POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022
MATHEMATICS
Paper-8B : GRAPH THEORY

Time : 2 hours]   [Full Marks : 50
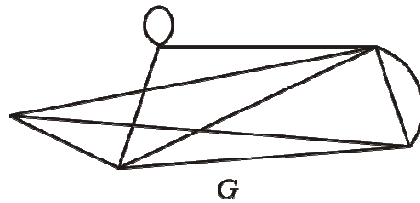
Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) State and explain Handshaking Lemma.

   b) Draw a graph having degree sequence (1, 2, 2, 4, 5).

   c) If G is a forest with n vertices and k components, find the number of edges of G.

   d) Define adjacency matrix with an example.

   e) A complete graph K_n is Eulerian iff n is odd — *True* or *False*. Justify your answer.

   f) Define Minimal spanning tree with example.

   g) Prove that if a graph contains exactly two odd vertices u and v, then there exists a path between u and v.

2. a) Establish Euler's formula for a connected planar graph.   5

   b) If a connected graph becomes disconnected when any edge is removed from it, then show that such a graph must be simple and acyclic.   5

3. a) Define degree of a vertex in digraph. What is a rooted tree ?   3

    b) Prove that a directed tree T can be represented as a rooted tree with root v_0 if and only if v_0 has in-degree 0 and all other vertices of T have in-degree 1.   7

4. a) A cycle on n vertices is isomorphic to its complement. Find the value of n.   5

    b) Show that in a connected simple planar graph with 6 vertices and 12 edges, each of the regions is bounded by 3 edges.   5

5. a) Define a binary tree. Prove that for a complete binary tree T of height h and with n vertices, $n = 2^{h+1} - 1$.   1 + 5

    b) Define Hamiltonian graph. Give a necessary condition for a graph to be Hamiltonian.   2 + 2

6. a) Define m-tree. Prove that an m-tree has at most m^p vertices at level p.   1 + 4

    b) Let G be a connected 4-regular planar graph with 8 vertices. How many faces does G have ?   3

    c) Show that the complete graph K_6 is non-planar.   2

7. a) Describe Kruskal's algorithm for finding a shortest spanning tree of a connected weighted graph with an example.   6

    b) Draw four spanning trees of the following graph.   4

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