POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Special Paper : Pure Mathematics
Paper - 9A(i) : Advanced Complex Analysis

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

(Symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Define a harmonic function. Give an example of it.

   b) Show that Poisson Kernel, $P(R,r,\phi-\theta) = \operatorname{Re}\left\{\dfrac{Re^{i\phi}+z}{Re^{i\phi}-z}\right\}$.

   c) Determine the branches and branch point of the function $f(z)=(z-4)^{1/2}$.

   d) State Mittag-Leffler theorem.

   e) Define order of an entire function.

   f) Find the order of e^{-z}.

   g) Show that the exponent of convergence of the zeros of $\sin z$ is 1.

2. a) Show that a necessary and sufficient condition for a function $f(z)=u(x,y)+iv(x,y)$ to be analytic in a domain D is that its real part $u(x,y)$ and imaginary part $v(x,y)$ are conjugate harmonic functions on D. 6

   b) Let $f(z)$ be a function regular in the closed disc $|z| \leq R$ and let $u(r,\theta)$ be its real part. If $u(r,\theta) \geq 0$ throughout the disc, then prove that $\frac{1}{2}u(0,0) \leq u(r,\theta) \leq 2u(0,0)$, $[0 \leq \theta \leq 2\pi, 0 \leq r < R]$. 4

3. a) Let $f(z)$ be an analytic function in a domain D containing z_0. If $f'(z_0) \neq 0$, then show that $f(z)$ is conformal at z_0. 6

   b) Prove that a direct analytic continuation, if it exists, is unique. 4

4. a) State and prove Weierstrass' factorization theorem. 7

   b) Prove that the infinite product $\prod_{n=1}^{\infty}(1+a_n)$ converges absolutely if and only if the series $\sum a_n$ converges absolutely. 3

5. a) State and prove Hadamard's three circles theorem. 7

   b) Show that $\log M(r)$ is a convex function of $\log r$. 3

6. a) Let $f(z)$ be a non-constant analytic function regular in $|z| \le R$ and let $M(r)$ and $A(r)$ denote the maximum value of $|f(z)|$ and $R|\{f(z)\}|$ respectively on $|z| = r$. Then show that $0 \le r < R$

$$M(r) \le \frac{2r}{R-r} A(R) + \frac{R+r}{R-r} |f(0)|.$$ 6

b) If $f(z)$ be an entire function with finite order ρ, then show that $n(r) = O(r^{\rho+\in})$ for $\in > 0$ and for sufficiently large values of *r*. 4

7. a) Show that

$$\operatorname{cosec} z = \frac{1}{z} + \sum_{-\infty}^{\infty}{}' (-1)^n \left(\frac{1}{z - n\pi} + \frac{1}{n\pi} \right),$$ where

' $'$ ' indicates that the term $n = 0$ is omitted from the sum. 6

b) State and prove Jensen's inequality. 4

POST-GRADUATE COURSE

Term End Examination — December, 2014 / June, 2015

MATHEMATICS

Special Paper : Applied Mathematics

Paper - 9A(ii) : Operations Research

Time : 2 Hours **Full Marks : 50**

(Weightage of Marks : 80%)

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Write the iterative scheme of cutting plane method to solve constrained optimization problem.

b) Write short notes on Wolfe's modified simplex method.

c) Write down Kuhn-Tucker conditions for quadratic programming problem.

d) Outline Gomory's cutting plane algorithm for all IPP.

e) Discuss the effects of addition of a new variable to the LPP.

f) Write the iterative scheme of steepest descent method.

g) Give the general iterative scheme to solve unconstrained optimization problem.

2. a) Write a short note on Davidon-Fletcher-Powell method. 3

b) Solve the following LPP : 7

Maximize $Z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \le 2$

$x_1 + 4x_3 \le 4$

$x_1, x_2, x_3 \ge 0$

3. a) Describe Golden section method to find the minimum of one dimensional unimodal function. 3

b) Minimize $f(x) = \begin{cases} \dfrac{2x}{3}, & x \le 3 \\ 5 - x, & x > 3 \end{cases}$

in the interval (1, 4) by Golden section method up to six experiments. 7

4. a) Using revised simplex method solve the following LPP : 7

Maximize $Z = x_1 + x_2$

subject to $3x_1 + 4x_2 = 7$

$4x_1 + 3x_2 = 7$

$x_1, x_2 \ge 0$

b) Solve using Kuhn-Tucker conditions : 3

Maximize $Z = 5 + 8x_1 + 12x_2 - 4x_1^2 - 4x_2^2 - 4x_3^2$

subject to $x_1 + x_2 \le 1$

$2x_1 + 3x_2 \le 6$

5. a) Consider the following LPP :

Maximize $Z = x_1 + x_2$

subject to $2x_1 + x_2 \le 6$

$x_2 \le 2$

$x_1, x_2 \ge 0$

Find the optimal solution. Using this optimal table find the optimal solution when the objective function is $Z = 3x_1 + x_2$. 7

b) Find the conjugate direction for the matrix $\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$. 3

6. a) Use dual simplex method to solve the following LPP : 7

Maximize $Z = -3x_1 - 2x_2$

subject to $x_1 + x_2 \ge 1$

$x_1 + x_2 \le 7$

$x_1 + 2x_2 \ge 10$

$x_2 \le 3$

$x_1, x_2 \ge 0$

b) Outline dual simplex algorithm to solve LPP. 3

7. a) Write short notes on Newton's method. 3

b) Using Wolfe's method solve the following QPP : 7

Maximize $Z = 2x_1 + x_2 - x_1^2$

subject to $2x_1 + 3x_2 \le 6$

$2x_1 + x_2 \le 4$

$x_1, x_2 \ge 0$