Question Bank for PG Course

অঙ্ক (Mathematics)

ষষ্ঠ(খ) পত্র (Paper - VIB)

Functional Analysis : VIB

1. Consider the set $X = \{1, \quad 2, \quad 3, \quad ...\}$ regarded as subspace of the set of real numbers R with usual metric. Then check the completeness, compactness and bounds (if any) of X is ....

2. Consider the set $Y = \{1, \quad \frac{1}{2}, \quad \frac{1}{3}, \quad ...\}$ regarded as subspace of the set of real numbers R with usual metric. Which subsets of Y are both open and closed?

3. In a metric space (X, d) , take $x_0 \in X$. For $x \in X$, let $f_x : \quad X \to R$ (space of real numbers with usual metric) be given as $f_x(y) = d(y,x) - d(y,x_0)$, for $y \in X$. Then what can be said about the continuity of f_x , for an arbitrary $x \in X$?

4. Consider the set $C[a,b]$ of all real valued continuous functions on the closed intervals $[a,b]$ with the *sup* metric. What can be said about compactness and bounds (if any) of $C[a,b]$?

5. What can be said about the subset $\{f_n\} \subset C[0,1]$ with respect to being uniformly bounded , where $f_n(t) = 1 + \frac{t}{n}; 0 \le t \le 1;$ and the set $C[0,1]$ of all real valued functions on the closed intervals $[0,1]$ is a metric space with the *sup* metric.

6. Give a necessary and sufficient condition for the subset $M \subset C[a,b]$ to be compact, where the set $C[a,b]$ of all real valued continuous functions on the closed intervals $[a,b]$ is a metric space with the *sup* metric.

7. Let T be a linear operator over a normed linear space X and T is continuous at $a \in X$. Then what can be said about the continuity of T at other points of X ?

8. Find the order of the representive matrix of a linear operator $T : \quad R^n \to R^m$.

9. Find the dimension of the space of bounded linear operators from R^n to R^n .

10. Let X be a Banach space. Then what can be said about the compactness and completeness of the subset $\{x \in X : \quad \|x\| = 1\}$ of X ?

11. For what values of p , the sequence space l_p of real sequences is a Hilbert space?

12. For what values of p , parallelogram law holds in the sequence space l_p of real sequences?

13. Let z be a fixed member of a Hilbert space H . Then what can be said about the norm of the bounded linear functional f over H given by $f(x) = <x, z>$, for all $x \in X$?

14. In a separable Hilbert space H , how many elements can an orthonormal system have?

15. Consider the quotient space $C[0,1]/L$, where $C[0,1]$ is the linear space of all real valued continuous functions over the closed interval $[0,1]$ and L consists of those members $f \in C[0,1]$ with $f(1) = 0$ i.e., vanishing at $t = 1$. Now if $h \in C[0,1]$ such that $h \notin (f + L)$, then find $(h + L) \cap (f + L)$.