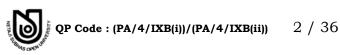


NETAJI SUBHAS OPEN UNIVERSITY

E SUBHAS OPEN UNC				শা	তকোৰ	হর পা	ঠক্রম	(P.	G.)						
		অনুশী	লন প		signn	nent)	: জুন	ં ૨૦	•	June	e, 20	20)			
	Spe	cial F	Paper	: Pui	MA e Ma	THE them			ppli	ed N	Iath	emat	ics		
-	9B(i) : Ād					-									
পূর্ণমান : ৫০		QUI	ESTI	ON P	APEI	R CU	M Al	NSW	ER	BOC					ম্ত্র : ২০%
(Full Marks				প্ৰহা হার		<u></u>	Z) (3 55	* F - 7	•		•	-	-		ks : 20%)
পরিমিত ও য				~	ণ্ড দেও ওয়া হল									<u>ମା</u> ଗ୍ଞାର	। ୧ଓ।শরের
	ecial cre	dit w	ill be	give	n for j	precis	se an	d co	rrec	t an	swei	r. Ma	rks w		:
Ċ	leducted				stake n the								dwrit	ing.	
Name (in Blo	ock Letter		-			-									
Enrolment No.		,		İ	<u> </u>	Í	1				ľ				
Study Centr	e Name ·											Co	nde ·		
To be filled	Serial No.				<u> </u>										
by the Candidate	question answere														TOTAL
For Evaluator's only	Marks awardeo	1													
Q.P. Code :	(PA/4/IXI	3(i))/(1	PA/4/	IXB(ii	i))										
		1													
PG-ScAP	-17114									Sıg	natu	re of	Evalu	lator	with Date
	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~								×						
		_		-				_							
ALLEN ALLEN	Ν	ET.	AJI	SU	BHA	IS C	)PE	N U	<b>UN</b>	IVI	ERS	SIT	Y		
ELBHAS OPEN USE	সাতকোত্তর পাঠক্রম ( P. G.)									STUDENT'S COP					
অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020) MATHEMATICS															
	Spe	cial F	Paper	: Pui	e Ma				ppli	ed N	Iath	emat	ics		
Paper - 9	9B(i) : Ad	vanc	ed To	polo	gy & 1	Paper	- 9B	(ii) :	Mat	hen	natic	al M	odels	In Ec	cology
Name (in Blo	ock Letter	r) :	•••••	•••••	•••••	•••••	•••••						•••••	•••••	
Enrolment No.															]
Study Centr	e Name :		••••		•••••			•••••				Co	ode : .		
Q.P. Code :	(PA/4/IXI	B(i))/(I	PA/4/	IXB(ii	i))										

PG-Sc.-AP-17114

Received Answer Booklet Signature with seal by the Study-Centre



# জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় (T.E. Exam.) নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা (QPAB) প্রবর্তন করা হবে। এই নতন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনশীলন পত্রে নির্দেশ অন্যায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

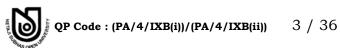
New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

#### Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1. Date of Publication : 20/06/2020 2. Last date of Submission of answer script by the student to the study : 19/07/2020 centre 3. : 16/08/2020 Last date of Submission of marks by the examiner to the study centre 4 Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.) :23/08/2020 Last date of submission of marks by the study centre to the 5. Department of C.O.E. on or before : 31/08/2020

এখানে কিছু লিখবেন না

# **Do Not Write Anything Here**



#### Special Paper : Pure Mathematics Paper - 9B(i) : Advanced Topology

(Symbols and Notations have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* of the following :

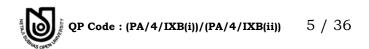
 $2 \times 5 = 10$ 

- a) Show that the space of rationals with the induced topology from the usual topology of reals is not locally compact.
- b) Prove that the continuous image of a sequentially compact set is sequentially compact.
- c) Give an example of a paracompact topological space which is not compact.
- d) For two compactifications (f,Y) and (g,Z) if  $(f,Y) \le (g,Z)$  and  $(g,Z) \le (f,Y)$ then show that (f,Y) and (g,Z) are equivalent.
- e) Is the real number space endowed with the cofinite topology metrizable ? Answer with reasons.
- f) In a metric space (X,d) prove that  $x \in \overline{A}$  iff d(x,A) = 0 where  $x \in X$ ,  $A \subset X$ .
- g) Show that any metric space is a uniform space.

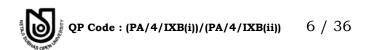
**First Answer :** 



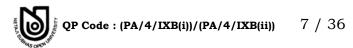
**Second Answer :** 



**Third Answer :** 



Fourth Answer :



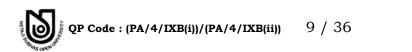
Fifth Answer :

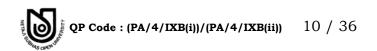
QP Code : (PA/4/IXB(i))/(PA/4/IXB(ii)) 8 / 36

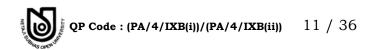
PG-Sc.-AP-17114

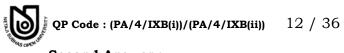
ONAS C	PENO							
2.	a)	Show that a filter $\mathfrak{R}^*$ is an ultrafilter in X iff any subset A of X which intersects						
		every member of $\mathfrak{F}^*$ belongs to $\mathfrak{F}^*$ . 5						
	b)	For a topological space $(X, \tau)$ , prove that following are equivalent :						
		i) X is compact						
		ii) Every net in X has a convergent subnet						
		iii) Every maximal net in X converges in X. 5						
3.	a)	Define countable compactness. Show that $(X, \tau)$ is countably compact iff every						
		infinite set $S \subset X$ has an $\omega$ -accumulation point in $X$ . 5						
	b)	Let X be a Tychonoff space. Prove that X is locally compact iff the remainder $\beta X \setminus X$						
		is closed. 5						
4.	a)	State and prove Stone-Cech theorem.						
	b)	Prove that every Hausdorff para-compact space is regular. 4						
5.	a)	If every open cover of a topological space $X$ has a closed locally finite refinement then						
		show that <i>X</i> is para-compact. 6						
	b)	Let $\tau_l$ be the lower limit topology on <i>R</i> . Assuming that ( <i>IR</i> , $\tau_l$ ) is Lindeloff, prove						
		that $(\mathbb{R}, \tau_l)$ is a paracompact space. 4						
6.	a)	Prove that $M \subset c[a,b]$ is relatively compact iff it is uniformly bounded and						
		equicontinuous. 6						
	b)	Show that the derived set of a countably compact set in a metric space is countably						
		compact. 4						
7.	a)	Prove that every uniformizable space is completely regular. 5						
	b)	Prove that the filter associated with a Cauchy net is a Cauchy filter and conversely						
		every net associated with a Cauchy filter is Cauchy. 5						

**First Answer :** 

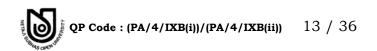


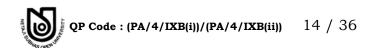


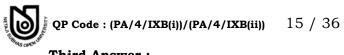




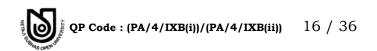
Second Answer :

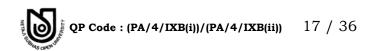


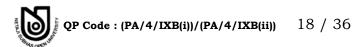




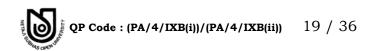
Third Answer :

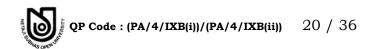


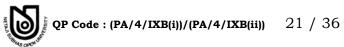




Fourth Answer :







# Special Paper : Applied Mathematics Paper - 9B(ii) : Mathematical Models In Ecology

Answer Question No. 1 and any four from the rest.

1. Answer any *five* questions :

- a) Define an ecosystem. Describe the different components of ecosystem.
- b) Define a dynamical model. What do you mean by a deterministic model ?
- c) Write down the Malthus model of population growth. What are the drawbacks of the model ?
- d) For the differential equation  $\frac{dx}{dt} = f(x)$ , define stability and asymptotic stability of

an equilibrium point  $x^*$  in the sense of Lyapunov.

e) Define a fixed point  $x^*$  of the difference equation  $x_{n+1} = f(x_n)$ . Write down a

condition for asymptotic stability of a fixed point  $x^*$  of this equation.

- f) Explain with an example the concept of time-delay in the mathematical modelling of a population growth.
- g) Write down the Lotka-Volterra predator-prey model.

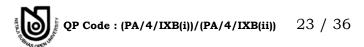
#### **First Answer :**

PG-Sc.-AP-17114

 $2 \times 5 = 10$ 



Third Answer :



Fourth Answer :

Fifth Answer :

### QP Code : (PA/4/IXB(i))/(PA/4/IXB(ii)) 24 / 36

# 2. Write down the logistic model equation of population growth and explain the different terms involved in the equation. Hence explain the concepts of carrying capacity and intraspecies competition. Investigate the asymptotic stability of the equilibrium points of the model. Solve the logistic equation for a given initial condition and discuss the behaviour of the solution if time approaches infinity.

3. a) Describe "Cobweb diagram" method of solution of the difference equation  $x_{n+1} = f(x_n)$ . 5

b) The growth of a population satisfies the difference equation  $x_{n+1} = \frac{kx_n}{b+x_n}$ , b, k > 0. When does the positive fixed point of the equation exist ? Discuss stability of the positive fixed point, when it exists.

4. a) Solve the second order difference equation  $x_{n+1} = x_n + x_{n-1}$  with  $x_0 = 0$  and  $x_1 = 1$ . What is Golden mean ? 6

- b) Find the equilibrium points of the Lotka-Volterra predator-pray model. Discuss their stability.
- 5. State and prove Bendixson's negative criterion for the existence of a closed orbit contained in a domain  $D \subset \mathbb{R}^2$ . 10
- 6. Find the equilibrium points of the following predator-prey system :

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x \left( 1 - \frac{x}{30} \right) - \frac{xy}{x+10}$$

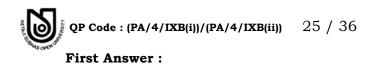
 $\frac{\mathrm{d}y}{\mathrm{d}t} = y\left(\frac{x}{x+10} - \frac{1}{3}\right).$ 

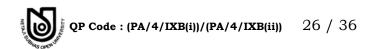
Investigate their stability.

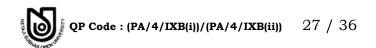
- 7. a) For the Lotka-Volterra two-species competition model, show that there is no closed orbit in the interior of the first quadrant.3
  - b) Define a cooperative system. Prove that the orbit of a cooperative system either converges to equilibrium or diverges to infinity. 7

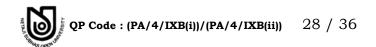
#### PG-Sc.-AP-17114

10

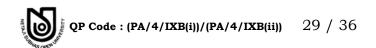


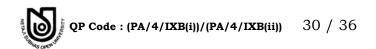


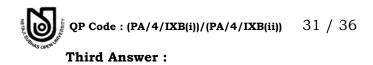


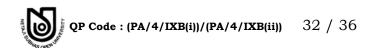


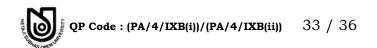
**Second Answer :** 

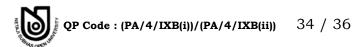












Fourth Answer :

