NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 9B(i) : Advanced Topology & Paper - 9B(ii) : Mathematical Models In Ecology

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

To be filled by the Candidate	Serial No. of question answered														**TOTAL**
For Evaluator's only	Marks awarded														

Q.P. Code : **(PA/4/IXB(i))/(PA/4/IXB(ii))**

PG-Sc.-AP-17114

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 9B(i) : Advanced Topology & Paper - 9B(ii) : Mathematical Models In Ecology

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

Q.P. Code : **(PA/4/IXB(i))/(PA/4/IXB(ii))**

PG-Sc.-AP-17114

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Special Paper : Pure Mathematics

Paper - 9B(i) : Advanced Topology

(Symbols and Notations have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* of the following : $2 \times 5 = 10$

   a) Show that the space of rationals with the induced topology from the usual topology of reals is not locally compact.

   b) Prove that the continuous image of a sequentially compact set is sequentially compact.

   c) Give an example of a paracompact topological space which is not compact.

   d) For two compactifications (f, Y) and (g, Z) if $(f, Y) \leq (g, Z)$ and $(g, Z) \leq (f, Y)$ then show that (f, Y) and (g, Z) are equivalent.

   e) Is the real number space endowed with the cofinite topology metrizable ? Answer with reasons.

   f) In a metric space (X, d) prove that $x \in \overline{A}$ iff $d(x, A) = 0$ where $x \in X$, $A \subset X$.

   g) Show that any metric space is a uniform space.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Show that a filter $\mathcal{F}^*$ is an ultrafilter in X iff any subset A of X which intersects every member of $\mathcal{F}^*$ belongs to $\mathcal{F}^*$. 5

   b) For a topological space (X,τ), prove that following are equivalent :

   i) X is compact

   ii) Every net in X has a convergent subnet

   iii) Every maximal net in X converges in X. 5

3. a) Define countable compactness. Show that (X,τ) is countably compact iff every infinite set $S \subset X$ has an ω-accumulation point in X. 5

   b) Let X be a Tychonoff space. Prove that X is locally compact iff the remainder $\beta X \setminus X$ is closed. 5

4. a) State and prove Stone-Cech theorem. 6

   b) Prove that every Hausdorff para-compact space is regular. 4

5. a) If every open cover of a topological space X has a closed locally finite refinement then show that X is para-compact. 6

   b) Let τ_l be the lower limit topology on R. Assuming that $(\mathbb{R}, \tau_l)$ is Lindeloff, prove that $(\mathbb{R}, \tau_l)$ is a paracompact space. 4

6. a) Prove that $M \subset c[a,b]$ is relatively compact iff it is uniformly bounded and equicontinuous. 6

   b) Show that the derived set of a countably compact set in a metric space is countably compact. 4

7. a) Prove that every uniformizable space is completely regular. 5

   b) Prove that the filter associated with a Cauchy net is a Cauchy filter and conversely every net associated with a Cauchy filter is Cauchy. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Special Paper : Applied Mathematics

Paper - 9B(ii) : Mathematical Models In Ecology

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Define an ecosystem. Describe the different components of ecosystem.

   b) Define a dynamical model. What do you mean by a deterministic model ?

   c) Write down the Malthus model of population growth. What are the drawbacks of the model ?

   d) For the differential equation $\frac{dx}{dt} = f(x)$, define stability and asymptotic stability of an equilibrium point x^* in the sense of Lyapunov.

   e) Define a fixed point x^* of the difference equation $x_{n+1} = f(x_n)$. Write down a condition for asymptotic stability of a fixed point x^* of this equation.

   f) Explain with an example the concept of time-delay in the mathematical modelling of a population growth.

   g) Write down the Lotka-Volterra predator-prey model.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. Write down the logistic model equation of population growth and explain the different terms involved in the equation. Hence explain the concepts of carrying capacity and intra-species competition. Investigate the asymptotic stability of the equilibrium points of the model. Solve the logistic equation for a given initial condition and discuss the behaviour of the solution if time approaches infinity. 10

3. a) Describe "Cobweb diagram" method of solution of the difference equation $x_{n+1} = f(x_n)$. 5

   b) The growth of a population satisfies the difference equation $x_{n+1} = \frac{kx_n}{b + x_n}$, $b,k > 0$.

   When does the positive fixed point of the equation exist ? Discuss stability of the positive fixed point, when it exists. 5

4. a) Solve the second order difference equation $x_{n+1} = x_n + x_{n-1}$ with $x_0 = 0$ and $x_1 = 1$.

   What is Golden mean ? 6

   b) Find the equilibrium points of the Lotka-Volterra predator-pray model. Discuss their stability. 4

5. State and prove Bendixson's negative criterion for the existence of a closed orbit contained in a domain $D \subset \mathbb{R}^2$. 10

6. Find the equilibrium points of the following predator-prey system :

$$\frac{dx}{dt} = x\left(1 - \frac{x}{30}\right) - \frac{xy}{x+10},$$

$$\frac{dy}{dt} = y\left(\frac{x}{x+10} - \frac{1}{3}\right).$$

   Investigate their stability. 10

7. a) For the Lotka-Volterra two-species competition model, show that there is no closed orbit in the interior of the first quadrant. 3

   b) Define a cooperative system. Prove that the orbit of a cooperative system either converges to equilibrium or diverges to infinity. 7

First Answer :

Second Answer :

Third Answer :

Fourth Answer :