

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম (P. G.)

অনুশীলন পত্র (Assignment) : জুন, ২০২০/ ডিসেম্বর, ২০২০ (June-2020/Dec.-2020)

MATHEMATICS

				Paper	: - 2B	8 : Co	mple	x Ana	alysis						
পূর্ণমান : ৫০	QUESTION PAPER CUM ANSWER BOOKLET মানের গুরুত্ব : ২০%														
(Full Marks : 50)				(Weightage of Marks : 20%											
পরিমিত ও য	থাযথ উত্ত	রর জ	ন্য বিশে	ণষ মূল	য দেও	য়া হবে	ৰ। অং	ণ্ডদ্ধ বা	নান, অ	পরিচ্ছ	নতা এ	বং অপ	শরিষ্কার	হন্তাক্ষরের	
S	মে) মেন্দ্র নির্দেশ	ত্ত্রে নং নংশ	রর কেরে না। ৮০	ট নেও ল'ন্য	য়া হে	ব। উপ সমহায়	ণান্তে হ	ধশে র ব	মূল্যমান	া সূচিত	চ আছে সামে	[] #1-0			
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PG-ScAP-17100								Received Answer Booklet							

Signature with seal by the Study-Centre





### জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় (T.E. Exam.) নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা (QPAB) প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

### Detail schedule for submission of assignment for the PG Term End Examination June-2020/Dec.-2020

1. Date of Publication : 20/06/2020 2. Last date of Submission of answer script by the student to the study : 19/07/2020 centre 3. : 16/08/2020 Last date of Submission of marks by the examiner to the study centre 4 Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.) :23/08/2020 Last date of submission of marks by the study centre to the 5. Department of C.O.E. on or before : 31/08/2020

এখানে কিছু লিখবেন না

## Do Not Write Anything Here



 $2 \times 5 = 10$ 

Answer Question No. 1 and any four from the rest.

- 1. Answer any *five* questions :
  - a) Show that  $u(x,y) = \frac{1}{2}\log(x^2 + y^2)$  is harmonic.

b) Find the radius of convergence of the power series  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} z^n$ .

- c) Examine the singularity of the function  $f(z) = \sin \frac{1}{z}$  at z = 0.
- d) Find a bilinear transformation which have fixed points 0 and  $\infty$ .
- e) Find where  $f(z) = \sin z$  is conformal and find the critical points if any.
- f) Find the residues of the function  $f(z) = \frac{e^{iz}}{z^2 + 1}$  at its singularities.
- g) Given that f(z) is analytic in a region R and f'(z) = 0 in R. Show that f(z) is constant in R.

#### **First Answer :**



**Second Answer :** 



Third Answer :



Fourth Answer :



Fifth Answer :

# **QP Code : PA/4/IIB** 8 / 20

PG-Sc.-AP-17100

- 2. a) State and prove Cauchy integral formula for derivative.
  - b) If C is a closed contour containing the origin inside it, prove that  $\frac{a^n}{n!} = \frac{1}{2\pi i} \oint_C \frac{e^{az}}{z^{n+1}} dz.$ 6+4
- 3. a) State and prove Liouville's theorem. From this deduce fundamental theorem of Classical Algebra. 4 + 2
  - b) Prove that the function  $f(z) = \frac{1}{z^2}$  is not uniformly continuous in  $|z| \le 1$ , but it is uniformly continuous in  $\frac{1}{2} \le |z| \le 1$ .
- 4. a) State and prove sufficient condition for analyticity of a complex valued function.
  - b) Expand  $f(z) = \frac{z}{(z-1)(2-z)}$  in a Laurent series valid for (i) |z-1| > 1, (ii) 0 < |z-2| < 1. 6+4
- 5. a) State and prove Riemann's theorem on removable singularity for an analytic function.

b) Find the region of convergence of the series  $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$ .

- c) If f(z) has an isolated singularity at  $z_0$  and  $f(z) \rightarrow \infty$  as  $z \rightarrow z_0$ , then show that f(z) has a pole at  $z_0$ . 5 + 3 + 2
- 6. a) State and prove Argument Principle.
  - b) Show by the method of contour integration  $\int_{0}^{\infty} \frac{\sin mx}{x} dx = \frac{\pi}{2}.$  5 + 5
- 7. a) Prove that a bilinear transformation transforms a circle into a circle and inverse points into inverse points.
  - b) Show that the transformation  $w = \frac{1-iz}{z-i}$  maps |z| < 1 into a circle in the *w*-plane whose centre is on the imaginary axis.
  - c) Examine the singularities of the function  $\sin \frac{1}{z-2} + \sin \frac{1}{z+2}$ . 4 + 4 + 2









**Second Answer :** 







Third Answer :







Fourth Answer :



