POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Paper - 3B : Partial Differential Equations

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Symbols / Notations have their usual meanings.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Define, with an example, a second order quasi-linear partial differential equation involving two independent variables.

   b) Form a partial differential equation by eliminating the arbitrary function *f* from the relation $z = f(xy/z)$.

   c) State the Cauchy-Kowalewski theorem.

   d) Show that the two-dimensional Laplacian operator is self-adjoint.

   e) What do you mean by a Pfaffian differential form and a Pfaffian differential equation ?

   f) State the interior Dirichlet's problem for a circle.

   g) Classify the partial differential equation $z_{xx} + xz_{yy} = 0$, considering various possibilities.

2. a) Describe the Charpit's method of solving a non-linear partial differential equation of the form $f(x, y, z, p, q) = 0$. 6

   b) Solve the partial differential equation

   $$x(x^2 + 3y^2)p - y(3x^2 + y^2)q = 2z(y^2 - x^2).$$ 4

3. a) Classify and reduce the following partial differential equation into canonical form :

   $$yz_{xx} + (x + y)z_{xy} + xz_{yy} = 0.$$

   Hence solve the equation. 7

   b) Solve the partial differential equation :

   $$r - 4s + 4t = e^{2x+y}.$$ 3

4. Using Riemann's method, obtain the solution of the partial differential equation $\dfrac{\partial^2 z}{\partial x \partial y} = F(x, y)$, given that (i) $z = f(x)$ on Γ, (ii) $\dfrac{\partial z}{\partial n} = g(x)$ on Γ, where Γ is the curve $y = x$ and $\dfrac{\partial}{\partial n}$ represents the normal derivative. 10

5. a) Show that the solution of the Dirichlet problem depend continuously on the boundary values. 3

b) Find the solution of the one-dimensional heat conduction equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, satisfying the following conditions :

$u(x,0) = x(a-x),\ 0 < x < a$

u is bounded as $t \to \infty$,

and $\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(a,t) = 0,\ \forall t$. 7

6. Obtain the various possible solutions of the one-dimensional wave equation in Cartesian form by the method of separation of variables. Which one of these solutions is appropriate with the physical nature of the problem ? Justify your answer. Hence find the solution of the one-dimensional wave equation $z_{tt} = c^2 z_{xx}$, $0 \le x \le l$, $t > 0$ subject to the conditions :

$z(0,t) = z(l,t) = 0$ for $t > 0$

$z(x,0) = f(x)$ and $z_t(x,0) = g(x)$ for $0 \le x \le l$.

10

7. a) Use Green's function method to solve the Dirichlet's problem for Laplace's equation for a sphere. 6

b) Solve the partial differential equation $px + qy = pq$, by using Jacobi's method. 4