NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 1B : Linear Algebra

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/IB**

PG-Sc.-AP-17098

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 1B : Linear Algebra

STUDENT'S COPY

Name (in Block Letter) : ....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/IB**

PG-Sc.-AP-17098

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June-2020/Dec.-2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

(Unexplained Notations and Symbols have their usual meanings)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Let T and U be linear operators on $\mathbb{R}^2$ defined by $T(x_1, x_2) = (x_2, x_1)$ and $U(x_1, x_2) = (x_1, 0)$. Describe T & U geometrically.

b) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ and U be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$. Prove that the linear transformation UT is not invertible. Generalize this result.

c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator on $\mathbb{R}^2$ defined by $T(x_1, x_2) = (-x_2, x_1)$. Prove that for any real number C the operator $(T - CI)$ is invertible.

d) Let $A = \begin{pmatrix} 2 & 1 & 0 & 0 & 3 \\ 0 & 2 & 2 & 0 & 4 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix} \in M_5(\mathbb{R})$.

Find the algebraic multiplicities and geometric multiplicities of the eigenvalues of A.

e) Let $a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$ be $\mathbb{R}^n$ be a unit vector. Find the minimal polynomial of the matrix aa^T and hence discuss its diagonalizability.

f) Let V be a vector space over a field F. Define (i) a symmetric bilinear form on V, (ii) a quadratic form on V.

g) Suppose V and W are two vector spaces over a field F and $T : V \to W$ is a linear transformation. Prove that if V is finite dimensional then rank T is finite.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Let V be a finite dimensional vector space and let T be a linear operator on V such that rank T^2 = rank T. Prove that the range and null space of T have only the zero vector in common.

b) Suppose T is a linear operator on a vector space V such that $T^2 = T$ (*i.e.,* T is idempotent). Prove that

i) $v \in Im\, T$ if and only if $Tv = v$,

ii) $V = Im\, T \oplus ker\, T$

iii) If V is finite dimensional then (i) and (ii) together implies that there exists an ordered basis $\mathfrak{B}$ of V such that the matrix of T with respect to $\mathfrak{B}$ is $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}_{n \times n}$ where I_r is the $r \times r$ identity matrix and 0's are null matrices of suitable orders $(n = dim\, V)$. 4 + (2 + 2 + 2)

3. a) Find a linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose image is spanned by $\{(1,0,-1),(2,1,3)\}$.

b) Does there exist a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,0,0) = (1,1)$? If yes, find such a linear transformation. Is it unique ? Answer with reasons.

c) Define an invariant subspace of a vector space V with respect to a linear operator T on V. Suppose S and T are two linear operators on a vector space V such that $S \circ T = T \circ S$. Prove that $ker\, S$ and $Im\, S$ are invariant under T. 3 + 3 + 4

4. a) Let $A = \begin{pmatrix} 1 & b & c \\ b & a & 0 \\ c & 0 & 1 \end{pmatrix}$ where a,b,c are positive real numbers satisfying $b^2 + c^2 < a < 1$.

Prove that all the eigenvalues of A are positive real numbers and hence conclude that the matrix is positive definite.

b) Let V and W be finite dimensional vector spaces over R. Let $T_1 : V \to V$ and $T_2 : W \to W$ be linear transformations whose minimal polynomials are respectively given by $f_1(x) = x^3 + x^2 + x + 1$ and $f_2(x) = x^4 - x^2 - 2$. Let $T : V \oplus W \to V \oplus W$ be the linear transformation defined by

$T(v,w) = (T_1(v), T_2(w))$

(*i.e.,* $T(v+w) = T_1(v) + T_2(w)$ for all $v \in V$, for all $w \in W$).

Find the minimal polynomial of T.

Is T diagonalizable ? Answer with reason. 4 + 4 + 2

5. a) Let A be a real square matrix. Prove that A^*A is a positive semi-definite matrix. Also prove that if A is invertible then A^*A is positive definite.

b) Does there exist an inner product $<, >$ on $\mathbb{R}^2$ such that $<(1,0),(0,1)> = -2$? If exists, find one such inner product. Is it unique ? Answer with reasons. 5 + 5

6. a) Let A be the companion matrix of the polynomial $f(x) = x^4 - 5x^2 + 4$. Find A.

Let $T : M_4(\mathbb{R}) \to M_4(\mathbb{R})$ be the linear transformation defined by $T(B) = AB$ for all $B \in M_4(\mathbb{R})$ ($M_4(\mathbb{R})$ denotes the vector space of all 4 × 4 real matrices). Prove that T is diagonalizable.

b) Find all possible Jordan forms and rational canonical forms of a matrix with characteristic polynomial $f(x) = (x+2)^5(x+3)^3$ and minimal polynomial $m(x) = (x+2)^2(x+3)^2$. (2 + 2) + (3 + 3)

7. a) Let V be a subspace of $\mathbb{R}[x]$ of real polynomials of degree at most 3. Equip V with the inner product $< f, g >= \int_0^1 f(t)g(t)\mathrm{d}t$. Apply the Gram-Schmidt process to the basis $\{1, x, x^2, x^3\}$.

b) Let W be the subspace of $\mathbb{R}^2$ spanned by the vector (3, 4). Using the standard inner product, let E be the orthogonal projection of $\mathbb{R}^2$ onto W.
Find
i) a formula for $E(x_1, x_2)$.
ii) the matrix of E in the standard ordered basis
iii) $W^{\perp}$ (*i.e.*, W perpendicular). 4 + 6

First Answer :

Second Answer :

Third Answer :

Fourth Answer :