Question Bank For PG Course

Mathematics

Paper-6B

FUNCTIONAL ANALYSIS : PGMT-VIB

Question 1

Consider the set $X = \{1, \quad 2, \quad 3, \quad ...\}$ regarded as subspace of the set of real numbers R with usual metric. Then check the completeness, compactness and bounds (if any) of X.

Question 2

Consider the set $Y = \{1, \quad \frac{1}{2}, \quad \frac{1}{3}, \quad ...\}$ regarded as subspace of the set of real numbers R with usual metric. Which subsets of Y are both open and closed?

Question 3

In a metric space (X, d), take $x_0 \in X$. For $x \in X$, let $f_x : \quad X \to R$ (space of real numbers with usual metric) be given as $f_x(y) = d(y,x) - d(y,x_0)$, for $y \in X$. Then what can be said about the continuity of f_x, for an arbitrary $x \in X$?

Question 4

Consider the set $C[a,b]$ of all real valued continuous functions on the closed intervals $[a,b]$ with the *sup* metric. What can be said about compactness and bounds (if any) of $C[a,b]$?

Question 5

What can be said about the subset $\{f_n\} \subset C[0,1]$ with respect to being uniformly bounded , where

$f_n(t) = 1 + \frac{t}{n}; 0 \le t \le 1;$ and the set

$C[0,1]$ of all real valued functions on the closed intervals $[0,1]$ is a metric space with the *sup* metric.

Question 6

Give a necessary and sufficient condition for the subset $M \subset C[a,b]$ to be compact, where the set $C[a,b]$ of all real valued **continuous** functions on the closed intervals $[a,b]$ is a metric space with the *sup* metric.

Question 7

Let T be a linear operator over a normed linear space X and T is continuous at $a \in X$. Then what can be said about the continuity of T at other points of X ?

Question 8

Find the order of the representive matrix of a linear operator

$T: \quad R^n \to R^m$.

Question 9

Find the dimension of the space of bounded linear operators from R^n to R^n .

Question 10

Let X be a Banach space. Then what can be said about the compactness and completeness of the subset $\{x \in X : \|x\| = 1\}$ of X ?

Question 11

For what values of p, the sequence space l_p of real sequences is a Hilbert space?

Question 12

For what values of p, parallelogram law holds in the sequence space l_p of real sequences?

Question 13

Let z be a fixed member of a Hilbert space H. Then what can be said about the norm of the bounded linear functional f over H given by $f(x) = <x, z>$, for all $x \in X$?

Question 14

In a separable Hilbert space H, how many elements can an orthonormal system have?

Question 15

Consider the quotient space $C[0,1]/L$, where $C[0,1]$ is the linear space of all real valued continious functions over the closed interval $[0,1]$ and L consists of those members $f \in C[0,1]$ with $f(1)=0$ i.e., vanishing at $t=1$. Now if $h \in C[0,1]$ such that $h \notin (f+L)$, then find $(h+L) \cap (f+L)$.

Question 16

The distance D(A,B) between two nonempty subsets of A and B of a metric space (X,d) is defined to be D(A,B)= inf d(a,b), a ϵ A, b ϵ B. What can you conclude about metric property of D?

Question 17

The distance D(A,B) between two nonempty subsets of A and B of a metric space (X,d) is defined to be D(A,B)= inf d(a,b), a ϵ A, b ϵ B. If D(A,B)=0 then what should be the actual relation between A and B?

Question 18

Let X be the set of all polynomials defined on [0,1]. Consider a metric d on X defined by d(p,q)=sup|p(t)-q(t)|, t ϵ [0,1] for all p,q ϵ X. Then what kind of metric space (X,d) is?

Question 19

Let $C[0,1]$ be the set of all real valued continuous functions on $[0,1]$. Consider two metric d_1 and d_2 defined by $d_1(f,g) = \sup_{t\epsilon[0,1]} |f(t) - g(t)|$, $f,g \,\epsilon\, C[0,1]$ and $d_2(f,g) = \int_0^1 |f(t) - g(t)| dt$. Let Y be the subset of $C[0,1]$ consisting of all $p \,\epsilon\, C[0,1]$ such that $p(0)=p(1)$. Then what should be the type of (Y, d_1) and (Y, d_2) as a metric space?

Question 20

What is the metric relation between C[0,1] and C[a,b] ?

Question 21

Let X be a non-empty vector space with a norm d. Then what kind of metric space (X,d) is?

Question 22

What are the topological properties of the closure of a unit ball on a finite dimensional normed space must have?

Question 23

Let X be a normed space such that the closed unit ball is compact, then which kind of linear algebraic property X must have?

Question 24

Let X and Y be normed linear spaces. What is the necessary and sufficient condition of a linear operator T : X→Y to be bounded?

Question 25

Let Y be a subspace of a Hilbert space H. What is the necessary and sufficient condition on Y to be complete?

Question 26

What is the topological property of every subset of a separable inner product space?

Question 27

If H is separable Hilbert space then what property every orthonormal set in H must have?

Question 28

Let H be a Hilbert space. If H contains an orthonormal sequence which is total in H , then what is a special topological property H must have?

Question 29

Let X be the inner product space of all real-valued continuous functions on $[0,2\pi]$ with inner product defined by $< x, y >= \int_0^{2\pi} x(t)y(t)dt$. Consider the sequence $\{u_n\}$ where $u_n = \cos nt$. What kind of sequence $\{u_n\}$ is?

Question 30

What is the necessary and sufficient condition for a subspace Y of a Hilbert space H to be closed in H?