QP Code: 22/PT/13/VIIB

POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-7B : INTEGRAL EQUATIONS AND GENERALISED FUNCTIONS

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(Notations and symbols have their usual meanings)

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

a) Define degenerate kernel.

b) Convert the differential equation

$$\phi''(x) = F(x, \phi(x)),\ 0 < x < 1$$

with $\phi(0) = \phi_0$, $\phi(1) = \phi_1$, to an integral equation.

c) Solve the integral equation

$$\phi(x) = 1 + \int_0^x (x-t)\phi(t)\,dt, \text{ with } \phi_0(x) = 0.$$

by the method of successive approximation.

d) Solve the integral equation

$$\phi(x) = \cos x + (x-2) + \int_0^x (t-x)\phi(t)\,dt.$$

e) Find the non-trivial solution of Fredholm integral equation

$$\phi(x) = \lambda \int_0^1 2t\,\phi(t)\,dt.$$

f) Reduce the following initial value problem to an integral equation :

$$\frac{dy}{dx} - y = 0,\ \ x > 0,\ y(0) = 1.$$

g) Define good function and fairly good function.

2. Find $D(\lambda)$ and solve the integral equation

$u(x) = e^x + \lambda \int_0^{10} x\,t\,u(t)\,dt$, by using the method of Fredholm's determinants. 10

3. a) Form an integral equation corresponding to the differential equation given by

$$\frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} + e^x y = x \ .$$

with the initial conditions $y(0) = 1,\ y'(0) = -1$. 5

b) Find the eigenvalues and the corresponding eigenfunctions of the integral equation

$$u(x) = \lambda \int_0^1 \cos(\pi t) \sin(\pi x) u((t)\ dt\ .$$ 5

4. a) Find Neumann series solution for

$$\phi(x) = 1 + \lambda \int_0^{\pi/2} \phi(t) \cos x\, dt\ .$$ 5

b) Find the interated kernel $K_2(x,t)$ of the kernel $K(x,t) = e^{|x|+t}$ defined on $R = \{(x,t);\quad -1 \le x,\ t \le 1\}$. 5

5. a) Reduce the following initial value problem to a Volterra integral equation :

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0,\ y(0) = 1,\ y'(0) = 1\ .$$ 5

b) Reduce the following boundary value problem to a Fredholm integral equation :

$$\frac{d^2y}{dx^2} + xy = 1,\ 0 < x < 1,\ y(0) = 1,\ y(1) = 1\ .$$ 5

6. If f is a continuous function on $[\ a,\ b\ ]$ and $K(x,t)\ (\neq 0)$ is a continuous function on $R = \{(x,\ t);\ a \le x,\ \ t \le b\}$ and $\phi_0(x)$ is any function continuous on $[\ a,\ b\ ]$ and for $x \in [a,\ b],\ \phi_n(x) = f(x) + \lambda \int_a^x K(x,\ t)\ \phi_{n-1}(t)\ dt\ \ (n = 1, 2, 3 \ldots)$, then show that the sequence $\{\phi_n(x)\}$ converges to the unique continuous solution of the integral equation

$u(x) = f(x) + \lambda \int_a^x K(x,\ t) u(t)\ dt$ for any finite value of λ. 10

7. a) Show that the eigenfunctions of a symmetric kernel corresponding to different eigenvalues are orthogonal. 5

b) Find the resolvent kernel and solve the integral equation:

$$\phi(x) = 1 + \lambda \int_0^{\pi} \sin(x+t) \phi(t)\ dt, \qquad 0 < x < \pi\ .$$ 5