Netaji Subhas Open University
Master's Degree Programme (PGMT) in Mathematics

Syllabus

Programme Objectives: The objective of the programme is to help the learners to achieve the deep concept of higher Mathematics. The M.Sc. degree of Mathematics in NSOU will not only help the student to concrete the abstract concept of Mathematics but also make them more aware of the current research of Mathematics. There are two specialization of M.Sc. Mathematics programme; Pure Mathematics and Applied Mathematics. Pure Mathematics is the theoretical part of Mathematics and Applied Mathematics is the application part of Mathematics. Both these specializations will enrich a student with deep mathematical flavor.

Expected Programme Outcome: After successful completion the students may increase their knowledge with the new tools and techniques of Mathematics. A student after completing this course may go either for various government jobs such as school service or research programme in various institutes and universities in India or abroad.

Course Structure

(Full Marks-1000)

Part-I

Paper I :	**Algebra**	
	Group A : Abstract Algebra	(Marks: 50)
	Group B : Linear Algebra	(Marks: 50)
Paper II :	**Analysis**	
	Group A : Real Analysis & Metric spaces	(Marks: 50)
	Group B : Complex Analysis	(Marks: 50)
Paper III :	**Differential Equations**	
	Group A : Ordinary Differential Equations	(Marks: 50)
	Group B : Partial Differential Equations & Special Functions	(Marks: 50)
Paper IV :	**Numerical Analysis & Computer Programming**	
	Group A : Numerical Analysis	(Marks: 50)
	Group B : Computer Programming & its applications to problems of Numerical Analysis	(Marks: 50)
Paper V :	**Mechanics**	
	Group A : Principles of Mechanics	(Marks: 50)
	Group B : Elements of Continuum Mechanics and Special Theory of Relativity	(Marks: 50)

Part-II

Paper VI :	**Topology & Functional Analysis**	
	Group A : General Topology	(Marks: 50)
	Group B : Functional Analysis	(Marks: 50)
Paper VII :	**Integral Transformations & Equations**	
	Group A : Differential Equations, Integral Transformations	(Marks: 50)
	Group B : Integral Equations	(Marks: 50)

Paper VIII : Differential Geometry & Graph Theory

Group A : Differential Geometry (Marks: 50)

Group B : Graph Theory (Marks: 50)

Paper IX & X: Special Paper –Any one of (i) Pure and (ii) Applied Mathematics)

(i) Pure Mathematics

Paper IXA:(i) Advanced Complex analysis (Marks: 50)

Paper IXB: (i) Advanced Topology (Marks: 50)

Paper XA : (i) Advanced Differential Geometry (Marks:50)

Paper XB: (i) Advanced Functional Analysis (Marks: 50)

(ii) Applied Mathematics

Paper IXA: (ii) Operations Research (Marks: 50)

Paper IXB: (ii) Mathematical Models in Ecology (Marks: 50)

Paper XA:(ii) Fluid Mechanics (Marks: 50)

Paper XB:(ii) Mechanics of solids (Marks: 50)

Detailed Syllabus

Paper -1

Group - A : Abstract Algebra (50 Marks)

General Concept : Classical Algebraic System, Algebraic structures , Morphisms.
Group: Morphisms of groups, normal sub-groups and quotient Groups, fundamental homomorphism theorem, isomorphism theorem s of groups, Conjugacy. Permutation groups.
Ring : Ideals of a Ring, Quotient Ring, Prime and Maxi mal ideals in a commutative ring with unity. Isomorphism of rings. Characteristic of a ring.
Fields : Integral domain and Quotient field s. Prime fields, Euclidean domain. Polynomial rings, Principal ideal domain. Extension of fields. Finite fields. Root fields of Polynomials, Splitting fields.
References :

1. I. N. Herstein - Topics in Algebra, Vikas Pub.
2. G. Birkhoff and S. Maclane - A survey of Modern Algebra.
3. Serge Lang - Algebra, Addition - Westey pub.
4. G. Birkhoffa nd T. C. Bartee-Modern Applied Algebra, Mc-Graw Hill.
5. D. S. Malik, J. N. Mordeson, M. K. Sen - Fundamentals of Abstract Algebra, Mc- Graw Hill.
6. P. B. Bhattacharya, S. K. Jain, S. R. Noyapai - Basic Abstract Algebra Cambridge.

Group — B: Linear Algebra (50 Marks)

Vector space: Normal and Unitary vector spaces. Euclidean vector space. Orthonormal basis. Isomorphism and inversion of Linear transformations, Dimension of a vector.
Inner Product Spaces: Inner product function, Norm of a vector, Pythagoras theorem, Gram-Schmidt Orthogonalization.
Linear transformations: Linear map and linear functional. Matrix representation of Linear transformations. Similar and Congruent matrices. Reduction of a matrix to Normal form. Jordan Canonical form. Characteristic polynomial. Minimal polynomial. Calley - Hamilton Theorem. Diagonalization of real symmetric matrices. Kernel and Image. Space spanned by Eigen vectors.
Reductions of matrices: Characterization of real quadratic form. Rank and Nullity. Invariant subspace. Equivalent Quadratic forms. Reduction to Canonical forms-Reduction of matrices to Diagonal or Normal Form
Quadratic Forms: Sylvester law. Simultaneous reduction of two quadratic forms and Classification of quadrics.
References :

1\. T . M. Apostol - Linear Algebra. John Wiley and Sons.
2\. K. Hoffman & R. Kunze - Linear Algebra, Prentice-Hall.
2\. V. A. Iyin & Poznyak - Linear Algebra, Mir Publication.
3\. S . Lang - Linear Algebra, Springer-Verlag.
4\. G. Hadly - Linear Algebra, Narosa.
5\. A. Kurosh - Higher Algebra, Mir Publication.
6\. A. R. Rao and P. Bhimasankaram -Linear Algebra, McGraw-Hill.

Paper—II

Group - A : Real Analysis and Metric Spaces (Marks 50)

Real Analysis: Open sets and closed sets and their properties. Bolzano-Weirstrass theorem. Heine—Borel Property. Monotone functions and Nature of their discontinuities. Functions of bounded variations. Lebesgue Measure of a bounded set. Measurable sets and their properties. Measurable functions. Convergence in Measure. Lebesgue theorem. Lebesgue integral of bounded measurable functions an d their proper ties. Comparison with Riemann-Integral. Lebesgue's criteria for Riemann integrability. Riemann - Stieltjes integrals, Simple properties. Fourier Series.

Metric space: Examples, c, S, c[a , b], Algebra of open and closed sets, closure, Interior and boundary of a set. Limit point, Housdroff property, Completeness. Connectedness. Important theorems in a metric space. Heine- Borel theorem. Continuous Functions over metric space –with applications. Uniform Continuity. Contraction theorem. Construction mappings Approximation theorem. Completeness in Matric spaces, examples, Baire theorem and Equivalent matrices. Compactness and Connectedness in metric space.

References:

1. G.F. Simmons- Modern Analysis
2. J. L. Kelley-Topology
3. P. Nathanson- Theory of functions of Real variable-I, II
4. Brown and Page - Functional Analysis
5. C. Goffman - Real functions

Group - B: Complex Analysis (50 Marks)

Complex Field: The algebra of complex numbers, extended complex plane, Riemann Sphere and Stereographic projection. Lines. Circles, Cross-ratio, Bilinear transformation.

Complex Functions : Functions of a complex variable, limit, continuity, differentiability, Cauchy-Riemann Equations, Sufficient condition for differentiability. Harmonic functions, Analytic functions.

Integration : Line integrals of a complex function, Cauchy's fundamental theorem and its consequences, Cauchy's integral formula. Maximum modulus theorem and its consequences, Morera's theorem, Liouville's theorem, Fundamental theorem of algebra .

Sequence and Series : Sequence and Series of Complex numbers and Complex functions, Uniform Convergence, Weierstrass' M-test, Weierstrass theorem on uniform convergence on Compact sets (statement only), term wise integration and differentiation. Power series. Cauchy-Hadarnard theorem. Uniqueness theorem.

Elementary Functions : Exponential function s, trigonometric functions, logarithm function. Many-valued functions. Branch Point.

Analytic Functions : Taylor's theorem, zeros of an analytic function, form of an analytic function near a zero, zeros are isolated point s, Schwarz's lemma. Open mapping theorem. Laurent series. Singularities : Pole, Essential singularity, Removable singularity. Form and behavior of a function near a pole, Casorati-

Weierstrass theorem, Riemann's theorem on Removable singularity, Simple examples.

Calculus of residues : Residue, Residue theorem, Meromorphic functions, Argument principle, Rouche's theorem . Contour integration.

References :

1. Theory of functions of a Complex variable-Vol I & II, A. I. Markushevich, Prentice-Hall, 1965.
2. Functions of one Complex Variable - J B. Conway, Springer-Verlag, 1973.
3. Complex variables and applications - R. V. Churchill & J. W. Brown, McGraw-Hill, International Edition (5th Edition), 1990.
4. Complex Analysis -1. V. Ahlfors, McGrow-Hill, 1953.
5. Foundations of Complex Analysis - S. Ponnusamy, Narosa Publishing House, 1995.

Paper - III

Group—A : Ordinary Differential Equations and Special Functions (50 Marks)

Existence and Nature of Solutions : Introduction, Order, Degree and Exactness of Differential equation, Principle of Duality, Picard's theorem.

General Theory of Linear Differential Equation : Basic concepts. Linear Differential Equation and its Properties, Existence and uniqueness theorems; Variation of parameters; Ordinary points: Regular singular points ; Tw o Space System; Auton omous System; Critic al points; Limit cycles.

System of linear differential equations : System of linear differential equations in Normal form, Homogeneous linear system. Wronskian; Characteristic Equation and Characteristic Values, Stability of solution of ordinary differential equation.

Second order linear differential equations: Uniqueness Theorem, Characteristic Equation and Characteristic Values, Boundary Conditions. Stourm-Liouville Systems, Fourier's Convergence Theorem.

Green's function : Green's functions and its properties, Sturm-Liouville theory; Boundary value problems.

Plain Autonomous Systems : Path of the system. Integral curves, Singular point. Critical point Node, Saddle point, Damped linear Oscillator.

Special Functions : Equation of Fuchsian type; Series solution by Frobenius method; Bessel , Legendre, Hermite, Laguerre and Hypergeometric differential equations: Simple properties of solutions; Asymptotic Expansions; Solutions in terms of contour integration.

Non-linear differential Equations: Fundamental Existence Theorem; Stability; Lyapunov's function; Differential Equations with periodic solutions; Method of Bogoliubev and Krylov.

References :

1. G. F. Simmons - Differential Equations
2. I. N. Sneddon - Special functions of mathematical physics
3. E. L. Ince — Ordinary Differential Equations
4. E A. Coddington and N. Levinson - Theory of Ordinary Differential Equations

Group — B : Partial Differential Equations (50 Marks)

Linear partial differential equations of the first order : Charpit method and Jacobi' s method or solutions; Lagrange's equation and its solution; Quasilinear partial differential equation of second order; Cauchy problem; Characteristic directions; Classification of equations; Normal form of equations; Adjoint and self-adjoint operators. Some important partial differential equations and their classifications; Wave equation: D'Alember t's solution, Riemann's method of solution of hyperbolic equations: Heat equation; Elementary solutions; Laplace's equation: Elementary solutions; Dirichlet and Neumann problems; The theory of Green's function for Laplace's equation

Elliptic differential equations: Laplace's equation, Poisson equation, Boundary value problems, Laplace's equation in spherical polar and cylindrical co-ordinates, Heat equation, Harmonic function and Mean value theorem.

Parabolic differential equations: Diffusion equation. Boundary conditions, Solution by method of separation of variables, Maxima-Minima Principle, Uniqueness Theorem.

Hyperbolic differential Equations: Wave equation and Helmholtz equation in spherical polar coordinates; Solution by the method of separation of variables: Solution by the method of Fourier series.

Green's function: Green's function method for solution of Laplace's equation, Green's functions for solving Diffusion equation, Green's functions for Wave equation-Helmholtz theorem.

References :

1. I. N. Sneddon - Elements of Partial Differential Equations
2. K. S. Rao - Introduction to Partial Differential Equations
3. H.F. Weinberg - A first Course in Partial Differential Equations
4. F. John - Partial Differential Equations

Paper - IV

Group - A : Numerical Analysis (50 Marks)

Introduction : Round-off errors an d Instability - inherent and induced, Control of Round-off errors , Hazards in Approximate Computations.

Solving System of n Linear Equations in n unknowns: LU Decomposition Methods, Determinant of a Matrix and Matrix Inversion, Least-Squares Solution Over-determined Linear Systems, ill- conditioned Matrix.

Eigen pair of an nxn Numerical Matrix: Power Method for Extreme Eigen values and Related Eigen vectors, Power Method with shifting.

Solution of Non- Linear Equations : Isolation or Bracketing of a Root (With odd multiplicity), Fixed Point Iteration, Newton-Raphson Meth od and Modified Newton-Raphson Method (for Real Roots only), Roots of Polynomial Equations with real Numerical Coefficients, Evaluation of Polynomials and their Derivatives. Bairstow's Method for Quadratic Factors of Polynomials, Quotient — Difference Algorithm for

Polynomial Roots, Nonlinear Systems, Newton 's Method, Quasi-Newton Method.

Polynomial Interpolation : Inverse Interpolation, Roots by Inverse Interpolation, Central Difference Interpolation Formulas - Gauss, Hermite Interpolation, Piecewise nominal Interpolation Cubic Spline Interpolation.

Approximation: Least Square Approximation to Discrete Data, Chebyshev Polynomials, Economized Power Series Approximation of Functions.

Numerical Integration: Newton-Cotes Integration formula. Romberg Method. Gaussian Quadrature Rules

Numerical Solution of Ordinary Differential Equations-Initial Value Problems: Taylor Series Method, Euler and Modified Euler Methods, Runge-Kutta Methods, Linear Multistep Methods: Adams-Bashforth, A darns-Moulton and Milne Formulae.

Two-point Boundary Value Problems of Ordinary Differential Equations: Finite Difference Method, Passage Method.

Elements of Finite Difference Method of Numerical Solution of Partial Differential Equations : Poisson Equation on a Rectangular Region, Parabolic Equation in One-space Dimension (Heat Equation) - Explicit Finite Difference Method, Crank- Nicolson Method (Implicit Method) Hyperbolic Equation in One- space Dimension (Wave Equation): Finite Difference Method, Method of Characteristics.

References :

1. H. R. Schwarz - Numerical Analysis, John Wiley and Sons, 1989.
2. C.E-Froberg -Introduction to Numerical Analysis, Addition Westley Publ. Co., Reading, 1979.
3. K.E.Atkinson -A n Introduction to Numerical Analysis. John Wiley and Sons, New York, 1978.
4. S, D. Conte, C. d e-Boor - Elementary Numeric al Analysis: An Algorithmic Approach, McGraw-Hill, 1981.
5. F. B. Hildebrand - Introduction to Numerical Analysis, McGraw-Hill, New York, 1982.
6. J. B. Scarborough - Numerical Mathematical analysis. Johns-Hopkins, 1978.
7. A. Ralston and P. Rabinowitz - A First Course in Numerical Analysis, McGraw-Hill, New York
8. L. Collate - The Numeric al treatment of differential equations. Springer, New York.

Group—B : Computer Programming and its Applications to Problems of Numerical Analysis (50 Marks)

Algorithms and Flowcharts: Algorithms, Objectives, Definition and Examples of Algorithms, Flowchart

Programming with C: Introduction to C Programming, Constants and variables, Operators and Expressions, Input and Output Statements, Control Statements, Arrays, Functions, Pointers : Address operators, pointer Declaration, Void Pointer, Passing Pointers to a Function, Pointers and One-Dimensional Array, Dynamic Memory Allocation. String Manipulation, Structure and Unions: Definition of Structure, Accessing a Structure, Nested Structure, Array of Structure, User Defined data type, Structure and pointers, Passing Structure and Function, Union. File Processing: File Pointer, Opening and Closing a File, File Handling Function, Writing to a File, Reading from a file, Operations on Data Files. Macro and Preprocessor : Macros, Macros with Arguments, The C Preprocessor.

Problems on Numerical Analysis: Solution of Algebraic and Transcendental Equations: Bisection method, Iteration Method or fixed Point Iteration, Newton-Raphson Method or Method of Tangent, Solution of System of Linear Equations: Jacobi's Iteration Method, Gauss-seidal's Iteration method, LU Decomposition Method. Integration: Trapezoidal rule, Simpson's 1/3 Rule. Ordinary Differential Equations: Euler's Method, Runge-Kutta Methods. Fitting of a Straight Line.

Data Structure: Asymptotic Notations, Time and Space Complexities, Data Structure, Arrays, Stacks, Evaluation of Expression: Postfix expression, Queues, Linked Lists.

REFERENCES:

1. E. Balagurusamy, Programming in ANSI C, 4th Edition, The Mcgraw-Hill Companies, New Delhi (2009).
2. B.Gottfried, Programming with C, Schaum's Outlines, The Mcgraw-Hill Companies, New Delhi (2001).
3. Horowitz and sahani, Fundamental of Data Structure, Galgotia, New Delhi (1995).

Paper-V

Group — A: Principles of Mechanics (50 Marks)

Preliminaries: Concepts of Inertial frame, Newton's law s of motions, Conservative forces. Conservation laws. Equations of motion of a particle in different systems of co-ordinates. Motion of a particle on smooth and rough surfaces.

D'Alembert's principle: Generalized co-ordinates. Constraints. Classification of

Hamilton's canonical equations of motion, Integral of energy. Poisson Bracket and its proper ties. Poisson bracket relations concerning linear and angular momentum. Action Principles: Hamilton's principle and the principle of least action. Verification of Hamilton's principle by D 'Alembert 's principle. Derivation of Lagrange's equations and Hamilton's equations from Hamilton's principle.

Symmetries and Constants of Motion: Noether' s theorem, applications on important physical problems like Brachistochrone. Shortest distance, Laws of Reflection and Refraction.

Canonical transformation: Concept of Phase space, Different kinds of Canonical transformations. Configuration space. Point Transformation and equivalency of Lagrangian mechanics. Hamilton-Jacobi equation, application to action-angle variables.

References :

1. Classic al Mechanics : H. Goldstein, N arosa, 198 0.
2. Classic al Mechanics : J. R. Taylor, University Science Bo oks, 2005.
3. Classic al Mechanics : Rana and Joag
4. Mathematical Methods of Classical Mechanics : V. I. Arnold, Springer-Verlag, 1978.
5. Principles of Mechanics : J. L. Synge and B. A. Griffith

Group - B : Elements of Continuum Mechanics and Special Theory of Relativity (without tensor) - 50 Marks

Special Theory of Relativity : Galilean transformation. Postulates of special theory of relativity. Lorentz transformation. Time Dilation. Length contraction and dilation. Velocity addition theorem. Einstein's Mass-Energy relation. Transformation formula for mass.

Kinematics of fluids : Lagrangian and Eulerian methods. The equation of continuity. Streamlines. Velocity potential.Rotational and irrotational motion. Euler's dynamical equations of motion. Integration of Euler's equation s. Steady motion. Bernoulli's theorem. Motion in two dimensions. Source Sink and Doublets. Constancy of circulation. Kelvin's theorem on minimum kinetic energy. Viscous flow theory. Navier-Stoke's equation. Circulation in viscous flow. Flow between parallel plates.

Deformation of Solid: Deformation of Elastic Solid, Strain tensor. Equations of compatibility. Analysis of stress. Stress equations of equilibrium an d motion. Stress-strain relations. Generalized Hooke's law. Equilibrium equations for an isotropic elastic solid. Simple applications. Strain energy function. Saint Venant's principle. Wave propagation. Isotropic elastic solid.

References :

1. Dynamics Part II: A. S. Ramsey (Cambridge University Press)
2. An Introduction to the Theory of Relativity-: P. Bergmann
3. Theory of Relativity : Special and General: M. Ray (S. Chand & Co)
4. Treatise on Hydrodynamics : A. S. Ramsey (G. Bell & Sons London)
5. Theoretical Hydrodynamics : L M . Milne-Thomson (Macmillan)
6. A Treatise on the Mathematical Theory of Elasticity : A. E. K. Love (Dover)
7. Mathematical Theory of Elasticity : I. S. Sokolnikoff (McGraw H ill)
8. Mathematical Methods of the Theory of Elasticity : V. Z. Parton & P. I. Perlin (MIR)

Part–II

Paper–VI

Group - A: General Topology (50 Marks)

Topological spaces. Examples, Base for a Topology. Sub-base. Neighbourhood system of a point, Neighbourhood base. Limit point of a set. Closed sets. Closure of a set, Ku ratowski closure operator; Interior and boundary of a set, Sub -space Topology,
First and Second Countable spaces. Continuous function over a Topological space. Homeomorphism; Nets, Filters, Their convergence, Product spaces, Projection function.Open and Closed function, Quotient spaces.
Separation axiom s T0 T1; T2; T3; T4 in Topological spaces. Product of T2-SPACES. Urysohns Lemma in Normal spaces, Tietze extension Theorem, Embedding in cube. Embed ding Lemma. Urysohn's metrization Lemma.
Open cover, Sub-cover, Compactness, Countable open cover, Lindeloff space, Compact sets, Finite Intersection property, Tychonoff Theorem on product of compact spaces , Continuous image of a comp act space s, Locally compact spaces, One point compactification.
Connected spaces, Separate d sets, Disconnection of a space, Union of connected sets, Closure of a connected sets , Connected sets of reals, Continuous image of connected spaces , Topological product of connected spaces, components, Totally disconnected spaces , Locally connected spaces. Uniformity in a set, Base, Sub-base of a Uniformity, Uniform spa ce. Uniform Topology. T_2—property of a Uniformity, Interior and closure of a s et in terms of uniformity, Uniformly continuous function. Product Uniformity.

References :

1.	Modern Analysis and Topology	: Simons
2.	General Topology	: Kelley
3.	Topological structure	: Thron
4.	Topology	: Du gundji
5.	General Topology	: Adhikary, Chatterjee, Ganguly
6.	General Topology	: K. K. Jha
7.	General Topology	: Vaidyanathaswami

Group — B : Functional Analysis (50 Marks)

Metric spaces: Metric Topology, Complete metric spaces, examples G, C [a, b]; Separable metric spaces, Continuous functions; Homeomorphism, Isometry; Compact metric spaces, Sequential compactness, Banach Contraction Principle Theorem, Ascoli-Arzela Theorem.

Normed Linear space (N LS) : Ban ach space. C [a, b] a s a Banach space. Quotient space of a NLS, Algebra of convex sets. Bounded Linear operators, their continuity, Unboundable Lin ear operator, Norm || T || if a bounded Linear operator T one NLS. Formulae for || T ||.

Equivalent norms, Riesz Lemma. Finite Dimensionality of NLS by compact unit ball, Boundedness of Linear operators over finite dimensional NLS, space BdL (X , Y) of bounded Linear operators ; its completeness. Bounded Linear Functional, Hahn - Banach Theorem; its applications, conjugate s paces of NLS ; Canonical mapping ; Embed ding of a NLS into its second conjugate spaces under a Linear Isometry ; Reflexive Banach spaces ; Open map ping Theorem : Close d Graph Theorem.

Inner product s paces (I.P.S.): Cauchy - Schwarz inequality, I.P. spaces as NL S, Law of Parallelogram, orthogonal (orthonormal) system of vectors ; Hilbert spaces ; Project ion Theorem in a Hilbert spaces H, Riesz representation for a bounded linear functional, Complete orthonormal system in H. Adjoint of bounded Linear operator in a Hilbert space H. Algebra of adjoint operators. Self-adjoint operators in H; their norms, every bounded Linear operator in H as a sum of self-adjoint operators ; eigen-values and eigen vectors of self-adjoi nt operators.

References :

1. B. K.Lahiri : Elements of Functional Analysis
2. Leierstermise and Sobolev : Introduction to Functional Analysis
3. Brown and Page : Functional Analysis
4. Kreyszig : Functional Analysis
5. Goffman and Pedrick : Functional Analysis
6. Taylor: Functional Analysis

Paper -VII

Group-A: Differential Equation s & Integral Transformations (50 Marks)

Fourier Transform: Its property if Continuity and Differentiability, Fourier transform derivatives. Riemann - Labesgue Theorem. Fourier Inversion Theorem; Convolution Theorem and Parseval's relation for Fourier Transform; Fourier Sine and cosine transform. Some applications like (i) Heat conduction in solids (ii) Wave equation.

Laplace's Transform : Laplace Transform of derivatives, Properties of Laplaces Transform, like (i) Linearity (ii) Shifting (iii) Translation (iv) Convolution Theorem. Differentiation and Integration of Laplace Tranform. Inverse Laplace Transform : Inversion by (i) use of linear and shifting property, (ii) use of formulas for derivative and Integral if a Laplaces Transfor m, (iii) use of convolution. Theorem. Heaviside series expansion. Applications in Linear ordinary and partial differential equations.

Hankel Transforms: Its inversion formula. Hankel Transform of derivatives. Finite Hankel Transform and inversion formula. Finite Hankel Transform of derivatives. Applications in problem of (i) free symmetric vibration of a stretched circular membrane (ii) conduction of heat in an infinite circular cylinder.

References :
1. I.N. Sneddon– The use of Integral Transforms, McGraw-Hill. Singapore 1972.
2. R.R. Goldberg, Fourier transforms, Cambridge University Press, Cam bridge, 1961.
3. D. Brain- Integral Transformation and their applications. Springer-Verlag, New York, 2002.
4. R. Brace wall- The Fourier transform and its applications, McGraw-Hill, New York, 1999.

Group-B : Integral Equations and Genera lized Functions (50 Marks)

Preliminary concepts : Integral Equation, Special types of kernels - symmetric kernel, kernel producing convolution integral, separable or degenerate kernel. Integral operator. Resolvent, resolvent kernel and resovent equation. Function space. Orthon ormal system of functions. Gram-Schmidt orthogonalisation. Approximation and convergence in the mean. The Riesz Fisher theorem.

Method of successive approximations : Neumann series, Iterated kernel. L_2— kernels and functions.

Fredholm Theory : The Fredholm theorems. Degenerate kernels. Method of approximation by degenerate kernels. Continuous kernels.

Hilbern-Schmidt kernel : expansion theorem, the Hilbert-Schmidt theorem, Hilbert s' formula, applications of Hilbert-Schmidt theorem. Expansion of the resolvent kernel. Positive kernels. Mercer's theorem. Fredholm integral equation of the first kind.

References :
1. Lectures on the theory of Integral Equations, Mir Pub.
2. First course in Functional Analysis, PHI.
3. A course in Mathematical Analysis, Vol-III, Part-II Dover Pub.
4. Integral Equations, Wiley, New York
5. Functional Analysis

Paper-VIII

Group - A : Differential Geometry (50 Marks)

Tenso rs : Transformation of Co-ordinates ; Summation conventions ; dummy index, free index, Kronecker delta ; Contravariant and Covariant vector. Invariants; Second order ten sors and higher ord er tensors; Algebra of Tensors ;
Contraction ; Symmetric and skew symmetric tensors. Quotient Law; Conjugate symme tric tensor. Curvilinear Co-ordinates.

Metric tensor : Linear element ds, Riemannian metric. Fundamental metric tensor, Riemannian space. Associated vectors, magnitude of vectors, angles. Christoffel symbols. First and Second kind; Relations
Covariant differentiation of vectors a nd Tensors. Riemann - Christof elf Tensor and its proper ties; Riem ann - Chris toffel tensor of first kind; Ricci Tensor ; Scalar Curvature, Einstein space.

Curves and Surfaces in spaces: Serret-Frenet formula; Helices, Surfaces : The element of length and metric tensor. First Fundamental form; Angle between two intersecting curves on a surface; Geodesies on a surface: Gaussian Curvature : Geode sic curvature. Necessary and sufficient condition for a curve on a surface to be geodesic.

Tensor derivative : Gauss formula: Weingarten's formula : Third Fundamental form of the surface. Equations of Gauss and Codazzi, Mensner's theorem ; Principal curvat ures, line of curvature.

References :

1. M. C. Chak i: A text book of Tensor Calculus
2. Sokolnicoff : Tensor Analysis
3. Weatherburn : Riemannian Geometry and Tensor Calculus
4. Eisenhart : Riemannian Geometry
5. S pain : Tensor Calculus
6. W. B. Boothby : An introduction to Differentiable manifold and Riemannian Geometry

Group - B : Graph Theory (50 Marks)

Graph s and Directed Graphs (Digraph) : Parallel edges. Adjacent edges. Loop ; Simple Graph ; Degree of a vertex ; Regular Graph, Odd and Even vertex; Pendant vertex. Properties of a graph like: sum of degrees of vertices in a graph equals to twice the number of edges i n G, Simple Graph has a pair of vertices of equal degrees. Directed Graphs (Digraph), Representation of binary relations on finite sets by Digraphs.

Subgraphs : isomorphism of graphs, walks, paths and cycles ; length of a walk ; closed walk ; Circuits and cycles.

Connected Graphs : Components of a graph ; A simple Graph of n vertices and m components has at most 1/2 (n–m) (n – m + 1) edges ; Complete Graph, Complement of a graph ; A Complete Graph of n vertices contain s 1/2 n (n –1) edges,

Eulerian and Hamiltonian Graphs : Eulerian graph, Hamiltonian Graphs, A Connected Graph of even degree vertices is Eulerian, Konigsberg Bridge problem; Hamiltonian Path .

Tree : Definition of Tree, Important properties like A tree of n vertices has (n - 1) edges ; A connected n - vertex graph with (n - 1) edges is a tree. Minimally connected tree ; Spanning tree ; Every connected graph has a spanning tree. Minimal s panning tree ; Kruskal's algorithm f or a minim al spanning tree ; Rooted tree, Binary tree.

Planar Graph : Imbedding of a Graph on a surface, Faces of a Planar Graph; Euler's polyhedral equation, Kuratowski's first Graph K_5 and Kuratowski second Graph $K_{3.3}$. Their properties.

Matrix representation of Graphs: Adjacency matrix of a Graph, Incidence matrix of a Graph.

Reference :

1. F. Harrary : Graph Theory
2. N. Deo : Graph Theory with applications to Engineering and Computer Science
3. M. K. Sen and B. C. Chakraborty : Introduction to Discrete M athematic s
4. M.K . Sen and D.S. Malik : Discrete Mathematical Structures
5. J. G ross and J. Yellen : Graph Theory and its applications

(i) Pure Mathematics

Paper-IXA (i): Advanced Complex Analysis (50 M arks)

Analytic continuation – The idea of analytic continuation. The analytic continuation of the exponential, trigonometric and hyperbolic functions. Direct analytic contin uation. Complete analytic function. Natural Boundary. Analytic continuation by power series. F unction el. Analytic continuation along a path. Monodromy theorem.

Conformal Mapping – I – Definition. Basic properties of con formal mapping. Conformal map pings by elementary functions. Schwarz Principal mappings by elementary functions. Schwarz cristoffel transformation. Some applications.
Entire function – Infinite product. Uniform convergenc e of infinite product s. Basic proper ties of entire function s. Factorization of entire functions.
Meromorphic fu nction – Mittag – Leffler theorem. Gamma function. Jensen's theore m. Poisson – Jensen theorem.C
C**onformal Mapping – II** – Univalent function. Normal families. The Riemann mapping theorem . The class Y.
Many valued function – The function log z. The power function. Branch points of an analytic function . Regular b ranches of analytic functions.
Riemann Surface – A few examples.

Paper IXB (i): Advanced Topology (50 Marks)

Compactness: More facts about nets and filters-subnets, clusters point, filter, ultrafilter. Characterization of compactness. Countable, Frechet , Sequential compactness, interrelationships, compactness in metric spaces, equivalence of the four types of compactness.
Compactification: More on Locally compact spaces, properties, compactification, more on one-point compactification, embedding Lemma, stone-Cech compactification, ordering in Hausdorff compactifications, Wallman's compactification.
Paracompactness: Locally finite family, paracompactness, basic properties, star operation, equivalent condition of paracompactness in respect of star operation, fully normal space, partition of unity.
Metrization: Metrization of topological space, Metrization of the product space R^j, Uryshon's metrization theorem, Nagata-Smirnov metrization theorem, Cartesian product of metrizable spaces, Two important results, namely, Arzela-Ascoli's theorem, Stone-Weirstrass.
Uniform space and proximity spaces: Definition of uniform spaces, basis, sub basis of a uniformity, topology induced by uniformity, uniformizable spaces, Metrizable spaces, uniformly continuous maps, Cauchy nets and filters, completeness in a uniform space, total boundedness and compactness.

Paper–XA (i) : Advanced Differential Geometry (50 Marks)

Differentiable Manifold: Differentiable mapping, Differentiable curves ; Integral Curve, Differential of a mapping, f-related vector field . One parameter group of transformations on a manifold. Co- tangent space, r-form and Exterior Product. Exterior differentiations, its existence and uniqueness. Pullback differential form. **Lie Group:** Left translation, right translation, Invariant Vector field. Lie algebra of the Lie Group G. Invariant Differential Form: Automorphism. Inner automorphism. One-parameter Sub Group of a Lie Group. Lie Transformation Group (Action of a Lie Group on a Manifold). Fundamental Vector field, Fundamental map.

Linear connection; Torsion tensor field and curvature tensor field on a Linear connection. Ricci Identity :

(i) for a 1-form w : (∇x ∇y_w -∇Y ∇X^w– $\nabla[X,Y]^w$) = -W(R(X, Y),Z,P)

(ii) for a 2-form w : (∇X ∇Y^w-∇Y ∇X^w–$\nabla[X,Y]^w$) (Z, P) =
-W(R(X,Y)Z,P) -W(Z,R(X,Y) P)

Riemanian Metric : Riemanian Connection (Levi - Civita Connection). Every Riemannian manifold (M, g) a of a unique. Riemannian Connection Manifold of constant curvature is an Einstein Manifold. A 3-dimensional Einstein manifold is a manifold of constant curvature.

Semi-symmetric Metric Connection. Weyl Conformal Curvature tensor. Goldberg's result :If (M, g) is a Riemannian manifold and A is the field of symmetric endom orphism corresp onding to Ricci tensor S i.e., g (AX, Y) = S (X, Y) for every Vector fields X, Y on M, then C (X, Y) Z = R (X, Y) Z- 1/n-2 {g (Y , Z) AX– g (X, Z) AY + S (Y, Z) X–S (X, Z) Y] + r/(n- 1) (n-2) {g(Y, Z) X– g) X, Z) Y}.

Confor mally sym metric. Ri emannian Manifold, A confor mally sym metric man ifold is of cons tant scalar curvature if (∇z S) (Y, W) = (∇w S) (Y, Z) for all Y, Z, W.

References :

1. N.J. Hicks : Differential Manifold
2. W.M. Boothby : Int. to Differential Manifolds and Riemannian Geometry
3. Y. Matsushima : D ifferentiable Manifold
4. P. M. Cohn : Lie Groups
5. B. B. Sinha : Int. to modern Differential Geometry
6. S. Helgason : Differential Geometry, Lie Group and Symmetric Spaces.

Group – XB (i) : Advanced Functional Analysis (50 Marks)

Convex hull of a set in a vector space : Its representation Theorem ; Symmetric sets, balanced sets, absorbing sets in a vector space ; Isomorphism in vector spaces. Topological vector spaces (TVS), translation and multiplication operators as self-homeomorphism, Bounded sets in TVS ; basic properties in TVS. Separation Theorem in TV S ; Linea r operators and their continuity in TV S ; Locally compact TVS, Minkowski functionals, semi-norms, Kolmogorov theorem on normability of a TVS.

Bounded Linear functionals and their representation over R_n, l_p (1< p <∞) and C [0,1]. Banach Steinhaus Theorem, Weak convergence in Normed Linear Space (NLS). Best approximation in N LS, strictly convex norms, uniqueness criterion of best approximation.

Resolvant set r(T) and spectrum s(T) of a bounded linear operator T over NLS, compact linear o perators, spectral properties of a bounded self-adjoin t operators T over Hilbert space, spectral radius formula ; Projection operators, their algebra and properties.

Eigen value, eigen vector of a linear operator over NLS X with Dim (X) <¥, characteristic equation : finite dimensional spectral theorem ; Banach algebra X, identity element; invertible and non - invertible elements o f X, Topological divisor of zero in X. Gelfand-Mazur Theorem.

Weak and weak* topology in conjugate space X* of a NL S X ; their properties, weak* compactness, Banach-Alaogulu Theorem.

References :

1. W. Rudin — Functional Anal ysis
2. B. K. Lahiri — Functional Analysis

3. Brown an d Page — Functional Analysis
4. Bachman and Narici — Functional Analysis
5. Kreyszig — Functional Analysis

(ii) Applied Mathematics

Paper IXA (ii) : Operations Research (50 Marks)

Classical Optimization techniques: Multivariable Optimization with no Constraints, equality constraint and inequality constraints. Method of constrained variation, Method of Lagrange multipliers. Kuhn-Tucker conditions. Revised Simplex meth od, Dual simplex method and modified dual simplex method.

Post optimality Analysis: Discrete changes in the Cost vector and Requirement vector. Addition of a single variable, Deletion of a variable and Addition of a new constraint.

Quadratic Programming: Wolfe's and Beale's method.

Integer Programming: Gomory's cutting Plane method, Branch and bound method.

One dimensional minimization method: Fibonacci method and Golden section method.

Unconstrained optimization technique: Steepest descent method, Quadratically convergent method, Newton's method & Dairdon-Fletcher-Powell method.

Constrained optimization technique: Cutting plane method.

Paper IX B (ii) : Mathematical Models in Ecology (50 Marks)

Introduction: Basic concept of ecology ecological systems. Mathematical models. Variables, Deterministic and Stochastic models. Modelling in discrete time and continuous time.

Continuous Sing e-Species Population Models: Basic Postulates, General Model equation. Malthus growth model, Logistic growth model, Allce effect. Qualitative analysis, Harvard model. Exercises.

Discrete Single-Species Population Models: Discrete models and difference equations. Differential Vs Difference equations. Equilibrium points and Stability. Graphical solution of Difference equations. Density dependent population growth. Equilibrium points and Criterion of Stability. Rabbit problem. Fibonacci sequence. Exercises.

Delay Differential equations Models: Introduction, Types of Delay equations, Discrete time Delay model, Distribute d Delay models of population.

Interacting Population Models: Qualitative analysis, Generalization and Stability. Periodic solutions and limit cycles. Classical Prey-Predator models. Realistic Lotka-voltera models. Co-operative systems. Ecosystem models. Functional groups and Nutrient Flows. Food-Chain model. Logistic primary production, Material cycling.

Paper – XA (ii): Fluid Mechanics (50 Marks)

Irrotational motion in 3D : motion of a sphere, flow around an ellipsoid, stream functio n for axis- symmetric flow, method of source and sinks, motion of a rigid body in an unbounded fluid, inertial motion of a body. Boundary conditions, motion of a circular cylinder, the unsteady flow, hydrodynamic reactions in a steady flow, Blastius-Chaplygin formulae, Kutta-Jouko uskii transformation, method of conformal mapping. Schwartz- Christofell formulae, Joukou skii's profile, flow around a flat plate and an elliptic cylinder. Vortex lines,vortex filaments, rectilinear and circular vortex, Stoke's formulae, Helmholtz's theorems, formulation of vortices, vortex layer, Karman vortex street.

Wave motion : plane waves, wave components, steady waves, progressive waves, energy of waves, group velocity, rate of trans mission of energy in simple harmonic surface waves, water at the common surface of two liquids, Long wave, capillary waves. Flow through pipes of circular, annular and elliptic cross-section s, boundary layer equations on a plane wall, Blasius solution for a flat plate.

References :

1. Milne-Thomson, L. M., Theoretical Hydrodynamics, Macmillan and Co. Ltd, London, 1955.
2. Ramsey, A. S., A Treatise On Hydromechanics, CBS Publishers and Distributors, New Delhi, 2000.
3. Chorlton, F., Textbook Of Fluid Dynamics, CBS Publishers and Distributors, New Delhi, 2003.
4. Lamb, H. Hydrodynamics, Cambridge University Press, 1932.
5. Kundu, P , K., Fluid Mechanic s. Academic Press, San Diego, 1990.
6. Landau. L. D., Liftshitz, E. M, Fluid Mechanics, Pergamon Press, London, 1959.
7. Kochin, N. E., KibeL I. A. & Roze, N. V. Theoretical Hydrodynamics, Interscience Publishers, 1964.

Paper X B (ii) : Mechanics of Solids (50 Marks)

Two Dimensional Elastostatic Problems: Introduction, Plane strain, Plane stress and Generalized plan stress, Plane elastotalic problem, Airy's stress function.
Extension and Torsion : Axial Extension of a Beam, Beam stretched by its own weight, Bending of a beam by terminal couples, Torsion of cylindrical bars of circular cross-section, Torsion of a cylindrical bar of any given section, Solution of the torsion problem for certain particular cases.
Semi- Infinite Solids With prescribed Displacements or Stresses on the Boundary: Semi-infinite solid with prescribed Displacements on the plane Boundary, Semi-infinite solid with prescribed surface traction on the plane boundary, Simple Solutions.
Variational Methods: Euler's Equation, Theorem of minimum potential energy, Theorem of minimum complementary energy, Reciprocal theorem of bethi and Rayleigh, Examples
Elastic Waves: Body Waves: Waves of dilatation and waves if distortion, Plane Waves, Surface Waves: Rayleigh wave, Love waves.
Transverse Vibration of Thin Elastic Plates: Basic Preliminaries, Differential equation of transverse vibration of thin plate, Vibration of a rectangular plate with simply supported edge, Free vibration of a circular plate: Clamped edge, Simply supported edge, Symmetrical vibration of a thin circular plate.
Plasticity : Basic Concepts : Relation between the stress and strain deviators, Stress-strain curve. Yield Criterion, Equation of Plasticity: Prandtl- Reuss Theory, Stress-strain relation of Von- Mises, Elasto-plastic problems.

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