

**Question Bank For PG Course**

**Mathematics**

Paper-7A

**DIFFERENTIAL EQUATIONS, INTEGRAL TRANSFORMATIONS : PGM-T-VIIA**

Question 1

The Laplace transform of a function  $f(t)$  defined for  $t \geq 0$  is defined by  $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ . If  $f(t) = e^{t^2}$ , then find  $F(s)$ .

Question 2

The Laplace transform of a function  $f(t)$  is denoted by  $\mathcal{L}[f(t), s] = \int_0^{\infty} e^{-st} f(t) dt$ . Then find  $\mathcal{L}\left[\frac{\sin \omega t}{t}, s\right]$ .

Question 3

Fourier transform of a function  $f(t)$  is denoted by  $\mathfrak{F}[f(t), \xi] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\xi t} dt$ . Then find  $\mathfrak{F}[e^{-a|t|}, \xi] (a > 0)$

Question 4

Find the inverse Laplace transform of  $\frac{s-2}{s^2+4s+13}$

Question 5

Find the inverse Laplace transform of  $\frac{1}{(s+a)^n}$

Question 6

If  $L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s)$ , then find  $L\{tf(t)\}$

## Question 7

If  $F_c\{f(x)\} = \int_0^\infty f(t) \cos(\omega t) dt = F_c(\omega)$ , then  $F_c\{f''(x)\} + \omega^2 F_c(\omega) + f'(0) = b$  implies

1.  $b < 0$
2.  $b > 0$
3.  $b = 1$
4.  $b = 0$

## Question 8

The Laplace transform of a function  $f(t)$  is denoted by  $\mathcal{L}[f(t), s] = \int_0^\infty e^{-st} f(t) dt$ .

If  $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$  then find the value of  $\mathcal{L}[f(t), s]$ .

## Question 9

The Laplace transform of a function  $f(t)$  is denoted by  $\mathcal{L}[f(t), s] = \int_0^\infty e^{-st} f(t) dt$ .

If  $f(t) = \frac{1 - \cos t}{t}$  then find the value of  $\mathcal{L}[f(t), s]$ .

## Question 10

If  $f(t)$  is a periodic function with period  $T$ , then find the Laplace transform  $\mathcal{L}[f(t), s]$

## Question 11

The Laplace transform of a function  $f(t)$  is denoted by  $\mathcal{L}[f(t), s] = \int_0^\infty e^{-st} f(t) dt$ .

If  $f(t) = \operatorname{erf}\left(t^{\frac{1}{2}}\right)$  then find the value of  $\mathcal{L}[f(t), s]$ .

Question 12

If  $F(\alpha)$  is the Fourier transform of  $f(x)$ , then find the Fourier transform of  $f(ax)$  ( $a > 0$ )

Question 13

If  $F(\alpha)$  is the Fourier transform of  $f(x)$ , then find the Fourier transform of  $f(x) \cos ax$

Question 14

If the Fourier sine transform of  $f(x)$  is  $\frac{\alpha}{1+\alpha^2}$ , thus find  $f(x)$

Question 15

For  $a > 0, b > 0$ , find the value of

$$\int_0^{\infty} \frac{d\alpha}{(a^2 + \alpha^2)(b^2 + \alpha^2)}$$

Question 16

The Laplace transform of a function  $f(t)$  defined for  $t \geq 0$  is denoted by  $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ . What is the Laplace transform of  $f(t) = t e^{-at} \cos \omega t$ ?

Question 17

The Laplace transform of a function  $f(t)$  is denoted by  $\mathcal{L}[f(t), s] = \int_0^{\infty} e^{-st} f(t) dt$ . Find  $\mathcal{L}\left[\int_0^t \frac{\sin x}{x} dx, s\right]$ .

Question 18

Fourier transform of a function  $f(t)$  is denoted by  $\mathfrak{F}[f(t), \xi] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\xi t} dt$ . Find  $\mathfrak{F}\left[e^{-\frac{t^2}{2}}, \xi\right]$ .

### Question 19

What is the inverse Laplace transform of  $\frac{s}{(s^2+a^2)^2}$ ?

### Question 20

What is the inverse Laplace transform of  $e^{-a\sqrt{s}}$ ?

### Question 21

If  $L\{f(t)\} = \int_0^\infty e^{-st} f(t) dt = F(s)$ , then find  $L\{f(at)\}$ .

### Question 22

What is the Mellin Transform of  $\frac{1}{1+x}$ ?

### Question 23

The Laplace transform of a function  $f(t)$  is denoted by  $\mathcal{L}[f(t), s] = \int_0^\infty e^{-st} f(t) dt$ .  
What is the Laplace transform of  $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$ ?

### Question 24

The Laplace transform of a function  $f(t)$  is denoted by  $\mathcal{L}[f(t), s] = \int_0^\infty e^{-st} f(t) dt$ .  
What is the Laplace transform of  $f(t) = \frac{1-\cos t}{t}$ ?

### Question 25

Find the zero order Hankel transform of  $\frac{e^{-ar}}{r}$  ( $a > 0$ ).

### Question 26

Find the zero order Hankel transform of  $e^{-ar^2}$ , ( $a > 0$ )?

### Question 27

Fourier transform of a function  $f(x)$  is denoted by  $F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-isx} dx$ .

What is the Fourier transform of  $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & \text{Otherwise} \end{cases}$ ?

---

Question 28

If  $F(s)$  is the Fourier transform of  $f(x)$ , find the Fourier transform of  $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & \text{Otherwise} \end{cases}$ .

---

Question 29

If  $F_c(s)$  is the Fourier cosine transform of  $f(x)$ , then find the Fourier cosine transform of  $f(x) = e^{-ax} \cos ax, a > 0$ .

---

Question 30

If  $F_c(s)$  is the Fourier cosine transform of  $f(x)$ , then find the Fourier cosine transform of  $f(x) = e^{-ax} \sin ax, a > 0$ .

---