POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-5A : PRINCIPLES OF MECHANICS

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(All symbols have their usual meanings)

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Distinguish between holonomic and non-holonomic constraints.

   b) Obtain the Lagrangian for a simple pendulum of length l and mass of the bob m with angle of deflection θ.

   c) Show that $\sum_{j=1}^{n} \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L$ is a constant of motion for a conservative, scleronomic system.

   d) If the Hamiltonian of a one dimensional dynamical system is given by $H = \frac{1}{2}(\alpha p^2 + \beta q^2 + 2\gamma pq)$,

      where α, β, γ are constants, then find the value of the Poisson bracket $\{p, H\}$.

   e) What is Coriolis force ? State the cause of this force.

   f) The linear transformation of a generalized coordinate q and corresponding momentum p is given by

      $$Q = p + 2iq, \; P = -\frac{1}{2}\left(q + \frac{ip}{\beta}\right)$$

      where β is a constant and $i = \sqrt{-1}$. Find the value of β for which this transformation will be a canonical transformation.

   g) Write down the Hamilton-Jacobi equation for a dynamical system.

2. Consider a mechanical system described by 'N' generalized coordinates $q_1, q_2, \ldots\ldots, q_N$. Show that the kinetic energy can be formulated as

   $$T = T_2 + T_1 + T_0, \text{ where } T_2 = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij}\,\dot{q}_i\,\dot{q}_j, \; T_1 = \frac{1}{2}\sum_{i=1}^{N} A_i\,\dot{q}_i, \; T_0 = A_0$$

   and the quantities A_{ij}, A_i, A_0 are to be determined by you.

   Find the form of the kinetic energy when the system is scleronomic.

   8 + 2

3. a) State *D*'Alembert's principle. Derive Lagrange's equation of motion for a scleronomic, holonomic system. 2 + 3

   b) A particle of mass m is projected with initial velocity u at an angle α with the horizontal. Use Lagrange's equations of motion to describe the motion of the projectile. The resistance of air may be neglected. 5

4. a) For a system described by generalized coordinates $q_1, q_2, \ldots, q_n$; define generalized momentum p_i corresponding to the generalized coordinate q_i. Establish the relation $\dot{p}_i = \frac{\partial L}{\partial q_i}$, $i = 1, 2, \ldots, n$. What are velocity-dependent potentials ? 1 + 2 + 2

   b) Obtain Hamilton's equations of motion for a system having n degrees of freedom from Hamilton's principle. 5

5. a) In polar coordinates (r, θ), the Lagrangian of the planetary problem is given by $L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu}{r}$

   where m is the mass of the planetary body and μ is a constant. Find the Routhian of the motion and write down the Routh's equations of motion. Hence solve the problem. 3 + 2

   b) If for a certain mechanical system $H = p^2 q^2 - \lambda pq$, where λ is a real constant, then show that pq is a constant of motion. Obtain the Hamilton's equations of motion for a simple pendulum. 2 + 3

6. a) Define action of a mechanical system. State and prove the principle of stationary action. 2 + 3

   b) Define the Poisson bracket of two dynamical variables $u = u(q_i, p_i, t)$ and $v = v(q_i, p_i, t)$ where q_i, p_i are canonical variables and t is time. If $F(q, p, t)$ and $G(q, p, t)$ are two constants of motion, then show that the Poisson bracket $\{F, G\}$ is also a constant of motion. 2 + 3

7. a) What is type I canonical transformation and what is the generating function of this transformation ?

   For the canonical transformation

   $Q = -p,\ P = q + \lambda p^2$

   where λ is a constant, obtain the type I generating function. 3 + 2

   b) Prove that the shortest distance between any two points on the surface of a sphere is along the great circle of the sphere through the respective points. 5