

# **NETAJI SUBHAS OPEN UNIVERSITY**

স্নাতকোত্তর পাঠক্রম ( P. G.)

অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

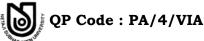
### MATHEMATICS

Paper - 6A : General Topology

							- <b>T</b> -	07					
পূর্ণমান : ৫০		QUE	STIO	N PAF	PER C	UM A	NSWE	ER BO	OKLE	T ?	যানের গুর	ম্ত্র : ২০%	
(Full Marks	: 50)								(We	eightage	e of Mar	ks : 20%)	
পরিমিত ও য	থাযথ উত্তরে	রর জন	য বিশে	ষ মূল্য (	দেওয়া হ	বে। অ	শুদ্ধ বাৰ	নান, অ	পরিচ্ছন	চা এবং 🔻	অপরিষ্কার	হণ্ডাক্ষরের	
	ক্ষে	ত্রে নম্বর	ৰ্বকট	ন ওয়া	হবে। উ	টপান্তে	প্রশ্নের ফ	মূল্যমান	াসূচিত আ	মাছে।			
	ecial cree											!	
	deducted								egible f marks		iting.		
Name (in Bl	ock Letter		-			-							
Enrolment No.			Т										
Study Centr	re Name :									. Code	:		
To be filled	Serial No.											-	
by the Candidate	question answered											TOTAL	
For Evaluator's	Marks												
only	awarded	l											
Q.P. Code :	PA/4/VIA	1											
PG-ScAF	P-17107							Si	ignatur	e of Eva	aluator v	with Date	
	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~						a	/					
							8						
	N		<b>TT</b> (	21101		Λπτ		Tarra		T/T\\\.			
	IN	ETA	JI i						<b>ERS</b>	[']'Y			
স্মৃতকোর্জ পাঠক্রম							•	· · · · · · · · · · · · · · · · · · ·					
		অনুশীৰ	ণন পত্র	(Assig	nmen	t) : জুন্	i, <b>২০</b> ২	o (Jur	ne, 202	0)			
				N	<b>IATH</b>	ЕМАТ	ICS						
			F	Paper -	6A : G	enera	1 Topo	ology					
Name (in Bl	ock Letter	):											
												1	
Enrolment	No.												
Study Cent	re Name :									. Code	:		
Q.P. Code :													
y.r. couc.	· // +/ v //	-											
PG-ScAF	P-17107							R	eceived	Answe	r Bookle	et	
L		1											

Signature with seal by the Study-Centre





## জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় (T.E. Exam.) নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুন্তিকা (QPAB) প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

#### Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1. Date of Publication : 20/06/2020 2. Last date of Submission of answer script by the student to the study : 19/07/2020 centre 3. : 16/08/2020 Last date of Submission of marks by the examiner to the study centre 4 Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.) :23/08/2020 Last date of submission of marks by the study centre to the 5. Department of C.O.E. on or before : 31/08/2020

এখানে কিছু লিখবেন না

# Do Not Write Anything Here

3 / 20

PG-Sc.-AP-17107

 $2 \times 5 = 10$ 

Answer Question No. 1 and any four from the rest.

1. Answer any *five* questions :

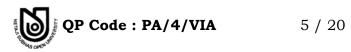
QP Code : PA/4/VIA

- a) Define basis of a topological space. Find a basis of ( $I\!\!R$ ,  $\tau_d$ ), where  $\tau_d$  is the discrete topology.
- b) If  $\{\tau_{\alpha}\}_{\alpha \in \Lambda}$  is a family of topologies on *X*, show that  $\bigcap_{\alpha \in \Lambda} \tau_{\alpha}$  is also a topology on *X*.
- c) Let *Y* be a subset  $[0,1) \cup \{2\}$  of *I*R endowed with standard topology. Show that  $\{2\}$  is open in the subspace topology on *Y*.
- d) Show that arbitrary intersection of open sets need not be open.
- e) In a Hausdorff space if  $x \in X$ , show that  $\bigcap \{\overline{N}_X : N_X \in N_X\} = \{x\}$  where bar denotes the closure and  $N_X$  is the nbd. system at x.
- f) Show that union of connected subsets need not be connected. State a sufficient condition for the union to be connected.
- g) Show that the real line  $\mathbb{R}$  with discrete topology is not compact.

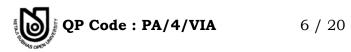
**First Answer :** 



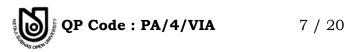
**Second Answer :** 



Third Answer :



Fourth Answer :

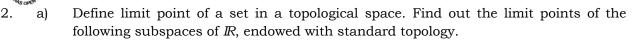


Fifth Answer :

#### 8 / 20

PG-Sc.-AP-17107

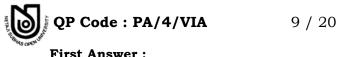
3



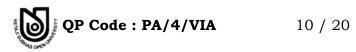
- i) The set of rational numbers, Q
- ii)  $\{1/n : n \in \mathbb{N}\}$

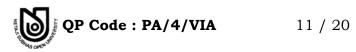
QP Code : PA/4/VIA

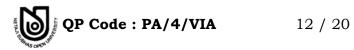
- iii)  $I\!R \{1, 2, 3, 4\}.$  1+2
- b) Write down the definition of a Hausdorff topological space  $(T_2)$ . Give an example of a topological space which is  $T_1$  but not  $T_2$  with reasons. 1+2
- c) Show that  $(X,\tau)$  is  $T_2$  iff the diagonal  $\Delta = \{(x \times x) | x \in X\}$  is closed in  $X \times X$ . 4
- 3. a) Define continuous function in topological spaces. Let  $(X, \tau_X)$  and  $(Y, \tau_Y)$  be two topological spaces and  $f: X \to Y$  be a continuous function. Show that for every subset A of X,  $f(\overline{A}) \subset \overline{f(A)}$ , bar denoting the closure. 1+2
  - b) Show by an example that in a topological space  $(X, \tau)$  for any two subsets  $A, B \subseteq X$ ,
    - i)  $\overline{A \cap B} = \overline{A} \cap \overline{B}$  may not be true
    - ii)  $int(A \cap B) = int(A) \cap int(B)$  holds. 1+3
  - c) Define a net. Prove that in a topological space  $(X, \tau)$ , a point  $u \in X$  is a limit point of  $A \subset X$  iff there is a net in  $A \setminus \{u\}$  such that the net converges to u. 3
- 4. a) Define a compact topological space. Prove that every compact subspace of a  $T_2$  space is closed. 1 + 4
  - b) Let (X, τ) be compact and (Y, τ') be T<sub>2</sub>-topological spaces. If f: X → Y is a bijective function, then prove that f is a homeomorphism.
    Show that arbitrary union of compact sets need not be compact. 3 + 2
- 5. a) Define a normal space. Show that a metric space is normal.
  - b) Let A be a connected subspace of a topological space  $(X,\tau)$ , if  $A \subset B \subset \overline{A}$ , then show that B is also connected.
  - c) Prove that continuous image of a connected set is connected. 3
- 6. a) Prove that a subset of  $\mathbb{R}$  with standard topology is connected iff it is an interval. 5
  - b) Denine a locally connected space. Prove that  $(X, \tau)$  is locally connected iff components of each open subspace of X are open in X. 5
- 7. a) Prove that the one point compactification  $(X_u, \tau_u)$  of a non-compact topological space  $(X, \tau)$  is  $T_2$  iff  $(X, \tau)$  is a locally compact  $T_2$ -space. 5
  - b) Define a uniform space (X, v). If  $\tau_v$  is the uniform topology on X induced by v then prove that the  $\tau_v$ -closure of  $A = \overline{A} = \bigcap \{ \bigcup (A) : \bigcup \in v \}$ . 5



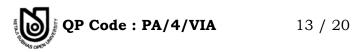
**First Answer :** 

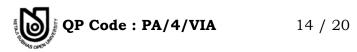


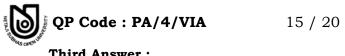




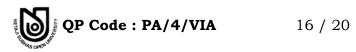
**Second Answer :** 

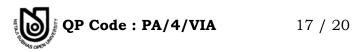


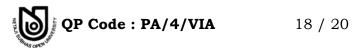




Third Answer :







Fourth Answer :

