NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 9A(i) : Advanced Complex Analysis & Paper - 9A(ii) : Operations Research

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

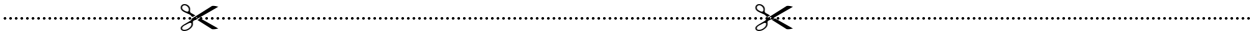
Study Centre Name : ..... Code : .....

To be filled by the Candidate	Serial No. of question answered														**TOTAL**
For Evaluator's only	Marks awarded														

Q.P. Code : **(PA/4/IXA(i))/(PA/4/IXA(ii))**

PG-Sc.-AP-17113

Signature of Evaluator with Date



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স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

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Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

Q.P. Code : **(PA/4/IXA(i))/(PA/4/IXA(ii))**

PG-Sc.-AP-17113

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Special Paper : Pure Mathematics

Paper - 9A(i) : Advanced Complex Analysis

(Symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Define the exponent of convergence of zeros of an entire function.

   b) Find the order of $e^{z/2}$.

   c) Show that for fixed R and ϕ, Poisson's Kernel $\frac{R^2 - |z|^2}{|Re^{i\phi} - z|^2}$ is harmonic in $|z| < R$.

   d) State Mittag-Leffler theorem.

   e) State Riemann mapping theorem.

   f) State Schwarz Reflection principle.

   g) Define an entire function. Give an example of it.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Prove that any harmonic function defined on a domain D has the mean value property in D. 4

   b) Let f be a function regular in the closed disc $|z| \le R$ and let $v(r,\theta)$ be its imaginary part. If $v(r,\theta) \ge 0$, then prove that $\frac{R-r}{R+r}v(0,0) \le v(r,\theta) \le \frac{R+r}{R-r}v(0,0)$, where $0 \le r < R$. 6

3. a) Let $u(x,y) \not\equiv$ constant be harmonic in a domain D. Then prove that $u(x,y)$ has neither a maximum nor a minimum at any interior point of D. 4

   b) Let $f(z)$ be an analytic function in a domain D containing z_0. If $f'(z_0) \neq 0$ then $f(z)$ is conformal at z_0. 6

4. a) State and prove Schwarz lemma. 6

   b) Test for convergence of the infinite product $\prod_{n=0}^{\infty}\left(1-\frac{z^2}{n^2}\right)$. 4

5. a) If $f(z)$ is an entire function and does not vanish on $\mathbb{C}$, then show that $f(z)$ is of the form $f(z) = e^{g(z)}$, where $g(z)$ is an entire function. 4

   b) State and prove Poisson-Jensen formula. 6

6. a) Let $f(z)$ be a non-constant analytic function in $|z| < R$. Then show that $M(r)$ is a strictly function of r in $0 \le r < R$. 4

   b) State and prove Jensen's inequality. 6

7. a) If $f(z)$ be an entire function with finite order ρ, then $n(r) = O(r^{\rho+\epsilon})$ for $\epsilon > 0$ and for sufficiently large r. 5

   b) Prove that if ρ is not an integer then $\rho = \rho_1$. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Special Paper : Applied Mathematics

Paper - 9A(ii) : Operations Research

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Discuss the post-optimality effect when a variable is deleted from an LPP.

   b) Write down the Kuhn-Tucker conditions for the following problem :

   Maximize $f(x_1, x_2, x_3) = -x_1^2 - x_2^2 - 2x_3^2 + 5x_1 + 3x_2$

   subject to $x_1 + 2x_2 \le 3$, $2x_1 + 3x_2 \le 12$.

   c) What are golden section, golden ratio and golden number ?

   d) What is the utility of mixed integer programming technique ?

   e) Write short notes on steepest descend method for solving non-linear unconstrained optimization problem.

   f) Under what conditions the Kuhn-Tucker necessary conditions are sufficient for maximization and minimization problems with ≤ type constraints.

   g) What are the differences between regular simplex method and Wolfe's modified simplex method ?

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Find the extreme points of the function

$f(x_1, x_2) = 2x_1^3 + 3x_2^3 + 5x_1^2 + 4x_2^2 + 6$

b) Find the dimension of a cylindrical tin (with top and bottom) made up of sheet metal to maximize its volume such that the total surface area is equal to 24π. 4 + 6

3. a) Use revised simplex method to solve the following LPP :

Maximize $Z = x_1 + 2x_2$

subject to $x_1 + x_2 \le 3$, $x_1 + 2x_2 \le 5$, $3x_1 + x_2 \le 6$, $x_1, x_2 \ge 0$.

b) Given the LPP

Maximize $Z = 3x_1 + 5x_2$

subject to $3x_1 + 2x_2 \le 18$, $x_1 \le 4$, $x_2 \le 6$ and x_1, $x_2 \ge 0$.

Discuss the effect on the optimality of the solution when the objective function is changed to $3x_1 + x_2$. 5 + 5

4. a) Discuss the criteria for selection of incoming and outgoing vectors in dual simplex method.

b) Using artificial constraint method, solve the following problem by dual simplex method and show that the problem has no feasible solution.

Maximize $Z = -x_1 + x_2$

subject to $x_1 - 4x_2 \ge 5$, $x_1 - 3x_2 \le 1$, $2x_1 - 5x_2 \ge 1$, $x_1, x_2 \ge 0$. 4 + 6

5. a) Write down the algorithm of cutting plane method for solving non-linear constrained optimization problem.

b) Using cutting plane method, solve the following problem :

Maximize $f(x_1, x_2) = 1 - 4x_1 - 2x_2$

subject to $2(x_1 - 2)^2 + (x_2 - 3)^2 \ge 12$, $2x_1 + x_2 \le 3$, $0 \le x_1, x_2 \le 5$, (use $\varepsilon = 0 \cdot 2$).

3 + 7

6. a) Minimize $f(x) = \begin{cases} 2\sqrt{x}, & x \le 1 \\ 3 - x, & x > 1 \end{cases}$

in the interval [0, 5] by golden section method up to six experiments.

b) Using Davidon-Fletcher-Powell (DFP) method minimize

$f(x_1, x_2) = 8x_1^2 + 4x_2^2 - 24x_1 + 16x_2 + 35$ with $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$ as the starting point. 4 + 6

7. a) Solve the following all integer programming problem :

Maximize $Z = 3x_1 + 4x_2$

subject to $3x_1 + 2x_2 \le 8$

$x_1 + 4x_2 \ge 10$

$x_1, x_2 \ge 0$ and are integers.

b) Applying Wolfe's method, solve the following quadratic programming problem :

Maximize $Z = 2x_1 + x_2 - x_1^2$

subject to $2x_1 + 3x_2 \le 6$, $2x_1 + x_2 \le 4$ and $x_1, x_2 \ge 0$. 4 + 6

First Answer :

Second Answer :

Third Answer :

Fourth Answer :