POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Special Paper : Pure Mathematics

Paper - 10B(i) : Advanced Functional Analysis

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

(Notations and symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) If A and B are two balanced sets in a vector space X, prove that $A + B$ is also a balanced set in X.

   b) Prove that convex hull of every open set in a topological vector space is open.

   c) Give an example with justifications of a set in a linear space which is symmetric but not balanced.

   d) If K be a convex absorbing set containing $\underline{0}$ of a topological vector space X, then show that
   $p_k(x) = \dfrac{1}{\sup\{b > 0 : bx \in k\}}, x \in X$, where p_k is the Minkowski functional for k on X.

   e) Prove that the weak* topology on X^* is Hausdorff.

   f) Let X be a Banach space. Let $I : X \to X$ be the identity operator. Find $\sigma(I)$, the spectrum of I.

   g) If $\{x_n\}$ be a weakly convergent sequence in a normed linear space X, then prove that $w - \lim\limits_{n \to \infty} x_n$ is unique.

   h) Is the space l_1 strictly convex ? Justify your answer.

2. a) When is a topological vector space X said to be locally compact ? If Y is a subspace of a topological vector space X and it is locally compact with respect to relativised topology, then prove that Y is a closed subspace of X. 1 + 5

   b) Prove that every neighbourhood of $\underline{0}$ in a topological vector space X contains an absorbing neighbourhood of $\underline{0}$. 2

   c) If A is any subset in a topological vector space X and G is an open set in X, then show that $A + G$ is open in X. 2

3. a) State and prove Gelfand-Mazur theorem. 1 + 4

   b) Let X be a Banach Algebra with identity e and let $x \in X$. Prove that $r_{\sigma(x)} = \lim\limits_{n \to \infty} \|x^n\|^{\frac{1}{n}}$. 5

4. a) Prove that the conjugate space of l_1 is l_∞. 7

b) In a Banach Algebra X with identity e if $x \in X$ satisfies $\|x\| < 1$, then show that $\left\|(e-x)^{-1} - e - x\right\| \le \dfrac{\|x\|^2}{1-\|x\|}$. 3

5. a) Prove that $T : L_2[0,1] \to L_2[0,1]$ given by $T(x) = y$, $x \in L_2[0,1]$, where $y(t) = tx(t)$, $0 \le t \le 1$ is a bounded linear self adjoint operator without having eigenvalues. 5

b) Let X be a normed linear space and Y be a Banach space over the same field of scalars and let $\{T_n : X \to Y\}$ be a sequence of compact linear operators such that $\underset{n \to \infty}{Lim} \|T_n - T\| = 0$, where T is a bounded linear operator from X to Y. Show that T is compact. 5

6. a) Prove that a non-empty closed convex subset C of a Hilbert space contains a unique vector of smallest norm. 6

b) Prove that a subspace M of a normed linear space X is weakly closed if and only if it is strongly closed. 4

7. a) Suppose A is a convex and absorbing set containing $\underline{0}$ of a topological vector space X. If $B = \{x \in X : p_A(x) < 1\}$ and $C = \{x \in X, p_A(x) \le 1\}$ then prove that $B \subset A \subset C$ and $p_A = p_B = p_C$. 6

b) Prove that the spectrum $\sigma(T)$ of a bounded linear operator $T : X \to X$, where X is a Banach space is compact and $|\lambda| \le \|T\|$ for all $\lambda \in \sigma(T)$. 4

POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Special Paper : Applied Mathematics
Paper - 10B(ii) : Magnetohydrodynamics

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Show that the circulation of the magnetic field $\vec{B}$ around any closed curve is equal to the product of the total current embraced by the curve and the magnetic permeability of the free space.

   b) Deduce the differential form of Faraday's law.

   c) Show that the normal component of the electric displacement is continuous across the interface.

   d) State Alfvén's theorem.

   e) Define magnetic Prandtl number.

   f) Show that for a polytropic gas, the coefficients $A(S)$ in the pressure-density relation $p = A(S)\rho^{\gamma}$, γ being the adiabatic exponent, is a function of the entropy S only and not on the nature of the gas.

   g) Define supersonic and subsonic flows.

2. Deduce the expression for the energy of the electromagnetic field and hence define Poynting vector. 10

3. Deduce the law of conservation of energy for MHD. 10

4. State and explain Maxwell's electromagnetic field equations governing the motion of conducting fluids. What is Lorentz force ? 10

5. Show that the magnetic flux linking and loop moving with a perfectly conducting fluid is constant. 10

6. Deduce the Prandtl MHD boundary layer equations for a strong interaction of the magnetic field. 10

7. Show that MHD shocks are compressive in nature. 10