NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 2A : Real Analysis & Metric Spaces

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/IIA**

PG-Sc.-AP-17099

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 2A : Real Analysis & Metric Spaces

Name (in Block Letter) : ....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/IIA**

PG-Sc.-AP-17099

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June-2020/Dec.-2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Find the total variation of $f : [\,0,1\,] \to I\!R$ defined as

$$f(x) = \begin{cases} x \cos\frac{\pi}{x}, & \text{if } 0 < x \le 1 \\ 0, & \text{if } x = 0. \end{cases}$$

b) If E is a bounded measurable set then show that there exists a subset $F \subseteq E$ such that F is a countable union of closed sets with $m(F) = m(E)$.

c) Show that the set of rational numbers Q is measurable.

d) Using definition of *R-S* integral, show that $\int_0^2 x^2 \mathrm{d}[x] = 5$; where $[x]$ denotes the largest integer $\le x$.

e) If $\{F_n\}$ is a sequence of sets of first category in a metric space (X, d) then show that $\bigcup_{n=1}^{\infty} F_n$ is also a set of first category.

f) If A and B are two sets in a metric space (X, d) such that $A \cap B \ne \phi$, then show that $Diam(A \cup B) \le Diam(A) + Diam(B)$
where $Diam(S)$ stands for the diameter of S, for any $S \subseteq X$. What happens if the condition $A \cap B \ne \phi$ is not given ? Justify.

g) If A and B are disjoint compact sets in a metric space (X, d) then show that there are disjoint open sets G_1, G_2 in (X, d) such that $A \subseteq G_1$ and $B \subseteq G_2$.

h) Is the set $A = [2,3] \cup \left\{ 3 + \frac{1}{2^n} : n = 1,2,\ldots \right\}$ with usual metric induced from $I\!R$ a compact set ? Justify.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Let $\{E_k\}$ be an increasing sequence of measurable sets such that $\bigcup_{k=1}^{\infty} E_k = E$ is bounded. Show that E is measurable and $m(E) = \lim_{n \to \infty} m(E_n)$. 4

b) If $f : [\,a, b\,] \to \mathbb{R}$ is a bounded measurable function and J_1, J_2 are open intervals in $\mathbb{R}$, then show that $f^{-1}(J_1 \cup J_2)$ is a measurable set in $[\,a, b\,]$. 3

c) Either prove or disprove :
Every measurable function is continuous but every continuous function need not be measurable. 3

3. a) If $f : [\,a,b\,] \to \mathbb{R}$ is summable and if $F(x) = \int_a^x f\,\mathrm{d}t$ in $a \le x \le b$, then show that $F'(x) = f(x)$ almost everywhere in $[\,a, b\,]$. 5

b) If $f(x) = \frac{1}{2} + \cos x$ in $0 < x < 2\pi$, then find f^+ and f^-. 2

c) Find the Fourier series for $f : [-\pi, \pi] \to \mathbb{R}$
given by $f(x) = \begin{cases} x, & \text{if } -\pi < x < 0 \\ \pi - x, & \text{if } 0 < x < \pi \end{cases}$ 3

4. a) If $f : [-\pi, \pi] \to \mathbb{R}$ is a bounded and R-integrable and f is periodic with period $= 2\pi$ and if f is increasing in $(0, \alpha)$ where $0 < \alpha < a$, then show that

$$\lim_{n \to \infty} \int_0^a f(t) \frac{\sin nt}{t} \mathrm{d}t = \frac{\pi}{2} f(0+).$$ 5

b) Show that Fourier series for $f(x) = x + x^2$ in $[-\pi, \pi]$ is

$$\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \left(\frac{\cos nx}{n^2} - \frac{\sin nx}{2n} \right).$$

What are the sums of the series at $x = \pm\pi$? Jusitify. 5

5. a) If two metrices ρ and d on a set $X (\neq \phi)$ are equivalent then show that a Cauchy sequence in (X, ρ) is a Cauchy sequence in (X, d) and conversely. 5

b) If $\sigma(x,y) = |\tan^{-1} x - \tan^{-1} y|$ for all $x, y \in \mathbb{R}$, then show that $(\mathbb{R}, \sigma)$ is a metric space. Is it complete ? Justify. 5

6. a) If a continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfies the property $f(x+y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$, then show that $f(x) = x f(1)$, for all $x \in \mathbb{R}$. 4

b) If T is a contraction in a complete metric space X and $x \in X$ show that $T\left(\lim_n T^n(x)\right) = \lim_n T^{n+1}(x)$. 4

c) Let (X, d) be a metric space and $x_0 \in X$. Show that $\{x \in X \mid 1 < d(x, x_0) < 5\}$ is an open set in (X, d). 2

7. a) Either prove or disprove :
   i) Continuous image of a connected space is connected.
   ii) Continuous image of a locally connected space is locally connected. 2 + 2

   b) Show that a continuous injective function of (0, 1) to real numbers with usual metric is a monotone function. 3

   c) If A is a compact subset of a metric space (X, d) and $x_0 \notin A$, then show that $d(x_0, A) = d(x_0, a_0)$ for some $a_0 \in A$. What happens if 'compactness' of A is suppressed ? 3

First Answer :

Second Answer :

Third Answer :

Fourth Answer :