POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Paper - 8A : Differential Geometry

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

(Notations have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : 2 × 5 = 10

   a) Show that for a space of dimension 5, $\delta_i^j = 5$.

   b) If ψ is a function of n coordinates, show that grad ψ is a covariant vector.

   c) For a V^2, in which $g_{11} = E$, $g_{12} = F$, $g_{22} = G$, find g^{ij}.

   d) Define the curl of a vector A_i.

   e) Define curvature tensor of a Riemannian space.

   f) Show that the intrinsic derivative of an invariant coincides with its total derivative.

   g) Define Gaussian curvature of a surface.

2. a) Prove that the inner product of two tensors A_q^p and B_l^{ij} is a tensor of type (2, 1). 5

   b) Prove that the length of a vector is an invariant. 5

3. a) Define Christoffel symbol of the 1st kind. For a space, where $g_{ij} = 0$, $i \neq j$, evaluate Christoffel symbol of the 1st kind. 5

   b) Deduce that $\dfrac{\partial g_{ik}}{\partial x^j} = -g^{pk}\begin{Bmatrix} i \\ pj \end{Bmatrix} - g^{im}\begin{Bmatrix} k \\ mj \end{Bmatrix}$. 5

4. a) Deduce that

$$\frac{\mathrm{d}}{\mathrm{d}t}(g_{ij}A^iB^j) = g_{ij}\frac{\delta A^i}{\delta t}B^j + g_{ij}A^i\frac{\delta B^j}{\delta t}.$$ 5

   b) Establish $\dfrac{\delta\lambda_i}{\delta s} = x\mu_i$. 5

5. a) When is a curve called helix ? Show that, when τ / x = constant, $\gamma^i = c\lambda^i + b^i$, where c, b^i are constants. 1 + 5

b) Find ds^2 for a right helicoid given by $r = (u^1 \cos u^2, u^1 \sin u^2, 0)$. 4

6. a) Prove that a geodesic is an auto parallel curve. 5

b) Determine whether the surface with the metric $ds^2 = (u^2)^2(du^1)^2 + (u^1)^2(du^2)^2$ is a developable or not. 5

7. Establish Gauss's formula for a surface in E^3. Hence show that $a^{\alpha\beta} \, x^r_{\alpha,\beta} = 2HQ^r$ where H is the mean curvature of the surface.

6 + 3 + 1