Question Bank For PG Course

Mathematics

Paper-2A

REAL ANALYSIS & METRIC SPACES : PGMT-IIA

Question 1

Let $f: [0,1] \to R$ and $g: [0,1] \to R$ be two real- valued functions given by $f(x)=x^2$ and $g(x)=x$. Then what is the relation between *sup* $\{f(x): x \in [0,1]\}$ and *sup* $\{g(x): x \in [0,1]\}$?

Question 2

If $E = \{1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots\}$, then find *inf* E.

Question 3

Find the limit point(s) of the set

$E = \{0, 1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots\}$.

Question 4

Find $\cap_{n=1}^{\infty} I_n$ where $I_n = (-\frac{1}{n}, \frac{1}{n})$, $n \in N$, N being the set of natural numbers.

Question 5

Let $[a_1, b_1]$, $[a_2, b_2], \ldots, [a_n, b_n]$ are finitely many disjoint closed intervals and $F = \cup_{k=1}^{\infty} [a_k, b_k]$. Then what is the Lebesgue measure of F, i.e., $m(F)$?

Question 6

Let F be a countable set. Then what is Lebesgue measure of F ?

Question 7

Let $E = \cup_{n=2}^{\infty}\left(n-\frac{1}{n}, n+\frac{1}{n}\right)$. Find $m(E)$.

Question 8

Find $L-\int \psi dx$, where $\psi:[0,1]\rightarrow R$ is given by

$$\begin{aligned}\psi(x) &= 0, \quad if \quad x \quad is \quad irrational \\ &= 1, \quad if \quad x \quad is \quad rational\end{aligned}$$

Question 9

Find $\int x^2 d[x]$

Question 10

. Obtain the fourier series of the function

$$\begin{aligned}f(x) &= 0, \quad if \quad -\pi \le x < 0 \\ &= 1, \quad if \quad 0 \le x \le \pi\end{aligned}.$$

Question 11

If A and B are two sets in the metric space (X,d) with $A \cap B \neq \Phi$, then what can be said about the upper bound of diameter of $A \cup B$?

Question 12

If G is any set in the metric space (X,d) and the closure of G is G, then what is relation between diameters of G, $\overline{G}$?

Question 13

Let (X,d) and (Y,ρ) be two metric spaces. Also let $f:(X,d)\to(Y,\rho)$ and $g:(X,d)\to(Y,\rho)$ be two continuous functions. Then what can be said about the set $\{x\in X:\ f(x)\neq g(x)\}$?

Question 14

What can be said about the continuity or uniform continuity of the function $f\ :\ (0,1]\to R$, given by $f(x)=\frac{1}{x}$, where R is the set of real numbers?

Question 15

Let (X,d) be any metric space and $\Phi\neq A\subset X$. If $u\in\overline{A}$, then what can be said about $dist(u,A)$?

Question 16

Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function. Then which of the following sets is/are open and which is/are closed? $S_1=\{x\in\mathbb{R}:f(x)>0\}$; $S_2=\{x\in\mathbb{R}:f(x)\geq 0\}$

Question 17

Find the derived set of the set $A=\{\frac{1}{m}+\frac{1}{n}:m,n=1,2,3,\ldots\}$

Question 18

Find the derived set of $S=\{m+\frac{1}{n}:m\in\mathbb{N},n\in\mathbb{N}\}$

Question 19

Which sets in $\mathbb{R}$ are both open and closed?

Question 20

If A and B are two non- empty sets in $\mathbb{R}$ such that $a<b,\forall a\in A$ & $\forall b\in B$, then what is the relation between $supA$ and $supB$?

Question 21

Find the Lebesgue measure of the set $\mathbb{Q}$ of rational numbers.

Question 22

Find $inf\, E$ when $E = \{1 + \frac{1}{n} : n \in \mathbb{N}\}$.

Question 23

What can be said about the union of the closed sets $\bigcup_1^\infty F_n$, where $F_n = \left[0, \frac{n}{n+1}\right], \forall n \in \mathbb{N}$?

Question 24

For a bounded set A in $\mathbb{R}$, if T={$|x-y|$:x,y∈A}, then find $supT$.

Question 25

Find the Lebesgue measure of zeros of the function $f(x) = \sin\frac{\pi}{x}, x \neq 0, x \in \mathbb{R}$.

Question 26

If A=(0,1], then find $dist(0,A)$.

Question 27

Let (X,d) and (Y,ρ) be two metric spaces. Also let f:X→Y and g:X→Y be two continuous functions. Then what can be said about the set {x∈X:$f(x)$=$g(x)$}?

Question 28

If $\xi = (\xi_1, \xi_2, \ldots) \in l_p$ and $\eta = (\eta_1, \eta_2, \ldots) \in l_q$ where p is a real number > 1 and $\frac{1}{p} + \frac{1}{q} = 1$, what does Hölder's inequality state?

Question 29

If $f(x)$=μ,∀x∈E, where μ∈$\mathbb{R}$ is constant and E is a set of finite measure, then find L−∫$f\,dx E$.

Question 30

Which of the following statement(s) represent(s) Bolzano-Weirstrass Theorem:
(i) Every infinite set has a limit point;
(ii) Every bounded infinite set has a limit point;
(iii) Every sequence has a convergent subsequence;
(iv) Every bounded sequence has a convergent subsequence;
(v) Every bounded set has a limit point.