Question Bank for PG Course

অঙ্ক (Mathematics)

তৃতীয় (ক) পত্র (Paper - IIIA)

Ordinary Differential Equations : PGMT-IIIA

1. Find a transformation so that the differential equation $(y - px)\, x = y$ where p=dy/dx≠0 can be transformed into a homogeneous equation.

2. What is the necessary condition for the existence of a singular solution of the differential equation (x,y,p)=0, p=dy/dx ?

3. By the substitution $x^2 = u,\ y^2 = v,\ P = \frac{dv}{du}$, the differential equation $(px - y)(x - py) = 2p$ where $p = \frac{dy}{dx}$ transforms to the clairaut's form. Write down the form.

4. Find two linearly independent solutions for the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0$$

5. Find a fundamental matrix of the homogeneous linear vector differential equation $\frac{dx}{dt} = \begin{pmatrix} -2 & 3 \\ 3 & -2 \end{pmatrix} x$ where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

6. In the homogeneous linear system $\frac{dx}{dt} = Ax$ where $A = \begin{pmatrix} 3 & 2 \\ -5 & 1 \end{pmatrix}$ and $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, find the characteristic roots of the matrix A.

7. For interval $0 \le x \le \pi$, the system of differential equation $\frac{d^2y}{dx^2} + \lambda x = 0$ with boundary conditions $y(0) = 0$ and $y(\pi) = 0$find the eigen functions and the eigenvalues.

8. Write down the condition for which a second order Sturn-Liouville equation $\frac{d}{dx}\left\{p(x)\frac{dy}{dx}\right\} + \{q(x) + \lambda\, r(x)\}y = 0$ is said to be regular in the interval $a \le x \le b$.

9. Find the Green's function for the equation $d^2u/dx^2=f(x)$ subject to the boundary condition $u(0)=u(1)=0$.

10. If $\delta(x)$ be the Dirac-delta function when a<x<b ; find the value of

$$\int_a^b f(x)\,\delta(x-x_0)dx$$

11. Find the critical point of the system $\frac{dx}{dt} = 5x - 6y + 2$, $\frac{dy}{dt} = 4x - 5y + 3$.

12. Determine the nature of the critical point $(0, 0)$ of the autonomous system

$$\frac{dx}{dt} = -3x + 2y$$

$$\frac{dy}{dt} = -x - 4y$$

13. In Bessel equation $z^2\frac{d^2y}{dz^2} + z\frac{dy}{dz} + (z^2 - \gamma^2)y = 0$,Find the indicial equation corresponding to the singularity $z = 0$.

14. In Laguerre equation $z\frac{d^2w}{dz^2} + (1 - z)\frac{dw}{dz} + rw = 0$, what is z=0 ?

15. The Legendre polynomials $P_m(z)$ and $P_n(z)$ are orthogonal in the interval $-1 \le z \le 1$, for positive integers *m* and *n* if $\int_{-1}^{1} P_m(z)P_n(z)dz = 0$ for $m \neq n$. Now for $m = n$ find the value of $\int_{-1}^{1} P_m(z)P_n(z)dz$.