NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 6A : General Topology

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/VIA**

PG-Sc.-AP-17107

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 6A : General Topology

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIA**

PG-Sc.-AP-17107

Received Answer Booklet

Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Define basis of a topological space. Find a basis of $(\mathbb{R}, \tau_d)$, where τ_d is the discrete topology.

b) If $\{\tau_\alpha\}_{\alpha\in\Lambda}$ is a family of topologies on X, show that $\bigcap_{\alpha\in\Lambda} \tau_\alpha$ is also a topology on X.

c) Let Y be a subset $[0,1)\cup\{2\}$ of $\mathbb{R}$ endowed with standard topology. Show that $\{2\}$ is open in the subspace topology on Y.

d) Show that arbitrary intersection of open sets need not be open.

e) In a Hausdorff space if $x \in X$, show that $\bigcap\{\overline{N}_x : N_x \in N_x\} = \{x\}$ where bar denotes the closure and N_x is the nbd. system at x.

f) Show that union of connected subsets need not be connected. State a sufficient condition for the union to be connected.

g) Show that the real line $\mathbb{R}$ with discrete topology is not compact.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Define limit point of a set in a topological space. Find out the limit points of the following subspaces of $\mathbb{R}$, endowed with standard topology.

   i) The set of rational numbers, $\mathbb{Q}$

   ii) $\{1/n : n \in \mathbb{N}\}$

   iii) $\mathbb{R} - \{1, 2, 3, 4\}$. 1 + 2

   b) Write down the definition of a Hausdorff topological space (T_2). Give an example of a topological space which is T_1 but not T_2 with reasons. 1 + 2

   c) Show that (X, τ) is T_2 iff the diagonal $\Delta = \{(x \times x) \mid x \in X\}$ is closed in $X \times X$. 4

3. a) Define continuous function in topological spaces. Let (X, τ_X) and (Y, τ_Y) be two topological spaces and $f : X \to Y$ be a continuous function. Show that for every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$, bar denoting the closure. 1 + 2

   b) Show by an example that in a topological space (X, τ) for any two subsets $A, B \subseteq X$,

   i) $\overline{A \cap B} = \overline{A} \cap \overline{B}$ may not be true

   ii) $int(A \cap B) = int(A) \cap int(B)$ holds. 1 + 3

   c) Define a net. Prove that in a topological space (X, τ), a point $u \in X$ is a limit point of $A \subset X$ iff there is a net in $A \setminus \{u\}$ such that the net converges to u. 3

4. a) Define a compact topological space. Prove that every compact subspace of a T_2 space is closed. 1 + 4

   b) Let (X, τ) be compact and (Y, τ') be T_2-topological spaces. If $f: X \to Y$ is a bijective function, then prove that f is a homeomorphism.

   Show that arbitrary union of compact sets need not be compact. 3 + 2

5. a) Define a normal space. Show that a metric space is normal. 3

   b) Let A be a connected subspace of a topological space (X, τ), if $A \subset B \subset \overline{A}$, then show that B is also connected. 4

   c) Prove that continuous image of a connected set is connected. 3

6. a) Prove that a subset of $\mathbb{R}$ with standard topology is connected iff it is an interval. 5

   b) Denine a locally connected space. Prove that (X, τ) is locally connected iff components of each open subspace of X are open in X. 5

7. a) Prove that the one point compactification (X_u, τ_u) of a non-compact topological space (X, τ) is T_2 iff (X, τ) is a locally compact T_2-space. 5

   b) Define a uniform space (X, ν). If τ_ν is the uniform topology on X induced by ν then prove that the τ_ν-closure of $A = \overline{A} = \bigcap \{\cup(A) : \cup \in \nu\}$. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :