NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 10A(i) : Advanced Differential Geometry & Paper - 10A(ii) : Fluid Mechanics

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No.

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered													**TOTAL**
For Evaluator's only	Marks awarded													

Q.P. Code : **(PA/4/XA(i))/(PA/4/XA(ii))**

PG-Sc.-AP-17115

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 10A(i) : Advanced Differential Geometry & Paper - 10A(ii) : Fluid Mechanics

Name (in Block Letter) : ....

Enrolment No.

Study Centre Name : .... Code : ....

Q.P. Code : **(PA/4/XA(i))/(PA/4/XA(ii))**

PG-Sc.-AP-17115

Received Answer Booklet

Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Special Paper : Pure Mathematics
Paper - 10A(i) : Advanced Differential Geometry

(Notations have their usual meanings.)

A manifold always means a differential manifold of class c^{∞}.
Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) When is a function said to be a diffeomorphism ?

b) Define a differentiable curve on a manifold.

c) Using $[X, X] = \theta$, show that $[X, Y] = -[Y, X]$.

d) When is a vector field said to be invariant under a smooth function ?

e) Define a 1-form on a manifold.

f) Show that $L_a L_b \neq L_b L_a$ in general.

g) When is a linear connection said to be symmetric ? Calculate $[X, Y]$ in such case.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Define a differentiable manifold of class c^{∞}. 3

 b) Show that S^1 is an one-dimensional differentiable manifold. 7

3. a) Define a tangent vector on a manifold. 3

 b) If $X = \frac{\partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2} - x^3 \frac{\partial}{\partial x^3}$, $Y = x^2 \frac{\partial}{\partial x^2}$, $f = x^1 x^2 + \sin^2 x^3$

 i) compute Xf

 ii) compute $[X, Y]$. 4 + 3

4. a) Show that a necessary and sufficient condition for two vector fields *X, Y* respectively on two manifolds *M, N* to be *f*-related, is that

 $f^*((f_* X) g) = X(f^* g)$

 where $f : M \to N$ is a c^{∞} map and $g \in c^{\infty}(f(p))$, $p \in M$. 6

 b) Does $X = -e^{x^1} \frac{\partial}{\partial x^1} + \frac{\partial}{\partial x^2}$ generates one parameter group of transformations ? Justify your answer. 4

5. a) Define the total differential of a differentiable function on a manifold *M*. Show that it is a 1-form of *M*. 2 + 2

 b) For any 0-form ω, prove that $d(f^* \omega) = f^*(d\omega)$. 6

6. a) Define a linear connection in the sense of Koszul. 3

 b) If $X = \xi^i \frac{\partial}{\partial x^i}$ and $Y = \eta^j \frac{\partial}{\partial x^j}$, find $\nabla_X^{\ Y}$ and $\nabla_Y^{\ X}$. 4 + 3

7. a) When is a Riemannian manifold said to be of constant curvature ? Define an Einstein manifold and show that every Riemannian manifold of constant curvaure is an Einstein manifold. 2 + 2 + 4

 b) Define a semi-symmetric metric connection. 2

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Special Paper : Applied Mathematics

Paper - 10A(ii) : Fluid Mechanics

(Notations / symbols have their usual meaning.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Show that for a two-dimensional irrotational motion velocity potential and stream function satisfy Laplace's equation.

   b) Find the velocity components for a liquid motion represented by the complex potential $\omega = A \log z$. Draw the stream lines.

   c) State Blasius theorem for the pressure thrust and its momentum on a cylindrical body in an incompressible homogeneous liquid.

   d) What is the vorticity vector in a fluid motion ? Define irrotational motion and vortex motion. Write down equation of vortex lines.

   e) Write down the constitutive relation for a viscous fluid. Then write down the Navier-Stokes equation for the flow of a viscous fluid.

   f) Define source, sink and doublets in a liquid motion.

   g) Show that for a vortex motion, strength of a vortex tube is constant along the length for all times.

   h) Explain standing waves and progressive waves along the surface of a water body.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Deduce the equation of motion of a sphere moving in an incompressible ideal fluid at rest at infinity with velocity U along the axis of 'Z'. Hence show that the effect of the presence of the liquid is to reduce the external force in the ratio $(\sigma - \rho) : \left(\sigma + \frac{1}{2}\rho\right)$, σ and ρ being the densities of the sphere and the liquid respectively.

b) A sphere of radius 'a' is made to move in an incompressible perfect fluid with non-uniform velocity 'u' along the x-axis. If the pressure at infinity is zero, prove that at a point 'x' in advance of the centre, pressure 'p' is given by the following equation :

$$p = \frac{1}{2}\rho a^3 \left\{ \frac{\dot{u}}{x^2} + u^2 \left(\frac{2}{x^3} - \frac{a^3}{x^6} \right) \right\}.$$ 5 + 5

3. a) Show that if there is a streaming past a fixed circular cylinder with uniform velocity U in the negative direction of X-axis and there is a circulation of strength k, then the cylinder experiences an upward lift amounting $\rho k U$, ρ being the density of the liquid.

b) State and prove Milne-Thomson circle theorem. Apply the theorem to find the complex potential of a uniform stream at incidence β with positive direction of X axis. 5 + 5

4. a) Show that a progressive wave in a finite depth of liquid with a free surface, liquid particle describes an ellipse about its mean position near the free surface.

b) Discuss the motion of capillary waves in a channel of uniform depth. 5 + 5

5. Describe Karman Vortex street. Discuss the motion of rectilinear vortices lying on such a street. Find out the velocity of lower row and show that it is equal to $\frac{k}{2a} \tan h \frac{\pi b}{a}$, where vortices '$k$' in the upper row are at the points $ma + \frac{i}{2}b$ and vortices '$-k$' in the lower row at $\left(m + \frac{1}{2}\right)a - \frac{1}{2}ib$, $m = 0, \pm 1, \pm 2$. 5 + 5

6. a) Consider a steady flow of an incompressible viscous fluid through a pipe with rectangular cross-section bounded by the plane $x = a$, $x = -a$, $y = b$, $y = -b$. Find the volume rate of flow at any cross-section.

b) Define axisymmetric motion and Stokes stream function. Obtain the differential equation satisfied by Stokes' stream function and velocity potential for an irrotational motion symmetric about an axis. 5 + 5

7. a) Calculate the energy which is dissipated in a viscous liquid in motion due to internal friction.

b) Obtain boundary layer approximation of Navier-Stokes equation for a flow of viscous incompressible liquid passing over a flat plate and discuss Blasius solution for boundary layer over a flat plate. 5 + 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :