POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Paper - 5B : Elements Of Continuum Mechanics & Special Theory Of Relativity

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) A rod has a length of 80 cm when it is moving with a speed of 0·75c along the direction of its length. Find the length of the rod with respect to an observer at rest.

b) Two particles come toward each other, each with speed 0·9c with respect to the laboratory. What is their relative speed ?

c) Prove that the determinant

$$\begin{vmatrix} a_{1j} & a_{2j} & a_{3j} \\ a_{1l} & a_{2l} & a_{3l} \\ a_{1n} & a_{2n} & a_{3n} \end{vmatrix}$$

is equal to 1 if *j, l, n* are in cyclic order and −1 if *j, l, n* are not in cyclic order.

d) The state of stress at a point is given by

$$(\tau_{ij}) = \begin{pmatrix} T & aT & bT \\ aT & T & cT \\ bT & cT & T \end{pmatrix}$$

where *a, b, c* are constants and *T* is some stress value. Determine the constants *a, b, c,* so that the stress vector on a plane normal to $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ vanishes.

e) Show that the principal directions of strain at each point of a linearly elastic isotropic body are coincident with the principal directions of stress.

f) For the velocity field given by $v_1 = kx_3$, $v_2 = kx_3$, $v_3 = k(x_1 + x_2)$, show that motion is irrotational.

g) The displacement in an elastic solid is given by

$$u_1 = a(X_1 + 2X_2 + 3X_3)$$

$$u_2 = a(-2X_1 + X_2)$$

$$u_3 = a(X_1 + 4X_2 + 2X_3)$$

where '*a*' is a small quantity. Find dilatation.

2. a) Discus briefly time dilation. 5

b) Find the transformation equations for the relativistic composition of velocities. 5

3. If Θ and θ be the angles between the line elements of lengths $\mathrm{d}L, \delta L$ and $\mathrm{d}l, \delta l$ respectively before and after deformation, then show that

$$\cos\theta - \cos\Theta \frac{\mathrm{d}L}{\mathrm{d}l}\frac{\delta L}{\delta l} = 2n_{ij}n_i m_j$$

where n_i and m_j are the direction cosines of the line elements $\mathrm{d}l$ and δl respectively and n_{ij} are Eularian finite strain tensor. 10

4. a) Show that all principal strains are real. 5

   b) Find the principal directions of strain and corresponding direction ratios of principal strains for the following strain tensor :

$$e_{ij} = \begin{pmatrix} a & b & 0 \\ b & -a & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ 5

5. a) Deduce the equations of equilibrium of a continuum. Also show that the stress tensor is symmetric. 7

   b) Prove that the normal stress across any plane through the centre of quadric surface is equal to the inverse of the square of the central radius vector of the quadric surface normal to the plane. 3

6. a) Given the following stress distribution

$$(\tau_{ij}) = \begin{pmatrix} x_2 & -x_3 & 0 \\ -x_3 & 0 & -x_2 \\ 0 & -x_2 & T \end{pmatrix}$$

   Find T such that stress distribution is in equilibrium with the body force $\overline{F} = -g\overline{e}_3$. 5

   b) What is compatibility relation for strain ? Derive the strain compatibility equations. 5

7. a) Find the streamline and path line of a continuum particle for the velocity field given by $v_1 = \dfrac{x_1^2}{1+t^2}$, $v_2 = x_2^2$, $v_3 = 0$. 5

   b) State and prove Kelvin's minimum energy theorem. 5