POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Paper - 5A : Principles Of Mechanics

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) What is meant by initial frame ?

   b) Show from Lagrange's equations that the orbit of a particle in a central force field lies in a plane.

   c) Show that if *q, p* are canonically conjugate to each other then the transformations

   $$Q = \log(1 + \sqrt{q}\cos p)$$

   $$P = 2(1 + \sqrt{q}\cos p)\sqrt{q}\sin p$$

   are canonical.

   d) What is Coriolis force ? What is the cause of this force ?

   e) Write down the Jacobi identity for Poisson brackets.

   f) Show that the quantity $E = \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L$ remains constant during the motion of a closed system.

   g) If for a certain mechanical system $H = p^2q^2 - \lambda pq$, where λ is a real constant, then show that pq is a constant of motion.

2. a) Deduce Lagrange's equation of motion for a conservative and unconnected holonomic system. 7

   b) Prove the virial theorem. 3

3. Derive Hamilton-Jacobi equation in terms of a suitable generating function. Solve the problem of free particle in one-dimension. 10

4. Discuss the problem of central force. Find the stability of circular orbits for the power law potentials

$$V(r) = -\frac{\gamma}{r^{\nu}}$$

where $\gamma \neq 0$ and $\nu \neq 0$. 5 + 5

5. a) Show that $\int_{t_0}^{t_1} L(q, \dot{q}, t)\,dt$ and

$$\int_{t_0}^{t_1} \left[L(q, \dot{q}, t) + \frac{dF}{dt} \right] dt$$ lead to the same equation of motion. 4

b) Derive Hamilton's principle from D'Alembert's principle. 6

6. a) Explain the Brachistochrone problem and draw the solution curve. 4 + 2

b) Find the equation of the curve which makes the surface area of revolution generated by rotating the curve $y = y(x)$ around the x-axis. 4

7. Define angle variables. A particle of mass m moves in two dimensions (x, y) in a simple harmonic oscillator having potential

$$V(x, y) = \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2 \quad (\omega_x \neq \omega_y).$$

Obtain the action variables expressing the energy in terms of these and hence find the angle variables. 2 + 5 + 3