## <u>Question Bank for PG Course</u> অঙ্ক (Mathematics)

প্রথম(ক) পত্র (Paper - IA) Abstract Algebra : PGMT-IA

- 1. What are the generators of the cyclic group Z of integers?
- 2. Determine whether, the ring of integers modulo p i.e.,  $Z_p$  (p is prime) is a field or skew filed or ring with unity .
- 3. How many disjoint cycles do yield the permutation  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$  as their product?
- 4. If Z be the set of integers, then number of elements of the quotient group Z/3Z is ......
- 5. Let G be a group and  $a \in G$ ,  $b \in G$ . Then the derived group of G is the subgroup of G generated by elements of some form .Find that form .....
- 6. Let  $R^*$  be the multiplicative group of non-zero real numbers. Find the kernel of the group homomorphism  $f: R^* \to R^*$  given by  $f(x) = |x|, \quad \forall x \in R^*$ .
- 7. What is the characteristic of an integral domain?
- 8. Let M be a maximal ideal of a non-zero commutative ring R with unity. Then determine whether R/M is an integral domain or field.
- 9. Let *R* be the ring of 2×2 matrices of the form  $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$ , where  $a \in F$ ,  $b \in F$ , *F* being a field. Then what may be said about the ideal  $M = \{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$ :  $b \in F \}$ ?
- 10. Let the field K be a finite extension of degree n over a finite field F with m elements. Then how many elements does K have?
- 11. What may be said about the multiplicative group of all non-zero elements of a finite field?
- 12. Let  $G_1$  be a finite extension field over a field F and  $G_2$  be a finite extension field over the field F. Then find  $[G_2: F]$ .

- 13. What is the degree of extension of the splitting field  $x^3 2 \in Q[x]$  over Q, where Q is the field of rational numbers?
- 14. Which of the following field is a prime field:
  - (a) field of rational numbers Q,
  - (b) field of real numbers R;
  - (c) field of complex numbers C;
  - (d) none of these.
- 15. Let *G* be a finite extension field over a field *F* of degree *m*. Then what is the dimension of the vector space *G* over the field F?