# NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম ( P. G.)

অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

## MATHEMATICS

### Paper - 7A : Differential Equations and Integral Transformations

পূর্ণমান : ৫০    **QUESTION PAPER CUM ANSWER BOOKLET**    মানের গুরুত্ব : ২০%

(Full Marks : 50)        (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

**Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.**

Name (in Block Letter) : ...................................................................................................

Enrolment No.

Study Centre Name : ................................................................... Code : ....................

| To be filled by the Candidate | Serial No. of question answered | | | | | | | | | | | | | **TOTAL** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| For Evaluator's only | Marks awarded | | | | | | | | | | | | | |

Q.P. Code : **PA/4/VIIA**

**PG-Sc.-AP-17109**

Signature of Evaluator with Date

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## জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।
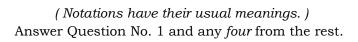
**New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.**

### Detail schedule for submission of assignment for the
### PG Term End Examination June, 2020

| | | |
|---|---|---|
| 1. | Date of Publication | : 20/06/2020 |
| 2. | Last date of Submission of answer script by the student to the study centre | : 19/07/2020 |
| 3. | Last date of Submission of marks by the examiner to the study centre | : 16/08/2020 |
| 4. | Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.) | : 23/08/2020 |
| 5. | Last date of submission of marks by the study centre to the Department of C.O.E. on or before | : 31/08/2020 |

## এখানে কিছু লিখবেন না
## Do Not Write Anything Here

*( Notations have their usual meanings. )*
Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions :        2 × 5 = 10

   a) Show that $L[\cos \omega t] = \dfrac{p}{p^2 + \omega^2}$.

   b) State and prove the shifting theorem of Laplace transform.

   c) If $F(p)$ is the Laplace transform of a function $f(t)$, which is piecewise continuous in any finite interval of $t$ and is of exponential order $O(e^{at})$ at $t \to \infty$, then show that $\underset{p \to \infty}{lim} \; F(p) = 0$.

   d) How Fourier transform of a function $f(x)$ of real variable $x$ is defined ?

   e) By the use of Fourier transform reduce the differential equation,

   $$6\frac{d^2 u}{dx^2} + 4\frac{du}{dx} + 4 = 0$$

   with the given initial conditions,

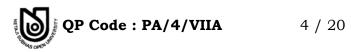   $$\left(\frac{du}{dx}\right)_0 = 0, \; (u)_0 = 1$$

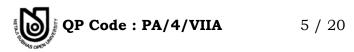   to an algebraic equation.

   f) State the convolution theorem of Laplace transform.

   g) Write the expression for the convolution of the two functions $f(x)$ and $g(x)$ and then state the convolution theorem.
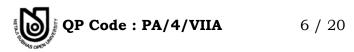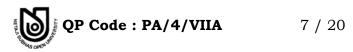
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   **First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. Considering the functions $f_1(x)$ and $f_2(x)$ defined by

$$\begin{aligned} f_1(x) &= 1,\ |x| \leq a \\ &= 0,\ |x| > a \end{aligned} \bigg\} \qquad \begin{aligned} f_2(x) &= 1,\ |x| \leq b \\ &= 0,\ |x| > b \end{aligned}$$

and using Parseval's relation, show that

$$\int_0^\infty \frac{\sin ax \sin bx}{x^2}\, dx = \frac{\pi a}{2},\ 0 < a < b.$$     10

3. Find the Fourier inversion of $|k|^{1/2}\, \overline{f}(k)$, where $\overline{f}(k)$ is the Fourier transform of $f(x)$.

     10

4. If $L[f(t)] = F(p)$ in the domain $Re(p) > a$ and $\phi(t) = \int_0^t f(\tau)\, d\tau$, then show that

$L[\phi(t)] = \dfrac{1}{p} F(p)$, which exists in the domain $Re(p) > a$. Here $L$ denotes the Laplace

transform.     10

5. Find $L^{-1}\left[\dfrac{p}{(p^2 + a^2)^2}\right]$ by the use of convolution theorem, where $L$ denotes Laplace

transform.     10

6. Use Laplace transform to find the solution of the equation $\dfrac{d^3 x}{dt^3} + \dfrac{dx}{dt} = \sin t$, satisfying the

initial conditions, $x(0) = 0$, $x'(0) = -2$, $x''(0) = 0$.     10
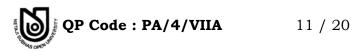
7. By the use of the function $f(x)$ defined by
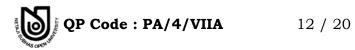
$f(x) = 1 - |x|$, when $|x| \leq 1$

$\quad = \quad$ 0, when $|x| > 1$

and using Fourier inversion theorem show that $\displaystyle\int_{-\infty}^\infty \frac{\sin^2 x}{x^2}\, dx = \pi$.     10
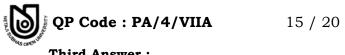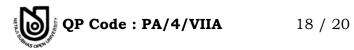
**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**