NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 6B : Functional Analysis

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														**TOTAL**
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/VIB**

PG-Sc.-AP-17108

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 6B : Functional Analysis

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIB**

PG-Sc.-AP-17108

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Notations and symbols have their usual meanings.
Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$
   a) Show that the closed unit sphere in a normed linear space is convex.
   b) Prove or disprove : In a normed linear space the norm function is uniformly continuous.
   c) Prove that in an inner product space every orthonormal set is linearly independent.
   d) Is l_2 a Hilbert space ? Justify.
   e) If A is a self-adjoint operator in a Hilbert space H, then show that A^n is self-adjoint for any natural number n.
   f) In an inner product space X, prove that the conditions $< x_n, x > \to < x, x >$ and $\| x_n \| \to \| x \|$ imply $x_n \to x$.
   g) Let H be a Hilbert space. Prove that for any bounded linear operator $A : H \to H$, $\left\| A^* A \right\| = \| A \|^2 = \left\| AA^* \right\|$.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) State Hahn Banach theorem in a normed linear space. Prove that for any $x(\neq 0)$ in a normed linear space, $\| x \| = sup\left\{ \frac{|f(x)|}{\|f\|}, f \in X^* \text{ and } f \neq 0 \right\}$.

   b) Show that a subspace of a separable metric space is also separable.

   c) Show that compactness is not a hereditary property in a metric space. 4 + 4 + 2

3. a) Given that X is a normed linear space and Y is a Banach space. Show that the space $B(X,Y)$ of all bounded linear operators from X into Y is a Banach space.

   b) Given that the conjugate space X^* of a normed linear space X is separable. Show that X is separable. 5 + 5

4. a) State and prove Riesz Representation theorem in a Hilbert space.

   b) If X is a normed linear space such that the unit sphere $\{x \in X : \| x \| = 1\}$ is compact, then prove that X is finite dimensional. 5 + 5

5. a) Let M be a proper closed subspace of a normed linear space X and let $x_0 \in X \setminus M$. If $d = \inf_{x \in M} \| x_0 - x \|$ then show that there exists an $f \in X^*$ such that $\| f \| = 1$, $f(x_0) = d$ and $f(x) = 0$ for all $x \in M$.

   b) Let $L_2[0, 2\pi]$ be the real Hilbert space of all square integrable functions f over $[0, 2\pi]$ with inner product given by $(f, g) = \int_0^{2\pi} fg \, dt$, $f, g \in L_2[0,2\pi]$.

   Show that $e_0(t) = \frac{1}{\sqrt{2\pi}}$, $e_n(t) = \frac{\cos nt}{\sqrt{\pi}}$, $n = 1, 2, \ldots$

   where $0 \leq t \leq 2\pi$ forms an orthonormal sequence in $L_2[0, 2\pi]$. 5 + 5

6. a) State and prove Bessel's inequality.

   b) Show that the collection of all self-adjoint operators forms a closed real linear subspace of the space of all continuous linear operators that map H into itself, where H is a Hilbert space.

   c) Show that the space $c[a, b]$ with sup norm is not a Hilbert space. 4 + 4 + 2

7. a) Suppose that $A : H \to H$ is a self-adjoint operator where H is a Hilbert space. Show that $\| A \| = sup\{< Ax, x > : \| x \| = 1\}$.

   b) Prove that in a separable Hilbert space H, every orthonormal system is countable.

   c) For two vectors x and y in an inner product space X, prove that $x \perp y$ if and only if $\| y \| \leq \| \alpha x + y \|$ for any scalar α. 5 + 3 + 2

First Answer :

Second Answer :

Third Answer :

Fourth Answer :