POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Paper - 8B : Graph Theory

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Prove that in any graph, the number of odd vertices is always even.

   b) Draw a tree having a degree-sequence (1, 1, 1, 1, 3, 3).

   c) State the problem of Ramsey.

   d) Define face of a planar graph and its size.

   e) What is the length of a Hamiltonian path in a simple n-vertex graph ?

   f) Define leaf and internal vertex of a rooted tree.

   g) Define Incidence matrix with an example.

2. a) Briefly describe the process of representation of binary relations on finite sets by directed graphs with examples. 7

   b) If T is a tree with an even number of edges, then prove that T must contain at least one vertex having even degree. 3

3. a) Define a bipartite graph. Let G be a connected bipartite planar graph with n vertices and e edges. Then show that $e \leq 2n - 4$. 6

   b) Prove that a connected graph with n vertices has at least ($n - 1$) edges. 4

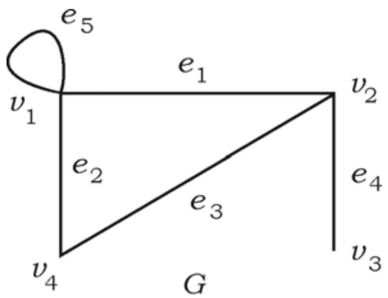
4. a) Define a binary tree. Prove that for a complete binary tree T of height h and with n vertices, $n = 2^{h+1} - 1$. 1 + 5

   b) Prove that a graph G is a forest if and only if $e - n + k = 0$, where e is the number of edges, n is the number of vertices and k is the number of components of G. 4

5. a) Define isomorphic graphs. Prove that any two simple connected graphs with n vertices, all of degree 2, are isomorphic. 4

   b) Describe Kruskal's algorithm for finding a minimal spanning tree of a connected weighted graph with an example. 6

6. a) Define induced subgroup by an edge-set. For the following graph G, find the induced subgraph $G(A)$ of G by the edge-set $A = \{e_1, e_2\}$. 5



b) Let G be a simple, connected, planar graph. Prove that there exists a vertex $v \in V_G$ such that $d(v) \le 5$. 5

7. a) Let G be a simple connected graph with 11 or more vertices. Show that either G or $\overline{G}$ is non-planar. 7

b) Define self-complementary graph with an example. 3