

## Question Bank For PG Course

Mathematics

Paper-9A(i)

Special Paper: Pure Mathematics

ADVANCED COMPLEX ANALYSIS: PGMT-IXA

### Question 1

If  $f(z) = u + iv$  is analytic in a region  $R$ ,  
then find the value of  $\frac{\partial(u,v)}{\partial(x,y)}$ .

### Question 2

Given that  $f(z)$  is analytic at  $z = \alpha$  and  
 $f'(\alpha) = f''(\alpha) = f'''(\alpha) = 0$  but  
 $f^{(iv)}(\alpha) \neq 0$ . Then find the magnifies  
angles of  $f(z)$  at  $z = \alpha$ .

### Question 3

What is the sufficient condition for convexity of a function ?

### Question 4

Find the order of the function  $f(z) = e^{z^n}$  ( $n$  is a positive integer)

### Question 5

Find the exponent of convergence of  
the zeros of  $f(z) = \cos z$ .

### Question 6

Given that  $f(z)$  is analytic in  $|z| < 1$ ,  
with a zero of order  $n$  at the origin. Also  
 $|f(z)| \leq 1$  for all  $z$  in  $|z| < 1$ . Then  
which of the following inequalities holds

1.  $|f(z)| \leq |z|^n, |z| > 1$
2.  $|f(z)| \geq |z|^n, |z| < 1$
3.  $|f(z)| \geq |z|^n, |z| > 1$
4.  $|f(z)| \leq |z|^n, |z| < 1$

### Question 7

Find the canonical product of the  
function  $f(z) = \sin z$ .

**Question 8**

What is Legendre's duplication formula?

**Question 9**

Given that  $f(z)$  is an entire function which never vanishes. Then there exists an entire function  $g(z)$  such that

1.  $f(z) = g(z)$  for all  $z$
2.  $f(z) = \frac{1}{g(z)}$  for all  $z$
3.  $f(z) = e^{g(z)}$  for all  $z$
4.  $f(z) = e^{-g(z)}$  for all  $z$ .

**Question 10**

An entire function  $f(z)$  is said to be of finite order if for some  $k$  and some  $R > 0$  and for all  $z$  with  $|z| \geq R$

1.  $|f(z)| \geq \exp(|z|^k)$
2.  $|f(z)| \leq \exp(|z|^k)$
3.  $|f(z)| \geq \exp(|z|^{2k})$
4.  $|f(z)| \leq \exp(|z|^{2k})$

**Question 11**

Find the branch points of the function  $f(z) = (z^3 - z)^{1/3}$

**Question 12**

Which of the following function is single valued?

1.  $\left(z^{\frac{1}{2}}\right)^3$
2.  $(z^3)^{\frac{1}{2}}$
3.  $(z^2)^{\frac{1}{2}}$
4.  $(z^{1/2})^2$

**Question 13**

Given that  $f(z)$  is a nonconstant analytic function in a domain  $D$ . Then examine the harmonicity of  $|f(z)|$ .

**Question 14**

Find the the harmonic conjugate of  
the function  $u(x, y) = \log \sqrt{x^2 + y^2}$

#### Question 15

Find the branch points of the function  
 $f(z) = \sin z^{1/2}$

#### Question 16

If  $f(z) = u + iv = z^2$  then what is the  
value of  $\frac{\partial(u,v)}{\partial(x,y)}$  ?

#### Question 17

Find the value of the infinite product  
 $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \dots$

#### Question 18

Determine the map which transforms  
the vertical semi-infinite strip  
 $-\frac{\pi}{2} < u < \frac{\pi}{2}, v > 0$  onto the upper half  
plane  $Im z > 0$ .

#### Question 19

What is the order of the function  
 $f(z) = \cos \sqrt{z}$  ?

#### Question 20

Find the exponent of convergence of  
the zeros of  $f(z) = \sin z$ .

#### Question 21

Find the residue of the Gamma function  
at  $-n$ .

#### Question 22

Find the critical points of the mapping  
 $f(z) = e^{2z} - 2iz + 3$ .

#### Question 23

What is the value of  $\Gamma(z)\Gamma(1-z)$ ?

**Question 24**

What is the order of the function

$$f(z) = e^{z^2} ?$$

**Question 25**

Which of the following function is single valued?

**Question 26**

Let  $f(z)$  and  $g(z)$  are entire functions having order 2 and 3 respectively.

Then what will be the maximum order of  $f(z) - g(z)$ ?

**Question 27**

Find the harmonic conjugate of the function  $u(x, y) = 2x - x^3 + 3xy^2$ .

**Question 28**

Find the branch points of the function

$$f(z) = z^{1/2} \cos z^{1/2}.$$

**Question 29**

Find the branch points of the function

$$f(z) = (z^3 + z^2 - 6z)^{1/2}.$$

**Question 30**

What is the Legendre's duplication formula?