Question Bank For PG Course

Mathematics

Paper-5A

PRINCIPLES OF MECHANICS : PGMT-VA

Question 1

Identify which of the following is not a feature of conservative forces:

1. A potential energy function V exists having a definite value at every point.
2. T+V=constant, where T is the kinetic energy and V is the potential energy.
3. The work done by the force is path dependent.
4. Around any closed path the work done is zero.

Question 2

Write the expression for kinetic energy(T) of a particle of mass m in spherical polar coordinate system (r, θ, ϕ)

Question 3

Consider the following constraint :

$(y + yz - 1)\dot{x} + (x + xz - 1)\dot{y} + xy\dot{z}$

Classify it.

Question 4

Which of the following statement is false about D'Alembert's principle :

1. It depends upon Newton's second law of motion.
2. It has the ability to get rid of the constraint forces.
3. It asserts that the work done by applied forces and inertial forces in an actual displacement is zero.
4. None of the above.

Question 5

For a conservative N-particle system having n degrees of freedom, write down the Lagrange's equations of motion with usual notations

Question 6

In which system the quantity $\sum_{j=0}^{n} \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L$(with usual notations) is a constant ?

Question 7

Which of the following is correct about Coriolis force?

1. It can change the speed of a particle.
2. It does not contribute to the energy equation.
3. It bends the path of a particle to the left in the Northern Hemisphere.
4. None of the above.

Question 8

Using standard notations, the relation between the Hamiltonian and Lagrangian of a system of particles with n degrees of freedom is given by

Question 9

Write the Hamilton's canonical equations of motion for a n-particle system $(i = 1,2, \dots, n)$in terms of Poisson Brackets(with usual notations) –

Question 10

With usual notations, which of the following is not a property of Poisson bracket?

1. $\{u_1 + u_2, v\} = \{u_1, v\} + \{u_2, v\}$
2. $\{u, v\} = \{v, u\}$
3. $\{u, vw\} = \{u, v\}w + v\{u, w\}$
4. $\{u, \{v, w\}\} + \{v, \{w, u\}\} + \{w, \{u, v\}\} = 0$

Question 11

The Lagrangian of a plane pendulum is $L = \frac{1}{2} ml^2\dot{\theta}^2 + mgl \cos\theta$, the notations having usual meaning. Write Its Hamiltonian .

Question 12

What is the Hamilton's principle of least action (notations having usual meaning)?

Question 13

From which equation the Brachistochrone is a path obtainable (notations having usual meanings)?

Question 14

Consider the canonical transformation $Q = -p,\ P = q + \lambda p^2$, where λ is a constant and (q,p), (Q,P) are old and new set of canonical variables respectively.
What is the Type 2 generating function for this transformation?

Question 15

If $S(q, E, t)$ is the type 2 generating function of canonical transformation and other notations have usual meaning, then write the Hamilton-Jacobi equation for a free particle .

Question 16

The expression for kinetic energy (T) of a particle of mass m in cylindrical polar coordinate system (r, θ, z) is given by

1. $T = \frac{1}{2}m(\dot{r}^2 + r\dot{\theta}^2 + \dot{z}^2)$
2. $T = \frac{1}{2}m(\dot{r}^2 + \dot{\theta}^2 + r^2\dot{z}^2)$
3. $T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2)$
4. $T = \frac{1}{2}m(\dot{r}^2 + \dot{\theta}^2 + r\dot{z}^2)$

Question 17

For a particle of mass m moving in one dimension under a potential $V(x) = A|x|^n$, the period of oscillation is given by

1. $T = \frac{2}{m}\sqrt{\frac{2\pi m}{E}}\left(\frac{A}{E}\right)^{1/n}\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$
2. $T = \frac{2}{n}\sqrt{\frac{2\pi m}{E}}\left(\frac{E}{A}\right)^{1/n}\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$
3. $T = \frac{2}{n}\sqrt{\frac{2\pi n}{E}}\left(\frac{E}{A}\right)^{1/n}\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$
4. $T = \frac{2}{m}\sqrt{\frac{2\pi n}{E}}\left(\frac{A}{E}\right)^{1/n}\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$

Question 18

A constraint of the form :

$$\vec{f}(\vec{r}_j, t) = 0, j = 1,2, \cdots, N$$

is called a

1. kinematical constraint
2. bilateral constraint
3. unilateral constraint
4. geometric constraint

Question 19

Kinetic energy of a holonomic and scleronomic system of N particles and n generalized coordinates may be written in the form:

$T = \frac{1}{2}\sum_{i,k=1}^{n} a_{ik}\dot{q}_i\dot{q}_k$,

where

1. $$a_{ik} = \sum_{j=1}^{n} m_j \frac{\partial \vec{r}_i}{\partial q_j} \cdot \frac{\partial \vec{r}_k}{\partial q_j}$$
2. $$a_{ik} = \sum_{j=1}^{N} m_j \frac{\partial \vec{r}_j}{\partial q_i} \cdot \frac{\partial \vec{r}_k}{\partial q_j}$$
3. $$a_{ik} = \sum_{j=1}^{N} m_j \frac{\partial \vec{r}_j}{\partial q_i} \cdot \frac{\partial \vec{r}_j}{\partial q_k}$$
4. $$a_{ik} = \sum_{j=1}^{N} m_j \frac{\partial \vec{r}_i}{\partial q_j} \cdot \frac{\partial \vec{r}_k}{\partial q_j}$$

Question 20

The Lagrangian for a simple pendulum of length l, mass of the bob m and angle of deflection θ, is given by

1. $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta)$
2. $L = \frac{1}{2}ml\dot{\theta} + mgl(1-\cos\theta)$
3. $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1+\cos\theta)$
4. $L = \frac{1}{2}ml\dot{\theta} - mgl(1-\cos\theta)$

Question 21

In a dynamical system the kinetic and potential energies are $T = \frac{1}{2}\frac{\dot{q}_1^2}{a+bq_2^2} + \frac{1}{2}\dot{q}_2^2, V = c + dq_2^2$ and $\beta = \frac{\dot{q}_1}{a+bq_2^2}$ $(a, b, c, d$ being constants and q_1, q_2 having usual meaning).

The Routhian of this system is given by

1. $R = \frac{1}{2}\dot{q}_2^2 + \left(d - \frac{1}{2}b\beta^2\right)q_2^2 - c - \frac{1}{2}a\beta^2$
2. $R = \frac{1}{2}\dot{q}_2^2 + \left(d + \frac{1}{2}b\beta^2\right)q_2^2 - c - \frac{1}{2}a\beta^2$
3. $R = \frac{1}{2}\dot{q}_2^2 + \left(d + \frac{1}{2}b\beta^2\right)q_2^2 + c + \frac{1}{2}a\beta^2$
4. $R = \frac{1}{2}\dot{q}_2^2 - \left(d + \frac{1}{2}b\beta^2\right)q_2^2 - c - \frac{1}{2}a\beta^2$

Question 22

Using standard notations, the expression for Coriolis force with reference to a frame rotating with angular velocity ω is given by

1. $m(\vec{r} \times \vec{\omega})$
2. $2m(\vec{v} \times \vec{\omega})$
3. $2m(\vec{v} \times \dot{\vec{\omega}})$
4. $m\vec{\omega} \times (\vec{r} \times \vec{\omega})$

Question 23

The Lagrangian for the motion of a particle in a rotating frame is given by $L = \frac{1}{2}m\dot{\vec{r}}^2 + \frac{1}{2}m\omega^2(x^2+y^2) + m\omega(x\dot{y} - y\dot{x})$.
The Hamiltonian of the particle will be

1. $H = \frac{1}{2m}(p_x + p_y + p_z) - \omega(yp_x - xp_y)$
2. $H = \frac{1}{2m}({p_x}^2 + {p_y}^2) + 2\omega(yp_x - xp_y)$
3. $H = \frac{1}{2m}({p_x}^2 + {p_y}^2 + {p_z}^2) + \omega(yp_x - xp_y)$
4. $H = \frac{1}{2m}({p_x}^2 + {p_y}^2) + \omega(xp_x + yp_y)$

Question 24

The Poisson bracket
$$\{x, xp_x - yp_y + ax^2 + by^2\}$$
with a and b as constants is equal to

1. x
2. p_y
3. y
4. p_x

Question 25

If the linear transformation of a generalized coordinate q and corresponding momentum p given by
$$Q = p + 2iq, \quad P = -\frac{1}{2}\left(q + \frac{ip}{\beta}\right)$$
is canonical (here $i = \sqrt{-1}$), then the value of the constant β is

1. 1
2. 2
3. 3
4. 4

Question 26

Hamiltonian of a one dimensional dynamical system is given by $H = \frac{1}{2}(\alpha p^2 + \beta q^2 + 2\gamma pq)$,
where $\alpha,\ \beta,\ \gamma$ are constants.
The Poisson bracket $\{p, H\}$is equal to

1. $-\alpha q + \gamma p$
2. $\beta q + \gamma p$
3. $-\alpha q - \gamma p$
4. $-\beta q - \gamma p$

Question 27

The number of degrees of freedom of two particles moving on a space curve and having constant distance between them, is

1. 3
2. 4
3. 1
4. 2

Question 28

The action of the path of a physical system is given by

1. $\int_{t_0}^{t_1} L(q_i, \dot{p}_i, t)dt$
2. $\int_{t_0}^{t_1} L(q_i, \dot{q}_i, t)dt$
3. $\int_{t_0}^{t_1} L(p_i, \dot{q}_i, t)dt$
4. $\int_{t_0}^{t_1} L(p_i, \dot{p}_i, t)dt$

Question 29

Consider the canonical transformation

$$Q = q\cos\theta - \frac{p}{m\omega}\sin\theta,$$

$P = m\omega q\sin\theta + p\cos\theta,$

where (q,p), (Q,P) are old and new set of canonical variables respectively. The Type 1 generating function for this transformation is given by

1. $G = \frac{1}{2}m\omega(q^2 + P^2)\cot\theta - m\omega qP\,\mathrm{cosec}\,\theta$
2. $G = \frac{1}{2}m\omega(q^2 + Q^2)\,\mathrm{cosec}\,\theta + m\omega qQ\cot\theta$
3. $G = \frac{1}{2}m\omega(q^2 + P^2)\cot\theta + m\omega qP\,\mathrm{cosec}\,\theta$
4. $G = \frac{1}{2}m\omega(q^2 + Q^2)\cot\theta - m\omega qQ\,\mathrm{cosec}\,\theta$

Question 30

The Hamilton-Jacobi equation for a dynamical system is given by

1. $H\left(q, \frac{\partial G_2}{\partial q}, t\right) + \frac{\partial G_2}{\partial t} = 0$
2. $H\left(p, \frac{\partial G_1}{\partial p}, t\right) + \frac{\partial G_1}{\partial t} = 0$
3. $H\left(q, \frac{\partial G_1}{\partial q}, t\right) + \frac{\partial G_1}{\partial t} = 0$
4. $H\left(p, \frac{\partial G_2}{\partial p}, t\right) + \frac{\partial G_2}{\partial t} = 0$