NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 3A : Ordinary Differential Equations

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/IIIA**

PG-Sc.-AP-17101

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 3A : Ordinary Differential Equations

Name (in Block Letter) : ....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/IIIA**

PG-Sc.-AP-17101

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June-2020/Dec.-2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Notations / symbols have their usual meanings.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Solve $\frac{dY}{dt} = AY$, where $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$ and $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.

b) In the following BVP, examine whether a Green's function exists and if it does then construct it :
$y'' + y' = 0$, $y(0) = y(1)$, $y'(0) = y'(1)$.

c) Find the regular singular point for the equation $t^2 y'' + ty' + (t^2 - 4)y = 0$. Is $t = 2020$ a regular singular point for the above equation ?

d) Compute the solution of $y''' + y'' + y' + y = 2020$ with the initial conditions $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.

e) Using the Picard's method of successive approximation, find the third approximation of the solution of the equatoin : $y' = t + y^2$, $y(0) = 0$.

f) Determine the nature of the critical points for the system $\dot{x} = 4x - 3y$, $\dot{y} = 8x - 6y$, where $\dot{x} = \frac{dx}{dt}$ etc.

g) Show that $L_n''(0) = \frac{n}{2}(n-1)$.

h) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Find the general solution of the equation $(t^2+1)y''-2ty'+2y=6(t^2+1)^2$, given that $y=t$ and $y=t^2-1$ are linearly independent solutions of the corresponding homogeneous equation. 5

b) If R is either a rectangle $|x-x_0|\le h$, $|y-y_0|\le k$ or a strip $|x-x_0|\le h, |y|<\infty$ and if $f(x,y)$ is a real-valued function defined on R such that $\frac{\partial f}{\partial y}$ exists, is continuous on R and $\left|\frac{\partial f}{\partial y}\right|\le \overline{K}\,\forall(x,y)\in R$ for a positive constant $\overline{K}$, then prove that $f(x,y)$ satisfies a Lipschitz condition on R with Lipschitz constant $\overline{K}$. Is the converse true ? Justify your answer. 5

3. a) Let the vector functions $\bar{\phi}_1,\bar{\phi}_2,\ldots\bar{\phi}_n$ be n solutions of the homogeneous linear vector differential equation $\frac{\mathrm{d}Y}{\mathrm{d}t}=AY$ on the interval $a\le t\le b$. Then prove that either

$W(\bar{\phi}_1,\bar{\phi}_2,\ldots,\bar{\phi}_n)=0\ \forall\ t\in[a,b]$ or $W(\bar{\phi}_1,\bar{\phi}_2,\ldots,\bar{\phi}_n)\ne 0\ \forall\ t\in[a,b]$,

where W is the Wronskian. 5

b) Find the nature and the stability of the critical points of the system $\frac{\mathrm{d}x}{\mathrm{d}t}=-ax+y$, $\frac{\mathrm{d}y}{\mathrm{d}t}=-x-ay$ for $a<0$ and $a>0$. 5

4. a) Find the Green's function and hence solve the following BVP :

$y''+k^2y=f(x)\ (k\ne\pi)$,

$y(0)=\alpha,\ y'(1)=\beta$,

where α,β are some real constants. 5

b) Show that the general solution of the equation $y''+3y'+2y=g(t)$ is bounded on $[0,\infty)$ if $g(t)$ is bounded for all t in $[0,\infty)$. 5

5. a) Solve the following system of ODEs :

$$\frac{\mathrm{d}Y}{\mathrm{d}t}=AY,\quad A=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & -1\\ 3 & 3 & -1\end{bmatrix},\quad Y=\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}.$$ 5

b) Find the eigenvalues and eigenfunctions for the following S-L equations :

$y''+\lambda y=0,\ -\pi\le t\le\pi$

subject to

$y(-\pi)=y(\pi)$,

$y'(-\pi)=y'(\pi)$. 5

6. a) Prove that $J_n(x)=\frac{1}{2}\left[J'_{n-1}(x)-J_{n+1}(x)\right]$. 5

b) Find the power series soluton of the differential equation $y''+ty'+t^2y=0$ in powers of x about $x=0$. 5

7. a) Prove that $\int_0^\infty e^{-rx} L_n(x)\,dx = \frac{1}{r}\left(1-\frac{1}{r}\right)^n$, $r \neq 0$. 5

b) Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :