

Question Bank for PG Course

অঙ্ক (Mathematics)

সপ্তম (ক) পত্র (Paper - VIIA)

Differential Equations, Integral Transformations : PGMT-VIIA

1. The Laplace transform of a function $f(t)$ defined for $t \geq 0$ is defined by $F(s) = \int_0^{\infty} e^{-st} f(t) dt$. If $f(t) = e^{t^2}$, then find $F(s)$.
2. The Laplace transform of a function $f(t)$ is denoted by $\mathcal{L}[f(t), s] = \int_0^{\infty} e^{-st} f(t) dt$. Then find $\mathcal{L}\left[\frac{\sin \omega t}{t}, s\right]$.
3. Fourier transform of a function $f(t)$ is denoted by $\mathfrak{F}[f(t), \xi] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\xi t} dt$. Then find $\mathfrak{F}[e^{-a|t|}, \xi] (a > 0)$
4. Find the inverse Laplace transform of $\frac{s-2}{s^2+4s+13}$
5. Find the inverse Laplace transform of $\frac{1}{(s+a)^n}$
6. If $L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s)$, then find $L\{tf(t)\}$
7. If $F_c\{f(x)\} = \int_0^{\infty} f(t) \cos(\omega t) dt = F_c(\omega)$, then $F_c\{f''(x)\} + \omega^2 F_c(\omega) + f'(0) = b$ implies
 1. $b < 0$
 2. $b > 0$
 3. $b = 1$
 4. $b = 0$
8. The Laplace transform of a function $f(t)$ is denoted by $\mathcal{L}[f(t), s] = \int_0^{\infty} e^{-st} f(t) dt$. If $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$ then find the value of $\mathcal{L}[f(t), s]$.
9. The Laplace transform of a function $f(t)$ is denoted by $\mathcal{L}[f(t), s] = \int_0^{\infty} e^{-st} f(t) dt$. If $f(t) = \frac{1-\cos t}{t}$ then find the value of $\mathcal{L}[f(t), s]$.
10. If $f(t)$ is a periodic function with period T , then find the Laplace transform $\mathcal{L}[f(t), s]$
11. The Laplace transform of a function $f(t)$ is denoted by $\mathcal{L}[f(t), s] = \int_0^{\infty} e^{-st} f(t) dt$. If $f(t) = \operatorname{erf}\left(t^{\frac{1}{2}}\right)$ then find the value of $\mathcal{L}[f(t), s]$.
12. If $F(\alpha)$ is the Fourier transform of $f(x)$, then find the Fourier transform of $f(ax) (a > 0)$
13. If $F(\alpha)$ is the Fourier transform of $f(x)$, then find the Fourier transform of $f(x) \cos ax$

14. If the Fourier sine transform of $f(x)$ is $\frac{\alpha}{1+\alpha^2}$, thus find $f(x)$
15. For $a > 0, b > 0$, find the value of $\int_0^\infty \frac{d\alpha}{(a^2+\alpha^2)(b^2+\alpha^2)}$