Question Bank For PG Course

Mathematics

Paper-3B

PARTIAL DIFFERENTIAL EQUATIONS AND SPECIAL FUNCTION : PGMT-IIIB

Question 1

Find a condition so that there exists a relation between two functions (x, y) and $v(x, y)$not involving the variables x and y explicitly.

Question 2

What is the necessary and sufficient condition for the Pfaffian differential equation $\vec{X}.d\,\vec{r} = 0$ where $\vec{X} = P\,\vec{i} + Q\,\vec{j} + R\,\vec{k}$ and $d\,\vec{r} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$ to be integrable?

Question 3

What is the primitive of the equation $ay^2z^2dx + bz^2x^2dy + cx^2y^2dz = 0$?

Question 4

What is the Lagrange's auxiliary equation of the partial differential equation $y^2p - xyq + x(2y - z) = 0$?

Question 5

Find Charpit's equations corresponding to the partial differential equation $2zx - px^2 - 2qxy + pq = 0$.

Question 6

Find the complete integral of the partial differential equation $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y}$.

Question 7

Find the general solution of
$(D^2 - DD' - 2D'^2 + 2D + 2D')z = 0$
where $D \equiv \frac{\partial}{\partial x}$ **and** $D' \equiv \frac{\partial}{\partial y}$.

Question 8

Find the particularl integral of
$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin(x + 2y)$.

Question 9

By the transformation $x = e^u,\ y = e^v$**,the partial differential equation**
$x^2 \frac{\partial^2 z}{\partial x^2} - 4y^2 \frac{\partial^2 z}{\partial y^2} - 4y \frac{\partial z}{\partial y} - z = 0$
reduces to which form?

Question 10

Examine the nature of the following partial differential equation

$$4y^2 z_{xx} + 2(1 - y^2) z_{xy} - z_{yy} - \frac{2y}{1+y^2}(2z_x - z_y) = 0.$$

Question 11

If a function $\emptyset$ **is harmonic in a closed region** V **and** $\frac{\partial \emptyset}{\partial n} = 0$ **on the boundary** S **of the closed region** V **thenwhat is the** $\emptyset$ **function?**

Question 12

If $\psi(x, y) = X(x)\,Y(y)$ **satisfies the two dimensional Laplace's equation**
$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ **then find** $\frac{1}{X}\frac{d^2 X}{dx^2}$ **and** $\frac{1}{Y}\frac{d^2 Y}{dy^2}$.

Question 13

If a function $\psi(x,y)$ satisfies the Laplace equation $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ at any point interior to the rectangle $0 \le x \le a$, $0 \le y \le b$ then state the boundary condition in case of an interior Dirichlet problem for a rectangle.

Question 14

Solve the one dimensional diffusion equation $\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$ if it has a solution of the type $T(x,t) = X(x)\,Y(t)$ where $\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{kY}\frac{dY}{dt} = -\alpha^2$, α being a nonzero real constant.

Question 15

Reduce the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0$ to its canonical form.

Question 16

What is the general solution of the following equation $\frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz}$?

Question 17

Find the primitive of the equation $yz\,dx = zx\,dy + y^2dz.$

Question 18

Determine the category of the following first order partial differential equation $xyp + x^2yq = x^2y^2z^2$ where $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}.$

Question 19

Form a partial differential equation by eliminating the arbitrary function f from $f(x+y+z,\ x^2+y^2-z^2)=0$.

Question 20

Find the complete integral of $zpq=p+q$.

Question 21

Solve the equation $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=x^2+xy+y^2$.

Question 22

Reduce the partial differential equation

$$x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}-y\left(\frac{\partial z}{\partial y}\right)+x\left(\frac{\partial z}{\partial x}\right)=0$$

by the transformation $x=e^u$ & $y=e^v$.

Question 23

By what transformation the partial differential equation $y^2\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x \partial y}+x^2\frac{\partial^2 z}{\partial y^2}=\frac{y^2}{x}\frac{\partial z}{\partial x}+\frac{x^2}{y}\frac{\partial z}{\partial y}$ transforms to its canonical form?

Question 24

What is the type of the following partial differential equation :- $(\cos^2 x)\frac{\partial^2 z}{\partial x^2}+(sin2x)\frac{\partial^2 z}{\partial x \partial y}+(\sin^2 x)\frac{\partial^2 z}{\partial y^2}=x$?

Question 25

What are the characteristic equations of the following partial differential equation :- $(1+x^2)\frac{\partial^2 z}{\partial x^2}+(1+y^2)\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$

Question 26

If a function ∅ is such that ∇2∅=0 in a closed region V and ∅=0 on the boundary S of the region V then what is the value of ∅ inside the region V?

Question 27

What will be the form of the Laplace's equation $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ in plane polar co-ordinate (r, θ)?

Question 28

If $T\,(x, t) = X(x)\tau\,(t)$ satisfies the heat equation $\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial x^2}$ then what is the relation between $\frac{1}{X}\frac{d^2 X}{dx^2}$ and $\frac{1}{K}\frac{1}{\tau}\frac{d\tau}{dt}$?

Question 29

Solve the following heat equation $\frac{\partial T}{\partial t} - k\frac{\partial^2 T}{\partial x^2} = 0,\ 0 \leq x \leq \pi,\ t \geq 0$ subject to the conditions

(i) $T(0, t) = T(\pi, t) = 0,\ t \geq 0$

(ii) $T(x, 0) = \begin{cases} x & if\ 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & if\ \frac{\pi}{2} \leq x \leq \pi \end{cases}$

(iii) $T(x, t)$ remains finite as $t \to \infty$

Question 30

What is the form of the general solution of the wave equation $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0$,
$-\infty < x < \infty,\ t \geq 0$?