NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 10B(i) : Advanced Functional Analysis & Paper - XB(ii) : Mechanics of Solids

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

To be filled by the Candidate	Serial No. of question answered														**TOTAL**
For Evaluator's only	Marks awarded														

Q.P. Code : **(PA/4/XB(i))/(PA/4/XB(ii))**

PG-Sc.-AP-17116

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

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Study Centre Name : ..... Code : .....

Q.P. Code : **(PA/4/XB(i))/(PA/4/XB(ii))**

PG-Sc.-AP-17116

Received Answer Booklet

Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Special Paper : Pure Mathematics

Paper - 10B(i) : Advanced Functional Analysis

(Notations and symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Give an example with proper justification of a symmetric set in a vector space X, which is not balanced.

   b) Let G be an open set in a topological vector space X and $A \subset X$. Prove that $A + G$ is open in X.

   c) When is a normed linear space said to be strictly convex ? Give an example of it.

   d) Give an example with proper justification of a linear operator which is bounded but not compact.

   e) Let H be a complex Hilbert space and $T : H \to H$ be a bounded self-adjoint linear operator. Prove that all the eigenvalues of T (if they exist) are real.

   f) Let X be a complex Banach algebra with identity e. If $x \in X$ and there are $y, z \in X$ such that $yx = e$ and $xz = e$, then show that x is invertible and $y = z = x^{-1}$.

   g) Let X be a complex Banach algebra with identity e and $x \in X$ be such that $\|x\| < 1$. Show that $\left\| (e - x)^{-1} - e - x \right\| \leq \frac{\|x\|^2}{1 - \|x\|}$.

   h) Let P_1, P_2 be two orthogonal projection operators of a Hilbert space H onto the closed subspaces Y_1, Y_2 of H respectively such that $P_1 P_2 = P_2 P_1$. Show that $P_1 P_2$ is also an orthogonal projection operator on H.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Prove that in a topological vector space X, the following statements are equivalent :

   i) A subset E of X is bounded.

   ii) If $\{x_n\}_{n \in \mathbb{N}}$ is a sequence in E and $\{\alpha_n\}_{n \in \mathbb{N}}$ is a sequence of scalars such that $\lim_{n \to \infty} \alpha_n = 0$, then $\lim_{n \to \infty} \alpha_n x_n = \underline{0}$ in X. 3 + 3

   b) Prove that every locally compact topological vector space is finite dimensional. 4

3. Let X be a topological vector space over $\mathbb{R}$ and f be a linear functional on X such that $f(x) \neq 0$ for some $x \in X$. Prove that the following statements are equivalent :

   i) f is continuous

   ii) The null space $N(f)$ of f is closed

   iii) $N(f)$ is not dense in X

   iv) f is bounded in some neighbourhood of $\underline{0}$ in X. 10

4. Let X be a vector space over $\mathbb{R}$, A be a convex and absorbing subset of X and p_A be the Minkowski functional of A. Then prove the following :

   i) $p_A(x+y) \leq p_A(x) + p_A(y)$, $\forall\, x, y \in X$

   ii) $p_A(tx) = t\, p_A(x)$ for all scalars $t \geq 0$ and for all $x \in X$

   iii) If A is balanced, then p_A is a seminorm

   iv) If $B = \{x \in X : p_A(x) < 1\}$ and $C = \{x \in X : p_A(x) \leq 1\}$, then $B \subset A \subset C$ and $p_A = p_B = p_C$. 10

5. a) Prove that a topological vector space X is normable if and only if there is a convex bounded neighbourhood of $\underline{0}$ in X. 5

   b) Prove that the dual space of l_p is isomorphic to the sequence space l_q, where $1 < p, q < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. 5

6. a) Prove that a bounded linear operator $P : H \to H$ on a Hilbert space H is an orthogonal projection operator if and only if P is self adjoint and idempotent. 6

   b) Let X be a complex Banach algebra with identity e and G be the set of all invertible elements of X. Show that the mapping $g : G \to G$ defined by $g(x) = x^{-1}$ is continuous. 4

7. a) Give an example with proper justification of a bounded self adjoint linear operator on a Hilbert space which has no eigenvalue. 3

   b) State and prove Banach-Alaoglu theorem. 1 + 6

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Special Paper : Applied Mathematics

Paper - XB(ii) : Mechanics of Solids

(Notations and symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) What is meant by the line of shear stress ?

   b) Write down the difference between plane strain and plane stress problem.

   c) Based on which principle the variational method is applicable ?

   d) Write down the displacements in terms of deflections of thin elastic plate.

   e) Find the bending and twisting moments of a thin elastic plate.

   f) Define plane wave.

   g) Explain Tresca's criterion.

   h) Define stress deviator.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. Deduce the Prandtl-Ruess stress-strain relations for plastic flow in an elasto-plastic medium in the form $\dot{s}_x = 2G\left(\dot{e}_x - \frac{\dot{W}}{2k^2}s_x\right)$ and two similar equations $\dot{\tau}_{yz} = G\left(\dot{\gamma}_{yz} - \frac{\dot{W}}{k^2}\tau_{yz}\right)$ and two similar equations. 10

3. Solve the problem of vibration of a thin rectangular plate with simply supported edge. 10

4. Find the displacement in a hemisphere if a constant force of magnitude P is applied to hemispherical surface along the direction of positive Z-axis. 10

5. What is Love waves ? Find the frequency equation for Love wave and show that it is dispersive in nature. 10

6. Show that in torsion problem, the torsion function ϕ satisfies the following Neumann problem $\nabla_1^2\varphi = 0$ in S and $\frac{\partial\varphi}{\partial n} = \frac{r\,dr}{ds}$ on L,
where the terms have their usual meaning. 10

7. a) State and prove the theorem of minimum potential energy. 6
   b) Explain how a plane problem of elasticity can be solved by using Airy's stress function. 4

First Answer :

Second Answer :

Third Answer :

Fourth Answer :