Question Bank for PG Course

অঙ্ক (Mathematics)

সপ্তম (খ) পত্র (Paper - VIIB)

Integral Equations And Generalised Functions : PGMT-VIIB

1. Find the solution of the homogeneous Fredholm integral equation u(x) = - $\int_0^1 u(t)dt$

2. Let φ satisfy $\varphi(x) = f(x) + \int_0^x sin(x-t)\varphi(t)dt$. Then find φ .

3. Consider the integral equation $y(x) = x^3 + \int_0^x sin(x-t)y(t)dt,\ 0 \le x \le \pi$. Then, find the value of $y(1)$

4. What is the value of λ for which $u(x) = 1 + \lambda x$ is a solution of $x = \int_0^x e^{x-t}u(t)dt$?

5. Find the resolvant kernel $R(x,t,\lambda)$ for the Volterra integral equation

   $\varphi(x) = x + \lambda \int_a^x \varphi(s)ds,$

6. For the integral equation $y(x) = 1 + x^3 + \int_0^x K(x,t)y(t)dt$, with kernel $K(x,t) = 2^{x-t}$, find the iterated kernel $K_3(x,t)$

7. Find the characteristic number homogeneous integral equation

   $\varphi(x) - \lambda \int_0^1 (3x-2)t\varphi(t)dt = 0$

8. Let λ_1, λ_2 be the characteristic numbers for the homogeneous integral equation

   $\varphi(x) - \lambda \int_0^1 (xt + 2x^2)\varphi(t)dt = 0$. Then find λ_1, λ_2.

9. Which of the following is correct?

   1. $u(x) = \int_0^1 e^{xt}u(t)dt$ is a Volterra integral equation
   2. $u(x) = \int_0^x xtu(t)dt$ is a Fredholm integral equation
   3. $u(x) = \lambda \int_a^b K(x,t)u(t)dt$ is a Fredholm integral equation
   4. $u(x) = \lambda \int_0^1 xtu(t)dt$ is a singular integral equation

10. The homogeneous Fredholm integral equation u(x) = $\lambda \int_a^b K(x,t)u(t)dt$ has only trivial solution. Then what could be said about the solution/solutions of the non-homogeneous equation u(x) = $f(x) + \lambda \int_a^b K(x,t)u(t)dt$?

11. Which of the following kernels are degenerate?

1. $K(x,t) = sin(xt)$
2. $K(x,t) = e^{x+t}$
3. $K(x,t) = e^{xt}$
4. $K(x,t) = cos(xt)$

12. Find the Volterra integral equation equivalent to the I.V.P, $\frac{dy}{dx} - 2xy = e^{x^2}$, $y(0) = 1$.

13. Find the kernel of the Fredholm Integral equation equivalent ton to the B.V.P,

$y''(x) + 9y(x) = \cos(x)$, $y(0) = y(1) = 0$, is ..... .

14. Find the eigenvalues of $y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)dt$

15. For the homogeneous Fredholm equation $\varphi(x) = \lambda \int_0^1 e^{x+t}\varphi(t)dt$, a nontrivial solution exists, when λ has the value ..... .