

## Question Bank For PG Course

Mathematics

Paper-10B(i)

Special Paper: Pure Mathematics

ADVANCED FUNCTIONAL ANALYSIS : PGM-T-XB

### Question 1

Which of the followings is/are true in a vector space?

1. Intersection of finite number of convex sets is convex
2. Intersection of any number of convex sets is convex
3. Union of finite number of convex sets is convex

### Question 2

*A balanced set is always symmetri-*  
is the statement true or false?

### Question 3

If a vector space  $X$  is isomorphic to the vector space  $R^n$  over the real field, then what is the dimension of  $X$ ?

### Question 4

Let  $A$  be any subset in TVS  $X$  and  $G$  be an open set in TVS  $X$ . Then which of the followings is/are true for  $A + G$ ?

1. Open
2. Closed
3. Neither open nor closed

### Question 5

Let  $B$  be a bounded set in TVS  $X$ . Is the closure  $\bar{B}$ , bounded in  $X$ ?

### Question 6

Every compact subset of a topological vector space is bounded-is this statement true or false?

### Question 7

A linear operator  $T: X \rightarrow W$  is continuous if it is continuous at the zero  $(0_X)$  of  $X$ - is this statement true or false?

### Question 8

What is the sufficient condition of normality for a TVS  $X$  in Kolmogorov theorem?

### Question 9

Consider the NLS  $\mathcal{P}$  of all real polynomials  $P = a_0 + ax + \dots + ax^n, \forall a_i \in R$  and  $\forall n \in Z^+ \cup \{0\}$  with the norm  $\|P\| = \max_i |a_i|$ . Is this space a Banach space?

#### Question 10

Let  $X$  and  $Y$  be two NLS with same scalar field and  $T \in Bd\mathcal{L}(X, Y)$ . If  $\{x_n\}$  converges weakly to  $x_0$  in  $X$ , then what is the weak limit of  $\{T(x_n)\}$ ?

#### Question 11

Is every Hilbert space,  $H$ , strictly convex?

#### Question 12

What is the resolvent set of a linear operator  $T$  on a NLS  $X$ ?

#### Question 13

What is the sufficient condition that the sum of two projection operators  $P_{Y_1}$  and  $P_{Y_2}$  is also a projection operator in a Hilbert space?

#### Question 14

Let  $T$  be a linear operator on finite dimensional Hilbert space and  $\lambda$  be an eigen value of  $T$ . Find a eigen value of  $T^*$ .

#### Question 15

Is every Cauchy sequence bounded in NLS  $X$ ?

#### Question 16

What kind of set theoretic property a convex sets must have?

#### Question 17

What kind of property a symmetric set must have in a vector space?

#### Question 18

Let  $T_a$  and  $M_\lambda$  be translation and multiplication operators on a TVS respectively and  $(M_\lambda \circ T_a)(x) = M_\lambda(T_a(x))$ . What property  $M_\lambda \circ T_a$  must have as an operator on  $X$ ?

#### Question 19

Let  $A$  be a closed set in a TVS  $X$ . Then what should be the topological property of  $a+A$  for any  $a \in X$ ?

#### Question 20

In a TVS  $X$ , let  $A$  be compact and  $B$  be closed. What can you conclude about  $A+B$ ?

#### Question 21

If  $Y$  is a convex set in a TVS  $X$  then what are the topological property of  $\text{Int}(Y)$  [interior of  $Y$ ] and  $\text{cl}(Y)$  [closure of  $Y$ ] ?

#### Question 22

Let  $f$  be a non-zero linear functional over a TVS  $X$ . Then what kind of equivalent topological property  $f$  must have?

- (i)  $N^c = \{x \in X | f(x) \neq 0\}$  is open
- (ii)  $f$  is continuous
- (iii)  $N(f) = \{x \in X | f(x) = 0\}$  is dense in  $X$

#### Question 23

Let  $X$  be a locally compact TVS. Then what should be the relation between any basis  $B$  of the vector space  $X$  and natural number set  $\mathbb{N}$ ?

**Question 24**

A TVS  $X$  has a convex bounded neighbourhood of additive identity in  $X$ . Then what special topological property  $X$  must have?

**Question 25**

Let  $X$  be a Banach space and  $Y$  a NLS over the same scalars and let  $T_n \in Bd\mathcal{L}(X, Y)$  such that for each  $x \in X$ ,  $\{T_n(x)\}$  is Cauchy in  $Y$ . Then what can you conclude about  $\sup_n \|T_n\|$ ?

**Question 26**

Let  $P_0(t) = 1, P_1(t) = t^1, \dots, P_n(t) = t^n$  in  $a \leq t \leq b$  and  $n$  be kept fixed. If  $Y = \text{lin.hull} [P_0, P_1, P_2, \dots, P_n]$  is the subspace of  $C[a, b]$  generated by  $[P_0, P_1, P_2, \dots, P_n]$ , then what can be the dimension of  $Y$  as a subspace of  $C[a, b]$ ?

**Question 27**

Identify the space of all real polynomials of degree not exceeding two using isomorphism.

**Question 28**

If  $T \in Bd\mathcal{L}(X, X)$  where  $X$  is a Banach space, if  $\|T\| < 1$ , then what can you conclude about the bounded linear operator  $I - T$ ?

**Question 29**

Which topological property, a spectrum of a bounded linear operator  $T: X \rightarrow X$  where  $X$  is a Banach space, must have?

**Question 30**

What is the necessary and sufficient property of a bounded linear operator

$P : H \rightarrow H$  ( $H$  being a Hilbert space) to be a Projection?