POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Paper - 4A : Numerical Analysis

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) What is the rank of a strictly diagonally dominant $n \times n$ real matrix ?

   b) What is the difference between the single step and multi-step methods for finding the solution of first order differential equation.

   c) If $T_n(x) = \cos(n\cos^{-1} x)$ represents n^{th} degree Chebyshev polynomial, then show that $T_{n+1} = 2xT_n(x) - T_{n-1}(x)$, $(n \geq 1)$.

   d) Define the condition number of a matrix A. When the matrix is called ill-conditioned ? Give an example.

   e) A clamped cubic spline $S(x)$ for function $f(x)$ is defined on [1, 3] by

$$S(x) = \begin{cases} 3(x-1) + 2(x-1)^2 - (x-1)^3, & 1 \leq x \leq 2 \\ a + b(x-2) + c(x-2)^2 + d(x-2)^3, & 2 \leq x \leq 3 \end{cases}$$

   given $f'(1) = f'(3)$, find *a, b, c* and *d*.

   f) Find absolute error, relative error in the following approximations :

      a) $x_{True} = 2 \cdot 718281$ $x_{approx} = 2 \cdot 718$

      b) $x_{True} = 0 \cdot 000023$ $x_{approx} = 0 \cdot 00002$

   g) Let (λ, X) be an eigenpair of $n \times n$ matrix A so that $AX = \lambda X$. What will be the corresponding eigenpair of the similar matrix $P^{-1}AP$?

2. Describe briefly Bairstow's method for finding quadratic factor of a real polynomial of degree $n(\geq 3)$. 10

3. Discuss the stability analysis of second order Runge-Kutta method and obtain the stability region of it. 10

4. Obtain an explicit finite difference scheme for solving the parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \; t > 0 .$$

$$u(x,0) = f(x), \quad 0 \leq x \leq 1$$

$$u(0,t) = 0 = u(1,t), \quad t > 0 .$$

   Under what condition the scheme is absolutely stable ? 10

5. Briefly describe the Romberg integration procedure for approximating an integral numerically. Write down its stopping criteria. 10

6. Describe Adams-Bashforth scheme for numerical solution of a well posed initial value problem

$$\frac{dy}{dx} = f(x,y), \; y(a) = y_0,$$

in a finite interval [a, b]. Explain how to start the scheme. 10

7. Describe cubic spline. Describe briefly the method of construction of cubic spline function. What are end point conditions of natural cubic spline ? 10