<u>Question Bank for PG Course</u> অঙ্গ (Mathematics)

দ্বিতীয় (খ) পত্র (Paper - IIB)

Complex Analysis : PGMT-IIB

1. Is the function
$$f(z) = |z|^4$$
 analytic at $z = 0$?

2. Given that f(z) is analytic. Then find the value of $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2$.

3. Given that C is the circle |z - 1| = 3. Then find the value of $\oint_C \frac{e^z}{(z+1)^2} dz$.

4. Find the value of $\oint_{\mathcal{C}} \frac{z}{(9-z^2)(z+i)} dz$ where C is the circle |z| = 2.

5. Find the radius of convergence of the power series $\sum_{n} \frac{(-1)^{n}}{n} (z - 2i)^{n}$.

6. Let P(x) be a polynomial of real variable x of degree $k \ge 1$. Consider the power series $f(z) = \sum_{n=0}^{\infty} P(n) z^n$ where z is a complex variable. Then find the radius of convergence of f(z).

7. Discuss the singularity of the function $f(z) = \log(z^2 + z - 2)$.

8. Given
$$f(z) = \frac{z^2 - 2}{z^3 + 3z + 1}$$
. Discuss role of the point at infinity.

9. Find the residue of the function $f(z) = \frac{e^{iz}}{z^{2}+1}$ at z = -i.

10. Find the residue of the function $f(z) = z^3 sin \frac{1}{z-1}$ at its singular point.

11. Discuss the effect of the transformation $f(z) = \frac{z-i}{z+i}$ maps on various half planes.

12. Find the bilinear transformation which maps points z = 0, -1, -i into w = i, 0, 1.

13. Find the fixed points of the transformation
$$f(z) = \frac{2z+3}{z+4}$$

14. When do the roots of the equation
$$z^5 + az + 1 = 0$$
 lie within $|z| = r$?

15. If
$$f(z) = \frac{(z^2+1)^2}{(z^2+5z+6)^3}$$
 then find the value of $\int_{|z|=4} \frac{f'(z)}{f(z)} dz$