Question Bank for PG Course

অঙ্ক (Mathematics)

তৃতীয় (খ) পত্র (Paper - IIIB)

Partial Differential Equations And Special Function : PGMT-IIIB

1. Find a condition so that there exists a relation between two functions (x, y) and $v(x, y)$not involving the variables x and y explicitly.

2. What is the necessary and sufficient condition for the Pfaffian differential equation $\overrightarrow{X}.d\,\vec{r} = 0$ where $\overrightarrow{X} = P\ \vec{\imath} + Q\,\vec{\jmath} + R\,\vec{k}$ and $d\,\vec{r} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k}$ to be integrable?

3. What is the primitive of the equation $ay^2z^2dx + bz^2x^2dy + cx^2y^2dz = 0$?

4. What is the Lagrange's auxiliary equation of the partial differential equation $y^2p - xyq + x(2y - z) = 0$?

5. Find Charpit's equations corresponding to the partial differential equation 2zx-px^2-2qxy+pq=0.

6. Find the complete integral of the partial differential equation $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{\partial z}{\partial x}.\frac{\partial z}{\partial y}$.

7. Find the general solution of
$(D^2 - DD' - 2D'^2 + 2D + 2D')z = 0$
where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

8. Find the particular integral of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin\,(x + 2y)$.

9. By the transformation $x = e^u,\ y = e^v$,the partial differential equation $x^2\frac{\partial^2 z}{\partial x^2} - 4y^2\frac{\partial^2 z}{\partial y^2} - 4y\frac{\partial z}{\partial y} - z = 0$ reduces to which form?

10. Examine the nature of the following partial differential equation
$4y^2z_{xx} + 2(1 - y^2)z_{xy} - z_{yy} - \frac{2y}{1+y^2}\left(2z_x - z_y\right) = 0.$

11. If a function $\emptyset$ is harmonic in a closed region V and $\frac{\partial \emptyset}{\partial n} = 0$ on the boundary S of the closed region V then what is the $\emptyset$ function?

12. If $\psi(x, y) = X(x)\,Y(y)$ satisfies the two dimensional Laplace's equation $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ then find $\frac{1}{X}\frac{d^2X}{dx^2}$ and $\frac{1}{Y}\frac{d^2Y}{dy^2}$.

13. If a function $\psi(x, y)$satisfies the Laplace equation $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ at any point interior to the rectangle $0 \leq x \leq a$, $0 \leq y \leq b$ then state the boundary condition in case of an interior Dirichlet problem for a rectangle.

14. Solve the one dimensional diffusion equation $\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$ if it has a solution of the type $T(x,t) = X(x)\,Y(t)$ where $\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{kY}\frac{dY}{dt} = -\alpha^2$, α being a nonzero real constant.

15. Reduce the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ to its canonical form.