Question Bank for PG Course

অঙ্ক (Mathematics)

দ্বিতীয় (ক) পত্র (Paper - IIA)

Real Analysis & Metric Spaces : PGMT-IIA

1. Let $f: [0,1] \to R$ and $g: [0,1] \to R$ be two real- valued functions given by $f(x)=x^2$ and $g(x)=x$. Then what is the relation between *sup* $\{f(x): x \in [0,1]\}$ and *sup* $\{g(x): x \in [0,1]\}$?

2. If $E = \{1, \frac{1}{2}, \frac{1}{3,} \cdots, \frac{1}{n}, \cdots\}$, then find *inf* E.

3. Find the limit point(s) of the set $E = \{0, 1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots\}$.

4. Find $\cap_{n=1}^{\infty} I_n$ where $I_n = (-\frac{1}{n}, \frac{1}{n}), \quad n \in N$, N being the set of natural numbers.

5. Let $[a_1, b_1], [a_2, b_2], \ldots, [a_n, b_n]$ are finitely many disjoint closed intervals and $F = \cup_{k=1}^{\infty}[a_k, b_k]$. Then what is the Lebesgue measure of F, i.e., $m(F)$?

6. Let F be a countable set. Then what is Lebesgue measure of F ?

7. Let $E = \cup_{n=2}^{\infty}\left(n - \frac{1}{n}, n + \frac{1}{n}\right)$. Find $m(E)$.

8. Find $L - \int \psi dx$, where $\psi : [0,1] \to R$ is given by

$$\begin{aligned} \psi(x) &= 0, \quad if \quad x \quad is \quad irrational \\ &= 1, \quad if \quad x \quad is \quad rational \end{aligned}.$$

9. Find $\int x^2 d[x]$

10. Obtain the fourier series of the function

$$\begin{aligned} f(x) &= 0, \quad if \quad -\pi \le x < 0 \\ &= 1, \quad if \quad 0 \le x \le \pi \end{aligned}.$$

11. If A and B are two sets in the metric space (X, d) with $A \cap B \neq \Phi$, then what can be said about the upper bound of diameter of $A \cup B$?

12. If G is any set in the metric space (X, d) and the closure of G is G, then what is relation between diameters of G, $\overline{G}$?

13. Let (X, d) and (Y, ρ) be two metric spaces. Also let $f: (X,d) \to (Y, \rho)$ and $g: (X,d) \to (Y, \rho)$ be two continuous functions. Then determine whether the set $\{x \in X : f(x) \neq g(x)\}$ is closed ,open or compact .

14. What can be said about the continuity or uniform continuity of the function $f : (0,1] \to R$, given by $f(x) = \frac{1}{x}$, where R is the set of real numbers?

15. Let (X,d) be any metric space and $\Phi \neq A \subset X$. If $u \in \overline{A}$, then what can be said about $dist(u,A)$?