QP Code: 22/PT/13/VIIIA

POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-8A : DIFFERENTIAL GEOMETRY

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(Notations and symbols have their usual meanings)

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) For a space of dimension 3, evaluate δ_i^i.

   b) If A_{ij} is a skew-symmetric tensor, prove that
   $\left(\delta_j^i \;\delta_l^k + \delta_l^i \;\delta_j^k\right) A_{ik} = 0$.

   c) Define Conjugate Symmetric tensor.

   d) Evaluate :
   $[ij, k] + [kj, i]$.

   e) State Ricci's Theorem.

   f) Define parametric curves on a surface.

   g) When is a point on a surface called hyperbolic ? Give an example of a surface of negative curvature.

2. a) If A_m is a covariant vector, determine whether $\dfrac{\partial A_m}{\partial x^j}$ is a tensor or not. 5

   b) If the equation $a_j^i A_i = \beta A_j$ holds for any covariant vector A_j, where β is a scalar, show that $a_j^i = \beta \delta_j^i$. 5

3. a) Prove that the length of a vector is invariant. 5

   b) Show that :

   $\left\{ {i \atop i\ j} \right\} = \frac{\partial}{\partial x^j} (\log \sqrt{g})$, where $g = |\, g_{ij} \,|$ 5

4. a) Define Riemann-Christoffel tensor of 1st kind and hence show that it is skew-symmetric in the first two indices. 5

   b) Show that for an Einstein Space of dimension $n \geq 2$

   $R_{ij} = \frac{R}{n} g_{ij}$. 5

5. a) Find the curvature at any point of the curve

   $c : x^1 = a, \quad x^2 = t, \quad x^3 = 0$, where

   $ds^2 = (dx^1)^2 + (x^1)^2 (dx^2)^2 + (dx^3)^2$, a being scalar. 5

   b) If A^i and B^i are two vectors of constant magnitudes and undergo parallel displacements along a given curve, then show that they are inclined at a constant angle. 5

6. a) Find the parametric curves of a surface given by

   $x^1 = a \sin u \cos v$

   $x^2 = a \sin u \sin v$

   $x^3 = a \cos u$

   and hence show that they form an orthogonal system. 5

   b) Prove that a geodesic is an auto parallel curve. 5

7. a) Determine whether the surface with the metric

   $ds^2 = (u^2)^2 (du^1)^2 + (u^1)^2 (du^2)^2$

   is developable or not. 5

   b) Prove that a surface is a sphere if and only if the second fundamental form is a non-zero constant multiple of its first fundamental form. 5