QP Code: 22/PT/13/VIA

POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-6A : GENERAL TOPOLOGY

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

a) Prove that the real number space $\mathbb{R}$ with lower limit topology is first countable space.

b) Obtain limit points (if any) of the following sets of reals in the space $\mathbb{R}$ of reals with usual topology.

(i) $A = \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\right)$

(ii) $\mathbb{Q} - \{ 1, 2, 3, \ldots, 1000 \}$, where $\mathbb{Q}$ is the set of all rationals.

(iii) $B = \{2^n \mid n \in \mathbb{N} \}$

(iv) The set of all irrationals $\mathbb{R} - \mathbb{Q}$.

c) Let (X, τ) be a discrete topological space. Find all the convergent sequences in (X, τ).

d) Show by an example that the continuous image of an open set is not always open.

e) Give an example of a compact topological space where every compact subsets need not be closed.

f) Find an example of a continuous bijection which is not a homeomorphism.

g) Find where the set of all rationals $\mathbb{Q}$ with indiscrete topology is connected or not.

2. a) Let X be an uncountable set and τ be the co-finite topology on X. Prove that every infinite subset of X is dense. 5

   b) Prove that $\mathbb{R}$ with lower limit topology is a first countable space. 5

3. a) Prove that a point x is a limit point of a subset A of a topological space (X, τ) if and only if $A \setminus \{x\}$ is a member of some filter converging to x. 5

   b) When a topological space is called connected ? Show that the union of connected sets is not always connected. 5

4. a) Show that in a T_2 topological space, every compact subset is closed. 5

   b) Prove that a 1 – 1 continuous function of a compact space onto a T_2-space is a homeomorphism. 5

5. a) In a topological space (X, τ) for any two sets A, B prove that

   (i) $\overline{(A \cup B)} = \overline{A} \cup \overline{B}$

   (ii) If $A \subset B$ then Int $A \subset$ Int B. 5

   b) Prove that every second countable space is separable. 5

6. a) Show that every second countable space (X, τ) is Lindelöff. 5

   b) Prove that every closed sub-space of a Locally compact space is locally compact. 5

7. a) Define a uniform space (X, u). Prove that every metric space is a uniform space. 1 + 3

   b) State and prove Tietze Extension theorem. 6