θ = Ɉ [From equation (3)] or, 2 l mr θ = Ɉ

182 NSOU l CC-PH-07 Hamiltonian r H P r P L θ = + θ- Ɉ Ɉ = ( ) 2 2 2 2 2 2 2 2 θ θ + - - + r P P P P r V r m m mr mr [Putting the values of equation (1), (2) and (3)] = 2 2 2 P 2 2 r P K m r mr θ + - [From equation (4)] ..... (5) Hamiltons's canonical equation are and 2 r r P H r P m P H P mr θ θ ? ∂ = = ? ∂ ? ? ∂ ? θ = = ? ∂ ? Ɉ Ɉ ... (6) 2 3 2 or, 0 θ θ ? ∂ = - = - ? ? ∂ ? ? ∂ = - = ? ∂θ ? Ɉ Ɉ r P H k P r mr r H P ... (7) From equation (6) we get P r r m = Ɉ or, 2 2 3 2 r P P k r m m r mr θ = = - Ɉ ɈɈ [from equation (7)] or, 2 3 2 P k mr mr r θ = - ɈɈ ... (8) 4. Hamiltonian for a free particle For a free particle, the potential energy is constant, and may be taken as zero. The kinetic energy

NSOU l CC-PH-07 183 ( ) 2 2 2 1 2

62%	**MATCHING BLOCK 102/115**	**SA**	Chapter 4 _Proposed Book.pdf (D113949991)

T m x y z = + + Ɉ Ɉ Ɉ ∴ Lagrangian L= T − V = ( ) 2 2 2 1 2 m x y z + +

Ɉ Ɉ Ɉ x x P L P mx x x m ∂ = = ⇒ = ∂ Ɉ Ɉ Ɉ y y P L P my y y m ∂ = = ⇒ = ∂ Ɉ Ɉ Ɉ z z P L P mz z y m ∂ = = ⇒ = ∂ Ɉ Ɉ Ɉ ∴
Hamiltonian
x y z H

36%	**MATCHING BLOCK 103/115**	**SA**	MPDSC 1.1 Classical Mechanics.pdf (D133919389)

P x P y P z L = + + - Ɉ Ɉ Ɉ = 2 2 2 2 2 2 2 2 1 2 y y x x z z P P P P P P m m m m m m n ? ? ? ? + + - + + ? ? ? ? ∴ ( ) 2 2 2 1 2 x y z H P P P

m = + + 6.8 Summary : 1. Constraints is defined with some examples. 2. Degrees of freedom is defined with some examples. 3. Generalised coordinates, generalised displacement, virtual work, D' Alembert's principle have been discussed. 4. Euler–Lagrange's equation, and its applications have been discussed. 5. For conservative system, potential function is only dependent on generalised coordinates whereas for non-conservative system the potentials are velocity dependent. 6. Concept of symmetry, homogeneity of space, isotropy of space, different conservation laws, Hamiltonian formulation, applications of Hamiltonian formulation have been discussed.

184 NSOU l CC-PH-07 6.9 Review Questions and Answer : 1. What are generalised coordinates? What is the advantage of using them? Ans. See the article no. (6.4). 2. What are constraints? Explain with examples. Ans. See the article no. (6.2). 3. It is not necessary for generalised force Q j to have the dimensions of force but it is necessary that the product Q j dq j must have the dimensions of work. Justify. Ans. We have from equation (6.4.3), the work done

90%	**MATCHING BLOCK 104/115**	**SA**	M. Sc. I Classical Mechanics all.pdf (D142231111)

j j i W Q q δ = δ ∑ where 1 · → → = ∂ = ∂ ∑ N i j i j i r Q F q

If we consider a particle defined by (r, θ) genaralised coordinat, the component of forces are ˆ ˆ → θ = + θ r F F r F The generlised force for r-coordinate ( ) ˆ ˆ ˆ · ·

71%	**MATCHING BLOCK 105/115**	**SA**	M. Sc. I Classical Mechanics all.pdf (D142231111)

r r r r r Q F F r F r F r r → →

θ ∂ ∂ = = + θ = ∂ ∂ Have the dimension for force But, the generalised force for θ-coordinate ( ) ( ) ˆ ˆ ˆ · · · r r r Q F F r F r r θ → → θ ∂ ∂ = = + θ ∂θ ∂θ = r F θ , have the dimension of torque. Hence the 1st statement is justified. Now, work done for Q θ is = Q θ θ = rF θ θ and for θ r is = rF r , Both have the dimension of work, hence 2nd statement is justified. 4. Obtain an expression for generalised acceleration. Ans. We have from equation (6.4.1) ( ) 1 2 , ,............. , i i

55%	**MATCHING BLOCK 106/115**	**SA**	M. Sc. I Classical Mechanics all.pdf (D142231111)

n r r q q q t → → = So, 1 n i i i i j j j r r r r q t

q t → → → → = ∂ ∂ ∂ = = + ∂ ∂ ∂ ∑ ɺ ɺ

NSOU l CC-PH-07 185 Here, j qɺ is the time derivative of generalised coordinate q j , is called the generalised velocity corresponding to the coordinate q j. Now, 1 n i i i j i

33%	**MATCHING BLOCK 107/115**	**SA**	M. Sc. I Classical Mechanics all.pdf (D142231111)

j r r d r q dt q t → → → = ? ? ∂ ∂ ? ? = + ? ? ∂ ∂ ? ? ? ? ∑ ɺɺ ɺ = 1 1 n n i i i j j i i j j r r r d q q dt q q t → → → = = ? ? ∂ ∂ ∂ ? ? + + ? ? ∂ ∂ ∂ ? ? ? ? ∑ ∑ ɺ ɺ ɺɺ = 1 1 n n i i i j j i i j j r r r q q q q

t → → → = = ∂ ∂ ∂ + + ∂ ∂ ∂ ∑ ∑ ɺ ɺ ɺ ɺɺ = 2 2 1 1 1 · → → = = = ∂ ∂ + ∂ ∂ ∂ ∂ ∑ ∑ ∑ ɺ ɺ ɺ n n n i i

32%	**MATCHING BLOCK 108/115**	**SA**	M. Sc. I Classical Mechanics all.pdf (D142231111)

j k j i k i j k j r r q q q q q q t + 2 1 → = ∂ ∂ ∑ ɺɺ n i j j j r q q + 2 1 n i j j j r q q dt → = ∂ ∂ ∑ ɺ + 1 n i j i j r q q → = ∂ ∂ ∑ ɺɺ + 2 2 i r t → ∂ ∂ = 2 2 1 1 1 2 · → → = = = ∂ ∂ + ∂ ∂ ∂ ∂ ∑ ∑ ∑ ɺ ɺ ɺ n n n i i j k j i k j j k j r r q q q q q q t + 1 2 → = ∂ ∂ ∑ ɺɺ n i j j j

r q q + 2 2 i r t → ∂ ∂ The above expression contains the terms ɺɺ j q , which is called generalised acceleration corresponding to the coordinate j q 5. Prove the laws of conservation of Linear momentum. angular momentum and energy of a system in configuration space. Ans. See article no. (6.6) 6. Derive Euler Lagrange equations of motion of a system using generatised coordinates.

186 NSOU l CC-PH-07 Ans. Let f be a function of generalised coordinate q j ; and their derivative j qɺ ; then path integral between two points an be written as ( ) ( ) ( ) 2 1 , , = ∫ ɺ x j x I f q x q x x dx [where x is the independent variable] For a small variation of this integral I, we can write 2 1 ? ? ∂ ∂ δ = δ + δ ? ? ∂ ∂ ? ? ∑ ∫ ɺ ɺ x

42%	**MATCHING BLOCK 109/115**	**SA**	M. Sc. I Classical Mechanics all.pdf (D142231111)

j j j j x f f I q q dx q q = 2 2 1 1 ? ? ? ? ∂ ∂ ∂ + δ ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ∑ ∫ ∫ ɺ x x j j j j j j x x f f d q q dx q dx q 2 1 ? ? ∂ - δ ? ? ∂ ? ? ∑ ∫ ɺ x j j j

x f d q dx dx q 2 2 1 1 x x j j j j j j x x

f df d q dx q dx q dx dq ? ? ∂ = δ - δ ? ? ∂ ? ? ∑ ∑ ∫ ∫ ɺ 2 1 | ∂ + δ ∂ ∑ ɺ x x j j j f q q At the end points [x 1 and x 2], all paths meet, so 3rd term should be zero,. = 2 1 ? ? ? ? ∂ ∂ ? ? - δ ? ? ∂ ∂ ? ? ? ? ? ? ? ∑ ∑ ∫ ɺ x i j j j j x f f d q dx q dx q For integral I, to be extrimum, δI = 0 0 ? ? ? = - = ? ? ? ? ? ? ? ∑ ɺ j j j df df d dq dx dq 0 ? ? ∂ ∂ - = ? ? ∂ ∂ ? ? ɺ j j f f d q dx q

NSOU l CC-PH-07 187 or, 0 ? ? ∂ ∂ - - = ? ? ∂ ∂ ? ? ɺ j j f f d dx q q This is the expression of Euler Language equation in terms of generalised coordinates. 7.

83% **MATCHING BLOCK 110/115** **SA** MSCPH-502.pdf (D165443984)

State Hamilton’s principle and derive Lagrange eqnation of motion from it.

Ans. See articles (6.7.3) and (6.7.4). 8. Derive the Lagrangian for a charged particle in an electromagnetic field. Ans. In an electromagnetic field, the force on a charged particle is given by F E v → → → → ? ? ? ? = + × ? ? ? ? ? ? ? ? ? ? ? q B ... (1) F → can be expressed in terms of vector and scalar potential A → and φ, and is given by F → → → → → → ? ? ? ? ∂ ? ? = -∇φ- + × ∇ × ? ? ∂ ? ? ? ? ? ? ? A q v A t ... (2) where, B A → → → = ∇ × and E A t → → → ∂ = -∇φ - ∂ Now, x x → ? ? ∂φ ∇φ = ? ? ∂ ? ? and ·

69% **MATCHING BLOCK 111/115** **SA** M. Sc. I Classical Mechanics all.pdf (D142231111)

x x x x y z x A A A v A v A v v v x x y z → → → → → ? ? ? ? ? ? ∂ ∂ ∂ ? ? ∂ × ∇× = - + + ? ? ? ? ? ? ? ? ? ∂ ∂ ∂ ∂ ? ? ? ? ? ? ? ? ? ? 188

NSOU l CC-PH-07 Total time derivative of
A x is

67% **MATCHING BLOCK 112/115** **SA** M. Sc. I Classical Mechanics all.pdf (D142231111)

x x x x x x y z dA A A A A v v v dt t x y z ∂ ∂ ∂ ∂ ? ? = + + + ? ? ∂ ∂ ∂ ∂ ? ? Hence, · x x x dA A v A v A x dt t → → → → → ? ? ? ? ∂ ∂ ×∇× = - + ? ? ? ? ? ∂ ∂ ? ? ? ?

The x-component of equation (2) is ·

25% **MATCHING BLOCK 113/115** **SA** M. Sc. I Classical Mechanics all.pdf (D142231111)

x x x x A dA A F q v A x t x dt dt → → ? ? ? ? ? ∂ ∂ ∂φ ∂ = - - + - + ? ? ? ? ? ∂ ∂ ∂ ? ? ? ? ? ? ? = ( ) . x dA d q v A dx dt ? ? - ϕ- - ? ? ? ? Now, · x x v A A v → → ? ? ∂ = ? ? ∂ ? ? , Since 0 x x A v ∂ = ∂ = · · x d q v A v A x dt v → → → → ? ? ? ? ? ? ∂ ∂ - φ- - ? ? ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ? ? As φ is independent of v x, Hence we can write . . → → → → ? ? ? ? φ- φ- ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∂ ∂ = - + ? ? ∂ ∂ ? ? ? ? v A v A x x d F q x dt v Let us put · V q v A → → ? ? = φ- ? ? ? ? , then x x V d V F q x dt v ? ? ∂ ∂ = - + ? ? ∂ ∂ ? ?

Equation (6.5.8) gives us ? ? ? ? ∂ ∂ ∂ ∂ - = = - + ? ? ? ? ∂ ∂ ∂ ∂ ? ? ? ? x x x

25% **MATCHING BLOCK 114/115** **SA** M. Sc. I Classical Mechanics all.pdf (D142231111)

x x d T T V d V F dt v v v dt v NSOU l CC-PH-07 189 or, ( ) ( ) 0 ?∂ - ? ?∂ - ? - = ? ? ? ? ∂ ∂ ? ? ? ? x T V T V d dt v x or, 0 ? ? ∂ ∂ - = ? ? ∂ ∂ ? ?x d L L dt v x Where L = T – V is the Lagrangian (as mentioned earlier) ∴ L = T –

V = T – qφ + q .v A → → ? ? ? ? ? ? The above expression is

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the Lagrangian for a charged particle in an electromagnetic field. 9.What is

the advantage of Lagrangian approach over Newtonian approach? Ans. (i) Lagrange's equations of motion are invariant in form, under the coordinate transformation. (ii) Energy and work are more fundamentals than force, in the motion of a system. Hence Lagrangian approach is more advantageous than Newtonian approach. (iii) In Lagrangian approach the choice of the coordinate is generalised, hence need not to tranform from centerian to polar, spherical, cylindrical etc. 10. Prove that shortest distance between two points in plane is straight line joining them using Hamilton's principle. Ans. For an element of small arc length ds in a plane can be represented by 2 2 2 1 ds dx dy dx y = + = + Then, ( ) 1 2 2 2 2 2 1 1 1 1 I ds y dx fdx = = + = ∫ ∫ ∫ J where 2 1 f y = + J
190 NSOU l CC-PH-07 For this curve to be extremum, δI = O and from Hamilton principle we get 0 f f y x y ∂ ∂ ? ? ∂ - = ? ? ∂ ∂ ∂ ? ? J [From equation. 6.6.3] 0 f y ∂ ⇒ = ∂ f = constant or, 2 1+ Jy = constant or, yJ = constant = m say y mx c = + [Integrating both sides] This is the equation of a straight line. 11. State and explain the Hamiltons principle of least action? Derive Lagrange's equations from the Hamilton's principle. Ans. See article no. (6.7.2) and (6.7.4) 12. Derive the canomnical equations of Hamilton. Ans. See articles no. (6.7.5). 13. The Lagrangian is given by ( ) 1/2 2 1 L q = - - J . Write down the Lagrarge equation of motion. Ans. we have, 0 ? ?∂ ∂ - = ? ? ∂ ∂ ? ? J d L L dt q q ( ) ( ) 1 2 2 1 1 2 2 - = - - - - J J J dL q q dq ( ) 1 2 2 1 q q = - J J
NSOU l CC-PH-07 191 and 0 dL dq = Lagrangian equation is given by ( ) 1/2 2 0 1 ? ? ? ? = ? ? ? ? - ? ? J J q d dt q or, ( ) ( ) ( ) 3/2 1/2 2 1 1 · 2 0 2 1 q q q q q q - - - - = - JJ J J J JJ J or, ( ) 2 2 1 0 q q q q - + = JJ J J JJ or, ( ) 2 2 1 0 - + = JJ J J q q q or, 0 q =JJ or, dq m dt = [Integrating both sides] or, = + q mt c 14. The Lagrangian is given by ( ) = - - + J J Jβ 2 2 2 3 2 1 1 , 2 2 L x x x w x x xx αααα , where ααααα, βββββ, ωωωωω are constants. Find the Hamiltonian. Ans. We have, ( ) 2 2 2 3 2 1 1 , 2 2 L x x x w x x xx = - -α +β J J J ∴ 2 ∂ = = + β ∂ J J J x L P x xx x 1 2 x P x x ⇒ = + β J

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is an expansion of a periodic function in terms of an
infinite sum

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x a c x a x a n x = + + + + + b 1 sin x + b 1 sin 2x + ...+ b
n sin nx = ( ) 0 1 1 cos sin 2 α = + + ∑ n n n a a nx b nx
......(1.1) It is a convergent series and a 0 , a n and b n

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x a a b i α - - = ? ? + - = + + ? ? ? ? ∑ = 0 1 1 2 2 2 in x in x in x in x n n e e e e a a ibn ∝ - - - = ? ? + - + - ? ? ? ? ∑ = 0 1 1 2 2 2 in x in x n n n n a ibn a ib a e e ∝ - = - + ? ? + + ? ? ? ? ∑ = 0 1 1 1 2 2 2 in x in x n n n n n n

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π -π = ∫ 2. π -π ∫ cos nx dx = 0 for 0 2 for 0 n n ≠ ? ? π = ? 3. 2 sin nx dx π -π = π ∫ 4. π -π ∫ cos 2 nx dx = for 0 2 for 0 n n π ≠ ? ? π = ? 5. π -π ∫ sin nx sin mx dx = 0 for n m≠ . 6. π -π ∫

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5/115 **SUBMITTED TEXT** 26 WORDS **27%** **MATCHING TEXT** 26 WORDS

π -π = π ∫ where, m = n and m,n < 0 0 1 ( ) 1 ( ) cos 1 ( ) sin n n a f x dx a f x nx dx b f x nx dx π -π π -π π -π ? ? = π ? ? ? = ? π ? ? ? = ? π ? ∫ ∫ ∫

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6/115 **SUBMITTED TEXT** 54 WORDS **32%** **MATCHING TEXT** 54 WORDS

n π π ? ? - - ? ? - π ? ? ? ? ∫ = [ ] 0 2 sin cos nx n n n π ? ? ? ? ? ? -π π + ? ? ? ? π ? ? ? ? ? ? ? = ( ) 2 1 n n - - = ( ) 1 2 1 n n + - The Fourier expansion is given by f(x) = x = ( ) 1 1 1 sin 2 n n nx n + ∝ = - ∑ = 2[sin x – 1 2 sin 2x + 1 3 sin 3x – 1 4 sin 4x +.........] 2.

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π -π ? ? ? ? + π ? ? ? ? ∫ ∫ = 0 1 0 h dx h π ? ? ? ? + = π ? ? ? ? ∫ Also, 0 1 1 ( )cos ( )cos n a f x nxdx f x nx dx π π -π = = π π ∫ ∫ 0 1 cos 0 h nxdx π = = π ∫ and b n ( ) 1 sin 1 cos h h nxdx n n π -π = = - π π π ∫ = 0, for even 2h for odd n ? ? ? ? π ? n n f (x) = ( ) 1 sin 2 is odd only 2 n nx h h n n ∝ = + π ∑ 2

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sin n b f x nxdx π = π ∫ = 2 0 sin sin h nxdx nxdx π π π ? ? ? ? - π ? ? ? ? ∫ ∫ = [ ] [ ] { } 2 0 cos cos h nx nx n π π π - + - π = ( ) ( ) { } 1 cos 1 cos - π + - π π h n n n = ( ) 2 1 cos h n n - π π = 0, 4 , n iseven h n isodd n ? ? ? ? π ? Hence, 1 4 sin ( ) n h nx f x n ∝ = = π ∑ (n is odd only) = 4 sin3 sin5 sin .... 3 5 ? ? + + + ? ? π ? ? h x x x 4. Half-wave rectifier : f(x) = sin x, 0 x≤ > π = 0, 2 x π ≤ ≤ π 20

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π − π 0 π 2π 3π x f x( ) fig. 1.8 Here, a 0 = 0 0 1 1 ( ) ( ) f x dx f x dx π -π + π π ∫ ∫ = 0 0 1 1 sin sin x dx xdx π -π - + π π ∫ ∫ = 0 2 4 sin x dx π = π π ∫ 0 0 1 1 sin cos sin cos n a x nxdx x nxdx π -π = - + π π ∫ ∫ 22 NSOU l CC-PH-07 0 2 sin cos x nx dx π = π ∫ = 2 0 for -odd 4 1 ,for -even 1 ? ? ? - ? π - ? n n n and 0 2 n b π = π ∫

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π π ∫ = 2 0 0 2 2 2 sin sin x x nx nxdx n n π π ? ? - ? ? π π ? ? ∫ = 0 4 sin x nxdx n π = - π ∫ = 0 0 4 4 1 cos cos π π ? ? - - - ? ? π π ? ? ∫ x x nxdx n n n n = 2 0 4 4 1 1 cos · sin π ? ? π - ? ? π ? ? n nx dx n n n n = ( ) 2 4 1 n n - and 3 2 2 0 0 1 2 2 3 3 π π -π ? ? π = = = ? ?

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We have, 1 ( ) sin 2 n n n x f x x b ∝ = π = = ∑ , where ( ) 2 0 2 sin 2 2 n n x b f x dx π = ∫ = 2 0 sin 2 n x x dx

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cos sin 2 2 2 2 ? ? π π - - ? ? ? ? - π ? ? π ? ? ? ? n x n x x n n = ( ) 4 4 cos 1 - π = - - π π n n n = 4 for is odd 4 for is even n n n n ? ? π ? ? - π ? Hence ( ) 1 1 4 ( ) sin 2 n n n x f x x n ∝ = - π = = - π ∑

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n l n x a f x dx l l = o 2 cos l n x x dx l l π ∫ = ( ) ( ) 2 sin cos 2 l o n x n x l l x l n n l l ? ? π π ? ? ? ? - ? ? ? ? ? ? ? ? ? ? ? ? - π ? ? π ? ? ? ? = ( ) 2 2 2 cos 1 l n n π- π = ( ) 2 2 2 1 1 n l n ? ? - - ? ? ? ? π = 2 2 0, is even 4 , is odd n l n n ? ? ? - ? π ? f(x) = |x| 2 2 1 4 1 cos 2 n n x l l l n ∝ =

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n n n n ∝ ∝ = = ? ? π - + ? ? ? ? ∑ ∑ = 2 2 1 1 2 3 n n ∝ = π - ∑ or, 2 2 1 1 6 n n ∝ = π = ∑ (ii) ( ) 2 2 2 2 1 1 1 1 1 .... 1 3 5 2 1 n n ∝ = = + + + - ∑ = 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 .... 1 3 5 2 4 2 n n ∝ = + + + + + - ∑ = 2 2 1 1 1 1 1 4 n n n n ∝ ∝ = = - ∑ ∑ = 2 1 3 1 4 n n ∝ = ∑ = 2 3 · 4 6

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15/115 **SUBMITTED TEXT** 204 WORDS **17%** **MATCHING TEXT** 204 WORDS

π π = = = π π π ∫ ∫ 0 0 sin cos n i a x nxdx π = π ∫ ( ) ( ) 0 0 sin 1 sin 1 2 i n x n x dx π = ? + - - ? ? ? π ∫ ( ) ( ) ( ) ( ) 0 cos 1 cos 1 2 1 1 o n x n x i n n π ? + - ? = - + ? ? π + - ? ? ( ) ( ) 0 1 cos 1 cos 1 1 2 1 1 n n i n n ? - + π - π- ? = - + ? ? π + - ? ? ∴ a 1 = 0, 0 2 2 3 i a = - π , a 3 = 0, o 4 2 15 = - π i a , a 5 = 0, 0 0 2 35 i a = - π , ... 0 0 sin sin n i b x nxdx π = π ∫ NSOU l CC-PH-07 33 = ( ) ( ) 0 0 cos 1 cos 1 2 i n x n x dx π ? - - + ? ? ? π ∫ = ( ) ( ) 0 sin 1 sin 1 2 1 1 o n x n x i n n π ? - + ? - ? ? π - + ? ? ( ) 2 0 0 1 0 0 sin 1 cos2 2 2 o i i i b xdx x dx π π = = - = π π ∫ ∫ ∴ 0 0 0 0 0 2 2 2 ( ) sin cos2 cos4 cos6 .... 2 3 15 35 i i i i i f x x x x x = + - - - π π π π . = 0 0 0 2 1 2

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cos sin , 2 n n n n a a nx b nx ∝ ∝ = = + + ∑ ∑ then prove that [ ] 2 1 ( ) f x dx

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

17/115 **SUBMITTED TEXT** 110 WORDS **25%** **MATCHING TEXT** 110 WORDS

π ∝ ∝ = = -π -π ? ? ? ? = + + π π ? ? ? ? ∑ ∑ ∫ ∫ n n n n a f x dx a nx b nx dx = 2 2 2 2 2 0 1 1 1 1 1 1 cos . sin 4 n n n n a dx a nx dx b nx dx π π π ∝ ∝ = = -π -π -π + + π π π ∑ ∑ ∫ ∫ ∫ 0 0 1 1 1 1 cos sin π π ∝ ∝ = = -π -π + π π ∑ ∑ ∫ ∫ n n n n a a nx dx a b nx dx + 1 1 2 cos sin π ∝ ∝ = = -π π ∑ ∑ ∫ n n n n a nx b nxdx = 2 2 2 0 1 1 1 1 2 0 0 0 4 ∝ ∝ = = ? ? ? ? π+ π + π + + + π π ? ? ? ? ∑ ∑ n n n n

SA 019E1130_Mathematical Physics.pdf (D165097245)

18/115 **SUBMITTED TEXT** 61 WORDS **17%** **MATCHING TEXT** 61 WORDS

π -π π - ∴ = = = π π π ∫ ∴ 2 2 1 1 cos sin cos . bx n b nx n nx a e nx dx b n π π -π -π ? ? + = = ? ? π π + ? ? ∫ 36 NSOU l CC-PH-07 = ( ) 2 2 1 2 · sin h. , n b b b n - π π + and b n = 2 2 1 1 sin cos sin x b b bx n nx e nx dx b n π π -π -π ? ? - = ? ? π π + ? ? ∫ = ( ) 1 2 2 1 2 sin - - π π + n n

SA 019E1130_Mathematical Physics.pdf (D165097245)

19/115 **SUBMITTED TEXT** 42 WORDS **37%** **MATCHING TEXT** 42 WORDS

π) is given by 0 1 ( ) cos , 2 ∝ = = + ∑ n n a f x a nx where 0 0 1 ( ) 2 a f x dx π = π ∫ ∴ ( ) /2 2 0 0 0 2 2 · 4 2 a x dx x dx π π ? ? π π ? ? = + π - = = π π ? ? ? ? ∫ ∫ and 0 1 ( ) cos 2 π = π ∫

SA 019E1130_Mathematical Physics.pdf (D165097245)

20/115 **SUBMITTED TEXT** 41 WORDS **29%** **MATCHING TEXT** 41 WORDS

x x A x x x = - = - - is finite and ( ) ( ) ( ) 1 2 2 1 1 =- - = + - x x A x x x is also finite Again, ( ) ( ) ( ) ( ) 1 2 2 1 1 1 = + = - - x n n B x x x is finite and ( ) ( ) ( ) ( ) 1 2 2 1 1 1 =- + = + - x n n B x x x

SA 019E1130_Mathematical Physics.pdf (D165097245)

21/115 **SUBMITTED TEXT** 66 WORDS **32%** **MATCHING TEXT** 66 WORDS

s= + Hence, ( )( ) ( ) 5 5 2 0 2 2 1 1 s s s s s s c x s s c x ∝ ∝ + = = + + - - ∑ ∑ ( ) 5 5 1 0 2 1 0 ∝ ∝ = = - + + = ∑ ∑ s s s s sc x n n c x ...→ (2.3.4) From equation (2.3.4) we get, ( )( ) ( )( ) 2 1 2 1 s s n s n s C C s s + - + + = - + + ...→ (2.3.5)

SA 019E1130_Mathematical Physics.pdf (D165097245)

22/115 **SUBMITTED TEXT** 55 WORDS **53%** **MATCHING TEXT** 55 WORDS

n n C C + = - ( )( ) 3 1 1 2 3! - + = - n n C C 48 NSOU l CC-PH-07 ( )( ) 4 2 2 3 4.3 n n C C - - + = - ( )( ) 5 3 3 4 5.4 n n C C - + = ( )( )( )( ) 0 2 1 3 4! n n n n C - + + = ( )( )( )( ) 1 3 1 2 4 5! n n n n

SA 019E1130_Mathematical Physics.pdf (D165097245)

23/115 **SUBMITTED TEXT** 51 WORDS **72%** **MATCHING TEXT** 51 WORDS

n n n n n n u x x x + ...→ (2.3.7) and u 2 (x) ( )( ) ( )( )( )( ) 3 5 1 2 3 1 2 4 3! 5! n n n n n n x x x - + - - + + = - + + ... → (2.3.8)

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

24/115 **SUBMITTED TEXT** 101 WORDS **70%** **MATCHING TEXT** 101 WORDS

n n n n C C n - - = - = ( ) ( ) ( ) ( ) 2 1 2 ! 2 2 1 2 ! n n n n n n - - [from equation (2.3.9)] = ( ) ( )( ) ( ) ( ) ( )( ) 1 2 2 1 2 2 ! 2 2 1 2 1 ! 1 2 ! n n n n n n n n n n n n - - - - - - - - = ( ) ( ) ( ) 2 2 ! 2 1 ! 2 ! n n n n - - - Similarly, ( )( ) ( ) 4 2 2 3 4 2 3 n n n n C C n - - - - - = - - - = ( ) ( ) ( ) 2 4 ! 2 2! 2 ! 4 ! n n n n - - -

SA 019E1130_Mathematical Physics.pdf (D165097245)

25/115 **SUBMITTED TEXT** 56 WORDS **43%** **MATCHING TEXT** 56 WORDS

n n n n n n n x P x x n n n + ... → (2.3.12) The first few of these function ( ) ( ) ( ) ( ) ( ) 0 2 2 4 2 4 1 1 3 1 2 1 35 30 3 6 P x P x x P x x x ? = ? ? ? = - ? ? ? = - + ? ? ( ) ( ) ( ) ( ) ( ) 1 3 3 5 3 5 1 5 3 2 1 63 70 15 8 P x x P x x x P x x x x ? = ? ? ? = - ? ? ? = - + ? ? .... (2.3.13)

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

26/115 **SUBMITTED TEXT** 92 WORDS **36%** **MATCHING TEXT** 92 WORDS

n n n n x D t C x D t C D t + + + + - + + { } 1 1 1 2 + + = + n n n n x D t C D t or, ( ) 2 2 1 1 1 1 1 2 1 1 2 2 0 n n n n n n x D t x C n D t C n C D t + + + + + ? ? ? ? - + - + - = ? ? ? ? or, ( ) ( ) 2 2 1 1 2 1 0 n n n x D t x D t n n

SA 019E1130_Mathematical Physics.pdf (D165097245)

27/115 **SUBMITTED TEXT** 12 WORDS **100%** **MATCHING TEXT** 12 WORDS

d y dy x x n n y dx dx - + - + = , [

SA 019E1130_Mathematical Physics.pdf (D165097245)

28/115 **SUBMITTED TEXT** 70 WORDS **50%** **MATCHING TEXT** 70 WORDS

x P x x x dx = = - = , ( ) ( ) 2 2 2 2 2 2 1 1 2 2! = - d P x x dx = ( ) ( ) 2 2 2 1 1 · 2 1 2 3 1) 2 2 .2 ? ? - = - ? ? ? ? d x x x dx ( ) ( ) ( ) 3 3 2 3 3 3 3 1 1 1 5 3 2 2 3! = - = - d P x x x x dx , ( ) ( ) ( ) 4 4 2 4 2 4 4 4 1 1 1 35 30 3 6 2 ·4! = - = - + d P x x x x

SA MPHS-21 MPII final 19.03.2022.pdf (D131037710)

29/115 **SUBMITTED TEXT** 66 WORDS **53%** **MATCHING TEXT** 66 WORDS

n n n xy y y P x .... (2.5.1) Proof : ( ) ( ) 1 1 2 2 2 1 2 1 2 - - - + = ? - - ? ? ? xy y y x y NSOU l CC-PH-07 53 = ( ) ( ) ( ) 2 3 2 3 1.3 1.3.5 1 2 2 2 2 2.4 2.4.6 y x y y x y y x y + - + - + - + ... + ( ) ( ) 1.3.5. 2 1 2 2.4.6..........2 - + -

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

30/115	**SUBMITTED TEXT**	69 WORDS	**64%**	**MATCHING TEXT**	69 WORDS

y n are ( ) ( ) ( ) ( ) 1 2 1 1.3.5.......... 2 1 1.3.5.......... 2 3 2 2 2.4.6.........2 2.4.6...........(2 2) n n n n n x x C n n - - - - - - + ... = ( ) ( ) ( ) 2 1.3.5.......... 2 1 1 .... ! 2 2 1 - - ? - ? - + ? ? - ? ? n n n n n x x n n = P n (x) ( ) ( ) ( ) ( ) ( ) 1 2 2 2 0 1 2 0 1 2 .... ∝ - = - + = = + + + ∑ n n n

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

31/115	**SUBMITTED TEXT**	32 WORDS	**61%**	**MATCHING TEXT**	32 WORDS

x) is a solution of ( ) ( ) 2 2 2 1 2 1 0 - - + + = d y dy x x n n y dx dx ... (2.6.1) P m (x) is a solution of 54

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

32/115	**SUBMITTED TEXT**	116 WORDS	**26%**	**MATCHING TEXT**	116 WORDS

or, ( ) ( ) ( ) ( ) 2 1 0 0 1 2 0 n n n n n n xy y ny P x y x y P x ∝ ∝ - = = - + + - - = ∑ ∑ ( ) 0 Put ∝ = ? ? ? ? = ? ? ? ? ∑ n n n t y P x The co-efficients of y n – 1 from both sides given ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 2 2 1 2 1 2 0 n n n n n nP x x n P x n P x P x xP x - - - - - - + - + - = or, ( ) ( ) ( ) ( ) ( ) 1 2 2 1 1 n n n nP x n xP x n P

SA 019E1130_Mathematical Physics.pdf (D165097245)

33/115	**SUBMITTED TEXT**	73 WORDS	**56%**	**MATCHING TEXT**	73 WORDS

m r r r x C m r m r x ∝ + - = + + - ∑ + ( ) ∝ + - = + ∑ 1 0 m r r r x C m r x + ( ) 2 2 0 0 m r r r x n C x ∝ + = - - = ∑ or, ( )( ) ( ) 2 0 1 m r r r C m r m r m r n x ∝ + = ? ? + + - + + - ? ? ∑ + 2 0 0 m r r r C x ∝ + + = = ∑ or, ( ) 2 2 0 m r r r C m r n x ∝ + = ? ? + - ? ? ∑ + 2 0 0 m r r r C x ∝ + + = = ∑ Putting, r = 0,1

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34/115	**SUBMITTED TEXT**	34 WORDS	**53%**	**MATCHING TEXT**	34 WORDS

n x x u x C x n n n ( ) ( )( ) ( ) 2 0 0 1 2 !..........2 1 2 .... r r n r r r x C x r n n n r ∝ = - - = + + + ∑ ... (2.8.4) The solution (2.8.4) is called

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

35/115 **SUBMITTED TEXT** 21 WORDS **66%** **MATCHING TEXT** 21 WORDS

Bessel function of the first kind of order n and is denoted by ( ) ( ) ( ) ( ) 2 0 1 2 ! 1 r n r n r x J x r n r ∝ + = - =

SA MPHS-21 MPII final 19.03.2022.pdf (D131037710)

36/115 **SUBMITTED TEXT** 94 WORDS **47%** **MATCHING TEXT** 94 WORDS

n x ′ Proof : We have, ( ) ( ) ( ) ∝ + = - ′ = Γ + + ∑ 2 0 1 J 2 ! 1 r n r n r x r n r Differentiating w. r. t. x, we get, ( ) ( ) ( ) ( ) ∝ + - = - + = x Γ + + ∑ 2 1 0 1 2 1 J 2 2 ! 1 r n r n r n r x r n r ′ ( ) ( ) ( ) ∝ + = - ∴ = Γ + + ∑ 2 0 1 J 2 ! 1 r n r n r x x n r n r ′ + ( ) ( ) ( ) ∝ + - = - Γ + + ∑ 2 1 0 1 2 2 2. ! 1 r n r r r x x r n r = ( ) ( ) ( ) ( ) ∝ + - = - + - Γ + + ∑ 2 1 1 1 2 1 ! 1 r n r n r x nJ x r n r = ( ) ( ) ( ) ∝ + + + = - +

SA 019E1130_Mathematical Physics.pdf (D165097245)

37/115 **SUBMITTED TEXT** 40 WORDS **44%** **MATCHING TEXT** 40 WORDS

n t n n e t x ∝ - =-∝ = ∑ ... (2.10.1) Proof : We have, ( ) - - = 1 2 2 2 · x x xt t t t e e e ( ) 2 1 1 .... 2 2! 2 xt xt ? ? = + + + x ? ? ? ? ( ) ? ? - + + ? ? ? ? 2 1 1 .... 2 2! 2 x x t t ... (2.10.2)

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

38/115 **SUBMITTED TEXT** 32 WORDS **57%** **MATCHING TEXT** 32 WORDS

J 0 (x) = ( ) ( ) ( ) 2 0 1 2 ! 1 r r r x r r ∝ = - Γ + ∑ ... (2.11.1) and J 1 (x) = ( ) ( ) ( ) 2 1 0 1 2 ! 2 r r r x r r ∝ + = - Γ + ∑ ... (2.11.2)

SA 019E1130_Mathematical Physics.pdf (D165097245)

39/115 **SUBMITTED TEXT** 36 WORDS **39%** **MATCHING TEXT** 36 WORDS

s s C s s x ∝ + = + + ∑ − 0 2 2 2 0 s s s s s s C sx nC x ∝ ∝ = = + = ∑ ∑ ... (2.13.4) From equation (2.13.4) we get, ( ) ( )( ) 2 2 2 1 S s s n C C s s + - = + + ... (2.13.5)

SA 019E1130_Mathematical Physics.pdf (D165097245)

40/115	**SUBMITTED TEXT**	119 WORDS	**44%**	**MATCHING TEXT**	119 WORDS

n C - = , ( ) 2 0 2 2 2 4.3.2.1 n n C C - = - or, ( ) ( ) ( ) ( )( ) - - - = - = - 2 3 4 0 6 0 2 2 4 2 , 2 4! 6! n n n n n C C C C Similarly, ( ) ( ) ( ) - - - = = - = - 1 3 1 1 1 2 1 1 2 2 2 3.2 3! 3! n n n C C C C ( )( ) 5 3 1 2 1 6 2 6 2 5.4 5.4.3! n n n C C C - - - = = - = ( ) ( )( ) 2 1 1 3 2 5! n n C - - - ( ) ( )( )( ) 3 7 1 5 3 1 2.5 2 2 7.6 7.6.5! n n n n C C - - - - = = - = ( ) ( )( )( ) - - - - 3 1 5 3 1 2 7! n n n C = ( ) ( )( )( ) 3 1 1 3 5 2 7! n n n

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41/115	**SUBMITTED TEXT**	102 WORDS	**37%**	**MATCHING TEXT**	102 WORDS

n n n n n x x C n ... (2.14.1) NSOU l CC-PH-07 67 Let us put c 0 = ( ) ( ) /2 ! 1 ! 2 n n n - , then co-efficient of x n of equation (2.14.1) given ( ) ( ) ( ) ( ) /2 /2 2 ......2 ! 1 · 2 ! ! 2 n n n n n n n - - - = ( ) ( ) ( ) ( ) - 2 /2 /2 2 2 ....2 2 ! 2 2 n n n n n n = ( ) ( ) ? ? - ? ? ? ? = · 1 ....2 2 2 2 2 ! 2 n n n n n Similarly, the co-efficients of 2n x - is given by ( ) 2 1 2 1! n n n - - - ∴ ( ) ( ) ( ) ( ) ( )( )( ) ( ) 2 4 1 1 2 3 2 2 2 .... 1! 2! n n n n n n n n n n

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42/115	**SUBMITTED TEXT**	41 WORDS	**47%**	**MATCHING TEXT**	41 WORDS

n (x). In general, ( ) ( ) ( ) ( ) /2 2 0 ! 1 2 ! 2 ! n s n s n s n H x x s n s - = = - - ∑ , n is even and ( ) ( ) ( ) ( ) /2 1 2 0 ! 1 2 ! 2 ! n s n s n s n H x x s n s - - = = - - ∑ , n

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43/115	**SUBMITTED TEXT**	11 WORDS	**66%**	**MATCHING TEXT**	11 WORDS

n (x) is given by 2 2 0 ( ) ! n tx t n n t e H x n ∝ - = = ∑ ... (2.15.1)

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44/115 **SUBMITTED TEXT** 84 WORDS **38%** **MATCHING TEXT** 84 WORDS

t x − t 2 + - + + 2 2 3 4 4 4 .... 2 t x t x t = 1+2tx − t 2 + 2t 2 x 2 −2t 3 x + 4 2 t + ... = 1 + 2xt + (2x 2 − 1)t 2 + ... Now, ( ) 0 ! n n n t H x n ∝ = ∑ = ( ) 0 1 ( ) 1! t H x H x + + ( ) 2 2 2! t H x + ... = ( ) 2 2 1 (2) 4 2 .... 2! t t

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45/115 **SUBMITTED TEXT** 106 WORDS **41%** **MATCHING TEXT** 106 WORDS

n n x x n n d H x e e dx - ? ? = - ? ? ? ? ... (2.16.1) NSOU l CC-PH-07 69 Proof : From equation (2.14.3) we obtain, H 0 (x) = 1 H 1 (x) = 2x H 2 (x) = (2x) 2 − 2! (2x) 2 − 2 [equation (2.14.3) is valid for s = 0 and 1] = 4x 2 − 2 H 3 (x) = (2x) 3 − 3! (2x) 1 = 8x 3 − 12x [

SA 019E1130_Mathematical Physics.pdf (D165097245)

46/115 **SUBMITTED TEXT** 126 WORDS **32%** **MATCHING TEXT** 126 WORDS

x x d e e H x dx for n = 1, ( ) - ? ? - ? ? ? ? 2 2 1 1 x x d e e dx = 2x = H 1 (x) for n = 2, (−1) 2 2 2 2 2 2 2 4 2 ( ) x x d e e x H x dx - ? ? = - = ? ? ? ? for n = 3, ( ) 2 2 3 3 3 1 x x d e e dx - ? ? - ? ? ? ? = 2 2 2 2 2 x x d e x e dx - ? ? - - ? ? ? ? = 2 2 2 2 4 2 x x x d e x e e dx - - ? ? - - ? ? ? ? = 2 2 2 2 3 8 8 4 x x x x e x e xe xe - - - ? ? - - + + ? ? ? ? 70 NSOU l CC-PH-07 = 8x 3 − 12x = H 3 (x)

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

47/115 **SUBMITTED TEXT** 245 WORDS **25%** **MATCHING TEXT** 245 WORDS

n n t e H x n ∝ - = = ∑ Differentiating both sides w. r. t x we obtain ( ) 2 2 0 2 ! n tx t n n t te H x n ∝ - = = ∑ ' ( ) ( ) ∝ ∝ = = ⇒ = ∑ ∑ 0 0 2 ! ! n n n n n n t t t H n H x n n ' ( ) ( ) ∝ + = ⇒ = ∑ ∑ 1 0 2 ! ! n n n n n t t t H x H x n n ' The co-efficients of t n gives ( ) ( ) - = - 1 2 ( ) ! 1 ! n n H x H x n n ' or, ( ) ( ) 1 ! 2 ( ) 1 ! n n n H x H x n - = - ' ( ) ( ) 1 2 n n nH x H x - ∴ = ' ... (2.17.1) NSOU l CC-PH-07 71 (2) ( ) ( ) ( ) 1 1 2 2 0 n n n H x xH x nH x + - - + = Proof : From equation (2.16.1) we obtain, ( ) ( ) 2 2 1 n n x x n n d H x e e dx - ? ? = - ? ? ? ? ∴ ( ) ( ) 2 2 1 2 n n x x n n d H x x e e dx - ? ? = - ? ? ? ? ' + ( ) 2 2 1 1 1 n n x x n d e e dx + - + ? ? - ? ? ? ? or, ( ) ( ) ( )( ) 2 2 2 2 1 1 1 2 1 1 1 n n n x x x x x n n n d d H x x e e e e dx dx + + - + ? ? = - + - - ? ? ? ? ' or, ( ) ( ) ( ) 1 1 2 2 n n n n n H x x H x H x - + = - ∴ ( ) ( ) ( ) 1 1 2 2 0 n n n H x x H x n H x + - - + = ... (2.17.2) 3. ( ) ( ) ( ) 2 2 0

SA 019E1130_Mathematical Physics.pdf (D165097245)

48/115 **SUBMITTED TEXT** 141 WORDS **38%** **MATCHING TEXT** 141 WORDS

n n H x x H x nH x - + = " ' Proof : We have equation (2.17.1), ( ) - = 1 ( ) 2 n n H x n H x ' ∴ ( ) ( ) ( ) - + ? ? = = - ? ? 1 1 ( ) 2 2 n n n n d H x n H x x H x H x dx " ' [From 2.17.2] or, ( ) ( ) ( ) 1 ( ) 2 2 n n n n H x H x xH x H x + = + - " ' ' = ( ) ( ) ( ) 2 ( ) 2 2 1 n n n H x x H x n H x + - + ' [From 2.17.1] 72 NSOU l CC-PH-07 = ( ) ( ) 2 2 n n xH x xH x - ' ∴ ( ) ( ) ( ) 2 2 0 n n n H x x H x n H x - + = ' ' [

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49/115 **SUBMITTED TEXT** 28 WORDS **39%** **MATCHING TEXT** 28 WORDS

x x n n dH x d e n e H x dx dx ... (2.18.2) and ( ) 2 2 2 ( ) x x x m d Hm d e m e H x dx dx - - ? ? ? ? = - ? ? ? ? ...(2.18.3)

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

50/115	**SUBMITTED TEXT**	116 WORDS	**21%**	**MATCHING TEXT**	116 WORDS

x m m n dHn x dH x d d H x e H x e dx dx dz dx - - ? ? ? ? - ? ? ? ? ? ? ? ? ? = ( ) ( ) ( ) 2 2 - - x m n m n e H x H x .... (2.18.4) Integrating both sides w. r. t. x from − ∝ to + ∝ of eqnation (2.18.4) we obtain, 2(m − n) ( ) ( ) 2 ∝ - -∝ ∫ x m n e H x H x dx = ( ) ( ) 2 ∝ - -∝ ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∫ n x m dH x d H x e dx dx dx − ( ) ( ) 2 ∝ - -∝ ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∫ m x m dH x d H x e dx dx dx = 0 − 0 = 0 Since, m ≠ n, then ( ) ( ) 2 0 ∝ - -∝ = ∫ x m n

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51/115	**SUBMITTED TEXT**	53 WORDS	**23%**	**MATCHING TEXT**	53 WORDS

x → a, but (x − a)P(x) and (x − a) 2 Q(x) remain finite as x → a, then x = a is called regular or nonessential singular point. If (x − a) P(x) and (x − a) 2 Q(x) remain infinite as x → a, then x = a

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52/115	**SUBMITTED TEXT**	71 WORDS	**44%**	**MATCHING TEXT**	71 WORDS

s s s s s C x + ( ) 0 1 ∝ = - ∑ s s s s s C x + 0 0 0 ∝ ∝ = = - = ∑ ∑ s s s s s s sC x C x The co-efficient of x s is (s + 2)(s+1) C s + 2 + (s 2 − s + s −1) C s = 0 ∴ ( )( ) 2 2 1 1 2 2 1 + - - = = - + + + s s s s s C C C s s s ... (4) From equation (4) we get

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53/115	**SUBMITTED TEXT**	55 WORDS	**68%**	**MATCHING TEXT**	55 WORDS
C C 4 2 0 1 1 4 8 = = - C C C ( ) 6 4 0 0 3 1 1 1 · 6 2 8 16 = - = + = + C C C C C 8 = ... and so on. C 3 = 0, C 5 = C 7 = ...0				c", ".1 c #4c8694", ".2 c #040e1c", ".3 c #2c525c", ".4 c #3c6e7c", ".5 c #0c262	

W http://web.mit.edu/ktmeow/Public/pic5

54/115 SUBMITTED TEXT 189 WORDS	36% MATCHING TEXT 189 WORDS
x x x or, 5x 3 = 2P 3 (x) + 5x ∴ f(x) = 5x 3 + x = 2P 3 (x) + 4x = 2P 3 (x) + 4P 1 (x) [. . . P 1 (x) = x] 6. Express f(x) = x 3 + 2x 2 – x – 3 interms of Legendre Polynomials. Ans. We have, ( ) ( ) 3 3 1 5 3 2 = - P x x x or, ( ) 3 3 2 3 5 5 = + x P x x ( ) ( ) 2 2 1 3 1 2 = - P x x or, ( ) 2 2 2 1 3 3 = + x P x P 1 (x) = x and P 0 (x) = 1 ∴ f(x) = x 3 + 2x 2 – x – 3 = ( ) ( ) 3 2 2 3 4 2 3 5 5 3 3 + + - + - P x x P x x = ( ) ( ) ( ) ( ) 3 2 1 0 2 4 2 7 5 3 5 3 + - - P x P x P x P x 7.	x+75(11-x)=920 | | 160 x+70(12-x)=1560 | | 170 x+70(26-x)=1920 | | 160x+50(18- x)=1120 | | 2x+5/5=3 x+1/2+(-x)+7/2 | | 150 x+65(11-x)=1990 | | 4(2 x+70(11-x)=1220 | | 150x+50(26-x)=1400 | | 150 x+65(13-x)=930 | | c^-8/x+0.05(4- x)=0.10(70) | | 170 x+60(29-x)=1960 | | -5/8 x=-7/8 | | 160x+70(14- x)=1700 | | x(3)=13(3)-2 | | x/3231248=.0175 | | 170x+50(15- x)=870 | | .75 x+14=20 | | 2-x=x+5 | | 170x+50(29- x)=1810 | | -x-6=-4 x-3 | | 160x+70(19- x)=1420 | | .02 x+.06y=1040 | | 25(1)=20(1)+20 | | 150x+70(19- x)=1810 | | 170x)=1720 | | 150

W https://www.geteasysolution.com/2+2x%5E2-2+2=

55/115 SUBMITTED TEXT 97 WORDS	24% MATCHING TEXT 97 WORDS
n n t t P t or, ( ) ( ) 1/2 2 2 0 1 1 α - = ? ? - = ? ? ? ? ∑ x n t P t or, ( ) ( ) 1 2 0 1 1 α - = ? ? - = ? ? ? ? ∑ n n n t P t or, 1 + t + t 2 +......... + t n + ... = 1 + P(1).t + P 2 (t)t 2 + ... + ...P n (t)t n + ... Equating the co-efficients of t n an either side we get, P n (1) = 1 8. Show that P n (– x) = (– 1)	

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56/115 SUBMITTED TEXT 78 WORDS	23% MATCHING TEXT 78 WORDS
n n xt t t P n Putting x = – x, we get ( ) ( ) ∝ - = + + = - ∑ 1 2 2 0 1 2 n n n xt t t P x ... (1) Also, putting t = – t we get, ( ) ( ) ( ) ∝ - = + + = - ∑ 1 2 2 2 0 1 2 1 n n n xt t t P x ... (2) 78 NSOU l CC-PH-07 From equation (1) & (2) we get, ( ) ( ) ( ) ∝ ∝ = = - - = - ∑ ∑ 0 0 1 n n n n n n n n t P x t	

SA 019E1130_Mathematical Physics.pdf (D165097245)

57/115 SUBMITTED TEXT 67 WORDS	70% MATCHING TEXT 67 WORDS
P n (– x) = (– 1) n P n (x) 9. If P n (x) is a Legendre Polynomial of degree n and **ααααα** is is such that P n (ααααα) 0, then show that P n – 1 (ααααα) and P n + 1 (ααααα) are of opposite signs. Ans. From	

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

58/115	SUBMITTED TEXT	115 WORDS	52%	MATCHING TEXT	115 WORDS

n (x) = (n + 1)P n + 1 (x) + nP n − 1 (x) ... (1) Putting, x = α is equation (1) we get, (2n + 1) α P n (α) = (n + 1)P n + 1 (α) + nP n − 1 (α) or, 0 = (n + 1)P n + 1 (α) + nP n − 1 (ααααα),. P n (α) = 0 or, ( ) ( ) 1 1 1 + - α = + α n n P n n P As n is positive integer so R. M. S is negative. Hence P n + 1 (α) and P n − 1 (α) are of opposite signs. 10.

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

59/115	SUBMITTED TEXT	7 WORDS	100%	MATCHING TEXT	7 WORDS

n r r n r x J x r n r

SA MPHS-21 MPII final 19.03.2022.pdf (D131037710)

60/115	SUBMITTED TEXT	54 WORDS	37%	MATCHING TEXT	54 WORDS

we get > f < = > x < + > y < and from equation (4.3.4) we get 2 2 Δ = Δ +Δ f x y Now for susbtraction f (x, y) = x − y ⇒ > f < = > x < − > y < and 2 2 Δ = Δ +Δ f x y [

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

61/115	SUBMITTED TEXT	58 WORDS	43%	MATCHING TEXT	58 WORDS

y x x y [From equation (4.3.4)] or, ( ) 2 2 2 2 2 2 ? ? Δ Δ Δ = + ? ? ? ? ? ? y x f x y x y = 2 2 2 2 y x f x y Δ Δ + For division say x f y = ⇒ x f y > < > <= > < and 2 2 2 2 4 Δ Δ = + Δ x x f y y y [From equation (4.3.4)] = 2 2 2 2 2 2 y x x y x y ? ? Δ Δ + ? ? ? ? = 2 2 2 2 y x f x y Δ Δ + 112

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

62/115	SUBMITTED TEXT	28 WORDS	41%	MATCHING TEXT	28 WORDS

y y y P y y y y y = 2 2 1 1 1 exp. 2 2 = ? ? ? ? ? ? − − ? ? ? ? σ π ? ? σ ? ? ? ? ∑ N i i y y y N y = 2 2 1 1 1 exp. 2 2 N i i i y y y

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

63/115	SUBMITTED TEXT	57 WORDS	50%	MATCHING TEXT	57 WORDS

N N N N i i i i i i i i i x y x x y = = = = – ∑ ∑ ∑ ∑ A = –––––––––––––––––––– 1 1 1 = = = ? ? – ? ? ? ∑ ∑ ∑ N N N i i i i i i i N x y x y and B = –––––––––––––––––––– ...(4.4.14) 2 2 1 1 = = ? ? – ? ? ? ? ∑ ∑ N N i i i i N x x

SA PG_M.Sc._Physics_345 12_Mathematical physics-I (1).pdf (D111988815)

64/115	SUBMITTED TEXT	28 WORDS	71%	MATCHING TEXT	28 WORDS

P N = ( ) 2 2 2 2 1 2 .... N P P P N P N + + + – ⟨ ⟩ = 2 2 1 1 N i i P N P N = ? ? – ⟨ ⟩ ? ? ? ? ∑ ...(4.5.2) Here ∆ P

SA MPDSC 1.2 Mathematical Methods of Physics.pdf (D133919731)

65/115	SUBMITTED TEXT	89 WORDS	39%	MATCHING TEXT	89 WORDS

we obtain 2 1 1 1 1 0 2 2 1 1 N N N N i i i i i i i i i N N i i i i x y x x y A R N x x = = = = = = – = = ? ? – ? ? ? ? ∑ ∑ ∑ ∑ ∑ ∑ 1 1 1 0 2 2 1 1 N N N i i i i i i i N N i i i i N x y x y B R N x x = = = = = – – = α = ? ? – ? ? ? ? ∑ ∑ ∑ ∑ ∑ Here, 5 2 1 i i x = ∑ = 22000 ⇒ 5 2 1 5 i i x = ∑ = 110000 2 5 5 1 1 300 90000 i i i i x x = = ? ? = ⇒ = ? ? ? ? ∑ ∑ 5 1 637 i i y = = ∑ 5 1 40160 i i i x

SA 019E1130_Mathematical Physics.pdf (D165097245)

66/115	SUBMITTED TEXT	32 WORDS	57%	MATCHING TEXT	32 WORDS

A A A ∆ = – 2 2 A A A ∆ = – .... n n A A A ∆ = – So, absolute error [ ] 1 2 1 ..... n A A A A n ∆ = ∆ +∆ + +∆

SA 019E1130_Mathematical Physics.pdf (D165097245)

67/115	SUBMITTED TEXT	12 WORDS	100%	MATCHING TEXT	12 WORDS

Conservation of Linear momentum 6.6.5 Conservation of angular momentum 6.6.6 Conservation of energy 6.7

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

68/115	SUBMITTED TEXT	15 WORDS	90%	MATCHING TEXT	15 WORDS

Motion along a specified path is the simplest example of a constrained motion. Imposing constraints

SA MSCPH-502.pdf (D165443984)

69/115	SUBMITTED TEXT	39 WORDS	74%	MATCHING TEXT	39 WORDS

dynamical system becomes smaller when constraints are present in the system. Hence the degree of freedom of a dynamical system is defined as the minimum number of independent coordinates required to simplify the system completely along with the constraints. Thus if

SA MSCPH-502.pdf (D165443984)

70/115	SUBMITTED TEXT	45 WORDS	55%	MATCHING TEXT	45 WORDS

to choose a suitable set of generalised co-ordinates in a given situation? By three principles, we can solve the problem : (i) Their values determine the configuration of the system. (ii) They may be varied arbitrarily and independently of each other, without violating the constraints on the system. (iii)

SA 019E1110_Classical & Statistical Mechanics_27-03-23.pdf (D165097242)

71/115	SUBMITTED TEXT	31 WORDS	100%	MATCHING TEXT	31 WORDS

x 1 , y 1 , z 1 ; ....x n , y n , z n , t)

SA MSCPH-502.pdf (D165443984)

72/115	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

This method is based on the principle of virtual work.

SA MSCPH-502.pdf (D165443984)

73/115 **SUBMITTED TEXT** 30 WORDS **77%** **MATCHING TEXT** 30 WORDS

δ - δ = ∂ ∂ ∑ ∑ j j i i i j i j j j i i r r F q P q q q or, · 0 → → ∂ ∂ - δ = ∂ ∑ ∑ j i j j i j j j i j r Q q P q q [

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

74/115 **SUBMITTED TEXT** 296 WORDS **31%** **MATCHING TEXT** 296 WORDS

j j j i i r r P q m r q q q = · · · → → → → ? ? ? ? ∂ ? ? ∂ ? ? δ - δ ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ? ? ∑ ∑ ɺ ɺ j j j i i i j i j j j i i r r d d m r q m r q dt q dt q ... (6.5.7) Now, · → → → ? ? ? ? ? ? ∂ ∂ ∂ ? ? ? ? ? ? δ = + ? ? ? ? ? ? ? ∂ δ ∂ ∂ ? ? ? ? ? ? ? ? ? ? ? ? ? ∑ i i i K j K j j K r r r dq d d dt q q q dt dt q = 2 → → ? ? ∂ ∂ ∂ ? ? + ? ? ∂ ∂ ∂ δ ? ? ? ? ∑ i k i k j j K r q r d q q t dt q = → → ? ? ∂ ∂ ∂ ∂ ? ? + ? ? ∂ ∂ ∂ ∂ ? ? ? ? ∑ i k i j k K r q r q q t t 158 NSOU l CC-PH-07 = → ∂ ∂ i j r q ɺ [from equation (6.5.4)] or, → → ? ? ∂ ∂ ? ? = ? ? ∂ ∂ ? ? ? ? i i j j r v d dt q q From equation (6.5.4) we get, → → → ∂ ∂ = + ∂ ∂ ∑ i i i j j j v r v q q t ɺ and, → → ∂ ∂ = ∂ ∂ ɺ i i j j v r q q Hence, equation (6.5.7) becomes · · · · → → → → → → ? ? ? ? ∂ ∂ ∂ ? ? ? ? ? δ = - δ ? ? ? ? ∂ ∂ ∂ ? ? ? ? ? ? ? ? ? ∑ ∑ ɺ ɺ ɺ j j j i i i i j i i i j j j i i v v v d P q mv m v m v q q dt q q = 2 2 1 1 2 2 ? ? ? ? ? ? ? ? ? ? ? ∂ ? ? - δ ? ? ? ? ? ? ? ? ? ? ? ∂ ? ? ? ? ? ? ? ? ? ? ? ? ∑ ∑ ∑ ɺ j i i i i j j j i i i d d m v m v q dt dq q = ? ? ? ? ∂ ∂ - δ ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ɺ j j j j i d T T q dt q q where 2 1 · 2 = ∑ i i T m v represents the kinetic energy of i the system.

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

75/115 **SUBMITTED TEXT** 35 WORDS **59%** **MATCHING TEXT** 35 WORDS

j j j j i j d T T Q q q dt q q NSOU l CC-PH-07 159 or, 0 ? ? ? ? ∂ ∂ - - δ = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ∑ j j j j j d T T q Q dt q q

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

76/115 **SUBMITTED TEXT** 24 WORDS **77%** **MATCHING TEXT** 24 WORDS

j j j d T T Q dt q q or, ? ? ? ? ∂ ∂ - = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ∑ ∑ j j j j d T T Q dt q q ... (6.5.8)

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77/115 **SUBMITTED TEXT** 41 WORDS **53%** **MATCHING TEXT** 41 WORDS

j i j j i i r r Q F V q q = · · → → ∂ ∂ ∂ - = - ∂ ∂ ∂ ∑ i j j i i r V V q q r Equation (6.5.8) becomes ? ? ? ?∂ ∂ ∂ - = - ? ? ? ? ∂ ∂ ∂ ? ? ? ? ∑ ∑ ᴊ j j i j d T T V dt q q

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78/115 **SUBMITTED TEXT** 56 WORDS **41%** **MATCHING TEXT** 56 WORDS

j j j V d V Q q dt q ....(6.5.10) Putting the equation (6.5.10) in equation (6.5.8) we obtain, ? ? ? ? ? ? ∂ ∂ ∂ ∂ - = - + ? ? ? ? ? ? ∂ ∂ ∂ ∂ ? ? ? ? ? ? ? ? ∑ ∑ ∑ ᴊ ᴊ ᴊ j j j j j j j d T T V d V dt q q q dt q

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79/115 **SUBMITTED TEXT** 9 WORDS **100%** **MATCHING TEXT** 9 WORDS

j j d L L dt q q ... (6.5.10) where L

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80/115 **SUBMITTED TEXT** 13 WORDS **100%** **MATCHING TEXT** 13 WORDS

The angle θ between rest position and deflected position is chosen as

SA MSCPH-502.pdf (D165443984)

81/115 **SUBMITTED TEXT** 50 WORDS **40%** **MATCHING TEXT** 50 WORDS

j j j j j j L L r L r r r = · · → → → → ? ? ? ? ∂ ∂ ∂ δ δ + ? ? ? ? ∂ ∂ ? ? ∑ ∑ ᴊ ᴊ j j j j j j L L r r dt r r = · → → → → → → ? ? ? ? ? ? ? ? ∂ ∂ ∂ φ× φ× + ? ? ? ? ? ? δ ? ? ? ? ? ? ∂ ? ? ∑ ∑ ᴊ j j j j j j

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

82/115 **SUBMITTED TEXT** 12 WORDS **100%** **MATCHING TEXT** 12 WORDS

n͡ is the unit vector along the direction of translation,

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83/115 **SUBMITTED TEXT** 36 WORDS **32%** **MATCHING TEXT** 36 WORDS

equation (6.6.3) we get, · → → ∂ ∂ = ∂ ∂ ∑ ∑ Ɉ i i j j r T P q q = ˆ · i i P n r → → ? ? × ? ? ? ? ∑ = · i i n r P → → → ? ? × ? ? ? ? ∑ = ˆ ·n L → Hence j T q ∂ ∂ ∑ Ɉ represent the total angular momentum along the

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84/115 **SUBMITTED TEXT** 17 WORDS **76%** **MATCHING TEXT** 17 WORDS

in equation (6.6.2) we get, ˆ · j j d L n L dt q → ? ? ∂ = ? ? ∂ ? ? ∑ If q j

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85/115 **SUBMITTED TEXT** 49 WORDS **53%** **MATCHING TEXT** 49 WORDS

j j j j dL L L q q dt q q = j j j j j j d L L q q dt q q ? ? ∂ ∂ + ? ? ∂ ∂ ? ? ∑ ∑ Ɉ ɈɈ Ɉ Ɉ ? ? ? ? ∂ ∂ = ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∵ Ɉ j j d L L dt q q = j j j d L q dt q ? ? ∂ ? ? ∂ ? ? ∑ Ɉ Ɉ = 0 j j j d T q dt q ? ? ∂ = ? ? ∂ ? ? ∑

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86/115 **SUBMITTED TEXT** 12 WORDS **95%** **MATCHING TEXT** 12 WORDS

sum of kinetic energy and potential energy i.e., total energy of a

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87/115 **SUBMITTED TEXT** 18 WORDS **61%** **MATCHING TEXT** 18 WORDS

is function of generalised coordinates, generalised momenta and time i.e., ( ) , , = j j H H q P t .

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88/115 **SUBMITTED TEXT** 13 WORDS **83%** **MATCHING TEXT** 13 WORDS

j i j j j j H H q P q P L

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89/115 **SUBMITTED TEXT** 14 WORDS **71%** **MATCHING TEXT** 14 WORDS

H may be a constant of motion but not the total energy of the

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90/115 **SUBMITTED TEXT** 20 WORDS **87%** **MATCHING TEXT** 20 WORDS

t 1 where T is the kinetic energy and V is the potential energy of the system.

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91/115 **SUBMITTED TEXT** 22 WORDS **85%** **MATCHING TEXT** 22 WORDS

that the motion of the system from time t 1 to time t 2 is such that lime integral 2 1

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92/115 **SUBMITTED TEXT** 21 WORDS **55%** **MATCHING TEXT** 21 WORDS

j j t l L q q dt = ∫ ⌡ = ( ) ( ) 2 1 , t j j j t T q q V q

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93/115 **SUBMITTED TEXT** 80 WORDS **34%** **MATCHING TEXT** 80 WORDS

j j j j t T T V q q q dt q q q or, 2 2 1 1 ( ) 0 ∂ - ∂ δ + δ = ∂ ∂ ∑ ∑ ∫ ∫ ⌡ ⌡ t t j j j j j j t t T V T q dt q dt q q or, 2 2 1 1 ( ) t t j j j j j j t t T V d T q dt q dt q dt q ? ? ∂ - ∂ ? ? δ + δ ∂ ∂ ? ? ? ? ∑ ∑ ∫ ∫ ⌡ – 2 1 0 ? ? ∂ ? ? ∂ = ∂ ? ? ? ? ∑ ∫ ⌡ t j j j t d t q dt dt q or, ( ) 2 1 t j j j t T V q dt q ∂ - ∂ ∂ ∑ ∫ + 2 1 2 1 0 | t j j j j j j t t T d T q q dt q dt q

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94/115 **SUBMITTED TEXT** 44 WORDS **35%** **MATCHING TEXT** 44 WORDS

t ? ? ∂ ∂ δ - δ = ? ? ∂ ∂ ? ? ∑ ∑ ∫ ⌡ ⌡ Since there is no co-ordiante variation at end points, so 2 0 1 | t t q j = δ and above equation reduces to 2 2 1 1 ( ) . 0 ? ? - ∂ δ - δ = ? ? ? ? ∑ ∑ ∫ ∫ j t t j j j j j t t d T

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95/115 **SUBMITTED TEXT** 18 WORDS **61%** **MATCHING TEXT** 18 WORDS

t j j j j t d T V d T q dt dq dt dq ? ? ? ? - ∂ - δ = ? ? ? ? ? ? ? ? ? ? ∑ ∫

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96/115 **SUBMITTED TEXT** 13 WORDS **75%** **MATCHING TEXT** 13 WORDS

j j j j j j H H H dH dq dP dt q P t ....(6.7.6)

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97/115 **SUBMITTED TEXT** 18 WORDS **80%** **MATCHING TEXT** 18 WORDS

j j j jj j j j j j j j j L L dH q dp P dq dq dq q t ∂ - ∂

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98/115 **SUBMITTED TEXT** 20 WORDS **82%** **MATCHING TEXT** 20 WORDS

j j j j j j j j j j j L dH q dP P dq P dq P dq dt

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99/115 **SUBMITTED TEXT** 28 WORDS **47%** **MATCHING TEXT** 28 WORDS

j j j j j j L q dP P dq dt t ... (6.7.8) Compairing the coefficients of equations (6.7.6) and (6.7.8) we get, and ∂ ? = ? ∂ ? ? ∂ ? = - ∂ ? ? ⌡ ⌡ j j j j H q P H P q ... (6.7.9) L H t t ∂ ∂ - = ∂ ∂ ... (6.7.10)

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100/115 **SUBMITTED TEXT** 68 WORDS **29%** **MATCHING TEXT** 68 WORDS

x y P P m P P k x k y m m + - + + + = 2 2 2 2 1 2 1 1 2 2 2 2 y x P P k x k y m m + + + Hence, Hamilton’s Canonical equations are ; x x P H x P m ∂ = = ∂ ⅃ ; y y P H y P m ∂ = = ∂ ⅃ 1 ; x x H p k x P ∂ = - = - ∂ ⅃ 2 y H p k y y ∂ = - = - ∂ ⅃ or, 1 = - = - ⅃ ⅃⅃ x p k x mx ; 2 = - = y p k y

SA ThesisPre.pdf (D14803032)

101/115 **SUBMITTED TEXT** 41 WORDS **30%** **MATCHING TEXT** 41 WORDS

L P ml θ ∂ = = θ ∂θ ⅃ ⅃ Now, ( ) 2 2 2 2 1 1 cos 2 H P L ml ml mgl θ = θ- = θ - θ + - θ ⅃ ⅃ ⅃ = ( ) 2 2 1 cos 2 ml mgl 1 θ + - θ ⅃ = ( ) 2 2 1 cos 2 θ + - θ P mgl ml Hamilton’s Canonical

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102/115 **SUBMITTED TEXT** 27 WORDS **62%** **MATCHING TEXT** 27 WORDS

T m x y z = + + ⅃ ⅃ ⅃ ∴ Lagrangian L= T – V = ( ) 2 2 2 1 2 m x y z + +

SA Chapter 4 _Proposed Book.pdf (D113949991)

103/115 **SUBMITTED TEXT** 50 WORDS **36%** **MATCHING TEXT** 50 WORDS

P x P y P z L = + + - ⅃ ⅃ ⅃ = 2 2 2 2 2 2 2 2 1 2 y y x x z z P P P P P P m m m m m m m n ? ? ? ? + + - + + ? ? ? ? ∴ ( ) 2 2 2 1 2 x y z H P P P

SA MPDSC 1.1 Classical Mechanics.pdf (D133919389)

104/115 **SUBMITTED TEXT** 11 WORDS **90%** **MATCHING TEXT** 11 WORDS

j j i W Q q δ = δ ∑ where 1 · → → = ∂ = ∂ ∑ N i j i j i r Q F q

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105/115 **SUBMITTED TEXT** 11 WORDS **71%** **MATCHING TEXT** 11 WORDS

r r r r r Q F F r F r F r r → →

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106/115 **SUBMITTED TEXT** 14 WORDS **55%** **MATCHING TEXT** 14 WORDS

n r r q q q t → → = So, 1 n i i i i j j j r r r r q t

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107/115 **SUBMITTED TEXT** 57 WORDS **33%** **MATCHING TEXT** 57 WORDS

j r r d r q dt q t → → → = ? ? ∂ ∂ ? ? = + ? ? ∂ ∂ ? ? ? ? ∑ ⅃⅃ ⅃ = 1 1 n n i i i j j i i j j r r r d q q dt q q t → → → = = ? ? ∂ ∂ ∂ ? ? + + ? ? ∂ ∂ ∂ ? ? ? ? ∑ ∑ ⅃ ⅃ ⅃⅃ = 1 1 n n i i i j j i i j j r r r q q q q

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108/115 **SUBMITTED TEXT** 45 WORDS **32%** **MATCHING TEXT** 45 WORDS

j k j i k i j k j r r q q q q q q t + 2 1 → = ∂ ∂ ∑ ⅃⅃ n i j j j r q q + 2 1 n i j j j r q q dt → = ∂ ∂ ∑ ⅃ + 1 n i j i j r q q → = ∂ ∂ ∑ ⅃⅃ + 2 2 i r t → ∂ ∂ = 2 2 1 1 1 2 · → → = = = ∂ ∂ + ∂ ∂ ∂ ∂ ∑ ∑ ∑ ⅃ ⅃ ⅃ n n n i i j k j i k j j k j r r q q q q q q t + 1 2 → = ∂ ∂ ∑ ⅃⅃ n i j j j

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

109/115 **SUBMITTED TEXT** 39 WORDS **42%** **MATCHING TEXT** 39 WORDS

j j j j x f f l q q dx q q = 2 2 1 1 ? ? ? ? ∂ ∂ ∂ + δ ? ? ? ? ∂ ∂ ? ? ? ? ? ? ∑ ∑ ∫ ∫ ⅃ x x j j j j j j x x f f d q q dx q dx q 2 1 ? ? ∂ - δ ? ? ∂ ? ? ∑ ∫ ⅃ x j j j

SA M. Sc. I Classical Mechanics all.pdf (D142231111)

110/115 **SUBMITTED TEXT** 10 WORDS **83%** **MATCHING TEXT** 10 WORDS

State Hamilton’s principle and derive Lagrange eqnation of motion from it.

SA MSCPH-502.pdf (D165443984)

111/115 **SUBMITTED TEXT** 49 WORDS **69%** **MATCHING TEXT** 49 WORDS

x x x x y z x A A A v A v A v v v x x y z → → → → → ? ? ? ? ? ? ∂ ∂ ∂ ? ? ∂ x ∇x = - + + ? ? ? ? ? ? ? ? ? ∂ ∂ ∂ ∂ ? ? ? ? ? ? ? ? ? ? 188

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112/115 **SUBMITTED TEXT** 53 WORDS **67%** **MATCHING TEXT** 53 WORDS

x x x x x x y z dA A A A A v v v dt t x y z ∂ ∂ ∂ ∂ ? ? = + + + ? ? ∂ ∂ ∂ ∂ ? ? Hence, · x x x dA A v A v A x dt t → → → → → ? ? ? ? ∂ ∂ x∇x = - + ? ? ? ? ? ∂ ∂ ? ? ? ?

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113/115 **SUBMITTED TEXT** 158 WORDS **25%** **MATCHING TEXT** 158 WORDS

x x x x A dA A F q v A x t x dt dt → → ? ? ? ? ∂ ∂ ∂φ ∂ = - - + - + ? ? ? ? ∂ ∂ ∂ ? ? ? ? ? ? ? = ( ) . x dA d q v A dx dt ? ? - ϕ- - ? ? ? ? Now, · x x v A A v → → ? ? ∂ = ? ? ∂ ? ? , Since 0 x x A v ∂ = ∂ = · · x d q v A v A x dt v → → → → ? ? ? ? ? ? ∂ ∂ - φ- - ? ? ? ? ? ? ∂ ∂ ? ? ? ? ? ? ? ? ? As φ is independent of v x, Hence we can write . . → → → → ? ? ? ? φ- φ- ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ∂ ∂ = - + ? ? ∂ ∂ ? ? ? ? v A v A x x d F q x dt v Let us put · V q v A → → ? ? = φ- ? ? ? ? , then x x V d V F q x dt v ? ? ∂ ∂ = - + ? ? ∂ ∂ ? ?

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114/115 **SUBMITTED TEXT** 64 WORDS **25%** **MATCHING TEXT** 64 WORDS

x x d T T V d V F dt v v v dt v NSOU l CC-PH-07 189 or, ( ) ( ) 0 ?∂ - ? ?∂ - ? - = ? ? ? ? ∂ ∂ ? ? ? ? x T V T V d dt v x or, 0 ? ? ∂ ∂ - = ? ? ∂ ∂ ? ?x d L L dt v x Where L = T – V is the Lagrangian (as mentioned earlier) ∴ L = T –

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115/115 **SUBMITTED TEXT** 11 WORDS **83%** **MATCHING TEXT** 11 WORDS

the Lagrangian for a charged particle in an electromagnetic field. 9.What is

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PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, generic, discipline specific, elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Material (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Ranjan Chakrabarti Vice-Chancellor
First Print : November, 2022 Printed in accordance with the regulations of the Distance Education Bureau of the University Grants Commission. Netaji Subhas Open University Under Graduate Degree Programme Choice Based Credit System (CBCS) Subject : Honours in Physics (HPH) Course : Electricity & Magnetism Course Code : CC-PH-08 Professor Kajal De (Chairperson) Director, School of Sciences NSOU Dr. Gahul Amin Associate Professor of Physics NSOU Dr. Goutam Kumar Mallik Associate Professor of Physics NSOU Mr. Pranab Nath Mallik Associate Professor of Physics NSOU Board of Studies Members Dr. Rupayan Bhattacharya Retd. Principal, Gurudas College Dr. Goutam Gangopadhyay Professor of Physics University of Calcutta Dr. Amit Kumar Chakraborty Associate Professor of Physics National Institute of Technology Dr. Subhratanu Bhattacharya Assistant Professor of Physics Kalyani University Dr. Manik Sanyal Associate Professor of Physics Barasat Govt. College Netaji Subhas Open University Under Graduate Degree Programme Choice Based Credit System (CBCS) Subject : Honours in Physics (HPH) Course : Electricity & Magnetism Course Code : CC-PH-08 : Course Editor : Dr. Goutam Kumar Mallik NSOU Mr. Pranab Nath Mallik NSOU Notification

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UNIT 1 : Electric Field and Electric Potential 1.1 Objective 1.2 Introductions 1.3 Electrostatiocs in Vacucm 1.4 Electrostatic potential 1.5 Multipole expansion of electrostatic potential 1.6 Gauss's Theorem and its application– 1.7 Laplace and Poisson's equation. 1.8 Electrostatic energy 1.9 Conductors in electric field 1.10 Capacitors 1.11 Electrical Image 1.12 Summary 1.13 Review question and answer 1.14 Problems and solution 1.1 Objective After completing this unit you will be able to understand 1. Electrostatic interaction between charges through Coulomb's law. 2. Electric field conception to explain the propagation of interaction by introducing field lines conception. 3. A vector presentation of electric field through the introduction of electric field intensity conception-a vector representation of electric field in space. 4. Electric flux, Gauss's theorem and its application. 5. Presentation of electric property by a scalar field conception through introduction of electric potential. 6. Presentation of electric field intensity ?? E as a gradient of electric potential V. . Equipotential surfaces.

NSOU ? CC-PH-08 ? 8 7. Conservative nature of electric field, Laplace's and Possion's Equations. 8. Energy assoicated with a symmertic charge distribution. 9. Capacitance of capacitors. 10. Electrical image and its application to some specific cases. 1.2 Introduction The term 'electricity' started its path from the experiment of Thales (600 BC)-Greek philosopher who rubbed Amber with silk and it was seen both of them developed the property of attracting small papers bits. As the Greeks called Amber as electron, so the term electricity boiled down. Electricity was in its rudimentary state still late 18 th century until, about 100 years after the introduction of Newton's Law of Gravitation (1687), Coulomb in 1785 AD, introduced the law governing the interaction between the charges – the subject electricity got its space. Atom, the basic ingredient of matter contains two charged particles called electron (negatively charged) and proton (positively charged) carrying equal but opposite charges of magnitude 1.6 × 10 19 Coulomb each, which is the smallest quantum of charge that can exist in nature. Obviously, Any charge that physically exists will be the integral multiple of the smallest quantum of charge - the magnitude of the charge of an electron, this is known as quantisation of charge. The charge also follows another law called conservation of charge which goes as, The total charge of an isolated system remains conserved. but this conservation is not like mass conservation law which changes with the speed of reference frame. Charge conservation law is independent of reference frame 1.3 Electrostatics in Vacuum (a) Coulomb's Law and Electric Field : Coulomb's law gives the interaction between the two static point charges. The law states that the force of interaction between two point charges separated by a distance

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is, i) directly proportional to the product of the charges, NSOU ? CC-PH-08 ? 9 ii) inversely proportional to the square of the distance of separation between the charges, iii) action along the line joining the

charges.
The
fig.-1.1
shows two static point charged particles q 1 and at position vectors 1 ?? r and ?? j r respectively. Then according to The Coulomb's law, the force on j th particle due to i th particle will be, ? ? 2 ˆ ... ... 13.1 ? ?? i j ji ij ij q q F

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r r Where ; ? ? ? ? ? ji j i r r r ? ? ? ? j i ji r r r and ? ? ˆ? ? ? ? ? ? ? j i j i ji r r r r r = unit vector along ? ? ? ? ? j i r r Similarly, the

force on i th particle due to j th particle will be 2 ˆ ? ? ? ? ?? ?? i j ij ji ij ij q q F F r r We can write eqn, (1.1) as, ? ? 2 ˆ, ... 1.13.2 ? ?? i j ij ij ij q q F k r r Where k is a constantthat, depends on the intervening space and choice of unit. In this book, we will use the SI unit for wide acceptance of this unit over the globe. In this unit k = 9 0 1 4 9 10 ?? ? ? Nm 2 c -2 . ? 0 is known the permittivity of vacuum. The value ? 0 = 8.8542 × 10 -12 C 2 N -1 m -2 . In SI unit the unit charge is that charge which when placed at 1m away from an identical charge in vacuum the force of interaction is 9 × 10 9 N. This unit charged is referred as Coulomb. If there are N number of particles bearing charge q 1 , q 2 , q 3 ......at poisiton vectors 1 2 3 , , ,.........., ? ? ? r r r respectively then, the total force on i th particle q i q j ?? i r ?? j r Fig. 1.1

NSOU ? CC-PH-08 ? 10 ? ? 2 1 ˆ... ... ... 1.13.3 ? ? ? ? ? ? ?? j N i j i ij j ij j i q q F k r r Equation (1.3) explains that, the superposition Principle is applicable for this electric interaction. This means that force on a charged particle is the vector sum of the forces due to all other charged particles. Problem - 1 A conductor possesses 80 ?C of positive charge. How many electrons does it have in deficit or in excess? Solution - 1 To get this positive charge it has to liberate electrons. As each electron has magnitude of charge 1.6 × 10 -19 C, so the deficit of number electron n = 80 × 10 -6 /1.6 × 10 -19 = 5 × 10 13 Problem - 2 Four point charges each of + 10 ?C is placed at (3m, 0, 0), (-3m, 0, 0) and (0, -3m, 0). Find the force on a charge 10 ?C placed at (0, 4m, 0) Problem - 3 (b) The Electric Field It is obvious that electric interaction is a distant force, which means that electric interaction may migrate through space without any physical contact. Now two questions arise i) Who is the carrier of this interaction? ii) With what speed the interaction travels. To resolve the first question, we introduce the conception, what is known as electric field? This field migrates with the speed of light. Due to the presence of charge, a quality in space is developed, which is known as electric field. In case of static charge, only electric field is developed but for dynamic charge magnetic field is also developed. The interaction between the charges takes place with this field obeying Coluomb's law without an material interaction. A space is said to possesses electric field

NSOU ? CC-PH-08 ? 11 The magnitude of the field is named as 'electric field intensity' ? ? ?? E and is defined as : The electric field intensity

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at a point is the force experienced by a unit positive charge placed at that point 1.

Field Due to A Point Charge The fig. (1.2) shows a point charge q is placed at the origin '0' of the reference frame. To calculate the field intensity atpoint p at position vector , ? r we place a test charge dq at p. (The charge dq is so small that it does not put any distortion of field pattern of q. Now from Coulomb's law the force on the charge dq is, ? ? 2 0 1 ˆ... ... 1.3.3 4 ? ?? ?? qdq F r r Therefore, the electric field intensity at p is, ? ? 2 0 1 ˆ... ... 1.3.4 4 ? ?? ?? ?? q F E r dq r This shows that the field pattern of a point charge is spherically symmetric but decreasing with square of the distance from the point charge. If 1 2 3 , , ... ?? ?? ?? E E E Calculation of electric potential and hence field intensity (a) For a point charge The fig. (1) shows a point charge q at origin 0 of referene frame. To find the potential at P at position r ? vector we proceed as follows.

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The electric field intensity at P due to + q charge at 0 0 1 ˆ , 4 2

q E r ? ?? ?? if V is the potential at P then, Then the work done in transferring a unit charge from ?? ? ? , P r toQ r dr ? ? ? ? 2 0 0 1 1 ˆ . . , 4 4 2 q q dV E dr r dr dr r ? ? ? ? ? ? ?? ?? ?? ? ? so the potential at P which is the work done to carry a unit charge from infinity to the point quasi-statically,

NSOU ? CC-PH-08 ? 12 ? ? 2 0 0 1 1 ... ... 13.5 4 4 r q q V dr r r ? ? ? ? ? ? ?? ?? ? As V is a function of r only 2 0 1 along 0P 4 q V r r ? ? ?? ? ? ?? or, 2 0 1 ˆ, 4 q E r r ? ?? ?? which is in exact coincidence with the previous result. (b) For a uniformly charged circular ring The fig (1.4) shows a uniform circular ring of radius a. + q Chargedistributed uniformly over the ring. We have to find out the potential at P at a distance x from centre 0 of the ring. ? be the charge per unit length on the ring. Consider an element charge ?dl at A (in fig) Then potential at P due this element of charge, 0 1 , 4 dl dV r ? ? ?? So the total potential at P du to the whole ring ? ? 2 2 0 0 0 0 1 1 1 1 ... ... 1.3.6 4 4 4 4 dl q q dl V r r r a x ? ? ? ? ? ? ? ?? ?? ?? ?? ? ? ? ? ? So the intensity along x axis (since V = V (x)) ? ? 3 2 2 2 0 1 along 0P 4 qx V E x a x ? ? ? ? ? ?? ? ? ? ? ? 3 2 2 2 0 1 ... ... 1.3.7 4 qx E a x ? ?? ? ? ?? (c) For uniformly charged disc The fig (1.5) shows a uniformly charged disc of radius R and of charge density ?Cm −2 .

To calculate

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the electric field intensity E, at point P at a distance x from 0 the centre of

the
disc we consider an elemental ring of radius r and thickness dr as in fig ( ). The area of the elemental ring = 2 ??r dr NSOU ? CC-PH-08 ? 13 The charge of the elemental ring dq = 2 ?r dr ? So the potential due this elemental ring at P, ? ? 1 2 2 2 0 0 1 2 4 4 rdr dV r x ? ? ? ? ?? ?? ? We put tan , r x ? ? ? 2 sec , dr x d ? ? ? so, ? ? ? ? 2 0 sin sec 2 dV x d ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 0 0 0 0 1 sin sec 1 ... ... 1.3.8 2 2 cos 2 x V x d R x x ? ? ? ?? As V is a function of x only, So ? ? ? ? 2 2 2 0 0 0 1 1 cos 1 cos 2 2 2 q V x E x R R x ?? ? ? ? ? ? along 0P ... ... (1.3.9) Alternative method Field at P due this elemental ring ? ? 3 2 2 2 0 1 2 4 rdr x dE r x ? ? ? ? ?? ? (ref. eqn no. (1.10)) ? ? 3 3 2 3 2 2 2 2 0 0 1 2 1 2 cos 4 4 rdr x rdr x x r x ? ? ? ? ? ? ? ? ?? ?? ? Now, tan , r x ? ? so, 2 sec dr x d ? ? ? So the above equation boils to, 2 3 0 0 1 1 2 tan sec cos 2 sin 4 4 dE d d ? ? ? ? ?? ? ? ? ? ?? ? ? ? ?? ?? Thus the total electric field at P, ? ? ? ? 1 2 0 0 0 1 cos 1 1 2 sin 2 1 cos 4 2 q E d a ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ? ?? ?? ? along 0P, same as eqn. .... (1.3.10).
NSOU ? CC-PH-08 ? 14 (d) For a uniformly charged spherical shell : The fig (1.6) shows a spherical shell of radius R carrying a charge q uniformly distributed over the sphere. We have to find out the potential at point P at a distane x as shown in fig (1.6) We consider an elemental ring within the angle ? and ? + d ? as in fig (1.6) The area of the elemental ring = 2 ?R sin ? (Rd ?), ? be the charge per unit area. Therefore the charge on the elemental ring ? ? 2 sin R Rd ? ?? ? ? The potential at P due to this ring ? ? 0 2 sin 1 4 R Rd dV r ?? ? ? ? ? ?? Now from the fig. (1.6) 2 2 3 2 cos r R x Rx ? ? ? ? or 2 2 sin rdr Rx d ? ? ? sind dr r Rx ? ? ? ? Therefore the above equation can be written as 0 1 2 4 R dV dr x ?? ? ?? So 0 1 2 , 4 R V dr x ?? ? ?? ? where the integration is to be carried out with proper limit. When the point P is outside sphere ? ? 2 0 0 0 1 2 1 4 1 ... 1.3.11 4 4 4 x R x R q R R V dr x x x ? ? ?? ?? ? ? ? ? ?? ?? ?? ? Be the intensity at a point due to a discrete distribution charges at a point, thenform principle of superposition the net field at that point will be 1 2 3 .... E E E E ? ? ? ?? ?? ?? ?? (a) Field Due To A Continuous Chage Distribution So far we have confined to discrete charge distribution, but in macroscopic world we frequently encounter the cases where charge distribution is continuous for example a charged metallic body. + q 0 P Fig. 1.3 r ? +
NSOU ? CC-PH-08 ? 15 To calculate the field intensity at the point p due to such a continuous charge distribution. We usually follow this procedure. We have taken an element of charge dq as in fig (1.7) The field at p due to this elemental charge, 2 0 1 4 dq dE r r ? ? ? ?? ?? So the net field at the point p will be, ? ? 2 0 1 ... ... 1.3.1 4 dq E d E r r ? ? ? ?? ? ? ? ?? ?? ? Where the intagration is to be carried out over the entire charge distribution. Now in case of line distribution of charge dq ? ??dl, where ??is the charge per unit length on the line element dl. In case of surface distribution of charge dq ? ??ds, where ??is the charge density over the surface element ds. For a volume distribution of charge dq ???? dv. Electric Lines Of Force Or Field Lines To give as visual representation of electric field the conception electric lines of force (now referred as field lines) was introduced by Michael Faraday. The electric field line is an imaginary line drawn in the space containing electric field such that tangent at any point on the line is along the direction of electric field at that point. The field lines bear the following properties : a) They emanate from a positive charge and end up to a negative charge or to in finity. b) They try to contract lengthwise and repel each other laterally. c) Field lines can't intersect each other. d) The number of field lines associated with a charge is finite and proportional to the magnitude of charge. e) The magnitude of field intensity at a point is the number of field lines passing per unit area when the area is placed perpendicular to the field lines. (c) Conservative Nature Of Electric Field & Electric Potential Now let us explore how this force field is related to energy and work. We have already seen how the electric interaction in a region can be represented by the vector ,E ?? the electric field intensity. Can we represent this field in terms of a scalar called electric dq dE P Fig. 1.1
NSOU ? CC-PH-08 ? 16 potential? The answer to this poser is yes. We have already seen that electric field due to a static point charge is given by the equation ? ? 0 3 0 0 1 4 q E

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r r r r ? ? ?? ? ?? ? ?? ? ? ? ? So, ? ? 0 3 0 0 1 4 q E r r r r ? ? ? ? ?? ? ?? ? ? ? ?? ? ? ? ? ? ?? ? ? ? ? So, ? ? ? ? ? ? 0 3 0 0 1 4 q E r r G R R G R R G r r ? ? ? ? ? ?? ? ?? ? ? ?? ? ?? ?? ? ? ? ?? ? ? ? ? ? ?? ? ? ? ?? ?? ? ? ? ?? , R R ? ?? where ? ? 0 R r r ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 0 0 0 i j k G R R G R R G R G R R

x y z x x y y z z ? ? ? ?? ?? ? ? ?? ? ? ? ? ? ? ? ? ?? ?? ?? So electric field is non-rotational. As curl of a gradient of a scalar is always zero i.e., 0 0 E V ?? ? ???? ? ?? ? ? Therefore, we can write ? ? ... 1.3.2 E V ? ?? ?? The negative sign is to carry on a logical convention that work is done in quest of electrostatic energy. From eqn. 1.3.2 It is obvious that E ?? remains same if V is replaced by V + c (const). Thus, an absoulate value of potential bears indeterminacy. It depends on the choice of origin. As the electric field is zero at an infinite distance from a charge, we usually refer this point to be of zero potential. With this choice we can put c ? 0. However, this constant is not so important as it does not affect the force field. Now we take a migraction of a unit + ve charge from the space point I along the loop i I f II i ? ? ? ? then the work done, O r ? ? ? r o ? q r ? r o ? ? P Fig. 1.2
NSOU ? CC-PH-08 ? 17 . . 0 ilflli W E dr dr d ? ? ??? ? ? ? ? ? ? ? ?? ? ? ? ? ? Thus log . . 0, f i i f a ilf along flli E dr E dr ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ?? ? So, . . f f i i alongilf along flli E dr E dr ?? ? ?? ? that shows work done along the path i i I f ? ? and i II f ? ? are same. Thus, the work done is independent of path and depends on the initial and final position. As work done is energy concerned, so in this force field, the work done depend on some energy function which solely depends on the energy of initial and final position. Such work which is a function of position is known as potential energy, here referred as electric potential or potential energy per unit charge and is represented by V.

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The electric potential at a point is the work done to bring a unit positive charge from infinity up to that point quasistatically. 1.4 The

electrostatic
potential It is defined as the amount of work energy needed to move a unit of electric charge from a reference point to a particular point in an electric field, precisely, it is the energy per unit charge for a test charge that is so small that the disturbance of the field under comsideration is negligible The electric potential at a point r in a static electric field E ?? is given by the integral ? ? · .... 1.4.1 E C V E dl ? ? ? ?? where C is an arbitrary path from some fixed reference point to ,r ? in electrostatics, the Maxwell-Faraday equation reveals that cure 0, E? ?? making the electric field conservative. So the integral above does not depend on any speafic path c chosen, but only an end points, implying V E is well defined at evey point. Therefor we can write ? ? ... 1.4.2 E E V ? ?? ?? ?? II f I i Fig. 1.3
NSOU ? CC-PH-08 ? 18 This states that the electric field points downhill towards lower voltage. The scalar potential can be visualized using equipotential, surfaces. An equipotential surface is a surface over which is a constant. The electric field is the negative of the gradient of the electric scalar potential. The electric field lines are every where normal to the equipotential surface and point in the direction of increasing potential. 1.4.1 Electric dipole Two equal but opposite point charges separated by a small distance consitutute an electric dipole. The dipole moment of a dipole has magnitrde, charge time the distance between the charges and is directed from + ve to − ve charge. If + q and − q be the charges shown. in fig (1.4). Then dipole moment of this dipole ? ? 2 , p q l ? ?? ? where l ? is taken along 2l q q ? ? ? ? Fig. 1.4 (a) Electric Potential due to dipote The fig (1.5) shows an electric dipole AB. The potential at point P at position Vector r ? from centre O of dipole, 0 1 4 q q V BP AP ? ? ? ? ? ? ?? ? ? 0 1 1 4 q r l r l ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? Taking r < < l we can write 2 2 2 2 0 1 1 4 2 cos 2 cos q r l rl r l rl ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 0 1 1 4 2 cos 2 cos q V r rl r rl ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? 0 1 1 4 2 cos 2 cos 1 1 q r l l r r ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? r P ?q A?l?? l B +q Fig. 1.5
NSOU ? CC-PH-08 ? 19 ? ? ? ? 0 cos cos 1 1 4 q l l r r r ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? 2 2 0 0 2 cos 1 cos 4 4 q l P r r ? ? ? ? ? ?? ?? or, ? ? 3 0 · 1 .... 1.4.3 4 p r V r ? ?? ?? ? So the electric field intensity at P 3 0 1 · 4 P r E V r ? ? ? ? ?? ? ?? ? ? ?? ? ? ?? ? ?? ? ? ? ? ? ? ? ? ? ? 3 3 0 1 1 1 · · .... 1.4.4 4 P r P r r r ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ?? ? ?? ? ? ? Calculation ? ? ? ?? ? ˆ ˆ ˆ ˆ ˆ ˆ · · x y z P r iP jP kP ix jy kz ? ? ? ? ? ? ? ?? ?? ? ? ˆ ˆ ˆ x y z i j k xP yP zP

x y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ˆ ˆ ˆ , x y z iP jP kP P ? ? ? ? ? ?? and ? ? 3 3 2 2 2 2 1 1 ˆ ˆ ˆ i j k

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x y z r x y z ? ? ? ? ? ?? 5 2 5 2 5 2 2 2 2 2 2 2 2 2 2 2 3 2 3 3 2 ˆ ˆ ˆ 2 2 2 y x Z i j k x y z x y z x

y z ?
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r r r ? ? ? ? ? ? ? So ? ? 3 5 0 3 · 1 4 P r r P E r r ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ?? ? ? ? ?? ?? ? ? ? 5 3 0 3 · 1 4 P r r P r r ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ?? ? ? ? ? ? 3 2 0 3 · 1 .... 1.4.5 4 P r r P r r ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ?? ? ? ? ??

So the radial component in (r, ?) co-ordinate 2 3 2 0 1 Pr cos 3 cos 4 r E P r r ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? 3 0 1 2 cos .... 1.4.6 4 P r ? ? ? ?? and the transverse component ? ? 3 0 1 sin 4 E O P r ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? 3 0 1 sin .... 1.4.7 4 P r ? ? ? ?? So E ?? makes an angle ? with E r ? ? ? 1 tan tan .... 1.4.8 2 r E E ? ? ? ? ? ? Thus along with same line when r < < l field lines are parallel to each other on same .r ? E ? y E 0 ? E r ? 0 x Fig. 1.6
NSOU ? CC-PH-08 ? 21 ? ? ? ? ? 2 2 2 3 0 1 3cos 1 .... 1.4.9 4 r P E E E r ? ? ? ? ? ?? ?? ?? field at end-on position (on the axis of dipole) along p ?? ? ? ? 3 0 1 2 ..... 1.4.10 4 end on P E r ? ? ? ?? ?? ?? field at broad side position (on perpendicular bisector) ? ? 3 0 ..... 1.4.11 4 P E r ? ? ? ? ?? ?? ?? (b) Dipole in a electric field. P.E of a dipole in electric field. The potential energy (P.E) of a dipole in a electric field is the work done to bring the dipole from infinity to the point concerned quasislatically Now if a dipole is brought to a point with its poles perpendicular to the direction of field then the work done will be zero since + ve and – ve charge will do same work but in opposite direction. So we can take the field normal position to be zero P.E Position. The fig. (1.7) shows a dipole AB of length ?? 2 . l V r ? P.E the potential energy at 0, the centre of dipole. Then the P.E of the dipole. ? ? ? ? U qV r l qV r l ? ? ? ? ? ? ? ? ? ? Taking l ? to be very very small ?? ? ? ? ? ? ?? ?? ? ? · · U q V r l V r q V r l V r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? 2 · · · ql V r P V r P E ? ? ? ? ? ? ? ? ? ?? ? ?? ?? ? ? ? ?? ? ? AsE V r ? ? ? ?? ? ? So the force on the dipole ? ? · F U P E ? ?? ? ? ? ?? ?? ?? ? 0 ? r ? ? ? ? ? ?q +q A?i?? B ?i? r ? 0 r ? ? ? ? Fig. 1.7
NSOU ? CC-PH-08 ? 22 Using the vector identity ? ? ? ? ? ? ? ? ? ? · · · P E E P P E E P P E ? ? ? ? ? ? ? ? ?? ? ? ?? ?? ?? ?? ?? ?? ?? ?? ?? ?? ?? ? ? ? ? ? ? We have ? ? · F P E ? ? ?? ?? ?? ....(1.4.18) since the rest of all terms are individually zero. (c) Torque on a dipole in uniform electric field. The fig (1.8) shows a dipole AB in uniform electric field .E ?? The total fore on the dipole is zero since poles are acted by equal and opposite forces. The torque on the dipole 21sin sin qE PE ? ? ? ? ? Taking direction of ? ? ..... 1.4.13 P E ? ? ? ? ? ?? ?? If the field is non-uniform apart from the torque there will be a translational force also on dipole. (d) Dipole–dipole interaction Now we shall calculate the potential energy of due to dipole–dipole interaction of two co-planar dipoles. The fig (1.9) shows two coplanar dipole of dipole moments 1 P ?? and 2 P ?? at position vector 1 r ? and 2 r ? respectively. ? ? 2 1 . r r r ? ? ? ? ? ? The potential energy of dipole of dipole moment 2 P ?? in the field of dipole of dipole moment 1 , P ?? 1 1 21 · , U P E ? ? ? ?? ?? where 1 E ?? is the field due to 1 P ?? at 2 . P ?? ? ? 1 2 1 3 2 0 3 · 1 · 4 P r r P P r r ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ?? ? ? ? ?? ?? ? ? ? ? ? 1 2 2 1 1 3 2 0 · · · 1 3 4 P r P r P P r r ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ?? ? ?? ?? ?? ?? 2 ? ?qE A +qE B 2 sin ? ? Fig. 1.8 0 ? ? r 2 ? r ? P ? ? ? ? ? ? r 1 ? 0 P 1 ? Fig. 1.9
NSOU ? CC-PH-08 ? 23 ? ? 1 2 1 1 2 2 3 0 1 cos 3 cos cos 4 P P P P r ? ? ? ? ? ? ? ?? ? ? ? 1 2 2 1 1 2 3 0 cos 3cos cos 4 P P r ? ? ? ? ? ? ? ? ? ? ? ?? 12 U? Now if the dipoles have same dipole moment i.e. 2 P P ? ?? ?? i.e they are oriented along same line, Then ? ? 2 2 3 3 0 0 1 3 . 4 2 P P U r r ? ? ? ? ? ?? ?? 1.5 Multiple Expansion of Electrostatic Potential It has been carlier seen that when the charge distribution has sufficient symnetry, or when the potential is to be calculated on symnetry axis of the distribution, one can obtain potential due to it a point outside the charge distribution by solving Laplace's equation with necessary boundary condition. It is a problem of finding potential due to an arbitrary charge distribution and one must take recourse to appronimation. One such proceedure, called multiple expansion of the potential, at a point far removed from the distribution. In this appronimation, the potential is expressed as a sum of contribution due to charge monopole, dipole, and quadrapole etc Let us consider a charge distribution shown in the figure (1.10). The potential at the point r ? is given by ?? ? ? 0 1 1 4 " V r r d r ? ? ? ? ? ?? ? ? ....(1.5.1) d? ´ r ´ ´ ? r ´ ? r ? o ? Fig. 1.10 P
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Using the law of cosines, ? ? 2 "2 2 2 cos r r r rr ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 cos r r r r ? ? ? ? ? ? ? ? ? ? ? ? where ??is the angle between r ? and r ? ? so we get, ? ? " 1 r r ? ? ? with ? ?? ? 2cos r r r r ? ? ? ? ? ? ? For points far from the charge distribution ??> 1, from binomial expansion. ? ? ? ? 1 2 2 3 1 1 1 1 3 5 1 1 .... " 2 8 16 r r r ? ?? ? ? ?? ? ? ? ? or interms of , and r r ? ? ? ?? ? ? ?? ? ? ?? ? 2 2 3 3 1 1 1 3 5 1 2cos 2cos 2cos ..... " 2 8 16 r r r r r r r r r r r r ? 2 3 2 3 1 3cos 1 5cos 3cos 1 cos 2 2 r r r r r r

r ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
In the last expansion, we sbserve that series comes with power of r r ? along with Legerdre polynomial as coefficients. we get ? ? ? ? ? ? 0 1 1 cos .... 1.5.3 " n n n r P r r r ? ? ? ? ? ? ? Substituting the equatim 1.5.3 in equation (1.5.1) we get the potential as ? ? ? ? ? ? ? ? ? 1 0 0 1 1 cos ' ' 4 n n n n V

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r r P r d r ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ?? NSOU ? CC-PH-08 ? 25 More explicitly, ? ? ? ? ? ? ? ? ? 2 2 3 0 1 1 1 1 cos 4 V r r d r r d r r r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? 2 3 1 cos .... 2 2 r d ? ? ? ? ?? ? ? ? ? ? ....(1.5.5) The

equation (1.5.5) is expression for multipole expansion of V in powers of 1 . r rearrange the term as follows– ?? ?? ?? ?? ? ? 0 1 2 ..... 1.5.6 V

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r V r V r V r ? ? ? ? ? ? ? ? ? where, ?? ? ? 0 1 ' ..... 1.5.7 4 V r d r ? ? ? ? ?? ? ? ?? ? ? ? ? 1 2 0 1 cos .... 1.5.8 4 V r r r d r ? ? ? ? ?? ? ?? ? ? ?? ? ? ? ? ? ? ? 2 2 2 3 0 1 1 3cos 1 ' .... 1.5.9 2 4 V r r r d r ? ? ? ?? ? ? ?? ? ?

The Monopole and Dipole Terms : Monopole term is defined as. ?? ? ? ? ? 0 0 1 ' ..... 1.5.10 4 V r r d r ? ? ? ? ? ?? ? ? It is the potential which would have at P if The whole charge is concentrated at the origin, and d ? ? ? ? is the monopole moment.

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If the total charge is zero, the dominant term in the potential will be the dipole. ?? ? ? ? ? 2 0 1 cos .... 1.5.11 4 dip V r r r d r ? ? ? ? ?? ? ?? ? ? Since ??is the angle between r ? ? and r ?

NSOU ? CC-PH-08 ? 26 ˆ cos · r r r ? ? ? ? ? So we can write the dipole term as ?? ? ? ? ? 2 0 1 1 ˆ. .... 1.5.12 4 dip V r r r r d r ? ? ? ? ? ? ?? ? ? This integral is called the dipole moment of the charge distribution, ? ? ? ? .... 1.5.13 P r r d ? ? ? ? ? ? ? ? ?? ? So the dipole contribution to the potential is given by ? ? ? ? 2 0 ˆ 1 · .... 1.5.14 4 dip P r V r r ? ?? ?? The dipole term plays the important role when all the monopole term vanishes, so ?? 1 V r ? is the potential as if a pure dipole is placed at the origin. The term ?? 2 V r ? is defined as potential contribution due quadrapole moment The figure 1.10a portrays gemetrical 1.6 The Gauss's Theorem (a) Electric flux : In science, flux, usually concerns to some flow of physical property. In case of fluid flow 'fluid flux' refers to the amount of fluid flowing through a specific area per unit time. 'Vehicular flux' often refers to the number vehicle crossing a specific gate area per unit time. However, in case of electric flux, no such transport physically exists. It refers to the crossing of electric field lines (which is an imaginary conception to give a visual presentation of field pattern) through specific area. It is defined as : The electric flux through an area the number of field lines passing perpendicular to the area. E ?? be the electric field at a space point. Then the flux through a surface, ds ? is, ... d E ds ? ? ?? ??? As d s ? is infinitesimal E ?? can be taken to be constant over the surface. The total flux z x P p r ? y Fig. 1.10 a

NSOU ? CC-PH-08 ? 27 on a finite surface can be obtained by integration as, . , E d s ? ? ? ?? ? where the integration is carried over the entire surface. (b) Surface area and solid angle : Surface area is treated as a vector whose magnitude is the area of the surface considered and direction is specified as follows. i) For closed surface the direction is + ve in outwards normal to the surface. ii) In case of open surface the direction is specified by right hand screw rule. The solid angle is three dimensional analogues to that of an angle in two dimension. Now an angle can be visualized physically as a two dimensional peeping from a point. Mathematically it is defined as the ratio of the arc by radius of a circle. In fig the angle subtended arc AB , d r dl r ? ? ? which is a dimensionless quantity. . Its unit is taken as radian which is defined as angle subtended by a arc of a circle of unit length and unit radius at the centre of the circle, so, the total angle about a point as 2 2 2 2 r r ? ? ? radian. Similarly, a solid angle can be visualized as a three dimessional peeping through a point and it is mathematically defined as three dimensional angle produced at the centre of a sphere due to an area boudary on the surface of the sphere. If ds be the elemental area in the surface of a sphere then, the solid angle suspended at the centre O is, 2 ds d r ?? ? If the area plane makes an angle ??with the tangent to the sphere at that point then solid angle 2 2 2 ˆ ˆ cos . . . ds ds r dsn r d r r r ? ? ? ? ? ? ? ? where ˆn and ˆr are unit vectors along d s ? and .r ? as explained in the figure. 0 r 1 d? A B dr Fig. 1.11 0 d?? r ds Fig. 1.12 d? ds ? d?? r ? ds dsn ? Fig. 1.13
NSOU ? CC-PH-08 ? 28 Unit of solid angle is named steradian, one steradian is the solid angle subtended at the centre of a sphere of unit radius by a unit surface on the sphere. Obviously the total solid angle about a point will 2 2 2 2 ˆ ˆ . cos 4 4 dsn r ds r r r r ? ? ? ? ? ? ? ? ? ? ? ? ? steradian When o is outside then referring the fig. Due to the orientation of 1 2 ˆ ˆ n and n the surface area vectors the total solid angle subtended at o will obviously be zero, because they will cast solid angle of same magnitude but in opposite sense and will cancel each other to make the yield null. (c) The Gauss's law : Now we consider a point charge q at a point o bounded by the surface S. The electric field on the surface ds is 2 0 1 ˆ, 4 q E r r ? ?? ?? so the flux through the surface element ds, 2 0 0 1 1 ˆ ˆ . . 4 4 q d E ds r d sn qd r ? ? ? ? ? ? ?? ?? ?? ? So the total flux outgoing the whole surface S, ? ? 0 0 0 1 1 .4 ... ... 1.6.1 4 4 S q qd q ? ? ? ? ? ? ? ?? ?? ? ?? The result in equation is independent of position of the charge and obviously when the point charge q falls outside the surface the yield integration is zero, in that case 0 ? ? 1.6 Application of Gauss's theorem Before going to the application of Gauss's theorem let us give a second look to what we have done in the previous discussion. We see that with Coulomb's law, if we know the charge, we are able to find out the field produced by the charge and from Gauss's law if the field in a region is known, we can work out the net charge responsible for creating the Fig. 1.14 n 2 ? n 1 ? o Fig. 1.15 q ds n ? r ? Fig. 1.11
NSOU ? CC-PH-08 ? 29 field if we can evaluate , cos E ds Eds ? ? ? ? ?? ? which is obviously not a function of single variable. So for evaluation of this integral the angle between dsand E ? ?? should remain constant throughout the surface. Such a hypothetical surface, on which such symmetry is maintained, so that ,E ds ? ?? ? can be evaluated, is called Gaussian Surface. (1) Field due to a uniformly charged spherical shell : The fig (1.17) shows a hollow spherical charged

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sphere of radius R and charge q, uniformly distributed over its surface.

To calculate the intensity

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at point P at a distance r form the centre o we consider a Gaussian surface

shown by dotted line in the fig (1.17). E be the intensity of field at a distance r from centre 0. Then using Gauss's law. 2 0 , cos 4 E ds Eds E r q ? ? ? ? ? ? ? ? ? ?? ? or, ? ? 2 0 1 ... ... 1.6.2 4 q E r ? ? ?? Inside the shell the right hand side is zero as there is no charge included within the Gaussian surface for any value of r (r > R) so E = 0 within
the shell. (2)

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Field due to a uniformly charged sphere : (a) At a point outside the surface : The fig (1.18) shows a uniformly charged sphere of radius R and total charge q uniformly distributed. To calculate the field intensity E at the point P outside the surface, we consider a Gaussian surface through the point P as shown by the dotted line. E ?? be the field intensity at point P and ds ? be an elemental area on the surface at point P, then by Gauss's theorem ? ? 0 , ... 1.6.3 q E ds? ? ? ?? ? ? Now from symmetry of charge and its consequent field distribution E remains same all over the Gaussian surface and is always on the surface. Hence, ? ? 2 0 4 , q E r ? ? ? or 2 0 4 q E r ? ?? Fig. 1.12 P P Fig. 1.13

NSOU ? CC-PH-08 ? 30 Taking direction into consideration ? ? 2 0 ˆ...... 1.6.4 4 q E r r ? ?? ?? (ˆr is a unit vector in the direction of .r ?) (b) At a point inside the surface : To calculate the field inside the surface, we consider the Gaussian surface represented by dotted line as shown in fig (1.14) R be the radius of the sphere and ? be the charge density taken to be uniform inside the sphere. E be the intensity over the Gaussian surface. Then, ? ? 3 0 4 . 3 E ds r ? ? ? ? ? ? ?? ? ? or ? ?? ? ? 3 2 2 0 3 4 4 3 R E r r R ? ? ? ? ? ? ? ? ? Thus ? ? 3 0 1 ... ... 1.6.5 4 q E r R ? ?? ? From the above equation it is obvious that electric field of a uniformly charged sphere is zero at the centre of the sphere and linearly increase up to the surface of sphere where it resumes its maximum value. For outside the surface the field falls ? ? ? ? 2 1 1 distance distance from the centre of the sphere. (3) A uniformly charged long cylinder : Consider an infinitely long cylinder having uniform linear charge density and radius a. Let P be a point located at a perpendicular distance or from the wire we construct the Gaussian surfaces, wehich in this case is a concetric cylinder of radius r and length ? . Applying Gauss's law for this surface, we get 0 1 . S E ds? ? ? ? ?? ? ? Where ? ? is the charge enclosed by the surface. Electrical field lines E ?? will be normal to the curved portion of the surface. Due to cylindrical symmeiry, E ?? will be of same magnitude all over it. Also E ?? will be tangential to the end faces, so .E ds ?? ? will be zero on these faces, we can write P Fig. 1.14 R Fig. 1.15 ? P E ? ? r a

NSOU ? CC-PH-08 ? 31 0 1 . .2 . S E ds E ds E rl ? ? ? ? ? ? ? ? ? ?? ? ? or, 0 0 1 1 2 2 4 E r r ? ? ? ? ? ?? ?? vector notation ? ? 0 1 2 ˆ... 1.6.6 4 E r r ? ? ? ?? ?? (b) Field inside cylinder : To find the electric field at any internal point, P at a distance r, we construct a cylinder cal Gaussian surface, of length l and radius r coaxial with it. The charge endosed by the surface is 2 . . encl Q r l s ? ? where ? is the charge density o to and is related to ? by 2 . .1 , a? ? ? ? 2 a a ? ? ? ? From Gauss's law 0 1 . encl E ds Q ? ? ? ? ?? ? Now . .2 E ds E rl ? ? ? ? ?? ? or, 2 0 0 1 1 2 encl E rl Q r l ? ? ? ? ? ? ? ? 2 2 0 1 r l a ? ? ? ? ? or, 2 0 1 2 4 r E a ? ? ?? In vector form, ? ? 2 0 1 2 ˆ... ... 1.6.7 4 r E r a ? ? ? ?? ?? (4) Unformly charged infinite plane : Consider an infinite plane sheet of charges with uniform surface charge density ?. To find out electric field at P at distance r, we construct a cylindrical Gaussian havig equal ? r P a Gaussian surface Fig. 1.16

NSOU ? CC-PH-08 ? 32 length on both sides From Gauss's law 0 1 . E encl S E ds Q ? ? ? ? ? ? ?? ? ? or ' o P P . = . . + . encl Q E ds E ds E ds E ds ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? curved surface The electric field is perpendicular to the area element at all points on the carved surface and is parallel to the surface P and P' o P P' . 0 E encl Eds Eds E ds ? ? ? ? ? ? ? ? ? ? ? ? ? ? curved Since the magnitude of the electric fild at these two equal surfaces is uniform, E is taken out of the intergration and , encl Q s ?? 0 2 s E ds ? ? ? ? Hence 0 2 S ES ? ? ? or, 0 2 E ? ? ? If ˆr is unitvect perpendicular to the plane, then in vector form ? ? 0 ˆ... ... 1.6.8 2 E n ? ? ? ?? Electrical Field Inside A Parrallel Plate Conductor : At the points P 2 and P 3 , he electric field due to both plates are equal in magnitude and opposite in direction. As a result, electric field at a point outside the plates is zero. But inside, electric fields are in the same direction ie. towards the right, the total electric field at a point P 1 E.ds ? ? P ´ ds ds ds E Gaussion surface Fig. 1.17

NSOU ? CC-PH-08 ? 33 ? ? 0 0 0 ... ... 1.6.9 2 2 inside E ? ? ? ? ? ? ? ? ? The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform every where inside the plate 1.7 Laplace and Pisson's Equations Consider a closed surface S enclosing a volume V and charge q. Then from Gauss's law we can write 0 0 . V q dV Eds ? ? ? ? ? ? ? ? ??? Where E ?? electric field intensity vector at the point ds ? and the ? is the charge density at the point concerned to .E ?? Using Gauss's divergence theorem in above equation we have 0 . V V dV EdV ? ? ? ? ? ? ? ?? ? or, ? ? 0 . ... 1.7.1 E ? ? ? ? ? ?? ? The equation (1.17) is known as differential form of Gauss's law of electrostatics. Now using potential – intensity relation, we have ? ? 0 . . ? ? ??? ? ? ? ?

NSOU ? CC-PH-08 ? 34 or, ? ? 2 0 ... ... 1.7.2 ? ? ? ? ? ? In free space ? = 0 the above equation takes the form 2 0 ... ... ? ? ? Equation (1.7.1) and (1.7.2) are respectively known as Poission and Laplace's equations, which play important role in solving out the potential of a charge distribution with given boundary condition, which is the basic motto of solving electrostatic problems. 2 ? in different co-ordinate system : (a) Rectangular co-ordinates (x, y, z) 2 2 2 2 2 2 2 n y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? (b) Spherical polar co-ordinates (r, ?, ?) ? ? 2 2 2 2 2 2 2 2 1 1 1 sin sin sin r r r r r ?? ? ? ? ? ? ? ?? ? ? ? ?? ? ? ? ? ? ? ? ? ?? ?? ? ? ? ? ? ?? (c) Cylindrical co-ordinates (r, ?, z) ? ? 2 2 2 2 2 1 1 s s z ? ? ? ? ? ? ? ? ? ?? ? ? ?? ? 1.7.1 Uniqueness Theorem Two solutions of Laplace's equation obeying the same boundary conditions diffon at best by a constant. In order to prove the The orem, let us assame that ? 1 and ? 2 are the two solutions of Lapalac's equation in volume V exterior to surface of different conductor S 1 , S 2 ....S n . bouded by on the outside surfaces. Assuming that ? 1 and ? 2 satisfy the same boundary condutions including the surfaces, S 1 ...S 2 ....S n , and specifically. These boundary conditions includes either the specificatons of the potential ?on the bounding surface which is known Dirichlet condition, or in other way, the specification of the normal derivati overs of ? i.e. . n ?? ? The bounding surface, known as Neuman condition.

NSOU ? CC-PH-08 ? 35 Let 1 2 , ? ? ? ? ?? As 2 1 0 ? ? ? ?? ? and 2 2 0 ? ? ??? ? so, 2 2 2 1 2 0 ? ? ? ? ? ? ?? ? ? inside V and 0 ? ?, an 0 n ?? ? ? on the surface S for Dirichlet and Neuman boundary conditions, respectively. Applying divergence Therem to the vector , ??? ?? we get ? ? ˆ· · S V nds dv ??? ? ? ??? ? ? ?? ?? ?? 2 2 2 V dv ? ? ?? ? ?? ?? ? ? ? ? \ 2 V dv ?? ? 2 0 ? ? ?? ?? ? ? The integration on the left hand side vanishes on both types boudary conditions so, we get 2 0 V dv ? ? ? ? ? It is clear that integrand is positive definite quantity, it must be zero at every point in V for the integral to vanish, where, 0, ?? ? ?? or, 1 2 ? ? ?? ?costant inside V V Now, for Diricihet boundary conditions 1 2 0 ? ?? ? on the surface S, so we have 1 2 ? ?? through out i.e. it is a unique solution. For Neuman's boundary conditions ? ? 1 2 ˆ· 0 n ? ? ?? ? ?? on. S or, 2 ??? ?Constant S As the constant is arbitrary, it may can be taken to be zero, and the solution is unique : 1 2 ? ??

NSOU ? CC-PH-08 ? 36 1.8 Electrostatic Energy The energy stored in accumulation of charge due to work done against the Coulomb'sforce, is called electrostatic energy. This is essentially potential energy in case of static charges. reference zero of potential energy can be set to be zero at that separation. Calculation of electric potential energy for a charge distribution Consider a point charge q i at position vector N i and all other charges, infinitely separated from each other. Then to bring a point charge q j at position vector j r ? the work done 2 2 0 0 1 ˆ· , 4 4 j j r r i j i j ij ij ij ij ij ij ij q q q q w r dr dr

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r r ? ? ? ? ? ? ?? ?? ? ? ? where ij j i r r r ? ? ? ? ? and ˆ j i ij j i r r r r r ? ? ? ? ? ? ? ? 0 0 1 1 4 4 r ij i j

i j ij ij ij q q q q w r r ? ? ? ? ?? ?? So the total work done for accumulating N charges 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 4 2 4 2 4 2 j N N N N N N N i j i j i j j j ij ij ij i j i j i j j q q q q q q U q r r r ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? ?? ?? ?? ? ..1.8.1 Where 0 1 1 4 N i j ij i q r ? ? ? ? ?? ? is the potential due to all particles at j th charge point. The multiplication of 1/2 is to avoid double counting like 1 2 q ? and 2 1 q ? which will have same effect on energy. Electrostatic energy for continuous charge distribution we can replace i) ?? j q r dV ? ? for volume distribution of charge, ii) ?? j q r ds ? ? for surface distribution of charge, iii) ?? j q r dl ? ? for line distribution of charge, Considering all type of distribution of charge the expression of electrostatic energy

NSOU ? CC-PH-08 ? 37 ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 V S L U r r dV r r dS r r dl ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? + the energy due to point charge e distribution. 1) Electostatic energy in terms of field vectors We start with a volume distribution of charge, then the electrostatic energy ? ? ? ? ? ? ? 1 1 . 2 2 V V U r r dV r DdV ? ? ? ? ? ? ? ? ? ?? Now using ? ? ? ? . . . D D D ? ? ? ? ?? ?? ? ?? ?? ?? In the above equation we have ? ? ? ? ? 1 1 . . . .] 2 2 V S V U D D dV D dS E D dV ? ? ? ? ? ? ? ??? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? ?? ?? ?? Now confining the charge to a finite region if the integration is extended to infinity the first term vanishes since 2 1 1 ,D r r ?? ? and 2 ds? r 2 so the first term falls as 1 . r Thus when we extend for all space the expression of electrostatic energy becomes. ? ? 1 . ... 1.8.2 2 V U E DdV ? ? ? ?? ?? We can obviously take electrostatic energy density ? ? 1 . 2 u E D ? ? ?? ?? 1.8.1 Electrostatic Energy of Uniformly charged sphere : Eectrostatic self energy of a charged sphere us given by– 2 2 0 0 1 1 . 2 2 inside U E dV E dV ? ? ? ? ? ? Assuming total charge of the sphere Q, ? its charge density and 'a' is the radius, then, electric field at

a dirtaue 'r' 3 2 2 0 0 4 3 1 1 · 4 4 r Q r E r r ? ? ? ? ? ?? ??

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Q r a ? ?? where r > a. and 2 0 1 4 Q E for

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r a r ? ? ?? ? 2 4 2 2 0 0 3 2 0 0 0 4 4 2 2 4 4 a a Qr Q U r dr r dr a r ?? ?? ? ? ? ? ? ? ? ? ? 2 2 0 0 3 1 1 1 1 · · 2 4 5 4 5 Q Q a a a ? ? ? ?? ??

Total energy of charged sphere, ? ? 2 0 3 1 ... ... 1.8.3 4 5 Q U a ? ?? 1.9 Conductors in electric field When a conductor is placed in electric field the free electrons in it move in opposite direction creating induced field in opposite direction as in fig. The electron migration continous untill the field inside the conductor vanishes and the conductor becomes equi-potential all through. Any charge given to the conductor will migrate to its surface. If not .E ? ? ? ? implies presence of electric field inside the conductor. Which contradicts the above discussion. 1) Field intensity on the surface of a charged conductor. Consider a cylinder a cylinder of plane surface area ds ? as in fig 1.2.9

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The electric field at any point of the surface is perpendicular to the

surface as ?? ? is perpendicular to constant potential surface. Now consider a Gaussian surface as denoted by a cylinder of each plane surface · ds E ? ?? be the intensity at the surface point at the element . ds ? The using Gauss's law we have . ds E ds ? ? ? ?? ? or, , E ? ? ? ? ?? acting normally outwards at the point considerd ... ... (1.9.3) Fig. 1.19
NSOU ? CC-PH-08 ? 39 2) Mechanical Force and presure on a charged conducting surface Consider a charged coductor as in fig. (1.20) then by Coulomb's Theorem the electric field at vicinity of the outside of the surface is E ? ? ? acting normally outwards. We can visualize the 1 2 ; E E E ? ? ?? ?? ?? where 1 E ?? is the field intensity due to the charge on the element ds and 2 E ?? is the field due to the rest of the charge at the element ds. Now the intensity inside the conductor is zero, so 1 2 0, E E ? ? ?? ?? thus 1 2 0 2 E E ? ? ? ? and at outside point the intensity will match with the Coluomb's theorem 1 2 0 E E E ? ? ? ? ? ? ?? ?? ?? acting outwards normal. Thus the elemental charged area ds ? will be under the outwards field intensity 1 0 E ? ? ? So the force on the charge elemental ? ? 2 0 ˆ ˆ . 2 ds F n n unitvector acting outwards perpendicular ly ? ? ? ? ? ? ?? So the electrostatic pressure 2 0 . P ? ? ? 1.10 Capacitors A capacitor is a device, which can store electrical energy. The capacitance of a conductor is defined, as the charge required increasing its potential by unity. If q charge is required to increase the potential of a conductor by V then its capacitance, , q C V ? which is found to be independent of charge. The capacity of a conductor increases in the presence of neighbouring conductors due to induction of opposite charge at proximity and similar charge at relatively apart. It further increases if the neighbouring conductor is earthed. This is what is known to be 'Principle of capacitor'. Practically a capacitor is combination of two conductors with equal and opposite charges at so proximity that their potential difference remains unaffected for the presence of other charges but depends on the shape, size and proximity of two conductors and the intervening medium. Fig. 1.20 ds
NSOU ? CC-PH-08 ? 40 Its practical unit is farad. Capacitance of a capacitor is said to be 1-farad if 1C if charge, given to the unearthed plate increases the potenrtial difference between the plates by 1V. 1) The parallel plate capacitor A set of two parallel conducting plates of same size with dielectrics or vacuum inside constitute a capacitor. To calulate its capacitance we take the separation to be very small compared to its lateral dimension, field inside the plates can be taken to uniform in between thr plates. The fig (1.21) shows a parallel plate capacitor of each plate area A separated by a distance d. Q be the charge given on plate- 1, The plate-2 is earthed and is carrying the bound charge –q as in fig (1.20) V be the potential difference between the plates. ? be the charge per unit area of plate-1. Then using Coluomb's theorem, the electric field inside the plates E? ? ? acting from plate-2 to plate-2 and it can be taken to be uniform considering d to be very small. Thus the potential difference . qd d A d V

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E d A A ? ? ? ? ? ? ? ? ? Thus the capacitance of a parallel plate capacitor can be written as ? ? 0 ... ... 1.10.1 r A q A C V d d ? ? ? ? ? ? Where 0 ?= permittivity of vacuum and r ? ? dielectric constant of intervening medium. In case of vacuum 1. r ? ? The eqn. Shows that capacitance is independent of charge. 2) Energy stored in a capacitor Consider a capacitor has to be charged Q, then its potential be V. In the course of charging q be the charge on the one plate of capacitor and ? be its potential. Then for further charging through an infinitesimal charge dq

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the work done. qdq dW vdq C ? ? So the total work done in charging the capacitor to the charge Q is + − + − + − + − + −

Fig. 1.21
NSOU ? CC-PH-08 ? 41 2 2 0 1 2 2 Q qdq Q W CV C C ? ? ? ? where V is the final potential difference between the plates. ... ... (1.10.3) Thus the above expression gives the electrostatic energy stored in a capacitor. 3) Capacitors with layer of dielectrics a) The dielectrics are parallel to plates : The fig (1.3.1) shows a parallel plate capacitor filled with two dielectrics with dielectric constants 1 ? and 2 ,q ? be the charge given on the upper plate which raise its potential by V. E 1 and E 2 be the electrostatic field intensities in medim 1 and 2. Then 1 1 2 1 1 2 2 2 1 2 0 1 0 2 0 1 0 2 0 1 2 x x x q V E x E x x x x A ? b) When the dielectrics are perpendicular to plates : Here we consider the plate separation is d and the area portion A 1 and A 2 are occupied by the dielectris of dielectric constants 1 ? and 2 ? respectively. q be the charge given to the upper plate. The potential difference ? ? 1 2 0 1 0 2 ... ... 1.10.4 V E d E d d d ? ? ? ? ? ? ? ? ? ? (2) Combination of Capacitors (a) Capacitors in series The fig (1.24) shows a set of capacitors C 1 , C 2 , C 3 ....C i ....C N , in series combination. ? ? ? ? x ? x ? ? ? Fig. 1.22 ? ? ? ? ? ? ? ? ? ? Fig. 1.23 C ? C ? C ? V ? ? V ? V ? V ? Fig. 1.24 +
NSOU ? CC-PH-08 ? 42 To calculate the equivalent capacitance We apply a potential difference V across the combination. V 1 , V 2 , V 3 .....V i .....V N be the Potential difference at steady state across C 1 , C 2 , C 3 .....C i .....C i ....C N respectively. q be the charge on the + plate of the capacitors, it will be same for all plates as same current flows for the same time through each capacitor. So, V = V 1 + V 2 ..... 1 2 1 2 3 1 1 1 .... q q q C C C C C ? ? ? ? ? ? ? ? ? ? ? So the equivalent capacitance eq q C will give V ? 1 2 3 1 1 1 1 1 1 ..... i N eq i i C C C C C ? ? ? ? ? ? ? ? ? (b) Capacitors in parallel The fig (1.25) shows a set of capacitors C 1 , C 2 , C 3 ....C i ...C N in parallel combination. N be the potential difference applied across the terminal of of the combination. q 1 , q 2 ....q i ....q N bethe charge at steady state on capacitors C 1 , C 2 , C 3 ,.....C N respectively.

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The total charge 1 2 .... i N q q q q q ? ? ? ? ? ? 1 2 3 ...... N C V C V C V C V ? ? ? ? ? If C

eq is the equivalent resistance of the combination then, 1 2 3 ..... eq N
q

71%	**MATCHING BLOCK 24/107**	**SA**	PHY3512 Physics for Physical Science II 376pg ... (D144373244)

C V C V C V C V C V ? ? ? ? ? ? Or, 1 2 3 ...... eq N C C C C C ? ? ? ? ? (c)

Energy loss due to sharing of charges of conductors Cosider two conductors having charges q 1 and q 2 , capacitances C 1 and C 2 and respective potentials V 1 and V 2 (V 1 < V 2). When they are make to touch or joined by a conducting wire, they will reach to common potential 1 1 2 2 1 2 arg CV C V TotalCh e V Commonpotential V V ? ? ? ? So the final charge on the conductors are

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C ? C ? C ? Fig. 1.25 V NSOU ? CC-PH-08 ? 43 1 1 2 2 1 1 1 1 2 C V C V q C V C C C ? ? ? ?? ? ? ? ? ? ? and 1 1 2 2 2 2 2 1 2 C V C V q C V C C C ? ? ? ? ?? ? ? ? ? ? ?

So the initial electrostatic energy stored 2 2 1 1 2 2 1 2 i E

72%	**MATCHING BLOCK 30/107**	**SA**	PHY3512 Physics for Physical Science II 376pg ... (D144373244)

C V C V ? ? ? ? ? ? stored ? ? ? ? 2 2 1 1 2 2 1 2 1 2 1 2 1 1 2 2 f C V C V E C C V C C C C ? ? ? ? ? ? ? ? ? ? ? ?

So the change in energy due to sharing of charges ? ? 2 2 2 1 1 2 2 1 1 2 2 1 2 1 2 1 1 2 2 i f

62%	**MATCHING BLOCK 29/107**	**SA**	PHY3512 Physics for Physical Science II 376pg ... (D144373244)

C V C V E E E CV C V C C C C ? 2 1 2 1 2 1 2 ... ... 1.10.5 2 C C V V C C ? ? ?

The above equation shows ?E to be always positive and thus there is always a loss of energy in sharing of charge. (d) Force of attraction between plates of charged parallel pate capacitor If ? is the surface charge density on one of the plates of parallel plate conductors then this charge is under the electric field ?/2? for the charge on the other plate. So force on the unit area acting between the plates is P = ? 2 /2?. For the plate area A, the force is 2 2 2 2 q A F A ? ? ? ? ? where q is the charge on one of the plate. This force in attractive as the plates bear opposite charges. (e) Breakdown Voltage A capacitor can withstand a limiting value of potential difference between its terminals, called break down voltage, which depends on the proximity of its plates, its structure and intervening dielectric. This is because, dielectric break down takes place at a limiting value of field intensity when the force on bonded electron crosses the bounding force of nucleus and the dielectric starts conducting. In case or air it is about 3 × 10 3 kV/m. Example-1 A parallel plate capacitor with place separation d and plate area A each is filled with two dielectries of dielectric constants K 1 and k 2 in such a way that half the length of the
NSOU ? CC-PH-08 ? 44 plates is covered by each dielectric. Show that the capacitance is given by ? ? 0 1 1 . 2 A C k k d ? ? ? Soln. The fig. (1.26) shows such an arrangement that can be visualized as a parallel combination of two capacitances, 0 1 1 2 k A C d ? ? and 0 2 2 2 k A C d ? ? So the total Capacitance ? ? 0 1 0 2 0 1 2 1 2 2 2 2 k A k A A C C C k k d d d ? ? ? ? ? ? ? ? ? hence proved. Example-2 Find the expression of force on a dielectric slab inserted partly in a parallel plate Capacitor maintained at constant potential difference ?. The fig. 1.27 Shows a parallel plate capacitor of length l and breath b. d be the distance between the plates. A dielectric slab of breath b is inserted through a distance x as in fig. (1.27). The system can be considered as a parallel combination of two capacitors C 1 (portion with dielectric) & C 2 (portion without dielectric). Thus ? ? ? ? 0 0 0 1 2 1 l x kxb b C C C kx x d d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? So the energy stored in the system ? ? 2 2 0 1 1 1 2 2 b U C kx x d ? ? ? ? ? ? ? ? ? ? ? So the force acting on the dielectric slab ? ? ? ? ? ? 2 2 0 0 1 1 1 1 2 2 b b U F kx x k x x d d ? 1.10 (a) Capacitance of Spherical Capacitor (i) Isolated Sphere The fig (1.28) shows an isolated conducting sphere of radius R. Q be the total charge on the surface. We consdier a gaussian surface given by dotted line. .E ?? be the electroic field intensity ? ?? ? ?? K d 1 K 1 Fig. 1.26 Fig. 1.27 + Fig. 1.28
NSOU ? CC-PH-08 ? 45 at a point on the surface. Then from Gauss's theorum 0 . S Q E dS? ? ? ?? ?? Or, 2 0 4 Q E r ? ? ? 2 0 , 4 r dV Q l E dr r ? ? ? ? ?? where V V r is The potenhal on Gaussian surface. So 2 0 1 4 r Q dV dr r ? ? ?? Considering V as the potential at R and the other plate to be at infinity, where the spotential is assumed to be zero we have 2 0 4 r V R Q dr dV r ? ? ? ?? ? ? 0 0 1 4 4 R Q Q O V R r ? ? ? ? ? ? ? ?? ?? ? 0 4 Q V R ? ? ?? So ? ? 0 4 ... ... 1.10.10 Q C R V ? ? ?? (ii) Two concentric spheres, outer sphere earthed The fig (1.29) shows a spherical capacitor consiting of two concentric spherical conducting shells of iner radius a and other radius b. Q be the charge given on the inner sphere, E be the intensity at point P, Through which passes a gaussian surface as shown dotted line. Then using Gauss's Theorem, 2 0 . 4 S Q E dS E r ? ? ? ? ? ? ?? ?? Or, 2 0 4 Q dV E dr r ? ? ? ? ?? + a r b P Fig. 1.29

NSOU ? CC-PH-08 ? 46 Or, 2 2 4 o b a v Q dr dv r ? ? ? ? ? ? ? ? ? 0 0 1 1 1 4 4 b a Q Q V r b a ? ? ? ? ? ?? ?? ? ? ? ? 0 0 1 1 4 4 b a Q Q V a b ab ? ? ? ? ? ?? ?? So its capacittance ? ? 0 4 ... ... 1.10.7 Q ab C V b a ? ? ? ?? ? As b? ? its gives the result of isolated capacitor. . (iii) Inner sphere earthed Let q the charge given on the outer sphere of radius b. a be the radius of inner sphere (conducting) q ? be the charge induced on inner sphere. Then from fig (1.30). As the potential on inner sphere is zero, Then 0 1 0 4 q q b a ? ? ? ? ? ? ? ? ?? ? ? Or, ? ? ... ... 1.1.8 a q q b ? ? ? Now considering the Gaussian surface Through P of radius r, We have using Gausse's theroem, . S q a E dS q r b ? ? ? ? ? ? ?? ?? 2 0 1 4 q a E b r ? ? ?? Or, 2 0 1 4 q dV dr r ? ?? Or, 2 0 1 1 4 a dV dr b r ? ?? Or, 2 0 4 V b O a q a dr dV b r ? ?? ? ? + – a r b P Fig. 1.30

NSOU ? CC-PH-08 ? 47 ? ? 2 0 0 1 1 4 4 q q a b a V b a b b ? ? ? ? ? ? ? ? ? ?? ?? ? ? ? ? ? ? ? 2 0 4 ...... 1.10.9 q b C b a v ? ? ?? ? (b) Capacitance of Cylindrical Capacitor. Outer Cylinder earthed The fig. (1.10.15) shows a cylindrical capacilor whose length L is much larger than its breadth so that electric field inside The cylinder can be considered to be axially symmeteric. We consider a Gaussian surface a cylinder of radius r (r > r > b) as shown by dotted cylinder. E ?? be the electric field at a point P on this Gaussian surface. Then applying Gauss's Theorem . S q E ds? ? ? ?? ? (? be The permittivity of medium in space between the cylinclers) ? = charge/unit length Or, 2 l E r l ?? ? ? ? ? ? 1 2 dV E r dr ? ? ? ? ? ?? 1 2 dr dV r ? ? ? ?? So potential difference between the cylinders 2 V a O b dr dV r ? ? ? ?? ? ? Or, ln ln 2 2 a b V b a ? ? ? ? ? ?? ?? ln 2 q b a L ? ?? (where q L ? ?) L P b a Fig. 1.31 dl

NSOU ? CC-PH-08 ? 48 ? ? 2 ...... 1.10.10 ln q L b C a V ?? ? ? (c) Capacitance of spherical capacitor with dielectrics The fig (1.32) shows

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a spherical Capacitor of inner radius a and outer radius

b with a dielectric of dieletric const K 1 Thickness t with inner sphere and The rest portion is filled in with dielectric constant K 2 . q be the charge given on inner sphere Then potential difference between inner and outer sphere ? ? 1 1 · · · ... ... 1.10.11 a t b a t a V E dr E dr ? ? ? ? ? ? ? ?? ? ? ?? ? Where 1 E ?? and 2 E ?? are electric field intensity at The specific point in inner an outer sphere respectively Now applying Gauss's Theorrein 1 1 0 · S q E dS k ? ? ? ? ?? ?? ? ? ?= permittirty of free space. Or, 2 1 0 1 4 q E r k ? ? ? Or, 1 2 0 1 1 ˆ 4 q E r a r a t k r ? ? ? ? ? ?? ?? Similarly ? ? 2 2 0 2 1 ˆ 4 q E r a t r b k r ? ? ? ? ? ?? ?? So 2 2 0 1 0 2 1 1 4 4 a t b a a t q q V dr dr k k r r ? ? ? ? ? ? ?? ?? ? ? ? ? ? ? ? 0 1 2 1 1 1 1 1 1 4 q k a a t k b a t ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? 0 1 2 1 4 b a t q l t k k a a t b a t ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? a b b-t K 2 K 1 K 2 K 2 Fig. 1.32

NSOU ? CC-PH-08 ? 49 ? ? ? ? ? ? ? 2 1 0 1 2 4 k bt k a b a t q k k ab a t ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? 0 1 2 2 1 4 ... ... 1.10.12 k k ab a t q C V k bt k a b a t ?? ? ? ? ? ? ? ? ? ? (d) Capacittance of Cylindrical Capacilor with dielectrics The fig (1.33) shows a cylindercal capacitor consisting of two co-axial cylinder of radius a and b. (b < a). A dielectric of Thickness t and dicelctric conat k i is attached with inner cylinder and the rest portion is filled in with dielectric of dielectric constant k 2 as shown in fig (1.33) E 1 and E 2 be the electric field in first and second dielectric Then applying Gauss's Thorein as pervious 1 0 1 ˆ 2 r E k r ? ? ? ?? and 2 0 2 ˆ ** 2 r E k r ? ? ? ?? So the potential difference between inner and outer cylinder ** ? = charge/unit length on inner cylinder 1 2 · · a t b a a t V E dr E dr ? ? ? ? ? ? ? ? ?? ? ?? ? 0 1 0 2 2 2 a t b a t a dr dr k r k r ? ? ? ? ? ? ? ?? ?? ? ? 0 1 2 1 1 ln ln 2 a t b k a k a t ? ? ? ? ? ? ? ? ? ?? ? ? ? ? 0 1 2 1 1 ln ln 2 q a t b L k a k a t ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? 0 1 2 2 ... ... 1.10.13 1 1 ln ln L q C V a t b k a k a t ?? ? ? ? ? ? ? ? ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 50 (c) Capacitance between two long parallel wires each of radius r separated by a distance d The fig (1.34) shows such an arrangement where length of wire L < < a and d < < r the radius of wire. Consider a point P at a distance at a distance x from wire A, ? be the charge per unit length of wire A and – ??be that of wire B. The inlensity of field at P 0 1 1 ˆ 2 E x x d x ? ? ? ? ? ? ? ?? ? ? ? ? ?? So the potentrial difference between A and B ? ? 0 1 1 · 2 r d r d r r V E dx dx x d x ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ? 0 ln ln 2 d r x d x ? ? ? ? ? ? ? ? ? ? ?? 0 0 ln ln ln 2 d r r d r r d r r ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ?? ? ? ? Thus the capacitance per unit length ? ? 0 ... ... 1.10.14 ln C V d r r ?? ? ? ? ? ? ? 1.11 Electrical Image Electrical imaging is a mathematical method of solving electrostatic problems introduced by Lord Kelvin. In this method the induced charge appeared on the surface (surfaces) due to the presence of point charge (charges) at proximity to the surface (surfaces) can be replaced by suitable point charge (charges) called image charge (charges), on one side of the surface. The field and potential on the other side of the surface can be evaluated by the superposition of field and potential due the charge (charges) and their image charge (charges). Here also the Lapalce's equation is satisfied as with the actual carges and hence uniqueness theorem is followed. Fig. 1.34 wire A B P x d-x d

NSOU ? CC-PH-08 ? 51 Thus, electrical image (images) can be defined as a ficitious point charge (charges) on one side of a surface (surfaces) that can replace the effect of induced charge (charges) on other side of the surface (surfaces). (1) Example–1 A point charge q is placed at (a, 0) in front of a infinitely extended earthed coducting sheet occupying (y, z) plane. Find i) the position of image charge ii) magnitude and nature of image charge iii) Potential at a point x < 0 iv) field intensity on the surface of the sheet v) force between the charge and the conductor. vi) charge density at any point on the surface of the conductor. vii) total induced charge on the surface. Solution The fig (1.35) shows a point charge q at position vector ˆ ia with respect to the origin O, in front of a large earthed coducting plane occupying y – z q i plane, be the image charge as shown in fig. (1.35). i) From the symmerty of the field q i lines is to be placed at ˆ ia ? ii) From the condition of zero potential, at any point C on conducting plane 0 1 0, 4 i q q AC BC ? ? ? ? ? ? ? ? ?? ? ? we have q i = –q iii) The potential at point P (x, y, z), V = V 0 + V i = potential due to source charge q + potential due to image charge. ? ? 0 1 ... ... 1.11.1 4 ˆ ˆ q q r ia r ia ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? Fig. 1.35 ? q ? q (-a,o a,o) O P(x,y,z) r 1 ? Fig. 1.36
NSOU ? CC-PH-08 ? 52 Where ˆ ˆ ˆ r ix jy kz ? ? ? ? is the position vector of point P, ˆ ˆ ˆ , , i j k are the unit vectors along x, y and z axis. Thus, ? ? ? ? ? ? 2 2 2 2 2 2 0 1 1 ...... 1.11.2 4 q x

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a y z x a y z ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? iv) The field intensity at point P (x, y, z) along x, y and z axis are ? ? ? ? ? ? ? ? 3 2 3 2 2 2 0 2 2 2 2 4 x q x a x a E x x a y z x a y z ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3 2 3 2 2 2 2 2 2 2 y y y E y x a y z x a y z ? ? ? ? ?? 3 2 3 2 2 2 2 2 2 2 z z z E y x a y z x a y z ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

On the surface of the conducting sheet x = 0, so only x-component of the field survive and ? ? 3 2 2 2 2 0 2 4 x aq E a y z ? ? ?? ? ? acting along –x axis. ... ... (11.11.3) v) The induced charge can be replaced by the image charge –q fig (1.37). So the force betwee the surface and the charge q is ? ? 2 2 0 4 2 q F a ? ?? and is attrative. v) ? be the surface density at a point (y, z) on the conductor face as in fig. (1.4.5) Then Fig. 1.37 ? q ? q (-a,o) (a,o) Y (y,z) x z
NSOU ? CC-PH-08 ? 53 ? ? ? ? ? ? 0 3 2 3 2 2 2 2 2 2 2 2 ... 1.11.4 4 2 aq aq E a y z a y z ? ? ? ? ?? ? ? ? ? ? ? ? ? ? vii) To calculate the total charge on the coductor we consider a ring of radius r and thickness dr about the origin 0 through the point (y, z). Then the area of the elemental ring dA=2 ?rdr, charges on the ring = 2 rdr ? ? so the total charge is reduced. ? ? ? ? ? ? 3 2 3 2 2 2 3 2 2 0 0 0 2 2 ... ... 1.11.5 2 2 aq dr aqrdr dr q a y z a r ? Problem–1 Find the work done in removing a charge q placed in front of earthed coductor at distance 'a' from the conductor. Solution. From the conception of electrical image, the induced charge can be replaced by –q charge on the other side of the surface at the same distance from the surface in which the charge q is present. The force of attraction between the charge q and the induced surface charge, when their distance of separation x is given by 2 2 0 16 q F x ? ?? Then the work to separate the charge to infinity is, 2 2 2 0 0 . , 16 16 d q q W F dr dx d x ? ? ? ? ?? ?? ? ?? ? which is the energy required for the separation. 1) Point charge placed between two large intersecting conducting sheet perpendicuar to each other. The fig (1.38) shows a z = constant section of a large coducting plane occupying (x – z) and (y – z) plane. Considering the conditions to be satisfied by the charges Y O Z M N (-a,b) P (a,b) ? q q ? q (a,-b) (-a,b) Fig. 1.38
NSOU ? CC-PH-08 ? 54 1. Laplaces eqn. at all places excipt at point P ; 2. Potential over the conductor to be zero 3. Potential at infinity to be zero, Thus proceeding in the same way as previous for infinite earthed conducting plane, the image charges will be –q at (–a, b) ; + q at (–a, – b) and –q at (a, –b) as shown infig. The potential at any point A (x, y) ? ? ? ?? ? ? ?? ? ? ? ?? ? ? ? ? 0 1 1 1 1 ˆ ˆ ˆ ˆ ˆ 4 q V i x a j y b i x a j y b i x a j y b i x a j y b ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? The intensity E ?? can be evaluated by eqn. ? ? . ... 1.11.5 E V ? ?? ?? (2) Point charge in front of an earthed conducting sphere. Let a point charge is placed infornt of an eirthed conducting sphere of radius a. Let the induced charges be replaced by image charge q ? at a distane d ? from centre O on the line OP due to symmetry of induced charge. We must follow the conditions i) Leplaces equation to be satisfied at all point r < a, except the point P ii) Potential over surface of sphere be zero iii) Potential at infinity from charges to be zero Now from condition (ii), potentia on surface of sphere, 0 1 0 4 S q q V PR MR ? ? ? ? ? ? ? ? ? ?? ? ? ? 2 2 2 2 0 1 0 4 2 cos 2 cos q q x d ad a d ad ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? Fig. 1.39 Q(r, ?? R r ? ? ? a d´ q´ L P d q d+a

NSOU ? CC-PH-08 ? 55 2 2 0 2 2 1 0 4 2 1 2 cos 1 cos q a q a d d a a a d a d ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? As this equation is valid for all values of ??? we can write & d q d q a a d a ? ? ? ? ? ? Or, ? ? 2 & ... ... 1.11.7 qa d d a d q q a d ? ? ? ? ? ? ? Thus we know the position, quantity and sign of image charge. Potential at point Q. 2 2 2 2 0 1 4 2 cos 2 cos Q a d q V r d rd r d rd ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Or, 2 2 2 2 4 2 0 1 4 2 cos 2 cos Q q a V r d rd r d a rda ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? Q r V E r ? ? ? and 1 Q V E rd ? ? ? ? ? ? ?? can be calculated ... ... (1.1.8) Or, ? ? 1 2 2 1 2 2 2 2 2 2 0 2 cos 2 cos 4 Q q d V r d rd a r rd a ?? ? ? ? ? ? ? ? when r = a i.e. wehen, Q is a point on the surface of the sphere, then V Q = O In order to find the The surface density on the surface of the sphere, we must know The radial component of the electric field. 0 4 Q r V q E r ? ? ? ? ? ? ? ??

NSOU ? CC-PH-08 ? 56 ? ? ? ? 2 2 3 2 3 2 2 2 2 2 2 2 cos cos ... ... 1.11.10 2 cos 2 cos d r d r d a d r r d rd a rd a ? when the Q point is on the surface of the sphere, when r = a, then ? ? ? ? 2 3 2 2 2 0 4 2 cos r r a d a q a E a d ad ? ? ? ? ?? ? ? ? ? ? ? 2 3 0 4 q a d a l ? ? ? ?? where ? = RP = ? ? 1 2 2 2 2 cos a d ad ? ? ? Or, we can write ? ? ? ? 2 3 0 ... ... 1.11.12 4 n r r a q E E a d a l ? ? ? ? ? ? ? ? ? ?? Induced charge density at on the sphere : If ??be the induced surface charge density at the point R on the spare. ? ? 2 0 3 4 n q E a d a ? ? ? ? ? ?? ? ? ? 2 2 3 ... ... 1.11.13 4 q d a a l ? ? ? ? ? ? ? ? ? ? Since d < a, equation (1.11.13) shows that ? is negative. The magnitude of the surface charge density is maximum when ? = O, i.e., l is minimum, and minimum when ? = ?. The ratio of manimum and the minimum surface charge density is given by,

NSOU ? CC-PH-08 ? 57 ? ? 3 3 min man l d a l d a ? ? ? ? ? ? ? ? ? Force exerted on the point charge + q at P. which is altractive, ? ? ? ? 2 2 2 0 0 4 4 q qa d qq F LP d a d ? ? ? ? ? ?? ?? ? ? ? ? ? ? ? 2 2 2 2 0 ...... 1.11.14 4 q ad d a ? ? ?? ? (3) Point charge in front of an unearthed condcting sphere. The fig (1.40) show an unearthed conducting sphere of radius a. Here to calculate the image charge and its position the following conditions must be satisfied : i) Laplaces equation for r < a except the point P, ii) Total charge induced is zero. iii) Sphere surface is at constant potential. iv) Potential at infinity is zero. So, this case will be similar to the earthed conducting sphere condition with a charge a q d ? is to be placed at the centre of conducting sphere to satisfy the condition (ii) as mentioned in fig (Fig.1.40). So the surface potential now will be 0 1 4 S qa d V a ? ?? 0 1 4 q d ? ?? instead of zero. The potential at point Q will be Fig. 1.40 a ? ? q +qa – d R r ? Q q d P -qa – d

NSOU ? CC-PH-08 ? 58 ? ? 2 2 2 2 4 2 0 1 ... ... 1.11.4 4 2 cos 2 cos Q q a a V rd r d rd r d a ra d ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? The radial component of field at ? ? , Q r? ? ? ? ? ? ? ? 2 3 2 3 2 2 2 2 2 4 2 0 cos cos 4 2 cos 2 cos Q r ad rd a V q r d E r r d rd r d a ra d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? 2 ...... 1.11.5 a r d ? ? ? On the surface of sphere (r = a) ? ? ? ? ? 2 3 2 2 2 0 1 1 ... ... 1.11.15 4 2 cos n q d E a a ad a d ad ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? So the induced charge per unit area on the surface, 0 n E ? ? ? will be maximum for 0. ? ? .... (1.11.16) ? ? 2 2 3 2 2 2 1 4 2 cos q d a a d a d ad ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Let us see the nature of nature of the induced charge, whether positive or negative when 0 ? ? ? ? ? 2 2 3 1 4 q d a a d d a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 3 4 q ad a a d a ? ? ? ? ?

NSOU ? CC-PH-08 ? 59 Which is clearly negative. Now when ? ? ? ? ? 2 2 3 1 4 q d a a d d a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 3 4 a q a d d d a ? ? ? ? ? ? ? ? ? ? ? ? Which is ? positive charge density There is no induced charges on the sphere separating the regions of positive and negative charge density : NOW, putting 0 ? ? in eqaution, we have for the line of non- electrification. ? ? 2 3 2 2 2 2 2 N a d d d a Cos ad ? ? ? ? ? ? ? ? ? Which is positive, it implies that 2 N ? ? ? Force experienced by the charge at P ? ? ? ? ? ? 2 2 2 3 2 2 2 2 2 3 2 2 2 0 0 0 2 4 4 4 ad q q a q a d a F d a d d d a ? ? ? ? ? ? ? ?? ?? ?? ? 1.12 Summary After studying the unit we should understand following : 1. Interaction between two charges through Coulomb's law and for a cluster of charges. 2. Conception of electric field electric field ?? E lines. Conservative nature of ?? E and introduction of electric potential V through . ? ?? ?? E V (a) Point charge 2 0 0 1 1 4 4 ? ? ? ? ? ? ?? ?? ? r q q V dr r r and 2 0 1 ˆ 4 ? ?? ?? q E r r

NSOU ? CC-PH-08 ? 60 (b) due to a uniformly charged ring on its axis at a distance x from its centre ? be The charge per unit length. 2 2 0 1 , 4 ? ?? ? q V a x ? ? 3 2 2 2 0 1 , 4 ? ?? ? ? ???? P qx E a x 2 . ? ? ? q a (c) due to a charged disc (? = charge per unit area) 2 2 0 ; 2 ? ? ? ? ? ? ? ? ? ? ? ? P V R x x ? ? 2 0 ˆ 1 cos 2 ? ? ? ? ?? ?? P q E x R (d) due to spherical distribution of charge (i) Uniformly charge spherical shell. (a) 1 0 1 , 4 ? ? ?? i x a V q a 0 ? i E (b) 0 0 1 , 4 ? ? ?? q x aV x 0 2 0 1 ˆ 4 ? ?? q E x x (ii) Uniformly charge sphere. P = charge density 3 2 0 ˆ 4 3 1 , 4 ? ? ? ?? i x px E x 0 2 0 ˆ 1 4 ? ?? q x E x Gausse's law and its and its application for various distribution of charge. Laplace and Poission's equations Poission's eqn 2 0 ? ? ? ? ? P Laplace's eqn 2 0 ? ? ? ? and their one dimensional soluns. Eletrostatic energy 0 1 1 ; 2 4 ? ?? ? qiqj U rij 1 · 2 ? ? ?? ?? V U E Ddv Study of capacitors Capacitance of parallel plate capacitor 0 ? ? ? ? C A x Capacitors in series ? ? 1 1 ? ? ceq c a O V P E x ? + ? R P x + a P ++ x – P

NSOU ? CC-PH-08 ? 61 Capacitors in parallel ? ? eq C c Energy loss in capacitor ? ? 2 1 2 1 2 1 2 1 2 ? ? ? c c v v c c due to sharing of charge 4. Capacitance of spherical apacitor 0 4 ? ?? c R Two concentric spheres, outer sphere grounded 0 4 ? ?? ? ab c b a Capactance of cylinder 2 ln ? ?? c L b a Spherical capacitor with dieteclric ? ? ? ? 0 1 2 1 2 1 4 k k a t k c k bt k a b a t ?? ? ? ? ? ? Capacitance of cylinderical capactor with dielectric, 0 1 2 2 1 1 ln ln L c a t b k a k a t ?? ? ? ? ? ? ? ? ? ? ? Capacitance of two parallel wires, 0 ln ?? ? ? c d r r 5. Electrical image Point charge in-fornt of a earthed unifinite cmducting plane. Point charge infornt of earhed conducting sphere Point charge in front of unearthed conducting sphere 1.13 Review question and answer QNO 1 Why the electric field inside a good condutor is zero in a steady state and any net charge on a good conductor must be entirdy on the surface? Answer : If there were field, charges would move, charges will move until they find the arrangement that makes the eletric field zero in the interior. if therby where charge in the interior, Then by Gaussn law there would be a field in the interior, which cannot be true.

NSOU ? CC-PH-08 ? 62 QNO 2 Why do electric field lines never cross each other? Answer : It is so because if they cross each other then at the point of interessection there will be two tangent's which is not possible. QNO 3 What is the net amount of charge on a charged capacitor? Answer : The net charge of a charge capactior is zero because the charge on its two plates are equal number and opposite in sign. Even when the capacitor is discharged net charge on the capacitor remains zero because each plate has zero charge. QNO 4. How does the field line and an equapotential surface behave? Answer : They are always at 90º QNO 5. What is the power dissipoted in a pure capacitor? Answer : Zero QNO 6. What will be the potential difference between the plates when a dielctric slab is introduced in parallel plate capacitor? Answer : decrease. QNO 7. A point charge q is held at a distance d infornt of an infinite grouded conducting plane what is the electric potential infornt of the plane? Answer : See Article 1.11. for answer. QNO 8. A point charge q is placed at a distance d from the centre of a grounded conducting sphere of radius a (a > d). Calculate the density of the induced surface charge on the sphere? Answer : See article 1.11 for answer. QNO 9. The concentric spheres of radii r 1 and r 2 (r 1 < r 2) carry electric charges + Q and –Q respectively. The region between The plates is filled with two insulating layors of dielectric constant ? 1 and ? 2 with widths d 1 and d 2 respectivly. Compute the capacitane of the system Answer : See article 1.10 C for answer. QNO 10. Which one of the following is an impossible in electrostatic field?– i) ? ? ? ? ˆ ˆ ˆ 2 3 E xyi yz j xz k ? ? ? ? ?? ii) ? ? ? ? ? 2 2 ˆ ˆ ˆ 2 2 E y i xy z j yz k ? ? ? ? ? ??

NSOU ? CC-PH-08 ? 63 Answer : For solution, if 0, E ?? ? ?? ?? Then That electric field exists in electrostatics correct answer is (1) QNO 11. There charges Q, + q and + q are placed at the vertices of right angled isoscles traingle as shown in the figure. What is the value of electrostatic energy? Solution : Length of the hypotenase 2 , a ? the net electro static energy is– 0 1 4 2 Qq Qq qq U a a a ? ? ? ? ? ? ? ? ?? ? ? QNO 12. Three infinite long plane sheets carrying unfirom charge densitics ? 1 = ?, ? 2 = + 2 ?, and ? 3 = + 3 ? are placed parallel to × z plane at z = a, z = 3a, and z = 4a as shown in the figure.... what is value of electric field at the point Q? Solution : The electric field a point Q due to an infinite long plane sheet carrying uniform charge density is given by– 0 2 E ? ? ? which is independent of the distance of point Q from the sheet and is, therefore uniform The direction of the electric field. is away from the sheet and prependicular to it if ? is positve and it towards the sheet and perpendicuar to it if ? is negative so ? ? 1 0 ˆ 2 E k ? ? ? ? ? ?? along – ve z direction ? ? 1 0 2 ˆ 2 E k ? ? ? ? ? ?? along –ve z direction and ? ? 3 0 3 ˆ 2 E k ? ? ? ? along – ve z direction From the superposition principle. The resultant electric field at point Q is 1 2 3 E E E E ? ? ? ?? ?? ?? ?? Q +q -q a Fig. 1.41 ? ????? ? ? ????? ? ? ????? ? Q Z Z=4a Z=3a Z=a X Fig.1.42

NSOU ? CC-PH-08 ? 64 ? ? ? ? ? ? 0 0 0 2 3 ˆ ˆ ˆ 2 2 2 k k k ? ? ? ? ? ? ? ? ? ? ? ? ? 0 3 ˆ k ? ? ? ? 1.14 Problems & Solutions QNO 1. The electric field in a certain region is given as 3 ˆ. E Ar r ? ?? Prove that charge contained within a spherical surface of radius 'a' centred at the origin is 5 0 4 . Aa ?? Solution : From The differential form of Gauss's in law 0 ·E ? ? ? ? ?? ?? So the charge density ? ? 2 3 0 2 1 r Ar r r ? ? ? ? ? ? ? ? ? ? ? (using spherical polar co-ordinate) 2 0 5 Ar ? ? ? Total charge within a sphere of radius 'a' is ? ? 2 0 4 a Q r r dr ? ? ? ? 4 5 0 0 0 20 4 a A r dr Aa ? ?? ? ?? ? QNO 2. The electrostatic potential due to a charge distribation is given by ? ? 0 4 r q e V r r ? ? ? ? ?? enclosed within a sphere of radius 1? given by 2 . q e Solution : Given ? ? 0 4 r q e V r r ?? ? ?? So the electrical field is ? ? 2 0 ˆ 4 r r re e q E V r r ?? ?? ? ? ?? ? ? ?? ? ? ? ? ?? ? ? ? ? ?? ??

NSOU ? CC-PH-08 ? 65 ? ? 2 0 ˆ 1 4 r q e r r r ?? ? ?? ?? So the total charge enclosed within a sphere of radius r is 0 . encl Q E ds ? ? ? ?? ? ? ? ? 2 2 0 2 0 0 0 ˆ ˆ 1 ·sin 4 r q e r r d d drr r ?? ? ? ? ? ?? ? ? ?? ? ? ? ? 1 r qe r ?? ? ? ? Thus the total charge enclosed within a sphere of radius 1 r? ? is ? ? 1 1 encl Q Qe ?? ? ? ? ? ? 2 q e ? QNO 3. Two large non-conducting sheet one with a fixed uniform positive charge and another with a fixed uniform, negative charge are placed a distance of 1 meter from each other. The magnitude of the surface charge densities are 2 10c m ? ? ? ? and 2 5c m ? ? ? ? for the positive plate and negative plate respectively. What is the electric field in the between the sheets? Solution : The electrical field between the sheet is 0 0 2 2 e ? ? ? ? ? ? ? ? ? 6 0 1 10 5 10 2 ? ? ? ? ? 6 12 15 10 2 8·86 10 ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 66 6 ·8465 10 ? ? 5 8·46 10 N C ? ? QNO 4. A long cylinder carries a charge density ? = Ar ; for some constant A. Find the electric field inside the cylinder. Solution : We draw Gaussian cylinder of length l, and radius r. For this surface, Gauss's law state that 0 1 · encle S E ds Q ? ? ? ? ?? ??? ? So the enclosed charge is ? ?? ? 2 0 0 0 e r encl Q dV Ar rdr d dz ? ? ? ? ? ? ? ? ? ? 2 3 0 2 2 3 r Al r dr Alr ? ? ? ? ? Now from symmetry, it is cleart that E ?? is radially out word, so for the curved portion for the Gaussian surface we get · 2 E ds E ds E rl ? ? ? ? ? ? ?? ??? Hence 3 0 2 2 3 E rl Ar ? ? ? ? ? ?? So finally 2 0 1 ˆ 3 E Ar r ? ? ? ?? QNO 5. A hollow, conducting sphereial shell of inner radius 'a' and outer radius be encloses a charge –'q' inisde, which is located at a distance d > a from The centre of the sphere what is the potential of the centre of the shell? Solution : charge induced on the inner sphere is + q and charge induced on the outer sphere is + q Thus the potential at the centre of the sphereical shell is

NSOU ? CC-PH-08 ? 67 0 1 4 q q q V d a b ? ? ? ? ? ? ? ? ? ?? ? ? 0 4 q q q q a d b ? ? ? ? ? ? ? ? ?? ? ? QNO 6. Given the potential 5 sin cos V r ? ? ? (a) Find the electric flun density D ?? at ? ? 1, 0 2 ? (b) Calculate the work done in maving a 100c? charge from a point (2, 30º, 120º) to B (4, 90º, 60º) Solution : 0 D E ? ? ?? ?? But 1 1 ˆ ˆ ˆ sin V V V E V r r r r ? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ? ?? ? ?? ? ? ? ?? ?? 2 2 2 ˆ sin cos 1 1 ˆ 5 cos cos sin r r r r ? ? ? ? ? ? ? ? ?? ?? ? ? ? ? At ? ? 1, ,0 2 ? 0 0 ˆ ˆ ˆ 5 0 0 D E r ? ? ? ? ? ? ? ? ?? ? ? ? ? ?? ?? 12 2 5 8·85 10 c m ? ? ? ? ? 2 44·25 c m ? work done . . B AB A W Q E dl QV ? ? ? ? ? ?? ??? ? ? B A Q V V ? ? 6 5 1 5 1 100 10 4 2 2 2 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 68 125J ? ? QNO 7. Consider two electric dipoles with their centres at fined distance of reparation. Show that if the angles of the dipoles make with the line joining ther centres are 1 ? and 2 ? and if 1 ? is held fined Then for equillibrium 1 2 tan 2tan 0 ? ? ? ? Solution : Configuration of the dipoles are show in the figure. We know that interaction energy between two dipoles, ? ? 2 1 21 1 2 1 2 3 sin sin 2cos cos P P W r ? ? ? ? ? ? ? The couple exerted by the first dipole on the second is ? ? 21 1 2 21 1 2 1 2 3 2 sin cos 2cos sin W PP r ? ? ? ? ? ? ? ? ? ? ?? At equilibrium 21 0 ? ? 1 2 1 2 sin cos 2cos sin 0 ? ? ? ? ? ? ? 1 2 tan 2tan 0 ? ? ? ? ? ? Proved QNO 8. Three electric charges

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are placed at the corners of an equilateral traingle ABC ? of length 'a'. Find the magnitude of the

dipole of the system. Solution : Taking B as the origin of two dimensional co-ordinate system B (O, O) 3 2 2 A a ? ? ? ? ? ? and ? ? ,0 c a The dipole moment the system as in equilateral traingle , ABC ? ? ? ˆ 3 ˆ ˆ 2 2 2 aj a P aql i q ? ? ? ? ? ? ? ? ? ? ?? ˆ 3aq j ? ? The magnitude of the dipole is 3qa ? r C 1 C 2 ? ? 2 ? ? 1 P 1 P 2 Fig.1.49 Fig.1.50 A (-2q) a a a q q

NSOU ? CC-PH-08 ? 69 QNO 9. A sphere of radius R centred at the origin carries charge density ? ? ,r ? ?, ? ? 2 · 2 sin R k R r r ? ? ? where k is a constant, and ? ? ,r ? ? are usual spherical co-ordinales. Find the appriomate potential for points on z axis far from the sphere. Solution : Monopole terme for potential ? ? 2 2 1 0 0 , sin R monopole V V dV r r d ?? ? ?? ? ? ? ? ? ? ? ? ?? ? ? ? Dipole term 2 cos 0 V r sdV ? ? ? ? ? and Qudruipole term ? ? 2 3 3 0 1 1 3cos 1 2 4 V r dV r ? ? ??? ? ?? ? ? ? 5 3 0 1 4 48 k R r z z ? ? ? ? ?? QNO 10. Find the capacity of concertric spheres with two dielectric. Solution : Let Q charge given on the inner sphere, so the electric field within the radius a to be 1 2 0 1 4 r Q E r ? ?? ? ? So the potential difference across 1 ? 1 2 0 1 · 4 · b b a a r Q dr V E dr r ? ? ? ? ?? ? ? ? ?? ??? 0 1 1 1 4 r Q b a ? ? ? ? ? ? ?? ?? ? Similary the potential difference across 2 ? ? 2 2 0 2 4 c b r Q dr V r ? ? ?? ? ? b a ? ? ? ? Fig.1.51

NSOU ? CC-PH-08 ? 70 0 20 1 1 4 Q b c ? ? ? ? ? ? ? ?? ? Total PD V = V 1 + V 2 Capacitance 1 2 Q C V V ? ? 0 1 2 4 1 1 1 1 1 1 r r a b b c ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Alternative method : Using spherical co-ordinate system, to find but the potential in at any point inside the dielectric, taking note that 0 V? ? ?? and, 0 V? ? ?? ? ? 2 2 1 0 V r r r r ? ? ? ? ? ? ? ? ? ? ? ? 2 0 V r r r ? ? ? ? ? ? ? ? ? ? ? ? 2 1 V r c r ? ? ? ? ? where c 1 is constant of integration Again intergating, in region I 1 1 2 c V c r ?? ? in 1 ? 1 2 2 C V c r ? ? ? ? ? in 2 ? Then from, 1 ˆ ,E grad v r ? ?? we get 1 1 2 ˆ , c E r a r b r ? ? ? ? ? ?? 1 2 2 ˆ , c E r b r c r ? ? ? ? ? ? ??

NSOU ? CC-PH-08 ? 71 1 1 0 1 2 ˆ r c D r a r b r ? ? ? ? ? ? ? ?? 1 2 0 2 2 ˆ r c D r b r c r ? ? ? ? ? ? ? ? ?? We observe that there are four unkowns, so we need four boundary conditions, to evaluate therm, They are, on , r a ? 1 1 2 a

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c V V c a ? ? ? on , r c ? 1 2 2 c c V V c c ? ? ? ? ? ? on 1 2 , ; r b V V ? ? i.e. 1 1 2 2 c c c c

b b ? ? ? ? ? ? ? on r = b, D n is continouons, so, nl nz D D ? or, 2 rl r D D ? i.e. 1 1 0 1 0 2 2 2 r r c c b b ? ? ? ? ? ? From, these boundary conditions, we obtain 'D' on the surface of the inner sphere of radius a ? ? ? ? ? ? 0 1 2 2 1 1 1 1 1 1 1 1 c a r r V V D a c b b a ? ? ? ? ? ? ? ? ? ? ? ? ? ? So the total charge on the inner sphere, will be 2 4a? times D on 'a' arg Capctance c a ch e C V V ? ? ? ? ? ? ? 0 2 1 4 1 1 1 1 1 1 c b r b r a ?? ? ? ? ? ? ?
NSOU ? CC-PH-08 ? 72 QNO 11. A dipole having a moment ? ? ˆ ˆ ˆ 3 5 10 i j k ? ? me m is loated at Q (1, 2, −4) in free space. Find V at P (3, 3, 4) Solution : Unit vector along the straight line ˆ ˆ 2 8 ˆ· 69 i j k PQ r ? ? ? Potential at the point P (3, 3, 4) ? ? ? ? ? ? 9 9 2 0 ˆ 3 5 10 · 2 8 10 ˆ· 10 69 69 4 i j k i j k P r r ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? 1·27 V m ? QNO 12. Three point charges are located as shown in the figure Fig (1.52) Find the approximate electric field at points far from the origin state your answer is spherical co- ordinates, and include The lowest orders in multipote expansion. Solution : Total charges Q = 3q − q − q = q 0 1 4 mono q V r ? ?? and dipole moment ˆ 3 p qaz ? so, 2 0 3 cos 4 dip qa V r ? ? ?? Therefore ? ? 2 0 1 3 cos , 4 q a V r r r ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? 2 3 0 1 3 ˆ ˆ ˆ , 2cos sin 4 q a E r r r r r ? ? ? ? ? ? ? ? ? ?? ? ? ? ?? ? ? ? ?? QNO 13 Consider an electric dipole ,P ?? which is is fined at a distance zo along the z-ams and at an orientation ? with respect to that axis, consider the xy plane as conductor at zero potential. what is the charge density on the conductor induced by the dipole Solution : As shown in the figure the dipole is ? ? sin ,0,cos P P ? ? ? ?? and its image dipole is ? ? sin ,0,cos P P ? ? ? ? ? ?? In the region z < 0, The potential at a point ? ? , r x yz ? Z 3q a a -q Y a -q X Fig.1.52
NSOU ? CC-PH-08 ? 73 is ? ? ? ? ? ? 3 2 0 2 2 2 0 0 sin cos 1 4 Px z z V r x y z z ? ? ?? 0 3 2 2 2 2 0 sin cos x z z P x y z z ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? The induced charge density on the surface of the coductor is given by 0 0 z z V z ? ? ? ? ? ?? ? ? ? 3 2 2 2 2 0 cos 2 P x y z ? ? ? ? ? ? ? ? ? 0 0 5 2 2 2 2 0 3 sin cos 2 Pz x z x y z ? ?? ? ? ? ? ? QNO 14. Two similar charges are placed at a distance 2d apart. Find appronimately, The minimum radius of a grounded conducting sphere placed midway between them that would neutralize their mutual repulsion. Solution : The electric field outside the sphere conresponds to the resultant electrical field of the two given charges + q and two image charges .q ? ? By the method electrical images. q a q d ? ? and they are to placed at the two sides of the centre of the sphere at the same distance 2 a d d ? ? from it For each charge + q, besides acted by repulsive force of + q, There is also the altraction exerted by the two image charge, For the resulant force to vanish, we must have 2 2 2 2 2 2 2 2 2 4 q a d q a d q d a a d b d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 4 8 3 2 1 3 5 .... q a a a d d d ? ? ? ? ? ? ? ? ? ? P′ ? P ? ? ? Z 0 Z 0 Fig.1.53
NSOU ? CC-PH-08 ? 74 2 3 2q a d ? The value of a (a > b) that satisfies the d bove requirement is given by, 8 d a? QNO 15. Chareges + q at points (q, 0 a) and −q at points (−a, 0, a) above a grounded conducting plane at z = 0, Find (a) The total force on charge + q (b) The work done against the electrostatic forces in arranging this distribution of charges (c) The surface charge density at the point (a, 0, 0). Solution : The method of image charges implies at + q (−a, 0, −a) and −q at (a, 0, −a). The resultant force exeted on +q at (a, 0, a) by other charges is ? ? ? ? ? ? 2 2 2 2 0 1 1 1 1 1 ˆ ˆ ˆ ˆ 4 2 2 2 2 2 2 q F i k i k a a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ?? 2 2 0 1 1 1 1 ˆ ˆ 4 4 8 2 8 2 4 q i k a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? Magintude of the force ? ? 2 2 0 2 1 32 F q a ? ? ?? Force is acting on × z plane and points to the origin along a direction at angle 45º to the x axis as shown in the figure. (b) we can build the system by bringing the charges + q and −q from infinity through the path 1 : , 0 L z x xy ? ? 2 : 0 L z xy ? ? ? Fig.1.54 a q′ q′ + q + q d d
NSOU ? CC-PH-08 ? 75 symmetrically to the points (a, 0, a) and (−a, 0 a) resectively. when the charges are at (?, o, ?) on path L 1 and (− ?, 0, ?) on Path L 2 respetively, each of the charges suffers a force ? ? 2 2 0 2 1 32 q l ? ?? whose direction is parallel to the direction of the path. so that total work done by the exernal forces is ? ? 2 2 0 2 1 2 32 a a q W Fdl dl l ? ?? ? ? ? ? ?? ? ? ? ? 2 0 2 1 16 q a ? ? ? ?? (c) Now take case of electric field at a point (a, 0, 0 +) just above the conducting plane; The resultant electric field intensity 1 E ?? produced by + q at (a, 0, a) and −q at (a, 0 −a) is 1 2 0 2 ˆ 4 q E k a ? ?? ?? The resultant field 2 E ?? produced by − q at (−a, 0, a) and + q at (−q, 0, a) is 2 2 0 2 1 ˆ 5 5 4 q E k a ? ?? ?? Hence the total field at (a, 0, 0 +) is 1 2 2 0 1 ˆ 1 5 5 2 q E E E k a ? ? ? ? ? ? ? ? ? ?? ? ? ? ?? ?? ?? So the surface charge density is 0 2 0 1 1 5 5 2 q E a ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? q (-a,o,a)? q (a,o,a) ? q (-a,o,a) ? ? q (a,o, a) Fig.1.55

UNIT 2 : Dielectric Properties of Matter 2.1 Objective 2.2 Introduction 2.3 Classification, of Dilectric Materials 2.4 Polarization 2.5 Gauss's Law in Dielectrics 2.6 Boundary Condition in Dielectric Medium 2.7 Energy Density within Dielectric Medium 2.8 Electronic Polarisation 2.9 Electric Field Inside a Cavity in Dielectric 2.10 Polar Dielectrics and the Langevin−Debye Formula 2.11 Some Speical Properties of Dielectric Material 2.12 Summary 2.13 Review question and answer 2.14 Problems and solutions 1.1 Objective In this unit you will be acquainted with microscopic as well as macroscopic properties of dielectric. Following topics will be converred : 1. Difference between polar and non polar dielectric 2. Explanation of polarisation and quantitative analys's of bound charges due to polarisation. 3. Idea of electric displacement vector and derivation of Gauss's Law in presence of dielectric. 4. boundary condition at the interface of two different dietectric medium. 5. To find the electric field in different structures/shapes of dielectric. 6. Moleular polarisation and its relation with dielectric constant. 7. Properties of differant types of dielectric.
NSOU ? CC-PH-08 ? 77 2.2 Introduction In electromagnetism,

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a dielectric is an insulator that can be polarised by an applied. electric field.

In our earlier study on electrostatic, we are acquainted with external featuring properties of them. The electric field become lessened, with introduction of dietectric media in place of vaccum, even the dietectic inside them. In this unit we will study The transformational properties of dielectric in presence of electric field. Basically. There are four mechanism of polarisation : (a) Electronic or atomic, polarisation This involves the displacement of the centre of the electron cloud around an atom with respect to the centre of its nucleus under the influence of electric field. (b)

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Ionic Polarisation The ionic polarzation occurs, when atoms formmolecules and is mainly due to a relative displacement of the atomic components of The molecules due to the influenece of electric field. (

c) Dipolar ar Orientation polarsation. This is due to orientation of the molecular dipoles in the direction of the field, which would otherwise to be distributed randomly due thermal agitation. (d) Interface or space charge polarisation This involves limited movement of charges resulting in allignent of charged dipoles under the electric field. It is usually observed at the grain boundaries or any other iterfae such as electrode material interface. Also we will study molecular level changes due to electric field, This changes are called polarization. Behaviour of bound charges along with the modified Gauss's law wil be explained in dietectric. 2.3 Classification Dielectric Material In our everyday experience, most of the materical, that we come into contact can be classified into two distinct branch−these are conductor and dielectric. There are many free electrons in conductor which are not attached to the atoms. They move freely every-where at random. Most of the metals have these properties and each atom has one on two such free electrons. There no free electrons in dielectric each electron is all attached to the atom/

NSOU ? CC-PH-08 ? 78 molecule. But the electrons can move slightly within the atom, as a consequence, negative and positive charges get slightly displaced. But in certain dielece trics centre of positive and negative charges of the atom donot coincide with the same point, and they tend to behave as electric dipole. Dielectric material can be classified into distinet catagories one is polar and the other is non-polar. (a) Polar dielectric : Dietetrics, in which each atom/molecule has permanent dipole moment even in absence of electric field is called polar dielectric. Let us illustrate an example of dipole moment behaviour of HCl,- Hydrogen and cholrine atom have one and seventeen electrons in their outer orbit, respectively. Their chage districbution is such that their centre of positive and negative charges concide with a single point location, but when they coalasce to form a HCL molecue, the one electron of Hydrogen atom goes to The surrouneling chlorine atom. So Hydrogen becomes positively charged and chlorine atom becomes negatively charged of HCL molecule. The molecule HCL is transformed into an electric dipole. Examples of dipolar molecules are water (H 2 O), Ammonia (NH 3), Carbondisulphide (CS 2) and Hydrogen sulphide etc. Dipole moment of different molecule is in the range of (1 – 20) coulmb/meter. (b) Non polar Dielectric : whose atom/molecule of Dielectric material does not have permanent dipole, is called non polar dielectric. In spite of having no permanent dipole, for non polar dielectric, atoms/molecules of the dielectric, or the dielectric as a whole, can be trans formed to have dipole moment under the influence of external electrical field. Examples of nonpolar molecules are H 2 , N 2 , CO 2 , CCl 4 , etc. 2.4 Polarisation When a dielectric material placed inside electrical field, then each atom/molecute becomes converted to dipole. In additon to this, if the material is polar in character, then atoms molecules become more polarised. Average dipole moment generated under the influence of the electric field ,E ?? alligned along the field, is termed as molecular polarsation, and denoted by the symbol P ?? If n is the number of molecule/atom, per unit volume, then, ? ? ... ... 2.4.1 P np ? ?? ?? P ?? is defined as polaraisation sation per unit volume of the dielectric. Its unit is c/m 2 . Let us take an infinetisimal parallelopiped of dielctric of length ' ?' and cross-section s? placed in an electric field E ?? alligned along the length (Fig. 2.1). Induced surface density

NSOU ? CC-PH-08 ? 79 charge ' and b b ? ?? will appear on the plane perpendicular to the direction of the electric field which is due polarsiation. Positive charges will mutually be neutralized by the negative charges inside the dielectric material. Total charges b S ?? ? and S b ?? ? ? will appear at the two terminal end surface of the dielectric material. So the total dipolemoment polarisation of parallelopiped will be . b Sl ? ? Again the volume of the dielectric is , Sl ? and its dipole moment is . P Sl ? ?? So, . . . b S l P S l ? ? ? ? ? ? ...... 2.4.2 b P ? ? ? So induced surface charge density on the surface perpendicular to the direction of the electric field is equal to the value of polarisation vector. Even if the plane surface is not perpendicular to the electric field, it can be shown that induced surface density of charge will be ? ? . ... 2.4.3 b P n? ? ?? ? Where n ? is the unit vector perper clicular to the surface. 2.4.1 Electrical field due to polarised dielectric When applied electric field causes polarisation in dielectric, dipole moment is developed in the dielectric. So these dipoles will certainly create electric field on its own. Let us try to find the potential due to this dipoles.

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Potential due to a single dipole P ?? is given by ? ? ? ? 2 0 ˆ 1 · " " ...... 2.4.4 4 " Pr V r r ? ?? ?? ??? Where r?? ? is the vector from the dipole to the point at which we are finding the potential

Fig. 2.1 P ? E ? ?

NSOU ? CC-PH-08 ? 80 (Fig. 2.2) Now the dipole moment P pdv ? ? ?? ?? in each volume element , d ? ? so the potential is ? ? 0 ˆ ('). " 1 ... ... 2.4.5 4 " P r r V d r ? ? ?? ? ? ? ? But we know from vector, ? ? ' 2 ˆ 1 " " " r r r ? ? ? where the differentiation is with respect to the source co-ordinates ?? , r ? so we have, ? ? 0 1 1 · ' ' 4 " V P d r ? ? ? ?? ? ?? ?? ? On integrating by part's 0 1 4 1 '. ' (') ' " " v v V P d P d r r ? ?? using Gauss's divegence theorem ? ? ? ? 0 0 1 1 1 1 . · ... ... 2.4.6 4 4 " V P ds P d r r ? ? ? ? ? ? ? ?? ?? ?? ? ? ? ?? ? ? ? ? ? The first term of integration, we get potential due to

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surface charge ? ? ˆ· ... ... 2.4.8 b P n ? ? ?? Where ˆn is the unit normal vector to the surface. The second term of the intgrand will give the potential of a volume charge P r

Fig. 2.2
NSOU ? CC-PH-08 ? 81 ? ? · ... ... 2.4.9 b P ? ? ?? ?? ?? Using the above defined term, equation (2.4.6) can be written as, ?? ? ? 0 0 1 1 1 ...... 2.4.10 4 " 4 4 " b b S V r ds d r r ? ? ? ? ? ? ? ? ?? ?? ? ? ? ? ? ? ? ? The experssions for V contains two terms, first is ˆ· b P n ? ? ? ?? which is the due to induced surface charge. Second term is from the distribution of charge throughout the volume of the dielectric, · , b P ? ? ? ?? ?? ?? produced due volume density of charge. In our earlier discussion we have assumed uniform distribution volume charge density, i.e. · , P O ? ? ? ?? ?? as a result, excluding the outer surface charge density of the dietetric, every- where in the volue + ? and – ? volume density of charges neutrilizes each other. 2.4.2 Bound Charges We have observed that polarisation causes the appearance of surface charge on the outer surfaces and charges inside the volume of the dielectric. These charges are bound or fixed with the individual atom or molecule, and cannot move freely. So no current is generated due to this. So they are callted bound charges. The character of free charges in a conduction is different from the bound charges of dielectric as they can move freely to conduct electricity on with drawal of the external field, bound charges are removed due to reunification of positive and negative charges. 2.5 Gauss's Law in Dielctrics Electrostatic field in the dielectric material is modified due to polarization and not like as in vaccum. In the Fig. 2.3 (a) a parallel plate capacitor, of metallic plate, each of area S, medium is vacuum inside them. In the Fig. 2.3 (b) An identical capacitor with a dielectric inside them. Both the plates of each capacitors are charged with +q and –q, respectively. A gaussian surface abcd encirculing the +q charges have been drawn. whose ab and cd surfaces are parallel to the plates of the capacitor, other planes are perpendicular to the plate. When the medium is vacuum as in Fig 2.3(a) and the electrical field intensity is 0 , E ?? then applying Gauss's Theorem in abcd plane, we get, S being their area of plate, 0 0 . E ds q ? ? ? ? ?? ???

NSOU ? CC-PH-08 ? 82 ? ? 0 0 · ... ... 2.5.1 E S q ? ? Observe that Gaussian integral is non zero only for cd plane. Induced charge will be developed due to polarisation adjacent to the plates of the capacitor and two terminal end of the dielectric, as shown in Fig. 2.3 (b) Let q ? ? be The induced bound charges near the positive plate of the capactior. E ?? is the electrical field intensity inside the dielectric. Applying Gauss's Theorem in abcd plane, as before, we get ? ? ? ? 0 · . ... 2.5.2 E ds E S q q ? ? ? ? ? ? ? ?? ??? ? From equation (2.5.1) and (2.5.2) we get 0 a q q E S S ? ? ? ? ? ? ? ? 0 ... ... 2.5.3 O q E E S ? ? ? ? ? If V 0 and V is the potertial difference between two plates of capactior with and without dielectirc, then 0 0 E V k E V ? ? where k is dielectric constant As E > E 0 , k is greater than 1. From equation (2.5.3) we get b c ? ? ? q d Electricfield Dielectric Fig. 2.3b Fig. 2.3a

NSOU ? CC-PH-08 ? 83 0 0 0 q q q k S S S ? ? ? ? ? ? ? Or, ? ? ? ? 1 1 ... ... 2.5.4 q q k ? ? ? From equation (2.5.4) it is clear That q q ? ? Now equation (2.5.2) can be written in the following way 0 · q E ds q q k ? ? ? ? ? ? ? ?? ??? ? Or, ? ? 0 · . ... 2.5.5 k E ds q ? ? ? ? ?? ??? ? Equation (2.5.5) is more advantageous form of Gauss's law to apply in dietectric. As the quantity k has been included in intagral of the left side, there is no need to include induced charges on the right hand side of the integral. 0 k ? is the permittivity of of the dielectric medium and denoted by .? In our earlier discussion. q ? ? and q ? have been mentioned to be the charges residing on the end surfaces of diectric, Their surface density of charges will be b P ?? ? ? and , b P ? ? respectively. . From equation (2.5.3) we get, 0 E kE P ? ? ? Or, ? ? 0 0 ... ... 2.5.6 KE E P ? ? ? ? Here E and P are all vector quantities, Equation (2.5.6) can be written, in the following way ? ? 0 0 ... ... 2.5.7 KE E P ? ? ? ? ?? ?? ?? Expression on the right hand side of the equation (2.5.7) is termed as Maxwell electrical displacement and denoted by D ?? From equation (2.5.5) we get .D ds q ? ? ?? ??? ? Again we know , f q dv ? ? ?? where ? f is free volume charge density, and integral has been covered all over the gaussian surfaces. Applying divergence theorem, we can show

NSOU ? CC-PH-08 ? 84 that ? ? · ... ... 2.5.8 f D ? ? ? ?? ?? Equation (2.5.8) is the differential form of the Gauss's law in dielectric medium. Equation (2.5.7) can be written in the following way, ? ? ? ? 0 0 1 ... ... 2.5.9 P k E E ? ? ? ? ? ? ? ?? ?? ?? As k < 1, 1 k ? ? ? is positive, and denoted as electrical susceptibility. . 2.5.1 Relation between ,E P ?? ?? and D ?? The equation (2.5.7) written as 0 , D E P ? ? ? ? ?? ?? ?? then we can get a clear relation between , . D E and P ?? ?? ?? Though D ?? depends only on free charges, E ?? and P ?? vectors depend both on free and bound charges. In linear isotropic dielectric, , D E ?? ?? and polarization vector are parallel To each other, i.e. , D KE ? ?? ?? where k is a scalar quantity In nonlinear, anisotropic dietectric , D E ?? ?? and P ?? are not parallel. Here K is represented by tensor quantity. . 2.6 Boundary Condition in Dielectric Medium (a) Let AB be boundary between two dietectric media, which is homogenous and isotropic. Consider The interface between 1 and 2, and imagine small pill box shaped Gaussian surface intersecting the interface [Fig. 2.4 (a)]. Its height and the area covered by the curved surface is very small. Let S? be area cut out by the pill box on the interface. If ? be the surface chage density of free charge on the interface, The application of Gauss's law to The pillbox yields– ? ? 2 1 ˆ ˆ · · ... ... 2.6.1 D n S D n S S ? ? ? ? ? ?? ?? ?? Where ˆn is the unit vector pointing from medium 1 to medium 2. Flux over the curved surface is negligible and does not contribute to the equation (2.6.1) Neglecting negligible volume of the pill box, we get, ? ? ? ? 2 1 ˆ· ... ... 2.6.2 D D n ? ? ? ? ?? ?? Or, ? ? 2 ... ... 2.6.3 n in D D ? ? ?

NSOU ? CC-PH-08 ? 85 It is clear from equation (2.6.2) or (2.6.3), that discontinuity in the normal component of the electric displacement in moving from one medim to other medium is given by the surface density of free charge on the interface between the media. Normal compant of D ?? is conlinous across it when there is no free charge at the interfaec. (b) Let AB be The boundary between two dielectric media 1 and 2. Take a closed path PQRS across The boundary AB [Fig. 2.4 (b)] Its height QR and SP are very small and neghigible and the length PQ = RS = dl. Let 1 E ?? and 2 E ?? are electric field vectors in media 1 and 2 at an inclination 1 ? and 2 ? with the norma to the boundary The work done in moving an unit positive charge around the path PQRSP is zero .E dl O ? ? ?? ??? ? sin 0 E dl ? ? ?? ? ? 1 1 2 2 sin · sin · 0 E dl E dl ? ? ? ? ? ? ? 1 1 2 2 sin sin 0 ...... 2.6.4 E E ? ? ? ? But 1 1 1 sin t E E ? ? and 2 2 2 sin t E E ? ? E 1t and E 2t are The tangertial component of electric fields on both sides dielectric boundary, So, E 1t = E 2t ... ... (2.6.5) Thus

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the tangetial component of The electric field is continuons across The interface between two media.			

D 2n ? D 2 ? ? 2 A B ? 1 ? D 1 D 1n 1 2 a 1 2 A B S R P Q ? 2 ? 1 ? E ? E 1t E 2t ? b Fig. 2.4

NSOU ? CC-PH-08 ? 86 (c) Refraction of Eelectical lines of Force : we know, for charge free interface between two dielectric medium D 1n = D 2n Or, 1 1 2 2 cos cos D D ? ? ? Or, 0 1 1 1 0 2 2 2 cos cos K E K E ? ? ? ? ? ? Or, ? ? 1 1 1 2 2 2 cos cos ... ... 2.6.6 K E K E ? ? ? and ? ? 1 1 2 2 sin sin ... ... 2.6.7 E E ? ? ? So we get from equations (2.6.6) and (2.6.7) ? ? 1 1 2 2 cot cot ... ... 2.6.8 K K ? ? ? This is the law of refraction for electrical lines of force when K 2 < K 1 Then 1 2 cot cot ? ? ? or, 2 1 ? ? ? implies that when the dielectric constant of medium two is greater than medium one, electrical lines of force in medium two, will move away from the normal at the interface. This is in contrast opposite to the normal refraction of light rays. 2.7 Energy Density within Dielectric Medium Consider a system free charges embedded in a dielectric medium. The inerease in the total energy when a small amount of free charge f ?? is added to the system is given by ?? ? ? ...... 2.7.1 f W r dV ? ? ? ? ?? ? ? Where the integral is taken over all space, and ?? r ? ? is the electrical potential. Here we have assumed that the original charges and the dielectric and held fixed, so that no mechanical work is done. From equation (2.7.1) we get, ? ? · f D ? ? ?? ?? ?? ? ? · ... ... 2.7.2 W D dV ? ? ? ?? ? ? ?? ??

NSOU ? CC-PH-08 ? 87 Where D? ?? is the charge in electric displacement due to increase in charge. Using the vector identity, ? ? ? ? . . · D D D ? ? ? ? ?? ? ? ? ? ?? ?? ?? ?? ?? ?? We get ? ? ? ? · · ... ... 2.7.3 W D d Dd ? ? ? ?? ? ? ?? ? ? ? ? ? ?? ?? ?? ?? giving, ? ? · · ... ... 2.7.4 W D ds D d ? ? ?? ? ?? ? ? ? ? ? ?? ??? ?? ?? If the dielectric medium is of finite spatial extent, then we can neglect the surface term to give, ? ? · · ... ... 2.7.5 W Pd E Pd ? ? ? ?? ? ? ? ? ? ? ? ? ?? ?? ? ? Assuming 0 D KE ? ? ?? ?? where K is the dielectric constant, the change in energy associated while D ?? has been increased from 0 to ?? D r ?? at all points in space is given by ? ? 0 0 · ... ... 2.7.6 D D W W E Pd ? ? ? ? ? ? ? ? ? ?? ?? Or, ? ? 2 0 2 0 0 1 2 2 E K E W d KE d ? ? ? ? ? ? ? ?? ? Which rediuces to ? ? 1 · ... ... 2.7.7 2 W E Dd ? ? ? ?? ?? So the electrostatic energy density inside a dietectric medium is given by ? ? 1 · ... ... 2.7.8 2 U E D ? ?? ?? 2.7.1 Potential Energy of Dipole in Electrical Field When a dipole placed in an electric field, two equal and opposite forces F ?? and F? ?? on the charges q and −q, which construtiles a couple (Fig. 2.5).

NSOU ? CC-PH-08 ? 88 The moment of the couple or toque = Force × perpendicular distance But F = qE and from the , ABC ? BC = ABSin θ 2 sin qE l ? ? ? ? [as AB = 2l = length of the dipole] Now, p = 2lq = dipole moment sin PE ? ? ? which forms a vector, ? ? ...... 2.7.9 p E ? ? ? ? ?? ?? This dipole will be rotated by the couple ? in the direction of the field. Let dw be the work done in rotating the dipole Through a angle , d? . dw d ? ? ? Total work done w in rolating the dipole from angle 1 ? to 2 . ? is 2 1 · w d ? ? ? ? ? ? The work done is stored in the dipole as potential energy U ? ? 2 1 cos cos U W pE ? ? ? ? ? ? Or, ? ? 2 1 cos cos U pE ? ? ? ? ? If the initial and final positions are 1 90º ? ? and 2 ? ? ? Then, cos U pE ? ? ? Fig. 2.5 ? E ? E ? F ? F 2 ? -q +q B C A ?
NSOU ? CC-PH-08 ? 89 Or, ? ? · ...... 2.7.10 U p E ? ? ?? ?? 2.8 Electronic Polarisation Consider an atom in an electric field of intensity ' E ?? ', since the nucleus of charge +ze and surronding encircling electron cloud of charge '–

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ze' of the atom have opposite charges and acted upon by Lorentz force.

As a consequence, nucleus moves in the direction of The field and electron cloud in the

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opposite direction. As electron cloud and nucleus gets displaced from their normal equillibrium positions, an allractive force between them is built and the separation continues until coulomb force F C is balanced

by The

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Lorentz force F 2 , until a new equillibrium state is ereated. Let ? be the charge density of the sphere. 3 4 3 ze

R ? ? ? ? where '–ze' is

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the total charges in the sphere. So the negative charge in the sphere of radius x, 3 4 3

x q x ? ? ? ? ? 3 3 4 4 3 3 ze x R ? ? ? ? ? ? ? 3 3 ... ... 2.8.1 x ze q x R ? ? Fig. 2.6 +ze x R
NSOU ? CC-PH-08 ? 90 Allractive coulomb force between nucleus and electrons 2 0 1 4 x p C q q F x ? ? ? ? ? ?? ? ? 3 2 3 0 1 4 zex ze x R ? ? ? ? ? ? ? ?? ? ? ? ? ? 2 2 3 0 ... ... 2.8.2 4 C z e x F R ? ? ?? Force experienced by displaced, nucleus in electric field internsity E ?? is ? ? · ... ... 2.8.3 p L q F E Ze E ? ? ?? ?? ?? Since, , L C F F ? we can write ? ? 2 2 3 0 ... ... 2.8.4 4 z e x zeE R ? ? ?? Or, ? ? 3 0 ... ... 2.8.5 4 zex E R ? ? ?? Again we know e dipolemoment E? ? where e ? is represented as electronic polarizability. . From equation (2.8.5), we get 3 0 4 e zex zex R ? ? ? ? ?? So, ? ? 3 0 4 ... ... 2.8.6

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e R ? ? ?? Hence electronic polarisibility is directly proportinal to the radius of the atom.

NSOU ? CC-PH-08 ? 91 2.9 Electrical Field inside a cavity in the Dielectric Spherical Cavity : Let us imagine a speherical cavity inside the dielectric whose centre at O and radius 'r'. The size of the cavity is small compared to the dielectric material, but large enough compared to size of the molecule. The electric field inside The dielectric is E. Electrical force on a unit positive charge placed at O is E where P E E E ? ? ? E P is electrical intensity due to the induced charge on the surface of the cavity. We use spherical co-ordinate system to find E P . If P ?? is the polarisation vector, then surface charge density on the surface of the cavity ˆ · , P n ? ? ?? where ˆ n is the unit vector perpendicur to the surface so, total surface charge on an elementray area ds is ˆ · cos dq ds P nds Pds ? ? ? ? ? ?? Electrical field, intensity due to this charges at 'O' 2 1 cos 4 P Pds dE r ? ? ? ?? We will take the component which is parallel to E ?? , Horizontal component is 2 2 0 cos ' cos 4 P P Pds dE dE r ? ? ? ? ?? Now elemental surface area between ? and , d ?? ? is 2 2 sin ds r d ? ? ? ? S o , 2 2 ·2 sin cos P dE P r d ? ? ? ? ? Or, 2 0 sin cos 2 P P dE d ? ? ? ? ? Intequrating 0 , to ? ? ? ? ? we get ? ? 2 0 0 0 0 2 sin cos · ... ... 2.9.1 2 2 3 3 P p P P E d ? ? ? ? ? ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 92 So the total intensity at the centre of the cavity ? ? 0 ... ... 2.9.2 3 P P E E E E ? ? ? ? ? ? ?? ?? ?? ?? If the dielectric material is kept inside a parallel plate capacitor, Then electrical field intensity at the centre of the cavity is ? ? 0 ...... 2.9.3 m i P E E E E ? ? ? ? ?? ?? ?? ?? where 0 E ?? is the itensity due to the chargeson the capacitor plates. This field induces potarization inside the dietetric, and induced surface charge density + P and –P on the terminal surface of the dielectric 0 i E P ? ? ? ? ?? Electrical field inside the dielectric 0 0 0 i E E E E P ? ? ? ? ? ? ?? ?? ?? ?? ?? So the electrical field intensity at the centre of the cavity is less than 0 , E ?? but it is greater than the field inside the dielectric. 2.9.1 Atomic and Molecular Polarisation : Clausious–Mossotti selation Now we will, find out the relation between relative permillivity and molecular/atomic polarisation. Let us now explore the field intensity at the centre of sphere of radius 'r' inside the dielectric material. All the molecules inside the sphere gets polarised along with the polarisation entire dielectric. But all the dipoles inside it contributing to the field at the centre gets neulralised or mitigated due to the vector sum of the evenly distributed dipoles (fields). So m E ?? is effective intensity of a molecule kept at the centre 'O'. We know 0 0 D K E E P ? ? ? ? ? ? ?? ?? ?? ?? where E ?? is The electrical intensity inside the dielectric. Or, ? ? 0 1 P k E ? ? ? ?? ?? From m P E E E ? ? ?? ?? ?? ? ? 1 2 3 3 k k E E E ? ? ? ? ? ? ?? ?? ?? Or, 3 2 m E E k ? ? ?? ?? So, ? ? ? ? 0 3 1 ... ... 2.9.4 2 m P k E k ? ? ? ? ? ? ?? ??

NSOU ? CC-PH-08 ? 93 Or, ? ? ? ? ? ? 0 3 1 ... ... 2.9.5 2 m k P E k ? ? ? ? ? ?? ?? If P m is the dipole moment of molecule generated due the electric field intensity E m , so the dipole moment per unit field strength, so we say molecular polarisibility m m P E ? ? If 'n' is the numbe of molecule per unit volume, Treating m P ?? as vector m m P nP nE ? ? ? ?? ?? ?? Hence, ? ? ? ? 0 1 3 ... ... 2.9.6 2 k n k ? ? ? ? ? Above relation of equation (2.9.6) is known as clausius-Mossotti relation. Physical implication of this relation is that we can get the entrie macroscopic propertics i.e. we can get the value of molecular polarsability from the relative permillivity. Thus, from a measurement of k, it is possible to get important quautitative information about molecular structures. From electronic polarisation, we know that molecular polarisibility from equation (2.8.6) is 3 0 4 a ? ? ?? Using classius-Mossotti relation, we get ? ? 3 0 0 0 4 1 4 · ... ... 2.9.7 2 3 3 3 n k n n a V k ?? ? ? ? ? ? ? ? ? ? ? where V is the volume of total one unit volume of molecules. From equation (2.9.7), we have 3 3 1 · 4 2 k a n k ? ? ? ? Or, ? ? 1 3 3 1 · ... ... 2.9.8 4 2 k a n k ? ? ? ? ? ? ? ? ? ? Equation (2.9.8) gives the relation between atomic radius and dielectric constant (k). 2.10 Polar Dielectrices and The Langevin Debye Formula Molecules like CH 3 CL, H 2 O, HCl, cthyl acetate carries electric dipole moment

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even in the absence of electric field. However, The net dipole moment is negligiby small since all the dipoles under continous thermal agitation, are oriented randomly when there is no			

external electric field. In the presence externally applied field, individual dipoles experience

NSOU ? CC-PH-08 ? 94 torques, which tend to allign them along the field direction. As a result the net dipole moment becoems large. Polarisability has been calculated based on the principle of statistical Thermodynamics. Under this principle, in thermal eqnillibrium, the probability of finding a molecule with potential energy U is proportional to , B U K T e ? where k B is The Boltzman constat and T is the absolute temperature. The potential energy of a dipole moment P ?? in an electric field is, ? ? · cos ... ... 2.10.1 V P E PE ? ? ? ? ? ?? ?? Assuming, local field is solely to be the electric field, the probability that a dipole will have orientation ? with respect to the field is cos . B PE K T e ? If > P < is The average polarisibility of dipolar molecule, at a particular temperature is given by the Langevin formula ? ? 1 coth ... ... 2.10.2 P P ? ? ? ? ? ? where B PE K T ? ? The fig. (2.8) shows The variation of P P ? ? as a function of .? At large electrical field strengths or at low temperatares i.e. where 1, B PE K T ? ? ?? Lengevin predicts 1. P P ? ? ? which states that nearly all the polar molecules have been alligned with the electric field. i.e. almost saturation P P ? ? Fig. 2.8 >P< P 1.0 ??

NSOU ? CC-PH-08 ? 95 while for small values of ? ? 1 , ? ?? i.e. normal fields and higher temperatures equation (2.10.2) reduces to 3 P P ? ? ? ? Or, ? ? 2 ... ... 2.10.3 3 B P E P K T ? ?? which indicates a linear relationship between > P < and E. Thus a polar dielectric is normally linear. Now the polarsibility ? is defined as the molecular dipole moment per unit field ? ? 2 ... ... 2.10.4 3 P P E KT ? ? ? ? ? Equation (2.10.4) shows the temperature dependence of polarisibility. This equation holds pretty well for small values of P and E and for large enongh T which we can presume as normal conditions. The total polarization for dilute gas can be written as ? ? 2 ... ... 2.10.5 3 e i P KT ? ? ? ? ? ?? ? ? ? ? ? where e ? and i ? are electronic and ionic polarisibility, respectively. . Equation (2.10.5) is known as Langevin–Debye equation. From clausius–Mossolti equation and equation (2.10.5), we get, ? ? 2 0 1 ... ... 2.10.6 2 3 3 e i k n P k KT ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? P o l a r nopolar

NSOU ? CC-PH-08 ? 96 This equation is known as Debye equation. From this equation we can find the value of dipole moments and polarization from measurement of gases. For polar molecules 1 vs T ? will be a straight line. But equation (2.9.6) shows that for nonpolar molecules ? versus 1T graph will be a straight line parallel to 1/T – axis (Fig. 2.9) The intercept of the straight line for polar molecules gives the value of ? ? e i ? ?? and the slope of the line gives P. The variation of dielectric properties with the frequency of an applied ac field also interesting. Due to interia of heavy polar molecules they cannot follow the rapid change in the direction of the applied ac field. For this at higher frequencies (in the micro wave region of above) the polar contribution to the dielectric constant begins to fall with frequency. But because of smaller incrtia of electric the electronic polarisibility remeins almost unchanged upto optical frequences. 2.11 Some special Properties of Dielectric Material Here we will discuss some specific propertis of dielectric material which is of immense use in engineering and as sensors, etc 1. Ferroelectric Materials : These are crystalline materials that displays electrical polarisations switehable by an external field. Ferro electric crystals have high dietectric constant and each unit cell of ferroelectric crystals carries reverisble electrical cell. Ferro electric property depends on temperature and this property vanishes at a certain critical temperature–dietectric property vauishes rapidly with temperature. Relation between dietectric constant, temperature and critical temperature is given by 0 e C K K T T ? ? ? Here T c is the critical temperature, c is constant and K 0 is the contribution to dielectric component from electronic dielectric constant. Examples are Barium Titanate (BaTiO 3) sodium nitrate and Rochelle salt. 2. Piezeo Electricity : The process of creating electrical polarisation by mechanical stress is called piezo electric effect. Contrary to this, inverse piezo electric effect is observed, when electric field is applied–The material gets strained and directry proportional to the strength of the electrial field.

NSOU ? CC-PH-08 ? 97 Examples are, quartz crystal, Rochelle saltet. Among The piezo electric semiconductor are Gatts, ZnO and Cds – which are mainly used utrasomic amplifiers, electromic watch, microphone etc. 3. Electret : This can be considered a piece of diectric materical with the presence of quasi-permanent real charges on the surface or in the bulk of the material or frozen-in-alligned dipole. Some dipole moment remains–even when the electrial field is removed. Some organic paraffin, and some plastic exhibits these propertics. When These materials are polarised in molten state, and There after solidefied, they retain Their dipolar characteris tics, and a permancent dipole generated. It is called thermal electret. Some materials are called photo electret when are transformed by light and electric field to dipole propertics. 4. Dielectric Break down : All dietectric material retain their property until high enongh field to destroy their characteristics, allowing large flow of current, this happens due to removal of electrons from the atomic orbit by strong electric field mainly. Also some breakdown is observed by the effect of following agent–intrivnsic, thermal, electrochemical, deffect and discharge, breakdown. 5. Dielectric Relanation Time : It takes a certain amount of time for a dietectric to be fully polarised when subjected to an electric field. It is observed that the electromic and ionic polarisation is attained instaneonsly, if we cosider high frequencies (10 7 – 10 17 /sec) and not The optical frequencies. Dielectric loss, at these frequencies, is mainly due to relaxation effect of the permanent dipoles. A molecule in dielectric, which tries to allign with the applied electric field, is effected by the opposing forces of adjacent molecule. This is the phenomenon of relaxation. Polarisation of the dielectric, when influenced by an atterating electric field, does not conform to proportionatc transformational gain. Rather a hysterisis is observed in polarisation. It has been observed that the platies of the capacitor gets charged again even after being discharged to meutralise the plates from the first follow up charging. Hence, sometimes a certain amount small current flow has been observed Electrical energy is lost due to hyserisis of the dielectric and flow of current, dielectric material gets heated. Motecules cannot orient harmoniously and swiftly with high frequency alternating field. So There is no loss of energy due to hysteris.

NSOU ? CC-PH-08 ? 98 2.12 Summary 1. Dielectrics are insulators that support charge. The dielectric constant K indicates polarisibility of dielectric. Each of the polarisation mechanism has a chargeteristics relaxation time (frequency). 2. Gauss's law for dielectric is relates free charge to the displacement vector. E ?? and D ?? Follow the boundary condition at the interface of two different dietectric media. Refraction law has been duduced. 3. Microscopic properties have been discussed after studying macroscopic properties. Inter relation between molecular polarisabiity and dielectric constant has been deduced by studying the electric field intensity in microscopic spherical cavity deep inside dielectric substance. 4. Clausius–Mossotti reation has been deduced as 0 3 1 2 k n k ? ? ? ? ? [see equation 2.9.6)] Again, Molecular polarisibility 3 0 4 a ? ? ?? (see quation (2.8.6). combining these two equation, we have establised the relation between atomic radius (a) and dielectri constant (k). 5. We have alo established the expression of molecular potarisibility ?? ? for polar dielectrics given by 2 3 e i P KT ? ? ? ?? ? [see equation (2.10.5)] which is known as Langevin–Debye equation. 6. ?versus 1/T for polar and nonpolar dielectrics have been plotted (see Fig. 2.9) and importance of the graph has been discussed. 2.13 Review Questions and Answers 1. What is polar and non polar dielectric? How They behave in an exeternally applied electric field. Answer : See article 6.3. 2. Find the electric potential inside a polarised dielectric. Answer : See article 2.4.

NSOU ? CC-PH-08 ? 99 3. Prove The continunity of normal and tangentral component of electrical field intensity at the interface two different media. Answer : See boundary condition article 2.6. 4. Establish The claussius–Mossotti relation. Answer : See claussium–Mosotti relation, article 2.9.1. 5. Define orientation polarisation. Answer : When an electric field is applied in a dielectric medium with polar molecules, the electric field tries to allign these dipoles along its field direction, due to that there is a resultant dipole moment in the dielectric material and this process is called orientation polarisation. 2 3 P P KT ? ? 6. Define local or internal or Lorentz field. Answer : In a dielectric material. The field acting at the location of an atom is called as local field or internal field 'E i '. The internal field E i must be equal to the sum of the applied field and the field due to The location of the atom by the dipoles of all other atoms. E i = E + The field due to all other atoms. 7. What is electric polarisation? Answer : It is defined as production of electric dipoles by the applied field. It is due to the shifting of charges in the elietectric by the applied electric field. 8. Mention The different break down mechanism in dietectric material. Answer : (i) Intrinsic and avalanche break down. (ii) Thermal break down (iii) Chemical and electrochemical break down (iv) Discharge break down (v) Defect break down 2.14 Problems and Solutions 1.

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The dielectric constants of a Helium gas at NTP is 1·0000685. Calculate the electric NSOU ? CC-PH-08 ? 100 polarizability of Helium atoms if the gas contains 2·7 × 10 26 atoms/m 3 . Calculate the radius of the Helium atom. [Given 12 1 0 8·854 10

Fm ? ? ? ? ?] Solution : Relative permitivity 1·0000685 r ? ? No of atoms of the Helim gas N = 2·7 × 10 26 atms/m 3 Permitivity of free space 12 0 8·854 10 F m ? ? ? ? Now, Polarization ? ? 0 1 r P E ? ? ? ? and . e P N E ? ? Where e ? is electromic polarizability of Helium atom From above two equation, we can write ? ? 0 1 e r N? ? ? ? ? Or, ? ? 0 1 r e N ? ? ? ? ? ? ? 12 24 8·854 10 1·0000685 1 2·7 10 ? ? ? ? ? Hence 42 2 2·245 10 e Fm ? ? ? ? Again 3 0 4 e R ? ? ?? Where R is the radius of Helium atom 1 1 42 3 3 12 0 2·245 10 4 4 3·14 8·854 10 e R ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? R = ·272 × 10 −10 meter. Radius of the Helium atom R = ·272 × 10 −10 meter 2. An electric field intensity of strength 10 kV/m is applied across a parallel plate capacitor filled with dietectric constant 2·5 The distance between the plate is 2 mm calculate. NSOU ? CC-PH-08 ? 101 (a) D, (b) P (c) The surface density of free charge on the plates (d) The surface density of polarization charge (e) The potetial difference between the plates Solution : (a) 9 4 2 0 10 2·5 10 220·98 36 r D E nc m ? ? ? ? ? ? ? ? ? (b) 9 4 2 0 10 1·5 10 132·58 36 e P E nc m ? ? ? ? ? ? ? ? ? (c) 2 ˆ · 220·98 s n D i D nc m ? ? ? ? ? ?? (d) 2 ˆ · 132·58 ps Pi nc m ? ? ? ? ?? (e) V = E.d = 10 4 × (2 × 10 −3) = 20 V 3. A dielectric cube of side 'a' centred at the origin carries a polarisation charge ?? P r kr ? ? ? ? , where K is constant. Find all the bound charges and prove that they all add up to zero. Solution : The bound volume charge density is equal to ? ? ? ? 2 2 1 · 3 b P r kr k r r ? ? ? ? ? ? ? ? ? ? ?? Since the bound volume charge density is constant. The total bound volume charge in a cube is equal to the product of the charge density and the volume q volume = – 3ka 3 The surface charge density b ? is equal to, ˆ ˆ · · b P n kr n ? ? ? ? Now 1 ˆ· cos 2 r n r a ? ? ? ? ? n P a a

NSOU ? CC-PH-08 ? 102 1 ˆ· 2 b kr n ka ?? ? ? ? The surface charge density is constant across the surface of the cube ? ? 2 3 1 6 3 2 surface q ka a ka ? ? Thus total bound charge on the cube is equal to q total = q volume + q surface 3 3 3 3 ka ka ? ? ? = 0 4. The sphere of radius R carries a polarisation ?? P r kr ? ?? ? ? where K is constant, and r is the radius vector from the centre. (a) Calculate bound charges b ? and b ? (b) Find the field inside and outside the sphere. Solution : The unit vector ˆn on the surface of the sphere is equal to the radial unit vector. The bound surface charge is equal to ˆ ˆ · · b r R r R P n Kr r KR ? ? ? ? ? ? ? ?? ? The bound volume charge density equal to ? ? ? ? 2 2 1 · 3 b P r kr k r r ? ? ? ? ? ? ? ? ? ? ? ?? ?? First consider the region outside the sphere. The electric field in this region due to the surface charge is equal to ? ? 2 3 2 2 0 0 4 1 ˆ ˆ 4 b surface R KR E r r r r r ? ? ? ? ? ?? ? ?? The electric field in this region due volume charge is equal to ? ? 3 3 2 2 0 0 4 1 3 ˆ ˆ 4 b volume R KR E r r r r r ? ? ? ? ? ? ?? ? ??

NSOU ? CC-PH-08 ? 103 Hennce the total electric field outside the sphere is equal to zero. To find the electric field inside the sphere : the electric field due to surface charge is equal to zero. The electric field due to volume charge is equal to ? ? 3 2 0 0 4 1 3 ˆ ˆ 4 b volume r Kr E r r r r ? ? ? ? ? ? ?? ? ?? 5. Two vast homogenous isotropic dielectrics are in contact in the plane z = 0. For 0, z? 1 1·5 r ? ? and for z > 0 2 1. r ? ? A uniform electric field 1 ˆ ˆ ˆ 4 2 4 E i j k kV m ? ? ? ?? exists for 0. z? (a) Find 2 E ?? for 0, z? (b) The angles E 1 and E 2 makes at the interface, (c) The energy densitics inJ/m 3 in both dielectrics. Solution : The problem is portrayed in Fig. 1... As, ˆ k is normal to The boundary plane of two dielectrics, normal components are as follows : 1 ln ˆ ˆ· · 4 E E n E k ? ? ? ?? ?? ln ˆ 4 E k ? ?? 2 2 ˆ ˆ · n E E k k ? ? ? ? ? ?? ?? Also, n t E E E ? ? ?? ?? ?? Hence, tangential component, ln ˆ ˆ 4 2 lt E E E i j ? ? ? ? ? ?? ?? ?? Applying boundary condition at the interface we have, 2 1 ˆ ˆ 4 2 t t E E i j ? ? ? ? ?? ?? Similary displacement vector, 1 2 1 2 1 2 n n n n r r D D E E ? ? ? ? ? ? ?? ?? ?? ??

NSOU ? CC-PH-08 ? 104 ? ? 1 2 2 ln 1·5 ˆ 4 1 r n r E E k ? ? ? ? ? ?? ?? 2 ˆ 6 n E k ? ?? Thus electric vector in dielectic 1 r ? ? is 2 2 2 t E E E n ? ? ? ?? ?? ?? ? ? ˆ ˆ ˆ 4 2 6 i j k kv m ? ? ? (b) Let 1 ? and 2 ? be The angles 1 E ?? and 2 E ?? make with the interfacing surface as shown in Fig. 1, while 1 ? and 2 ? are the angles they make to the interface as in figure, we have, 1 1 90º ? ? ?? 2 2 90º ? ? ?? Since ln 4 E ? and 2 2 4 2 2 5 lt E ? ? ? 1 2 5 5 tan 4 2 ? ? ? 1 1 5 tan 48·1888 2 ? ? ? ? 1 41·8112º ? ? Similarly 2 2 2 2 5 5 tan 6 3 n E t E ? ? ? ? 1 2 5 tan 36·6991º 3 ? ? ? ? 2 53·3009º ? ?

NSOU ? CC-PH-08 ? 105 (c) The energy densties are given by ? ? 1 9 2 6 1 1 1 1 10 1·5 16 4 16 10 2 2 36 E W E ? ? ? ? ? ? ? ? ? ? 3 238·7 J m ? ? ? ? 9 2 2 2 6 2 1 10 1 4 2 6 10 2 36 W ? ? ? ? ? ? ? ? ? 3 247·57 J m ? ? 6. An electric field vector ? ? ˆ ˆ ˆ 4 2 4 E i j k mv m ? ? ? ?? is incident at a particular point on the interface between air and conducting surface. Find D ?? and s ? at that point. Solution : Electric displace ment vector is given by 9 3 10 ˆ ˆ 4 2 4 10 36 D i j k ? ? ? ? ? ? ? ? ? ? ? ?? 2 ˆ ˆ ˆ 0·353 0·1768 0·353 i j k pc m ? ? ? ? ? ? ? 9 2 2 2 3 10 4 2 4 10 36 s D ? ? ? ? ? ? ? ? ? ? ? ? ? 2 ·00593 pc m ? 7. A sphere of radius R has a dielectric constant r ? and uniform charge density of 0 ? (a) Find

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the potential at the centre of the sphere. (b) Find the potential at the surface of the sphere.

Solution : Given O v O r R O r R ? ? ? ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 106 For r > R, ? ? 3 2 0 4 4 3 r r E r ? ? ? ? ? Or, 0 3 r r E ? ? ? Hence potential, at any point inside the sphere 2 0 1 . 6 r V E dr c ? ? ? ? ? ? ? ? For r < R, ? ? 3 2 0 0 4 4 3 r R E r ? ? ? ? ? Or, 3 0 2 0 3 r R E r ? ? ? Potential at external point out side the sphere, 3 0 2 0 . 3 R V E dr

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c r ? ? ? ? ? ? ? As , r? ? V = 0 and C 2 = 0 At r = R, ?? ?? V r V R ? ? ?? 2 2 0 0 1 0 0 6 3 r R R c ? ? ? ? ? ? ? ? ? ? 2 0 1 0 2 1 6 r r R c ? ? ? ? ? ? ? (

a) So the potential at the centre, V (r = 0) V (r = 0) ? ? 0 0 2 1 0 2 1 6 r c R ? ? ? ? ? ? (b) At r = R The surface of the sphere, ? ? 2 0 0 3 R V r R ? ? ? ? 8. A sphereical shell is filled with dielectric material 0r ? ? ? ? for a > r > b and 0 ? for 0 > r >

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a. If a charge q is placed at the centre of the

shell, find, (a) P ?? for a > r > b

NSOU ? CC-PH-08 ? 107 (b) v? ? for a > r > b (c) ? ? at r = a, and r = b Answer : Applying Gauss's law, 2 0 0 4 r q Dr E b r a r ? ? ? ? ? ? ? ? ? ?? ? ? 2 4 r D q r ? ? ?? for a > r > b (1) Now, Polarization is given by, 0 1 r P D ? ? ? ? Hence, ? ? 2 1 4 r r r P q r ? ? ? ? ? for a > r > b (2) ? ? 2 2 1 · 0 v r P r P r r ? ? ? ? ?? ? ? ? ? ? ?? ?? Surface density of charge, ? ? 2 1 ˆ tan 4 r s r r q P n r a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? 2 1 ˆ · r 4 r s r r q P n for b b ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? 9. The electric polarizability of Ar atom is 1·7 × 10 −40 F/m 2 . What is the dielectric constant of solid Ar if its density is 1·8 g/cm 3 Solution : Relative atomc mass of Argan at to atom is 39·95 gm/mole. If N A is the Avaggadra's member. If N is the no of atom per cc. ? ?? ? ? ? 23 1 3 1 6·02 10 1·8 39·95 A mol mol cm cm N d N M gm mol ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 108 N = 2·71 × 10 22 cm −3 with, N = 2·71 × 10 28 m −3 and 40 1 1·7 10 e Fm ? ? ? ? ? 0 1 1·55 e r N? ? ? ? ? ? If we use the clausius-Mossdti equation we get 0 0 1 3 1·87 2 1 3 e r e N N ? ? ? ? ? ? ? ? ? 10. At normal pressure and temperature of 35ºc dielectric constant is k = 1·000516 for Argon atom. Find atomic polarisability and volome. Solution : At normal pressure and temperature (0ºc) atomic density is 2·687 × 10 25 /m 3 So density at 35ºc is 25 273 2·687 10 273 35 n? ? ? ? 25 3 n 10 m 2.382 ? ? From Clausius Mossolti equation, 12 0 25 3 k 1 3.854 10 .000516 n k 1 2.382 10 3.000516 ? ? ? ? ? ? ? ? ? ? 4 2 1.917 10 Fm ? ? ? Molecular volume is given by, 3 25 3 3 4 1 k 1 1 .000516 a 3 n k 2 2.382 10 3.000516 7.221 10 m ? ? ? ? ? ? ? ? ? ? 11. A diaelectric cube of side 'l' centred at the origin carries a frozen-in-polarization P cr ? ? ? . Find all the bound charges and total charges.

NSOU ? CC-PH-08 ? 109 Solution : Given, ˆ ˆ ˆ P cr c(xi yj zk) ? ? ? ? ? ? Bound volume charge density b .P c(1 1 1) 3c ? ??? ?? ? ? ?? ?? Total volume charge of the cube 3 3c ?? ? Bound surface charge density, at the top, b c P.n 2 ? ? ? ? ? ? (on all six surfaces) Hence at the top of the cube total charge C along +ve Z direction, 2 3 b c 2 ? ? ? ? ? ? ? 1 · cos 2 r n r l ? ? ? ? ? Hence 1 ˆ· 2 b cr n cl ? ? ? ? The total surface charges comprising all the six faces of the cube of are l 2 is equal to n ? ? ? Fig. 2

NSOU ? CC-PH-08 ? 110 ? ? 2 3 1 6 3 2 surface q cl l cl ? ? ? Total bound charges compring volume and surfaces q = q surface + q volume = = 3cl 3 − 3cl 3 = 0 12. The space between two parallel plate copacitor is filled with two slabs of linear dielectric naterial as shown in Fig. 3. Each slab has thickness b, so that the total distance between two plates is 2b. Slab 1 has a dieclectric material of dielectric constant 1 ? and slab 2 has dielectric constant 2 . ? The free charge density on the top plate is ? and on the bottom plate is ?? (a) Find the electric displacement Dineach slab. (b) Find the electric field E in each slab. (c) Find the polarization in each slab (d) Find the potential difference between the plate (e) Find the location and amount of all bound charges (f) Now knowning all charges recalculate the field in each slab (a) Applying Guass's law ? ? · free D ds Q ? ? ? ?? ??? enclosed From the Gaussians surface we get. DS s D ? ? ? ? ? Note that D = 0 in the metal. Similarly for the second slab D? ?? (b) 1 D E E ? ? ? ? ? ? ? ?? ?? in slab 1, 2 E ? ? ? in slab 2. Again we, know 0 , r ? ? ? ? So, 1 0 2 ? ? ? and 2 0 0 5 1·25 4 ? ? ? ? ? E 1 E 2 b b s +?
NSOU ? CC-PH-08 ? 111 1 0 2 E ? ? ? and 2 0 4 5 E ? ? ? (c) 0 e P E ? ? ? ?? ?? so, 0 0 e e r r P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? Now, 1, e r ? ? ? ? ? ? ? ? 1 1 1 1 r r P ? ? ? ? ? ? ? ? ?? ? 1 2 P? ? and 2 5 P ? ? (d) Now potential 1 2 0 1 4 5 2 b V E b E b ? ? ? ? ? ? ? ? ? ? ? ? 0 13 10 b? ? ? (e) Volume charge density 0 b ? ? Now bound charges is slabs : 1 b P ? ? ? at the bottom of slab 1 2 ? ? 1 b P ? ? ? at the bottom of slab 1 2 ? ?? 2 b P ? ? at the bottom of slab 1 2 5 ? ? 2 b P ? ? ? at the bottom of slab 2 5 ? ? ? (f) In slab 1 total surface charge above 2 2 ? ?? ? ? total surface charge below 2 5 2 5 ? ? ? ?? ? ?? ? ??
NSOU ? CC-PH-08 ? 112 ? ? ? ? ?? ? 0/2 ? 0/2 ?? Fig. 4 ? ? ? ? which implies 1 0 2 E ? ? ? Inslab 2 total surface charge above 4 0 2 2 5 5 ? ? ? ? ? ? ? ? total surface charge below 4 5 5 ? ? ?? ? ? which implies 2 0 4 5 E ? ? ?
UNIT 3 : Magnetic Field 3.1 Objective 3.2 Introductions 3.3 A brief capitulation on magnetism 3.4 Unit of B ?? 3.5 Track of a charged particle in a magnetic field 3.6 Biot Savart Law 3.7 Torque on a current loop 3.8 Ampere's law and its application 3.9 Properties of B ?? 3.10 Summary 3.11 Review Question and Answer 3.12 Problems and solution 3.1 Objective After completing this unit you will be able to understand– 1. The force due to magnetic field over a moving charge and trajectory of charge in a magnetic field. 2. The origin of magnetic field due to flow of charge through two laws, Biot- Savart's law and Ampere's law. 3. Application of Biot-Savart's law to find magnetic induction for
NSOU ? CC-PH-08 ? 114 a) straight current carrying finite and infinite one dimensional conductor. b) circular loop, c) solenoid. 4. Application of Ampere's law to find the magnetic field in some symmetric cases of current distribution. 5. Vector magnetic potential. 3.2 Introduction The history of magnetic effect has been known from the ancient time; from the discovery of loadstone. The relation between electricity and magnetism was first established experimentally by Oersted in 1819, though such connection had been hinted in a book by Gilbert in 1600. The quantitative relation between current and magnetic field was established by Biot-Savart and Ampere during the period 1820-1825. 3.3 A brief recapitulation on Magnetism We have already come across the existence of magnetic field which is prouduced by permanent magnet or by moving electric charge and so they also under go magnetic interaction when placed in a magnetic field according to Newtonian law of action and reaction. The magnetic field is described by magnetic field lines which provides its direction (along the tangent to the field line at the point concerned) and the magnetic field induction B ?? at a point is the number of field lines corssing per unit area through the point, when the area is held perpendicular the field lines at the point concerned. Unlike the electric field lines the magnetic field lines are closed which leads to the conclusion of nonexistence of free magnetic poles in nature. The force on a moving charge q in electric field E ?? and magnetic induction B ?? is given Lorentz force, dF qE qv B ? ? ? ?? ?? ? ?? Thus the force on a moving charge due to static magnetic field ? ? ... ... 3.1.1 m dF qv B ? ? ?? ? ?? So both static and moving charge experiene electric interaction but only a moving

NSOU ? CC-PH-08 ? 115 charge may undergo magnetic interaction. The direction of force is perpendicular to both v ? and .B ?? The magnetic force is a no work force as . . . 0. dr dw F dr F dt F vdt dt ? ? ? ? ? ?? ? ?? ?? ? The direction of the force can be obtained easily by right hand thumb rule shown in fig (3.1). The thumb finger gives the direction of force when the other fingers specify the direction of rotation from v ? to .B ?? 3.4 The Unit of .B ?? The magnetic field induction at a point is said to be unity if 1C of charge moving at 1m/ s, perpendicular to the field, experiences a force of 1 Newton. This unit is called Tesla. ? ? 1 1 1 1 sin 2 1 N C ms B T ? ? ? 1 N N T Am Cms ? ? ? 3.5 Track of charged particle in uniform magnetic field 1. When v ? is perpendicular to B ?? Here F = qvB sin 90º = qvB and is directed along . v B ? ? ?? So the motion is confined in a plane with radius of rotation r, such that centripetal force qvB = mv 2 /r, r r = mv/qB. If ? is the is the angular frequency of rotation, then v = ?r, or r = m ?r/qB or ? = qB/m, so T = 2 ?m/ qB. Thus we see that the time period of rotation is independent of velocity of particle and depends on the q/m of the particle. Motion is shown in fig. (3.2) 2. When v ? makes an angle ? with the direction of .B ?? In this case the term v cos ? remains unaffected by magnetic field, since this component is along the line of .B ?? For the component v sin ? the magnetic force F m = qvB sin ? = Fig. 3.1 × B ? ? ? Fig. 3.2
NSOU ? CC-PH-08 ? 116 mv 2 sin 2 ?/r, or r = mvsin ?/qB. So the time period of rotation T = 2 ? m/qB. The motion is as shown in fig (3.3) So the path is helical with pitch (= the distance moved in the net direction of motion in each rotation = vcos ? (T) = 2 ?mvcos ?/qB) The

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force on a current carrying conductor in a uniform magnetic field.			

we consider a segment dl of current carrying conductor carrying current i as shown in fig. (3.4) dq be the charge flowing through the element dl in time dt. Then the force on the element m dq dl dF dqv B dq B dl B idl B dt dt ? ? ? ? ? ? ? ? ? ?? ? ?? ?? ? ?? ? ?? ? ? ... ... 3.5.1 m dF idl B ? ? ?? ? ?? So the force is perpendicular to both dl ? and ,B ?? the direction of force can easily be obtained from Fleming's left hand rule as in fig. (3.5). The total force on the conductor ? ? ... ... 3.5.2 m F idl B i dl B iL B iAC B ? ? ? ? ? ? ? ? ? ? ?? ? ?? ? ?? ?? ?? ?? ?? Fig. 3.3 × Fig. 3.4 i A B Fig. 3.5 ? Magnetic Field B Force F ? Current
NSOU ? CC-PH-08 ? 117 3.6 Biot-Savart Law So far we have discussed the force on charge or current element in magnetic field. Now we shall discuss how the current produce the magnetic field. This is given by Biot-Savart law. The law states that if i be the current through an element length dl ? then the magnetic field produced by the current element idl ? at position vector r ? with current element as origin in SI unit is given by (please refer the fig. (3.6) ? ? 0 3 ... ... 3.6.1 4 dl r dB i r ? ? ? ? ? ? ? ?? The direction of magnetic field at a point can be obtained from dl r ? ? ? or by right hand thumb rule. It states that if we stretch our right thumb in the direction of current element and curl our other fingers through p specify the direction of magnetic field. Application of

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Biot-Savart Law. 1) Magnetic field intensity due to a straight current carrying conductor.			

The fig. (3.7) shows a current carrying conductor with current i. To calculate magnetic field intensity at the point p at a distance a from the conductor we take an current element id y ?? at point y as in fig. (3.7). The magentic field due to this elemental current at P, using Biot-Savart law 0 3 4 d y r dB i r ? ? ? ? ? ?? ? ?? 0 3 cos 4 dy dB i r ? ? ? ? directed vertically inside the plane of paper. So the total magnetic field intensity at P 2 1 0 2 cos 4 dy B i r ? ? ? ? ? ? ? d ? Fig. 3.6 i O ? r ? P Fig. 3.7 O P B A y i dy r ? a ? ? ? ? ?

NSOU ? CC-PH-08 ? 118 We put tan , y a ? ? Then 2 sec . dy a d ? ? ? Again cos a r? ? Thus 2 2 1 1 2 2 0 0 2 i i sec cos cos cos 4 4 a d B d a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Or, ? ? 0 2 1 sin sin , 4 i B a ? ? ? ? ? ? ... ... (3.6.2) acting down The plane of paper. . Thus we see that at a distance a from the wire the field is constant in magnitude and is tangent to the circle of radius a following right hand cork-screw rule and the field lines are closed. For a straight infinitely long conductor 2 1 2 2. and ? ? ? ? ? ?? So from eqn. (3.6.2) we have ? ? 0 2 ... ... 3.6.3 4 B i a ? ? ? ?? Vertically inside the plane of paper. . 2. Field due to a current carrying circular loop. The fig. (3.8) shows a circular loop of radius a carrying current i in anticlockwise direction when viewed from the right side of the fig. (3.8). We have to find out the magnetic field intensity B
at
a point P
on

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the axis of the coil at a distance x from the centre O of the

ring. We consider an elemental length dl at A. Then magnetic field at p due to the current element idl ? at A is 0 2 4 dl dB i r ? ? ? ?? along PC (Since dB ?? is perpendicular to both r ? and the ring wire at A) A) Resolving dB ?? along op and perpendicular to op as sin dB ? and cos , dB ? we see that, the component cos dB ? vanishes on summation over the entire wire by symmetry So the neat field is along OP and is, 0 0 H 2 2 sin 2 B 4 4 idl i a a r r r ? ? ? ? ? ? ? ? Fig. 3.8 i O ? dBcQ? dB dBSin ? x r A l ?
NSOU ? CC-PH-08 ? 119 ? ? ? ? 2 0 0 3 2 3 2 2 2 2 2 2 2 4 4 ai a i a x a x ? ? ? ? ? ? ? ? ? ? So, ? ? 2 0 3 2 2 2 2 4 H a i B a x ? ? ? ? ? ?? along OP. . For a coil with Nturens ? ? 2 0 3 2 2 2 2 4 H a Ni B a x ? ? ? ? ? ? ?? along OP ... ... (3.6.4) 3.7 Torques and Forces in Magnetic Dipole A magnetic dipole experiences a torque in a magnetic field, just as an electric dipole in an electric field. Let us calculate The torque on a rectangular current loop in a uniform magnetic field. Loop is placed at the origin and at inclination of ? from The z-axis towards the y axis. Let B ?? point in the z-direction. The forces on the two sloping sides cancel. The forces on the horizantal sides likewise equal and opposite, so the net force is zero, but they form a torque ˆ sin N aF i ? ? ??? The magnitude of the force on each of these segment is F = IbB and therefore ˆ sin N Iab i ? ? ??? Or, ? ? ... ... 3.7.1 N m B ? ? ??? ?? ?? Where m = Iab is the dipole moment of the loop. ? B ? m z ? I o a b ? y Fig. 3.9 F ? B z ? ? m r a F Y Fig. 3.10
NSOU ? CC-PH-08 ? 120 The torque N ??? tends to dearease the angle ,? If U is the potential energy of the loop. Then, sin U N mB ? ? ? ? ? ?? so that cos constant. U mB ? ? ?? Taking 0 2. U at ? ? ? ? we get ? ? cos · ... ... 3.7.2 U mB m B ? ? ? ? ? ? ?? ?? The equation (3.7.2) is similar to the expression for the potential energy of an electric dipole of moment P ?? placed in an electric field. E ?? is · U P E ? ? ?? ?? 3.8
Ampere's Law The

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Ampere's law states that the line integral of magnetic field vector about a closed path is equal to ? times the current

through the surface enclosed by the
closed path, mathematically ? ? 0 . ... 3.8.1 B dl I ? ? ? ?? ? ? were 0 ? stands for the permeability of of free space, in case of medium it should be replaced by permeability of the medium. If J ?? is the current density then, . , S I J ds ? ? ?? ? (where ds ? stands for an elemental surface area) So the Amere's law takes the form, 0 0 . . S B dl I J ds ? ? ? ? ? ? ?? ? ?? ? ? Using Stoke's theorem 0 0 . . S B ds I J ds ?? ? ? ? ? ? ? ?? ? ?? ? ? Since this equation is true for all value of s ? so, Fig. 3.11 ? B i

NSOU ? CC-PH-08 ? 121 ? ? 0 ... ... 3.8.2 B J ?? ? ? ?? ?? Application of Ampere’s law 1) Field at a point due to a long current carrying conductor The fig. (3.12) shows long current carrying conductor carrying current i. We have to find out the expression of magnetic field intensity B at point P at a distance a from the conductor as in fig. (3.12). In the mode of calculation we draw a circle of radius a around the conductor, then by symmerty field on it wil be of sme magnitude and acting tangent to the circle. (Direction of field is given by Biot-Savart law) Now by Ampere’s law 0 ·B dl i ? ? ? ?? ? ? Or, 0 2B a i ? ? ? Or, ? ? 0 ... ... 3.8.3 2 B a ? ? ? 2) Field due to a long solenoid We consider a solenoid of n-turns per unit length. The length of the solenoid be very very long compared to its radius. i be the current flowing through it in a anticlockwise direction when viewed from the right. The fig. (3.13) shows the solenoid and fig. (3.14) shows a vertical section of The solenoid. i a P Fig. 3.12 Fig. 3.13 × × × × × × × × × × × × × × × × × d c ? a b c f h g d Fig. 3.14
NSOU ? CC-PH-08 ? 122 We draw two loops abcd and efghe each of length l as shown in fig. (3.14) We consider the loop efghe . Here . 0 efghe B dl? ? ?? ? ? as no current is entrapped in the loop. As this valid for any such loop so B = 0. Thus the magnetic field intensity outside the loop is zero. Now we take the loop abcda 0 . abcda B dl inl ? ? ? ?? ? ? Or, 0 . . . . b c d a a b c b B dl B dl B dl B dl nil ? ? ? ?? ? ? ? ? ?? ? ?? ? ?? ? ?? ? Or, 0 Bl O O O nil ? ? ? ? ? ? ? 0 ... ... 3.8.4 B ni ? ? ? Here we have cosidered the field inside the solenoid is axial and uniform. 3) Field due to a toroid A toroid is a device consisting of a ring (a torus) wrapped with insulated conducting wire. × Fig. 3.15 O C 3 Fig. 3.16 c 3 c 1 o r P
NSOU ? CC-PH-08 ? 123 The fig. (3.15) shows a toroid of inner radius a and outer radius b with n turns per unit length, carrying current i. The fig. (3.16) shows a section of the toroid in a plane through the plane of the ring passing through its Centre O of the ring.
To find

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the magentic field intensity at a point P at a distance r from centre		

please refer fig. (3.16). i) When r < a, . 0 B dl? ? ?? ? ? as no current is entrapped in the loop. So B = 0. ii) For r < b, . 0 B dl? ? ?? ? ? as current enclosed in the loop is again zero, since each turn passes twice throgh it carrying equal but opposite current. iii) For a > r < b, within the core of the toroid 0 . 2 B dl B r Ni ? ? ? ? ? ? ?? ? ? according to ... Amper’s law. . So, ? ? 0 0 2 . ... 3.8.5 B Ni r ni ? ? ? ? ? Thus we see the field inside the core of a toroid is not constant. However the field reamains fairly constant when the inner radius and outer radius are close to each other. 4. Magnetic field due to a current carrying cylinder The fig. (3.18) shows a long cylinder carrying current i. J be the current density which is taken to be constant. From symmetry, field lines are closed circles, co-centric with the axis of wire. We are calculating magnetic inducution at point P at a distance r from Tha axis. (a) When r < a, P is outside the wire. We consider the dotted loop C 1 . 0 B ?? be the magnetic inducution on C 1 . Then 2 0 0 0 . 2 C B dl B r i i ? ? ? ? ? ? ? ?? ??? ? ? ? 0 0 2 ... ... 3.13 B i r ? ? ? (b) When P is inside The cylinder r > a. Refer to loop C 2 . 2 2 2 0 · C B dl J r ? ? ? ? ? ?? ? ? 2 2 2 2 0 0 2 2 i a B r J r ir a a ? ? ? ? ? ? ? ? ? 0 2 ... ... 3.8.6 2 i ir B a ? ? ? ? c 2 Fig. 3.17 c 1 r
NSOU ? CC-PH-08 ? 124 5. Force between two parallel current carrying wires. The fig. (3.19) shows two long Parallel current carrying conductors A and B, separated by a distance a, carrying current i 1 and i 2 respectively. Then using Ampere’s law the magnetic induction B ?? at point P due to the conductor A is, 0 1 2 , 4 i B a ? ? ? ?? directed normally into the plane of paper. . The force on a current element 2 i dl ? at P due to this B ?? is 2 dF i dl B ? ? ?? ? ?? 2 dF i dlB ? ?? acting along PQ 0 2 2 4 i i i B a ? ? ? dl acting along PQ So the force per unit length on wire B is 0 1 2 2 4 i i B a ? ? along PQ ... ... (3.8.7) Now using Netwon’s third law the force per unit length on A due to current in B will be same but will act along B. So the wires will attract each other. If the currents are oppositely directed the wires will repel each other. 3.9 Properties of B ?? To know a vector completely, we must explore its curl and divergence (Helmholtz criterion). We have already explored the curl of B ?? through Ampere’s law as 0 B J ?? ? ? ?? Now to explore . , B ? ?? we define magnetic flux through an elemental surface ds, as the number of field lines passing through the surface perpendicular to the surface. The magnetic field intensity B at a point is defined as the number of field lines passing through the point per unit area, when the area C B i 1 i 2 d ? P a Fig. 3.19 Fig. 3.20 d? r B ? ds ?

NSOU ? CC-PH-08 ? 125 is placed perpendicular to the direction of the field. So magnetic flux through an elemental area ds, at a point where the magnetic field intensity vector is B ?? as shown in the fig. (3.20) . d B ds ? ? ?? ? So the total magnetic flux through a surface S, . . S B ds ? ? ? ?? ? ? Now for a closed surface . 0, S B ds ? ? ? ? ?? ? ? since the magnetic field lines are closed, as isolated pole does not exist in nature. Now using Gauss's divergence theorem we can write. . . . 0 S V B ds B dv ? ? ? ? ? ? ? ?? ? ?? ? ? ? as this is applicate for any volume. So ? ? . 0. ... ... 3.9.1 B ? ? ? ?? As the divergence of curl of a vector is always zero, so we can write ? ? ... ... 3.9.2 B A ? ?? ?? ?? Where A ?? is called megnetic vector potential. It is to be mentioned that that vector potential A ?? is not uniquely defined through the equation (3.9.2) as it B ?? remains unchanged with addition of a function whose curl is zero. (1) Vector magnetic potential for a current loop The fig (3.21) shows a current loop carrying current i. Then for a current element , idl ? the magnetic field at a point P of position vector r ? with current element at origin is given by Biot-Savart law, 0 3 4 r dB idl r ? ? ? ? ? ? ?? ? Now ?? 2 3 1 1 1 ˆ r r r r r ? ? ? ? ? ? ? ? Fig. 3.21 d ? ? dB ? P r ?

NSOU ? CC-PH-08 ? 126 So we can write ? ? ?? 0 1 4 dB idl r ? ? ? ?? ? ?? ? ?? ? ? ? ? 0 1 ... ... 3.9.3 4 idl r ? ? ? ? ? ??? Now ? ? , A A A ?? ? ? ??? ???? ?? ?? ?? so using this identity in the above equation we have ? ? 0 1 4 idl dB idl r r ? ? ? ? ? ? ? ?? ? ?? ? ? ? ? ? ? ? ? ? ? ? ?? ??? 0 , 4 idl r ? ? ? ? ? ?? ? ? ? ? ? ??? since 0 idl ?? ? ??? 0 0 . 4 4 idl idl B r r ? ? ? ? ? ? ? ?? ? ?? ? ? ? ? ? ? ? ? ? ??? ??? ?? ? ? Compering this equation with , B A ? ?? ?? ?? ? ? 0 ... ... 3.9.4 4 idl A r ? ? ? ? ? ??? ?? ? Again ? ? ? ? 0 0 · ... ... 3.9.5 4 4 V J ds dl Jdv A r r ? ? ? ? ? ? ? ? ? ?? ? ? ? ?? ?? ? As A ?? can't define B ?? uniquely, , A ?? can be considered as a mathematical interstep for computer of .B ?? 2) Multipole Expansion of the vector potential In order to find the appromimate value of vector potential due to a localized current distribution, method of multipole expansion of potential can offer appromimate value at large distance from the source, which can be expressed in powers of 1/ r, 1/ r 2 . Higher order terms with negligible non-zero value in the series is the one important aspect of this method, ensuring appronimately fair value of potential. We get the expansion as follow from the figure–

NSOU ? CC-PH-08 ? 127 " 2 12 1 1 2 cos r r r rr ? ? ? ? ? ? ? ? ? ? ? ? ? 0 1 cos ... ... 3.9.6 n n n r P C r r ? ? ? ? ? Where ? is the angle between r ? and r ? ? Accordingly vector potential of a current loop can be written as ?? ? ? ? ? ? ? 0 0 1 1 cos ' ... ... 3.9.7 4 " 4 n n n I A r dl r P dl r r ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? Or, ? ? ? ? ? ? ? ? 2 2 0 2 3 1 1 1 3 I cos ' cos 1 2 ...... 3.9.8 4 2 A r dr r dl r dl r r r ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ?? ? ? ? ? Now the magnetic monopole term is always zero, for the integral is just the total vector displacement around a closed loop : ? ? 0 ... ... 3.9.10 dl ? ? ?? Dipole term plays important role as monopole term is zero, ? ? ? ? 0 0 2 2 ˆ cos . '. ...... 3.9.10 4 4 dipole I I A r r dl r r dl r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ?? ? ? ? ? This integral can be written elegant way if we use the following relation– ? ? ? ? ˆ ˆ . ... 3.9.11 r r dl r ds ? ? ? ? ? ? ? ? ?? ? ? Then ? ? ? ? 0 2 ˆ ... ... 3.9.12 4 dip m r A r r ? ? ? ? ? ?? ?? ? where m ?? is the magnetic dipole moment ? ? ... ... 3.9.13 m I ds Is ? ? ? ?? ? ? P r´ ´ r ? Fig. 3.22 r´ ? dr´=d ´ ?

NSOU ? CC-PH-08 ? 128 where s ? is the vector area of the loop. If the loop is flat, s ? is the ordinary area enclosed, with direction followed by the usual right hand rule. In reality, the dipole potential is suitable appronimation whenever the distance r greatly exceeds the size of the loop. The magentic field of a perfect dipole is easiest way to calculate if we allign m ?? at the origin and in the z-direction. ?? ? ? 0 2 sin ˆ ... ... 3.9.14 4 dip m A r r ? ? ? ? ? ?? ? so, ?? ? ? 0 3 ˆ ˆ 2cos sin 4 dip m B r A r r ? ? ?? ? ? ? ?? ? ?? ? ?? ?? Astonishingly, this is identical in structure to the field of a electric dipole If we write ? ? ? ? ˆ ˆ ˆ ˆ . · m m r r m ? ? ? ? ?? ?? ?? ˆ ˆ cos sin m r m ? ? ? ?? Then ? ? ˆ ˆ ˆ ˆ ˆ 3 . 3 cos cos sin m r r m m r m r m ? ? ? ? ? ? ?? ?? ?? ˆ ˆ 2 cos sin m r m ? ? ? ?? So can write ?? dip B r as ?? ? ? ?? ? ? ? ? 0 3 1 ˆ ˆ 3 . ... 3.9.15 4 dip B r mr r m r ? ? ? ? ? ? ? ? ?? ? ?? ?? Fig. 3.24 z y Field of a pure dipole Field of a physical dipole Fig. 3.25 Fig. 3.23 z ? m Q Y

NSOU ? CC-PH-08 ? 129 Also magnetic dipole can be written in the following way– we know 1 I d ' 2 m Is r ? ? ? ? ? ?? ? ? ? ? ? ? ? 1 ... ... 3.9.16 2 m r J d ? ? ? ? ?? ? ?? 3.10 Summary After studying the unit we should understand following. 1. The magnetic force on a moving charge . F qv B ? ? ?? ? ?? It is a no work force. 2. The trajectory of charge in magnetic field is circular for v B ? ? ? ?? and helical for other angles of projection. 3. Biot-Savart's law 0 2 cos 4 dl dB i r ? ? ? ? and its application. 4. Amperes law 0 .B dl I ? ? ? ? ?? ? ? and its application. 5. Study of nature of .B ?? Introduction of vector potential A as B A ? ?? ?? ?? ?? ? 6. Magnetic dipole ? ? 1 2 m r J d ? ? ? ? ?? ? ?? 7. Forque on a current loop N m B ? ? ??? ?? ?? 8. Vector potential of magnetic field B A ? ?? ?? ?? ?? 3.11 Review Questions and Answer 1. What represents The line integral of magnetic vector potetial A ?? about the boundary of surface in a magnetic field? 2. If the flux density at a point in space is ? ? ˆ ˆ ˆ 2 4 , B xi ayj k a consant ? ? ? ? ?? ˆ ˆ ,i j and

NSOU ? CC-PH-08 ? 130 k are unit vectors along x, y and z directions. The find the value of a. 3. Show that magnetic force on a charge is a no work force. 4. Is megnetic field is a conservative field? 5. A beam of charge undergoes deflection in a space. Can you identify which field electric or magnetic field is present in the field. 6. A current is sent through a hanging coiled spring. What change do you expect when the current is suitehed off. 7. Two long parallel conducting wires carrying current i 1 & i 2 are kept separated parallel to each other at a distance d. Will the force between the wires increase if the diameter of one wire is donbled. 8. A straight wire carrying current i 1 is placed along the centre of loop carrying current i 2 as shrown in figure. Is there any force of interaction between the coil and straight wire. (Neglect gravitational interaction). 9. Starting from the expression of magnetic vector potential 0 4 I dl A R ? ? ? ? ? ?? ? obtain. The expression for magnetic induction .B ?? Also show that · 0. B ? ? ?? ?? 10. Find the force between two ideal magnetic dipoles of moments 1 m ?? and m 2 separated by a distance r. Assume that m 1 and m 2 point in the direction of the vector joining them. (Ch–13) 11. Find the vector potential of inside and out side a sole noid with n turns per unit length givn current I and dradius R 1. The line integral of A ?? over a boundary of surface S is given by . . Adl ? ?? ? ? Now applying stokes law ? ? . . S Adl A dS ? ?? ? ? ?? ? ?? ?? ? ? Now B A ? ?? ?? ?? by definition. . . Adl B dS ? ? ? ?? ? ?? ?? ? = flux through the surface S. 2. B ?? to be a magnetic flux density . 0. B ? ? ?? c 2 c 1 r Fig. 3.26
NSOU ? CC-PH-08 ? 131 ? ? 2 4 . 0 ay x B x y z ? ? ? ? ? ? ? ? ? ? ? ? ?? 1 2 0, a ? ? Or, 1 2 a? 3. The magnetic force on charge is given by , F qv B ? ? ?? ? ?? now work done for a displacement dr ? ? ? ? ? ? ? . . · . 0, dr dw F dr q v B dr q v B dt q v B vdt dt ? ? ? ? ? ? ? ? ? ?? ? ? ?? ? ? ?? ? ?? ? as ? ? v B ? ? ?? is perpendicular to .v ? Thus magnetic force is a no work force. 4. No. Please see differential form of Ampere’s law. 5. Consult text. 6. When the current flows each spiral attracts the neighbours turn and the coil turns become closer. When current is switched of the distance between the turns increases. 7. No. The force depends on current and mean distance of separation. 8. No. The magnetic field produced by each of them is along the direction of other, So 0. F idl B ? ? ? ? ?? ? ?? 9. Current density is a function of source co-ordinate, while here all the differential operatars act m field co-ordinaltes. Assuming source co-ordinate as ? ? , , x y z ? ? ? and field co-ordinates as ? ? , , , x y z so R r r ? ? ? ? ? 0 4 I dl B A R ? ? ? ? ?? ? ?? ? ? ? ? ? ? ?? ?? ?? ?? ? ? Idl J r d ? ? ? ? ? ? ?? ? ? 0 ... ... 1 4 Jd R ? ? ? ?? ? ? ?? ?? Now, ? ? 1 1 J R J r J r R R ? ? ? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? ?? ? ?? ?? ?
NSOU ? CC-PH-08 ? 132 Now, ?? 0 J r ? ?? ? ?? ?? as ? ?? is an operator of ?? ? ? , , f r f x y z ? ? ? 3 ... ... 2 J J

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R R R ? ??? ? ?? ?? ?? ?? From (1) and (2) 0 3 4 J R B d R ? ? ? ? ? ? ? ?? ?? ?? Now 0 3 . . 4 J R B d R ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? ?? ?? ? ? 0 3 3 0 4 R R J r J d R R ? ? ? ? ? ? ? ? ?? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? ?? ?? ?? (so 3 0, R R ?? ? ?? ??

and ? ?? acts only on field co-ordinates) 10. Solution The force on the dipole m 2 due to m 1 is given by ? ? 1 2 . , F m B ? ? ?? ?? ?? ?? where ? ? 1 0 1 1 3 5 . 3 4 m r m B r r r ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ?? ?? ? ˆ ˆ ˆ ˆ r xi yj zk ri ? ? ? ? 1 1 ˆ , m m i ? and 2 2 ˆ m m i ? Sol n 10. ? ? ? ? 0 2 1 1 3 ˆ 3 4 F m m m i x r ? ? ? ? ? ? ? ? ? ? ? ? ? ??
NSOU ? CC-PH-08 ? 133 ? ? 3 0 1 2 ˆ 1 2 m m i x x ? ? ? ? ? 0 0 1 2 1 2 4 4 3 6 ˆ ˆ 2 4 m m m m i i x x ? ? ? ? ? ? ? ? ? which is the force of allraction along the line joining the dipole. Solution 11. Magnetic field is uniform inside a role roid, of n turms, and carying current I is equal to 0 ˆ 0 nIz r R B r R ? ? ? ? ? ? ? ? ? ?? ? ? . . . S Adl A da B da ? ?? ? ? ? ? ?? ? ?? ?? ? ?? ? ? Now considering a closed loop of radius r insicle the solenoid. ? ? 2 0 .2 A r nI r ? ? ? ? ? ?? when r R ? 0 ˆ 2 nI A r ? ? ? ? ?? Direction of A ?? along the direction of I. Now considering a closed circular loop of radius r. Outside the solemoid, 2 2 · A r B R ? ? ? ? ?? ?? 2 0 ˆ 2 nI A R r ? ? ? ? ? ?? (Since the field extends up to r = a) Hence 0 2 0 ˆ ˆ 2 In r R A In R r R r ? ? ? ? ? ? ? ? ? ? ? ?? r m 1 m 2 ? ? x-axis

NSOU ? CC-PH-08 ? 134 3.12 Problems and Solutions 1. A proton and a deuteron have equal kinetic energies. Compare the radii of their paths when a magnetic field is applied normal to their orbits. 2. A particle of charge q and mass m is projected with a velocity v perpendicular to a uniform megnetic field of field intensity B. Find the angle of deviation ? when i) d > r (radius of rotation of charge) ii) d = r iii) d > r 3. A square loop of side 'a' carries a current in anti-clockwise direction when viewed normally. Calculate the magnetic field at the center of the coil. 4. An electron is rotating of charge e is rotating frequency n around the nucleus in a circular orbit of radius r. Find the magnetic field B at the position of necleus. r = 5.1 × 10 -11 m and n = 6.8 × 10 15 Hz. 5. A particle with charge q is projected successively along x-axis and y-axis with same velocity .v ? The force on the particle in these situations is given by 3 1 ˆ 2 2 vqB k ? ? ? ? ? ? ? ? and 1 ˆ 2 vqB i ? ? ? ? ? ? ? respectively. Find the direction of magnetic induction .B ?? 6. Show that the vector potential for a uniform magnetic field B ?? along z-direction is given by ? ? 1 2 A r B ? ? ? ? ?? ? ?? 7. The fig (1) and fig. (2) shows two circuits with loop radius 2r and r but with different orientation of loop. Find the ratio of magnetic field produced at the centres o 1 and o 2 . 8. The magnetic vector potential in a region is defined by ˆ sin . y A e xk ? ? ? ?? An infinitely long coductor, having a cross section area, a = 5 mm 2 and carrying a dc current I = 5A in the y-direction, passes through the region as is fig (3) Determine the expression for (a) B ?? and (b) the force density f ?? exerted on the conductor. c ´ 2 2r r Fig. 1 Fig. 2 r O 2 2r

NSOU ? CC-PH-08 ? 135 9. The fig (4) shows a conducting wire carrying current i placed perpendicular to a uniform magnetic field B ?? acting verfically into the plane of paper. AB and CD have length l and circular are BC has radius l. Find the force on wire. 10. The fig (6) shows a very very long wire carrying current i, with its middle portion bent in the form of a circular are of radius r as shown in fig (6). Find the magnetic induction B ?? at the centre O of arc. 11.

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A steady current I flows down a long cylindrical conductor of radius a. The			

carrent density at a distance r from the axis of the conductor is proportional to r. Calculate the magnetic field both, inside and out side the wire as a functor of r. 12. (a) A particle of mass m and charge q is rotating in a circle of radius a with an angular velocity w. Show that the ratio of its magnetic moment to mechanical moment angular momentum is q/2m (b) If the magntiude of the angular momentum of a electron rotating in a circular orbit is L find its magnetic moment. Solution of numerical problems 1. Let v p and v d be the veolcities of proton and deuteron. m be the mass of proton, then the mass of deuteron will be 2m. Their respective radius be r p and r d . Then in a magentic field B normal to their path, Conductor Y X I I B Fig. 3 × o ? ? ? a Fig. 3.32 i i A B o 45 0 45 0 r r c Fig. 6

NSOU ? CC-PH-08 ? 136 Bqv p = mv p 2 /r p ..... (3.12.1) Bqv d = 2mv d 2 /r p ..... (3.12.3) So r p /r p = v p /2V d ..... (3.12.3) By the condition mv p 2 /2 = 2mv d 2 /2, so v p /v d = 1 2. Comparing with equation 3.12.3 we have 2 d p r r ? 2. Here r = mv\qB. So from the fig (3.32) sin ??= d/r = Bqd/mv. So, i) if r < d, ??is acute and the particle will migrate in next medium. ii) if r = d, ??= ?/2 and the particle will graze the surface of separation. iii) if r > d, the particle will describe a half circle and return back to the previous medium parallel to itself. 3. The fig (3.33) shows a square loop carrying current i. The magnetic induction at centre o due to arm DA 0 4 2 DA i B d ? ? ? ? ? sin45º sin45º ? 0 2 2 4 i d ? ? ? perpendicular to ABCD and directed normally up the plane of paper. So the total magnetic induction 0 0 2 2 4 2 2 4 ABCD i i B B d d ? ? ? ? ? ? ? acting normally up the plane of paper. 4. Here 19 15 4 1.6 10 6.8 10 10.22 10 i en A ? ? ? ? ? ? ? ? ? ? 7 4 11 4 10 10.22 10 14.4 ... ...3.23 2 5.1 10 B T ? ? ? ?? ? ? ? ? ? ? ? 45 0 45 0 i i o D A C B Fig. 3.33 d/2

NSOU ? CC-PH-08 ? 137 5. We know magnetic force 2 ˆ ˆ ˆ x y x y z i j k F qv B q v v v B B B ? ? ? ?? ? ?? Now for first condition magnetic field should be in y-z plane So, ˆ ˆ ˆ y z i j k F q v o o o B B ? ?? That gives qv By 3 2 qv B ? (Comparing the z-compones of force) So By 3 2 ? B.... (3.12.4) Imposing the second condition similarly we have B 2 = B/2 .... (3.12.5) So ? ? ˆ ˆ 3 2 B B j k ? ? ?? Let ? be the angle with y-axis. 2 2 3 3 ˆ . cos 2 4 2 B B j B B ? ? ? ? ? ? ? ? ? ? ? ?? 3 cos 30º 2 ? ? ? ?? ? 6. Here ˆ , B Bk ? ?? So Ax Az Ay y x z i j k A A A A gives x y z ?? 0 y x z z A A A A y z z x ? ? ? ? ? ? ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 138 & Ay Ax B x y ? ? ? ? ? ? So A ?? will be either proportional to B ?? and a linear function of r. We can take two choice 1. 1 A C Br ? ?? ? 2. ? ? 2 A C B r ? ? ?? ?? ? As for first choice 0, B A ? ?? ? ?? ?? so it is the The second choice 2 0 0 i j k A C B x y z ? ?? yields A x = –C 2 yB & A z = 0 Ay = C 2 Bx Now 2 2 ˆ i j k B A KB x y z c yB C Bx o ? ? ? ? ?? ? ? ? ? ? ? ? ?? ?? Or B = C 2 B + C 2 B = 2BC 2 Or, C 2 = 1/2 Thus ? ? 1 2 A r B ? ? ? ? ?? ? ?? from 2nd choice 7. In fig (1.21), The magnetic is O 1 ? ? 0 0 1 4 2 4 2 B i i r r r ? ? ? ? ? ? ? ? ? ? ? ?? up the plane of paper. . In fig (2), The magnetic induction at O 2 ? ? 0 0 2 3 4 2 4 2 B i i r r r ? ? ? ? ? ? ? ? ? ? ? ?? up the plane of paper. . 2 1 : 3. B B ? ? ?? ??
NSOU ? CC-PH-08 ? 139 8. 0 0 sin y i j k B A x y z e x ? ? ? ? ? ? ?? ? ? ? ? ? ?? ?? Or, ˆ ˆ sin cos y y B e xi e xj ? ? ? ? ? ?? ? ? ˆ ˆ sin cos y e i x y x ? ? ? ? For exerted on a elemental length dl ? on conductor ? ? ˆ F I dl B Ijdl B ? ? ? ? ? ?? ? ?? ?? ? ? ˆ ˆ ˆ sin cos y Idlj i x j x e ? ? ?? ? ? ? ˆ ˆ sin 5 sin y y Idle xk e xk dl ? ? ? ? ? force density ˆ 5 sin y F e xk N m dl ? ? ?? 9. We set x & y axis as in figs The force on arm AB, 0 ˆ 4 AB F ilBi ? ? ? ?? The force on arm CD, 0 ˆ 4 CD F ilBj ? ? ? ?? The force on arm BC, ? ? 0 ˆ ˆ 2 & 4 2 BC i j F i B ? ? ? ? ? ?? 45 0 r i o Fig. 3.32 i r
NSOU ? CC-PH-08 ? 140 ? ? 0 ˆ ˆ 4 ilB i j ? ? ? ? So the total force ? ? 0 2 ˆ ˆ 4 F ilB i j ? ? ? ? ? ?? 10. Magnetic induction at O due to straight portions of wire ? ? 0 2 sin90º sin 45º 4 sin 45º stright i B r ? ? ? ? ? ? ? ? ? ? ? ?? ? ? 0 2 2 1 4 i r ? ? ? ? acting normally into the plane of paper. . 0 2 4 arc i B r ? ? ? ? ? ?? acting normally into the plane of paper. . Total magnetic induction ? ? 0 2 2 1 4 2 straight arc i B B B r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? acting normally into the plane of paper. 11. Solution Considuring an Amperian loop in the form of a circle of radius r (r < a) with its axis on the axis of the cylinder, B is is tangential to the loop be cause of symmetry and constant over it 0 . encl B dl I ? ? ? ? ?? ? ? Or, ? ? 0 0 0 .2 2 . B r rdr J r ? ? ? ? ? ? 3 2 0 0 0 2 2 3 r r k r dr k ? ?? ? ?? ? where k is portionality constatin J = Kr 2 0 . 3 r B k for r a ? ? ? ?
NSOU ? CC-PH-08 ? 141 For any external point r < a, I end = I and then, 0 2B r I ? ? ? Or, 0 2 I B for r a r ? ? ? ? Now the total current ? ? 2 0 0 2 . 2 a a I rdr J r k r dr ? ? ? ? ? ? 3 2 3 a k ? ? or, 3 3 2 I k a ? ? Thus 2 0 3 2 Ir B for r a a ? ? ? ? a solution (12.b) current flow due to charge q rotating in a circular orbit is 2 2 2 q q qa I T ? ? ? ? ?? ? Magneitc moment = current x area of the loop 2 M I a ? ? 2 2 q M a m m ? ? ? ? 2 2 q mwa m ? 2 q m ? ? Angular momentum. Magnetic moment Angularmomentum 2 q m ? From above, taking q = e as charge of electron. Then Magnetic moment 2 e L m ? 2 eL M m ? ?
Unit 4 Magnetic Properties of Matter Structure 4.1 Objectives 4.2 Introduction 4.3 Magnetisation (M) and its Measurement 4.4 Auxiliary Magnetic Field (H ??) 4.5 Magnetic Permeability and Susceptibility 4.6 Classification of Magnetic Materials 4.7 Relation between B and H of Magnetic Material in Magnetic Field 4.8 Hysteresis or Magnetisation Cycle 4.9 Importance of Hysteresis Loop 4.10 Summary 4.11 Review Questions and Answers 4.12 Problems and Solutions 4.1 Objectives You will know from this unit— What is magnetisation and its measurement (M) Behaviour of closed circulating current and its relation to non-uniform magnetisation.
NSOU ? CC-PH-08 ? 143 Relation between auxiliary magnetic field H ? , magnetic induction vector B ? , and intensity of magnetisation M ? . Hysteresis loss for ferromagnetic material and its importance. Classification of magnetic material according to their property mainly pare dia and ferromagnetic material. 4.2 Introduction In our earlier unit we have studied the primary properties of magnetism. Apart from the directive, and altractive/repulssive properties, the fundamental property of a magnetic field is that its flux through any closed surface vanishes. Mathematically it is expressed as .B=0 ? ?? ?? i.e. these field lines close on themselves. The most common source of magnetic fields is the electric current loop. It may be an electric current in a circular conductor or the motion of an orbiting electron in an atom. Associated with both types of current loops is a magnetic dipole moment, the value of which is iA, the product of current (i) and area of the loop (A). Besides these, electrons protons, and neutrons in atoms have a magnetic dipole moments for their intrinsic spin property. At present, we will study more about properties of magnetism, and intensity of magnetisation. The nature of circulating current related to non-uniform magnetisation and the relation between current density and intensity of magnetisation will be studied in detail here. A simple relation between auxiliary magnetic field (H ?) and magnetic induction (B ?). their relation will be established here. Magnetic material is classified into three main category—para, dia and ferromagnetism. Their general properties are included here particularly, ferromagnetic material with their hysteresis property are relevant in fabricating temporary or permanent magnet. Ferromagnetic material is of immense use in industry, i.e. in transformer design. 4.3 Magnetisation and its Measurement In an atom, electron revoles around the nucleus in different orbits, so we can say that each orbit is closed electrical circuit, which acts as a magnetic dipole. Magnetism of this closed electrically orbital circuit or magnetic dipole can be expressed in terms of Magnetic polarisation. It is the active current flow and a is it surface area, then magnetic dipole

NSOU ? CC-PH-08 ? 144 polarisation of each orbit, m = Ia ....(4.3.1) Normally, these orbits or dipoles are randomly distributed, so in effect, resultant magnetic effect of these get neutralised. They try to orient themselves in order under the infulence of external magnetic field. So the material retains magnetic properties. Total number of dipoles oriented along the externally applied field is defined as the intensity of magnetisation M ? . If V is the volume and total number of dipoles ?m i in it, then magnetisation M is given by i m M V ? ? .... (4.3.2) where m i is the ith magnet dipole value. 4.3.1 Equivalency between Magnetic Circuit and Electrical Circuit : Let us take a piece of magnetic material. This piece can be imagined to be assembly of small mesh structures. As current is the source of magnetisation, so the magnetic behaviour of every micro mesh can be considered due to the flow of current in one direction. This is portrayed in Fig 4.l. This current flow is same for every network for uniform magnetisation. Fig. 4.1 It is clear from the figure that current in adjacent orbital circuit or mesh is equal and in opposite direction, is neutralised by each other, only the current flow left out in external boundary or periphery of the collected mesh does not vanish and remain active. It can be concluded that a magnetic material is an arrangement of equally structured numbers of orbitally current curcuit mesh work for uniform magnetisation. This active current around the periphery is called circulating current. The characteristics feature of this current is that it is not due to freely moving electrons. It is the current produced by electrons revolving M NSOU ? CC-PH-08 ? 145 in atoms of different structural configuration of magnetic material. So this current is called bound current. A relation ship between magnetisation and bound current can be derived as follows. Fig. 4.2 In Fig. 4.2 a small sample of magnetic material is displayed, whose area is 'a' and its breath is d. M is intensity of magnetisation and m its dipole moment, then we get, m = Mad ....(4.3.1.3) As the magnetic material is like an electrical circuit of equally spaced block, and I is the governing current then magnetic dipole moment will be, m = Ia ....(4.3.1.4) coparing equations (4.3.3) and (4.3.4) we get, M = I – d = K ....(4.3.1.5) K is known as surface current density. It is clear from equation (4.3.5) that magnetisation intensity and surface current density are identical. The direction of the current on each surface is given by K ? = M ? × n ^ ....(4.3.1.6) This equation is very important. Here n ^ is the unit surface vector. Now surface vector K ? directed externally outward. M ? and n ^ , being parallel to each other, current flow in upper and lower surface have no existence. 4.3.2 Relation between Magnetisation and Current Density in Non Uniform Magnetisation : Magnetisation current is active in adjacent boundary of mesh block of magnetic material in non-uniform magnetisation. Two adjacent block of magnetic material is shown in Fig. 4.3. Magnetisation is not uniform everywhere, so intensity of magnetisation is different in two blocks. Let M Z (y) and M Z (y+?y) are magnetic intensity of two blocks, respectively. Equation (4.3.5) gives the current density. So the current flow in each block will be different. Arrow sign indicates the direction of flow of current. Let I x (1) and I x (2) M I d }

NSOU ? CC-PH-08 ? 146 be the current flow of the blocks, respectively. From equation (4.3.5) and Fig. 4.3, We get, I x (1) = M z (y) ?z and I x (2) = M z (y + ?y)?z Fig. 4.3 At the junction of the two block I x (1) is –ve along x-axis and I x (2) is tve along X- axis and active. As I x (2) is grea ter than I x (1) so we can understand that current will be more active along positive X-axis. Remainder of the current flow will be ?I x = I x (2) – I x (1) or,?I x = [M z (y+?y) – M z (y)] ?z ......(4.3.7) Now, M z (y+?y) = M z (y) + z M y ? ? ?y + ... other negligible terms So, ?I x = z M y ? ? ?y ....(4.3.8) J is the current density per unit area and it's direction along per pendicular to the area, so current density due to unequal magnetisation along y-axis will be 1 x m x I Mz (J) y z y ? ? ? ? ? ? ? ? ....(4.3.9) This current density is due magnetisation, (that is why m is used as subscription). The reasons given above is responsible for the origin of (J m) x1 . It is clear from Fig. 4.3.(b) that residual x component of current will be due to the variation in magnetisation along z-axis and will remain active. Hence M (y) z 1 dz } M (y+dy) z 2 dy M (y+dy) z M (y) z dy dz (a) (b)

NSOU ? CC-PH-08 ? 147 y y m x2 M (z z) M (z) (J) y y z ? ? ? ? ? ? ? ? ? ? ? ? ? ? = My z ? ? ....(4.3.10) So the total current density at any point due to uneven magnetisation will be ? ? ? ? ? ? m m m x x1 x2 J J J ? ? ??? ??? ??? = z y M M y z ? ? ? ? ? ???? ???? ....(4.3.11) = (? ? × M ?) x i.e. ? ? m x J ??? is x component of (? ? × M ?) x In this way we can derive the flow of current along, y and z axis, so the resultant current density will be m J M ??? ??? ?? ??? ....(4.3.12) Equation (4.3.12) is the relation between current density and intensity of magnetisation. ? ? × M ? = 0, for uniform magnetic field or J m = 0, the current flow remain active only along the of periphery. There will be no influence inside the magnetic material. 4.3.3 Alternative Method to Find ? ? × J ? = M ? To find a quantitative relation between M ? and J ? , let us consider magnetic vector potential due to a magnetised body as shown Fig. 4.4. The vector potential due to a single current loop of magnetic moment m ? is given by 0 2 ˆ m r A(

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r) 4 r ? ? ? ? ? ? ....(4.3.13) where r ? is a radius vector from the loop to the point

of observation. In a magnetised object, each volume element dz carries a dipole moment M ? dz, so the total vector potential is
NSOU ? CC-PH-08 ? 148 ? 0 2 M(r) r A(r) d 4 r ?? ? ? ? ? ? ? ?? ??? ? ? ? ......(4.3.14) From vector algebra, 2 ˆ 1 r r r ?? ? ? ?? ?? ?? with this ? ? 1 A(r) M(r) d r ? ? ? ? ? ? ? ? ? ? ? ? ? ??? ? ? ? Integrating by parts ? ? 0 M 1 A(r) M(r) d 4 r ? ? ? ? ? ? ? ? ? ? ? ??? ? ? ? ? M(r) d r ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ??? ? or, 0 0 1 1 A(r) M(r)d M(r)da 4 r 4 r ?? ?? ? ? ?? ??? ??? ? ? ? ? ....(4.3.15) Fig. 4.4 The first term looks just like the potential of a volume current. J ? = ? ?? × M ? ......(4.3.16)

100% **MATCHING BLOCK 55/107** SA MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

while the second term looks like the potential of a

surface current K ? = M ? × n ^ ....(4.3.17) r I r r II ? d? I P
NSOU ? CC-PH-08 ? 149 where n ^ is the unit normal vector, with this definition 0 0 J(r)d ' K(r)da A(r) 4 4 r r ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ?? ? ? ? ? ....(4.3.18) Equation (4.3.18) shows that potential of magnetised object is the same as would be produced by a volume current J ? = ? ?? × M ? throughout the magnetic material, plus a surface current K ? = M ? × n ^ on the boundary. 4.4 Auxiliary Magnetic Field (H ?) Now we place a magnetic material inside a solenoidal coil and a current I f is flown across it from a battery. If total current density is J ? , then J ? = J ? f + J ? m Here J ? f and J ? m are free current density and bound current density. From Ampere's circuital law ? ?? × B ? = ? 0 J ? = ? 0 (J ? f + J ? m) .... (4.4.1) As J ? m = ? ?? × M ? So, ? ?? × B ? = ? 0 J ? f + ? 0 ? ?? × M ? or, ? ?? × B ? / ? 0 – ? ?? × M ? = J ? f or, ? ?? × f 0 B M J ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ....(4.4.2) is denoted as H ? , equation (4.4.2) becomes ? ?? × H ? = J ? f ....(4.4.3) H ? is known as auxiliary magnetic field or magnetisation. In reality H ? is very important as it is directly related to the current flow from battery. If we study equation (4.4.1) and (4.4.3), we can conclude that magnetic induction vector B ? is related to total current flow, but cannot be measurable easily otherwise H ? can be measured easily as it is related free flow of current.
NSOU ? CC-PH-08 ? 150 Applying stoke's theorem in equation (4.4.3) we get ? ? f f ˆ ˆ H .nds H.dl J .nds J ? ?? ? ? ? ? ? ? ?? ?? ?? ?? ? ? ....(4.4.4) Equation (4.4.4) states that an integration of H ? around a closed loop is linked with free current flow from EMF/other sources. This equation is frequently used to find H ? . We understand from the above analysis that application of auxiliary magnetic field H ? in material initiates the evolution of magnetic field, which is known as magnetic induction vector B ? . Now, 0 B M H ? ? ? ? ?? ??? ?? or, B ? = ? 0 (H ? + M ?) ....(4.4.5) Here ? 0 is the permeability of free space. Equation (4.4.5) is the relation between B ? , H ? and M ? . 4.5 Magnetic Permeability and Susceptibility Magnetic permeability of a material is the ability of a material to support the formation of a magnetic field inside itself. So it is known as degree of magnetisation standard unit of magnetic permeability is Hm –1 . The magnetic permeability is a relative measurement that it is taken with respect to the magnetic permeability of vacuum. A diamagnetic material has a relative permeability less than 1, Where as a paramagnetic material has a value slightly greater than one which means that when a paramagnetic material is placed in external magnetic field, it becomes slightly magnetised. But a ferromagnetic materials have relative permeability. Magnetic susceptibility is the measure of magnetic properties of material which indicates whether the material is attracted or repell from external field. This is quantitative measurement of the magnetic properties. It is denoted as ? m . For a isotropic linear magnetic material. M ? = ? m H ? .... (4.5.1) ? m is a dimensionless quantity the values of ? m for common para magnetic and diamagnetic materials are given below.

NSOU ? CC-PH-08 ? 151 Table 4.1 Paramagnetism and Diamagnetism: Magnetic Susceptibilities Paramagnetic substance ? m Diamagnetic substance ? m Aluminum 2.3×10 −5 Copper −9.8×10 −6 Calcium 1.9×10 −5 Diamond −2.2×10 −5 Magnesium 1.2×10 −5 Gold −3.6×10 −5 Oxygen (STP) 2.1×10 −6 Lead −1.7×10 −5 Platinum 2.9×10 −4 Nitrogen (STP) −5.0×10 −9 Tungsten 6.8×10 −5 Silicon −4.2×10 −6 Now we can estalish a simple relation between B ? and H ? from equations (4.4.5) and (4.5.1) B ? = ? 0 (H ?? + M ?) = ? 0 (H ? + ? m H) = ? 0 H ? (1+? m) ....(4.5.2) or, B ? = ?H ? .... (4.5.3) Here ? = ? 0 (1 + ? m) ....(4.5.4) where ? is the permeability of the medium. Also we get relative permeability, as r m 0 1 ? ? ? ? ?? ? ....(4.4.5) ? r is a dimensionless quantity. In vacuum, ? m = 0, ? r = 1. 4.6 Classification of Magnetic Materials Magnetic materials can be classified according to the behaviour of magnetic moments of electron of an atom react to applied magnetic field diamagnetic, paramagnetic and ferromagnetic materials. Diamagnetic Materials : They are weakly magnetised in a direction opposite to the applied magnetic field. Examples are hydrogen, nitrogen, gold, silver, copper, antimony etc. Its behaviour with applied field and temperature are shown in Fig. 4.5

NSOU ? CC-PH-08 ? 152 Explanation : Dia magnetic substance are composed of atoms that have no net magnetic moments. When it is placed in an external magnetic field, the substance as a whole acquires net magnetic moment in a direction opposite to the applied field. They do not have unpaired electrons. Characteristics : 1. Diamagnetic is universal, all materials when exposed to external magnetic field, tend to develop magnetic moments opposite to the direction of the applied field. 2. No parmanent dipoles. 3. Relative permeability is less than one but positive. 4. Susceptibility is negative and small, independent of temperature. (Fig. 4.5) 5. Weak repulsion is its main features. Fig. 4.5 Paramagnetic Materials : It is the phenomenon by which the orientations of the magnetic moments are mainly dependent on temperatrure and applied field. The number of orientations of orbital and spin magnetic moments be such that the vector sum of the magnetic moments is not zero. Resultant magnetic moment in each atom is not zero even in absence of field. Paramagnetic property vanishes in the absence of external field. Its behaviour with applied field and temperature are shown in Fig. 4.6. Examples are Aluminium, platinum, chromium, sodium, calcium, oxygen etc. Paramagnetism Characteristics : a. Paramagnetic materials have an unpaired electron in their valence shell b. These unpaired electrons are in constant spinning motion. c. This incessant spin of the electron form a dipole moment, they act as small magnets themsleves M H M= H ? ??? slope = ? ? T ???constant

NSOU ? CC-PH-08 ? 153 Fig. 4.6 d. However the dipoles are in random directions and donot interact with one another, so the total magnetic field created by paramagnetic material is zero. e. In a magnetic field, the individual dipole moments of the atoms get alligned in a single direction, along the applied magnetic field. This produces a magnetic field in the direction of the applied field. Ferromagnetic Materials : Ferromagnetism is the basic mechanism by which certain materials form permanent magnets or attracted to magnets. They are strongly magnetised in the same direction as that of applied field and retains its magnetic moment ever after removal of the applied field. Examples are Iron, cobalt, nickel. This property is due to the contribution of spin magnetic moment to the magnetic dipole moments is very large. It posseses strong magnetic properties due to the presence of magnetic domains. In these domains, large numbers of atomic moments (10 12 to 10 15 per unit volume) are alligned parallel, so that magnetic force within is strong. In unmagnetised state, the domain are nearly randomly organised, and the net magnetic field for the part as a whole is zero. The domains are oriented to produce a strong magnetic field under the influence of magnetising force. Fig. 4.7 : Ferromagnetic Material Properties saturation level M H ? Tc ? Ferromagnetic Paramagnetic Feromagnet ic Material Properties M H M= H ? ??? slope = ? ? T ?? 1 T Paramagnetism

NSOU ? CC-PH-08 ? 154 Characteristics : 1. They have large and positive susceptibility. 2. They have strong tendency from weaker to the stronger parts of the non-uniform magnetic field. 3. B ? = ? 0 (H ? + M ?). Magnetisation is not proportional to the applied field. They exhibit property called hysteresis. 4. Susceptibility depends on the temperature. 5. It retains their magnetic property even after the external field is removed. Fig. 4.8 : Allignment ferromagnetic domain with magnetic field 4.7 Relation between B ? and H ? of Material in Magnetic Field We know that relation between magnetic induction vector B ? and auxiliary magnetic field vector H ? as B ? = ?H ? . In Fig. 4.9, an experimental setup to find out the relation between Fig. 4.9 ? B=0 B<0 ? ? Saturation Point ? I N nos Turns to galvanometer B H Diamagnet Paramagnet (a) (b)

NSOU ? CC-PH-08 ? 155 B ? and H ? . A magnetic material is introduced inside a torroid. Two coils have been wound on the torroid, one is primary coil, through which current is passed. Other is secondary coil, which has been connected to galvanometer. The magnetic field is produced by the current flow in the primary, which results in magnetic flux inside the torroid. Magnetic flux is varied by the change in current flow. Induced emf can be measured. Thus B ? can be measured. In this way, we can find the auxiliary magnetic field H ? and induced B ? . Thus a graph can be plotted from this data. B ? and H ? graphs can be drawn for different material by placing it inside the torroid. A straight line graph is observed for paramagnetic and diamagnetic material. The gradient of the straight gives the susceptibility ? m [Fig. 4.9(b)]. 4.8 Hysteresis or Magnetisation Cycle Magnetic properties of ferromagnetic material is different from other materials, which is clear from Fig. 4.10. B ? and H ? graph. Behaviour of M ? and H ? is not directly proportional and graph of B ? – H ? is like a loop or cycle. Characteristic features of B ? – H ? from the graph in Fig 4.10 are detailed below : Fig. 4.10 : B – H Graph of ferromagnetic material i) When I = 0, then H = 0, B = 0 and M = 0, so the point 'O' shows the unmagnetised state of the material. Initially, we increase current slowly i.e. B increases as H increases. Magnetisation M has reached saturated state at the point C, i.e. H also is Magnetizing Force in Opposite Direction -M M -B B Saturation in Opposite Direction Flux Density in Opposite Direction Magnetizing Force Saturation Flux Density Retentivity Coercivity F G E D K C O b P

NSOU ? CC-PH-08 ? 156 saturated. As B ? = ? 0 (H ? + M ?), so B increases very little even if H is increased. OPC portion of the graph shows such variation. ii) Now we decrease the current slowly after achieving saturated M at the point. We observe that B – H graph, instead of retracing the path OPC, it traverses along path CD i.e. B does not become zero value even when H = 0. Section CD portrays the residual magnetisation. iii) Now if we reverse the flow of current and increase it slowly, H is also increased along the DEF portion of the graph. Residual magnetisation gets lost if the intensity of magnetic field is in the opposite direction along OE. The value of this applied magnetic field is called co-elective force. The point F describes the sate of saturated magnetisation in opposite direction. iv) H is gradually increased from the point F in the reverse direction, and gets zero value. As a consequence we get the curve FGKC portion of the graph. Thus the material achieves previous value of magnetisation at the point C. The path CDEFGKC will be repeated again and again if I or H is changed repeatedly in forward and reverse direction, but the traversing path OPC of the graph is not traced at all. This closed path CDEFGKC is called magnetisation cycle or Hysteresis loop. At each step M or B lags behind the corresponding magnetising field H. It will be shown latter that the area of B – H loop equals to energy loss per unit volume per cycle of magnetisation. 4.8.1 Hysteresis Loss : When a sample of ferromagnetic material is subjected to magnetisation and demagnetisation, in a hysteresis cycle, some amount of energy dissipated. As the magnetic domains allign with the magnetising field first in one way and then the other it produces mechanical stress and consequent heating. The energy spent during magnetisation is not totally recovered due to the irreversible changes in domain structure. Let us take a typical domain of magnetic material of magnetic moment m ? which makes an angle ? with the field H ? at any instant of time when the magnetisation of the material is M ? . ? The sum of the components of m ? perpendicular to H ? over unit volume is zero ?m sin ? = 0

NSOU ? CC-PH-08 ? 157 Again, M = ?mcos ? dM = – ?msin ?.d? Now the torque acting on the domain of moment m ? when it makes an angle ? with H ? is 0 m B m Hsin ? ? ? ? ? ? ?? ? ....(4.8.1) where B ? = ? 0 H ? = magnetic induction due to H ? . The work done by the external source in alliging the domain in a unit volume through a additional angle is dw = ?m? 0 H sin ?d? = ? 0 HdM ......4.8.2) So the work done per unit volume in traversing the specimen through a complete cycle of magnetisation is W = 0 0 HdM ? ?? ?? × area enclosed by the M – H loop. (4.8.3) or, W = 0 0 HdM ? ?? ?? 0 0 HdB ? ? ? ?? area enclosed by the B–H loop. 4.8.2 Increase in Temperature Due to Hysteresis Loss : Let A be the area of B – H loop and n is the number of hysteresis cycle, m is mass and ? the density. ? is the specific heat T is the temperature. Energy lost per sec is = volume of the material × number of cycle × area of the loop = m nAJ /s ? Heat energy produced = mnA J? Calories/s. By calore metric principle = msT = mnA J? So T = c nA o Js?

NSOU ? CC-PH-08 ? 158 4.9 Importance of Hysteresis Loop We can understand the nature of magnetic behaviour of a material by studying the structure of a hysteresis loop and identity its utility in manufacturing magnet. Hysteresis loop of soft iron and steel is shown in Fig 4.11 certain conclusion can be drawn are as follow : (i) Retentivity of steel is more than solf iron. (ii) Coercivity of steel is more than soft iron i.e. much greater coercive force is necessary to demagnetise steel-magnet. (iii) Area of hysteresis loop of iron is much lesser than steel i.e. energy spent per cycle for soft iron is much less than steel. Fig. 4.11 So, we can understand why soft iron core is required in the manufacturing of electromagnet of transformer. Because transsient magnetism requires smaller area of loop and lesser coercive force. Larger coercivity is the necessity to have strong magnet. A strong magnet does not undergo a complete magnetic cycle. So energy lost due to hysteresis cycle in strong magnet, even though area is having much large area. 4.9.1 Demagnetisation of Magnetic Material Ferromagnetic material retains some magnetism when it undergoes a hysteresis cycle. Magnetic transformation is unaltered even after with drawal of the applied magnetic field. H B Hard Steel Soft Iron O 2

NSOU ? CC-PH-08 ? 159 In order to demagnetise it, the magnet is placed in a gradually diminishing field and undergoes few hysteresis cycle. Area of the loop decreases gradually until it becomes zero. Thus the material has reached the state complete demagnetic state. (Fig. 4.12) Fig. 4.12 4.10 Summary 1. Magnetisation of material is measured by M ? . Numbers of magnetic dipole produced per unit volume, if material, is defined as magnetic moment. 2. Magnetic material behaves as composed of huge numbers of equal area circulating current loop, in uniform magnetisation. And the result at current flows only through peripheral region of the material. 3. Bound current exists in non-uniform magnetisation. Density of the current flow is given by J ? m = ? ?? × M ? . ? ?? × B ? = ? 0 J ? = m 0 (J ? f + J ? m) where J ? f is the free current density sourced from battery/other sources and J ? m is bound current density due magnetisation. 4. ? ?? × H ? = J ? f and, f H.dl NI ? ? ?? ?? ? where 0 B H M, ? ? ? ?? ?? ??? the auxiliary magnetic field only related to the free current If. B ? = ? 0 (H ? + M ?) for paramagnetic and diamagnetic material. 5. Hysteresis is the characteristics properties of feromagnet. Total energy spent in a hysteresis cycle is the area of the loop in SI. 6. Structure of the loop helps in identifying certain material with a specific purpose. 4.10 Review Questions and Answers 1. An electron (charge e mass m) revolves around the nucleus in a circular

NSOU ? CC-PH-08 ? 160 orbit with radius r and velocity v. The electrostatic force provides the necessary centripetal force. a) Calculate the current and magnetic dipole moment due to the orbital motion of the electron. b) Write down the force equation when the electron is placed in a uniform magnetic field B ? perpendicular v ? , how is the force equation modified? Answer : a) Orbital current due to the electron. e e ev I T 2 2 r ? ? ? ? ? ? , where T is time period, ? its augular velocity, , The magnetic dipole moment m = I ?r 2 2 ev evr . r 2 r 2 ? ? ? b) The force equation will be, 2 2 2 0 1 e mv 4 r r ? ? ? c) The electron will experience a force evB and its velocity rises from v to v ? in the presence of magnetic field. The equation of motion will be— 2 2 2 0 1 e v evB me 4 r r ? ? ?? 2. Determine the magnetisation current density due to non-uniform magnetisation current. Answer : See article 4.3 3. If the magnitude of augular momentum of an electron rotating in a circular orbit is ‘L’ find the magnetic moment. Answer : Orbital current due to electron, with time period T is e e ev I T 2 r 2 r v ? ? ? ? ? So the magnetic moment 2 2 ev evr | m| I r r 2 r 2 ? ? ? ? ? ?

NSOU ? CC-PH-08 ? 161 We know, v = ?r and augular momentum L = m ?r 2 2 eωr eL | m | 2 2m ? ? ? 4. Using the concept of bound current density for non-uniform magnetisation, establish f H J ?? ? ? ? ? , where f J ? is free current density. . Answer : See article 4.4. 5. Explain what you mean by free current and bound current in magnetisation of matter. Answer : Free current is produced by the electric charges, like electron when they move. It produces Joule’s heating effect. Bound current is produced by the orbital motion and spinning of electron in atom. It does not produces Joule’s heating effect. In magnetised matter atomic loops of current circuit are distributed at random. In uniform magnetisation, produced by the adjacent current loops cancel each other. Hence net effective current inside the material vanishes. Only we get some amount of surface current. In non-uniform magnetisation of matter, cancelletion will be partial and donot vanish. This residual current inside the maternal is called volume current. Thus we get formation of a current, which we call magnetisation current on bound current. 6. Discuss why soft iron is suitable for use as the core of transformer where as steel is preferred for making permanent magnet. Answer : The core of a transformer is made of soft iron because it has high permeability so it provide complete linkage of magnetic flux of the primary coil to the secondary coil. Therefore it has high coercivity and low retentivity. Soft iron provides the best material for the core of a transformer as its permeability (?) is very high. Its hysteresis curve is of small area and its coercivity low. A permanent magnet requeres high retentivity and high coercivity. Steel magnet has this property and is able to resist loss of magnetism due to improper handling. 4.11 Problems and Solutions 1. An infinitely long cylinder of radius R carries a frozen-in magnetisation, parallel to the axis. M ? = krz ^

NSOU ? CC-PH-08 ? 162 where K is a constant, and r is the distance from the axis (there is no free current here). (a) Find the bound current. (b) Find the magnetic field inside and outside. (c) Use Ampere's law to find H ? and B ? . Solution : a) Given the magnetisation of the material along z-axis and is equal to M ? = kr ? z ^ The bound volume current is given J ? = ? ?? × M ? b ˆ ˆ ˆ r r z 1 J r r z 0 0 kr ? ? ? ? ? ? ?? ? ? = 1 1 ˆ ˆ (kr) r (kr) r r r r r ? J b = −k ? ^ The bound surface current is given by −K ? b = M ? × n ^ K ? b = kr (z ^ × r ^) ? K b = Kr ? ^ b) A sobenoidal field is produced due to bound current. The field outside the cylinder will be directed along the z-axis Applying Ampere's law we get, B.dl BL ? ? ? ?? ? ? The current intercepted by the Ampere's loop is given by R in b r I K L KLdr ?? ? ? = −(KLR + KL(R−r) = −KLr Ampere's law can now be used to find the magnetic field. 0 n 0 J ˆ ˆ B z Krz L ? ? ?? ? ?
NSOU ? CC-PH-08 ? 163 (c) Now ? ?? .M ? = 0, it implies Ampere's law uniquely defines H ? . Now the H ? field is painting in z-direction. Using Ampere's law, in terms of the H ? field, we certainly conclude that for the Ampere's law in tercepted H.d l HL J 0 ? ? ? ? ? ? ? Since there is no free current, which can be only true if H ? = 0, which implies that 0 1 H B M 0 ? ? ? ? ? ? ? So the magnetic field B ? is given by B ? = ? 0 H ? Magnetisation outside the cylinder is zero and therefore magnetic field is zero B ? = 0 For the region inside the cylinder M = krz ^ So internal magnetic field B ? = ? 0 krz ^ which is identical to earlier solution 2. An iron rod (density 7.7×10 3 Kgm −3 and specific heat 470 JKg −1) is subjected to cycles of magnetisation having frequency 50 cycles. If the area of B−H loop of the specimen is 6×10 3 Jm −3 . Calculate the rise in temperature per min. Solution : Hysteresis area enclosed by the B−H loop = Energy lost per unit volume per cycle = 6×10 3 Jm −3 ? Energy lost per min = 6×10 3 ×50×60× 3 m 7.7 10 ? where mis the mass of the sample. Let T be the rise in temperature, we get,
NSOU ? CC-PH-08 ? 164 mST = m × 470 × 7 = 6 × 10 3 × 50 × 60 3 m 7.7 10 ? or, T = 3 0 1 3 6 10 50 60 4.97 C(min) 470 7.7 10 ? ? ? ? ? ? ? 3. Compute the intensity of magnetisation of the bar magnet whose mass, magnetic moment and density are 400g, 2 Am 2 and 8gCm −3 , respectively. Solution : Volume of the magnet = Mass density = 3 3 6 400 10 (8 10) 10 ? ? ? ? ? = 50 × 10 −6 m 3 Magnitude of the magnetic moment P m = 2Am 2 ? So the intensity of magnetisation, = Magnetic moment I volume ? = 6 2 50 10 ? ? M = 0.4 × 10 5 Am −3 4. Region 0 ? z ? 2m is occupied by an infinite stabs of permeable material (? r = 3.5). If B ? = 2 ˆ ˆ 10yi 5xjmwb / m ? within the slap determine (a) J ? (b) J ? b (c) M ? (d) K ? b on z = 0 Solution : By definition (a) 7 0 r By B 1 Bx ˆ J H k x y 4 10 (3.5) ? ? ? ? ? ? ? ??? ??? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? = −3.410k ^ KA/m 2 (b) Bound current density, J ? b = ? m J ? = (3.5−1) (−3.410) × 10 3 = −8.525k ^ KA/m 2
NSOU ? CC-PH-08 ? 165 (c) M ? = ? m H = 3 7 ˆ ˆ 2.5(10yi 5xj) 10 4 10 3.5 ? ? ? ? ? ? = (5.676 yi ^ − 2.840j ^) KA/m (d) K ? b = M ? × n ^ , since z = 0, is the lowerside of the slab occupying 0 ? z ? 2, n ^ = −k ^ Hence, K ? b = (5.676yi ^ −2.840 × g) × (−k ^) = (2.840xi ^ + 5.676yj ^) KA/m 5. The volume of the core of a transformer is 1000cc. It is fed with ac if 50 HZ. The loss of energy due to hysteresis. Calculate the area of the B−H loop. CU-13 Solution : The energy loss per second in the transformer core = 36J 3600s The energy loss per cycle 36J 1s 3600s 50cycles ? ? So the energy loss per m 3 = J 3 6 36 3600 50 m 1000 10 cycle ? ? ? Now, energy loss (in ergs) percycle per cc 3 7 6 1 36 10 10 (loop area) 4 3600 50 10 ? ? ? ? ? ? ? So, loop area = 25.13 cm 2 .

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Unit 5 Electromagnetic Induction Structure 5.1 Objectives 5.2 Introduction 5.3 Faraday's law of Electromagnetic Induction 5.4 Self-inductance 5.5 Mutual Inductance 5.6

Neumann's Formula 5.7 Calculation of Mutual Inductance 5.8 Series and Parallel Combinations of Inductances 5.9 Magnetic Energy 5.10 Summary 5.11 Review Questions and Answers 5.12 Problems and Solutions 5.1 Objectives In this unit, you will study the nature of electromagnetic induction through different related phenomenon as detailed below : Concept of magnetic flux. Faraday and Neuman's law, and Lenz's law, its application, its importance and specific characteristics. Motional electromotive force and Faraday's electromotive force, its quantitative significance. What is self inductance and mutual inductance, the ways to measure it and detail aspects to understand.
NSOU ? CC-PH-08 ? 167 Idea about Neuman's expression. Arrangement of inducetance in series/parallel, different values inductance to achieve for specific uses. Coupling phenomenon in inductance and its application. The nature of electromagnetic energy, what is its application in different areas of study. 5.2 Introduction In 1820, Oersted had shown that an electric current generates a magnetic field. But can a magnetic field generate an electric current? This was almost simultanceously and independently in 1831 by Joseph Henry and Michael Faraday. Faraday showed experimentally that

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whenever the magnetic flux linked with a closed circuit changes

with time an electric current is induced in the circuit. The reason behind the generation of current flow in a closed circuit without any current generating source in it, flow of electric current by changing magnetic flux with time across the loop, is called electromagnetic induction. Faraday's law along with Lenz's law, which follows from conservation of energy, comprise the governing laws of inductive current and its direction. Scientist Neuman, had further elaborated the spectrum of any form of electromagnetic flux flow. This is known as Faraday. Neuman law of electromagnetic induction. Farady explained electromagnetic induction using the concept of lines of force later on Maxwell used Faraday's ideas and build the foundation of his quantitative electromagnetic theory. Faraday's law has played an important role in the technological transformation as we find today. 5.3 Faraday's Laws of Electromagnetic Induction Fig. 5.1 Magnet plunged into coil (induced current makes near end a S pole) G G Magnet pulled out of coil (induced current makes near end a N pole)
NSOU ? CC-PH-08 ? 168 In Fig. 5.1 experiment of Faraday when the bar magnet is moved with respect to the coil following observations have been seen. i) The galvanometer shows a deflection when ever there is a relative motion between the coil and the magnet. Deflection indicates that an induced current has been set- up in the coil. ii) Faster movement of the magnet induces more deflection and less when movement of the magnet is slowed. iii) Reverse deflection in the galvanometer when the same pole is moved in opposite direction or opposite pole of magnet is move in the same direction. The observations led to the inculcation of the following two laws of electromagnetic induction. a)

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Induced emf in a circuit is proportional to the rate of change of magnetic flux linked with the circuit. b) The

direction of induced emf is such that it tries to oppose the cause
of generation i.e. the variation of magnetic flux inducing it. The second law is known as Lenz's law, which specifies the direction of current Lenz's law follows from the principle of conservation of energy.

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If ?is the flux linked with a circuit at any instant t, then d ? is the

time rate of change of flux. The combination of the two laws of electromagnetic induction reveals ? = – d? dt ....(5.3.1) where ? is the induced emf. The negative sign indicates that the emf ? opposes the changes of flux. If R is the resistance of the circuit, we get the induced current as i = e – R = – 1 – R d? df ....(5.3.2) If the electric field in space is denoted by E ? then emf acound a closed path or curve c is e E.dl ? ? ? ?? ? ? .....(5.3.2) If S is an open surface bounded by the curve placed in magnetic field B ? , then the magnetic flux through the surface
NSOU ? CC-PH-08 ? 169 s B.ds ?? ? ??? ? ....(5.3.4) Now, using equations (5.3.1), (5.3.2) and (5.3.4) we can write as c s d E.dl B.ds dt ?? ? ? ?? ??? ? ? ? ....(5.3.5) which is the intgegral form of Faraday's law when the

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circuit is fixed, the time derivation can be taken outside the integral, when it becomes partial derivative.

Now using stokes theorem, we get, ? ? ? ? s B E .ds .ds t ? ?? ?? ? ? ??? ? ? ? ....(5.3.6) since this must be true for any arbitrary surface s, then we get B E t ? ?? ?? ? ? ? ? ....(5.3.7) which is the differential form of

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Faraday’s law. 5.4 Self-inductance The induced emf ? in a coil is proportional to the rate of change of magnetic flux

passing through
it due to its own current. This emf is termed as self induced EMF. Magnetic flux produced by the current depends on the geometry of the circuit for non-ferromagnetic material. The induced emf

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is proportional to the rate of change of the current through the coil

and its proportionality constant is called self-inductance L. If I is the current flowing in a circuit, then associated magnetic flux can be written as, ? = LI .....(5.3.8) d d dI dt dI dt ? ? ? ....(5.3.9) The induced emf in the circuit is given by d LdI dt dt ? ? ?? ?? ....(5.3.10) where L = d? dI
NSOU ? CC-PH-08 ? 170 The SI

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unit of self inductance is henry (H). One henry is the

value of self-inductance in a closed circuit or coil in which one volt is produced by a variation of the inducing current of one ampere per second. Otherwise a circuit is said to have a self inductance of 1 henry if 1 weber of flux is linked with the circuit never 1 ampere of current flows through it. In SI unit ?, I, t, ? and L are expressed in weber, ampere, volt and henry 1 vall, second 1 weber 1 henry = = 1 ampere 1 ampere = 1 V.A −1 .S As ??has the dimensions [ML 3 T −2 I −1] and L are of dimensions [ML 2 T −2 I −2] 5.4.1 Calculation of Self-inductance 1. A solenoid : If I be current flow along aircored long solenoid of length containing N number of turns the axial magnetic field at any inside point. 0 N.I B= l ? ? ....(5.4.1) If A is the area of cross-section of the solenoid the flux linking each turn is 0 1 NIA BA l ? ? ? ? ......(5.4.2) and the total flux linking N turns 2 0 1 N AI N l ? ?? ? ? ....(5.4.3) Now the self-inductance L is defined as the flux linkage per unit current. So the self- inductance of the solenoid is 2 0 N A L I l ? ? ? ? ......(5.4.4) If the solenoid is wound on a materials of permeability ?, then 2 N A L l ? ? ....(5.4.5)
NSOU ? CC-PH-08 ? 171 Fig. 5.2 If the solenoid is not very long then the anial magnetic field at any anial point p as show in Fig. 5.2 is given by 0 2 1 NI B (cos cos) 2 ? ? ? ? ? ? = ? ? 0 2 2 2 2 NI x x 2l (x) a x a ? In a length dx about P, there are N – L dx number of turns and hence the flux linking these turns is ? ? N d dx BA l ? ? So the total magnetic flux through the solenoid is l 0 BAN d dx l ?? ? ?? ? = 2 0 2 2 2 2 2 0 N AI x x 2 (x) a (x a) ? or, 2 l 2 2 2 2 0 2 0 N AI (x) a x a 2 ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? X dx P ? ? ? ?
NSOU ? CC-PH-08 ? 172 = 2 2 2 0 2 N A (a) a .I ? ? ? ? ? ? ? ? ? ? ? ? So, 2 2 2 0 2 N A L a a ? ? ? ? ? ? ? ? ? ? ? ? ....(5.4.6) = 2 2 0 N A 1 (a /) a / l ? ? ? ? ? ? ? ? ? ? ? Note that for ? <<a, equation (5.4.6) reduces to equation (5.4.5) 2. Long Caonial Cable : Consider a long coaxial cable consisting of two concentric

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cylinder of inner radius a and outer radius b as shown in Fig. 5.3. The

two cylinder carry the same current I in the opposite directions; then they form a coaxial cable. Fig. 5.3 Applying Ampere’s circuital law, it can be shown that the magnetic field outside the cable is zero, while at an internal point at a distance r from the axis (a>r>b) the magnetic field is given by 0 I B 2 r ? ? ? If we imagine two coaxial cylinders of radii r and r+dr and of unit length, the flux in the region between the two cylinders is B×(1×dr) = Bdr so the total flux is b b 0 0 n r a a I dr B.dr b / a 2 r 2 ? ? ? ? ? ? ? ? ? ? ? ? ? The inductance per unit length is 1 r dr b a

NSOU ? CC-PH-08 ? 173 0 b L ln a 2 ? ? ? ....(5.4.7) In the above discussion we have neglected the flux within the materials of the two cylinders. This is justified when b<<a. If instead of air, the space between the two cylinder is having a medium of magnetic per meability ?, then from equation (5.4.7) will be modified with inductance b L ln a 2 ? ? ? ......(5.4.8) 3. Two-wire transmission lines : Two parallel wire transmission line is shown in Fig. 5.4, given the separating distance d, a its radius, and ? is the permeability of the medium in which they reside. Fig. 5.4 We assume that the radius a of each wire is much less than d, so that the flux inside the material of the wires may be neglected. The two wires carry the same current in the opposite directions. The flux is concentrated between the two wires. Total magnetic field at any point at a distance x from one wire is I 1 1 B 2 x d x ? ? ? ? ? ? ? ? ? ? ? So the flux through an elemental area of width dx and length unity is I 1 1 d B.dx 1 dx 2 x d x ? ? ? ? ? ? ? ? ? ? ? ? ? ? Therefore the total flux through the entire area between the two wires of unit length is d a 0 a I 1 1 d dx 2 x d x ? ? ? ? ?? ?? ? ? ? ? ? ? ? ? ? I d a a ln ln 2 a d a ? ? ? ? ? ? ? ? ? ? ? ? 2a dx x 1 I I
NSOU ? CC-PH-08 ? 174 I d a ln a ? ? ? ? So, self-inductance per unit length is ? ? d a h ln I a ? ? ? ? ? ? ......(5.4.9) Assuming d<<a or, ?? d H L ln m a ? ? ? 4. Toroidal Coil : The magnetic field inside a toroidal coil having mean length L, N being the number of turns of cross-sectional area A, and carrying current I is given by N B nI I L ?? ? ? where ? is the permeability of inside medium. Therefore, the flux through the N turn, neglecting the variation over the cross section, is given by 2 N A NBA I L ? ? ? ? So the self-inductance is 2 N A L I L ? ? ? ? ....(5.4.10) 5.5 Mutual Inductance Fig. 5.5 C 2 C 1 I 1 B 1 ?
NSOU ? CC-PH-08 ? 175 Two coils C 1 and C 2 are two fixed coils placed sufficiently close to each other, as shown in Fig. 5.5. If I 1 is current passed through the coil C 1 then magnetic field B 1 will be produced around the coil C 1 . Magnetic flux ? 21 will be passed through the coil C 2 due to B 1 . Alternatively, we can say that ? 21 flux linkage due to B 1 the magnetic field of coil C 1 . B 1 will change if I 1 changes, then ? 1 and ? 21 will also change. An induced emf will be produced in coil C 2 due to this. This phenomenon is called mutual induction. Again, there will be change in ? 1 , due to the change in current I, as a consequence an induced emf will also be induced in C 1 . This is called self-induction. For a number of turns in both the coils, we can write 2 2 21 1 s B .ds ? ? ? ??? ? where ds ? 2 is the elemental area in the coil 2. By Bio t-Savart law we can write 0 1 2 I ˆ d r B 4 r ? ? ? ? ? ? ??? ? ? ? So for coil C 1 , 0 1 1 1 2 I ˆ d r B 4 r ? ? ? ? ? ? ??? ? ? ? Taking into consideration that, other features of the coil as intact, B ? 1 depends only on I, so we can write, 2 1 0 1 1 2 21 2 s l I ˆ d r.ds 4 r ? ? ? ? ? ? ? ? ??? ??? ? ? ? ?? 21 = M 21 I 1 ....(5.5.1) nowhere 2 1 0 1 2 2 2 s ˆ d r.ds M 4 r ? ? ? ? ? ? ? ? ? ??? ??? ? ? ......(5.5.2) M 21 is proportionality constant between ? 21 and I 1 . Induced emf in the coil C 2 , according to Faraday's law, will be 2 2 2 d dI M dt dt ? ? ?? ?? ? ? ? ....(5.5.3)
NSOU ? CC-PH-08 ? 176 It is observed from the relation in equation (5.5.3) that induced emf in coil C 2 is related to the current flow changes in coil C 1 M 21 is defined as co-efficient of mutual inductance. It will be kept in mind that this will remain unchanged if the configuration of the two coil remain fixed. 5.6 Neumann's Formula Determination of mutual inductance is very complex, depending on the set up of two coils Neumann has formulated a relation to simply the calculation. We know that flux linkage is given by 2 2 21 1 s B .ds ? ? ? ??? ? and 1 1 B A ??? ? ? ? where A ? 1 is the magnetic vector potential corresponding to B ? 1 . Also, we know that vector potential is given by 0 c I dl A 4 r ? ? ? ? ?? ? ? or, 1 0 1 1 1 c I dl A 4 r ? ? ? ? ? ?? ? ? Hence ? ? 2 2 2 2 21 1 s c A .ds A .dl ? ? ?? ? ? ? ??? ?? ? ? ? ? or, 1 2 2 1 0 1 21 c c dl I .dl 4 r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ? ? But we know, ? 21 = M 21 I 1 ? 1 2 2 1 0 21 c c dl .dl M 4 r ? ? ? ? ? ? ?? ?? ? ? .... (5.6.1) Since the order of integration may be interchanged we can write 0 1 2 21 12 dl .dl M M 4 r ? ? ? ? ? ? ? ?? ?? ? ? .... (5.6.2)
NSOU ? CC-PH-08 ? 177 This is known as Neumann's formula for the mutual inductance of two arbitrary coils or loops. The double integral (5.6.2) is not easy to work with except for circuits. With simple geometry but it does illuminate two important points : i) M 12 =M 21 =M. This signifies that in any case the flux ? 1 through loop C 1 when a current I flows around C 2 is exactly equal to the flux ? 2 through loop C 2 when the same current I flows around C 1 . This is called as reciprocity theorem. ii) M 12 or M 21 is depends on the structure of the coil, configuration and relative position of the two coils. 5.7 Calculation of Mutual Inductance 1. Two solenoids : Fig. 5.6 Two coaxial solenoids are shown in the Fig. 5.6 where P is a long primary solenoid and S is short secondary solenoid. There is alomost no magnetic field outside the long solenoid. If a current I flows through the primary, the magnetic induction produced at the centre would be 1 0 0 N B nI I L ?? ?? where N 1 , and L were the total number of turns and length of the primary solenoid, respectively. If A be cross-sectional area of P, then flux linked with the secondary coil of total number of turns N 2 would be 0 1 2 2 N N A B.A.N I L ? ?? ? So the mutual inductance will be 0 2 IA N N M I L ? ? ? ? ? ......(5.7.1) P S

NSOU ? CC-PH-08 ? 178 2. Two parallel circular coaxial coil Let C 1 and C 2 are two parallel circular coanial coils with the centres O 1 and O 2 and radii a and b (Fig. 5.7). x is the axial separation O 1 and O 2 . The flux through C 2 can be assumed to be uniform taking into account C 2 is small compared to C 1 .

90%	**MATCHING BLOCK 64/107**	**W**
If I is the current in C 1 , magnetic induction at O 2 is given by 2 0 1 3 2 2 2 2 N Ia		

B 4 (a x) ? ? ? ? ? where N 1 is the number of turns in coil C 1 . Total flux linked with the coil C 2 is ? 2 = B. ?b 2 N 2 Fig. 5.7 or,? 2 = 2 2 0 1 2 3 2 2 2 N N a b I 2(a x) ? ? ? ....(5.7.2) So the mutual inductance between the coil is 2 2 0 1 2 2 3 2 2 2 IN N a b M I 2(a x) ? ? ? ? ? ....(5.7.3) If the coils are coplanar then x = 0 and 2 0 1 2 N N b M 2a ? ? ? Value of M for large circular loop C 2 can be determined by using Neumann's formula. 5.8 Inductance in series and parallel combinations of Inductances 1. Series connection : Fig. 5.8 O 2 C 2 b x O 1 a C 1 I
NSOU ? CC-PH-08 ? 179 Fig. 5.8 (a) shows two coils of self-inductances connected in series.

48%	**MATCHING BLOCK 65/107**	**W**
The induced emf in coil 1 due to self-inductance when current I flows through it, 11 1 dI L dt ? ? while the emf induced in coil 2 due to		

the

59%	**MATCHING BLOCK 66/107**	**W**
current I in coil 1 is 21 dI M dt ? ?? where M is the mutual inductance of the two coils. The emf induced in		

the coil due to self inductance is 22 2 dI L dt ? ?? and the emf in coil 1 due to the current in coil 2 is 12 dI M dt ? ?? Total emf due to the flux aiding me and another is 11 22 12 21 1 2 dI (L L 2M) dt ? ? ? ?? ?? ?? ?? ? ? .....(5.8.1) Again, eq dI L dt ? ?? ....(5.8.2) Comparing equations (5.8.1) and (5.8.2) the equivalent self-inductance, L eq is L eq = L 1 + L 2 + 2M .... (5.8.3) In Fig. 5.8(b) mutual flux opposes the self-flux of the two coil in series, then we get, 11 22 12 21 ? ? ? ?? ?? ?? So the equivalent self-inductance is L eq = L 1 + L 2 − 2M ......(5.8.4) 2. Parallel connection Fig. 5.9 ? ? ? L 1 L 2 I I 1 I 2 I M
NSOU ? CC-PH-08 ? 180 Fig. 5.9 shows

95%	**MATCHING BLOCK 67/107**	**W**
two coils of self-inductances L 1 and L 2 connected in parallel.		

Total I gets divided into branches as I 1 and I 2 . Assuming that the

44%	**MATCHING BLOCK 68/107**	**W**
mutual flux aids the self-flux, the total emf induced in coil 1 is 1 1 dI dI L M dt dt ? ?? ? Similarly 2 1 2 2 dI dI L M dt dt ? ?? ? Since the		

two coils have the same emf i.e. ? 1 = ? 2 = ? for being parallel, we have 1 2 1 dI dI L M dt dt ? ??? ......(5.8.5) and 2 2 2 MdI L dI dt dt ? ??? ......(5.8.6) Solving this two equations, we get 1 2 2 1 2 dI (L M) dt

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L L M ?? ? ? ? and 2 1 2 1 2 dI (L M) dt L L M ?? ? ? ? Therefore, 1 2 1 2 2 1 2 dI dI (L L 2M) dI dt dt dt L L M ?? ? ? ? ? ? ? If L eq be the equivalent self-inductance, then eq dI L dt ? ?? ? 2 1 2 eq 1 2 L L M L L L 2M ? ? ? ? ......(5.8.7) If

there is no magnetic coupling between the coils then M = 0 and we have 1 2 eq 1 2 L L L L L ? ? or, eq 1 2 1 1 1 L L L ? ? ....(5.5.8) 3. Coefficient of coupling : In order to find mutual inductance, there is necessity for
NSOU ? CC-PH-08 ? 181 two disconnected coil, so that current flow in one coil can induce emf on other coil. Mutual flux between them can be less than or at best equal to the self-fluxes of the two loops. It implies that 12 22 ? ?? and 21 11 ? ??. So we can write ? 12 = K 1 ? 22 and ? 21 = K 2 ? 11 , where K 1 and K 2 are two numbers less than or equal to one. So we can write, MI 2 = ? 12 = K 1 ? 22 = K 1 L 2 I 2 since ? 22 = L 2 I 2 Hence, M = K 1 L 2 ....(5.8.9) M.I 1 = ? 21 = K 2 ? 11 = K 2 L 1 I 1 or, M = K 2 L 1 ....(5.8.10) From equations (5.8.9) and (5.8.10) we get, M 2 = K 1 K 2 L 1 L 2 or, M = 1 2 K L L .... (5.8.11) where K = K 1 K 2 , and 0 K 1 ? ? . This geometrical constant is known as coefficient of coupling of the loops. This coupling coefficient depends on verying geometry, which can be designed according to one's criteria. 5.9 Magnetic Energy 1. Energy in an inductor : When a electric current flows in an inductor it will store energy in the form magnetic field. For a pure conductor power which must be supplied at any instant of time to initial current through the inductor is di P iv Li dt ? ? Hence the energy in put to have a final current i is given by— Energy stored (E) = t 1 0 0 Pdt Lidi ? ? ? 2 1 E LI 2 ? ? ....(5.9.1)
NSOU ? CC-PH-08 ? 182 Self-inductance of a circuit can be defined as the two times the magnetic energy stored in a circuit when a unit current is established in it. So the self-inductance is thus a measure of the magnetic energy stored in the circuit for a given current. 2. Energy stored in a magnetic field : Energy is required to establish a magnetic field, which is stored as a magnetic field energy. Let us take a number of current carrying loops in a finite region of a medium. Magnetic flux associated with the ith circuit is given by i i i i s c ˆ B.nds A.dl ? ? ? ? ? ?? ? ? ? ......(5.9.2) where A is the magnetic vector potential associated to B ? by B ? = ? ?? × A ? . The magnetic energy of the system is n i i i 1 1 U I 2 ? ? ? ? i i i il 1 I A .dl 2 ? ? ? ? ? ? ? ....(5.9.3) Assuming each circuit is a closed path in the medium which is conducting. i i I dl ?? should be replaced by J ? dv and i i c v by so ? ? ? ? v 1 U J.AdV 2 ? ? ? ? ? ....(5.9.4) where J ? is the volume current density. Now using the relation ? ?? ×M ? = J ? , we can write equation (5.9.4) as v 1 U (H).AdV 2 ? ? ?? ? ? ? ? ....(5.9.5) From, vector identity .(A H) H.(A) A.(H) ? ? ? ?? ? ?? ? ? ? ? ? ? ? ? ? ? Now, using the vector identity and divergence theorem, we get
NSOU ? CC-PH-08 ? 183 V S 1 1 ˆ U H.(A)dV A H.nds 2 2 ? ?? ? ? ? ? ? ? ? ? ? ? ....(5.9.6) where S

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is the surface bounding the volume V. Note that integration is to be carried out over the entire volume occupied by the current

distribution. For convenience, the surfaces can be moved to infinity. This will not affect the integration (5.9.4) because J ? = 0 outside the region occupied by the current distribution. Now at large distances A ? and H ? will fall at least as rapidly as those of dipoles, hence 2 3 1 1 A ~ , H ~ r r . So the integrand A ? × H ? falls off at least as 1 /r 3 or faster. As the surface element ds ? goes as r 2 , the surface integral vanishes as 1/r or faster as r goes to infinity. Therefore, equation (5.9.6) becomes 1 U H.Bdv 2 ? ? ? ? ? ......(5.9.7) So we conclude from equation (5.9.7) that magnetic energy stored in a magnetic field with energy density as 1 – 2 (H ? .B ?). So we can write, energy density as 2 2 1 B 1 U H.B H 2 2 2 ? ? ? ? ? ? ? ? ...... (5.9.8) 5.10 Summary We have learned following topics on electromagnetic induction. 1. Idea about magnetic flux : BS B.ds ?? ? ? ? ??? ? Faraday and Neumann's laws d dt ? ? ? and Lenz's law. . 2. Differential form of Faraday's law B B t ? ?? ?? ? ? ? and integral form d E.dl dt ? ? ?? ? ? ? ?? ? ? 3. Self-inducetance and mutual inductance. Inducetance L = ? – I , and mutual inductance 2 1 1 2 M . I I ? ? ? ?

NSOU ? CC-PH-08 ? 184 4. Neumann's expression far mutual inductance : 1 2 2 1 0 21 12 c c dl .dl M M 4 r ? ? ? ? ? ? ? ?? ?? ? ? We have studied self and mutual inductance of current loop with different geometrical shapes. 5. Magnetic energy : 2 mag 1 1 U LI A.Jdv 2 2 ? ? ? ? ?? 2 0 1 B dv 2 ? ? ? 6. Energy density in magnetic field 2 2 0 0 H H.B B u 2 2 2 ? ? ? ? ? ? ? ? 5.11 Review Questions and Answers 1. Obtain the integral form of Faraday's law and then show that B E t ? ?? ?? ? ??? ? ? Answer : See article 5.3. 2. Starting from energy consideration prove that M 2 ? L 1 L 2 . Answer : For two fixed closed circuit with positive coupling, with currents C 1 and C 2 in the respective circuits EME equations at any instant. 1 2 1 1 1 1 di di R i L M dt dt ? ? ? ? ....(5.9.1) 2 1 2 2 2 2 di di R i L M dt dt ? ? ? ? ....(5.9.2)

NSOU ? CC-PH-08 ? 185 From energy conservation consideration, the rate of energy supplied from the source must be equal to the rate of Joule heat dissipation plus the rate of energy stored in the magnetic field. so, 2 2 1 1 2 2 1 1 2 2 dU i i i R i R m ag dt ? ? ? ? ? ? ? ? 1 1 2 2 1 1 2 2 1 1 2 2 2 2 1 1 2 2 1 2 2 dU mag i i i R i R dt i e i (ik i k) ? ? ? ? ? ? ? ? ? ? ? From (5.9.1) and (5.9.2), We get mag 1 2 2 1 1 1 2 2 1 1 2 dU di di di di L i L i M i Mi dt dt dt dt ? ? ? ? 2 2 1 2 1 2 1 2 di di 1 1 d L L M (i i) 2 dt 2 dt dt ? ? ? Taking i 1 = i 2 =0 at t = 0, and integrating the equation from t = 0, to t = t We find 2 mag 1 1 2 1 2 1 1 U L i L i Mi i 2 2 ? ? ? ....(5.9.3) or, 2 2 mag 2 2 1 2 2 1 1 1 1 2U L M M i i i L L L L ? ....(5.9.4) which is valid for all i 1 and i 2 and let i 1 = 2 1 M i L ? . Then since U mag is positive or zero for all values of i 1 and i 2 we must have M 2 ? L 1 L 2 . 3. Show that the equivalent inductance of

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the two coils of self-inductances L 1 and L 2 , connected in parallel

is 2 1 2 eq 1 2 L L −M L = L +L ±M Answer : See articale (5.8) for answer.

NSOU ? CC-PH-08 ? 186 4. Show that the self-inductance of a long solenoid length l radius a and with n turns per unit length is approximately given by L = 2 2 2 2 0 n . a l a a ? ? ? ? ? ? ? ? ? ? Answer : See Article 5.4. 5. State and prove the reciprocity theorem in mutual inductance. Derive Neumann's formula for the mutual inductance between two arbitrary loops. Answer: See article 5.6. 6. Two long parallel wires carrying the same current I in the opposite direction and separated by a distance d in the air. The length of the wire are much larger than d. Find the self-inductance per unit length. Answer : See article 5.4. 7. Two coils with self-inductance L 1 and L 2 respectively, have mutual inductance M. Find an expression for their coefficient of coupling. Answer : See article 5.8. 8. Obtain a formula for the mutual induction between two loops carrying current. Answer : See article 5.7. 5.12 Problems and Solutions 1. A wire of length 1m moves at right angle to its length at a speed of 100 m/ s in a uniform magnetic field 1 wb/m 2 which is also acting at right angle to the length of the wire. Calculate the emf induced in the wire when the direction of motion— (i) right angles to the field, (ii) inclined at 30° to the field. Solution : Induced electric field due to motion in magnetic field is equal E ? = (? ? × B ?) Induced emf E.dl ? ? ? ?? ? ? (B).dl ? ?? ? ? ? ? For a length 'L' of the rod, induced emf will be

NSOU ? CC-PH-08 ? 187 ? = vBLsin ? (? is the angle between ? ? and B ?) (i) Here ? = 90° ??? = vBL which v = 100 ms −1 , B = 1wbm −2 , L = 1m ??? = 100volt (ii) When ? = 30° ? = vBL 50V 2 ? ? 2. A conducting metallic disc is rotating about an axis passing through its centre, perpendicular to its own plane. An external magnetic field is applied in a direction perpendicular to the plane of the disc. What will induced emf ? What will be the current flow it a metallic wire is connected between periphery and the axis? Solution : Let P be point where the wire is connected at the periphery. Let the disc rototates, it position at t is covers a distance dr in time. t + ?t. i.e. PQ = dr. So the area POQ = 1 − 2 rddr. Intercepted magnetic flux d ? = 1 − 2 Brdr. Therefore the induced emf between O and P d 1 dr 1 Br Brv dt 2 dt 2 ? ? ? ?? ?? ? or, ? = 1 − 2 Br 2 w The direction of the induced emf cannot be determined in this specifi case. Current flow 2 Br I R 2R ? ? ? ? 3. Suppose a square loop of side a is placed in the plane of a long straight wire carrying current I. The nearest side of the loop is at a distance r from the wire. Find the magnetic flex through the loop. If someone pulls the loop directly away from the wire at a constant speed, what should be the emf generated in the loop? What is the value of emf generated when the loop is pulled parallel to the wire? Solution : O r P Q dr Fig. 3

NSOU ? CC-PH-08 ? 188 Fig. 4 The magnetic induction at a distance x from the wire is 0 2I B . 4 x ? ? ? This magnetic field is in a direction normal to plane of the loop. So the flux through an elemental area adx within the loop as shown in Fig. 4. 0 Iadx d Badx 2 x ? ?? ? ? Total flux across the loop is r a 0 0 r Ia r a d x Ia 2 x 2 a ? ? ? ? ? ? ? ? ? ? The induced emf in the loop is d d dr . dt dr dt ? ? ? ?? ?? 0 Iav vd 1 1 r dr 2 r a ? ? ? ? ?? ?? ? ? ? ? ? ? ? ? 2 0 Iva 2 r(r a) ? ? ? ? 4. A square loop of wire of side 'a' lies on a table near a very long straight wire which carries a current I. Find the flux through the loop if it moves away from the wire at a speed 'v'. What is the emf generated ? What is the emf generated in case the loop is pulled to the right with the same speed ? ?? r a a a I x dx

NSOU ? CC-PH-08 ? 189 Fig. 6 Solution: The magnetic induction at a distance x from the long wire which carries current I is 0 I B 2 x ? ? ? which is directed normally on the loop the flux through an elementary area adx within the loop is given by 0 Ia dx d Badx 2 x ? ?? ? ? or, ? ? a r 0 0 n r a I Ia a r dx l 2 x 2 r ? ? ? ? ? ? ? ? ? ? ? or, ? ? 0 Ia a ln 1 2 r ? ?? ? ? ?Induced emf d d dr . dt dr dt ? ? ??? ?? 0 2 Iv d 1 a v dt 2 a r 1 r ? ? ? ? ? ? ? ?? ?? ? ? ? ? ? ? ? ? ? ? 2 0 va 2 r(r a) ? ? ? ? ? ? The current is in anti clockwise direction, when the flux is pulled right, there is no change in flux, so ? = 0. a x r dx v I

Unit 6 Maxwells Equations And Electromagnetic Wave Propagation Structure 6.1 Objectives 6.2 Introduction 6.3 Displacement current and Maxwell's Equations 6.4 Maxwell's Equations 6.5 Poynting Theorem 6.6 Maxwell Stress Tensor 6.7 Potential Formulation and Gauge Transformations 6.8 Boundary Conditions 6.9 Wave Equation 6.10 Propagtion of EM Waves in Free Space 6.11 Plane EM Waves in an Isotropic Dielectric Medium 6.12 Reffection and Refraction at the plane interface of two Dielectrics: Normal Incidence 6.13 Reflection and refraction at oblique incidence at the Interface Between two Dielectrics 6.14 Summary 6.15 Review Questions and Answers 6.16 Problems and Solutions

NSOU ? CC-PH-08 ? 191 6.1 Objectives After the completion of the Unit, learners will be able to under stand : 1 Nature of electromagnetic waves and its source 2 Unified classical theory of electromagnetism forwarded by Maxwell and prediction of oscillating nature of electromangnetic waves 3 Electromagnetic spectrum 4 Boundary condition,behavior of electromagnetic wave throgh free space, throgh different media like dielectric, metal etc. Its behavirol changes while interfacing different media. 5 Different optical phenomen, like total internal reflection, Brewester's law and evanascent waves etc. 6 Reflection and Refraction of Electromagnetic of waves. 7 Gauge Theories 6.2 Introduction Electromagnetic waves EM are synchrorized oscilliations of electric and magnetic fields. These waves are created due periodic change of electric and magnetic fields. The creations of electromagnetic waves are formed when a cherged particle is accelerated as a part of oscillatory motion, the charged particle creates ripples on oscillations in its electric field and also produces a magnetic field. The electric and magnetic of the wave are perpendicular to each other and also, perpendicular to the Magnetic field Electric field Propagation of electromagnetic wave Y Z X Electromagnetic Wave Fig. 6.1

NSOU ? CC-PH-08 ? 192 direction of the EM waves. EM waves carry energy, momentum and angular momentum away from their source particle and can impart those quantites to matter with which they interact them. EM waves travel with a constant velocity of C=3x10 ? m/s in vacuum. They are diflected neither by the electric field nor the magnetic field, they exhibits, interference diffraction. An EM wave can travel through anything be it vacuum air and solid material. It requaires no medium to propagale from one space to another space. This tranverse wave are measured by their amplitude, and wave length.

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The highest point of a wave is known as 'Crest', whereas the longest point is known as 'Trough'.

The EM wave can be split into a range of frequencies, which is known as electromagnetic spectrum-examples are radiowaves, microwaves infrared, x-rays and gamma rays etc. In quantum mechanics, EM radiowaves, are termed as photons, uncharged elementary particles with zero rest mass which are the quanta of the electromagnetic field responsible for all electromagnetic interactions Quantum effects generates additinal source of radiation such as the transition of electrons to lower energy in an atom and black body radiation photons have energy of 'h ?' where h is Planek's constant, the higher the frequency the higher the energy. 6.3 Displacement Current and Maxwell's Equations Displacement Current From Ampere's law we can calculate the magnetic field due to steady current flow. We know that the integral ?H.dl ? ? ? around any closed path or loop is equal to i' where i is current passing an area bounded by the closed curve C (Fig. 6.2) If Ampere's law is true all the time, then the i determined should be independent of the surface chosen. Let us consider the case of charging of a capacitor by (Fig.6.3) charging current i (t) = V R e -t/Rc , which leads to a magnetic field H ? as observed, with the Ampere's Flat surface loop C Curved surface Ƨ . Fig. 6.2 i

NSOU ? CC-PH-08 ? 193 law ?B.dl = i ?? ? If we look at i = i (t) If look at i = O This is because there is no charge flowing between the capacitor plates. It points out that Ampere's law is either wrong or incomplete. Also from Ampere's law in differential form x H = J ? ? ? ? ....(6.3.1) Where J ? is the current density. Taking the divergence of the above equation. . x H = . J = O ?? ? ? ? ?? ? Fig. 6.3 This reflects that . J = O ? ? ? which violates the continuty equation. As the electric charge is piling up on the plate of the capacitor contained whithin the volume enclosed by the surface S 1 and S 2 , the continuity equation is ? O t ? ? ? . J + ? ? ? ....(6.3.2) Where is the charge density on the capacitor which varies with time. So, some quantity must be added to equation (6.3.1) on the right hand side, which must be consistent with the equation (6.3.2). In order to find this, quantity, which must be consistent, an electric displacement vector D ? related to the charge density by . D = ? ? ? ? ....(6.3.3) From equation (6.3.2) and (6.3.1) We find . (. . t ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ? D J + D) = J+ O t ....(6.3.4) Now if we add ? D? t? to the right hand side of equation (6.3.1) then its divergence

NSOU ? CC-PH-08 ? 194 will satisfy equation (6.3.2) with the inclusion of ? D? t? , in Ampere's law of differential form, we have, D H J + ? ?? ? ? ? ? ? ? ? t ....(6.3.5) The quantity ? D? t? was first introduced by Maxwell and is called displacement current density. For a very slowly varying field ? D? t? is negligible. We can use equation (6.3.1) of unomodified Ampere's law of steady field. We learnt earlier that the electric field can be generated by charges and changing magnetic flux. So we see from Ampere Maxwell that a magnetic field can be generated by moving charges (current) and changing electricflux. Thats is a change in electric flux throgh a surface bounded by C can lead to an induced magnetic field along the loop ie: induce magnetic field is along the same direction at caused by the changing electric flux, without the term t ? ? ? D D J = electromagnetic wave propagation would be impossible. Based on the displacement current density, we define the displacement current, as I = d ∫J d . ? ds = ∫ ? D? t? ? . ds We must bear in mind that displacement current is a result of time varying electric field. 6.4 Maxwell's Equation Here we sumarize the laws associated with electromagnetic field : .? ?? D = ? ....(6.4.1) .? ?? B = O ....(6.4.2) ? ? ? ? ? ? B x E = - t .... (6.4.3) ? ? ? ? ? ? ? D x H = J+ t ....(6.4.4)

NSOU ? CC-PH-08 ? 195 Equation (6.4.1) and (6.4.2) express Gauss's law for the electric and the magnetic field respectively. Equation (6.4.1) is a mathematical statement of Coulomb's law, while the physical significance of equation (6.4.2) is the absence of free magnetic monopole Equation (6.4.3) states the Faraday-Henry law of electromagnetic induction, while equation (6.4.4) is the Ampere-Maxwell law containing the factor displacement current density. t ? ? D All the equations comprising (6.4.1), (6.4.2), (6.4.3) and (6.4.4) represent Maxwell equations. It is also highlighted that the term ? (? r, t) and ? J (? R, t) in all the above equation contain all charges and current respectively, whether free or bound. There is however, a more convenient form of the set of general equation of Maxwell, suitable for the study of electromagnetic fields inside material subtances that are subject to electrical polarization and magnatizetion, let (? B ? E,) represents electromagnetic field inside the material of subtance having both electric and magnetic properties assuming P as polarization vector and M magnetization vectors respectively Introducing the auxillary fields we have. ? ? ? ? ? ? o I D = ε B - M ? 0 E + P , H = ...(6.4.5) For a linear medium, ? ? ? ? m e E , M = ? H P = ε x ....(6.4.6) so that ? ? ? ? ? 1 D = E , H = B ? ....(6.4.7) where,) ? m e ε = o (1+and = (1+) ? ? ? ? o ....(6.4.8) Modified Maxwell equation taking into account ? t (? r, t) and J t (? r, t) as the free charge and current densities respectively inside the meterial take the form .? ?? D = ? f ....(6.4.9) .? ?? B = O ....(6.4.10) ? ? ? ? ? ? B x E = - t ....(6.4.11)

NSOU ? CC-PH-08 ? 196 ? ? ? ? ? ? ? D x H = J + t t ....(6.4.12) with proper set of different boundary condition depending on the different material media at their interface, solution representing their properties, for the above set of equation can be obtained. The Maxwell equations are also associated with certain conservation laws, such as the conservation of charge and the conservation of energy which is explaned by Poynting's theorem. Conservation of charge can be demonstrated with the help of equation (6.4.1) and (6.4.4). Taking the divergence of equation (6.4.4) ? ? ? ? ? ? ? D x H = J+ ?? ?? t ?? ? ? ? ? ? ? ?? J + ? ? ? t ???D) ?? ? ? ? ? ?? J + ? ? ?t ?? ....(6.4.13) Equation (6.4.3) represents the equation of continuity. Taking the divergence of both side of equation (6.4.3) ? ? ? x E = O - ?? ? ?? ? ? ? ? B t = = ? ?t ? ???B) ? , so div ? B = O, which is compatible with equation (6.4.2) 6.5 Poynting's Theorem It states that in a given volume, the stored energy changes at a rate given by the work done on the charges within the volume, minus the rate of which energy laves the volume. From equation (6.4.4) and (6.4.5) we can obtain, ? ? ? x E - E. ?? ? ? ? ? ??? x ? ? ? ? B t -E. J ? ?? ?? ? ? ? D t ? ? ? ?? ....(6.5.1) using the vector identity ? ? ? ? ? ?? ? ? x B) = B. x A -A. x B ? ? ? ? in the LHS of equation (6.4.13) we have,

NSOU ? CC-PH-08 ? 197 ? ? ? ? ? ? ? ? ?? ? ?? E x H) = - H J ? ? ? ? ? ? B t ? ? ? D t ....(6.5.2) For a linear and non-dispersive media, we can write, D = Ɛ.? ? ? and B = μ ? ? ? where Ɛ and μ are permitivity and permeability of the media, so we have ? ? ? ? B t H. = ? ?t ( ) 1 2 ? H. ? B and ? ? ? ? D t E. = ? ?t ( ) 1 2 ? E. ? D Putting this in equation (6.4.14) and integrating over finite volume V we get ∫ d dt ?? ? (E x H) dv = ? ∫ v [E.D + H.B] dv ? ∫ v J.E.dv ? ? ?? ?? ?? v .... (6.5.3) Applying Gauss's divergence theoreom to the left hand side of equation (6.5.3) ∫ d dt (E x H).n ds = o ? ∫ v (E.D + H.B) dv ? ∫ v J.E.dv ? ? ?? ?? ?? s 1 2 ....(6.5.4) where S is the surface bounding the volume V as, ∫ (E. D+H.B)dv= ∫ v J.E.dv+ ?? ?? v d dt ? 1 2 ?? ∫ s (E x H) ? ? .n ds o ....(6.5.5) The term E.D ? 1 2 ? is energy stored in electrical fields and the term (H.B) ? 1 2 ? is the energy stored in magnetic field. The left hand side of equation (6.5.5) points out the rate at which the electromagnetic energy stored in the volume V decreases with time. The rate of work done by the electromagnetic force on an infinetisimal charge dq = dV is given by dw dt = dq(E x ? ? x B). ? ? ? ? = dq E. ? ? = E. (dV)=E.J.dV ? ?? ?? ?? where ? is the velocity of the charge element and J = ?? ? ? , So the term ∫ E. ?dV ?? indicates the rate of doing work on the charge in the volume V by electromagnetic field, in other words it is Joule heat. The last term on the right hand side of equation (6.5.5) gives the rate which energy flows out of the bounding volume V.

NSOU ? CC-PH-08 ? 198 The vector S = E x H ? ? ? called Poynting vector has the unit of Joule m s -2 -1 and so it can be described as the energy flowing out through unit area per unit time. Poynting's Theorem can also be represented in differential form ?? S + ?? ? ? W t = P 2 ....(6.5.6) where P = J.E = 2 ?.E 2 as the Joule heat and 1 2 W = (H +) ? 2 ?? ? electromagnetic energy stored in per unit volume. 6.5.1 Time Average Value of The Poynting Vector Now assuring the electric and magnetic fields is given by: 2 Im)?? Re Re Re Now taking the average of the above equa tion we get ? o >S< = >(E x B Cos?t<->(E x B 2 Im) 1 [0 0] 2] 1 2 so ?? ?? ?? ?? ? ? ? ? ? ? ? ? ? ? ? Im Re Im Re Re IM IM Re Re IM IM ? o Cos <>Sin < -(>E x B Cos Sin <+>E x B Sin 1 >S< = >E x B >E x B ? o = [>E x B E x B 1 2 ?t ?t ?t ?t ?t Re))(( ) ? ? ? ? ? ?? ?? ?? ?? ? ? ? ?? ?? IM Re o Re IM Re IM E =E 0 e jwt = (E jECos?t+jSin ?t) = (E cos -E im Sin) and B B = (B jB cos +jSin) Now the Poynting vector S is given by, S (E x B) 1 ?o = E Cos -E Sin x (B ? ? 2 2 (( )Cos) Re IM Re Re Re IM IM Re IM IM cos -B sin) = [E x B)Cos - E x B)Sin Cos -(E x B .Sin +(E x B Sin] (?t ?t e jwt ?

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t ?t 1 ?o ?t ?t 1 ?o ?t ?t ?t ?t ?t ?t ?t ?t

NSOU ? CC-PH-08 ? 199 But to compute the Poyting vector the simplest way to use a real form for both field ?? E and ?? B rather than complex exponential representation. 6.5.2 Energy of Electromagnetic Waves A plane monochromatic EM waves propagating in Z direction ie: K direction is given by ?? E = E o cos (kz- ?t) and ?? B = J B o cos(kz- ?t) ....(6.5.7) where B o = E o c The total energy associated with the electromagnetic wave fields is U = U E +U M = 1 2 ∫ v B 2 ? o + E ? o 2 dV) (as B = E / C and c = o 0 ? ? The electric and magnetic energy contributions to the total energy are equal and electromagnetic energy density for a polarised wave is U EM = ? o E 2 = ? o E o 2 Cos 2 (kz-) The Poynting vector becomes ?? S = 1 ? o ?? (E x B) = c Cos (kz- t) ? ? ? o o 2 2 ?? k = k u c EM The time average density is defined as the average over one period T of the EM wave, ∫ o >u < = em ? o o E 2 T T Cos (kz- t)dt 2 ? = ? o o E 2 T T 2 = 1 2 ? o o E 2 = 1 2 B o 2 ? o ....(6.5.8) It follows that energy density of EM wave is proportinal to the square of the amplitude of the electric (or magnetic) field. 6.5.3 Momentum of Electromanetic Radiation
NSOU ? CC-PH-08 ? 200 Due to The wave-particle duality of radiation, as stated in Quantum Mechanics, radiation or photonos travelling with speed c, the energy of each photon is given by ? ? ? ? = h? Momentum of a single photon is P ? ? K = ? ? c k So for n photons per unit volume we can relate average Poynting vector to n , multiplied by velocity vector kc >S< = n ck = >u < c k ϵ em Now P ? is defined as momentum of EM waves carried across a plane normal to propagation vector k per unit area per unit time P ? S/c ? ? when all the momentum of EM wave is absorbed in normal incidence, it exhibits a force per unit area equL to the normal incoming flux of radiations. The radiation pressare is P = P.n = S/c =< P >u< rad rad EM In case of diffuse radiation ie. radiation bouncing around in all direction, the pressure is given by P rad = >u EM </3 6.6 Maxwell Stress Tensor It is a symmetric second order tensor used in classical electromagnetics to represent the interaction between electromagnetic forces and mechanical momentum. A second rank tensor whose product with unit vector to a surface reveals the force per unit area transmitted across the surface by an electromagnetic field. Its easy to calculate the Lorentz force on the charge moving freely in homogeneous electromagnetic field, which is simple situation. In complex situation of interaction of particle and electromagnetic field, Maxwell stress tensor lays the way to use tensor arithmatic to find an answare to the problem at hand. Momentum conservation is rescued by the realization that fields themselves carry momentum, also its attributed energy. As we know The Lorentz force on a moving charge particle is given by F = q (E+ x B) ʋ ? ?? ? ....(6.6.1)
NSOU ? CC-PH-08 ? 201 So the force per unit volume acting on charge density distribution in a volume V f = (E+ x B) ? ʋ ? ?? ? ....(6.6.2) =f = ?E + J x B ? ? ? ? ....(6.6.3) From Maxwell's Electromagnetin equation ? = ? ? ?? E ? ? ....(6.6.4) and J = ? 1 ? o ? x B - ? ? ? ? ? E t ? ? ....(6.6.5) Substituting ? and J ? from equation (6.6.4) and (6.5.5) in equations (6.6.3) we get, f = ? ? ?????? ? ?

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x B - ? ? ? 1 ? o ? ? E t ? x B ??? ? ? ? ....(6.6.6) Now, (E x B) = ? ? ? ? t ? ? E t ? x B + E x ? ? B t ? ? ? ....(6.6.7) or, ? ? E t ? x B = ? ? ? t (E x B) - E x ? ? ? ?

B t ? ? ....(6.6.8)
Also from Maxwell's third equation ? ? ? ? B t ? E x ? = - ....(6.6.9) Subsituting above in equation (6.6.8) we get ? ? ? ? t ? ? E t ? E) x ? x B ? = (E x B) + E (? ? ? ....(6.6.10) From, equation (6.6.6) and (6.6.10), we get after rearraing f = ? 0 ? ? [(?? ? E) E-Ex(xE)] + 1 ? o [(?xE)xB]- [(x E)] ? ? t B ?? ?? ? ? ? ? ? ? .....(6.6.11) Introducing a term (x E)B B ? ? ? in the equation (6.6.11) to make it more symnetrical ? ? f = ? [(?? ? E) E-Ex(xE)] + 1 ? o [(? ? .B) B-Bx xB]- ?? ?? ? ? ?? ? ?[? ? t (ExB)] ? ? ? ?? ....(6.6.12) From the property of gradient, we know that,

NSOU ? CC-PH-08 ? 202 ??? ? ? ? ? ?? ? ? ??? ? ????? ? A.B) = A x x B + B x x A + (A. ???? ? ? ? ?????? ? ? x?x E + E x ? x E + (E. ??? ? ?????? ??? ? ??? x x E + 2(E. ? ? ? ??? ?? so, ?? ? ? ? ?????? ?x?x E = ? ? ? ? 1 2 ? ??? ....(6.6.13) 2 1 B B (B) (B.)B 2 ??? ? ? ? ? ? ? ? ? ? ? ? ....(6.6.14) Subsituting, (6.6.13) and (6.6.14) in equation (6.6.12) ??? ? ? ??????? ? ? ? f = ? ? ????????? ? 1 ? o ?? ? 1 2 ???????? ??? ????????? ? 1 2 ?? ? ? ? ? ? t (E x B)] ? ? ? ??? ? ??? ? ? or ??????? ? f = ?????? ?? ? 1 ? o ??? ??????? ? ??????? ??? ??? ??? ?? ? ? 1 2 ? ?? ? ? ? ? 1 2 ? o ??? ??? ? ??? ? ? ? ? ? x ? ? t ? ? ? ? Equation (6.6.13) can be simplilfied with the introduction of Maxwells tress tensor T ij = ?o (E E - i j 1 2 δijE) + 2 1 ? o (B B - i j 1 2 δ ij B) 2 ....(6.6.15) The indices i' and j' refer to the co-ordinates x, y ans z, so the stress tensor has a total nine components (T xx , T xy , T xz , T yx , T yy , T yz , T zx , T zy ,T zz) Thus, T xx = 1 2 ?o (E -E -E) + x y z 2 2 2 1 2 1 ? o (B -B -B) x y z 2 2 2 ....(6.6.16) T yy = 1 2 ?o (E -E -E) + y x z 2 2 2 1 2 1 ? o (B -B -B) y x z 2 2 2 ....(6.6.17) T zz = 1 2 ?o (E -E -E) + Z x y 2 2 2 1 2 1 ? o (B -B -B) z y x 2 2 2 ....(6.6.18) and, T xy = T = yx ? o E E + x y 1 ? o B B x y ....(6.6.19)

NSOU ? CC-PH-08 ? 203 T yz = T = zy ? o E E + y z 1 ? o B B y z ....(6.6.20) Tzx = T = xz ? o E E + z x 1 ? o B B z x ....(6.6.21) T ij is represented as a tensor by T ? a rank 2- tensor. It is represented by a 2-dimensional, (3x3) matrix T xx T yx T zx ? T xy T yy T zy T xz T yz T zz T = ...(6.6.22) We can form the dot product of T ? with a vector a ? ? a. T j = (a T)) ? x,y,z j ij (? ....(6.6.23) the out-coming object, which has one remaining index, is itself a vector. Now if we take the divergence of T ? has as its jth component. ?? ? j = (??) ?o (j) Ej + j+(B. j] [ ] [?? ?? ? ????? ?? ?? ? ?? ? (1 ? o ?? ?? ?? ?? 1 2 ? o -[?o j (E) + j (B) ? ? 2 2] 1 2 ....(6.6.24) So the force per unit volume in equation (6.6.24) take the form, ? ? S t ? f = ????? ? ? ?? ? o ? o ....(6.6.25) So The total force on the charges in volume V is given by, F = ? ∫ fdv = ∫ v v ? ? S t ????? ? dv ?? ? o ? o ∫ ? dv ....(6.6.26) or F = ? ∫ fdv = T.ds - o ? ? ? ? ?o ? o ∫ ? ? S t ? dv ....(6.6.27) Here S o represents the surface. In the static case, ∫ ? ? S t dv is to be dropped.

NSOU ? CC-PH-08 ? 204 Physical signticance of T ? is the force per unit area (or stress) acting on the surface. Here Tij is the force (per unit area) in the ith direction acting on an element of surface alligned in the j th direction. The diagonal elements represent pressures , and off diagonal elements are shears. 6.6.1 Conservation of Momentum According to Newton's second law the force on an object is equal to its momentum F = ? dP mech t ? d ....(6.6.28) So equation (6.6.27) can be writen in the form given below ? o ? o F = ? d(P) mech t ? d = ∫ ? ? S t ? dv + T.ds o ? ? ? ? ....(6.6.29) where P ? mech is the total mechanical momentum of the particles in volume V. This expression in equation (6.6.29) is similar to the representation of Poynting theorem. The first integral represents momentum stored in the electromagnetic fields themselves ?o? o g em = ? ∫ S dv ....(6.6.30) while the second integral is the momentum per unit time flowing in throgh the surface equation (6.6.29) is the general statement of conversation of momentum in electrodynamics. Any increase in the total momentum (mechanical plus electromagnetic) is equal to the momentum brought in by the fields when V encompass all space then, no momentum flows in or out, and P mech + g em is constant. If the mechanical momentum is V is not changing ie in region of empty space, then ∫ ? d g en t dv = T.ds o ? ? ? ? = ∫ ?? T dv ?? and hence ? d g en t = ?? T ? ? ....(6.6.31) This is the "continuity equation" for electromagnetic momentum, with g em (momentum density) in the role of ? (charge density) and T ? playing the role of J ? ; it expresses the total local conservation of field momentum. But in general charges and fields exchange momentum and only the total is conserved. Here we note that S ? plays the energy per unit area per unit time transported bythe

NSOU ? CC-PH-08 ? 205 fields, while S ? ? o Ɛo is the momentum per unit volume stored in those fields. Similarly T ? is the electromagnetic stress acting on thesurface and- T ? represent's the flow of momentum ie momentum current density, carried by the fields. 6.6.2 Angular Momentum The angular momentum of EM wave is a vector that expresses the amount of dynamical rotation present in the electromagnetic while travelling approximately in a straight line. The beam of light can also be taking, the two distinct forms of rotation of light beam are its polarization and its wave front shape. Two forms of rotation are identified as light spin angular momentum. Now the energy density of electromagnetic fields carry energy, 1 2 u = (Ɛo E 2 + B) 2 1 ? o ....(6.6.32) and momentum, g = em Ɛo (E xB) ? ? ? ....(6.6.33) The Angular momentum L = r x g = em Ɛo [r x(E xB)] ? ? ? ? ? ? ....(6.6.34) In case of static fields, it can have angular momentum as long as E xB ? ? is non zero and it is only when these field contributions are incorporated that the conversation laws are prevailed. 6.7 Potencial Formulation and Gauge Transformations In Maxwell's theory, the basic field variable are the strenghts of electric and magnetic fields which may describe in terms of auxillary variable like scalar and vector potentials. The gauge (theory) transformations in this theory consists of certain changes in value of these potentials that do not yield in a change of the value of electric and magnetic fields. Thus the invariance is preserved as we look forward to the formulation of modern theory of electrodynamics. In electrostactics we know x 0 ? E = ? ? it enables one to write E = ? ? ? ?, where ? is a scalar function. In electrodynamics E ? x ? ? ? ? But B ?? ? ?? ? ? . So it demands for certain generalisation time dependent solution of the problems.

NSOU ? CC-PH-08 ? 206 So we can write variable B ? as B = ? ? x ? ? ? where is ? ? vector potencial. Now = ? x [E +] = 0 ? ? ? ? A t ? ....(6.7.1) Above equation (6.7.1) can be transformed with the introduction of gradient of a scalar function ? to the expression (E +) ? ? ? A t ? ....(6.7.2) In static case ? ? A t ? ? ? , so, E ? ? ? ? ? ? from Gauss's law ??? ? ? ? o ? ? we have, 2 0 (.A) / t ? ? ?? ? ??? ? ? ? ? ....(6.7.3) substituting equation (6.7.2) in modified Ampere's law in electrodynamincs, ? ? ? ? ?? x ? J + o ? o ? ? E t ? ? ? ? ?? ? ?? ? ? ? ? ? ? ? x J - x ? o o ? ? o o (? ? ? t ? ? ? A t 2 ? ? ? ? ? ? ....(6.7.4) from vector identity ? ? ?????? ? ? ? ? ??? ? x ? ? ? ?? ??? ? ? we have from equation on (6.7.4) ?? ? ?? ? o ?o) ??????? ? o ?o) ? ?? ? J ? ? ? ? ? A t 2 ? ? ? ? t ??? ? ....(6.7.5) So from equation (6.7.3) and (6.7.5) carry all the informations in Maxwell's equations. We conclude that potential formulation of Maxwell's electromagnetic formulation reduce the six variable of ? ? and ? ? (three of each) to four variables are three values of vector potencial ? ? and one value to sealar function ?.

NSOU ? CC-PH-08 ? 207 Equation (6.7.2) and the equation ? ? ? ? x ? ? ? do not uniqely define the potentials. Let us introduce two sets of potential (?, ? ?) and (?' , ? A '), gives the same electirc and magnetic fields, so writing ? ? ? ? + ? ? ? and ? ? ? ' + , We have, ? ? ? ? ? ? ? ????? ? ? ? ? x A = x ' x or x ? ? ? ? ? ? ? ? ? ? or ? ? = S ? ? where S is scalar Again, E = +) = (? ??? ? ?? ? ? ´ ? ? ? ? A t ? ? ? A´ t ? = O ? ? ? ? ? ? ? t or ? ? ? ? ? ? B ? ? ? s t So the term ? B? ? ? ? S t is independant of position co-ordinates, it is only function of time, taking it as g(t), thus ? ? S t ? g (t) B ? ? or ? ? ? ? ? P t where P = S- ∫ g(t)dt t ´ ´ O The function P can replace S in the defination of ?. since = P = S, so ? ? ? ? ? A = P ´ ? ? ? ? ? ....(6.7.6) and ? ? ? ´= ? ? P t ....(6.7.7) Thus, we observe that addition of ? P ? to ? ? and the subtraction of ? ? P t from ? do not alter the ? ? and ? ? . these changes in ? and ? ? are called gauge transformations.

NSOU ? CC-PH-08 ? 208 We can also choose set of potential ? ? , ? , such that, ???? ? o ? o ? ? ? ? ? A t 2 ?? ? ....(6.7.8) This choice is called Lorentz gauge under this transformation, equation (6.7.3) and (6.7.5) becomes, ? ? ?? ? o ? o ? ?? ? J ? ? ? A t 2 ? ? ? ? ....(6.7.9) ? ? ? ? ? o ? o ? ???? o ? ? ? ? t 2 ....(6.7.10) We can choose another set of gauge called coulomb gauge, where ??? ? ? ?? , Equation (6.7.3) and (6.7.5) become, ? ? ? ? ? o ? o = - ? ? J + ? ? ? ? A t 2 ? ? o ? o ?? ? t ....(6.7.11) and, ?? ? ? ???? o ....(6.7.12) Equation (6.7.12) can easyly be solved to find ? , as in electrostatics ? (r, t) = 1 4?? o ∫ (r , t) ´ ? 1 r- r 1 ´ ? ? dv 6.8 Boundary Conditions We can use Maxwell's equations to derive the boundry condition on the magnetic field across a surface. Consider a "pillbox" across the surface taking Maxwell's equation ?? ? ? ? ? ? ....(6.8.1) intergate over the volume of the pillbox, apply Gauss's the orem : ∫ ?? ? ? dv v ? ? s ?? ? ? ds ? ? ? ? ....(6.8.2) S 1 S 3 S 2 B 1 B 2 Medium 1 Medium 2 n l Fig. 6.4

NSOU ? CC-PH-08 ? 209 where V is the volume of the pillbox, and S its surface. We can break the integral over the surface into three part's over the flat ends (S 1 and S 2) and over the curved wall (S 3) [Fig.6.4] ∫ ?? ? ds s 1 ? ? ∫ ?? ? ds s 2 ? ? ∫ ?? ? ? ds s 3 ? ? ....(6.8.3) In the limit that the lenght of the pillbox approaches zero the integral over the curved surface also approaches zero. It each end has small area "A" then equation (6.8.3) becomes -BA + B A = O in 2n ....(6.8.4) B = B in 2n ....(6.8.5) it impiles the normal component of the magnetic field ?? ? must be continuous across the surface. Boundary condition 2: Tangential Component of E ? consider a loop spanning the surface (Fig. 6.5) Maxwell equation : E ? x ? ? ? ? ? B t ? ....(6.8.6) Integrate over the surface bounded by the loop and apply stoke's theorem to get ∫ ? ?? ? x ds s ? ? c ?? ? ?? dl ds ∫ ? ? ? ? ? ? t s ? ? ? ....(6.8.7) Now take the limit, in which the width of the loop becomes zero. The contributions to integral around the loop C from narrow ends become zero; as does integral of the magnetic field across the area bounded by the loop, so from equation (6.8.7) ? ? 1t 2t ? ? - = O ....(6.8.8) so, ? ? 1t 2t = ...(6.8.9) E 1 E 2 h 1 h 2 Fig. 6.5 e S 1 S 3 S 2 D 1 D 2 Medium 1 Medium 2 h Fig. 6.6 h
NSOU ? CC-PH-08 ? 210 Hence

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the tangential component of the electric field is continuous across the

boundary. Boundary Condition 3 : Normal Component of D , consider a pillbox crossing the boundary (Fig. 6.6) From Maxwell's equation, ?.D = ? ?? ....(6.8.10) Integrating over the volume of pillbox, apply Gauss's Theorem ∫ ?? D dv = D.ds =∫ dv ? ?? ? ? ? ....(6.8.11) Assuming the height of the pillbox to zero, and surface charge density s 3 area of the pillbox being small A, then -

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D 1n A + D 2n A = ? s A or D 2n - D 1n = ?

s ....(6.8.12) When surface charge density is zero. The normal component of D ? is continuous across the boundary, However it is not true for the normal component of E ? unless the two materials have identical (permittvities). From continuitiy equation, ?.J = ?? ? ? ? t ....(6.8.13) Integrating equation (6.8.13) over the volume pullbox having approximately zero height, we get from (6.8.10) J - J = 1n 2n ? ? s ? ?t ....(6.8.14) For monochromatic electro magnetic wave, s s will vary as e -jwt then ƐE - Ɛ E = 1 1n 2 2n s ? ....(6.8.15) and, ? 1 E 1n - ? 2 E 2n = jw ? s ....(6.8.16) Now consider the following cases It ? s = O. from equation (6.8.15) and (6.8.16) i) ? c1 Ɛ 1 = ? c2 Ɛ 2 ....(6.8.17)
NSOU ? CC-PH-08 ? 211 Which can be satisfield for properly chosen materials, ii) If ? s ? o, eleminating ? s from equation (6.8.15) and (6.8.16), [Ɛ Ɛ 1 1n 2 2n +] E = [+ j] E j? c1 ? ? c2 ? ....(6.8.18) iii) It ? c2 = ? , then E 2n = O since the electric field inside a perfect conductor must be zero. From equation (6.8.15), we have E = 1n ? s Ɛ 1 as D = 1n ? s when electromagnetic waves pass into a conductor, the field amplitudess fall exponentially with a decay lenght given by the skin depth ? 2 ? ? ?? ....(6.8.19) As conductivity increases, the skin depth gets smaller. Since both static and oscillating electric fields vanish within a good conductor, the boundary condition is given by: E 1t ? ? ? ? ? ?t D 1n s ? ? ? ? D 2n ....(6.8.20) Tangential Component of ? ? consider a loop across the boundary (Fig. 6.7) From Maxwell's equations ? x? ? ? J ? ? ? D t ? ? ? ,we intergrate over the surface bounded by the loop, and apply Stoke's theorem to obtain H 1 H 2 h1 h2 Fig. 6.7 i

NSOU ? CC-PH-08 ? 212 ∫ ? ?? ? x ds s ? ? c ?? dl ? ∫ D. ds ? ? ? ? ? ? t ? ? ∫ s J? ds? ?? ....(6.8.21) As before taking the limit where the lengh of the narrow edges of the loop become zero then we have, H 1t l - H 2t l = J s^ ....(6.8.22) Where J s^ represents a surface current density perpendicular to the direction of the tangential component of ? ? that is being matched. 6.9 Wave Equation Formulation of complete and symmetric theories of electricity and magnetism, together with Lorentz force law, by Maxwell, have culminated in the prediction of wave theory of light identified and discovered as electrimagnetic wave, which travells with the speed C. Let us assume that the medium is linear permittivity e, the permeability ? and electrical conductivity are constant. The wave equation for magnetic intersity is obtained by taking curl of ? x? ? ? J ? ? ? D t ? ? ? ? ? ? ? x x x x ? ? ? J ? ? ? D t ? ? ? ? ? ? ....(6.9.1) As the current density, ? ? ? J c ? ? and electric displacement, ? ? D Ɛ ? ? , so from equation (6.9.1) ? ? ? ? x x x x ? ? ? ? c E Ɛ (E) ? ? ?t ? ? ? ? ? ? ....(6.9.2) Putting the value of ? x E ? ? from Maxwell's equation and given ? ? ? B ? ? simplitying equation (6.9.2) ? x? x? ? ??? ?? c Ɛ ? ? ? ? ? ?t ? ? ? ?t ? ? ....(6.9.3) using the vector identity ? x? x ? ? ? ????? ? ? ? ? ? ? ? ? ?? ? From equation (6.9.3) then NSOU ? CC-PH-08 ? 213 ? ?? ? ? ? ? ? ? ? ? ? ? c ? ? Ɛ ?? ? ? ? ? ? ?t ? ?? ? ? t 2 ? ....(6.9.4) using, ?? ? ? ??? ? ? ? ? ? ? ?? , Equation (6.9.4) becomes ? ? ? ? ?? ? ? ? ? -Ɛ c ? ?? ? ? t 2 ? ? ? ?t ? ....(6.9.5) Equation (6.9.5) is the wave equation. Again from Maxwell's equation, ? ? ? ? x ? ? ?t ? ? ? we can obtain, ? ? ? ? ? ? x x x ? ? ?t ? ? ? ? ? ....(6.9.6) ? ? ? ? ? ?? ? ? x x Ɛ c ? ? ?t ? ? ? ? ? ? ? ? ?t 2 ? ....(6.9.7) or, ? ??? ? ? ? ? ? ? ? ? ? ? = Ɛ c ? ? ?t ? ??? ? ? ? ?t 2 ? ? ....(6.9.8) If the medium contains no charge ? = O, so that ??? ? ?? ? ? D ? ? ?? ?? , so equation (6.9.8) becomes, ? ? ? ? ? ? ? ? ? ? Ɛ c ?? ? ?t 2 ? ? ? ? ?t ? ....(6.9.9) Which is a wave equation. 6.10 Propagtion of EM Waves in Free Space ie ? = 0, and ? = 0, J = 0 An EM wave

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unlike mechanical waves which requires the presence of material media to transport energy from one location to another

space, carries the energy throngh a vacuum at a spaced of C = 3 x 10 8 m/s which will be proved here from all the electromagnetic equations of Maxwell. In free space, Maxwell's equations become
NSOU ? CC-PH-08 ? 214 ??? ? ? ?? ....(6.10.1) ??? ? ? ?? ....(6.10.2) ? ? x ? ?? ? ? ? ? ? ? ?t ? ....(6.10.3) ? ? x ? ?Ɛ ? ? ? ? ? ? ?t ? ....(6.10.4) Taking the curl of equation (6.10.3) we obtain, ? ????? ? ? ? ? ? ? ? ? ?? ? ?t ?? ?? x ? ? or ? ? ? ? Ɛ ? ? ? ? ? ?? ? ?t 2 ? ?? ....(6.10.5) Similary taking curl of equation (6.10.4) and using (6.10.3) we get, ? ? ? ? ? ? ? Ɛ ? ? ? ?? ? ?t 2 ? ....(6.10.6) Thus it appears that both ? ? and ? ? satifiys the well known wave equation - ?? ? ? ? ? ? c ? ? ? ?t 2 ....(6.10.7) So the velocity of propagetion of EM wave is 8 0 0 1 C = = 3x10 m/s ε μ Which is exactly the speed of light in free space, there is a corelation can be drawn that light is a form of EM waves. Let us seek a simple solutions concerning ? ? or ? ? E r t and H r, t ??? ? ? ? ?? ? ? ? ? ? j r) j r t ??? ?? ??? ?? ? ?? ? ?? ? ?? ?? ....(6.10.8) Where E ? ? and H ? ? are complex amplitudes. Which are constants in space and time ? ? is the were vector determining the direction of prepagation of wave, ? ? is defined as
NSOU ? CC-PH-08 ? 215 ? ? n = n ? ? ĥ ? ? c ....(6.10.9) Where n is the unit vector along the direction of propagation. Therefore, ? ? ? ? ? ? ? ? ? ? and = E 2 d ? ? ? t 2 ? ? ? ? ....(6.10.10) subsituting equation (6.10.10) in equation (6.10.5) we get ? or ? ? ? ? ? ? ? ? ? ? ....(6.10.11) Now plugging in the value of ? 0 = 4 x 10 -7 H/m and ? 0 = 8.85 x 10 -12 F/m, we get v = 3x10 10 m/s = speed of light(c) Relative directions of ? ? ? ? and ? ? ? From equations (6.10.8), (6.10.1) and (6.10.2) it can be show that. ? ? ? ??? ? ? and ? ?? ? ? ? ....(6.10.12) so both ? and ? ? ? are perpendicular to the propagation vector ? ? , which implies the transverse characteristic of EM wave or light wave. From equation (6.10.8), (6.10.3) and (6.10.4) it can be shown that j x j or x ? ? ? ? ? ????? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ....(6.10.13) and j x j or x ? ? ? ? ? ????? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ....(6.10.14) Equation (6.10.13) shows that ? ? is both perpendicular to both ? and ?? ? ? Equation (6.10.14) shows that ? ? is both perpendicular to both ? and ? ? ? . Hence field vectors ? ? and ? ?

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are mutually perpendicular and also both are perpendicular to the direction of propagation

vector ? ? As ? ? ? ?? ? ? ? ? , thus in vaccum, K is real quantity, it proves that both ? ? and ? ? are in phase.
NSOU ? CC-PH-08 ? 216 Wave Impedance : The ratio of the absolute value ? ? and ? ? is defined as wave impendance. ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? The value of Z 0 comes around 376.6 ?. 6.11 Plane EM Waves in an Isotropic Dietectric Medium Let us consider a linear homogeneous and isotropic dietectric medium where ? = 0, Maxwell's equation then becomes ? ? ? ? . ? ? ....(6.11.1) ? ? ? ? . ? ? .... (6.11.2) ? ? x ? ? ? ? ?t ? ? ? ? ....(6.11.3) ? ? x ? ?ϵ ? ? ? ? ?t ? .... (6.11.4) Taking curl of equation (6.11.3) and using (6.11.4), we obtain ? ????? ? ? ? ? ?? ? ? ?? ? ?t ?? ?? ? ? x ? ? ϵ??? ? ?t 2 ? As ? ? ? ? . ?? for no charge present in the dielectric ? ? ? ? ? ? ? Ɛ ? ? ? ? ?t 2 ? ....(6.11.5) In the same way, taking curl of equation (6.11.4) and ? ? ? ? . ? ? , we get ? ? ? ? ? ? ? Ɛ ? ? ? ? ?t ? ....(6.11.6)
NSOU ? CC-PH-08 ? 217 So both ? ? and ? ? follow the standrad differential wave equation. So we get the velocity of electromagnetic waves in dielectric medium, 1 C 0 0 r r r r 1 v = = (ε μ ε μ) = ε μ μ ε ....(6.11.7) Where 1 0 0 C = ε μ is the speed of EM waves in free space. ? r is the permittivity or deelectric constant K. ? r is the relative permeability of the medium For a nonmagnetic dielectric medium ? r = 1 so, C v = k Solutions of the wave equations (6.11.5) and (6.11.6) are given by ?

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E (r, t) = E e 0 J(k. r - t) ? ? ? ? ? H (r, t) = H e 0 J(k. r - t) ? ? ? ? ? ? ....(6.11.8)

Where E 0 ? and H 0 ? are complex amplitudes, which are constants in both space and time and, wave vector is given by ? ? ? ? n ˆ ? ? ^ ¯ λ n ˆ ? ? c n ˆ ....(6.11.10) It shows that both ? E and ? H are prependicular to the direction of propagation vector ? ? , which reveals that nature of electromagnetic waves. subtituting the solution given in equation (6.11.8) to (6.11.4) we obtain ? K x E = H ?? ? ? ....(6.11.11) ? K x H = E ??? ? ? ....(6.11.12) From equations (6.11.11) and (6.11.12) we can conclude that both ? E and ? H are prependicular to each other and also both of them are perpendicular to the direction of propagation vector ? K .
NSOU ? CC-PH-08 ? 218 From equation (6.11.8) and from the wave equation we can get K 2 = ??? 2 ....(6.11.13) It states that in this stated dielectric properties wave vector ? K is a real quanlity and from equations (6.11.11) and (6.11.12) it can be shown that both ? E and ? H are in phase. Wave impedance is found to be ? ? ? ? ? ? ? ? ? ?? ? ? ? ....(6.11.14) or ? ? ? ? ? r ? r ? ? ? ? Poynting Vector : Poynting Vector ? S is given by ? S = E x H ? ? ....(6.11.15) sustituting the volume of ? H from equation (6.11.11) in equation (6.11.15) we get, ? 1 ?? S = E x (K x E) ? ? ? ? 1 ?? [K (E. E)-E (E. K)] ? E 2 ?? K ? ? ? ? ? ? ? So, S = ? E 2 ?? K ? ....(6.11.16) Equation (6.11.16) shows that energy flows in the direction of propagation vector K ? , we can write the magnitudes as K E = ??H or ε E = ? H

NSOU ? CC-PH-08 ? 219 or 1 ? ϵ = E 2 1 ? ? ? ? ....(6.11.17) Therefore energy is equally shared between electric and magnetic field in linear isotropic and homogeneous dietecric medium. Total electromagnetic energy density is u = 1 ? ϵ + E 2 1 ? ? ? = = H E H 2 2 2 ϵ ....(6.11.18) we can also write Poynting vector as S ? ? n ˆ ? nˆ u ϵ?? uv ....(6.11.19) Where nˆ is the unit vector in the direction of propagation. So the magnitude of Poynting vector is velocity of the wave multiplied by the energy density and propagate in harmony with the electric and magnetic field. 6.12 Reffection and Refractron at the plane interface of two Dietectrics: Normal Incidence The interface separating the two different dielectrics is taken to be the X-Y plane at Z = 0. A plane monochromatic EM wave of angular frequency w incident normaly at the interface from medium with electric field vector along the X-axis. The wave will be partly reflected into the medium 1 and partly transmitted into the medium 2. The frequency of the reflected and transmitted wave will be same as the boundary condition must be the same at all times. The incident and transmitted waves will move along positive X-diretion. While reflected wave along negative X-axis. The incident reflected and transmitted wave have electric and magnetic field vector are as follows: Incident wave : ? ? ? E 1x e j(k 1 z -?t) ? ? ? ? J ˆ H 1y e j(k 2 z ?t) ? Reflected wave : ?´ ? ? ? c ˆ ?´ ?x e - (k z 1 + ?t) ? NSOU ? CC-PH-08 ? 220 ?´ ? ? ? ˆ ?´ ?y e - j(k z 1 + ?t) ? Transmitted wave : ? ? ? E 2x e j(k z 1 - ?t) ? ? ? ? J ˆ H 2y e j(k z 2 - ?t) ? Taking the magnetic permeability of the medium to be ? 0 , we can write magnetic field vectors as follows : H = E , H = E and ´ = ´ 1y 1x 2y 2x y x ? ? ? ? n 1 ? 0 c n 2 ? 0 c n 1 ? 0 c The propagation constants are given by k 1 = n 1 w/c and k 2 = n 2 w/c From the boundary condition follows that tangential component of electric field is continuous at z = 0. so we have E 1x - E' 1x = E 2x ....(6.12.1) We have from the boundary condition that tangential component of magnetic field is continuous at z = 0, then H 1y - H' 1y = H 2y or n 1 (E 1x + E' 1x) = n 2 E 2x ....(6.12.2) Solving equations (6.12.1) and (6.12.2) we get, E = ' 1x E 1x n n 2 1 - n n 2 1 + ...(6.12.3) ? ? ? ? ? ? ? ? ? ? ? ? ? ? Medium 1 Medium 2 ? ? ? ? ? ? ? X O Y ? Fig. 6.8 NSOU ? CC-PH-08 ? 221 E = 2x E 1x 2 n 1 n n 2 1 + ...(6.12.4) Fresnel co-efficient is defined as the ratio of reflected and wave and incident wave and ratio of transmitted wave and incident waves. These co-efficients are denoted by r and t. We get, r = ?´ ?x ? ?x = n n 2 1 - n n 2 1 + ...(6.12.5) t = ? ?x = ? ?x 2 n 1 n n 2 1 + ...(6.12.6) Now, the reflectance (or reflecting power) is defined as R = >S 1 ? >S´ ? ? ....(6.12.7) and transmittance as T = >S 1 ? >S< 2 ....(6.12.8) where >S 1 <, >S' 1 < and >S 2 < are the average energy. flows per unit area per unit time for the reflected and transmitted waves. So they represent the time averaged Poynting vectors or intensities of three waves. Also we know that E >S< = 2 0 0 2 ? ? E n = c 2 0 0 2 ? where, n = 0 ϵ ? is refractive index and E 0 is the amplitude of electric fields. So reflectance and transmittance in normal reflection is given by R = n ?´ ?x 2 ? ?x 2 = r 2 =

62% **MATCHING BLOCK 81/107** **W**

n n 2 1 - n n 2 1 + 2 ....(6.12.9) and T = n = n 2 t 2 4n n 1 2 (n n 2 1 +) 2 n E 2 2n 2 n E 1 1n 2 n 1 = ...(6.12.10)

NSOU ? CC-PH-08 ? 222 From equations (6.12.9) and (6.12.10) we get R n + T n = 1...................(6.12.11) which proves that energy is conserved

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at the interface. 6.13 Reflection and refractron at oblique incidence at the Interface Between two Dietectrics The reflection and refraction of

EM waves

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at a plane surface between two media of different dielectric properties are well known phenomena. The different aspects of the phenomena divide themselves into two

classes. 1) Kinematatic Properties : a) Angle of reflection equals angle of incidence. b) Sneall's law : sini sin ɤ = n 2 n 1 where i and r are the angle of incidence and reflection, while n 2 and n 1 are the corresponding indices of refraction. 2) Dynamic properties : a) Intensities of reflected and refracted radiation. b) Phase changes and polarization. 3) Polarization : Consider the incident, the reflected and the transmitted waves as shown in the Fig. 6.9. Here K I , K T , and K R be the propagation vecitors for the incident, reflected and transmitted waves, respectively. From boundary conditions, we know that all rays must have the same angular frequency. The electric and the magnetic field vectors can be written as follows : For parallel polarization (P) Incident wave : ? ? (r. t) i(k.r-wt) I ? ? ? ? ?? e ? ?? ? ? (r. t) (? ? ? n 1 ? ? c k 1 ˆ x[? ?? e j(k.r-wt) I ?? Refected wave : ?

21%	MATCHING BLOCK 84/107	SA	MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

R (r. t) j(k .r-wt) R ? ? ? ? ?R e ? ?? ? R (r. t) (? ? ? n 1 ? ? c k´ 1 ˆ x[? ?R e j(k .r-wt) R ?? } ....(6.13.1) } .... (6.13.2) NSOU ? CC-PH-08 ? 223 Transmitted wave : ? T (r. t) j(k .r-wt) T ? ? ? ? ?T e ? ?? ? T (r. t) (? ? ? n 2 ? ? c k´ 2 ˆ x[? T0 e

j(k .r-wt) T ?? Where k , 1 ˆ k´ 1 ˆ , k 2 ˆ are the unit vectors along ? ? ? ? R ? ? and ? T ? respectively

70%	MATCHING BLOCK 85/107	SA	167E2440-Electricity & Magnetism.docx (D165105106)

let ? I , ? R and ? T be the angles between the normal to the interface and the

propagation direction. The angles ? I , ? R and ? T are called angle of incidence, reflection and refraction, respectively. Reflection and Retraction for oblique incidence: The plane of incidence XZ plane, The incident Electric field as in the xz plane (P polarization). All three waves have the same frequency that is determined once and for the source then three wave numbers are related by K I ? I = K R ? I = K T ? 2 or ? ? = ? R = ? ? ? ? ? T = n ? n ? ? T .... (6.13.4) The existence of boundary conditions at Z = 0, which must be satisfield at all points at all times, implies that the spatial (and time) variation of all the fields must be the same at z = 0, consequently, we must have the phase factors all equal at z = 0. For the spatial terms, evidently ? ? ? r ? ? = ? R ? ? r ? = ? T ? ? r ? when z = 0 } ....(6.13.3) K R E R H R H T K T E T H I K I E I Z = 0 n 1 n 2 ? ? ? R ? T Z X Fig. 6.9
NSOU ? CC-PH-08 ? 224 which is known as phase matching ....(6.13.5) Explicitily,

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x (K I) x + y (K I) y = x (K R) x +y (K R)y = x (K T) x + y (K T) y ....(6.13.6) for all x and all y

Equation (6.13.6) can only hold if the components are separately equal, for if x = 0 , we get (K I) y = (K R) y = (K T) y ....(6.13.7) while y = 0, gives (K I) x = (K R) x = (K T) x ....(6.13.8) let us orient all

83%	MATCHING BLOCK 87/107	SA	MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

our axes so that ? ? ? lies in the xz plane [ie.(K I) y =0];

so from equation (6.13.7) it follows that ?

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R ? and ? T ? also lies in the same plane. Thus we conclude that First law : The incident, reflected and transmitted wave vectors form a plane

From equation (6.13.8) it follows that K I sin ? I = K R sin ? R = K T sin ? T ....(6.13.9) Second law :

89%	**MATCHING BLOCK 89/107**	**SA**	PHY3512 Physics for Physical Science II 376pg ... (D144373244)

The angle of incidence is equal to the angle of reflection ? I = ? R .... (6.13.10) which is

the law of reflection Now for the transmitted angle Third law, Sin? ? Sin? ? = n ? n ? ....(6.13.11) which is the snell's law of refraction. The boundary condition

100%	**MATCHING BLOCK 90/107**	**SA**	019E1230-Electromagnetic Theory.pdf (D165097241)

that the tangential component of the electric field is continuous across the interface (

z = 0) gives E 0I cos ? I - E 0R cos? R = E 0T cos ? T ....(6.13.12) The boundary condition

76%	**MATCHING BLOCK 91/107**	**SA**	019E1230-Electromagnetic Theory.pdf (D165097241)

that the tangential component of the magnetic intensity is continuous across the interface (

z =0) gives n 1 E 0I + n 1 E 0R = n 2 E 0T n 1 (E 0I + E 0R) = n 2 E 0T ....(6.13.13) NSOU ? CC-PH-08 ? 225 Equations (6.13.12) and (6.13.13) can be solved for E 0R E 0I and E 0T E 0I which gives the Fresnel's reflection and transmition co-efficients r = p (n cos - n cos) 2 I 1 T ? ? (n cos + n cos) 2 I 1 T ? ? ....(6.13.14) t = p 2n cos 1 I ? (n cos + n cos) 2 I 1 T ? ? ....(6.13.15) Eliminating the refracting index from the above equations (6.13.14) and (6.13.15) we get r = p tan (?? ?? I T tan (? ? ? ? I T ....(6.13.16) t = p 2cosSin ? ? I ? Sin (Cos (? ? ? ? ? ? ? ? I T I T .... (6.13.17) Note : r p given by equation (6.13.16) is drawn r p verses ? I [Fig. 6.12 (a)] S Polarization: for S polarization electric field of the incident wave is nornal to the plane of incidence consequently, magnetic field in the plane of incidence. All the reflected and transmitted waves are shown in fig. 6.10. Frequencies of the three waves are the same, Like P wave, the boundary conditions remains the same at z = 0, the spatial and time variation of all fields must be the same. Consequently phase factor are all equal at z = 0 i.e. (K. r) = (K .r) =(K .r) I z=0 R z=0 T z=0 ? ? ? ....(6.13.18) Thus the law of reflection (? I = ? R) and Snell's law (n 1 Sin ? I = n 2 Sin ? T) are obtained for S- polarization. Also phase matching confirms that all the wave vectors ? ? ? , ? R ? and ? T ? are coplanar. * Reflection and refraction for oblique incidence, the plane of incidence (plane of the paper xz). The incident electric field is perpendicular K R H R E R E 2 K T H T E I K I H I ? R ? I z ? ? z ? ? n 1 n 2 Fig? ???? X NSOU ? CC-PH-08 ? 226 to the x z plane (S polarization) The continuity of the tangential components of ? ? and ? ? gives (at z = 0)

31%	**MATCHING BLOCK 92/107**	**SA**	PHY3512 Physics for Physical Science II 376pg ... (D144373244)

E 0I e j ?? ? ? r) ? ? + E 0R e j ?? ? R r) ? ? = E 0T e j ?? ? T r) ? ? ? E 0I + E 0R = E 0T ....(6.13.19) and n E cos e 1 0I I ? j ?? ? I r) ? ? ? ? n E cos e 1 0R R j ?? ? R r) ? ? ? ? n E cos e 2 0

T T j ?? ? T r) ? ? nE cos 1 0I I ? ? ? nE cos E = 1 0R I 0R nE cos 2 0T T ? ....(6.13.20) Solving (6.13.19) and (6.13.20) for E 0I E 0R and E 0I E 0T We get the Fresnel's reflection and transmission coefficients. ? E 0I E 0R ? r s ncos 1 I ? n cos 2 R ? ? n cos 1 I ? n cos 2 R ? ? ....(6.13.21) and ? E 0I E 0T ? t s 2ncos 1 I ? n cos 1 I ? n cos 2 T ? ? ....(6.13.22) Again utilising Snell's law, equation (6.13.21) and (6.13.22) can be written as ? r s Sin( ) ?? ? I T Sin( ) ? ? ? I T ....(6.13.23) ? t s 2cos Sin ? ? I T Sin( ) ? ? ? T T ....(6.13.24) Note r s given by equation (6.13.23) is drawn r s verses ? I [Fig. 6.12 (a)] Reflectance is the amount of flux (radiation) reflected by a surface, normalised by the amount of flux incident on it. Transmittance is the amount of flux (radiation) transmitted by a surface, normalised by the amount of flux incient on it. So if S IS ? is the time averaged Poynting vector for the incident wave for s polarization and S RS ? for the reflected wave, reflectance is given by

NSOU ? CC-PH-08 ? 227 ? ?S < RS ? R = S n ˆ n ˆ ?S < IS ? ....(6.13.25) If S TS ? is time - averaged Poynting vector for the transmitted wave, then transmittance ? ?S < TS ? T = S n ˆ n ˆ ?S < IS ? ....(6.13.26) Similarly, the reflectance and the transmittance for P polarization are as follows- ? ?S < RP ? R = P n ˆ . n ˆ . ?S < IP ? ....(6.13.27) ? ?S < TP ? T = P n ˆ . n ˆ . ?S < IP ? ....(6.13.28) So the Fresnel's coefficients are ? r S 2 ? R s ncos 1 I ? n cos 2 T ? ? n cos 1 I ? n cos 2 T ? ? ....(6.13.29) ? ? T s 4nncos cos 1 2 I T ? ? (n cos 1 I ? n cos) 2 T ? ? ncos 2 T ? n cos 1 I ? t S 2 ....(6.13.30) ? ? R P ncos ncos 2 I 1 T ? ? ? n cos 2 I ? n cos 1 T ? ? >r < P 2 ....(6.13.32) ? ? T P 4nncos cos 1 2 I T ? ? (n cos 2 I ? n cos) 1 T ? 2 ? ncos 2 T ? n cos 1 I ? t P 2 ....(6.13.33) Note : See Fig. 6.12 (b) for R S , R P , T S , T P plotted against angle of incidence. Key points to be take away : 1. Both the coefficients (R & T) are only independant of the material properties i.e permeability (as per second form the equations), throgh still have same implications of the reflective index. 2. Both the coefficients (R & T) are only dependent on the angle of incident ? I and angle of refraction ? R

NSOU ? CC-PH-08 ? 228 Some interesting results to observe : 1. Normal incidence : Here ? I = ? T so from equations (6.13.14), (6.13.15), (6.13.21) we get, ? r? S ? r P n n 2 1 ? n n 2 1 ? ....(6.13.33) and ? t S ? t P 2n 1 n n 2 1 ? ....(6.13.34) Also R S = R P and T S = T P 2. Grazing angle of incidence ; In this case incident waves touches the interface at on angle ? I ? , then r P = - r S = -1..(6.13.35) and t P = t S = 0 .... (6.13.36) so that R P = R S = 1..(6.13.37) T P = T S = 0..(6.13.38) This reaveals that there is total reflection for both S and P polarization. Just think of a beam of light shinning on a flat surface 3. Brewoters law : A reletionship for light waves stating that

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the maximum polarization (vibration in one plane only) of a ray of light may be acheived by letting the ray incident on a surface of transpent medium in such away that the refracted ray makes

Transmitted Beam Reflected Beam Incident Beam n 1 n 2 ? ? ? ? ? ? Transmitted Beam Reflected Beam Incident Beam n 1 n 2 ? ? ? ? ? ? ? 90 0 Fig. 6.11

NSOU ? CC-PH-08 ? 229 on angle of 90 0 with the reflected ray. From equation (6.13.16), we find that for (? I + ? T) = r P = 0 which implies that for (? I

NSOU ? CC-PH-08 ? 230 + ? T) = , the electric field polarized parallel to the plane of incidence is not reflected at all. Under this condition reflection coefficients r S ? ? ie the electric field polarized normal to the plane of incident is partly reflected.Thus an unpolarized light consiting of both types of vibration of E fields incident at angle. ? B satisfying the condition (? I + ? T) = , will be plane polarized normal to the plane of incidence. This angle of incidence ? B for which r P = 0 is known as Brewsteis angle under this condition, from Snell's law, we have ....(6.13.39) Example of Breswter law application is polarized sunglasses, These glasses use the principle of Brewster angle. The polarized glasses reduce glare that is directly from the sun and also from the horizental surface like road and water. Total Internal Reflection, Evanescent Wave : According to Snell's law, Sin ? T Sin ? I = n 2 n 1 So when light wave passes from a optically denser medium into a rarer one ie. n 1 < n 2 the wave vector ? K bends away from the normal. Specifically, if the light is incident at the ceitical angle ? C defined as ? C = n 1 Sin n -1 2 we get Sin? T = 1 , or ? T = 2 ? , which implies that, the trasmitted ray just grazes the surface. It ? I ? ? C , then Sin ? T ? 1 , which implies that it dies not correspond to any possible ? T . Here no rays are reflected, rather the whole light wave reflected back to the denser medium. This phenomenon is called total internal reflection. In spite of no reflection into the denser medium, The fields are not zero in that medium, which is called evanescent-wave. It altenuates rapidly and it transports no energy into the rarer medium. Transmitted wave vector can be written as ? K = K (Sin i + cos K) T T T T ? ? ....(6.13.40) with ? K = T ? v 2 ? ? ?n 2 C ....(6.13.41)

NSOU ? CC-PH-08 ? 231 As sin ? T ?1, obviously we can write cos ? T 2 T = 1-Sin θ , which is maginary number. Now, for the transmitted wave ? ? ? = ? ?? c ? j(k .r- t) T ? ?? where x = = = K x + j k z * Where K * ? ?n 1 C Sin? I and K = ? C 2 I n 1 2 Sin θ - n 2 2 So, we can write the tranmittes wave as ? T (r. t) j(k .r-wt) T ? ? ? ? ?T e ? ?? or ? T (r. t) i(k x- t) * ? ? ? ? ?T e e ? -k 2 ....(6.13.41a) This is the wave defined as evanescent wave propagating in x-radiation ie. parallel Fig. 6.13 1.0 0.8 0.6 0.4 0.2 n 1 n 2 R I R P 0 0 30 0 60 0 ? ? 90 0 ? ? successive internal reflections. I n 1 /n 2 R I R P 0 0 ? ? 90 0 ? ? ? C Reflectance (a) (b)

NSOU ? CC-PH-08 ? 232 to x direction with a penetration depth of K -1 . It decays rapidly and becomes negligible beyond a distance of few wavelengts. Reflectances for S and P polarizations when n 1 >n 2 and n 1 <n 2 Let 2 T = Sin θ 1- cos? T = jD. Now for parallel and perpendicular, electric field vectors, reflection coeffcient becomes from, equation (6.13.14) and (6.13.21) ....(6.13.41) and r ´ = S ncos jn 1 I 2 ?? ? n cos jn 1 I 2 ? ? ? ....(6.13.42) Then Reflectance is given by, R´ r´ p p ? 2 ....(6.13.43) R´ r´ S S ? 2 ....(6.13.44) It folloes fro above equations that R´ p ? R´ S ? ? ....(6.13.45) From Fig. 6.13 (a) it is clear that the reflectance for S and P polarizations when n 1 >n 2 ; it shows that there is no total internal reflection. From Fig. 6.13 (b) it is clear that when n 1 <n 2 : there is a critical angle qc and total internal reflection. From, equations (6.13.41) and (6.13.42), it can be written in phase from, r P ' = e (-j2 ?) ....(6.13.46) and r S ' = e (-j2 ?') ....(6.13.47) where tan ? = n 1 ?/n 2 cos? 1 and tan ?' = n 2 ? n cos 1 I ? . Here, the electric field lags that incident wave by 2 ?, for P polarization and 2 ?' for S-polarization respectively. elearlly elliptically polarized light will be observed it the incident wave is polarized is a palne making on angle (? 90) 0 with the plane of incidence. 6.14 Summary 1. We have studied that how Maxwell's equation and its solution proved the secmingly disparate phenomen of electricity magnetism, and optics are all related aspect

NSOU ? CC-PH-08 ? 233 of the larger phenomenon of electro magnetics. Solutions to the fundamental euations of electrialy and magnetism are electromagnetic waves. Most importants findings of the solution os Maxwell's equations is the revelation that all forms of electromagnetic wave be it, visible light x-rays, r-ray, infrared on ultraviolet light, propagle at the speed of light in vaccum, and transference of energy from one space to another without any medium. 2. We have also seen, how Maxwell's moclified the Ampere's law of steady flow current to the case of varying current with the introduction of defination of displacement current due to changing electric field. Instantaneous magnetic field generated due to changing electric field has led to the propagation of radiation, which carries energy from one place to another. The EM radiation which propagaties energy is a major discovery by Maxwell's due to his prediction of displacement current Modern age comminacation is impossible with out EM radiation. 3. We have also studied the introduction of gauge transformation. We have discussed the six variable of EM fields can be represented. 4.

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The tangential componet of the electric field (? ?) is continuous across the interface.When the

medium conductivity infinity, the tangential component of magnetic intennity (? ?) is continuous accross the interface 5. We have discussed reflection and refraction at the plane interface of two non conducting (di electric) media (i) Normal inciedence and (ii) oblique incidence. We have calculated the Fresnel reflection coefficient and Fresnel transmission cofficient for both normnal and oblique incidence . We have also evaluated reflectances for s and p polarizations when (a) n 1 >n 2 (when there is no total internal reflection) and (b) n 1 <n 2 (when there is critical angle ?c and total internal reflection). 6. We have derived Brewster's law tan 6.15 Review Questions and Answer Question : 1. S now that the dispalcement current in a parallel plate capacitor is equal to the conduction current in the connecting leads. Answer : The capcitance of a parallal plate capacitor is C ? ? A d
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where A is the area of plate d is the distance between them,

and ? is its permittivity. The conducting current in the conecting leads is i e ? dq dt ? C dv dt or The electric field in the (capator) dielectric E = n/d. Now electric displacement is D? ? ? ? d ?E , Hence the displacement current density dD dt ? ? d dv dt The displacement current is i d ? dD dt ? dv dt A A d ? Hence i D = i C 2. State Poynting vector Answer : See article 6.2 3. Define Brewster's Law Answer : See article 6.13 4. Define critical angle penetrating depth. Answer : See section 6.13 5. Define, momentum, pressure and angular momentum of electromagnetic radiation. Answer : Radiatopn pressure is the mechanical pressure exerted upon any surface due to the exchange of momentum between the object and the electromagnetic field. This includes the momentum of EM radiation of any wave lenght is obsoerd reflected or emitted by matter on any scale. For further follow-up answer, see article 6.5 6. State the boundary conditions between two interfacing different dielectric. Answer : See article 6.8 id t ~ ? l i
NSOU ? CC-PH-08 ? 235 7. State the boundary conditions between dielectric and conductind media Answer : See article 6.8 8. Prove that the momentum density stored in an electromagnetic field is given g ? ? s / c 2 ? in free space where Poynting vector. Answer : Force F ? ? d(P) mechanical dt from this we can show that P ? electromagnetic = P em ? = = momentum related to electromagnetic wave ? momentum density Pem ? ? ? ? ? S ? ? Again 1 ? ? ? ϵ ? 1 c So, momentum density P em ? S ? ? c 2 ? g ? 6.16 Problems and Solutions A Steady current I is flowing through a metallic wire of length L and radius R throght a potential difference V calculate (a) Poynting vector, (b) Total energy delivered to the system and, (c) derive the value of the resistance of the wire R using J, ? ∫ E, d ? ? . Solution : Assuming electric field E is parallel to the wire then, E? V L the magnetic field is circumtanential at the surface B? ?? ? 2 R ? Hence the Poynting vector magnetude is (a) S 0 ? ? ? ? E x B ? ? ? ? ? ? L V ? ? ? ? 2 R ? ? VI 2 RL ?
NSOU ? CC-PH-08 ? 236 and it shows that Poynting vector is inward (b) The energy passing throngh the surface of the wire S . ds 0 ? ∫ ? ? - S 0 2 RL ? ? -VI 2 RL ? 2 RL ? ? -VI (c) Now ∫ (J. E) d = ? ∫ d ??? ? ? ? ? ?? V 2 L 2 ∫d? ? ? V 2 L 2 2 R L 2 ? ? ? V 2 L R = 2 ? R V 2 ? ? ? L ? ∫ (J. E) d = ? ? ? As ∫ (J. E) d ? ?? = total Joule loss per unit time due to the flow os current V ? R ∫ (J. E) d ?= ?? ? L = R R 2 ? ? 2. A plane electromagnetic wave has the magnetic field given by Where K is the ware number, where k j, are the cartensian unit vectors in x, y and z directions respectively. (a) Find the electric field E ? (x,y,z,t) (b) Find the average Poynting vector Solution :
NSOU ? CC-PH-08 ? 237 (a) (b) The average Poynting vector is given by 3. The space time dependence of the electric of a linearly polarzed light in free space is given by E 0 Cos (?t-kz). Find the time average density associate with electric field. Solution : u E = ½ e 0 E 2 = ½ e 0 E 2 cos 2 (?t-kz) ? >u' E < = ¼ e 0 E 0 2 4. A plane polarized electromagnetic wave in free space at time t = 0, is given by E ? (x,z,t) = j exp[j(6x+8z)] Solution : Magnetic field vector is given by 5. It the vector potential satisfied the coulomb gauge find the value of the constant.

NSOU ? CC-PH-08 ? 238 Solution : condition for Coilomb gauge is ??? ? ? ?? 6. A vector potential ? ? ? ke r r -at (where a and k are constants) corresponding to an electromagnetis field is changed to ? ? ? ? ke r r -at . Prove that. This will be a gauge transformation if the corresponding change ?' - ? in the scalar potential is - akr 2 e -at Solution : Gauge transformation 7. The intensity of sunlight reaching the earthis surface is about 1300 wm-2. Calculate strenght of the electric and magnetic fields of the incoming sunlight. Solution : The time average Poynting vector >S< = ½ Re(E x H) ? ? ? Taking incoming wave variation E = E e 0 -jwt ? and H = H e 0 -jwt ? . we get or, H = rms E rms So, S av = E rms 2 c? ? on δ -7 -1 rm s E = cμ S av = 3×10 ×4 10 1300 Vm =7000Vm -1 Again B rms = E rms = 700 c 3 x 10 8 T 2.33 x 10 T -6 =

Unit 7 Network Theorems Structure 7.1 Objectives 7.2 Introduction 7.3 Thevenin’s Theorem 7.4 Norton’s Theorem 7.5 Superposition Theorem 7.6 Maximum Power Transfer Theorem 7.7 Reciprocity Theorem 7.8 Summary 7.9 Review Questions and Answers 7.10 Problems and Solutions 7.1 Objectives You will know from this unit— To learn teachniques of solving circuits for bilinar network comprising passive elements, Application of KVL-KCL, in series, parallel, voltage and current divider rule, source transformation techniques, Study of Thevenin, Norton, Superposition, Reciprocity Theorem and Power Transfer Theorem, and their equivalent circuits, to simply the evaluation process, Necessity of Thevenin’s and Norton’s Theorem in A.C. circuit behaviour and analysis. 7.2 Introduction Network theorems give a more simple way to analyse electrical circuits than Ohm's law or Kirchhoff's laws. They are not basic theorems and are deducible from Kirchhoff's laws.

NSOU ? CC-PH-08 ? 240 To begin with, the details we focus on some relevant definitions. Electric network : Electric network is combination of electric elements like cells, resistances, capacitors, inductances, diodes, transistors etc. An active network is that which carries source (sources) of emf, like cell, transistor etc. A passive network work does not carry active elements —the source of emfs. Electric circuit : Electric circuit is a closed path through which electric current flows or intended to flow. A closed circuit is that through current flows and when no current flows through, it is an open circuit. Two points are said short circuit when zero impedance joins the two points. A linear circuit is that where in the circuit elements do not change with voltage or current, otherwise it is a non-linear circuit. Node is point at junction of two or more circuit elements. The Sources: Voltage Source A voltage source is an emf generator. The figure (7.1) shows an emf generator with emf E, internal resistance r i and load resistance R L . i be the current flowing through the circuit. Then E = i(r i +R L) = ir i + V 0 , so the output voltage is less than the input due to internal potential drop V i =ir i . To make the output voltage independent of current r i should tend to zero. A voltage source with zero internal resistance is an ideal voltage source. Current Source : A current source is a current generator. Reference to fig (???), we can write i L L i i E E i r R R r (1) r ? ? ? ? Fig 7.2 So the source current become independent of load resistance as r i tends to infinity. ~ Ideal voltage source Ideal ac voltage source Fig 7.1

NSOU ? CC-PH-08 ? 241 An ideal current source has infinite internal resistance. Thevenin's theorem The theorem states that any two terminal linear, bilateral network with impedances and energy sources can be replaced by an open circuit voltage V Th (called Thevenin's voltage) generator across the terminals with an internal impedance Z Th (Thevenin's impedance) measured across the terminals replacing the energy sources by their respective internal impedances. Then the current through the load resistance Z L will be Th i th i V i Z Z ? ? Fig. (7.3) illustrate the procedure of Thevenin's theorem. z i E z 1 z 2 z L i L a z i E b z 1 z 2 B A V TH z i I z 1 z 2 c Z TH Z TH V TH B A d z L B A Fig. 7.3 a) The circuit for which i L is to be determined, z i is the internal impedance of the voltage source E. b) Load z L is removed and the Thevenin's voltage 2 Th i 1 2 Ez V z z z ? ? ? is calculated about AB. c) The E is short-circuited and 2 i 1 Th i 1 2 z (z z) z z z z ? ? ? ? is calculated across the terminal AB. d) The Thevenin's equivalent circuit. Proof of Thevenin's theorem (using Kirchhoff's laws) The adjoining figure is a replica of Fig.(7.4) with the currents flowing through the Given impedances). Using KVL and KCL in the circuit, we have, Fig 7.4 z i E z 1 z 2 i L F L z 1 (i - i) L

NSOU ? CC-PH-08 ? 242 i 1 1 2 i(z z) (i i)z E ? ? ? ? ....(7.3.1) i 1 1 2 i(z z) i z E (7.3.2) ? ? ? ....(7.3.2) Eliminating i from above equations we have 2 2 1 1 2 Th L L i 1 2 2 i 1 2 i 1 L Th i 1 2 Ez Ez (z z z) V i z (z z z) z (z z) z (z z) z z (z z z) ? ? ? ? ? ? ? ? ? ? ? ? ? ? .....(7./3.3) Which is same as the yield of the Thevenin's theorem thus proving the theorem. Procedure of Theveniirs Theorm: Find the open circuit voltage at the terminals, Voc. Find the Thevenin's equivalent resistance, R TH at the terminals when all independent sources are zero. Replacing independent voltage sources by short circuit Replacing independent current sources by open circuit Reconnect the load to the Thevenin equivalent circuit Example-1 : Find the current i L through 5 ? resistor in the adjoining circuit.\ Solution : Th 10 2 2 40 20 10i 5i 10, i.e. i , SoV 5. 10 V 15 3 3 3 ? ? ? ? ? ? ? ? From Fig (7.6) Th5 L 40 5 10 10 40 8 3 and R So i 1.6A 10 15 3 25 5 5 3 ? ? ? ? ? ? ? ? ? ? Example-2 : The four arms of a Wheatstone bridge have the following resistances: AB=100 ?, BC=10 ?, CD=4 ?, DA=50 ?. A galvanometer of 20 ? resistance is connected across BD. Use 20 V 10 ? 5 ? 5 ? 10 V 10 ? i L Fig 7.5 20 V 10 ? 5 ? 10 V 10 ? V TH Fig 7.6 10 ? 5 ? 10 ? R TH

NSOU ? CC-PH-08 ? 243 thevenin's theorem to compute the current through the galvanometer when a p.d of 10V is maintained across AC. Solution : We proceed to apply Thevenin's theorem in the Circuit. G 100 ? 10 ? 50 ? 4 ?

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A C B D 100 ? 10 ? 50 ? 4 ? A C B D 10 V 10 V 100 ? 10 ? 50 ? 4 ? A C B D 10 V a b c d 10 ? 4 ? 100 ? 50 ? C A B

D

Fig 7.7 b) Galvanometer is removed and the V BD =V Th is calculated ? ? ? ? Th 10 10 1 2 V .10 4 10 0.168V 110 54 11 27 ? ? ? ? ? d) The voltage source is short-circuited and R Th is calculated across BD. Th 100 10 50 4 R 12.79 100 10 50 4 ? ? ? ? ? ? ? ? So the current through the galvanometer Th G Th L V 0.168 i 5mA R R 12.79 20 ? ? ? ? ? 7.4 Norton's Theorem The theorem states that any two terminal linear bilateral network with energy sources and resistances can be replaced by current source with current I N (Norton's current), obtained by short circuiting the chosen terminals and an resistance R N (Norton's resistance), in parallel to it obtained across the terminal by replacing the energy sources by their respective internal resistances. Fig (7.8) gives an illustrative presentation of Norton's theorem.

NSOU ? CC-PH-08 ? 244 Fig. 7.8 a) The circuit for which i L is to be determined; r i is the internal impedance of the voltage generator. b) Load R L is short-circuited and short circuit current called Norton's current is calculated through the terminal AB N i i i r R ? ? ? c) E is short-circuited and Norton's impedance 2 i 1 N i 1 2 R (r R) R r R R) ? ? ? ? is calculated across AB. d) Gives the Norton's equivalent circuit with current N N i L N R i i R R ? ? Proof of Norton's theorem (using Kirchhoff's laws) The proof is similar to the proof of Thevenin's theorem. Here in Norton's theorem we have 2 i 1 N N i 1 2 i 1 2 L L N 2 i 1 1 i 1 2 L 2 i 1 L i 1 2

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R (r R) E R i (r R R) (r R) ER i , R R r (r R) R (r R R) R (r R) R (r R R) ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

which

is same as the result from Kirchhoff's laws, thus proves the Norton's Theorem. Procedure of Norton's Theorm : 1. Find the short circuit current at the terminals, I SC . 2. Find Thevenin's equivalent resistance, R TH (as before). 3. Reconnect the load to Norton's equivalent circuit. r i E R L i L a E b B A i N c R N B A d R 1 R 2 r i R 1 R 2 R 1 R 2 i N R L i L

NSOU ? CC-PH-08 ? 245 Example Let us solve Example-1 using Norton's theorem. From Kirchhoff's laws, 20V−10i i = 0, or i 1 = 2 A and 10V−5i 2 =0 ? i 2 =2A. So i N = i i + i 2 = 4A From the adjoining figure (7.9) N 50 10 R 5ll10 15 3 ? ? ? ? ? i 10 4 3 i 1.6A 10 5 3 ? ? ? ? ? 7.5 Superposition Theorem The theorem states that any linear bilateral network with several energy sources, the current/voltage in any element will be the algebraic sum of contribution from each source replacing the other sources by their respective internal impedances. Fig. 7.10 Proof : The Fig. 7.10(a) hows a circuit to analyze, Fig. 7.10(b) and Fig. 7.10(c) are the circuits for analyzing the problem using Superposition theorem. E i =(r i1 + R l)i' + R 3 i 1 ? ......(7.5.1) 20 V 10 ? 5 ? 10 V 10 ? Fig 7.9 10 ? 5 ? 10 ? R N i 1 i 2 i N r i1 E 1 Fig.10(a) Fig. 10(b)

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R 3 R 1 R 2 i 1 r i2 E 2 r i1 E 1 R 3 R 1 R 2 i 1 r i2 r i1 Fig. 10(c) R 3 R 1 R 2 i 1 r i2 E 2 I NSOU ? CC-PH-08 ? 246 Then from Fig. 7.5(b) 3 i2 1 2 2 3 i2 3 i2 2 1 0 (R r)(i i) i R (R r)i (R r

R)
i ? ? ? ? ? ? ? ? ?? ? ? ? ? ......(7.5.2) Solving the above two equations by Cramer's
rule, we have n i1 1 1 i2 3
r

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r R E 1 0 (r R 1 i ? ? ? ? ? ? ? Where il 1 2 i2 3 i2 2 3 r R R (r R r R R ? ?? ? ? ? ? Similarly from Fig. 7.10(c) i1 1 1 i2 3 2 r R) 0 1 i (r R) E ? ?? ? ? ? ? So i1 1 1 1 2 i2 3 2 r R) E 1 i i (r R)

E ? ? ?? ? ? ? ? is the total current through the Resistance R 2 , according to superposition theorem. Now we find out the same solution using Kirchhoff's laws. Please refer Fig. 7.10(a) From Kirchhoff's laws we have, E 1 = (

95% **MATCHING BLOCK 100/107** W

R 1 + r i1)i + i 1 R 2 E 2 = (r i2 + R 3)(i 1 − i) + i 1 R 2 = −(r i2 + R 3)i + (r i1 + R 3 + R 2)

i 1 From above two equations eleminating i we have, i1 1 1 1 i2 3 2 (r R) E 1 i (r R)
E ? ? ? ? ? which shows i 1 = 1 1 i i ? ?? ? proving superposition theorem from Kirchhoff's laws. Procedure : 1. Dependent source are Never deactivated (always active) 2. When an independent voltage source is deactivated, it is set to zero, replaced by short circuit 3. When an independent current source is deactivated, it is set to zero, replaced by open circuit NSOU ? CC-PH-08 ? 247 Example Please refer Example-1 : The efm source 10V is short circuited. Then equivalent resistance about 20V emf generator is Fig. 7.11(a) 1 125 R 10|| (10 5||5) 10||12.5 22.5 ? ? ? ? ? So, 1 10 10 1 i .72A 125 12.5 2 ? ? ? ? Fig. 7.11(b) From the principle of superposition theorem we now, activate the source 10 volt and deactivate 20 volt source, and find out the current in the are EF having 5 ?. Applying Kirchoff's law in ABCD network, −10i 1 − 5(i 1 − i 2) − 10 − 10i 1 = 0 ...... (a) And, applying Kirchoff's law in the BEFC network, −5i 2 + 10 − 5(i 2 − i 1) = 0 ....(b) Solving equations (a) and (b) we find i 2 = ·88A 10? A D B C E F i 2 5 ? 5? 10? 10V i 1 20V 10? 10? P Q A D B C E F i 1 5 ? 5?
NSOU ? CC-PH-08 ? 248 Total current in the arm EF having 5 ? resistance i = i 1 + i 2 i = ·72 + ·88 = 1·6A 7.6 Maximum Power Transfer Theorem The theorem states that in a linear bilateral with resistances the energy supplied to the load resistance reaches maximum when the load resistance equals the energy source resistance. Proof : Consider the circuit, Fig. (7.12) with a generator of emf E having internal resistance r i connected to external load R L . Current through the load resistance L L i L E R ,i (r R) ? ? So the power delivered to the load resistance, 2 2 L L L 2 i L E R P i R (r R) ? ? ? For maximum power 2 L 2 3 L i L i L 2R dP 1 0 E dR (r R) (r R) ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? So, r i = R L , which proves the theorem. The percentage of power efficiency of a circuit is defined as L i L R power delevered to the external load % 100 100 total power consumed in the circuit r R ? ? ? ? ? ? Thus efficiency of a circuit becomes 50% when the circuit deliver maximum power to the external load. The variation of ?% with R L is shown in Fig (7.6.3) The variation of power dissipated across the load with P across the load resistance R L is shown in Fig. (7.6.4) Fig. 7.12 r i E 1 R L i 1
NSOU ? CC-PH-08 ? 249 Fig. 7.14 Example : 1) In the Fig. 7.1.5 Find the value of R for maximum power transfer in 3 ? resistance. Solution : Using maximum power theorem we have for this condition R 6 3, R 6 ? ? ? R = 6 ? 7.7 Reciprocity theorem The theorem states that in a linear bilateral network with sources and resistances, if the source of emf in one mesh produce a current in another mesh then the transfer of the emf source to the second mesh keeping its internal resistance in the previous position will produce same current in the first mesh. Proof: The Fig. 7.16(a) shows the given network and the Fig. 7.16(b) shows the changed 10 V 3 ? Fig 7.15 6 ? R 100% η% R L Fig. 7.13 r = R i L P P max RL r i E Fig.7.16(a) R 2 R 1 I R L r i E Fig.7.16(b) R 2 R 1 I R L
NSOU ? CC-PH-08 ? 250 network following Reciprocity theorem. Consider Fig. 7.16(a), here 2 2 i 1 2 L 2 L i 1 2 L 2 L

R ER E I (

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r R) R || R R R (r R)(R R) R R ? ? ? ? ? ? ? ? ? ? Now from Fiq. 7.16(b) we see that 2 2 L 2 i 1 2 1 i i 1 2 L 2 L R ER E I R R ||(r R) R (R r) (r R)(R R) R R ? ? ? ? ? ? ? ? ? ? ? ?

So, I = I ', that

proves Reciprocity theorem. Proceedure : Conditions to be met for the application of reciprocity theorem : (i) The circuit must have a single source. (ii) Initial conditions are assumed to he absent in the circuit (iii) Dependent sources are excluded even if they are linear (iv) When the positions of source and response are interchanged, their directions should he marked same as in the original circuit. Example : 1. Two cells of emf e 1 and e 2 (e 1 <e 2) and internal resistances r 1 and r 2 are connected in parallel to the ends of a wire of resistance R. Find the current through R using i) Thevenin's theorem ii) Norton's theorem iii) Superposition theorem. Solution : The circuit diagram is shown in the Fig. 7.17. i) i R using Thevenin's theorem. Step-1 To find V Th Remove R and find the open circuit p.d V Th Current through the loop i R r 1 e 1 r 2 e 2 R Fig 7.17

NSOU ? CC-PH-08 ? 251 1 2 1 2 e e i , r r ? ? ? So the potential across the ab 1 2 2 1 Th 1 1 1 2 e r e r V e ir r r ? ? ? ? ? Step-2 To find R Th Short circuit the emf source and find the resistance across ab. 1 2 Th 1 2 1 2 rr

R rllr r r ? ? ? So the current through

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R 1 2 2 1 Th 1 2 1 2 2 1 R Th 1 2 1 2 1 2 1 2 e r e r V r r e r e r i R R r r r r r R r R R r r ? ? ? ? ? ? ? ? ? ? ? ii) i R

using Norton's theorem. Step-1 To find i N . KT Short circuit the resistance R and find the current through ab. Norton's current 1 2 1 2 2 1 N 1 2 1 2 e e e r e r i r r r r ? ? ? ? Step-2 To find R N Here the Norton's resistance 1 2 N 1 2

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r r R r r ? ? Fig. 7.20 So current through the resistance R R Th r 1 r 2 Fig 7.19 a b r 1 e 1 r 2 e 2 a Fig 7.18 b

R N r 1 r 2 a b r 1 e 1 r 2 e 2 i N a b Step 2 Step 1
NSOU ? CC-PH-08 ? 252 1 2 1 2 2 1 N N 1 2 1 2 1 2 2 1
R N 1 2 1 2 1 2 1 2
r

87%	**MATCHING BLOCK 103/107**	**W**

r e r e r R i r r r r e r e r i R R r r r r R r R R r r ? ? ? ? ? ? ? ? ? ? ? iii) i R

using Super Position theorem. Step-1 Short circuit e 1 and evaluate i 1 . 2 1 2 1 1 1 1 1 2 1 2 2 1 e
r e
r i r

44%	**MATCHING BLOCK 105/107**	**SA**	MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

R r R r r r R r R r r R ? ? ? ? ? ? ? Fig. 7.21 Step-2 Short circuit e 2 and evaluate i 2 1 2 1 2 2 2 1 2 1 2 1 2 e r e r i r R r R r r r R r R r r R ? ? ? ? ? ? ? ? The total current through R, i R = i 1 +i 2 = 1 2 2 1 1 2 1 2 e r e r r r r R r R ? ? ? 7.8

Summary (1) Network circuit Theory is a useful procedure to analyze and simply the complex circuit in different configuration. Equivalent circuits are drawn by applying different theorems like Thevenin, Norton superposition and Reciprocity theorems. Also we have shown that r 1 r 2 e 2 R Fig 7.21 (a) i 1 r 1 r 2 Fig 7.21 (b) e 1 a a i 2
NSOU ? CC-PH-08 ? 253 Thevenins and Nortons circuit theory are all equivalent in analyzing circuit having bilinear port. Different types of problems have been discussed using Kirchoff's laws, nodal theory and mesh network to simply Thevenin and Nortons circuit. We have seen the condition for maximum power transfer from source to the load when source impedence is equal to the load impedence. (2) Determination of Voltage Sign : In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs., otherwise results will come out to be wrong. Following sign conventions is suggested : (a) Sign of Battery E.M.F. A rise in voltage should be given a + ve sign and a fall in voltage a −ve sign. Keeping this in mind, it is clear that as we go from the −ve terminal of a battery to its +ve terminal (Fig. 7.2.3), there is a rise in potential, hence this voltage should be given a +ve sign. If, on the other hand, we go from +ve terminal to −ve terminal, then there is a fall in potential, hence this voltage should be preceded. Limitation of Thevenin's and Norton's Theorem (1) These theorems used only in the analysis of linear circuits (2) The power dissipation of the Thevenin's equivalent is not identical to the power dissipation of the real system. Super position theorem limitation—the requisite of linearity indicates that this theorem is only applicable to determine voltage and current but not power. 7.9 Review Questions and Answers 1. What are the steps to follow Thevenin's Theorem? Ans. See section (7.3) for answer. ? + A A E Rise in Voltage +E A A E Fall in Voltage ?E A +? B V Fall in Voltage ? ? V=IR Motion Current A ? ? B V Rise in Voltage ? ? V=IR Motion Current
NSOU ? CC-PH-08 ? 254 2. What are the steps to follow Norton's Theorem? Ans. See section (7.4) for answer. 3. Convert the voltage source of figure to a current source Fig. 7.24 Solution : E 100 0 I 20 53.13 Z 5 53.13 ? ? ? ? ?? ? ? ? 4. Convert the current source of figure to a voltage source. Solution : Fig. 7.25 c L c L z z (4 90)(6 90) z j12 12 90 z z j4 j6 ? ?? ? ? ? ? ? ?? ? ??? ? ? ? ? E = IZ = (10 ?60°)(12 ?−90°) = 120 ?−30° 7.10 Problems and Solution 1. In the diagram given in Fig 7.2.6 determine the Norton's equivalent source current and resistance with respect to the terminals a,b. Fig. 7.26(a) Solution : Step-1 : Short circuit ab, then the short circuit current I=10 53 0 c ? a ? X L 6? 4? X c E=120 -30 0 X 12 C ? N ? ? E=100Lo 3 ? R X = 4 L ? C? I=20A 53.13 0 3 ? R X 4 L ? a a? 6V a b 3? 3?
NSOU ? CC-PH-08 ? 255 N 6 3 3 i 2 2 0.67 2 2.67A 3 3||3 3 3 4.5 ? ? ? ? ? ? ? ? ? ? Step-2 : Short circuit 6V and measure the resistance across ab. Fig. 7.26(b) R N = 3+3||3=4.5 ? Fig. 7.26(c) Problem-2 : Determine the current through 5W r esistor in Fig. 7.27(a) Solution : Step-1 : Short circuit 5 ? resistance and find i N . Fig. 7.27(b) N 2 17 i 2 5 12 6 ? ? ? ? Step-2 : Open the 5 ? resistance and find R N . Fig. 7.27C Equivalent circuit of Fig. 7.24(d) is Fig. 7.24(d) So R N = 12 ? So current through 5 ? resistance 5 17 12 6 i 2A 12 5 ? ? ? Problem-3 : Find the i) Thevenin's and ii) Norton's equivalent circuit of the adjoining Fig. 7.25 between

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a and b. Fig. 7.28 6V a b 3? 3? 3? R N B A 3? 3? 5A 2? 2A 10? R N 5A a b 2? 2A 5? 2? 5A 2? 2A 10? R N 5A 2? 2A 10? i N 10V a b 10

V 10? 10? 10? 10?
NSOU ? CC-PH-08 ? 256 Solution : Since no current R Th = 10 + 10||10 Thevenin's Norton's flows through pq, =10+5=15 ? equivalent equivalent so V ab = 10V circuit circuit Fig. 7.29 Problem-4 Find the current through 5k ? resistance in the circuit in Fig. 7.30(a) using Thevenin's theorem. Solution : Fig. 7.30(a) Step-1 : Open circuit 5k ? and find V Th . From the adjoining circuit 10 30 20 6i 20i or i mA 26 ? ? ? ? Fig.7.30(b) So, Th 10 30 30 12 V 30 6 30 V 26 13 13 ? ? ? ? ? ? ? Step-2 : Replace the emf generator by their respective internal resistances and calculate R Th . Fig. 7.30(c) From adjoining circuit Th 6 20 60 R 6|| 20 k 26 13 ? ? ? ? ? 10V a b 10V 10? 10? 10? 10? q p 10? 10? 10? 10? R N a b 10V 15? a b ? ? A 20V 20k? 10 k ? ? 6k? 30V b a 20V 20k? 10 k ? ? 5k? 6k? 30V 20k? 10 k ? ? 6k? b a R

NSOU ? CC-PH-08 ? 257 So, Th 5 Th 30 12 V 30 12 13 i 2.88mA R 5 60 125 5 13 ? ? ? ? ? ? ? ? Problem 5 : Solve for the power delivered to 20 ? resistor in the circuit diagram shown in Fig. 7.31(a). All the resistances are in ohms. Fig. 7.31(a) Fig. 7.31(b) Solution : 4A source and its parallel resistance can be converted into a voltage source (15×4)=60 volt in series with a resistance as shown in Fig. 7.31(a). Now using superposition theorem to find the current through the 20 ? resistor, when 60V source is removed, the total resistance as seen by 2V baliery is 1 + 2011 (15+5)11 ?. The battery current is = 2 — 11 A. At point P, the current is divided into two parls. The current passing throng 20 ? is 1 2 1 11 I 20 A .09A (20 20) 11 ? ? ? ? ? When 2 Volt battery removed, the resistance as seen by the battery 60 Volt is 20.95. The current from the battery is 28693A ~ ~ 2.87A This current divides at point A, the current through the 10 ? is 2 2.87 I 1 0.14A 20 1 ? ? ? ? Total current flows through 20 ? is = I 1 + I 2 = 0.09 + 0.14 = .23A 2V 20? 1? 5? 15? ?? 2V L0? 1? 5? 15? 60V

Unit 8 Electrical Circuits Structure 8.1 Objectives 8.2 Introduction 8.3 Alternating Current and Its Characteristics 8.4 Representation of Sinusoidal ac by Complex Number 8.5 Kirchoff's Laws 8.6 A.C. Responses of A Resistance, An Inductance and A Capacitance. 8.7 Series LCR Circuit 8.8 Parallel LCR Circuit 8.9 Summary 8.10 Review Questions and Answers 8.11 Problems and Solutions 8.1 Objectives You will know from this unit— All the parameters of AC—voltage and current, its average value, root mean square value (RMS) Application of complex number Kirchoff's Laws

NSOU ? CC-PH-08 ? 259 Behaviour of Resistance, inductance and Capacitor Series LCR Circuit, its unique resonant properties Parallel LCR Circuit its unique resonant properties and uses. 8.2 Introduction Alternating current (AC) is an electric current which periodically reverses direction and changes magnitude continuously with time in contrast to direct current (DC) which is unidirectronal. The most common form of wareform of alternating current in most electric power circuits is a sinewave, whose positive half period corresponds with positive direction of the current and vice versa. We will study the physical properties of resistor R, inductor L, and Capacitor C under the impact of AC, and the nature of current flow and wattless current. Use of complex number is essential as it is convenient to represent and calculate both AC, signals and impedance. Two dimension length and angle allows as to calculate amplitude and phase together, and keep them consistent. Unique properties of combinational circuit like series LCR and parallel LCR, culminating to the concept of 'Band Width', Quality Factor, which is of practical importance in physics and engineering will also be studied in detail. 8.3 Alternatring Current and Its Characteristics Alternating Current (AC) or voltages is current or voltage which periodically reverses direction and changes its magnitude continuously with time. The most common form of AC is sinusoid. Even, if it is nonsinusoid it can be resolved into many sinusoid by Fourier Transform. For symmetric AC its average over a complete cycle is zero. The most common form of generation of AC works on the principle of Faraday's law of electromagnetic induction. Whenever a coil is rotated in a uniform magnetic field about an axis perpendicular to the field,

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the magnetic flux linked with the coil changes and an induced emf is set up

across its ends. A pure sinusoidal voltage is represented in Fig. 8.1 can be written as v(t) = V 0 sinwt ......(8.3.1) Fig. 8.1 Here v(t) is the instanceous value of the voltage and V 0 its amplitude, w its angular ? Time 90 0 V m I m V I

NSOU ? CC-PH-08 ? 260 frequency time period T is relatred to frequency w 1 f 2 T ? ? ? . When this alternating voltage applied in a circuit, current flows through it is given by i(t) = I 0 sin (wt + ?) ....(8.3.2) 2. RMS value of AC Waveforms : The rms value of an alternating current is given by that steady (dc) current which when flowing through a given circuit for a given time produces the same joule heat as produced by the alternating current when flowing through the same circuit for the same time. The root mean square value of an alternating current with period T is given by T 2 rms 0 1 I i (t) dt T ? ? .... (8.3.5) It can be related to the heating effect of ac. Total Joule heat produced by ac in a resistance R overf a time peiod T is T 2 2 rms 0 i (t).R.dt I RT ? ? ....(8.3.6) For a simple sinusoidal current i = I 0 sinwt the rms value is 2 T 2 2 2 0 rms 0 0 I 1 I I sin wtdt T 2 ? ? ? So, 0 rms I Peak value of current I 2 2 ? ? ....(8.3.7) Similarly for a sinusoidal voltage 0 rms V V . 2 ? I rms or V V rms is a measurable value as all measuring instruments based on the heating effect of current and calibrated accordingly. The peak value of the domestic AC, mains supply of rms voltage 220V is 220V2 = 311V. Because of this high peak value of 311V from AC mains is more shocking than same value of DC supply of 220V. 3. Form Factor : The ratio of the rms value to the average value over a half cycle of a periodic function is defined to be the form factor of the periodic waveform. For a sinusoidal ac, the form factor is given by.

NSOU ? CC-PH-08 ? 261 Form factor 0 f 0 I 2 K 1.11 2I ? ? ? ......(8.3.8) From factor gives an idea about the wave shape. Any deviation in the value of K f from 1.11 indicates deviation from sinusoidal nature. 4. Power in AC circuits : According to Joule heat energy generated is proportional to the square of current flow. In alternating current rate of electrical energy P spent in a circuit varies with time and though, at any instant rate energy spent being the product of voltage and current, but in reality average P is effective parameter. for electrical energy spent. So the average value of P is T 0 1 P v(t)i(t)dt T ? ? where v(t) = V 0 sinwt and i(t) I 0 sin(wt+ ?), here ? is the phase angle. Now, T 0 0 0 V I P sin wt.sin(wt)dt T ? ?? ? or, 0 0 V I P . cos 2 2 ? ? So, P – = V rms × I rms cos ? ......(8.3.9) The term cos ? is known as power factor. The product V rms I rms does not, except for the case cos ? = 0, gives the true power dissipated in the circuit as does the product in d.c. circuit. Here V rms I rms is called apparent power. While P – gives the real power in the circuit. So we have, Real power = Apparent power × Power factor ......(8.3.10) In A.C. circuit instantaneous power is given by p(t) = i(t).v(t) and i(t) have same sign. The positive value of p(t) imdicates that the source of A.C. supply is delivering energy to the circuit. Again when v(t) and i(t) have opposite sign, implying that the source is receiving energy from the circuit when phase angle ??? ? ? or 90°, so no power is dissipated in the circuit. The current flowing in such curcuit is called wattless current. It will be seen latter that when current flowing through pure inductor or pure capacitor is a wattless current. 5. Peak factor : The ratio of the peak value to the rms value of any ac waveform is called its peak factor. For a pure sinusoidal ac,

NSOU ? CC-PH-08 ? 262 peak factor (k p) = 0 0 I 2 I / 2 ? ....(8.3.11) Since waves of same rms value may have different peak values a knowledge of peak factor is necessary to get an idea if peak value. Again a knowledge of peak value is important while testing dielectric insulation or hysteresis loss. 8.4 Representation of sinusoidal ac by complex numbers Two main reasons that make the use of complex numbers suitable to model AC, circuits and many other sinewave phenomena in several branches of science and technology are described below. 1. The AC signals are characterised by a magnitude and phase that are, respectively very similar to the modulus and argument of complex numbers. 2. The basic operations such as addition subtraction multiplication and division of complex number are easier to carryout. We know that when a vector is multiplied by –1, though its magnitude remains unaltered, but the direction changes in opposite direction i.e. 180°. As –1 = 1 1 ? ? ? , so we multiply or operate 1 ? two times in 180° polar angle. Hence operating 1 ? one time on a vector it will rotate in 90° in anticlocwise direction. In a two dimensional co-ordinate system (XY), let X-axis represents real number and imaginary number along Y-axis (Fig. 8.2). It a vector A ? is along the positive X-direction then vector quantity will be perpendicular to A ? . Denoting A ? as jB ? , then the resultant of A ? and jB ? will be another vector P ? as shown in Fig. 8.3 Fig. 8.2 Fig. 8.3 imaginary j 2 Real +ve imaginary +ve Real

NSOU ? CC-PH-08 ? 263 Clearly for A + jB, its magnitude XO = 2 2 A B ? which is real quantity and the phase angle with respect to real axis ??= tan –1B / A is inclined. So if A = X 0 cos? and B = X 0 sin ? are taken, then A and B are replaced by X and ? real numbers and consequently A + jB is expressed as complex number A + jB = X 0 cos? + jX 0 sin ? = X 0 e j? . Here 2 2 0 X A B ? ? is magnitude of A + jB and e j? is its phase term. In case of alternating current, if ? = V 0 sinwt or V 0 coswt is taken, then they are real and imaginary part of v = V 0 e jwt . So v or i can be explained in complex plane whoe vector disposition or phasor X = A + jB = X 0 e jwt , the phasor X will be rotating with time at the angular speed w and its value X 0 remains unchanged. 8.4.1 Impedence and Reactance In AC, circuit, current flow, which faces resistance is called impedence or reactance when voltage v and current flow i are expressed in complex number, the impedance can be expressed by the ratio of v and i let ? = Ve jwt and i = I 0 e v(wt+ ?) , then by the analogy of ohms law in DC circuit, we have v = zi, here z is called the imp-edance or reactance, so, j t 0 0 V v z e , i I ? ? ? ? the real part of z is called resistance R and imaginary part is called reactance X. So, z = R + jX From the above equation, we get 0 0 V R cos I ? ? Earlier in this unit, we have shown that P = V rms I rms cos? = I rms 2 |z|cos ? = I rms 2 R Comparing this, above average power dissipated with the Joule's law of heat in steady flow of current, we can say that real part of z is to denoted as resistance of the circuit.

NSOU ? CC-PH-08 ? 264 8.5 Kirchoff's Laws Kirchoff's laws of AC circuits are as follows : 1. The phasor sum of the currents at any point in the circuit is zero. 2. The phasor sum of the voltages around any closed loop is zero. The current and voltage equations are derived in the same way as in DC circuits. The algebric simplifications of phasor quantities is no different from that of DC quantities until numerical quantities are introduced. For KVL, let v 1 , v 2 ....v n be the voltages around a closed loop. Then v 1 + v 2 + ...+ v n = 0 ......(8.5.1) In the sinusoidal steady state, each voltage may be written in cosine form, so that, Equation (8.5.1) becomes V 01 cos (wt + ? 1) + V 02 cos(wt + ? 2) + ... + ...+ V 0n cos(wt+ ?n) = 0 ....(8.5.2) This can be written as, Re(V 01 e j?i e jwt) + Re(V 02 e j?2 e jwt) .....+Re(V 0n e j?n e jwt) = 0 ......(8.5.3) or, Re[V 01 e j?1 +V 02 e j?2 + ...+ V on e j?n]e jwt = 0 If we let V s = V 0s e j? s , then Re[(V 1 + V 2 + V 3 +......+V n e jwt] = 0 ......(8.5.4) Since e jwt ? 0 V 1 + V 2 + ... + V n = 0 ....(8.5.6) indicating that kirchoff's voltage law holds for phasors. In the same way, we can show that Kirchoff's current law holds for phasors. If we let i 1 , i 2 ....i n be the current entering or leaving closed circuit in a network at time t, then i 1 + i 2 + ...+ i n = 0 ......(8.6.7)

NSOU ? CC-PH-08 ? 265 If I 1 , I 2 ....I n are the phasor forms of the sinusoids i 1 , i 2 ....i n then I 1 + I 2 + ... + I n = 0 ....(8.5.8) which is Kirchoff's current law in the AC domain. 8.6 AC Responses of a Resistance, An Inductance and a Capacitance 1. Resistance R Fig. 8.4 Fig. 8.5 In the above AC circuit potential difference across the resistor / conductor is v(t) = i(t)R, here R is the resistance of the conductor, which follows from ohms law. As R is a real quantity, so, v(t) and i(t) will be in the same phase, and they are both indicated in the phasor diagram along the positive real number axis by two straight line (Fig. 8.5). 2. Pure inductance 'L' Fig. 8.6 Pure inductive circuit Alternating voltage v(t) is applied across inductor L. We know that when a time ~ Current Flow ? ? AC Power Source ????? ? o Sin t Resistive Load Voltage Current ? ? ? ? ?? ? ? ??? ? i L V=VoSin t ? Fig. 8.6 v.i? ? ? ? ? ???? t ? ?

NSOU ? CC-PH-08 ? 266 varying current flows through an inductor, a back emf is produced if the coil is having inductance L, the back induced emf is Ldi dt ? , which mitigates the supplied AC voltage source v(t) so the voltage equation of the circuit Ldi v 0, dt ? ? To find solution, we apply complex analysis as discussed earlier in this unit. Let i(t) = I 0 e j?t is the instantaneous current in the circuit, then j t 0 0 Ldi v(t) j LI e j LI dt ? ? ? ? ? ? ....(8.6.1) so the impedence of the circuit z = j ?L, which is an imaginary number, so 2 j z Le ? ? ? Now we get, v(t) = ?LI 0 e j(?t+ ? / 2 ....(8.6.2) Taking V 0 as the maximum potential difference, then V 0 = ? LI 0 and alternating voltage source is ahead of the current flow in phase by 2 ? . In Fig 8.7 phasor representation of v and i is shown. (3) Pure Capacitance Fig. 8.8 Fig. 8.9 When an alternating voltage is supplied across capacitor charged first in one direction and then in the opposite direction. Let v(t) = V 0 sin ?t Instantaneous charge at the capacitor plate Q t = C V 0 sin ?t ....(8.6.3) So, the charging current at any instant i(t) = t 0 dQ ωεV cosωt dt ? ~ ? V Capasitor (C) Current Flow I ? ? ? ? ????? t ?? ? ? Votage Current

NSOU ? CC-PH-08 ? 267 = 0 ωc V sin(ωt) 2 y ? ? or, i(t) = I 0 sin(?t + ? / 2) .....(8.6.4) Here I 0 is the peak current given hy = o I 1 c ?? obviously, current i(t) through a pure capacitor leads over the applied voltage by 90°. Hence power dissipated by the circuit containing pure capacitar is zero as power factor cos ? = 0. Rate of heat dissipation in the circuit at any instant P(t) = v(t) i(t) = 2 0 v cosωt cos(ωt) 2 ωL ? ? = 2 0 V sin 2 ωt 2ωL Average rate of dissipation of heat energy is 2 T 2 0 0 V 1 P sinωtdt 0 2ωL T ? ? ? So, there is no dissipation of energy in a circuit containing capacitance. Continually at an interval of T 4 time source current electrical energy is transformed to magnetic energy returns it to the source as electrical energy. Such mutual exchange of energy is non- dissipated. 8.7 Series LCR Circuit Resistance R, inductance L and a capacitance C are connected in series with an AC source. So source emf. V = V R + V L + V C and separately resistance are as follows R the resistance, impendance of L is Z L = j ?L and impedence of C, z c = ? j / ?c Fig. 8.10

NSOU ? CC-PH-08 ? 268 So, at any instant relation between the voltage and current is given by v = (R + j ?L – j / ?c) i = [R + J(?L – 1 / ?c)]i ......(8.7.1) Here circuit resistance is R and reactance is X = (?L – 1 / ?c) Hence i = j 0 v e | z | ? ? ....(8.7.2) Now the total impedence of the circuit is ? ? 0 2 2 1 | z | 1 R ωL ωc ? ? ? ? ? ? ? ? ? ? ? ......(8.7.3) Phase relation between v and i, ?= tan –1 X R as shown in Fig. 8.11(a). Let source emf at any instant is v(t) = V 0 cos?t is real part of V 0 e j?t . So the current flow at any instant is the real part of i(t) = j(ωt) 0 0 V e | z | ?? where ? = tan –1 1 ωL ωc R ? ......(8.7.4) Phasor diagram (a) ?L < 1 / ?c , (b) ?L > (1 / ?c) and (c) ?L = 1 / ?c Fig. 8.11 From eq n (8.7.4) it appears that phase angle is not alway positive, so the phase ? jwLI j(wL-l/wl)I RI j wc ? I jwLI R I j wc ? I

NSOU ? CC-PH-08 ? 269 relationship of v(t) and i(t) varies with the circuit parameter. i) [?L – 1 / ?c] < 0 is positive then current flow legs behind source emf v(t). ii) [?L – 1 / ?c] > 0, ? is negative, v(t) lags behind i(t). iii) ?L = 1/ ?c then, ??= 0, v(t) and i(t) are in the same phase. Under third condition, mentioned above, impendence of the circuit becomes z 0 = R, that is it is purely resistive and lowest value maximum current flows though the circuit (with maximum value) under this condition, frequency of AC source becomes 0 1 ω ω LC ? ? ......(8.7.5) This state of AC circuit is called resonant and ? 0 is resonant frequency. In reality resonant circuit behaves as purely resistive even in the presence of L and C. 8.7.1 Phasor relation graph Here, phasor such as v(t), Ri, j ?Li and –ji / ?c are shown in the graph with direction, in Fig.12 Fig. 8.12 8.7.2 Different types of Resonance in Series LCR We have shown that under resonant condition, maximum current flows through the circuit, when maximum root mean square current us given by rms rms V I R ? ......(8.7.6) Root mean square current, other than resonant condition is given by ? ? rms rms 2 2 V I 1 R ωL ωc ? ? ? ? ? ? ? ? ? ? ? .....(8.7.7) ? ?t V L ? V C ? V C ? V 2 ? ? V R ? V ?

NSOU ? CC-PH-08 ? 270 and phase angle 1 1 L c tan R ? ? ? ? ?? . In reality current resonant condition can achieved either by changing the frequency of the source or changing value of capacitance may bring different values of I rms . Also it can brought a voltage resonant condition by other than series resonant state in the circuit. Fig. 8.13 Fig. 8.14 8.7.3 Quality Factor and Shapness of Resonance in Series LCR Circuit : The sharpness of resonance relates to the reapdity of the fall in current on eigherside of the resonance frequency. The current falls to a very low values depending on the sharpness of the resonant. The smaller the value of the resistance the greater the current at resonance and the shasper the resonance. Behaviour of R in circuit in forming sharper resonance is shown in Fig. 8.13. Fig. 8.15 If we now reduce or increase the frequency until the average power absorbed by the resistor R in series with resonance circuit is half that of its maximum value at resonance, we produce two frequency points called the half power points which are –3d B down from maximum taking OdB as the maximum current reference the point corresponding to the Irms ? ? ? ? ? R 1 R 2 R 3 R <R <R < 1 2 3 Z(f) Impedance fr Capacitive Inductive X < X C L X < X L C Z=R X = X and cancel each other L C Frequency f Series Resonance - Impedance vs. Frequency. Frequency f Band w ithCurrent 0.707I MAX I MAX f o f L f o f H

NSOU ? CC-PH-08 ? 271 lower frequency at the half power point is called the lower cutoff frequency, denoted as f L the point corresponding to the upper frequency for half power point called higher frequency cult of f H . The difference f H – f L is called the bandwith (Fig. 8.15). At the half power point, impedence z 0 = 2R . or, ? ? 2 1 R L 2R c ? ? ? ? ? ? So, ? ? 1 L R c ? ? ? ? ? ? 2 LC RC 1 0 ? ? ? ? ? Only the positive roots are acceptable, Rools are 2 2 1 (R C 4LC) RC 2LC ? ? ? ? and 2 2 2 (R C 4LC) RC 2LC ? ? ? ? ....(8.7.8) The bandwidth is 0 0 2 1 0 0 R R 1 L L Q CR ? ? ? ?? ? ? ? ? ? ? or, 0 2 1 Q ? ? ? ? ?? .... (8.7.9) This equation relates the Q to the bandwidth. Sharpness of resonance increases with the increase in Q. Quality factor increases with the decrease in R, as there is no change in resonant frequency. Graph (Fig. 8.13) shows the dependency of sharpness with variation in R. The circuit can store energy in the form of magneticfield or electrical energy across the condenser. The performance efficiency is also given by the Q-factor maximum energy stored Q 2 energy dissipated per cycle ? ? 2 0 2 r 1 2 LI 2 I RT ?? ?

NSOU ? CC-PH-08 ? 272 where T is the time period, and 0 r I I 2 ? 0 0 L 1 Q R CR ? ? ? ? ......(8.7.10) Another term, used in resonant circuit is selectivity. It is defined is its ability to respond more readily to signals than to signals of other frequencies. This response becomes progressively weaker as the frequency departs from the resonant frequency it is mathematically defined as 2 1 0 f f Bandwidth Selectivity Resonant frequency f ? ? ? ......(8.7.11) 8.7.4 Voltage Resonance in Series LCR Circuit : Here we will study the behaviour of changing C and L in a series LCR circuit and show other types resonance. Let us take potential diffference across the capacitor. rms c I V C ? ? ....(8.7.12) But, ? ? rms rms 2 V I 1 R L C ? ? ? ? ? ? ? ? ? ? ? ? ? ? Therefore, ? ? rms c 2 2 2 2 2 V V C R LC 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ....(8.7.13) The variation of |Vc| with ? is shown in the figure Fig. 8.16 Vc I I ? c ?

NSOU ? CC-PH-08 ? 273 Therefore, for the maximum |V c | we must show 2 2 2 2 2 d R C (LC 1) 0 d ? ? ? ? ? ? ? ? ? ? ? ? After differentiation 1 2 2 C 1 R C 1 LC 2L ? or, 1 2 C 0 2 1 1 2Q ? ? ? ? ? ? ? ? ? ? ? ? ....(8.7.14) Equation (8.14) shows that ? c > ? 0 The reason is that current peaks at resonance and capacitor voltage is the product of current and (1 / ?c) which decreases with increasing frequency. c 0 ? ? ? when Q is very large. 8.8 Parallel Resonant Circuit When an alternating voltage source is connected to an inducetor having small resistance combinedly in parallel with a capacitor, which is shown in Fig. 8.17. Two current component from the source is divided into two branches of parallel resonant circuit. These are phasor current I RL and I C so total phasor current I = I RL + I C and impedences Z c = – j /? C and z RL = R + j ?L source phasor voltage V = V RL + V C . When R is small, I L and I C will be almost 180° out of phase, since the capacitor current I C leads the source voltage V by 90° and while the inductor current I L lags the source voltage V by nearly 90°. At resonance V and I are in phase. V V I jωCV R JωL z ? ? ? ? ......(8.8.1) where z is the complex impedence of the circuit. Fig. 8.17 ? ????????????? ? ? R L ? ? ?(t) c I C

NSOU ? CC-PH-08 ? 274 2 2 2 R jωL 1 1 jωC jωC z r jωL R ω L ? ? ? ? ? ? ? ? 2 2 2 2 2 2 R ωL j ωC R ω L R ω L ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? The source voltage V and the source current I will be in phase when the imaginary part of 1 / z varishes. Then 2 2 2 ωL ωC R ω L ? ? or, 2 0 2 1 R ω ω LC L ? ? ? .....(8.8.3) the source voltage and current are in phase, power factor is unity, this condition of the circuit is said as parallel resonant frequency. For ? 0 to be real, the condition must be 2 2 R 1 LC L ? or, L R C ? ....(8.8.4) The impedence of the circuit is ? ? 2 2 2 2 2 0 d 2 R L L 1 R R R R R LC L ?? ? ? ? ? ? d L R CR ? ......(8.8.5) R d is called dynamic resistance. At the resonance peak current from the supply is called make up current and is given by 0 op d V I R ? ......(8.8.6) The cupacitor current at parallel resonance is 0 cp 0 V I 1 w c ? ....(8.8.7) So, cp 0 0 op I ω L ω CRd I R ? ? ....(8.8.8)

NSOU ? CC-PH-08 ? 275 Q of the circuit is defined as the ratio of the capacitor current to the line current at parallel resonance. Hence, cp 0 op I ω L Q I R ? ? ......(8.8.9) the capacitor current I cp is much large than the supply current due to high value of Q. At resorce, capacitor or inductor current is Q times the supply current. Such highly magnified current of parallel resonance can be compared to magnified voltage of series resonance circuit. From equation (8.8.2) the impedence z or admittance Y can be written as 2 2 2 2 2 1 R ωL j ωC z R ω L (R ω L ? ? ? ? ? ? ? ? ? ? ? ? ? ? or, 2 2 2 2 2 2 2 2 2 1 R ωL Y ωC z (R ω L) (R ω L ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? .....(8.8.10) Maximum impedence or minimum admittance at a frequency Q m is given by 2 d Y 0 dω ? which gives ? ? 1 1 2 2 2 m 2 1 2CR R ω 1 LC L L ? ? ? ? ? ? ? ? ? ? ? ? ? ......(8.8.11) comparing eq h (8.8.3) and (8.8.11) we find that ? m < ? 0 , i.e. maximum impedence is achieved at higher frequency than the resonant frequency ? 0 . If Q is large as R is low, then we can assume that ? n ~ ~ ? 0 . This circuit is called anti resonant because current is minimum at resonance which is in contrast to the series resonant current. Sometimes it is called rejector circuit. 8.8.1 Selectivity of Parallel Resonant Circuit : (Fig. 8.18) shows the variation of z with frequency. For R ? 0, then ? m ~ ~ ? 0 , which is the condition for high Q circuit. Let 0 0 ω ω ? ? then if the impedence of the circuit z becomes 0 z 2 , we say that resonance has become very sharp. Then select band of frequency which is allowed blocked or filtered, which is the condition for high selectivity of parallel resonant circuit.

NSOU ? CC-PH-08 ? 276 Fig. 8.18 From equition (8.8.2) 1 1 j c. z R j L ? ? ? ? ? .....(8.8.11) when R ? 0, The above equation can be written as 2 j L z (1 LC) J CR ? ? ?? ? ? ......(8.8.12) At resonance ? = ? 0 and z = z 0 = L CR From equation (8.8.12) ? ? 0 z z j 1 1 L c R ? ? ? ? ? ? or, ? ? 0 2 2 z Z 1 1 1 L c R ? ? ? ? ? ? .....(8.8.13) For high selectivity we have chosen 0 z z 2 ? (8.8.13) becomes after simplification. 1 L R c ? ? ? ? ? ? or, 2 R 1 0 LC L ? ?? ? ? ....(8.8.14) t Z Leading p.f. Lagging p.f. ? o ? impedance Z = L CR Current I =V L CR 0 ure: Resonance Curve for Parallel Resonance

NSOU ? CC-PH-08 ? 277 Writing 2 0 1 LC ? ? for assuming low value of R 2 0 1 0 0 Q ? ? ? ? ? ? ? ......(8.8.15) Real roots are, 1 0 2 1 1 1 2Q 4Q ? ? ? ? ? ? ? ? ? ? ? ? ? ? and 2 0 2 1 1 1 2Q 4Q ? ? ? ? ? ? ? ? ? ? ? ? ? ......(8.8.16) So the band width is 0 2 1 Q ? ? ? ? ? ? ? ? ? ......(8.8.17) From equation (8.8.17) we conclude that for high selectivity, preferred choice remains for high value of Q. Fig. 8.8.4 Parallel Resonance In LCR Circuit. Fig. Resonance Curve for Parallel Resonance Fig. From the eq, we have rms C 2 2 2 2 2 V V C R (Lc 1) ? ? ? ? ? ? ? ? ? ? ? The value of C for the maximum value of V c can be found in the following way Let x = V[? 2 C 2 R 2 + (? 2 Lc–1) 2] 2 2 2 2 dx 2 R C 2 L(Lc 1) 0 dc ? ? ? ? ? ? ? f Band with 0.707Z r Z r f 1 f r f 2 Z BW

NSOU ? CC-PH-08 ? 278 C = C 0 = 2 2 2 L R L ? ? So, this resonance can be spoiled with the variation of R. When C = C 0 , maximum of value of Vc is 2 2 2 rms c(max) V (R L) V R ?? ? 8.9 Summary 1. In AC circuits, it is seen that impedance of reactive components, like inductor or capacitor is expressed in terms of imaginary number, or, phasor qualysis of impedence of inductance by jL ? or for capacitor by 1 j C ? . Total electrical energy dissipated in this component is zero. 2. The frequency at which impedence becomes minimum in series LCR circuit is called resonant frequency. At resonant, angular frequency 0 1 (LC) ? ? and quality factor is 0 L Q R ? ? . 8.10 Review Questions and Answers 1. Define bandwidth. Ans. It is defined as the breadth of the resonant curve upto frequency at which the power in the circuit is half if its maximum value. The difference between two half power frequencies is called the bandwidth. 2. Define selectivity. Ans. The selectivity of a RLC circuit is the ability of the circuit to respond to a certain frequency and discriminate against all other frequencies. If the band of frequencies to be selected or rejected is narrow, the quality factor of the resonant circuit must be high H L 0 f f Bandwidth Selectivity Resonant frequency f ? ? ?

NSOU ? CC-PH-08 ? 279 3. Define phasor and phase angle. Ans. A sinusoidal wareform can be represented in terms of phasor. A phasor is a vector with definite magnitude and direction. From the phasor, the sinusoidal waveform can be constructed. Phase angle is the angular measurement that specifies the position of the alternating quantity relative to a reference. 4. Define power factor. Ans. Power factor is defined as the cosine of the angle between voltage and current. If ? is the angle between voltage and current that cos ? is called as the power factor. Other definition is the ratio between real power and apparent power. 5. Define Apparent power and power factor. Ans. The apparent power (inVA) is the product of the rms values of voltage and current, S = I rms V rms . The power factor is the cosine of the phase difference voltage and current. It is also the cosine of the load impedence. Power factor = cosf. The power factor is leading when current loads voltage (capacitative) and lagging when current lags voltage (inductive load) 8.11 Problems and Solutions 1. A coil takes a current of 1A from 6v dc supply, when connectexd to a 120v 5 OHZ supply the current is 10A. Calculate the resistance, impedence, inductive reactance and inductance of the coil. Solution : Resistance dc voltage 1 R 6 dc current 6 ? ? ? ? impedence = ac voltage 120 12 ac current 10 ? ? ? V I ?

NSOU ? CC-PH-08 ? 280 Since 2 2 L z R X ? ? So, inductive reactance 2 2 L X (z R 10.39 ? ? ? ? Again, X L = 2 ?fL inductance L X 10.39 L 33.1mH 2 f 2l 50 ? ? ? ? ? ? 2. A coil of resistance 5 ? and inductance 120mH in series with 100 mF capacitor, is connected to a 220V, 50 Hz supply. Calculate (a) the current flowing, (b) the phase difference between the current and supply voltage (c) the voltage across the coil and (d) the voltage across the capacitor. Solution : Circuit diagram is shown in the fig X L = 2 ?fL = 2 ?(50)(120×10 −3) = 37.7 ? C 6 1 X 31.83 2 fc 2 (50)(100 10) ? ? ? ? ? ? ? ? ? Since X L is greater than X C the circuit is inductive X L − X C = 5.87 ? Inpedence 2 2 L C z R (X X) 7.71 ? ? ? ? ? ? ? ? ? ? ? (a) Current v 220 I 28.53A z 7.71 ? ? ? (b) Phase angle ? = arctan L C X X 5.87 arctan R 5 ? ? ? ? ? ? ? ? ? ? ? ? = 49°35´ (c) Impedence of the coil = 2 2 L (R X) ? =38.03 ? (d) Voltage across the coil. = IZ coil = 38.03 × 28.53 = 1085V

NSOU ? CC-PH-08 ? 281 Phase angle of the coil = arctan 1 L X 37.7 arctan R 5 ? ? =82°45´ Voltage across the capacitor V C = IX C = 28.53 × 31.83 = 90 ?.10V 3. A coil of inductance 0.1 H and resistance 30W is connectred in parallel with a 10 ?F capacitor across a 50V, variable frequency AC supply calculate (a) the resonant frequency, (b) the dynamic resistance, (c) the current at resonance and (d) the circuit Q-factor at resonance. Solution : (a) Parallel resonant frequency, ? ? 2 2 r 2 6 2 1 1 1 1 30 R f 2 LC 2 L 0.1 (10 10 (0.1) ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? =152 Hz (b) Dynamic resistance Rd = 6 L 0.1 333.33 RC 30 (10 10) ? ? ? ? ? ? (c) Current at resonance r 50 I 0.15A 333.33 ? ? (d) Circuit Q-factor at resonance 2 frL 3.183 R ? ? ? Capacitor current at resonance C C V I X ? r V 0.477Amp 1 2 f c ? ? ? ? ? ? ? ? ? ? ? ? ? Hence Q factor = C r I 0.477 3.183 I 0.15 ? ?

NSOU ? CC-PH-08 ? 282 4. A complex voltage (20 + j.O)V is applied to a series LR Circuit of complex impedence (1+ ?3j) ?. Calculate the power factor and the power consumed by the circuit. Solution : Complex current j 2 2 2 0 j1 0 20 I 1 0e 1 3 j (3) 1 ? ? ? ? ? ? ? ? ? where tan ? = 3 So the current lags behind the emf by an angle ? = tan −1 3 = 60° ? Power factor cos ? = cos60° = 0.5 Power consumed P = VI cos ? = 20 × 10 × 5 = 100 watt. 5. An electric lam p which runs at 100V and 10A current is connected across 220 Volt 50 cycle AC main. Calculate the value of the choke to be connected in series for the lamps safety. Solution : Here the resistance of the circuit or lamp V 100 R 10 I 10 ? ? ? ? Impedance 2 2 2 2 2 2 z R L 10 (2 50) L ? ?? ? ? ?? We have 220 z 22 10 ? ? So, 2 2 2 22 10 L 0.0623H. (2 50) ? ? ? ??

Unit 9 ? Ballistic Galvanometer Structure 9.1 Objectives 2.2 Introduction 9.3 Moving Coil Galvanometer 9.4 Summary 9.5 Review Questions and Answer 9.6 Problems and Solution 9.1 Objectives After the completing this chapter the learner will understand - ? The construction and operation of electrical measuring instrument- the galvanometer. ? How a ballastic Galvanometer is used to mesure charge ? How a ballastic Galvanometer can be converte it into dead-beat galvanometer to mesure current and voltage. ? What is CDR and its rate to current a galvanometer from ballastic to dead- beat and V.C.V.S ? The current sensitivity, voltage sensitivity and charge sensitivity of galvanometer and their radiation. 9.2 Introduction A ballastic galvanometer is a type of instrument, commonly a miror galvanometer, unlike a current-measureing glavanometer. The moving part has a large moment of inertia and hence giving it a long period of ocillation period. The glavanometer works on the principle of permanent magnet moving coil. The force is generated on the coil, due to Lorentz Force law. "Due to interaction of fluxes, the pointer in the meter or miror is deflected. As it is the deffected different torqes to make the pointer stop at its steady state motion. The different torques are deffeching NSOU ? CC-PH-08 ? 284 control torques and damping torquesis almost zero. For that reason.it is called a ballastic galvanometer. It is really an integrator measuring the quantity of charge or discharge throught it. It can be used ad voltmeter and ammeter. 9.3 Introduction The fig. 9.1 shows a line diagram of a moving coil galvanometer. It consists of a) A rectangular frame with insulated copper wire round on it. b) The frame is suspended symametrically with a thin phosphor bronze fibre with its end connected to the one end of copper wire. c) C is a nonconducting or conducting core palced symametrically inside the frame. d) N-S represent concave magnetic poles placed co- contri at the mid point of the gap. e) The bottom end of the copper wire connected to a phosphor- bronze thread to measure the deflection of coil using lamp and, scale arrangement. f) The small mirror M, attached to due suspended phosphor-bronze thread to measure the deflection of coil using lamp and scale arrangement. Theory- We consider the coil c to have n number of turns with area A (Vertical lenght l and horizental width b, A=lb) suspended in an uniform magnetic field B dq be the amount of charge flow through the coil in time dt at an instant t, so i = dq/dt at instant t. The torque on the coil (where ???niA). N Moving Coil Core (C) Permanent Magnets S Phosphor-Broze Strip Mirror Lower Suspension Spring Fig. 9.1 NSOU ? CC-PH-08 ? 285 Taking the magnitude of torque we have ....(9.3.1) If I is the moment of inertia of the coil with core along the axis of suspention, then [w being the angular speed at instant t] or I d ? = nABdq If we consider the moment of inertia of the coil-core system is sufficiently high such that the coil-core system does not move during the passege of charge q 0 , Then intergrating the eqn. (9.2) we have I ? 0 = n A B q 0 ....(9.3.2) If there is no disipitive force, then from conservation of energy we can write, ½ I ? 0 2 = ½ c ? ? 2 ....(9.3.3) Where ? ? is the angular amplitude of first throw. After migration of charge the equation of motion of the coil at an instant t will be I d 2 ? = -C ?, where C represents the torque per unit turist (or torsional rigidity) of the suspension fiber. So the time period of osciallation of the coil T = 2 ? I /c or I = 4 ? ....(9.5) Elimininting ? ? from eqns. 9.3 & 9.4 and putting the value of I from eqn. 9.5, we have q 0 = T 2 ? C nAB ? 0 ....(9.3.4) Theory of moving Coil galvanometer with damping forces. When a coil moves in a galvanometer two damping forces play important role to oppose the motion. 1. Mechanical damping force, mostly due to the air friction which at low angular velocity of coil can be taken to be proportinal to the angular speed. dt 2 NSOU ? CC-PH-08 ? 286 ? ??d? dt , say F a ?? d? dt 2. Electromagnetic damping force, due to rotation of coil in magnetic field. To proceed with the calculation of e.m damping, please refer Fig. 9.2. Due to rotation of coil through dq in a inform magnetic field B, that lies always in the plane of the coil, the vertical section passes through the area d.s = 2 b 2 d?? ? ? x l = bld ?? so the number of time of forces intercepted by a single turn of coil = bld? B. so the rate of change of magnetic lines through the coil of n turns d? dt ?? nblB ?? nAB d? dt d? dt ?? G Where G = nAB is called golavanometer constant. So the induced end generated in the coil to oppose the motion d? dt e??? G ....(9.3.5) so the induced current d? dt i d ??? G R ....(9.3.6) Where R = R G (Galvanometer resistance) + Re (External resistance in the circuit) The opposing torque developed due to this induced current d? dt G 2 ?? ni AB= d R ....(9.3.7) d? ? Q b Fig. 9.2

NSOU ? CC-PH-08 ? 287 so the equation of motion of coil considering the damping force takes theform, C? d 2 ? dt 2 I ?? - a d? dt - G 2 R d? dt C? ?? - (a +) G 2 R d? dt or ....(9.3.8) wherw 2b = (a + G 2 /R)/I and ? o 2 = C/I To seek for a solution we put q = Ae ?t , then we have from eqn.(9.3.8) ?2 + 2b ? + ? o 2 = O or ? = b ± Thus the solution of eqn. (9.3.8) coils down to ....(9.3.9) Now for b< ?o, The first term exponentially decreases with time, the first term within braces increase exponentilly with time but less effectively then the outside term; the second term descreses exponentially with time. Hence the motion is a damped non-oscillatory motion. Similar logic leads that when b = ? o motion is non- oscillatory and is known critically damped motion. When b> ? o the eqn. (9.3.9) can be written as or we can write the above eqn. as ? = ? o e -bt sin (?t + ?) .... (9.3.10) where A 1 +A 2 = ? o sin ??? i (A 1 -A 2) = ? o cos?, ? = and i - ? .

NSOU ? CC-PH-08 ? 288 So the resultant motion will be an oscillatory motion with decresing time dependent amplitude and with of angular frequency ? o 2 2 - b = ? . Now at t = o, ? =o, the above equation gives ? = o, so we have ? = ? o e -bt sin ?t..(9.3.11) So the maximum deflection on either side of central position of coil will be at t = T 4 , 3T 4 5T 4 ? , The variation of deflection with time is as show in the following fig. (9.3) The successive maximum deflections are ? ? ???? ? e -bT/4 , ? ? ???? ? e -b3T/4 , ? ? ???? ? e -b5T/4 ....(9.3.12) The ratio of successive amplitude of deflections ?? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ??????????? e +bT 2 ? d. d is known to be decrement per half cycle or simply decrement. l n d = ??= bT/2 is called logrithmic decrement. Now from the eqn (9.3.12) ? ? ???? ? e -bT/4 or ? ? ???? ? e bT/4 ??? ? e ?/2 As the damping factor is small of l is also very small. So we can write ? o = ? 1 (1+ ?/2) T e -bt (O,O) 10 ? ? ? = e Sin (wt) 0 -bt t Fig. 9.3

NSOU ? CC-PH-08 ? 289 So the correct relation of change flowing and the first throw becomes q T ?? ? ^ ¯ C n?? ? ? ? (1+ /2) ? ....(9.3.13) Procedure for Calculation of ?? To find a resonable average value of ?, the first throw and the eleventh throw is noted them, ?? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ? ? ? ? ?? ? ?? ? ? ??????? ?? ?? ? ? ? ?? ? e 10bT/2 ?e 10? or ? = ?? ?? en ? ? ?? ? ?? ( ) ....(9.3.14) Similarly two other sets of such readings are taken and the average value of ? is calculated. Critical damping resistance (CDR) We have already seen that the condition of ballastic galvanometer to be b> ? 0 , where b represents half the damping torque per unit moment of inertia (2b often reffered as damping factor) and ? o is the angular velocity of the coil at t = 0, without damping. Now b = ?? ?? (a+ G) 2 R , where R = R G (galvanometer resistance) + R e (External resistance with galvanometer circuit) Thus we can write the condition for a galvanometer to be ballastic, or R < e 2 Ic - a G 2 -R G the limiting value of R s . R = e Ic - a G /(e) 2 -R G ....(9.3.15) This value called CDR of the galvanometer. However the air damping factor due to air resistance is normally much less compared to the electromagnetic damping

NSOU ? CC-PH-08 ? 290 so we can neglect a and the CDR takes the form Rc = 2 Ic G 2 -R G ....(9.3.16) when the external resistance Re>Rc the damping force is sufficient to make the motion non-oscillatory and the galvanometer acts as dead-beat galvanometer. Measurement of steady current Now it the galvanometer circuit contains an external emt source E to supply a steady current i then the general equation of motion of galvanometer coil becomes I d 2 ? dt 2 = - C - ? (a+ G) 2 R d? dt + niAB ....(9.3.17) Then the resultant motion will be a super position of a damped harmonie motion about a steady deflection ? s (say). Inthe mesurement of current we choose the external resistance less than the C.D.R so that the motion is over damped. In this case ni s AB = c ? s (? s = steady diflection for current is) i s = c nAB ? c ....(9.3.18) To increase the electro-magnetic damping the core of the dead beat galvanometer is made of soft iron which has large permiability. Under damped (I) Critically damped Over damped ? S ? O Time Over damped Vs Critically damped Fig. 9.4

NSOU ? CC-PH-08 ? 291 Lamp and scale arrangement. (Fig. 9.5) The deflection of coil is useally measured by using a lamp and scale arrangement. The arrangement consist of a lamp L mounted on a vertical stand ST. The lamp (collimates) a beam of light onthe mirror M attached to the suspension wire of galvanometer. The reflected beam is received on a semi-transparent scale S held on same vertical stand ST and held horizontally and paralled to the mirror plane M. D is the distance between mirror and scale. It due to passage of current the coil reflects through ? and the deflection of light spot on scale is d, then tan2 ? = d/D or ? = ½ d/D (when ??is very small) Sensitivity of Galvanometer The quality of a galvanometer to respond towards charge / current and voltage measurement is the measure of its sensilivity. Accordingly a galvanometer may have three types of sensitivity. 1. Current Sensitivity. The current sensitivity of a galvanometer is the deflection in mm of the light spot on a scale placed 1m away from the galvanometer mirror intitially perpendicular mirror to the mirror palne of galvanometer due to the passage of 1 ?A current through the glavanometer. So it a current of i s ?A produce a deflection d mm on the scale placed 1m away, then, the current sensititivity. S i ???? d i s mm ? ?? 2. Voltage Sensitivity. The voltage sensitivity of a galvanometer is the deflection in mm of the light spot on a scale placed 1mm away from the galvanometer mirror, initially M M D S L 2? ST L-Lamp S-Scale M-Morror Fig. 9.5

NSOU ? CC-PH-08 ? 292 perpendicular to the mirror-plane of the galvanometer due to 1 ?V potential differnce across the galvanometer. So if the potential differnce of Vs ?V produced a diflection of dmm on the said scale then, the voltage sensitivity. S v ???? d V s mm ? ?V ???? d ? i s R G ???? R G S ....(9.3.19) 3. Charge Sensitivity. (This is concerned to the ballastic galvanometer in practice) The charge sensitivity of a galvanometer is the deflection in mm of a galvanometer is the away from the galvanometer mirror, initially perpendicular to the mirror plane of the galvanometer due to the passage of 1mc of charge through the galvanometer such that, the during the flow of charge the coil does not move. So the charge sensitivity Sq???? d q mm / ?c ....(9.3.20) Comparing the expression of q and i we have Sq???? T S i ? ^ ¯ (T = period of oscillation of galvanometer coil)...(9.3.21) 9.4 Summary We have learned the following lessons : 1. Basic principle of construction of ballastic and dead-beat galvanometer, differance between them. 2. Its operatinal physical parameter are 1. Charge sensitivity Sq???? d? ? dq ???? ?? T NAB C ???? ?? T S i where S i is the charge sensitivity

NSOU ? CC-PH-08 ? 293 2. The current sensitivity S i S 1 ???? d? d 1 ???? NAB C 3. Voltage sensitivity (S v) S V ???? d? dv ???? 1 Rg d? di ???? S i Rg As S i ? Rg therefore Sv ? 1 VR g ??? CDR is given by R C ???? G 2 2 IC ???? Rg 9.5 Review Questions and Answer 1. Plot a neat diagram of Ballastic galvanometer write the names of its various components. Ans : See the text 2. Give the theory of moving coil galvanometer. Explain the conditions under which the galvanometer works a) ballastic b) dead-beat. Ans : See the text 3. The 1st and 11th throw of a ballastic galvanometer are 5cm and 12cm respectively. Calculate the value of logarithmic decrement. So in ? ???? 1 10 ln ? 1 ? 11 4. Define charge sensitivity, current sensitivity and voltage sensitivity of ballastic galvanometer. Ans : See Article 9.3 5. what are difference between ballastic and Dead-beat Galvanometer?

NSOU ? CC-PH-08 ? 294 Ans : Ballastic Deat-beat 9.6 Problems and Solution Q. 1. A moving coil galvanometer has the following characteristics _ number of turns of the coil = 50; Area of coil = 70 mm 2 ; Resistance of coil = 30 ?? tlux density of redial field = ??I.T. Torsional constant of the suspension wire = 7 x 10 ? ?m/rad. Calculate the current and voltage sensitivity. Solution: Given ?N=50, A = 70mm 2 , B = O.I.T, C = 7 x 10 ? Nm/rad, R = 30 ? 1. Damping is small and the motion is oscillatory. 2. It measure charge 3. The transient flow of charge causes an impluse while the coil has not moved sufficently from its rest position. This condition is achieved by enhancing the movement of inertia of the coil to have larges time period of oscillation to about 10-20 seconds. The external driving torque is zero when the coil rotates 4. The coil frame is non-metalic to reduce electromagnetic damping. . 5. The ballastic throw measure the charge. 6. The external resistance of the glavanometer circuit must be greater than CDR to ensure oscillatory motion. 1. Damping is large and the motion is non-oscillatory. 2. It measure current 3. The coil rotates under the action of torque. 4. The coil is around on a metalic frame to increase electromagnetic damping. 5. The steady deflection measure the current. 6. The total external resistance must be less than CDR to obtain non- oscillatory motion.(aperiodic)

NSOU ? CC-PH-08 ? 295 Current sensitivity (Is) C NAB 50 x 70 x 10 x 0.1 x 10 -6 +8 ?? ?? 7 ??350 x 100 7 = 5 x 10 div / amp = 5div / mA. Voltage sensitivity (Vs) C NAB ?? R ?? Is R ?? 30 5 ?? 6 6 = 0.167 div/mV. Q.2. What is galvanometer constant? Solution : In a moving coil galvanometer the current (I) passing through the galvanometer is directly propertional to its deflection(?). i = G ? where G ?= C NAB = galvanometer constant.

NSOU ? CC-PH-08 ? 296 Further References 1. Electricity and Magnetism - Chattopadhayay & Rakshit New Central Book Agency, Kolkata 2. Electricity and Magnetism - David J. Griffiths Cambridge University Press UK 3. Electricity and Magnetism - Sadiku OUP, USA 4. Electricity and Magnetism - Hayt & Buck, Mc.Graw Hill

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is, i) directly proportional to the product of the charges, NSOU ? CC-PH-08 ? 9 ii) inversely proportional to the square of the distance of separation between the charges, iii) action along the line joining the

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r r Where ; ? ? ? ? ? ji j i r r r ? ? ? ? j i ji r r r and ? ? ˆ? ? ? ? ? ? ? j i j i ji r r r r r = unit vector along ? ? ? ? ? j i r r Similarly, the

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at a point is the force experienced by a unit positive charge placed at that point 1.

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The electric field intensity at P due to + q charge at 0 0 1 ˆ, 4 2	the electric field intensity at point P due to +q charge at

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	the electric field intensity E, at point P at a distance x from 0 the centre of	The electric field intensity at a point P at a distance r (r <) from the centre of

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7/107	SUBMITTED TEXT 92 WORDS	44% MATCHING TEXT 92 WORDS
	r r r r ? ? ?? ? ?? ? ? ? ? So, ? ? 0 3 0 0 1 4 q E r r r r ? ? ? ? ?? ? ?? ? ? ? ?? ? ? ? ? ? ?? ? ? ? ? So, ? ? ? ? ? ? 0 3 0 0 1 4 q E r r G R R G R R G r r ? ? ? ? ?? ? ?? ? ? ?? ? ?? ?? ? ? ?? ? ? ? ? ? ?? ? ? ?? ?? ? ? ?? , R R ? ?? where ? ? 0 R r r ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 0 0 0 i j k G R R G R R G R G R R	R eq = R 1 R 2 R 3 + R 1 R 4 + [?] +R 2 R 3 R 4 + R 5 R 1 R 3 + R 2 R 3 + [?] + R 2 R 4 R 1 R 2 + R 1 R 4 + [?] + R 3 R 4 + R 5 R 1 + R 2 + R 3 + R 4 . (4.48)

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8/107	SUBMITTED TEXT 24 WORDS	58% MATCHING TEXT 24 WORDS
	The electric potential at a point is the work done to bring a unit positive charge from infinity up to that point quasistatically. 1.4 The	

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	x y z r x y z ? ? ? ? ? ?? 5 2 5 2 5 2 2 2 2 2 2 2 2 2 2 2 3 2 3 3 2 ˆ ˆ ˆ 2 2 2 y x Z i j k x y z x y z x	x;y;z ∈ X: (x → (y → z) = y → (x → z); (I2) (x → y) → ((y → z) → (x → z)) = 1; (I3) x →

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10/107 SUBMITTED TEXT 171 WORDS	**35%** MATCHING TEXT 171 WORDS
Using the law of cosines, ? ? 2 "2 2 2 cos r r r rr ? ? ? ? ? ? ? ? ? ? 2 2 2 cos r r r r r ? ? ? ? ? ? ? ? ? ? ? ? where ??is the angle between r ? and r ? ? so we get, ? ? " 1 r r ? ? ? with ? ?? ? 2cos r r r r ? ? ? ? ? ? ? For points far from the charge distribution ??> 1, from binomial expansion. ? ? ? ? 1 2 2 3 1 1 1 1 3 5 1 1 .... " 2 8 16 r r r ? ?? ? ? ? ?? ? ? ? ? or interms of , and r r ? ? ? ?? ? ? ?? ? ? ?? ? 2 2 3 3 1 1 1 3 5 1 2cos 2cos 2cos ..... " 2 8 16 r r r r r r r r r r r r r r ? 2 3 2 3 1 3cos 1 5cos 3cos 1 cos 2 2 r r r r r r	Using the law of cosines, R 2 = r 2 + r′ 2 − 2rr ′ cosθ′ = r 2 [1 + (r′ r) 2 − 2(r′ r) cosθ′] (2.56) or R = r√1 + ϵ (2.57) Where ϵ = (r′ r)[r′ r − 2 For points well outside the charge distribution, ϵ is much less than 1, and this invites a binomial expansion of R = 1 r (1 + ϵ) −1 2 / = 1 r (1 − 1 2 ϵ + 3 8 ϵ 2 − 5 16 ϵ 3 + ⋯) 59 Or, in terms of r,r ′ , and θ′ 1 R = 1 r [1 − 1 2 (r′ r)(r′ r − 2cosθ′) + 3 8 (r′ r) 2 (r′ r − 2cosθ′) 2 − 5 16 (r) 3 (r′ r − 2cosθ′) 3 + ⋯] (2.58) 1 R = 1 r [1 − (r′ r)(cosθ′) + (r′ r) 2 (3

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r r r ? ? ? ? ? ? ? So ? ? 3 5 0 3 · 1 4 P r r P E r r ? ? ? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ?? ?? ? ? 5 3 0 3 · 1 4 P r r P r r ? ? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ?? ? ? ? ? 3 2 0 3 · 1 .... 1.4.5 4 P r r P r r ? ? ? ? ? ? ? ? ? ?? ? ? ? ?? ? ? ? ??	

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12/107 SUBMITTED TEXT 38 WORDS	**76%** MATCHING TEXT 38 WORDS
If the total charge is zero, the dominant term in the potential will be the dipole. ?? ? ? ? ? ? 2 0 1 cos .... 1.5.11 4 dip V r r r d r ? ? ? ? ? ?? ? ? ?? ? ? Since ??is the angle between r ? ? and r ?	If the total charge is zero, the dominant term in the potential will be the dipole V dipole (r) = 1 4πε 0 r 2 ∫r ′ ρ(r ′)dτ′cosθ′ v (2.63) Since θ′ is the angle between r′ and r

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13/107 SUBMITTED TEXT 64 WORDS	**37%** MATCHING TEXT 64 WORDS
r r P r d r ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ?? NSOU ? CC-PH-08 ? 25 More explicitly, ? ? ? ? ? ? ? ? ? 2 2 3 0 1 1 1 1 cos 4 V r r d r r d r r r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? 2 3 1 cos .... 2 2 r d ? ? ? ?? ? ? ? ? ? ....(1.5.5) The	

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14/107 **SUBMITTED TEXT** 54 WORDS	**38%** **MATCHING TEXT** 54 WORDS
r V r V r V r ? ? ? ? ? ? ? ? ? where, ?? ? ? 0 1 ' ..... 1.5.7 4 V r d r ? ? ? ? ?? ? ? ?? ? ? ? ? 1 2 0 1 cos .... 1.5.8 4 V r r r d r ? ? ? ? ?? ? ?? ? ? ?? ? ? ? ? ? ? 2 2 2 3 0 1 1 3cos 1 ' .... 1.5.9 2 4 V r r r d r ? ? ? ?? ? ? ? ?? ? ?	

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sphere of radius R and charge q, uniformly distributed over its surface.	sphere of radius R with a charge Q uniformly distributed over its surface

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16/107 **SUBMITTED TEXT** 16 WORDS	**62%** **MATCHING TEXT** 16 WORDS
at point P at a distance r form the centre o we consider a Gaussian surface	

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17/107 **SUBMITTED TEXT** 16 WORDS	**76%** **MATCHING TEXT** 16 WORDS
Field due to a uniformly charged sphere : (a) At a point outside the surface : The	

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18/107 **SUBMITTED TEXT** 53 WORDS	**63%** **MATCHING TEXT** 53 WORDS
r a r ? ? ?? 2 4 2 2 0 0 3 2 0 0 0 4 4 2 2 4 4 a a Qr Q U r dr r dr a r ?? ?? ? ? ? ? ? ? ? ? ? 2 2 0 0 3 1 1 1 1 · · 2 4 5 4 5 Q Q a a a ? ? ? ?? ??	r < a) – ∫ E (r >a) = – ∫ a Q 4πε 0 r r 2 – ∫ dr ' a r2 ⌉ 1 Q 1 Q 1 2 1 Q⌈ 2 r –a = = – ⌊3– 2 ⌊ a ⌋ 4πε 0 a 4

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19/107 **SUBMITTED TEXT** 14 WORDS	**80%** **MATCHING TEXT** 14 WORDS
The electric field at any point of the surface is perpendicular to the	The electric field at any point on the flat parts of the Gaussian surface is perpendicular to the

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r r ? ? ? ? ? ? ?? ?? ? ? ? ? where ij j i r r r ? ? ? ? ? ? and ˆ j i ij j i r r r r r ? ? ? ? ? ? ? ? 0 0 1 1 4 4 r ij i j

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E d A A ? ? ? ? ? ? ? ? ? Thus the capacitance of a parallel plate capacitor

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22/107 **SUBMITTED TEXT** 18 WORDS **50%** **MATCHING TEXT** 18 WORDS

the work done. qdq dW vdq C ? ? So the total work done in charging the capacitor to the charge Q is + − + − + − + − + −

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23/107 **SUBMITTED TEXT** 32 WORDS **57%** **MATCHING TEXT** 32 WORDS

The total charge 1 2 .... i N q q q q q ? ? ? ? ? ? 1 2 3 ...... N C V C V C V C V ? ? ? ? ? If C

the total capacitance really easy! Q tot = Q 1 +Q 2 +Q 3 +... C tot V = C 1 V +C 2 V +C 3 V +... C

W http://docshare.tips/intro-physics-2a4_58c369f7b6d87fe4528b594d.html

24/107 **SUBMITTED TEXT** 30 WORDS **71%** **MATCHING TEXT** 30 WORDS

C V C V C V C V C V ? ? ? ? ? ? Or, 1 2 3 ...... eq N C C C C C ? ? ? ? ? (c)

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25/107 **SUBMITTED TEXT** 44 WORDS **55%** **MATCHING TEXT** 44 WORDS

C ? C ? C ? Fig. 1.25 V NSOU ? CC-PH-08 ? 43 1 1 2 2 1 1 1 1 2 C V C V q C V C C C ? ? ? ?? ? ? ? ? ? ? and 1 1 2 2 2 2 2 1 2 C V C V q C V C C C ? ? ? ?? ? ? ? ? ? ?

SA PHY3512 Physics for Physical Science II 376pg (1).pdf (D144373244)

26/107 SUBMITTED TEXT 11 WORDS	**100%** MATCHING TEXT 11 WORDS
a spherical Capacitor of inner radius a and outer radius	a spherical capacitor of inner radius a and outer radius

W https://pdfhall.com/electric-charge-exvacuo_5b36bbe9097c476d3d8b456d.html

27/107 SUBMITTED TEXT 136 WORDS	**26%** MATCHING TEXT 136 WORDS
a y z x a y z ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? iv) The field intensity at point P (x, y, z) along x, y and z axis are ? ? ? ? ? ? ? ? 3 2 3 2 2 2 0 2 2 2 2 4 x q x a x a E x x a y z x a y z ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 3 2 3 2 2 2 2 2 2 2 y y y E y x a y z x a y z ? ? ? ? ?? 3 2 3 2 2 2 2 2 2 2 z z z E y x a y z x a y z ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?	A y ∂z)î+ (∂A x ∂z – ∂A z ∂x)ĵ+ (∂A y ∂x – ∂x ∂y)x î+ ∂ ∂y ĵ+ ∂ ∂z kˆ) · [(∂A z ∂ y – ∂A y ∂î+ (∂A x ∂A z ∂x)ĵ+ (∂A y ∂ x – ∂A x ∂y)kˆ] 21 = [∂ ∂x (∂A z ∂y – ∂A y ∂y (∂A z – ∂A z ∂x) + ∂ ∂z (∂A y ∂x – ∂A x ∂y)] = ∂ ∂x (∂A z ∂y) – ∂ ∂A y ∂z) + ∂ ∂

W http://www.gdcpattan.com/wp-content/uploads/2020/04/PhysicsSemester-II.pdf

28/107 SUBMITTED TEXT 19 WORDS	**55%** MATCHING TEXT 19 WORDS
are placed at the corners of an equilateral traingle ABC ? of length ‘a’. Find the magnitude of the	are located at the corners of an equilateral triangle of side 0.2m. Exercises 51 (a) Find the magnitude in newtons of the

W https://edisciplinas.usp.br/pluginfile.php/5086475/mod_folder/content/0/Extras/Electricity%20and% ...

29/107 SUBMITTED TEXT 37 WORDS	**62%** MATCHING TEXT 37 WORDS
C V C V E E E CV C V C C C C ? 2 1 2 1 2 1 2 … … … 1.10.5 2 C C V V C C ? ? ?	

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30/107 SUBMITTED TEXT 37 WORDS	**72%** MATCHING TEXT 37 WORDS
C V C V ? ? ? ? ? ? stored ? ? ? ? 2 2 1 1 2 2 1 2 1 2 1 2 1 1 2 2 f C V C V E C C V C C C C ? ? ? ? ? ? ? ? ? ? ? ?	

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31/107	**SUBMITTED TEXT**	31 WORDS	**43%**	**MATCHING TEXT**	31 WORDS

c V V c a ? ? ? on , r c ? 1 2 2 c c V V c c ? ? ? ? ? ? on 1 2 , ;
r b V V ? ? i.e. 1 1 2 2 c c c c

SA PHY3512 Physics for Physical Science II 376pg (1).pdf (D144373244)

32/107	**SUBMITTED TEXT**	14 WORDS	**82%**	**MATCHING TEXT**	14 WORDS

a dielectric is an insulator that can be polarised by an applied. electric field.

SA PHY18R171_PPT_FULL.pdf (D109295126)

33/107	**SUBMITTED TEXT**	28 WORDS	**61%**	**MATCHING TEXT**	28 WORDS

Ionic Polarisation The ionic polarzation occurs, when atoms formmolecules and is mainly due to a relative displacement of the atomic components of The molecules due to the influenece of electric field. (

SA PHY18R171_LM.pdf (D109119436)

34/107	**SUBMITTED TEXT**	35 WORDS	**42%**	**MATCHING TEXT**	35 WORDS

Potential due to a single dipole P ?? is given by ? ? ? ? 2 0
ˆ 1 · " " ...... 2.4.4 4 " Pr V r r ? ?? ?? ??? Where r?? ? is the vector from the dipole to the point at which we are finding the potential

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35/107	**SUBMITTED TEXT**	31 WORDS	**35%**	**MATCHING TEXT**	31 WORDS

surface charge ? ? ˆ· ... ... 2.4.8 b P n ? ? ?? Where ˆn is the unit normal vector to the surface. The second term of the intgrand will give the potential of a volume charge P r

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36/107	SUBMITTED TEXT	14 WORDS	70%	MATCHING TEXT	14 WORDS

the tangetial component of The electric field is continuons across The interface between two media.

SA 019E1230-Electromagnetic Theory.pdf (D165097241)

37/107	SUBMITTED TEXT	13 WORDS	100%	MATCHING TEXT	13 WORDS

ze′ of the atom have opposite charges and acted upon by Lorentz force.

SA PHY18R171_PPT_FULL.pdf (D109295126)

38/107	SUBMITTED TEXT	30 WORDS	62%	MATCHING TEXT	30 WORDS

opposite direction. As electron cloud and nucleus gets displaced from their normal equillibrium positions, an allractive force between them is built and the separation continues until coulomb force F C is balanced

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39/107	SUBMITTED TEXT	18 WORDS	71%	MATCHING TEXT	18 WORDS

Lorentz force F 2 , until a new equillibrium state is ereated. Let ? be the charge density of the sphere. 3 4 3 ze

SA PHY18R171_LM.pdf (D109119436)

40/107	SUBMITTED TEXT	17 WORDS	71%	MATCHING TEXT	17 WORDS

the total charges in the sphere. So the negative charge in the sphere of radius x, 3 4 3

SA PHY18R171_LM.pdf (D109119436)

41/107	SUBMITTED TEXT	16 WORDS	75%	MATCHING TEXT	16 WORDS

e R ? ? ?? Hence electronic polarisibility is directly proportinal to the radius of the atom.

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42/107 **SUBMITTED TEXT** 28 WORDS	**77%** **MATCHING TEXT** 28 WORDS
even in the absence of electric field. However, The net dipole moment is negligiby small since all the dipoles under continous thermal agitation, are oriented randomly when there is no	

SA PHY18R171_PPT_FULL.pdf (D109295126)

43/107 **SUBMITTED TEXT** 17 WORDS	**58%** **MATCHING TEXT** 17 WORDS
the potential at the centre of the sphere. (b) Find the potential at the surface of the sphere.	the sphere. At the centre of the sphere r = 0 E = 0 At the surface of the sphere

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44/107 **SUBMITTED TEXT** 41 WORDS	**55%** **MATCHING TEXT** 41 WORDS
The dielectric constants of a Helium gas at NTP is 1·0000685. Calculate the electric NSOU ? CC-PH-08 ? 100 polarizability of Helium atoms if the gas contains 2·7 × 10 26 atoms/m 3 . Calculate the radius of the Helium atom. [Given 12 1 0 8·854 10	

SA PHY18R171_LM.pdf (D109119436)

45/107 **SUBMITTED TEXT** 41 WORDS	**47%** **MATCHING TEXT** 41 WORDS
c r ? ? ? ? ? ? ? As , r? ? V = 0 and C 2 = 0 At r = R, ?? ?? V r V R ? ? ?? 2 2 0 0 1 0 0 6 3 r R R c ? ? ? ? ? ? ? ? ? ? ? 2 0 1 0 2 1 6 r r R c ? ? ? ? ? ? ? (	

SA MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

46/107 **SUBMITTED TEXT** 13 WORDS	**95%** **MATCHING TEXT** 13 WORDS
a. If a charge q is placed at the centre of the	

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47/107 **SUBMITTED TEXT** 11 WORDS **100%** **MATCHING TEXT** 11 WORDS

force on a current carrying conductor in a uniform magnetic field.

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48/107 **SUBMITTED TEXT** 12 WORDS **84%** **MATCHING TEXT** 12 WORDS

Biot-Savart Law. 1) Magnetic field intensity due to a straight current carrying conductor.

SA Test submission.pdf (D119125515)

49/107 **SUBMITTED TEXT** 15 WORDS **100%** **MATCHING TEXT** 15 WORDS

the axis of the coil at a distance x from the centre O of the

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50/107 **SUBMITTED TEXT** 15 WORDS **82%** **MATCHING TEXT** 15 WORDS

the magentic field intensity at a point P at a distance r from centre

The electric field intensity at a point P at a distance r (r <) from centre

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51/107 **SUBMITTED TEXT** 21 WORDS **77%** **MATCHING TEXT** 21 WORDS

Ampere’s law states that the line integral of magnetic field vector about a closed path is equal to ? times the current

SA EC 6403 EMF even 15.12.16 MAIN COPY-1.docx (D129223756)

52/107 **SUBMITTED TEXT** 51 WORDS **46%** **MATCHING TEXT** 51 WORDS

R R R ? ??? ? ?? ?? ?? ?? From (1) and (2) 0 3 4 J R B d R ? ? ? ? ? ? ?? ?? ?? Now 0 3 . . 4 J R B d R ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? ?? ?? ? ? 0 3 3 0 4 R R J r J d R R ? ? ? ? ? ? ? ? ?? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ?? ?? ?? ?? ?? (so 3 0, R R ?? ? ?? ??

SA MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

53/107 **SUBMITTED TEXT** 15 WORDS	**78%** **MATCHING TEXT** 15 WORDS
r) 4 r ? ? ? ? ? ? ....(4.3.13) where r ? is a radius vector from the loop to the point	r r 2 (11.10) where ˆr is a unit vector in the direction from the loop to the point

W https://edisciplinas.usp.br/pluginfile.php/5086475/mod_folder/content/0/Extras/Electricity%20and% ...

54/107 **SUBMITTED TEXT** 15 WORDS	**84%** **MATCHING TEXT** 15 WORDS
A steady current I flows down a long cylindrical conductor of radius a. The	

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55/107 **SUBMITTED TEXT** 11 WORDS	**100%** **MATCHING TEXT** 11 WORDS
while the second term looks like the potential of a	

SA MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

56/107 **SUBMITTED TEXT** 16 WORDS	**68%** **MATCHING TEXT** 16 WORDS
Unit 5 Electromagnetic Induction Structure 5.1 Objectives 5.2 Introduction 5.3 Faraday's law of Electromagnetic Induction 5.4 Self-inductance 5.5 Mutual Inductance 5.6	

SA E & M full.pdf (D165275368)

57/107 **SUBMITTED TEXT** 10 WORDS	**100%** **MATCHING TEXT** 10 WORDS
whenever the magnetic flux linked with a closed circuit changes	Whenever , the magnetic flux linked with a closed circuit changes,

W http://www.hillagric.ac.in/edu/cobs/studymaterial/Physics%20121.pdf

58/107 **SUBMITTED TEXT** 20 WORDS	**81%** **MATCHING TEXT** 20 WORDS
Induced emf in a circuit is proportional to the rate of change of magnetic flux linked with the circuit. b) The	

SA PHY18R171_LM.pdf (D109119436)

59/107 **SUBMITTED TEXT** 19 WORDS	**71%** **MATCHING TEXT** 19 WORDS
Faraday’s law. 5.4 Self-inductance The induced emf ? in a coil is proportional to the rate of change of magnetic flux	Faraday’s law of induction states the induced emf ε in a coil is proportional to the negative of the rate of change magnetic flux:

W https://pdfhall.com/electric-charge-exvacuo_5b36bbe9097c476d3d8b456d.html

60/107 **SUBMITTED TEXT** 18 WORDS	**65%** **MATCHING TEXT** 18 WORDS
If ?is the flux linked with a circuit at any instant t, then d ? is the	

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61/107 **SUBMITTED TEXT** 17 WORDS	**58%** **MATCHING TEXT** 17 WORDS
circuit is fixed, the time derivation can be taken outside the integral, when it becomes partial derivative.	

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62/107 **SUBMITTED TEXT** 16 WORDS	**78%** **MATCHING TEXT** 16 WORDS
cylinder of inner radius a and outer radius b as shown in Fig. 5.3. The	cylinder of length L and inner radius a and outer radius b , as shown in Figure 6.5.3. The

W https://pdfhall.com/electric-charge-exvacuo_5b36bbe9097c476d3d8b456d.html

63/107 **SUBMITTED TEXT** 13 WORDS	**76%** **MATCHING TEXT** 13 WORDS
is proportional to the rate of change of the current through the coil	

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64/107 **SUBMITTED TEXT** 17 WORDS	**90%** **MATCHING TEXT** 17 WORDS
If I is the current in C 1 , magnetic induction at O 2 is given by 2 0 1 3 2 2 2 2 N Ia	If I is the current in C 1 , the magnetic induction at O 2 is given by B = μ 0 N 1 Ia 2 2(

W http://www.gdcpattan.com/wp-content/uploads/2020/04/PhysicsSemester-II.pdf

65/107 **SUBMITTED TEXT** 26 WORDS **48%** **MATCHING TEXT** 26 WORDS

The induced emf in coil 1 due to self-inductance when current I flows through it, 11 1 dI L dt ? ? while the emf induced in coil 2 due to

The induced emf in coil 2 due to its self inductance is: e 22 = −L 2 di dt (4.11) The mutual induced emf in coil 1 due to

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66/107 **SUBMITTED TEXT** 23 WORDS **59%** **MATCHING TEXT** 23 WORDS

current I in coil 1 is 21 dI M dt ? ?? where M is the mutual inductance of the two coils. The emf induced in

current in coil 2 is: e 12 = −M di dt (4.10) 113 Where M is mutual inductance between the coils. The induced emf in

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67/107 **SUBMITTED TEXT** 11 WORDS **95%** **MATCHING TEXT** 11 WORDS

two coils of self-inductances L 1 and L 2 connected in parallel.

two coils of self inductances L 1 and L 2 be connected in parallel

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68/107 **SUBMITTED TEXT** 27 WORDS **44%** **MATCHING TEXT** 27 WORDS

mutual flux aids the self-flux, the total emf induced in coil 1 is 1 1 dI dI L M dt dt ? ?? ? Similarly 2 1 2 2 dI dI L M dt dt ? ?? ? Since the

mutual flux aids the self flux of two coils. Then the total emf induced in the coil 1 and coil 2 is given by L 1 di 1 dt – M di 2 dt (4.17) e 2 = −L 2 di 2 dt – M di 1 dt (4.18) Since the

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69/107 **SUBMITTED TEXT** 47 WORDS **44%** **MATCHING TEXT** 47 WORDS

L L M ?? ? ? ? and 2 1 2 1 2 dI (L M) dt L L M ?? ? ? ? Therefore, 1 2 1 2 2 1 2 dI dI (L L 2M) dI dt dt dt L L M ?? ? ? ? ? ? ? If L eq be the equivalent self-inductance, then eq dI L dt ? ?? ? 2 1 2 eq 1 2 L L M L L L 2M ? ? ? ? ......(5.8.7) If

L 1 L 2 – M 2 and di 2 dt = – e(L 1 – M) L 1 L 2 – M 2 di dt = – e(L 1 + L 2 – 2M) L 1 L 2 – M 2 (4.21) 115 If L is the equivalent inductance of the system, then e = −L di dt (4.22) Comparing eq (4.21) and (4.22) L = L 1 L 2 – M 2 L 1 + L 2 – 2M (4.23) If

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70/107 **SUBMITTED TEXT** 12 WORDS **90%** **MATCHING TEXT** 12 WORDS

unit of self inductance is henry (H). One henry is the	

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71/107 **SUBMITTED TEXT** 11 WORDS **95%** **MATCHING TEXT** 11 WORDS

the two coils of self-inductances L 1 and L 2 , connected in parallel	the two coils of self inductances L 1 and L 2 be connected in parallel

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is the surface bounding the volume V. Note that integration is to be carried out over the entire volume occupied by the current	

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73/107 **SUBMITTED TEXT** 17 WORDS **72%** **MATCHING TEXT** 17 WORDS

The highest point of a wave is known as 'Crest', whereas the longest point is known as 'Trough'.	

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74/107 **SUBMITTED TEXT** 7 WORDS **78%** **MATCHING TEXT** 7 WORDS

t ?t 1 ?o ?t ?t 1 ?o ?t ?t ?t ?t ?t ?t ?t ?t	

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75/107 **SUBMITTED TEXT** 42 WORDS **40%** **MATCHING TEXT** 42 WORDS

x B - ? ? ? 1 ? o ? ? E t ? x B ??? ? ? ? .... (6.6.6) Now, (E x B) = ? ? ? ? t ? ? E t ? x B + E x ? ? B t ? ? ? ....(6.6.7) or, ? ? E t ? x B = ? ? ? t (E x B) - E x ? ? ? ?	

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76/107	**SUBMITTED TEXT**	11 WORDS	**100%**	**MATCHING TEXT**	11 WORDS

the tangential component of the electric field is continuous across the

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77/107	**SUBMITTED TEXT**	14 WORDS	**76%**	**MATCHING TEXT**	14 WORDS

D 1n A + D 2n A = ? s A or D 2n - D 1n = ?

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78/107	**SUBMITTED TEXT**	18 WORDS	**55%**	**MATCHING TEXT**	18 WORDS

unlike mechanical waves which requires the presence of material media to transport energy from one location to another

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79/107	**SUBMITTED TEXT**	13 WORDS	**88%**	**MATCHING TEXT**	13 WORDS

are mutually perpendicular and also both are perpendicular to the direction of propagation

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80/107	**SUBMITTED TEXT**	20 WORDS	**66%**	**MATCHING TEXT**	20 WORDS

E (r, t) = E e 0 J(k. r - t) ? ? ? ? ? ? H (r, t) = H e 0 J(k. r - t) ? ? ? ? ? ? ....(6.11.8)

SA 07170532.pdf (D104224069)

81/107	**SUBMITTED TEXT**	88 WORDS	**62%**	**MATCHING TEXT**	88 WORDS
n n 2 1 - n n 2 1 + 2 ....(6.12.9) and T = n = n 2 t 2 4n n 1 2 (n n 2 1 +) 2 n E 2 2n 2 n E 1 1n 2 n 1 = ...(6.12.10)				n , 15 n , 17 n , 19 n , ...] {\displaystyle \tanh(1/n)=[0;n,3 n,5n,7n,9n,11n,13 n,15n,17 n,19n,\	

W https://en.wikipedia.org/wiki/Continued_fraction

82/107 **SUBMITTED TEXT** 19 WORDS **52%** **MATCHING TEXT** 19 WORDS

at the interface. 6.13 Reflection and refractron at oblique incidence at the Interface Between two Dietectrics The reflection and refraction of

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83/107 **SUBMITTED TEXT** 24 WORDS **50%** **MATCHING TEXT** 24 WORDS

at a plane surface between two media of different dielectric properties are well known phenomena. The different aspects of the phenomena divide themselves into two

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84/107 **SUBMITTED TEXT** 132 WORDS **21%** **MATCHING TEXT** 132 WORDS

R (r. t) j(k .r-wt) R ? ? ? ? ?R e ? ?? ? R (r. t) (? ? ? n 1 ? ? c k´ 1 ˆ x[? ?R e j(k .r-wt) R ?? } ....(6.13.1) } ....(6.13.2) NSOU ? CC-PH-08 ? 223 Transmitted wave : ? T (r. t) j(k .r-wt) T ? ? ? ? ?T e ? ?? ? T (r. t) (? ? ? n 2 ? ? c k´ 2 ˆ x[? T0 e

SA MPDSC 2.2- Electrodynamics SLM.pdf (D160853593)

85/107 **SUBMITTED TEXT** 17 WORDS **70%** **MATCHING TEXT** 17 WORDS

let ? I , ? R and ? T be the angles between the normal to the interface and the

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86/107 **SUBMITTED TEXT** 21 WORDS **83%** **MATCHING TEXT** 21 WORDS

x (K I) x + y (K I) y = x (K R) x +y (K R)y = x (K T) x + y (K T) y ....(6.13.6) for all x and all y

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87/107 **SUBMITTED TEXT** 15 WORDS **83%** **MATCHING TEXT** 15 WORDS

our axes so that ? ? ? lies in the xz plane [ie.(K I) y =0];

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88/107 **SUBMITTED TEXT** 28 WORDS **52%** **MATCHING TEXT** 28 WORDS

R ? and ? T ? also lies in the same plane. Thus we conclude that First law : The incident, reflected and transmitted wave vectors form a plane

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89/107 **SUBMITTED TEXT** 15 WORDS **89%** **MATCHING TEXT** 15 WORDS

The angle of incidence is equal to the angle of reflection
? I = ? R
……………………………………………………………………………………
(6.13.10) which is

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90/107 **SUBMITTED TEXT** 13 WORDS **100%** **MATCHING TEXT** 13 WORDS

that the tangential component of the electric field is continuous across the interface (

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91/107 **SUBMITTED TEXT** 13 WORDS **76%** **MATCHING TEXT** 13 WORDS

that the tangential component of the magnetic intensity is continuous across the interface (

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92/107 **SUBMITTED TEXT** 78 WORDS **31%** **MATCHING TEXT** 78 WORDS

E 0I e j ?? ? ? r) ? ? + E 0R e j ?? ? R r) ? ? = E 0T e j ?? ? T r) ? ? ? E 0I + E 0R = E 0T ....(6.13.19) and n E cos e 1 0I I ? j ?? ? I r) ? ? ? ? n E cos e 1 0R R j ?? ? R r) ? ? ? ? n E cos e 2 0

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93/107 **SUBMITTED TEXT** 33 WORDS **80%** **MATCHING TEXT** 33 WORDS

the maximum polarization (vibration in one plane only) of a ray of light may be acheived by letting the ray incident on a surface of transpent medium in such away that the refracted ray makes

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94/107 **SUBMITTED TEXT** 14 WORDS **78%** **MATCHING TEXT** 14 WORDS

The tangential componet of the electric field (? ?) is continuous across the interface.When the

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where A is the area of plate d is the distance between them,

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96/107 **SUBMITTED TEXT** 41 WORDS **88%** **MATCHING TEXT** 41 WORDS

R (r R) E R i (r R R) (r R) ER i , R R r (r R) R (r R R) R (r R) R (r R R) ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

R 1 R 2 R 3 + R 1 R 4 + [?] +R 2 R 3 R 4 + R 5 R 1 R 3 + R 2 R 3 + [?] +R 2 R 4 R 1 R 2 + R 1 R 4 + [?] +R 3 R 4 + R 5 R 1 + R 2 +

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97/107 SUBMITTED TEXT 22 WORDS	**50%** MATCHING TEXT 22 WORDS
A C B D 100 ? 10 ? 50 ? 4 ? A C B D 10 V 10 V 100 ? 10 ? 50 ? 4 ? A C B D 10 V a b c d 10 ? 4 ? 100 ? 50 ? C A B	

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R 3 R 1 R 2 i 1 r i2 E 2 r i1 E 1 R 3 R 1 R 2 i 1 r i2 r i1 Fig. 10(c) R 3 R 1 R 2 i 1 r i2 E 2 I NSOU ? CC-PH-08 ? 246 Then from Fig. 7.5(b) 3 i2 1 2 2 3 i2 3 i2 2 1 0 (R r)(i i) i R (R r)i (R r	R 2 R 3 + R 1 R 3 , I 2 = E 2 R 1 + E 2 R 3 + E 1 R 3 R 1 R 2 + R 2 R 3 + R 1 I 3 = E 1 R 2 – E 2 R 1 R 2 + R 2 R 3 + R 1 R 3 . (4.33)

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r R E 1 0 (r R 1 i ? ? ? ? ? ? ? Where il 1 2 i2 3 i2 2 3 r R R (r R r R R ? ?? ? ? ? ? Similarly from Fig. 7.10(c) i1 1 1 i2 3 2 r R) 0 1 i (r R) E ? ?? ? ? ? ? So i1 1 1 1 2 i2 3 2 r R) E 1 i i (r R)	R 2 + E 1 R 3 + E 2 R 3 R 1 R 2 + R 2 R 3 + R 1 R 3 , I 2 = E 2 R 1 + E 2 R 3 + E 1 R 3 R 1 R 2 + R 2 R 3 + R 1 R 3 , I 3 = E 1 R 2 – E 2 R 1 R 1

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100/107 SUBMITTED TEXT 23 WORDS	**95%** MATCHING TEXT 23 WORDS
R 1 + r i1)i + i 1 R 2 E 2 = (r i2 + R 3)(i 1 – i) + i 1 R 2 = –(r i2 + R 3)i + (r i1 + R 3 + R 2)	R 3 + R 1 R 3 , I 3 = E 1 R 2 – E 2 R 1 R 1 R 2 + R 2 R 3 + R 1 R 3 . (4.33)

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101/107 SUBMITTED TEXT 65 WORDS	**52%** MATCHING TEXT 65 WORDS
r R) R || R R R (r R)(R R) R R ? ? ? ? ? ? ? ? ? Now from Fig. 7.16(b) we see that 2 2 L 2 i 1 2 1 i i 1 2 L 2 L R ER E I R R ||(r R) R (R r) (r R)(R R) R R ? ? ? ? ? ? ? ? ? ? ?	R 1 R 2 R 3 + R 1 R 2 R 4 + [?] +R 2 R 3 R 4 + R 5 R 1 R 3 + R 2 R 3 + [?] +R 2 R 4 R 1 R 1 R 4 + [?] +R 3 R 4 + R 5 R 2 + R 3 +

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R 1 2 2 1 Th 1 2 1 2 2 1 R Th 1 2 1 2 1 2 1 2 e r e r V r r e r e r i R R r r r r r R r R R r r ? ? ? ? ? ? ? ? ? ? ? ? ii) i R	R 2 + E 1 R 3 + E 2 R 1 R 2 + R 2 R 3 + R 3 , I 2 = E 2 R 1 + E 2 R 3 + E 1 R 3 R 1 R 2 + R 2 R 3 + R 1 R 3 , I 3 = R 2 − E 2 R 1 R 2 R 3 + R 1

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r e r e r R i r r r r e r e r i R R r r r r r R r R R r r ? ? ? ? ? ? ? ? ? ? ? iii) i R	R 2 + E 1 R 3 + E 2 R 3 R 1 R 2 + R 2 R 3 + R 1 R 3 , I 2 = E 2 R 1 + E 2 R 3 + E 1 R 3 R 1 R 2 + R 2 R 3 + R 1 R 3 , I 3 = R 2 − E 2 R 1 R 2 R 3 + R 1

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104/107 **SUBMITTED TEXT** 11 WORDS	**38%** **MATCHING TEXT** 11 WORDS
r r R r r ? ? Fig. 7.20 So current through the resistance R R Th r 1 r 2 Fig 7.19 a b r 1 e 1 r 2 e 2 a Fig 7.18 b	

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105/107 **SUBMITTED TEXT** 69 WORDS	**44%** **MATCHING TEXT** 69 WORDS
R r R r r r R r R r r R ? ? ? ? ? ? ? Fig. 7.21 Step-2 Short circuit e 2 and evaluate i 2 1 2 1 2 2 2 2 1 2 1 2 1 2 e r e r i r R r R r r r R r R r r R ? ? ? ? ? ? ? The total current through R, i R = i 1 +i 2 = 1 2 2 1 1 2 1 2 e r e r r r r R r R ? ? ? 7.8	

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a and b. Fig. 7.28 6V a b 3? 3? 3? R N B A 3? 3? 5A 2? 2A 10? R N 5A a b 2? 2A 5? 2? 5A 2? 2A 10? R N 5A 2? 2A 10? i N 10V a b 10	

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107/107	SUBMITTED TEXT	15 WORDS	80%	MATCHING TEXT	15 WORDS
	the magnetic flux linked with the coil changes and an induced emf is set up				

SA 167E2440-Electricity & Magnetism.docx (D165105106)

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PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, generic elective, discipline Specific, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assesment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English/Bengali. Eventually,
the English version SLMs
will be translated into Bengali too, for the benefit of learners. As always,
all of our teaching faculties contributed in this process. In addition to this
we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success.
Professor (
Dr.) Subha Sankar Sarkar
Vice-Chancellor
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