NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 1A : Abstract Algebra

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/IA**

PG-Sc.-AP-17097

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 1A : Abstract Algebra

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/IA**

PG-Sc.-AP-17097

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June-2020/Dec.-2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Let G and H be two groups and $f : G \rightarrow H$ be an epimorphism. If G is cyclic, prove that H is also cyclic.

b) Let $Z(G)$ denote the centre of a group G and $N(a)$ denote the normalizer of a in G. Prove that $a \in Z(G)$ iff $N(a) = G$.

c) Prove that a group of order 38 has a non-trivial normal subgroup.

d) Let D be a Euclidean domain and $a, b, c \in D$. If $a \mid bc$ and $(a, b) = 1$, prove that $a \mid c$.

e) Consider the ring of integers Z. Let $p \in Z$ be a prime. Prove that ideal $< p >$ in Z generated by p is a maximal ideal of Z.

f) Let F be a field consisting of p^n elements where p is a prime and n is a positive integer. Prove that every element of F is a root of the polynomial $x^{p^n} - x$ over Z_p.

g) Prove that the field extension $Q(\sqrt{2}, \sqrt{3})/Q$ is a simple extension where Q is the field of rational numbers.

h) Let $f : R_1 \rightarrow R_2$ be a ring epimorphism. If I is an ideal of R_1, prove that $f(I) = \{ f(i) : i \in I \}$ is an ideal of R_2.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Let G_1 and G_2 be two groups and N_1 and N_2 be normal subgroups of G_1 and G_2 respectively. If $G_1 \cong G_2$ and $N_1 \cong N_2$, is it true that the quotient groups G_1/N_1 and G_2/N_2 are isomorphic ? Justify your answer. 5

b) Let H and K be two normal subgroups of a group G with $K \subseteq H$. Prove that $\frac{G/K}{H/K} \cong G/H$. 5

3. a) Let G be a group and G' be its derived subgroup. Prove that

i) G' is a normal subgroup of G ;

ii) If H is a normal subgroup of G,

Prove that G/H is commutative iff $G' \subseteq H$. 2 + 3

b) Let G be a group. Prove that $Inn(G)$ is a normal subgroup of $Aut(G)$ where $Inn(G)$ is the set of all inner automorphisms of G and $Aut(G)$ is the group of all automorphisms of G. 5

4. a) Let G be a group of order p^n where p is a prime and n is a positive integer. Prove that $Z(G) \neq \{e\}$ where e is the identity element of G. 5

b) Let G be a finite group. Prove that $|G| = \sum_{a} [G : N(a)]$ where the summation is over a complete set of distinct conjugacy class representatives. 5

5. a) Define prime ideal and maximal ideal in a commutative ring R with identity. Prove that in a Boolean ring every prime ideal is a maximal ideal. 1 + 1 + 3

b) Let R be a commutative ring with identity. Prove that every proper ideal R is contained in a maximal ideal of R. 5

6. a) Prove that $Z[x]/\langle x \rangle \cong Z$ where Z is the ring of integers and $\langle x \rangle$ is the ideal of $Z[x]$ generated by x. 4

b) Let R be a principal ideal domain and $p \in R$. Prove that p is irreducible iff p is prime. 6

7. a) Let F/K be a finite field extension and L be an intermediate field of F/K. Prove that $[F:K] = [F:L][L:K]$. 5

b) Let $f(x)$ be a non-constant polynomial of degree n over a field K and S be a splitting field of $f(x)$ over K. Prove that $[S:K] \leq n!$. Is S/K an algebraic extension ? Justify your answer. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :