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W	URL: http://staff.ustc.edu.cn/~jingxi/Lecture%209_10.pdf Fetched: 2/9/2021 1:06:42 AM	2

Entire Document

1 D:\Suvendu\Netaji Subhas Open University\CC-2 (Mathematics : Analytical Geometry) Preface \ 7th Proof PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, generic elective, discipline Specific, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assesment and Accreditation Council of India (NAAC) with grade ''A''. UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English/Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor

2 D:\Suvendu\Netaji Subhas Open University\CC-2 (Mathematics : Analytical Geometry) Preface \ 7th Proof First Print : November, 2021 Printed in accordance with the regulations of the Distance Education Bureau of the University Grants Commission. Netaji Subhas Open University Under Graduate Degree Programme Choice Based Credit System (CBCS) Subject : Honours in Mathematics (HMT) Course : Analytical Geometry Code : CC-MT-02

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5 D:\Suvendu\Netaji Subhas Open University\CC-2 (Mathematics : Analytical Geometry) Preface \ 7th Proof UG : Mathematics (HMT) Unit 1 ❒ Techniques for sketching parabola, ellipse and 7 hyperbola and their reflection properties Unit 2 ❒ Transformation of co-ordinates 21 Unit 3 ❒ General equation of second degree 28 Unit 4 ❒ Tangent, Normal, Pole, Polar, Conjugate diameters 43 Unit 5 ❒ Equation of a chord of a conic in terms of its 59 middle point Unit 6 ❒ Polar Equations 69 Unit 7 ❒ Introduction to three dimensional geometry 93 Unit 8 ❒ Planes 101 Unit 9 ❒ Straight lines 118 Unit 10 ❒ Sphere, Cylinder, Cone 141 Unit 11 ❒ Central Conicoids, Conicoids and Tangent, Normal 187 Unit 12 ❒ Triple product of vectors 201 Unit 13 ❒ Vector equation and application to geometry 213 Netaji Subhas Open University Course : Analytical Geometry Code : CC-MT-02

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Matching text As the text appears in the source.

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5/29	SUBMITTED TEXT	117 WORDS	30%	MATCHING TEXT	117 WORDS

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6/29	SUBMITTED TEXT	102 WORDS	29%	MATCHING TEXT	102 WORDS

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7/29	SUBMITTED TEXT	55 WORDS	34%	MATCHING TEXT	55 WORDS

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8/29	SUBMITTED TEXT	20 WORDS	61%	MATCHING TEXT	20 WORDS

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9/29	SUBMITTED TEXT	12 WORDS	76%	MATCHING TEXT	12 WORDS

θ + θ • ⇒ θ π α β α β β α + = = β – α ⇒ – β

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10/29	SUBMITTED TEXT	58 WORDS	56%	MATCHING TEXT	58 WORDS

θ − − θ ∠ α θ α ⇒ θ α α α θ π α = θ θ α ⇒ θ α ⇒ θ α π θ α 72 NSOU ?CC-MT-02 ∴ θ π α+ θ α θ α ' θ α π ? ? ' θ−α− = ? ? ? ? θ α ' θ θ θ θ π π ? ? ? ? +θ + +θ = ? ? ? ? ? ? ? ? ⇒ θ θ α θ ∆ θ α ⇒ θ α ⇒ θ α

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11/29	SUBMITTED TEXT	20 WORDS	57%	MATCHING TEXT	20 WORDS

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12/29	SUBMITTED TEXT	12 WORDS	89%	MATCHING TEXT	12 WORDS

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W http://users.sussex.ac.uk/~tafb8/ssa/prob_solution_sheet2004_1.pdf

13/29	SUBMITTED TEXT	44 WORDS	41%	MATCHING TEXT	44 WORDS

θ θ θ θ 75 NSOU ?CC-MT-02 ⇒ ? ? + ? ? ? ? ? ? = ? ? ? ? ? ∴ θ ? ? = θ ? ? ? ? ? ? θ ⇒ ∠ θ ∠ π θ ∴ ⇒ θ α θ ∠ θ α ∠ π θ α ∴ ⇒ θ 76 NSOU ?CC-MT-02 l θ ∴ = π−θ ⇒ = − θ ⇒ θ θ ? ? θ + θ − θ θ. ' ' ' ' θ ' 77 NSOU ?CC-MT-02 θ α β θ α β θ α β α β θ θ θ

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14/29	SUBMITTED TEXT	10 WORDS	83%	MATCHING TEXT	10 WORDS

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15/29	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

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W https://www.docsity.com/en/solved-problems-for-assignment-4-dynamics-and-vibrations-meen-363/6373468/

16/29	SUBMITTED TEXT	38 WORDS	52%	MATCHING TEXT	38 WORDS

θ α β ⇒ θ α ∴ θ β θ θ θ θ θ θ θ θ α β θ α β θ 81 NSOU ?CC-MT-02 l ' θ' θ θ α θ α θ ' θ' θ' α θ' θ α θ α ⇒ θ' α θ α θ' α θ θ' θ θ' α θ θ' θ α θ' α

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17/29	SUBMITTED TEXT	18 WORDS	59%	MATCHING TEXT	18 WORDS

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18/29	SUBMITTED TEXT	44 WORDS	61%	MATCHING TEXT	44 WORDS

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19/29	SUBMITTED TEXT	101 WORDS	26%	MATCHING TEXT	101 WORDS

β α β ⇒ α β θ α β θ α β 78 NSOU ?CC-MT-02 ⇒ β α θ θ α θ ⇒ β θ α θ θ α β θ α β θ α β α β α β θ γ β θ α α β α β β α α θ α θ θ α θ θ α θ 79 NSOU ?CC-MT-02 α π π ? ? ? ? = +θ–α+ +θ ? ? ? ? ? ? ? ? ⇒ θ α θ ∴ θ α ⇒ θ α θ ⇒ θ α θ θ α θ θ α θ θ α θ α β α β θ 80 NSOU ?CC-MT-02 θ β θ α α β α β θ θ α β θ θ α β θ θ θ α β θ θ α β θ α β θ α

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W https://www.yumpu.com/en/document/view/19198436/pdf-itempdf-university-of-oxford/38

20/29	SUBMITTED TEXT	39 WORDS	56%	MATCHING TEXT	39 WORDS

α – + α + θ α θ α θ α θ θ α θ α θ θ α θ ∴ θ α θ ⇒ α θ α θ 85 NSOU ?CC-MT-02 θ θ α θ β θ β ⇒ β θ θ β θ β β α θ β α

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21/29	SUBMITTED TEXT	27 WORDS	41%	MATCHING TEXT	27 WORDS

α α α ∴ α θ β 86 NSOU ?CC-MT-02 θ θ α α α ? ? = θ– ? ? ? ? = + θ θ γ γ θ α θ α α + α θ θ α γ α + α γ γ α θ α ∴ + α

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22/29	SUBMITTED TEXT	30 WORDS	48%	MATCHING TEXT	30 WORDS

α α γ ⇒ γ α θ α γ = + θ α γ 87 NSOU ?CC-MT-02 + α α–γ α α α ∴ α ? ? θ– ? ? α ? ? ∴ α α ? ? θ– ? ? ? ? ' ' + ' ' θ ' ' α ' ' π ? ? α+ ? ? ? ? α

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23/29	SUBMITTED TEXT	40 WORDS	43%	MATCHING TEXT	40 WORDS

θ θ θ α θ β α–β α+β ? ? ? ? ? ? ? θ α β α β θ α β θ θ θ θ θ θ γ γ γ θ θ θ θ α θ α θ θ α α θ α θ α 91 NSOU ?CC-MT-02 θ α β α β γ γ γ θ – θ α θ

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24/29 **SUBMITTED TEXT** 29 WORDS **34%** **MATCHING TEXT** 29 WORDS

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25/29 **SUBMITTED TEXT** 60 WORDS **61%** **MATCHING TEXT** 60 WORDS

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W https://www.pa.uky.edu/~kwng/spring2009/lecture/L2%20in%20spherical%20coordinates.pdf

26/29 **SUBMITTED TEXT** 21 WORDS **66%** **MATCHING TEXT** 21 WORDS

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27/29 **SUBMITTED TEXT** 12 WORDS **66%** **MATCHING TEXT** 12 WORDS

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28/29 **SUBMITTED TEXT** 20 WORDS **56%** **MATCHING TEXT** 20 WORDS

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W http://staff.ustc.edu.cn/~jingxi/Lecture%209_10.pdf

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α= α + α + α α α α α − + + + ? ? −α= + ? ? ? ? + − + − + + + + = + − = + − = + + − + + − + + + − ∴ + − + − − + + + − + − + + + − + × − + ??? − + − × − + − − − − 233 NSOU ?CC-MT-02 = +α = +β α β α×β= ++ −− + α×

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PREFACE In a bid to standardise higher education in the country,
the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS)
based on five types of courses: core, generic discipline specific elective, and ability/ skill enhancement for graduate students of all programmes at Elective/ Honours level. This brings in the semester pattern, which finds efficacy in tandem with credit system, credit transfer, comprehensive and continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry acquired credits. I am happy to note that the University has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English. Eventually, these will be translated into Bengali too, for the benefit of learners. As always,
we have requisitioned the services of the best academics in each domain for the
preparation of new SLMs, and I am sure they will be of commendable academic support. We look forward to proactive feedback from all
stake-holders who will participate in the teaching-learning
of
these study materials. It has been a very challenging task well executed, and 1 congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor Printed in accordance with the regulations of the Distance Education Bureau of the University Grants Commission. First Print : November, 2021 Netaji Subhas Open University Choice Based Credit System (CBCS) Honours in Mathematics (HMT) Course : Dianamical System Course Code : GE-MT-21
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[1] PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, discipline specific / generic elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive and continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the University has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes of all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this, we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Prof. (Dr.) Subha Sankar Sarkar Vice-Chancellor
[2] Printed in accordance with the regulations of the Distance Education Bureau of the University Grants Commission. First Print : April, 2022 Netaji Subhas Open University Under Graduate Degree Programme Choice Based Credit System (CBCS) Subject : Honours in Mathematics (HMT) Course Code : CC-MT-05 Course : Numerical Methods

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Integration 87-97 Unit 7 ❑ Computer Language 98-102 Unit 8 ❑ Number System 103-111 References 112 Course Code : CC–MT-05 Course : Numerical Methods UG : Mathematics (HMT)
[6]
Unit 1 rrrrr Error Analysis Structure 1.0 Objectives 1.1 Introduction 1.2 Reason of Numerical Errors 1.3 Measurement of Errors 1.4 Summary 1.5 Exercises 1.0 Objectives After going through this unit one can able to learn about l types of errors l measurment of errors 1.1 Introdution The process of solving physical or any scientific problems can be roughly divided into three phases. The first consists of constructing a mathematical model for the corresponding problem. This model could be in the form of differential equations or algebraic equations. In most cases, this mathematical model cannot be solved analytically, and hence a numerical solution is required. In which case, the second phase in the solution process usually consists of constructing an appropriate numerical model or approximation to the mathematical model. For example, an integral or a differential equation in the mathematical formulation will have to be approximated for numerical solution appropriately. A numerical model is one where everything in principle can be calculated using a finite number of basic arithmetic operations. The third phase of the solution process is the actual implementation and solution of the numerical model.
NSOU l CC-MT-05 8 1.2 Reason of numerical Errors It can be the combined effect of two kinds of error in a calculation. l the first is caused by the finite precision of computations involving floating- point or integer values called Round off error l The second usually called Truncation error is the difference between the exact mathematical solution and the approximate solution obtained when simplifications are made to the mathematical equations to make them more amenable to calculation. The term truncation comes from the fact that either these simplifications usually involve the truncation of an infinite series expansion so as to make the computation possible and practical, or because the least significant bits of an arithmetic operation are thrown away. 1.3 Measurement of Errorss Numerical Errors usually measured in three ways, Absolute Error, Relative Error and Percentage Error.

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Absolute Error : Absolute Error is the magnitude of the difference between the true value and the

approximate value x
a . Therefore absolute error is defined as the error between two values is defined as , a a E x x = - where x denotes the exact value and x a denotes the approximation. Relative Error: The relative error of x is the absolute error relative to the exact value. Look at it this way: if your measurement has an error of ± 1 inch, this seems to be a huge error when you try to measure something which is 3 inch long but when measuring distances on the order of miles, this error is mostly negligible. The definition of the relative error is . a r x x E x - = Note : Consider you try to measure a rod of length 10 cm, and found length as 9.98 cm from your scale. Here True value or actual value of the rod 10 cm and approximate value of the length of the rod is 9.98 cm. So, the absolute error will be (10 – 9.98) cm = 0.02 cm and the relative error will be 10 9.98 0.002. 10 - =
NSOU l CC-MT-05 9 Percentage error : One can express this error in percentage as 100, a p x x E x - = ´ which gives the value 0.002 × 100 = 0.2 for the example taken here. This is called percentage error. Example 1.3.1 : If 22 7 p =

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is approximated as 3.14, find the absolute error, relative error and relative percentage error.

Solution: Absolute error 22 3.14 7 a E = - 22 21.98 7 - = 0.02 0.002857 7 = = Relative error 0.002857 22/ 7 r E = = 0.0009 Relative percentage error = E p = E r × 100 = 0.0009 × 100 = 0.09% Example 1.3.2 : Compute the percentage error in the time period for l = 1 if the error in the measurement of l is 0.01. Solution : Given the 2 . l T g = p Taking log of both sides we have, 1 1 log log2 log log 2 2 T l g = p+ - 1 2 dT dl T l \ = 1 0.01 100 100 100 0.5% 2 2 1 ´ = ´ = ´ = ´ dT dl T l Now we will discuss some important types of Numerical Errors
NSOU l CC-MT-05 10 l Loss of significance l Inherent errors l Round-off error l Truncation errors : (i) Loss of significance is an undesirable effect in calculations using finite- precision arithmetic such as floating-point arithmetic. It occurs when an operation on two numbers increases relative error substantially more than it increases absolute error, for example in subtracting two nearly equal numbers (known as catastrophic cancellation). The effect is that the number of significant digits in the result is reduced unacceptably. Ways to avoid this effect are studied in numerical analysis. Example: As an example, consider the behavior of ( ) 2 1 1 as = + - f x x x approaches to 0. Evaluating this function at 9 1.89 10 x - = ´ using Matlab incorrectly returns the answer 0, which shows that too many significant digits have cancelled. (ii) Inherent errors: This type of errors

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is present in the statement of the problem itself, before determining its solution.

Inherent errors occur due to the simplified assumptions made in the process of

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mathematical modelling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the proposed problem.

Inherent errors
can be minimized by taking better data on by using high precision computing aids. High

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precision refers to the number of decimal positions, i.e. the order of magnitude of the last digit in a value.

For example the number 46.398 has a precision of 0.001 or 10 −3 . Example 1.3.3 : Which of the following numbers have greatest precision? 3.1201, 2.42, 5.320205. Solution: In 3.1202, the precision is 10 −4 , In 2.42, the precision is 10 −2 , In 5.320205, the precision is 10 −6 . Hence the 5.320205 has the greatest precision. (iii) Round-off errors: Generally, the numerical methods are carried out using calculator or computer. In numerical computation, all the numbers are represented by decimal fraction. Some numbers such as 1/3, 2/3, 1/7 etc. can not be represented by decimal fraction in finite numbers of digits. Thus, to get the result, the numbers should be rounded-off into some finite number of digits. NSOU l CC-MT-05 11 Again, most of the numerical computations are carried out using calculator and computer. These machines can store the numbers up to some finite number of digits. So in arithmetic computation, some errors will occur due to the finite representation of the numbers; these errors are called round-off error. Thus, round-off errors occur due to the finite representation of numbers during arithmetic computation. These errors depend on the word length of the computational machine. Method of

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rounding off: To round off a number to n significant digits first truncate it to n digits: if truncated part is less than half a unit

at last significant place then ignore it, if it is greater than half a unit at last significant place then add one to last significant digit: if it is exactly half a unit at last significant place then add one to it if it is odd. So absolute error is always minimum by this process which is less than or equal to half a unit at last significant figure (s.f) i.e. 1 10 2 m- £ ´ if approximation is done to m places after decimal. Sign of equality holds in the case when truncated part is exactly half a unit at last s.f .Reader may think that can't we do the reverse in this case i.e. if last s.f is even the we add one to it and ignore the other case? Because in this case also 1 10 . 2 m a E - = ´ But on a closure look we can identify that this make the last digit of the approximated number odd which attract more error in further calculation.

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Example 1.3.4 : Round off the following numbers, to four significant digits i) 23.4251 ii) 32.4250 iii) 24.87500 iv) 19.995 v) 437.261 vi) 19.36235 Solution: i) 23.43 ii) 32.42 iii) 24.88 iv) 20.00 v) 437.3 v) 19.36 Example 1.3.5 : Round off the number 54762 to four significant digits and

then calculate absolute error, relative error and percentage error. Solution: i) The given number is 54762 (= N) After round off to four significant figures, The given number would be 54760 (= N 1) Absolute error 54762 54760 2 a E = - = Relative error 5 2 3.652 10 54762 r E - = = ´ Relative percentage error = E p = E r × 100 = 3.652 × 10 −5 × 100 = 3.652 × 10 −3 %

NSOU l CC-MT-05 12 Exercise 1.3.6 : Round off the following numbers to four significant digits and then calculate absolute error, relative error and percentage error. i) 437.261 ii) 19.36235 (iv) Truncation errors: These errors occur due to the finite representation of an Inherently infinite process. For example, the use of a finite number of terms in the infinite series to compute the value of cos ,sin , , x x x e etc. The Taylor's series expansion of sin x is 3 5 sin .... 3 5 x x x x x= - + - This is an infinite series expansion. If only first five terms are taken to compute the value of sin x for a given x, then we obtain an approximate result. Here, the error occurs due to the truncation of the series. Suppose, we retain the first n terms, the truncation Error is given by ( ) 2 1 2 1 ! n trunc x E n + £ + It may be noted that the truncation error is independent of the computational machine. Example 1.3.7 : Find

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the number of terms of the exponential series such that their sum gives the value correct to six decimal places

at Solution: We know, ( ) ( ) 2 3 1 1 .... 2! 3! 1 !
n

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x n x x x e x R x n - = + + + + + - Where ( ) , 0 . ! n n x R x e x n q = > q > Maximum absolute error (at) ! n x x x e n

q = = and maximum relative error is . ! n x n Hence ( ) max 1 at 1is . ! = x e x n For a six decimal accuracy at 1, x = we have 6 1 1 10 ! 2 n - > ´ or, 6 ! 2 10 n < ´ which gives n = 10.
NSOU l CC-MT-05 13 1.4 Summary In this unit, the concept of Numerical errors, measurement of errors like absolute errors, relative errors, percentage error, loss of significant, inherent , round off and truncations errors are discussed with different examples. 1.5 Exercises 1)

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If 0.333 is the approximate value of 1 , 3 find absolute, relative and percentage

errors. (Ans: .00033, 0.00099, 0.99) 2) If 2 3 5xy u z = and error in x, y, z be 0.001,0.002 and 0.003. Compute the relative error in u when x = y = z = 1. (Ans: .14) 3) Find the difference of 2.01 2 - correct to three digits. (Ans: 3.53 × 10 −3) 4) If 0.005, xD = 0.001 yD = be the absolution errors in x = 2.11 and y = 4.15, find the relative error in the computation of x + y. (Ans: 0.001 (approx.)) 5) Use the series of ( ) 3 5 1 log 2 .... 1 3 5 e x x x x x x ? ? + = + + + + ? ? - ? ? to compute the value of log ( ) log 1.2 e correct to seven deciamal places and find the number of terms retained. (Ans : 2,0.1823215) n ^{3} 6) What do you understand by Inherent errors occurs in numerical computation? 7) Write process of rounding off?
Unit 2 rrrrr Transcendental and Polynomial Equations Structure 2.0 Objectives 2.1 Introduction 2.2 Iteration method or Fixed point iteration 2.3 Bisection method 2.4 Regula-falsi method 2.5 Newton-Raphson method 2.6 Summary 2.7 Exercises 2.0 Objectives After going through this unit one can able to learn about l how to find the roots of non-linear equation by using different methods. l the covergence of methods are also discussed. 2.1 Introduction Determination of roots of algebraic and transcendental is a very important problem in science and engineering. A function f (x) is called algebraic if, to get the values of the function starting from the given values of x, we have to perform arithmetic operations between some real numbers and rational power of On the other hand, transcendental functions include all non-algebraic functions, i.e. , ,log , x x e a x sin ,cos , x x 1 1 sin ,cos x x - - etc. And others. An equation f (x) = 0 is called algebraic or transcendental as f (x) is algebraic or transcendental.
NSOU l CC-MT-05 15 The equations 7 2 3 7 1 0, x x x + + + = 3 8 7 0 x x+ + = etc. are the examples of algebraic equations and on the other hand 3log cos 0, x e x x + + = 4 cot 0 x e x x - + + = etc. are the examples of transcendental equation. Though we know some methods like Cardan's method, Euler's method, Ferrari's method, Descartes' method in algebra to solve algebraic equation up to fourth order. In general there is no closed form formula to evaluate the algebraic equation of degree greater than two. The definition of roots of an equation can be given in two different ways: Algebraically, a number c is called a root of an equation ( ) 0 f x = iff ( ) 0 f c = and geometrically, the real roots

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of the equation ( ) 0 f x = are the values of x where the graph of ( ) y f x = meets the x-axis.

Throughout our discussion, we assume that I. The function f (x) is continuous and continuously differentiable up to a sufficient number of times. II. ( ) 0 f x = has no multiple root i.e., if a is a real root of ( ) 0, f x = in a sufficiently small interval (a, b), then f (a) = 0 and either ( ) 0 f x ¢ > or ( ) 0 f x ¢ < in ( ) , . a b Most of the numerical methods, used to solve an equation are based on iterative techniques. Different numerical methods are available to solve the equation f (x) = 0. But each method has some advantage and disadvantage over another method. Generally, the following aspects are considered to compare the methods: Convergence or divergence, rate of convergence, applicability of the method, amount of pre-calculation needed before application of the method. etc. The process of finding the approximate values of the roots of an equation can be divided into two stages: I. Location of the roots. II. Computation of the values of the roots with the specified degree of accuracy. The interval [a, b] is said to be the location of a real root c if f (c) = 0 for a > c > b. There are two methods used to locate the real roots of an equation I. Graphical method II. Method of tabulation which is an analytic method.
NSOU l CC-MT-05 16 Graphical method l In this method the graph of y = f (x) is drawn in rectangular co-ordinate system. Then the points at which graph meets the −axis are

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the location of the roots of the equation f (x) = 0. As an example, we consider the equation 2 1 0. x x+ - =		

We draw the graph of 2 1 y x x = + - with respect to 0 , x x y oy ¢ ¢ as rectangular axes, which meets the x-axis at A and .A ¢ Thus the equation has two real roots, one is positive and other is negative. From the graph it is clear that the co-ordinate of A is lies between 0.6 and 0.7 and that of A ¢ is between -1.6 to -1.7. Thus 0.6 is an approximate value of the positive root ( ) say . ¢ a and −1.6 is an approximate value of the negative root ( ) say . ¢ a l If f (x) is not simple, rather complicated in form, we rewrite the equation f (x) as ( ) ( ) 1 2 , x x q = q where ( ) 1 x q and ( ) 2 x q are simple functions such that, we can draw conveniently the graphs of ( ) 1 y x = q and ( ) 2 y x = q with respect to rectangular axes.

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Then the x-co-ordinate of the point of intersection of the graphs give the			

location of the real roots of the equation ( ) 0. f x = As an example, we consider an equation 3 4 2 0, x

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x- - = we rewrite the equation as 3 4 2. x x = + The graphs 3 and 4 2 y x y x = = - are		

drawn with respect to the rectangular axes. From the graph it is seen that the roots are in [−2, −1], [−1, 0], [2,3]. DISADVANTAGE : The graphical method to locate the roots is not very useful. Because the drawing of the location of the function y = f (x) is itself complicated. But it makes possible to roughly determine the interval of the roots. Then an analytic method is used to locate the root. METHOD OF TABULATION This method depends on the continuity of the function f (x). Before applying the tabulation method, the following nature should be noted.

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Theorem 2.1.1 : If f (x) is continuous in the interval (a, b) and if f (a) and f (b)		

NSOU l CC-MT-05 17 have the opposite signs, then

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at least one real root of the equation f (x) = 0 lies within the interval (			

a, b). Geometrically we can explain the theorem as: Let, f (x) < 0 and f (b) > 0. Then from the graph we can say that there must be a point in (a, b) such that f (x) = 0 If the curve y = f (x) touches the x−axis at some point, say at x = c then c is a root of f (x) = 0, though f (a) and f (b) may have same sign where a > c > b. For example f (x) = (x − 3) 2 , touches the x−axis at x = 3. Although f (2.5) < 9 and f (3.5) < 0 but x = 3 is a root of the equation f (x) = 0. A trial method for tabulation is as follows: From the table of signs of (x), setting x = 0, ,1, 2,.......... x = ± ± If the signs of f (x) changes its signs for two consecutive values of then at least one root lies between these two values.

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Example 2.1.2 : Find the location of the roots of the equation 2 1 0. x x+ - = Solution:

we form a table : x 0 -1 1 0.5 -0.5 -1.6 -1.7 f (x) - - + - - - + Since ( ) deg 2, f x = the ( ) f x has two roots. Since ( ) 1 0 f < and ( ) 0.5 0, f > then the location of one root is (0.5, 1). Also ( ) 1.6 0 f - > and ( ) 1.7 0. f - < Then the location of the other root is (−1,6, −1.7). Example 2.1.3 : Find the number of real roots of the equation 3 3 2 0 x x- - = and locate them. Solution : ( ) 3 3 2. x f x x = - - The domain of definition of the function is ( ) , . -¥ ¥ we form a table : x -¥ 0 1 ¥ Sign of f (x) + - - + f (x) = 0 has two real roots, since the function has twice changes sign, among them one is negative root and other is greater than one.

NSOU l CC-MT-05 18 A new table with small intervals of the location of the root is constructed in the following: x 0 -1 1 2 Sign of f (x) - + - + Then the roots are in (−1, 0) and (1, 2). ORDER OF CONVERGENCE: Assume that the sequence {x n } of numbers to a and let n n x Î = a - for 0. n ^{3} If there exists two positive constants A & p such that 1 lim . + ®¥ Î = Î n n p n A Then the sequence is said to converge to a wth the order of convergence p. The number A is called the asymptotic error constant. If p = 1, the error of convergence of {x n } is called linear and if p = 2, the error of convergence of {x n } is called quadratic etc. 2.2 Iteration method or Fixed point iteration Let ( ) f x be a continuous function on the

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interval { } ,a b and the equation ( ) 0 f x = has at least one root on { } , . a b

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The equation ( ) 0 f x = can be written in the form ( ) ( ) ...... 1 x x =

j Thus a root x of the given equation satisfies ( ) . x = j x Therefore the point x remains fixed under the mapping j and so a root of the equation is a fixed point of .j ( ) xj is called the iteration function. Here we also assume that ( ) xj is continuously differentiable in { } , . a b Using graphical or tabulation method, we first find a location or crude approximation { } 0 0 ,a b of a real root x(say) of ( ) 0 f x = and let { } 0 0 0 0 x x a x b = £ £ be the initial

NSOU l CC-MT-05 19 approximation of .x Thus xsatisfies the equation ( ) ( ) ..... 2 . x = j x Putting 0 x x= in (1), we get first approximation of x as ( ) 1 0 ,

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x x = j and then the successive approximations are calculated as: ( ) 2 1 , x x = j ( ) 3 2 ,......., x x = j ( ) ( ) 1 .... 3 n n x x + =

j The above iteration is generated by the formula ( ) 1n n x x + = j and is called the iteration formula, where x n is the n-th approximation of the root x of ( ) 0. f x = These successive iterations are repeated till the approximate numbers n x s ¢ converges to the root with desired accuracy, i.e. 1 , n n x x + - >Î where Î is a sufficiently small number. The sequence { } n x of iterations or the successive better approximations may or may not be converge to a limit. If { } n x converges, then it converges to x and the number of iterations required depends upon the desired degree of accuracy of the root x. CONVERGENCE OF METHOD OF ITERATION: The presentation of ( ) 0 f x = as ( ) x x = j is not unique, therefore the convergence of { } n x depends upon the nature of ( ) .xj Now we investigate about the nature of ( ) xj which yields a convergent sequence { } . n x By Lagrange's mean value theorem
we
get, ( ) ( ) ( ) 1 0 0 1
x x

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x ¢ x - = j x -j = x - j e where 0 1 x > e > x ( ) ( ) ( ) 2 1 1 2 x x x ¢ x- = j x -j = x- j e where 0 2 x > e > x .... ( ) ( ) ( ) 1n n n n x x x + ¢ x- = j x -j = x- j e where 0 n x > e > x Thus, ( ) ( ) ( ) ( ) ( ) 1 2 1 0 .... n n n n x x x e e e + ¢ ¢ ¢ ¢ x- = x- j e = x- j j j Assuming , ( ) x¢j > r in 0 0 a x b £ £ we have NSOU l CC-MT-05 20 1 0 n n x x + x- £ x- r Thus, 1 0 lim lim 0 + ®¥ ®¥ x- £ x- r ® n n n n x x

if p > 1, i.e. ( ) 1 j > x ( ) 1, . . 1 ¢ ® ¥ r < j < if i e x Therefore the method is convergent for ( ) { } 0 0 1 in , . x a b ¢j £ r > ESTIMATION OF ERROR: Let, x be an exact root of the equation ( )

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x x = j and ( ) 1 . n n x x + = j Therefore, ( ) ( ) ( ) 1 1 , n n n x x x c - - ¢ x- = j x -j = x - j , where 1n x c - > > x 1 , n l x - £ x- ,[where, ( ) 1] c l ¢j £ > { } 1- £ x- + - n n n l x x x After rearrangement, this relation becomes 1 1 0 1 1 n n n n l l x x x x x

l l -
x- £ - £ - - - Let the maximum number of iteration needed to achieve the accuracy e be ( ) . N e Then ( ) ( ) 1 0 1 0 1 log , . . 1 log N l x x l x x i e N l l e - - - £ e e 3 - For 0.5, l £ the estimation of the error is given by the following simple form : 1 n n n x x x - x- £ - ORDER OF CONVERGENCE:

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The convergence of an iteration method depends on the suitable choice of the

iteration function ( ) xj and the initial guess 0 .x Let, { } n x converges to the exact root ,a so that ( ) . x = j x NSOU l CC-MT-05 21 Thus ( ) ( ) 1 . n n x x + -x = j -j x Let, 1 1 . n n x + + e = -x Note that ( ) 0. x ¢ j 1 Then the above relation becomes ( ) ( ) 1n n + e = j e +x -j x ( ) ( ) 2 1 .... 2 n n ¢ ¢¢ = e j x + e j x + ( ) ( ) 2 ¢ = e j x + e n n o i.e. ( ) 1 0 n n + e ¢ = j x 1 e hence the order of convergence of the iteration method is linear. GEOMETRICAL INTERPRETATION : The geometrical meaning s of the fixed-point iteration in different cases are illustrated by Figure. Convergent for (a) Stair case solution, (a) Divergent for (b) Spiral case solution, O O O O

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x 1 x 1 x 0 x 0 x 3 x 1 x 0 x x 2 4 x 0 x 2 x 3 x 1 x 2 y x = y x = y x = y x = x x x x x x x x y x ( ) = f y x ( ) = f y x ( ) = f y x ( ) = f f x ( ) f x ( ) f x ( ) f x ( ) 0 ( ) 1 ¢ > f x > 1 ( ) 0 ¢ - > f x > ( ) 1 ¢ f x > ( ) 1 ¢ f x < (

b) Divergent for ( ) 1 ¢ f x > -

Fig 2.1 : Illustration for Fixed-point iteration ADVANTAGE AND DISADVANTAGE:
NSOU l CC-MT-05 22 The disadvantage of this method is that a pre-calculation is required to re-write ( ) 0 f x = to ( ) x x = j in such a way that ( ) 1. x¢j > The advantage of this method is that the operation carried out at each stage are of same kind, and this makes easier to develop computer program. 2.3 BISECTION METHOD It is an iterative method and is based on a well-known theorem which states that if ( ) f x be a continuous function in a closed
interval { } ,a b and ( ) ( ) 0,

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f a f b > then $ at least one real root of the equation ( ) 0, f x = between a and b.

If further ( ) f x ¢ exists and ( ) f x ¢ maintains same sign in { } , , a b i.e. ( ) f x is strictly monotonic, then there is only one real root of ( ) 0 f x = in { } , . a b This method is nothing but a repeated application of the above theorem. First we consider a sufficiently small interval { } 0 0 , , a b by graphical or tabulation method , in which ( ) ( ) 0 0 0 f a f b > and ( ) f x ¢ maintains same sign in { } 0 0 , , a b then there is only one real root of ( ) 0, f x = in { } 0 0 , . a b Now divide the interval { } 0 0 ,a b into two equal intervals { } 0 ,a c and { } 0 ,c b where 0 0 . 2 a b c + = If ( ) 0, f c = then c is an exact root of the equation. If ( ) 0 f c ^{1} then the root lies either in { } 0 ,a c or in { } 0 , . c b If ( ) ( ) 0 0 f a f c > then we take the interval { } 0 ,a c as the new interval, otherwise we take { } 0 , . c b Let the new interval be { } 1 1 ,a b and use the same process to select the next new interval. In the next step, let the new interval be { } 2 2 , . a b The process of bisection is continued until either the midpoint of the interval is a root, or the length ( ) n n b a- of the interval { } , n n a b is sufficiently small. The number a n and b n are approximate roots of the equation ( ) 0. f x = Finally 2 n n n a b x + = is taken as the approximate value of the root .a
NSOU l CC-MT-05 23 y a c b x f b( ) f c( ) f a( ) Fig 2.2 : Illustration for Bisection method Now the length of the interval { } 1 1 ,a b is 0 0 2 b a- and the length of the interval { } 2 2 ,a b is 0 0 2 2 b a- and at the n-th step the length of the interval { } , n n a b is 0 0 . 2 n b a- In the final step 2 n n a b+ a = is chosen as root, then the length of the interval being 0 0 1 2 n b a + - and hence the error does not exceed 0 0 1 . 2 n b a + - Thus, if e be the error at the n-th step then the lower bound of n is obtained from the following relation 0 0 1 . 2 n b a + - £ e CONVERGENCY: let 1n+ e be the error in approximating a by 1 , n x + then 0 0 1 1 0 2 n n n n n b a x b a + + - e = a- > - = ® as . n ®¥ Thus the iterative method must be convergent. To get a root of ( ) 0 f x = correct up to p-significant figures, we are to go up to q-th iteration so that q x and 1q x + have same p-significant figures. DISADVANTAGE : This method is very slow, but it is very simple and will converge surely to the exact root. So the method for any function only if the function is continuous within the interval [a, b], where the root lies.
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Example 2.3.1 : Find a root of the equation 2 7 0 x x+ - = by bisetion method, correct up to two decimal places.
Solution. Let ( ) 2 7. f x x x = + - ( ) 2 1 0 f = - > and ( ) 3 5 0. f = < So, a root lies between 2 and 3.

Left end point Right and point Midpoint n a n b n x n+1 f (x n+1) 0 2 3 2.5 1.750 1 2 2.5 2.250 0.313 2 2 2.250 2.125 -0.359 3 2.125 2.250 2.188 -0.027 4 2.188 2.250 2.219 0.143 5 2.188 2.219 2.204 0.062 6 2.188 2.204 2.196 0.018 7 2.188 2.196 2.192 -0.003 8 2.192 2.196 2.194 0.008 9 2.192 2.194 2.193 0.002 10 2.192 2.193 2.193 0.002 Therefore, the root is 2.19 correct up to two decimal places. Another popular method is the

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regula falsi method. This method was developed because the bisection method converges at fairly slow speed.

In general regula falsi method is faster than bisection method. 2.4 Regula Flasi

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Method This method is also known as method of false position, Method of chords, method of linear interpolation.

Let a root of the equation ( ) 0 f x = be lies in the interval { } , , a b i.e. ( ) ( ) 0. f a f b > The idea of this method is that on a sufficiently small { } , , a b the arc of the ( ) y f x = is replaced by

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the chord joining the points ( ) ( ) ,a f a and ( ) ( ) , . b f b

The abscissa
of

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the point of intersection of the chord and the x-axis is

taken as the approximate value of the root.
NSOU l CC-MT-05 25 Let, 0

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x a= and 1 . x b= The equation of the chord joining the points ( ) ( ) 0 0 ,x f x and ( ) ( ) 1 1 ,x f x is ( ) ( ) ( ) ( ) 0 0 0 1 0 1 ...... 1 y f x x x x x f x f x - - = - - To find the point of

intersection,
set 0 y = in (1) and let ( ) 2 ,0x be the point.
Then, ( ) ( ) ( ) 0 2 0 0 1 0 1 0

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f x x x x x f x f x - - = - - Therefore, ( )( ) ( ) ( ) ( ) 0 1 0 2 0 1 0 ..... 2 f x x x x x f x f x - = - - This is the second approximation of the root. Now if ( ) 2 f x and ( ) 0 f x are opposite signs then the root lies between 0 x and 2 x and replace 1 x by 2 x in (2). Then the next approximation

is obtained as : 0 2 0 3 0 2 0 ( )( ) ( ) ( ) - = - -

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f x x x x x f x f x If ( ) 2 f x and ( ) 1 f x are opposite signs then the root lies between 1 x and 2 x and the new approximation is obtained as: ( )( ) ( ) ( ) 2 1 2 3 2 1 2 - = - - f x x x x x f x f x The procedure is repeated till the root is obtained to the desired accuracy. If the

n-th approximate root n x lies between n a and , n b then the approximate root is thus obtained as : ( )( ) ( ) ( ) ( ) 1 3

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n n n n n n n f a b a x a f b f a + - = - -

GEOMETRICAL INTERPRETATION : The illustration of the method is shown Figure where x is the root of the equation ( ) 0. f x =
NSOU l CC-MT-05 26 f x( ) O x 0 = a x 1 x 2 x b x Fig 2.3 : Illustration for Regula-falsi method CONVERGENCE OF REGULA FALSI METHOD: As ( ) ( ) 0, n n f a f b > considering the proper sign of ( ) n f a and ( ) n f b we can write the equation (3) as follows: ( )( ) ( ) ( ) 1

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n n n n n n n f a b a x a f b f a + - = - - or ( )( ) ( ) ( ) ( ) 1 .... 4 n n n n n n n f b b a x b f b f a + - = - - Since, n n x

a= or , n b we have for both relation of (4) as ( )( ) ( ) ( ) 1

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n n n n n n n f x b a x x f b f a + - = - - Or, ( ) ( ) ( ) ( ) ( )( ) 1+ - - = - n n n n n n n x x f b f a f x b a Or, ( )( ) ( ) ( )( ) 1+ ¢ - - a = - n n n n n n n n n x x b a f f x b a when > a > n n n a b Or, ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 , + ¢ ¢ ¢ ? ? a - - a - a = - a = -a a ? ? n n n n n

n x x f f x f x
f ( )
since, 0 , a = ? ? ? ? f where { } { } Min , , ¢

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a > a > a n n n x Max x NSOU l CC-MT-05 27 Or, ( ) ( ) ( ) ( ) ( ) 1 , + ¢ ¢ ¢ a - a a - = a - ¢ a n n n n f f x x f where ( ) 0 0 , ... 5 ¢ > a a > n n

a b The approximation lies in { } 0 0 ,a b and ( ) f x ¢ is continuous, then there exist two numbers m, M such that ( ) 0 n m f x M ¢ > £ £ for all { } 0 0 , . x a b Î Then from (5) we get, ( ) ( ) 1+ - a - £ a - n n M m x x m Now putting 1, n n= - 2,....,2,1,0 n - for n successively and multiplying ( ) 1 n+ relations we get : ( ) ( ) ( ) 1 1 1 0 + + + - e = a - £ a - n n n M m x x m If we choose the interval { } 0 0 ,a b such that 1, . . 2 , M m i e M m m - > > Then 1 1 lim lim ( ) 0 + + ®¥ ®¥ e = a - = n n x x x Therefore the method is convergent. Thus for the convergence of the Regula Falsi Method, the interval { } 0 0 ,a b must be very small. ADVANTAGE: The advantage of this method is that it is very simple and the sequence { } n x is sure to converge. The another advantage of this method is that it does not require the evaluation of derivatives and pre-calculation. DISADVANTAGE: The method is very slow and not suitable for hand calculation.

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Example 2.4.1 : Find a root of the equation 3 2 2 0 x x+ - = using Regula-Falsi method, correct up to three decimal places. Solution. Let ( ) 3 2 2. f x x x = + - ( ) 0 2 0 f = - > and ( ) 1 1 0. f = < Thus, one root lies between 0 and 1.

The calculations are shown in the following table.
NSOU l CC-MT-05 28 left end right end n point a n point b n f (a n) f (b n) x n+1 f (x n+1) 0 0.0000 1.0 −2.0000 1.0 0.6700 −0.3600 1 0.6700 1.0 −0.3600 1.0 0.7570 −0.0520 2 0.7570 1.0 −0.0520 1.0 0.7696 −0.0072 3 0.7696 1.0 −0.0072 1.0 0.7707 −0.0010 4 0.7707 1.0 −0.0010 1.0 0.7709 −0.0001 Therefore, a root of the equation is 0.771 correct up to three decimal places. 2.5 Netwon-Raphson Method This is also an iterative method and is used to find isolated roots of an equation ( ) 0. f x = The object of this method is to correct the approximate root 0 x (say) successively to the exact root a. Initially, a crude approximation of a small interval { } 0 0 ,a b is found out in which only one root a (say) of ( ) 0 f x = . Let, ( ) 0 0 0 0 x x a x b = £ £ is an approximation of the root a of the equation ( ) 0. f x = Let, h be a small correction on 0 ,x then 1 0 x x h = + is the correct root. Using Taylor’s series expansion, ( ) ( ) ( ) ( ) 1 0 0 0 0,

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f x f x h f x hf x ¢ = + = + + = since 1 x is a root of ( ) 0 f x = Neglecting the second and the higher order derivatives, the above equation reduces to- ( ) ( ) 0 0 0 f x hf x ¢ + = Or, ( ) ( ) 0 0 = - ¢ f x h f x Therefore, ( ) ( ) ( ) 0 1 0 0 0 ...... 1 f x x x h x f x = + = - ¢

Further if 1 h be the correction on 1 ,x then 2 1 1 x x h = + is the correct root of ( ) 0. f x =
NSOU l CC-MT-05 29 Then using the previous process we get, ( ) ( ) 1 1 1 = - ¢

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f x h f x Therefore, ( ) ( ) 1 2 1 1 1 1 f x x x h x f x = + = - ¢

Processing in this way, we get ( ) 1 n+ th corrected root as ( ) ( ) ( ) 1 2

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n n n n f x x x f x + = - ¢ This

expression generates a sequence of approximate values 1 2 3 , , ,......., ,.... n x x x x each successive term of which is closer to the exact value of the root a. The method will terminate when 1n n x x + - becomes very small. In this method the arc of the curve is replaced by the tangent to the curve, hence this method is sometimes called method of tangent. Note : the Newton Raphson method may also used to find a complex root of an equation when the initial guess is taken as a complex number. GEOMETRICAL INTERPRETATION: The geometrical interpretation of this method is shown in the figure 1. In this method, a tangent is drawn at ( ) ( ) 0 0 ,x f x to the curve ( ) . y f x = The tangent cuts the x-axis at ( ) 1 ,0 . x Again the tangent is drawn at ( ) ( ) 1 1 , , x f x which cuts the x-axis at ( ) 2 ,0 . x This process is continued until as . n x n = x ® ¥. The choice of initial guess of this method is very important. If the initial guess is near the root then the method f x( ) O x 1 x 0 x x 2 x Fig 2.4 : Geometrical interpretation of Newton-Raphson method
NSOU l CC-MT-05 30 converges very fast. If it is not so near the root or if the starting point is wrong, then the method may lead to an endless cycle. This illustrated in figure2. In this figure the initial guess 0 x gives the fast convergence to the root, the initial guess 0 y leads to an endless cycle and the initial guess 0 z gives a divergent solution. Even if the initial guess is not close to the exact root, the method may diverge. To chose the initial guess the following rule may be followed. If ( ) ( ) 0 f b f x ¢¢ > the initial guess be 0 x b= and if ( ) ( ) 0 f a f x ¢¢ > then 0 x a= be the initial guess. f x( ) O x 0 y 0 z 0 x Fig: Illustration of the choice of the initial guess of the Newton-Raphson method. CONVERGENCE OF NEWTON RAPHSON METHOD: Comparing with the iteration method, we may assume the iteration function as: ( ) ( ) ( )
f
x x x f x j = - ¢ Thus the above sequence will be convergent, if and only if ( ) ( ) ( ) ( ) ( ) 2 2 1

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f x f x f x x f x ¢ ¢¢ - ¢j = - ¢ NSOU l CC-MT-05 31 i.e. ( ) ( ) ( ) ( ) ( ) ( ) 2 2 1, . . f x f x i e f x f x f x f x ¢¢ ¢ ¢¢ > < ¢

RATE OF
CONVERGENCE OF N-R METHOD: Let, x be a
root of the equation ( ) 0. f x = Then, ( ) 0. f x = The iteration scheme for
NR-method is ( ) ( ) 1

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n n n n f x x x f x + = - ¢ Let, . n n x = e +x Then from the above relation we get- ( ) ( ) 1+ e +x e +x = e +x- ¢ e +x n n n n f f Or, ( ) ( ) ( ) ( ) ( ) ( ) 2 1 2 ..... 2 ..... 2 n n n n n n f f f f f f + e ¢ ¢¢ x +e x + x + e = e - e ¢ ¢¢ ¢¢ x +e x + x + Or, ( ) ( ) ( ) ( ) 2 1 .... 2 1 ..... + ¢¢ e x e + + ¢ x e = e - ¢¢ x +e + ¢ x n n n n n f f f f Or, ( ) ( ) ( ) ( ) 2 1 .... 1 .... 2 + ? ? ¢¢ ¢¢ x x ? ? e e = e - e + + -e + ? ? ? ? ? ? ¢ ¢ x x ? ? ? ? n n n n n

f
f f f Or , ( ) ( ) ( ) 2 3 1 0 2 n n n f f + ¢¢ x e e = + e ¢ x Neglecting the terms of order 3 n e and higher power
the expression becomes 2 1 , n n A + e = e where ( ) ( ) 2 f A f ¢¢x = ¢ x
NSOU l CC-MT-05 32 This relation shows that NR method has quadratic convergence or second order convergence.
Example 2.5.1 : Use Newtwon-Raphson method of
find a

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root of the equation 3 1 0. x x+ - = Solution. Let ( ) 3 1. f x x x = + - Then ( ) 0 1 0 f = - > and ( ) 1 1 0. f = < So one root lies between 0 and 1.

Let 0 0 x = be the initial
root. The iteration
scheme is ( ) ( ) 1

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n n n n f x x x f x + = - ¢ 3 3 2 2 1 2 1 . 3 1 3 1 + - + = - - = + + n n n n n n x x x x x x The sequence { } n x

for different values of n is shown below. n x n x n+1 0 0 1 1 1 0.7500 2 0.7500 0.6861 3 0.6861 0.6823 4 0.6823 0.6823 Therefore, a root of the equation is 0.682 correct upo to three decimal places. Example 2.5.2 : Find an iteration scheme to find the kth root of a number a. Solution. Let x be the kth
root of a. That is 1 k x a= or 0. k

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x a- = Let ( ) . k f x x a = - The iteration scheme is ( ) ( ) 1 n n n n f x x x f x + = - ¢ or, 1 1 1 k k k n n n n n k k n n x a kx x a x x kx kx + - - - - + = - = NSOU l CC-MT-05 33 ( ) 1 1 1 . n k n a k x k x - ? ? = - + ? ? ? ? ? ? 2.6

Summary In this unit we have studied how to calculate the roots of a transcendental equations and polynomial equations by the methods of tabulation, graphical, fixed point iteration, bisection ,Regula Falsi and Newton-Raphson. Their convergence analysis have also been studied. 2.7 Exercises 1. Solve the equation tan 1 x x = - by Regula falsi method starting with 0 2.5 x = and 1 3.0 x = correct upto three decimal places. 2. Obtain the a root for each of the following equations using bisection method, regula-falsi method and Newto-Raphson method i) 3 2 2 7 0 x x x + - + = ii) ( ) sin 10 1 x x = - iii) cos 0 x x - = 3. Describe Newton-Raphson method for computing a simple real root of an equation ( ) 0. f x = Give a geometrical interpretation of the method. Prove that the Newton-Raphson method converges quadratically. 4. Use Newton-Raphson method to find the value of the following terms i) 35 ii) 3 24 Ans. i) 5.916080, ii) 2.884499
Unit 3 rrrrr System of linear algebraic equations Strucure 3.0 Objectives 3.1 Introduction 3.2 Gaussian elimination method 3.3 Gauss-Jordan method 3.4 Gauss-Jacobi method 3.5 Gauss-Siedel mthod 3.6 Successive over Relaxation (SOR) method 3.7 Summary 3.8 Exercises 3.0 Objectives After studying this unit one can l get an idea of finding the solutions of system of linear equations by using direct methods and iterative methods. 3.1 Introduction A linear equation in variables 1 2 , ,......, n x x x is an

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equation of the form 1 1 2 2 .... n n a x a x a x b + + + = where 1 2 , ,...., n a a a

and are
constant real or complex numbers. The constant is called the coefficient of ; i x and b is called the constant term of the equation. A system of linear equations (or linear system) is a finite collection of linear equations in same variables. For instance, a linear system of n equations in n variables 1 2 , ,...., n x x x can be written as
NSOU l CC-MT-05 35 1 11 1 12 2 1 1 21 1 22 2 2 2 1 22 2 ....

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n n n n n nn n n a x a x a x b a x a x a x b a x a x a x b ? + + + = ? + + + = ? ? ? ? ? + + + = ? ? (3.1.1)

The above system can be written in
the form AX = B where ( ) , 1,2,3..... ij n n A a
i j n ´ ? ? = = ? ? is a non-singular matrix and { } ( ) 1,2,3,..., i B b i n ¢ = = = Two types of methods are availavle. i) Exact methods or Direct method ii) Iterative methods When A is of moderate order with co-efficients most non-zero, then usually exact or direct methods are used. Order of A is usually > 200 and the linear system is called dense. When A is of large order and most co-efficients zero, then iterative methods are used. A is sparse and order of A is sometimes as large as 10 6 . Exact or direct methods : Cramer’s rules, Gaussian elimination method, Gauss Jordan Method etc Iterative methods : Method of simple iteration, Gauss-Seidal iteration method Theorem 3.1.1 : Any system of linear equations has one of the following exclusive conclusions. (a) No solution. (b) Unique solution. (c) Infinitely many solutions. A linear system is said to be consistent if it has at least one solution; and is said to be inconsistent if it has no solution. Geometric interpretation The following three linear systems
NSOU l CC-MT-05 36 (
a) 1 2 1 2 1 2 2 3 2 0 2 4

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x x x x x x + = ? ? - = ? ? - = ? (b) 1 2 1 2 1 2 2 3 2 5 2 4 x x x x x x + = ? ? - = ? ? - = ? (c) 1 2 1 2 1 2 2 3 4 2 6 6 3 9 x x x x x x + = ? ? - = ? ? - = ? have no solution, a unique solution, and infinitely many solutions, respectively. See Figure1. x 1 x 1 x 1 x 2 x 2 x 2

o o o (c) infinitely many solutions (a) No solution (b) a unique solution Figure : 3.1 Note : A linear equation of two variables represents a straight line in R 2 . A linear equation of three variables represents a plane in R 3 . In general, a linear equation of n variables represents a hyperplane in the n-dimensional Euclidean space R n . Matrices of a linearsystem Definition 3.1.2 The augmented matrix of the general linear system (3.1.1) is the table 11 1 1 1 ... ... ? ? ? ? ? ? ? ? ? ? n m mn m a a b a a b (3.1.2) and the coefficient matrix of (3.1.1) is 11 1 1 ... ... ? ? ? ? ? ? ? ? ? ? n m mn a a a a (3.1.3) Systems of linear equations can be represented by matrices. Operations on equations (for eliminating variables) can be represented by appropriate row operations on the corresponding matrices. For example,
NSOU l CC-MT-05 37 1 2 3 1 2 3 1 2 3 2 1 2 3 8 3 4 7

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x x x x x x x x x + - = ? ? - + = - ? ? + + = ? The corresponding

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augmented matrix is 1 1 2 1 2 3 1 8 3 1 4 7 - ? ? ? ? - - ? ? ? ? ? ? Now we will do the needful row operations. Operating 2 1 2 R R - and 3 1 3 R R - on the above, we get 1 1 2 1 0 5 5 10 0 2 10 4 - ? ? ? ? - - ? ? ? ? - ? ? Operating ( ) 2 1/ 5 - R and ( ) 3 1/ 2 - R on the above, we get 1 1 2 1 0 1 1 2 0 1 5 2 - ? ? ? ? - ? ? ? ? - - ? ? Operating 3 2 R R- on the above, we get 1 1 2 1 0 1 1 2 0 0 4 4 - ? ? ? ? - ? ? ? ? - - ? ? Operating ( ) 3 1 4 R - on the above, we get 1 1 2 1 0 1 1 2 0 0 1 1 - ? ? ? ? - ? ? ? ? ? ?

NSOU l CC-MT-05 38 Operating 1 3 2 R R + and 2 3 R R+ on the above, we get 1 1 0 3 0 1 0 3 0 0 1 1 ? ? ? ? ? ? ? ? ? ? Operating 1 2 R R- on the above, we get 1 0 0 0 0 1 0 3 0 0 1 1 ? ? ? ? ? ? ? ? ? ? That is, we get the solution as 3 2 1, 3 x x = = and 1 0. x = Elementary row operations Definition 3.1.3 : There are three kinds of elementary row operations on matrices: (a) Adding a multiple of one row to another row; (b) Multiplying all entries of one row by a non zero constant; (c) Interchanging two rows. Another method for solving system of linear algebraic equations is Cramer's Rule. Cramer's Rule : To solve a system of linear equations, a simple method (but, not efficient) was discovered by Gabriel Cramer in 1750. Let the system of linear algebraic equations are 1 , 1,2, , = = = ∑ ... n ij j i j a x b i n (3.2.1) Let the determinant of the coefficients of the system (3.2.1) be of oder n i.e., , , 1,2, , = = ⋯ ij D a i j n. In this method, it is assumed that 0. D ¹ The Cramer's rule is described in the following. From the properties of determinant

NSOU l CC-MT-05 39 11 12 1 1 11 12 1 21 22 2 1 21 22 2 1 1 1 2 1 1 2 ... ...

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n n n n n n nn n n nn a a a x a a a a a a x a a a x D x a a a x a a a = = 11 1 12 2 1 12 1 21 1 22 2 2 22 2 1 1 2 2 ... ... 2 ... + + + = + + + + + + n n n n n n n n nn n nn a x a x a x a a a x a x a x a a a x a x a x an a [Using operation 1 1 2 2] ¢ = + + +⋯ n n C C x C x C 1 12 1 2 22 2 2 ... ... n n n n nn b a a

b a a b a

a = [

Using (3.1.1)] Therefore, 1 1 . = x D x D Similarly, 2 2 ,..... . n x x n D D x x D D = = Ingeneral, i x i D x D = where ( ) 11 12 1 1 1 1 1 1 21 22 2 1 2 2 1 2 1 2 1 1 ... ... 1,2,...., ... i i i

n i i n x

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n n ni n ni nn a a a b a a a a a b a a D i n a a a b a a - + - + - + = = Inverse of a Matrix From the theory of

matrices, it is well known that every square non-singular matrix has unique inverse. The inverse of a matrix A is defined by 1 . adjA A A - = The matrix adj A is called adjoint of and defined as

NSOU l CC-MT-05 40 11 1 1 ... ... , ... n n nn A A adjA A A ? ? ? ? = ? ? ? ? ? ? where A ij being the cofactor of ij a in .A The main difficulty of this method is to compute the inverse of the matrix A. From the definition of adj A it is easy to observe that to compute the matrix , adj A we have to determine n 2 determinants each of order ( ) 1 . n- So, it is very much time consuming. Many efficient methods are available to find the inverse of a matrix, among them Gauss-Jordan is most popular. 3.2 Gaussian elimination method We assume that the set of linear equations given by 11 1 12 2 1 1 21 1 22 2 2 2 1 1 22 2 ... ...

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n n n n n nn n n a x a x a x b a x a x a x b a x a x a x b + + + = + + + = + + + = (3.2.1) has a

unique solution and we proceed as follows. ( ) ( ) 1 1 , , = = ij i ij i a a b b ( ) , 1,2,3,....., i j n = Let ( ) 1 11 0. a ¹

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Multiply the 1st equation of (1) by ( ) ( ) 1 1 1 1 11 i i m a a = - and add to the ith equation

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when 1 x is eliminated from that equation ( ) 2,3,...., i n = giving the following equivalent equations ( ) ( ) ( ) ( ) 1 1 1 1 1 2 11 12 1 1 ...

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n n a x a x a x b + + + = ( ) ( ) ( ) 2 2 2 2 22 2 2 .... n n a x a x b + + = (3.2.2) .... ( ) ( ) ( ) 2 2 2 2 2 ... nn n n n a x a x b + + =

NSOU l CC-MT-05 41 where ( ) ( ) 1 1 1 1 11 i i m a a = - and ( ) ( ) ( ) 2 1 1 1 1 , i ij ij j a a m a = - ( ) ( ) ( ) 2 1 1 1 1 i i i b b m b = - ( ) , 2,3,....., i j n = (3.2.3) Assuming again ( ) 2 22 0. a [1] We note that the set of equations (3.2.2) except the 1 st

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is a system of 1 n- linear equations in the 1 n- unknowns 2 3 , ,....

n x x x and applying the above eliminations procedure to this system 2 x is eliminated from the last 2 n- equations of the set giving the equivalent system ( ) ( ) ( ) ( ) 1 1 1 1 1 2 11 12 1 1 ....

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n n a x a x a x b + + = (3.2.4) ( ) ( ) ( ) 2 2 2 2 22 2 2 ..... n n a x a x b + + = ( ) ( ) ( ) 3 3 3 2 33 3 3 ... n n a x a x b + + = ... ( ) ( ) ( ) 3 3 3 2 3 .... nn n n n a x a x b + + =

where ( ) ( ) 2 2 2 2 22 i i m a
a = -
and ( ) ( ) ( ) 3 2 2 2 2 , i ij ij j a a m a = - ( ) ( ) ( ) 3 2 2 2 2
i i i b b m b = - ( ) , 3,4,....., i j n = (3.2.5) Continuing this process, we finally obtain equivalent

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system of equations at the ( ) 1 n thstep - ( ) ( ) ( ) ( ) 1 1 1 1 1 2 11 12 1 1 .... n n a x a x a x b + + = (3.2.6) ( ) ( ) ( ) 2 2 2 2 22 2 2 ..... n n a x a x b + + = ( ) ( ) ( ) 3 3 3 2 33 3 3 ... n n a x a x b + + = ... ( ) ( ) n n nn n n a x

b=
NSOU l CC-MT-05 42 where ( ) ( ) k k ik ik kk m a a = and ( ) ( ) ( ) 1 , k k k ik ij ij kj a a m a + = - ( ) ( ) ( ) 1k k k ik i i k b b m b + = - ( ) , 1......, , .1,2,3,.... i j k n k n = + (3.2.7) The upper triangular system (6) may easily be solved as follows. From the last equation ( ) ( ) ; n n n n nn x b a = then substituting this value of n x in the last but one equation we get the value of 1 , n x - and then again substituting the values of 1 , n n x x - in the last but two equation we compute 2n x - and so on. Finally we get 1 .x This process of solving an upper triangular system of linear system of equations is often called back substitution. When the diagonal coefficient there is unity, the last term of the constant vector contains the value of . n x This can be used in the ( ) 1 n- th equation represented by the second to the last line to obtain 1n x - and so on right up to the first line which will yield the value of 1 .x The name of this method simply derives from the elimination of each unknown from the equations below it producing a triangular system of equations represented by 1 1 12 1 2 2 2 1 ... 0 1 ... ... 0 0 ... 1 n n n n x c a a x c a x c ¢ ¢ ¢ ? ? ? ? ? ? ? ? ? ? ? ? ¢ ¢ ? ? ? ? ? ? = ? ? ? ? ? ? ? ? ? ? ? ? ¢ ? ?? ? ? ? (3.2.8) which can then be easily solved by back substitution where 1 1 n n n j ij j j i x c x c a x = + ¢ = ? ? ? ¢ ¢ = - ? ? ∑ One of the disadvantages of this approach is that errors (principally round off errors) from the successive subtractions build up through the process and accumulate in the last equation for . n x The errors thus incurred are further magnified by the

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NSOU l CC-MT-05 43 process of back substitution forcing the maximum effects of the round-off error into . i x A simple modification to this process allows us to more evenly distribute the effects of round off error yielding a solution of more uniform accuracy. In addition, it will provide us with an efficient mechanism for calculation of
the inverse of the matrix A.

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Example 3.2.1 : Solve the eqations by Gauss elimination method. 1 2 3 2 4, x x x + + = 1 2 3 2 2, x x x - + = 1 2 3 2 2 3. x x

x + - =

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Solution. Multiplying the second and third equations by 2 and 1 respectively and subtracting them from first equation we get 1 2 3 2 4

x

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x x + + = 2 3 3 3 0 x x- = 2 3 2 1. x x - + = Multiplying third equation by −3 and subtracting from seond equation we obtain 1 2 3 2 4 x x x + + = 2 3 3 3 0 x x- = 3 3 3. x = From the third equation 3 1, x = from the second equations 2 3 1 x x= = and from the first equation 1 2 3 2 4 2 x x x = - - =

or, 1 1. x = Therefore the solution is 1 1,
x = 2 1, x = 3 1. x = 3.3
Gauss-Jordan method Let us begin by writing the system of linear equations as we did in Gauss elimination method but now include a unit matrix on the right hand side of the expression. Thus, 11 12 1 21 22 2 1 2 ... ...

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n n n n nn a a a a a a a a a ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 ... n b b b ? ? ? ? ? ? ? ? ? ? ? ? ? 1 0 ... 0 0 1 ... 0 ... ... 0 0 ... 1 ? ? ? ? ? ? ? ? ? ? ? ? ....(3.3.1)

NSOU l CC-MT-05 44 We will treat the elements of this matrix as we do the elements of the constant vector b i . Now proceed as we did with the Gauss elimination method producing zeros in the columns below and to the left of the diagonal element. However, in addition to subtracting the line whose diagonal element has been made unity from all those below it, also subtract from the equations above it as well. This will require that these equations be normalized so that the corresponding elements are made equal to one and the diagonal element will no longer be unity. In addition to operating on the rows of the matrix A and the elements of , we will operate on the elements of the additional matrix which is initially a unit matrix. Carrying out these operations row by row until the last row is completed will leave us with a system of equations that resemble 11 22 0 ... 0 0 ... 0 ... ... 0 0 ... ¢ ? ? ? ? ¢ ? ? ? ? ? ? ¢ ? ? nn a a a 1 2 ... n b b b ¢ ? ? ? ? ¢ ? ? ? ? ? ? ¢ ? ? 11 12 1 21 22 2 1 2 ... ... n n n n nn b b b b b b b b ? ? ? ? ? ? ? ? ? ? ? ? (3.3.2) If one examines the, it is clear that so far we have done nothing to change the determinant of the original matrix A so that expansion by minors of the modified matrix represent by the elements a ij a ¢ is simply accomplished by multiplying the diagonal elements ii a together. A final step of dividing each row by ij a¢ will yield the unit matrix on the left hand side and elements of the solution vector i x will be found . The final elements of B will be the elements of the inverse matrix of A. Thus we have both solved the system of equations and found the inverse of the original matrix by performing the same steps on the constant vector as well as an additional unit matrix. Perhaps the simplest way to see why this works is to consider the system of linear equations and what the operations mean to them. Since all the operations are performed on entire rows including the constant vector, it is clear that they constitute legal algebraic operations that won't change the nature of the solution in any way. Indeed these are nothing more than the operations that one would perform by hand if he/she were solving the system by eliminating the appropriate variables. We have simply formalized that procedure so that it may be carried out in a systematic fashion. Such a procedure lends itself to computation by machine and may be relatively easily programmed. The reason for the algorithm yielding the matrix inverse is somewhat less easy to see. However, the product of A and B will be the unit matrix I, and the operations that go into that matrix-multiply are the inverse of those used to generate B.

NSOU l CC-MT-05 45 Example 3.3.1 : To see specifically how the Gauss-Jordan method works,
consider

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the following system of equations: 1 2 3 1 2 3 1 2 3 2 3 12 3 2 24 2 3 36 x x x x x x x x x + + = ? ? + + = ? ? + + = ? (3.3.3)			

If we
put this in the form required by expression (3.3.1) we have 1 2 3 3 2 1 2 1 3 ? ? ? ? ? ? ? ? ? ? ? 12 24 36 ? ? ? ? ? ? ? ? ? ? ? 1 0 0 0 1 0 0 0 1 ? ? ? ? ? ? ? ? ? ? ? (3.3.4) Now normalize the all rows by factoring out the lead elements of the first column so that ( )( )( ) 1 2 3 1 2 3 2 1 1 3 3 1 3 1 2 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? 12 8 18 ? ? ? ? ? ? ? ? ? ? ? 1 0 0 1 0 0 3 1 0 0 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.3.5) The first row can then be subtracted from the remaining rows (i.e. rows 2 and 3) to yield ( ) 6 1 2 3 1 0 0 12 4 8 1 0 4 1 0 3 3 3 6 3 3 1 0 1 0 2 2 2 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? - - - - ? ? ? ? ? ? ? ? ? ? ? ? ? ? - - ? ? ? ? - ? ? ? ? (3.3.6) Now repeat the cycle normalizing by factoring out the elements of the second column getting ( ) ( )( ) ( ) 1 1 3 0 0 1 2 6 2 2 4 3 3 1 6 2 0 1 2 3 0 3 2 4 4 0 1 1 4 2 1 0 3 3 ? ? ? ? ? ? ? ? ? ? ? ? ? ? - ? ? - - ? ? ? ? ? ? ? ? ? ? - ? ? ? ? ? ? ? ? - ? ? ? ? ? ? (3.3.7)

NSOU l CC-MT-05 46 Subtracting the second row from the remaining rows (i.e. rows 1 and 3) gives ( ) 1 1 1 1 0 0 4 4 3 2 2 3 1 24 0 1 2 3 0 4 4 0 0 1 7 1 1 1 12 4 3 ? ? - - ? ? ? ? ? ? ? ? ? ? ? ? ? ? - ? ? ? ? ? ? ? ? ? ? - - ? ? ? ? ? ? ? ? - - ? ? ? ? ? ? (3.3.8) Again repeat the cycle normalizing by the elements of the third column so ( ) ( ) ( )( ) 1 1 0 1 0 1 6 2 2 1 1 3 3 1 24 2 1 0 1 0 2 2 2 8 8 0 0 1 7 1 1 1 12 4 3 ? ? - - - ? ?? ? ? ? ? ? ?? ? ? ? - - - ? ?? ? ? ? ? ? ?? ? ? ? ? ?? ? ? ? ? ? ? ?? ? - ? ? ? ? (3.3.9) and subtract from the remaining rows to yield ( ) 5 1 1 1 0 0 13 12 4 3 1 11 7 1 1 24 0 0 2 2 24 8 3 0 0 1 7 1 1 1 12 4 3 ? ? - - - - ? ?? ? ? ? ? ? ?? ? ? ? - - ? ?? ? ? ? ? ? ?? ? ? ? ? ? ?? ? ? ? ? ? ? ?? ? - ? ? ? ? (3.3.10) Finally normalize by the remaining elements so as to produce the unit matrix on the left hand side so that ( )( ) ( ) ( ) 5 1 1 12 4 3 1 0 0 13 1 7 1 2 24 1 1 0 1 0 11 2 24 4 3 0 0 1 7 1 1 1 12 4 3 ? ? - ? ? ? ? ?? ? ? ? ? ? ?? ? - - - ? ? ? ? ?? ? ? ? ? ? ?? ? ? ? ?? ? ? ? ? - ? ? ? ? (3.3.11) The solution to the equations is now contained in the center vector while the right hand matrix contains the inverse of the original matrix that was on the left hand side of expression (3.3.4). The scalar quantity accumulating at the front of the matrix is the determinant as it represents factors of individual rows of the original matrix. The

NSOU l CC-MT-05 47 row subtraction shown in expressions (3.3.6), (3.3.8), and (3.3.10) will not change the value of the determinant. Since the determinant of the unit matrix on left side of expression (3.3.11) is one, the determinant of the original matrix is just the product of the factored elements. Thus our complete solution is { } 13 11 7 , x = - where ( ) 12 Det A = - and 1 5 1 1 12 4 3 7 1 2 12 4 3 1 1 1 12 4 3 A - ? ? - ? ? ? ? = - ? ? ? ? - ? ? ? ? ? ? (3.3.12) Pivoting : We have assumed in each step fo the Gaussian elimination that ( ) 0. k kk a [1] To remove this restriction, begin each step of elimination process by switching rows to put a non-zero elemnt in the pivot posion. Since A is non-singular, this is always possible. Sometimes it may happen that the pivot element is small (actually zero, but due to roundoff it becomes vary small). To guard against this, pivoting is used. Let at stage ( ) 1 1 k k n £ £ - ( ) max k k ij c a = Let 0 i be smallest row index i k< for which the maximum is attain. If 0 , i k< then switch rows k and 0 i in ( ) k A and ( ) ; k b and proceed with step k of the elimination process. All multipliesrs will now satisfy 1, 1,...., ik m i k n £ = + (remember ( ) ( )) k k ik ik kk m a a = And this ensures the groth in the elements of ( ) k A and thus eliminating the possibility of loss of significant errors. The pivoting is used in the solving in the linear system of equation is shown in the example given below.
NSOU l CC-MT-05 48
Example 3.3.2 :

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Solve the following system of equations by Gauss elimination method (use partical pivoting). 2 3 2 5 x x+ = 1 2 3 2 4 11 x x x + + = 1 2 3 3 5 12. x x x - + - = -

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Solution. The largest element (the pivot) in the coefficients of the variable 1 x is −3, attained the third equation. So we interchange first and third equations 1 2 3 3 5 12

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x x x - + - = - 1 2 3 2 4 11 x x x + + = 2 3 2 5. x x+ = Multiplying the second equation by 3 and adding with the first equation we get, 1 2 3 3 5 12 x x x - + - = - 2 3 3 x x+ = 2 3 2 5 x x+ = The

seond pivot is 1, which is at the position 22 a and 32 .a Taking 22 1 a = as pivot to avoid interchange of rows. Now, subtracting and third equation from second
equation,
we obtain 1 2 3 3 5 12
x x

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x - + - = - 2 3 3 x x+ = 3 2. x- = - Now by back substitution, the values of 3 2 1 , , x x x are obtained as ( ) 3 2 3 1 2 3 1 2, 3 1, 12 5 1. 3 x x x x x x = = - = = - - - + = Hence the solution is 1 2 3 1, 1, 2. x x x = = =

Some prelimary concepts Let V be the vector space.
NSOU l CC-MT-05 49 Norm of a Vector is defined as a real valued function N (x) satisfying the conditions i) ( ) 0 , , 0 if 0 [3]
" Î = =
N

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x x V x f x ii) ( ) ( )( ) .a = a a " Î N x N x isascalar x V iii) ( ) ( ) ( ) N x y N x N y + £ + (1) ( ) ( ) def 1 2 1 1 , ,...., = ¢ = = ∑ n i n i N x x x where x x x x (2) ( ) { 2 1 2 def 2 1 } = = = ∑ n i i N x x x (3) ( ) def max £ £ ¥ = = k i n i N x x x Example 3.3.3 : ( ) 1,0, 1,2 x ¢ = - Then 1 4, x = 2 6, x = 2

x ¥ =
Norm of a Matrix : By a norm of a matrix ( ) , 1,2,3..... ij n n A a i j n ´ ? ? = = ? ?
is defined as a real number A which
satisfies the following conditions i) 0,

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A ³ 0 A iff A = is a null matrix ii) ( ) a = a a A A isascalar iii) A B A B + £ + iv) AB A B £ n n A A \ £ (1) def 1 max = ∑ ij j i A A a (2) 1 2 2 def 2 . ? ? ? ? = ? ? ? ? ? ? ∑ ij i j A A a NSOU l CC-MT-05 50 (3) def max ¥ = ∑ ij i j A A a Example 3.3.4 : 1 2 3 4 5 6 7 8 9 A ? ? ? ? = ? ? ? ? ? ? Then ( ) 1 max 12,15,18 18 A = = ( ) 1 2 2 2 2 2 1 2 ....9 285 16.88 A = + + = = ( ) max 6,15,24 24 A ¥ = = 3.4

Gauss-Jacobi interation method Consider the

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system of linear equations 11 1 12 2 1 1 21 1 22 2 2 2 1 1 22 2 ... ... n n n n n nn n n a x a x a x b a x a x a x b a x a x a x b + + + = + + + = + + + = (3.4.1)

Intially the given equations
of the systems are so arranged the 0 ii a ¹ for 1,2,...., , i n = and suppose that this rearrangement is (3.4.1). Now (3.4.1) is reset in the form 1 1 12 1 2 11 11 11 ...

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n n a b a x x x a a a = - - - 2 2 21 2 2 22 22 22 ... n n a b a x x x a a a = - - - .... NSOU l CC-MT-05 51 1 . 1 2 1 ... n n n n n n nn nn nn b a a x x x a a

a - - = - - -
Or in
brief ( ) 1 1,2,...., i i ij j j i ij x b a x i n a ¹ ? ? = - - = ? ? ∑ (3.4.2) In the Gauss-Jacobi method the iteration is generated by the formula ( ) ( ) 1 1 k k i ij i j j i ii x b a x a + ¹ ? ? = - ? ? ? ? ∑ ( ) 1,2,...., i n = (3.4.3) The initial guess ( ) ( ) 0 1,2,....., i x i n = being chosen arbitrarily. To examine the convergence of the process, set max ij j i ii a K i a ¹ = ∑ (3.4.4) From (3.4.3) for every i, ( ) ( ) 1 1 + ¹ ? ? e = - e ? ? ? ? ∑ k k ij i j j i ii a a and so ( ) ( ) ( ) ( ) 1 1 1 k k k k ij ij i j ii ii j i j i a a K a a + ¹ ¹ e £ e £ e £ e ∑ ∑ And so ( ) ( ) 1k k K + e £ e (3.4.5) Hence for every ( ) ( ) 0 k k K e £ e (3.4.6) This shows that if ( ) 1, 0 > e ® k K as , k ® ¥ i.e., the iteration converges. The system of linear equations (1) is said to be srtictly diagonally dominant if ( ) 1,2,...., ii ij j i a a i n ¹ < = ∑
NSOU l CC-MT-05 52 i.e. if 1. K > Thus the Gauss-Jacobi iteration converges if the given system of linear equations is strictly diagonally dominant. Let 1.

37% **MATCHING BLOCK 74/158** SA Numerical Analysis Dr RSM.pdf (D144415232)

K > By (3.4.5) ( ) ( ) ( ) ( ) ( ) { } 1 1 1 k k k k k K h K h + + + e £ e + £ e + where ( ) ( ) ( ) ( ) ( ) 1 1 k k k k k h x x + + = - = e -e Or ( ) ( ) 1 1 + e £ - k k K h K

which gives the estimation of error. Smaller the value of K, more rapid will be the convergence. Also note that the above condition of convergence is sufficient but not necessary.

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Example 3.4.1 : Solve the following system of linear equations by Gauss-Jacobi's method correct up to four decimal places and calculate the upper bound of absolute errors. 27 6 54

x y z + - = 6 15 2 72 x y z + + = 54 110. x y z + + = Solution. Obviously,

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the system is diagonally dominant as 6 1 27 , + - > 6 2 15 , + > 1 1 54 . + > The Gauss-Jacobi's iteration scheme is ( ) ( ) ( ) ( ) 1 54 6 27

x
k k x y z + 1 = - + ( ) ( ) ( ) ( ) 1 27 6 2 15 k k k x x z + 1 = - - ( ) ( ) ( ) ( ) 1 110 . 54 k k k x x y + 1 = - -
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Let the initial solution be (0, 0, 0). The next iterations are shown in the following table. k x y z 0 0 0 0 1 2.00000 4.80000 2.03704 2 1.00878 3.72839 1.91111 3 1.24225 4.14167 1.94931 4 1.15183 4.04319 1.93733 5 1.17327 4.08096 1.94083 6 1.16500 4.07191 1.93974 7 1.16697 4.07537 1.94006 8 1.16614 4.07488 1.93999 9 1.16632 4.07477 1.93998 10 1.16632 4.07477 1.93998 11 1.16635 4.07481 1.93998 Fig. : 3.1 The solution correct up to four decimal places is 1.1664,

x = 4.0748, y = 1.9400. z = Here { } 1 1 7 8 2 8 max max , , . 27 15 54 15 = [1] ? ? ? ? = = = ? ? ? ? ? ? ∑ n ij i ii j j i A a a ( ) ( ) 0 5 5 3 10 ,4 10 ,0 . e - - = ´ ´ Therefore the upper bound of absolute error is ( ) ( ) 0 0 5 5.71 10 . 1 A e e A - £ = ´ - 3.5 Gauss-Seidel iteration method A slight variant of the Gauss-Jacobi iteration is the Gauss-siedel method in which the system is also written in the form (2) with 0 1,2,3,.... , [1] = ii a for i n but the iteration is carried out successively by the formulae
NSOU l CC-MT-05 54 ( ) ( ) ( ) ( ) 1 1 1 12 1 2 11 1 ...

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k k k n n x b a x a x a + = - - - ( ) ( ) ( ) ( ) 1 1 2 2 21 2 1 22 1 ... k k k n n x b a x a x a + + = - - - .... ( ) ( ) ( ) ( ) 1 1 1 1 1 , 1 1 1 ... k k k n n n n n n nn x b a x a x a + + + - - = - - - ( ) 1,2,3,....

i n = (3.5.1)
The initial guess ( ) ( ) 0 1,2,..., = i x i n
being chosen arbitrarily. ( ) ( ) ( ) ( ) 1 1 1 1,2,3,.... k k k i ij ij i j j ii j i j i x b a x a x i n a + + > < ? ? = - - = ? ? ? ? ? ? ∑ ∑ We Assert that Gauss-Seidel iteration also converges if 1>K where K is defined in (3.4.4). Assume the K > 1. For every i ( ) ( ) ( ) 1 1 1 k k k i ij ij i j j j i j i ii b a a a + + > < ? ? e = - e - e ? ? ? ? ∑ ∑ (3.5.2) Define temporarily ij j i i ii a K a > = ∑ for ( ) 1,2,3,..... i n = (3.5.3) 0 1 i K K £ > > and ( ) ( ) ( ) 1 1 1 + + > > ? ? e £ e + e ? ? ? ? ? ? ∑ ∑ k k k ij

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ij i j j ii j i j i a a a ( ) ( ) 1 1 k k ij ij j i j i ii a a a + > < £

e + e ∑ ∑ ( ) ( ) ( ) 1k k i i K K K + £ e + - e So that for some i,
NSOU l CC-MT-05 55 And so ( ) ( ) ( ) ( ) 1 1 + + e £ e + - e

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k k k i i K K K Or ( ) ( ) ( ) 1 1 + - e £ e - k k i i K K K (3.5.4) Since ( ) 1, 1 - £ > - i i K K K as K K we have Which leads to ( ) ( ) 1 k k K + e £ e (3.5.5) Hence for every k ( ) ( ) 0 k k K e £ e (3.5.6) So that ( ) 0 sin 1. e ® ® ¥ > k as k ce K If K > 1, an estimate of the error is given by ( ) ( ) 1 1 + e £ - k k K h K where ( ) ( ) ( ) ( ) ( ) 1 1 . k k k k k h x

x + + = - = e -e It may appear the Gauss-Seidrel method is more rapidly convergent than the Gauss-Jacobi method. Here also the condition that the given system is strictly diagonally dominant is sufficient for the convergence of the method but not necessary. 3.6 Successive Overrelaxation (S.O.R) Method We have to sove the linear system AX = b where ( ) , 1,2,3... ij n n A a i j n ´ ? ? = = ? ? is a non-singular matrix and { }( ) 1,2,3,.... . i b b i n ¢ = =
NSOU l CC-MT-05 56 Assume that the diagonal elements of matrix A are non-zero. If some 0, ii a = then by interchanging some rows , we can make all 0. ii a ¹ This is possible as is non-singular. The matrix A can always be written as = + + A D L U Where ij ij D a ? ? = d ? ? L ® Lower triangular matrix with diagonal elements zero U ® Upper triangular matrix with diagonal elements zero So , AX = b (3.6.1) becomes ( ) + + = D L U X b (3.6.2) Now multiplying by some non-zero scalar on bothside of equation (3.6.2) we have ( ) w + + = w D L U b or, ( ) w = w -w + LX D U X b (3.6.3) DX DX = (3.6.4) Adding (3.6.3) and (3.6.4) we get, ( ) ( ) 1 +w = w + -w -w D L X DX UX b (3.6.5) The iteration scheme is ( ) ( ) ( ) ( ) 1 1 , 0 1 + +w = w + -w -w = ¥ i i i D L X DX UX i b (3.6.6) (3..6.6) – (3.6.5) gives, ( ) ( ) ( ) ( ) ( ) ( ) 1 1 , 1 + +w = -w -w = ¥ i i i D L e De Ue i Where ( ) ( ) 1 1 i i e X X + + = - where ( ) 1i e + iis the error in the ( ) 1i th + stage of approximation. Or, ( ) ( ) ( ) ( ) 1 1 1 - + = +w -w -w ? ? ? ? ? i i e D L D U e ( ) ( ) ( ) 1 0 2 1 ... - + = = = = i i i Me M e M e where ( ) ( ) 1 1 M D L D U - = +w -w -w ? ? ? ?
NSOU l CC-MT-05 57 Suppose 1 2 , ,....., n

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l l l are eigen values of the matrix M and 1 2 , ,...., n X X X are corresponding eigen-vectors

such that they are linearly independent. Let ( ) 0 1 1 2 2 , ...

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n n e X X X = a + a + + a ( ) ( ) ( ) ( ) 1 1 1 1 1 1 1 2 2 2 , ... i i i i n n n e X X X + + + + = a l + a

l + + a l 0 , asi ® ® ¥ (if all eigen values are > 1 numerically or spectral radius 1 1 . max 1 £ £ > l > j j n i e ( ) 1 0 i X X as i + \ - ® ® ¥ Now, ( ) ( ) 1 det det .det 1 M D L D U - = +w -w -w ? ? ? ? ( ) 1 det det 1 D D - = -w ( ) 1 det det 1 - = -w D D I ( ) 1 n = -w Now, 1 2 det , ,......, n M = l l l ( ) 1 2 , ,......, 1 n n \l l l = -w i.e. max 1 i i l ³ -w or, 1 max 1 i i -w £ l > therefore, equation (3.6.6) will converge if ( ) 0 2 > w> where isreal. w This method is called overrelaxation method when 1 2, > w > and is called the underrelaxation method when 0 1. > w> When 1, w = the method becomes Gauss – Seidel's

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method. Example 3.6.1 : Solve the following system of equations 1 2 3 3 2 6 x x x + + = 1 2 3 4 2 5 x x x - + + = NSOU l CC-MT-05 58 1 2 3 2 4 7 x x x + + =

by
SOR method taken w = 1.01 Solution. The iteration scheme for SOR method is ( ) ( ) ( ) ( ) ( ) 1 11 11 11 12 13 1 1 1 1 2 3
k k k k
k
a

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x a x w a x a x a x b + ? ? = - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 22 22 21 22 23 2 2 2 1 2 3 k k k k k a x a x w a x a x a x b + + ? ? = - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 1 33 33 31 32 33 3 3 3 1 2 3 k k k k k a x a x w a x a x a x b + + + ? ? = - + + - ? ? ? ? or ( ) ( ) ( ) ( ) ( ) 1 1 3 1 1 2 3 3 1.01 3 2 6 k k k k k x x x x x + ? ? = - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 2 3 2 1 2 4 4 1.01 4 2 5 k k k k k x x x x x + + ? ? = - - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 1 3 3 3 1 2 4 4 1.01 2 4 7 k k k k k x x x x x + + + ? ? = - + + - ? ? ? ? Let ( ) ( ) ( ) 0 0 0 1 2 3 0. x x x = = =

The
detail calculatios are shown in the following table. k x 1 x 2 x 3 0 0 0 0 1 2.02000 1.77255 0.29983 2 1.20116 1.39665 0.80526 3 0.99557 1.09326 0.98064 4 0.98169 1.00422 1.00838 5 0.99312 0.99399 1.00491 6 0.99879 0.99728 1.00125 7 1.00009 0.99942 1.00009 8 1.00013 0.99999 0.99993 9 1.00005 1.00005 0.99997 Therefore the required solution is
NSOU l CC-MT-05 59 1 1,0000, x = 2 1,0000, x = 3 1,0000 x = correct up to four decimal places. Example : 3.6.2 Consider a linear system Ax = b, where 3 1 1 1 1 3 1 , 7 1 1 3 7 A b - - ? ? ? ? ? ? ? ? = - - = ? ? ? ? ? ? ? ? - - ? ? ? ? (a) Check, that the SOR method with value 1.25 w= of the relaxation parameter can be used to solve this system. (b) Compute the first iteration by the SOR method starting at the point ( ) ( ) 0 0,0,0 . T x = Solution : (a) Let us verify the sufficient condition for using the SOR method. We have to cheek, if matrix A is sysmmetric, positive definite (spd) : A is symmetri, so let us check positive definitness : det (3) = 3 < 0, det 3 1 8 0, 1 3 - ? ? = < ? ? - ? ? det 3 1 1 1 3 1 20 0 1 1 3 - ? ? ? ? - - = < ? ? ? ? - ? ? All leading principal minors are positive and so the matrix A is positive definite. We know, that for spd matrices the SOR method converges for values of the relaxation parameter w from the interval 0 > w > 2. Conclusion : the SOR method with value w = 1.25 can be used to solve this system. (b) The iterations of the SOR method are easier to compute by elements than in the voctor form : 1. Write
the system as
equations : 1 2 3 3 1

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x x x - + = - 1 2 3 3 7 x x x - + - = 1 2 3 3 7 x x x - + = - 2. First, write down the equations for the GS interations : NSOU l CC-MT-05 60 ( ) ( ) ( ) ( ) 1 1 2 3 1 /3 k k k x x x + = - + - ( ) ( ) ( ) ( ) 1 1 2 1 3 7 / 3 k k k x x x + + = + + ( ) ( ) ( ) ( ) 1 1 1 3 1 2 7 / 3 k k k x x x + + + = - - + 3.

Now multiply the right hand side by the parameter w and add to it the vector ( ) k x from the previous interation multiplie by the factor of ( ) 1 :w- ( ) ( ) ( ) ( ) ( ) ( ) 1 1 1 2 3 1 1 / 3

70% **MATCHING BLOCK 86/158** **SA** S41641 Mathematics 06.pdf (D164869290)

k k k k x w x w x x + = - + - + - ( ) ( ) ( ) ( ) ( ) ( ) 1 1 2 2 1 3 1 7 / 3 k k k k x w x w x x + + = - + + - ( ) ( ) ( ) ( ) ( ) ( ) 1 1 1 3 3 1 3 1 7 / 3 k k k k x w x w x x + + + = - + - - + 4.

For k = 0, 1, 2,.... compute ( ) 1 k x + from these equations, starting by the first one. Computation for

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k = 0. ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 0 0 0 1 1 2 3 1 1 3 1 1.25 ·0 1.25· 1 0 0 3 0.41667 x w x w x x = - + - + - = - + - + - = - ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 0 1 0 2 2 1 3 1 7 3 0.25·0 1.25· 7 0.41667 0 3 2.7431 x w x w x x = - + + - =- + - + = ( ) ( ) ( ) ( ) ( ) ( ) 1 0 1 1 3 3 1 2 1 7 x w x w x x = - + - - + ( ) 3 0.25·0 1.25· 7 0.41667 2.7431 3 1.6001 = + - + + = - The

next three interations are ( ) ( ) 2 1.4972,2.1880, 2.2288 , T x = - ( ) ( ) 3 1.0494,1.8782, 2.0141 , T x = - NSOU l CC-MT-05 61 ( ) ( ) 4 0.9428,2.0007, 1.9723 , T x = - the exact solution is equal to ( ) 1,2, 2 . T x = - 3.7 Summary The system of linear equations has been solved by using direct approach and iterative approach. In the direct approach Gauss elimination method and Gauss- Jordan method have been studied in detail where as the iterative approach Gauss Jacobi, Gauss Seidal methods are studied and their convergence are also studied. In SOR method also the convergence analysis has been studied. 3.8 Exercises 1. Using Gauss
elimination method with pivoting,

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solve the system of linear equations 1 2 3 2 4 3, x x x + + = 1 2 3 3 2 2 2, x x x + - = 1 2 3 6. x x x - + = (Ans: 1 3 2.8, 1.16, 2.04 x x = = - =) 2. Solve the following system of

equations
with and without pivoting and compare the result with exact solution (1, 1, 1). 3. Solve
the following system of equations by Gauss-Jacobi methos: i) 1 2 3 10 12,

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x x x + + = 1 3 2 210 13, x x + + = 1 2 3 2 2 10 14. x x x + + = (Ans: 1 2 3 1, 1, 1 x x x = = =) ii) 1 2 3 8 3 2 20, x x x - + = NSOU l CC-MT-05 62 1 2 3 4 11 33, x x x + - = 1 2 3 6 3 12 35. x x x + + = (Ans: 1 2 3 3. 168, 1.9858, .9117 x x x = - = =) 4.

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Solve the following system of equations by Gauss-Seidel method correct upto four decimal places: i) 12 6 9, x y z + + = 8 3 2 13, x y z + + + 5 7 + + = x y z (Ans : x = 1, y = 1, z = 1) ii) 8 18, x y z - + = 2 5 2 3, x y z + - = 3 16 x y z + - = - (Ans : x = 2, y = 0.9998, z = 2.9999) 5. Solve the following system of equations by S.O.R method correct upto four decimal places: 6, x y z + + = 4, x y z - - = - 2 2 1. x y z + - = - (Ans: x = 1, y = 2, z = 3)

Unit 4 rrrrr Interpolation Structure 4.0 Objectives 4.1 Introduction 4.2 Polynomial Interpolation 4.3 Newton's Forward Interpolation 4.4 Newton's Backward Interpolation 4.5 Central difference Interpolation 4.6 Lagrange's Interpolation 4.7 Finite difference operator 4.6 Exercises 4.7 Summary 4.0 Objectives After studying this unit one can be able to l construct different forms of interpolation polynomial l some knowledge of finite difference operators are also discussed. 4.1 Introduction
The method

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of obtaining the value of the function for any intermediate value of the argument when the values of

a functions are known for a set of values of the arguments is known as interpolation. Mathematically, if the values of the function ( ) y f x = at , , 2 ,....., x a a h a h a nh = + + + be known then finding the value of the function at x b= where a b a nh > > + is known as interpolation. If x lies outside the above said range, then the corresponding process is called extrapolation.
NSOU l CC-MT-05 64 4.2 Polynomial Interpolation Let ( ) ( ) , . ¥ Î -¥ ¥ f x C The principle of interpolating polynomial is "the selection of a function ( ) xj from a given class of functions such that the graph ( ) y x = j passes through a finite set of given points". When the function ( ) y x = j is a polynomial, the process of representing ( ) f x by ( ) xj is called polynomial interpolation. The polynomial interpolation is based on the following theorem known as Weierstrass theorem: Theorem 4.2.1 : Let a function ( ) { } , Î f x C a b and let 0 e < be any preassigned small number. Then, $ a polynomial ( ) xj for which ( ) ( ) ; f x x -j > e { } , Î x a b i.e. any continuous function can be uniformly approximated by a polynomial of sufficiently high degree within any prescribed tolerance on the finite interval. Theorem 4.2.2 : Given any real valued function ( ) f x and ( ) 1 n+ distinct points 0 1 2 3 , , , ,....

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n x x x x x there exist unique polynomial of maximum degree n which interpolates ( ) f x at the points 0 1 2 3 , , , ,.... . n x x x

x x Exersise: Prove the above theorem. In a polynomial interpolation the approximation function ( ) xj is taken to be a polynomial ( ) n y x of degree n£ given by ( ) 2 0 1 2 ....
n n n y

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x a a x a x a x = + + + + (4.1) and it is given ( ) ( ) ( ) 0,1,2,...., = = n i i y x f x i n (4.2) i.e. ( )( ) 2 0 1 2 ... 0,1,2,....., n i i n i i a a x a x a x f x i n + + + + = =

Now (4.2) is a system of ( ) 1
n+ linear equation with ( ) 1 n+ unknowns 0 1 2 , , ,...., . n a a a a Since the co-efficients determinant
NSOU l CC-MT-05 65 ( ) 0 0 1 1 1 ..... 1 ..... 0 1 .....

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n n i j i j n n n x x x x x x x x x < = - 1

Õ by Vandermonde's determinant as the points 0 1 2 , , ,...., n x x x x are distinct the values of 0 1 2 , , ,....., n a a a a can be uniquely determined so that ( ) n y x exists and is called interpolating polynomial. The given points 0 1 2 , , ,...., n x x x x are called interpolating points or nodes such that 0 1 2 ,...., n x x x x > > > and also we shall write ( )( ) 0,1,2,...., i i y f x i n = = 4.3 Newton's Forward Interpolation Formula Let ( ) y f x = be a continuously differentiable function. Given set of ( ) 1 n+ values ( ) ( ) ( ) 0 0 1 1 , , , ,.... , n n x y x y x y of x and y, it is required to find ( ) , n y x a polynomial of degree n, so that y and ( ) n y x coincide at tabulated points. Let the values of x be equidistant so that 0 , i x x ih = + (0 h < is the step length, 0,1,2,....). i n = Since ( ) n y x is a polynomial of degree , this
can be written in
the

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form ( ) ( ) ( )( ) ( ) 0 1 0 2 0 1 0 .... n n y x a a x x a x x x x a x x = + - + - - + + - ( ) ( ) 1 1 ... n x x x x - - - (4.3.1) We now determine the coefficient 0 1 2 , , ,..... n a a a a

using the notation ( ) ( ) 0,1,2,....,
n i i

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y x y i n = = We have 2 1 0 0 0 2 1 0 0 0 0 1 2 2 2 1 0 1 0 2 . , 2 2! y y y y y y y y a y a a x x x x

h
h h - D D - + D = = = = = = - - By continuing this method of calculating the coefficients we shall find that 3 4 0 0 0 3 4 3 4 , ,...... . 3! 4! ! n n n y y y a a a h h n h D D D = = =
Substituting these values of 0 1 2 , , ,....., n a a a a
in equation (4.3.1), we get ( ) ( ) ( )( ) ( ) 2 0 0 0 0 0 1 0 2 .... 2!
n
y
y
y

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x y x x x x x x x x h h D D = + - + - - + + - NSOU l CC-MT-05 66 ( ) ( ) 0 1 1 .... ! n n n y x x x x

n h - D - - (4.3.2) Setting 0 , x x u h - =
we have from equation (4.3.2) ( ) ( ) ( )( ) 2 3 0 0 0 0 1 1 2 ... 2! 3! n

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u u u u u y x y u y y y - - - = + D + D + D + + ( )( ) ( ) 0 1 2 ... 1 ! n u u u

u u y n - - - + D (4.3.3) Equation (4.3.3) is Newton's forward interpolation formula. The error term

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is given by ( ) ( )( ) ( ) ( ) ( ) ( ) 1 1 1 1 2 ... 1 ! n n n u u u u n R x h f n + + + - - - = x + { } 0 , , n mim x x x > x { } 0 max , , n x x x >

Note:

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Newton's forward interpolation formula is used to interpolate the values of near the beginning of a set of tabulator values.

The difference table used in Newton's forward formula is

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as follows : x y Dy D 2 y D 3 y D n y x 0 y 0 Dy 0 x 1 y 1 D 2 y 0 Dy 1 D 3 y 0 x 2 y 2 D 2 y 1 .... D n y 0 ..... D 2 y

n–2 Dy n–1 x n y n
NSOU l CC-MT-05 67

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Example 4.3.1 : The following table gives the values of e x for certain equidistant values of x. Find the value of

e x when x = 0.612 using Newton's forward differene formulae. x : 0.61 0.62 0.63 0.64 0.65 y : 1.840431 1.858928 1.877610 1.896481 1.915541 Solution. The forward difference difference table is x y Dy D 2 y D 3 y 0.61 1.840431 0.01897 0.62 1.858928 0.000185 0.018682 0.000004 0.63 1.877610 0.000189 0.018871 0.0 0.64 1.896481 0.000189 0.019060 0.65 1.915541 Here, 0 0.61, x = 0.612, x = 0.01, h = 0 0.612 0.61 0.2. 0.01 x x u h - - = = = Then, ( ) ( ) ( )( ) 2 0 0 0 1 1 2 0.612 0.2. 2! 3! u u u u u y y u y y - - - = + D + D + = ( ) 0.2 0.2 1 1.840431 0.2 0.018497 0.000185 2 - = + ´ + ´ ( )( ) 0.2 0.2 1 0.2 2 0.000004 6 - - + ´ 1.840431 0.003699 0.000015 0.0000019 = + - + 1.844115. =
NSOU l CC-MT-05 68 4.4 Newton's Backward Interpolation Formual Let ( ) y f x = be a continuously differentiable function. Given set of ( ) 1 n+ values ( ) ( ) ( ) 0 0 1 1 , , , ,...., , n n x y x y x y of x and y, it is required to find ( ) , n y x a polynomial of degree n, so that y and ( ) n y x coincide at tabulated points. Let the values of x be equidistant so that 0 , i x x ih = + (h < 0 is the step length, 0,1,2,....). i n = Since ( ) n y x is a polynomial of degree n, this can be written in the form ( ) ( ) ( )( ) ( ) 0 1 2 1 ....
n
n n

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n n n y x a a x x a x x x x a x x - = + - + - - + + - ( ) ( ) 1 0 .... - - - n x x x x (4.4.1) We now determine the coefficient 0 1 2 , , ,...., n a a a a

using the notation ( ) ( ) 0,1,2,.....,
n i i

70%	MATCHING BLOCK 103/158	W

y x y i n = = We have 2 1 1 2 0 1 2 2 2 1 2 , , 2 2! n n n n n n n n n y y y y y y y a y

a a x x h h h - - - - - Ñ - + Ñ = = = = = - By continuing this method of calculating the coefficients we shall find that 3 4 4 3 4 3 4 4 , ,...., . 3! 4! ! n n n n y y y a a a h h n h Ñ Ñ Ñ = = = Substituting these values of 0 1 2 , , ,...., n a a a a in equation (4.4.1), we get ( ) ( ) ( )( ) ( ) 2 0 1 2 ... 2!

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n n n n n n y y y x y x x x x x x x x h h - Ñ Ñ = + - + - - + + - ( ) ( ) 1 1 ... ! n n n n y x x x x n h - Ñ - - -1 (4.3.2) Setting , n x x v h - = we have from equation (4.3.2) ( ) ( ) ( )( ) 2 3 0 1 1 2 .... 2! 3! n n n

n v
v v v v y x y v y
y y + + + = +
Ñ + Ñ + Ñ + + ( )( ) ( ) 1 2 ... 1 ! n n v v v v n y n + + + - Ñ (4.3.3)
NSOU l CC-MT-05 69 Equation (4.3.3) is Newton's backward interpolation formula. The error term is given by ( ) ( )( ) ( ) ( ) ( ) ( ) 1 1 1 1 2 .... 1 !

38%	MATCHING BLOCK 109/158	SA	Numerical Analysis Dr RSM.pdf (D144415232)

n n n v v v v n R x h f n + + + + + + = x + 0 1 1 0 min{ , , , , } max{ , , } > x > ⋯ ⋯ n n x x x x x x x

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Note : Newton's backward

66%	MATCHING BLOCK 106/158	W

interpolation formula is used to interpolate the values of near the end of a set of tabulator values.

The difference table used in Newton's backward formula is as follows x y Ñy Ñ 2 y Ñ 3 y Ñ n y x 0 y 0 Ñy 1 x 1 y 1 Ñ 2 y 2 Ñy 2 Ñ 3 y 3 x 2 y 2 Ñ 2 y 3 .... Ñy 3 .... Ñ n y n ..... Ñ 2 y n Ñy n x n y n
Example 4.4.1 :

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From the following table of values of x and f (x) determine

the value of f (0.29) using Netwon's backward interpolation formula.

48%	MATCHING BLOCK 108/158	W

x : 0.20 0.22 0.24 0.26 0.28 0.30 f (x) : 1.6596 1.6698 1.6804 1.6912 1.7024 1.7139 Solution. The difference table is x f (x) Ñf (x) Ñ 2 f (x) Ñ 3 f (x) 0.20 1.6596 0.22 1.6698 0.0102 0.24 1.6804 0.0106 0.004 0.26 1.6912 0.0108 0.0002 –0.0002 0.28 1.7024 0.0112 0.0004 0.0002 0.30 1.7139 0.0115 0.0003 –0.0001 Here, 0.30, n x = 0.30, x = 0.02, h = 0.29 0.30 0.5. 0.02 n x x v h - - = = = -

NSOU l CC-MT-05 70 Then, ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 3 1 1 0.29 ... 2! 3! n n n n u u u f f x u f x f x f x u + + = + Ñ + Ñ + Ñ + ( ) 0.5 0.5 1 1.7139 0.5 0.0115 0.0003 2 - - + = - ´ + ´ ( )( ) ( ) 0.5 0.5 1 0.5 2 0.0001 6 - - + - + + ´ 1.7139 0.00575 0.0000375 0.00000625 = - - + 1.70811875 1.7081. = ≃ 4.5 Central Interpolation formula Stirling's Interpolation formula : For this formula the number of nodes will be taken to be odd, i.e. 2 , n m = The nodes being 0 1 2 , , ,....., . m x x x x ± ± ± The Gauss

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forward interpolation formula is given by ( ) ( ) ( ) 2 3 0 0 1 1 1 2! n u u y x y u y

y y - - - = + D + D + D ( )( ) ( ) 3 4 1 1 1 2 ..... 3! u u u y y - - - - + D + D where u lies 0 and 1 And Gauss Backward formula is given by ( ) ( ) ( )( ) 2 3 3 2 1 2 2 1 0 0 1 1 ... 2 2! 3! n u u y y y y u y x y u y - - - - -
D + D D + D ? ? = + + D + + ? ? ? ? where u lies between -1 and 0 Taking mean of the above two Gauss's formulas, we get ( ) ( ) ( ) ( ) ( )( ) 2 2 2 3 0 1 1 1 1 1 1 2 2! 3! n u u u u u y x y u y y y y - - - - - - - - = + D + D + D + D + ( ) 3 4 1 1 ... y y - - D +D
NSOU l CC-MT-05 71 The above equation is called Stirling's interpolation formula. 4.5.2 Bessel's formula is for n is odd and is given by ( ) ( ) ( ) ( ) 2 2 1 0 0 1 0 1 1 1 2 2 2! 2 n u u y y y x y y u y - ? ? - D + D = + + - D + ? ? ? ? ? ? ( ) ( ) 3 1 1 1 2 .... 3! u u u y - - - + D + The above relation is Bessel's formula. Exercise: Obtain the difference table for Stirling's and Bessel's formula. Example 4.5.1 : Use the central difference interpolation formula of Stirling of Bessel to

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find the values of y at (i) x = 1.40 and (ii) x = 1.60 from the following table x : 1.0 1.25 1.50 1.75 2.00 y : 1.0000 1.0772 1.1447 1.2051 2.2599 Solution.

The central difference table is i x i y i Dy i D 2 y i D 3 y i −2 1.00 1.0000 0.772 −1 1.25 1.0772 −0.0097 0.0675 0.0026 0 1.50 1.1447 −0.0071 0.0604 0.0015 1 1.75 1.2051 −0.0056 0.0548 2 2.00 1.2599 (i) For 1.40, x = we take 0 1.50, x = then ( ) 1.40 1.50 0.25 0.4. u = - = - The Bessel's formula gives ( ) ( ) ( ) 2 2 0 1 0 1 0 1 1 1.40 2 2 2! 2 u u y y y y y y u y - - + + D + D = + - D
NSOU l CC-MT-05 72 ( ) ( ) 3 1 1 1 1 3! 2 u u u y - + - - D ( ) 1.1447 1.2051 0.4 0.5 0.0604 2 + = + - - ´ ( ) 0.4 0.4 1 0.0071 0.0056 2! 2 - - - - - + ( )( )( ) 1 0.4 0.5 0.4 0.4 1 0.0015 6 + - - - - - ´ 1.118636. = (ii) For 1.60, x = we take 0 1.50, x = then ( ) 1.60 1.50 0.25 0.4. u = - = Using Stirling's formula ( ) ( ) 2 2 3 3 2 2 1 0 2 1 0 1 1 1.60 2 2! 3! 2 s s y y y y y s y y s y - - - - - D + D D + D = + + D + ( ) ( ) 2 0.4 0.675 0.0604 1.1447 0.4 0.0071 2 2 + = + + ´ - ( ) 0.4 0.16 1 0.0026 0.0015 6 2 - + + 1.1447 0.02558 0.000568 0.0001148 1.1695972. = + - - = 4.6 Lagrange's Interpolation Let ( ) y f x = be a continuously differentiable function. Given set of ( ) 1 n+ values ( ) ( ) ( ) 0 0 1 1 , , , ,.... , n n x y x y x y of x and y, it is required to find ( ) , n y x a polynomial of degree n, so that y and ( ) n y x coincide at tabulated points. Here the values of ( ) 0,1,2,.... i x i n = are not equispaced. Since ( ) n y x is a polynomial of degree n, this can be written in the form ( ) ( )( ) ( ) ( )( ) ( ) 0 1 2 1 0 2 ....
n
n

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n y x a x x x x x x a x x x x x x = - - - + - - - ( )( ) ( ) ( )( ) ( ) 2 0 1 0 1 1 .... n n n a x x x x x x a x x x x x x - + - - - + + - - - (4.5.1) NSOU l CC-MT-05 73 where 0 1 2 , , ,...., n a a a a are coefficient to be determined

from the relation ( ) ( ) ( ) 0,1,2,...., . = = =

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n i i i y x y f x i n Putting 0 x x= in equation (4.5.1), we get ( ) ( )( ) ( ) 0 0 0 1 0 2 0 .... n f x a x x x x x x = - - - Putting 1 x x= in equation (4.5.1), we get ( ) ( )( ) ( ) 1 1 1 0 1 2 1 .... n f x a x x x x x x = - - - Similarly putting 2 3 , ,.... n x x x x = in equation (4.5.1), we get ( ) ( )( ) ( ) 2 2 2 0 2 1 2 .... n f x a x x x x x x = - - - .... ( ) ( )( )( ) ( ) 0 0 1 1 .... n n n n n n n f x a x x x x x x x x - = - - - - Substituting the values of 0 1 2 , , ..., n a a a a in (4.5.1) we get ( ) ( )( ) ( ) ( )( ) ( ) ( ) 1 2 0 0 1 0 2 0 .... n n n x x x x x x y x f x x x x x x x - - - = - - - ( )( ) ( ) ( )( ) ( ) ( ) 0 2 1 1 0 1 2 1 .... n n x x x x x x f x x x x x x x - - - + + - - - ( )( ) ( ) ( )( ) ( ) ( ) 0 1 2 2 0 2 1 2 .... n n x x x x x x f x x x x x x x - - - + - - - ( )( ) ( ) ( )( ) ( ) ( ) 0 1 1 0 1 1 .... n n n n n n x x x x x x f x x x x x x x - - - - - - + - - - which is Lagrange's interpolation formula.

The above formula may be
written in the following way as
NSOU l CC-MT-05 74 ( ) ( ) ( ) ( ) ( ) ( ) 0
n i
n i i i
f
x

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f x y x x x x x = » = w ¢ - w ∑ where ( ) ( )( ) ( ) 0 1 ..... w = - - - n x x x x x x x ( ) ( ) ( ) ( ) ( ) ( ) 1 0 n i n i i i f x f x x R x x x x + = = w + ¢ - w ∑ Where ( ) ( ) ( ) ( ) ( ) { } { } 1 1 0 0 min , ,...., max , ,...., 1 ! + + x = w > x > + n n n n f R x x x x x x x x

n
Example 4.6.1 : A
function ( ) f x

defined on the interval (0, 1) is such that ( ) 0 0, f = ( ) 1/ 2 1, f = - ( ) 1 0. f = Find the quadratic polynomial ( ) p x which agrees with f for 0,1/ 2,1. x = If 3 3 1 d f dx £ for 0 1, x£ £ show that ( ) ( ) 1 12 f x p x - £ for 0 1. x£ £ Solution. Given 0 0, x = 1 1/ 2, x = 2 1 x = and ( ) 0 0, f = f ( ) 1/ 2 1, f = - ( ) 1 0, f =
From Lagrange's interpolating formula, the required quadratic polynomial i s ( ) ( )( ) ( )( ) ( ) ( )( ) ( )( ) ( ) 0 2 1 2 0 1 0 1 0 2 1 0 1 2

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x x x x x x x p x f x f x x x x x x x x x - - - - = + - - - - ( )( ) ( )( ) ( ) 0 1 2 2 0 2 1 x x x x f x x x x x - - + - - ( )( ) ( )( ) ( )( ) ( )( ) ( ) ( )( ) ( )( ) 1/ 2 1 0 1 0 1/ 2 0 1 0 0 1/ 2 0 1 1/ 2 0 1/ 2 1 1 0 1 1/ 2 x x x x x x - - - - - - - = ´ + ´ - + ´ - - - - - - ( ) 4 1 . x x = - The error ( ) ( ) ( ) = - E x f x p x is given by ( ) ( )( )( ) ( ) 0 1 2 3! f E x x x x x x x ¢¢¢ x = - - - NSOU l CC-MT-05 75 or, ( ) ( ) 0 1 2 3! f E x x x x x x x ¢¢¢ x = - - - 3 3 1 0 1/ 2 11. 1 in 0 1 . 3! d f x x x as x dx ? ? £ - - - £ £ £ ? ? ? ? ? ? Now, 0 1, x- £ 1/ 2 1/ 2 x - £ and 1 1 in 0 1. - £ £ £ x x Hence, ( ) 1 1 1 1. . . 2 6 12 E x £ = That is, ( ) ( ) 1 . 12 - £ f x p x Example 4.6.2 : Find the missing term in the following table x : 0 1 2 3 4 y : 1 2 4 ? 16 Solution. Using Lagrange's formula ( ) ( )( )( ) ( )( )( ) 3 2 0 1 2 4 7 14 8 8 0 1 0 2 0 4 x x x x x x L x - - - - + - = = - - - - ( ) ( )( )( ) ( )( )( ) 3 2 1 0 2 4 6 8 . 3 1 0 1 2 1 4 x x x x x x L x - - - - + = = - - - ( ) ( )( )( ) ( )( )( ) 3 2 2 0 1 4 5 4 . 4 2 0 2 1 2 4 x x x x x x L x - - - - + = = - - - - ( ) ( )( )( ) ( )( )( ) 3 2 3 0 1 2 3 2 . 24 4 0 4 1 4 2 x x x x x x L x - - - - + = = - - - Therefore, ( ) ( ) ( ) ( ) ( ) 0 0 1 1 2 2 3 3 + + + ≃ y x y L x y L x y L x y L x 3 2 3 2 7 14 8 6 8 1 2 8 3 x x x x x x - + - - + = ´ + ´ - NSOU l CC-MT-05 76 3 2 3 2 5 4 3 2 4 16 4 24 x x x x x x - + - + + ´ + ´ - 3 2 5 1 11 1. 24 8 12 x x x = - + +

Thus, ( ) 3 8.25. y =
Hence the
missing tern is 8.25. Example 4.6.3 :

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Using the following data, find by Lagrange's formula, the value of ( ) 10= f x at x ( ) 0 1 2 3 4 9.3 9.6 10.2 10.4 10.8 11.40 12.80 14.70 17.00 19.80 i i i i x y f x =

Also find the value of x where ( ) 16.00. f x = Soluion : To compute ( ) 10 , f we first calculate the following products : ( ) ( ) 4 4 0 0 10 j j
j j x x x = = - = - Õ Õ ( )( )( )( )( ) 10 9.3 10 9.6 10 10.2 10 10.4 10 10.8 0.01792, = - - - - - - = - ( ) 4 0 1 0.4455,

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j j x x = - = Õ ( ) 4 1 0, 1 0.1728, j j j x x = 1 - = - Õ ( ) 4 2 0, 2 0.0648, = 1 - = + Õ j j j x x ( ) 4 3 0, 3 0.0704, j j j x

x = 1 - = - Õ and ( ) 4 4 0, 4 0.4320. = 1 - = + Õ j j j x
x Thus, ( ) ( ) ( ) 11.40 12.80 14.70 10 0.01792 0.7 0.4455 0.4 0.1728 0.2 0.0648 f ? » - ´ + + ? ´ ´ - - ´ ? ( ) ( ) ( ) 17.00 19.80 0.4 0.0704 0.8 0.4320 ? + + ? - ´ - - ´ ? 13.197845. =
NSOU l CC-MT-05 77 4.7 Finite difference operator Shift Operator E : Let h be a non-zero constant is the step length. The shift operator E for any arbitrary function ( ) f x defined in ( ) ,-¥ ¥ is represented by ( ) ( ) .
Ef
x f x h = + Now ( ) ( ) ( ) ( ) 2 . 2
E f x

50%	MATCHING BLOCK 119/158	SA	M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

E Ef x Ef x h f x h = = + = + and in general ( ) ( ) . n E f x f x nh = + Forward difference operator :D It is defined by ( ) ( ) ( ) f x f x h f x

D = + -
where h is the
step length D is a linear operator and 1, E D = - 1. E = D + Putting 0 x x= we get ( ) ( ) 0 0 0 1 0 , y f x h f x y y D = + - = -
The second order
difference is given by ( ) 2 0 1 0 2 1 1 0 2 1 0 2

55%	MATCHING BLOCK 118/158	W

y y y y y y y y y y D = D - D = - - - - = - + Similarly the 3 rd order difference is represented by 3 2 2 0 1 0 3 2 1 0 3 3 y y y y y y y

D = D -D = - + - and k-th order difference is given by ( ) 0 0 1 - = ? ? D = - ? ? ? ? ∑ k i k k i i k y y i Exercise: i) Prove that first order difference of a constant is 0. ii)

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The first order difference of a polynomial of degree n is a polynomial of degree 1. n- Backward difference operator ÑÑÑÑÑ : The first order backward difference

operator is defined by ( ) ( ) ( ) f x f x f x h Ñ = - - The central difference operator :dThe central difference operator d is defined by
NSOU l CC-MT-05 78 ( ) ( ) ( ) ( ) 1 1 2 2 1 1 2 2

59%	MATCHING BLOCK 121/158	SA	M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

f x f x h f x h E E f x - ? ? d = + - - = - ? ? ? ? ( ) ( ) ( ) ( ) 1 2 f x h f x h f x f x d + = + - = D ( ) ( ) ( ) ( ) 1 1 1 2 2 2 f x f x h f x h f x h d = + - - = D - Thus

we have the result 1 1 2 2 E E - d º - Example: i) Show that 1 1 . E -
º -Ñ Proof : We know that ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 1 1

31%	MATCHING BLOCK 122/158	SA	M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

f x f x f x h f x E f x E f x - - Ñ = - - - = - = - 1 1 E - ⇒ º -Ñ (proved) (ii) Show that 2 . D -Ñ º d Proof : We know that ( ) ( ) ( ) ( ) 1 1 2 2 1 1 2 2 f x f x h f x h E E f x - ? ? d = + - - = - ? ? ? ? 1 1 2 2 E E - ⇒ d º - ( ) ( ) 2 1 2 1 2 1 - ⇒ d

º - + = + D - + -Ñ = D -Ñ E E (proved) 4.8 Summary In this Unit we have studied Newton's forward, backward interpolations, Central Interpolation, Bessel's and Striling's interpolation, Lagrange's interpolation and the related problems. We have also studied the some operators like shift, forward difference, backward difference and central difference and relations between them.
NSOU l CC-MT-05 79 4.8 Exercise 1. Determine ( ) f x as a polynomial in

61%	MATCHING BLOCK 124/158	SA	nm-27-06-2017.doc (D29511457)

x for the following data : x : -4 -1 0 2 4 f (x) 1245 33 5 9 1335 Ans : ( ) 4 3 2 3 5 6 4 5 5 f x x x x x = - + = - + - 2. Given the values : x : 5 7 11 13 17 f (x) 150 392 1452 2366 5202

Evaluate f (9)

using Lagrane's interpolation forula. (Ans : 810) 3. The following table gives the sales of a concern for five years. Estimate the sales for the year (i) 1986 (ii) 1992 : Year 1985 1987 1989 1991 1993 Sales 40 43 48 52 57 Ans : (i) 41.02 (ii) 54.46 4. Find the seventh and the general terms of the series 3, 9, 20, 38, 65,.... Ans : (i) ( ) 7 154 f = (ii) ( ) ( ) 3 2 1 2 3 13 6 f x x x x = + + 5. Using the Stirling's formula to find 32 u from the following table x i 20 25 30 35 40 45 xi u 14.035 13.674 13.257 12.734 12.089 11.309 Ans : 32 13.059 u = 6. Prove that (i) . E E D = D (ii) hD E e= (iii) 1 . E - Ñ = D (iv) ( ) 2 2 1 D = + D d Unit 5 rrrrr Numerical differentiation Structure 5.0 Objectives 5.1 Introduction 5.2 Newton's Forward Differentiation Formula 5.3 Newton's Backward Differentiation Formula 5.4 Lagrange's Differentiation Formula 5.5 Summary 5.6 Exercises 5.0 Objectives After studying this unit one can be able to l find numerical differentiation of a function by using different methods. 5.1 Introduction Numerical differentiation is connected with the computation of derivatives of a function whose values are known at a tabular points. The fundamental operation of differentiation is applied to the interpolating polynomial to evaluate the derivatives of the given of the given function whose values are known at some tabular points. 5.2 Netwon's Forward Differentiation Formula

64%	MATCHING BLOCK 123/158	W
Let ( ) y f x = denote a continuously differential function which takes the values 0 1 2 3 , , , ,..... n y y y		

y y for the equidistant values 0 1 2 3 , , , ,.... n x x x x x of the independent variables

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x, then we have from Newton's Forward Interpolation formula as ( ) ( ) ( )( ) 2 3 0 0 0 0 1 1 2 ... 2! 3! u u u u u		

f x y u y y y - - - » + D + D + D +
NSOU l CC-MT-05 81 ( )( ) ( ) 0 1 2 ... 1 ! n u u u u n y n - - - + + D Where ( ) 0 , , i i i y f x x x ih = = + (0 h < is the step length, 0,1,2,....) i n = and 0 x x u h - = so that 1 · df df df du dx du dx h du = = ( ) 2 2 3 0 0 0 1 2 1 3 6 2 ... 2! 3! dy u u u f x u y y y dx h ? ? - - + ¢ \ = » D + D + D + ? ? ? ? ( ) 2 2 3 0 0 2 2 1 6 6 .... 3! d y u f x y y dx h - ? ? ¢¢ = » D + D + ? ? ? ? And so on In particular for 0 x x= i.e. for 0, u = them 0 2 3 0 0 0 1 1 1 ... 2 3

54%	MATCHING BLOCK 125/158	W
x x dy y y y dx h = ? ? ? ? » D - D + D + ? ? ? ? ? ? ? ? ? 0 2 2 3 0 0 2 2 1 ... = ? ? ? ? » D -D + ? ? ? ? ? ? ? ? ? x x d y y y dx h		

The above formulae are applicable for numerical differentiation at a point x near the beginning of the tabulated values. 5.3 Netwon's Backward Differentiation Formula

64%	MATCHING BLOCK 126/158	W
Let ( ) y f x = denote a continuously differential function which takes the values 0 1 2 3 , , , ,..... n y y y		

y y for the equidistant values 0 1 2 3 , , , ,.... n x x x x x of the independent variables x, then we have from Newton's Forward Interpolation formula as ( ) ( ) ( )( ) 2 3 1 2 3 1 1 2 ... 2! 3!

71%	MATCHING BLOCK 127/158	W
n n n n u u u u u f x y u y		

y y - - - + + + » + D + D + D + ( )( ) ( ) 0 1 2 ... 1 ! n u u u u n y n + + + - + D

NSOU l CC-MT-05 82 where ( ) 0 , , i i i y f x x x ih = = + (0 h < is the step length, 0,1,2,.....) i n = and - = n x x u h so that 1 · = = df df df du dx du dx h du ( ) 2 2 3 1 2 3 1 2 1 3 6 2 .... 2! 3! - - - ? ? + + + ¢ \ = » D + D + D + ? ? ? ? n n n dy u u u f x y y y dx h ( ) 2 2 3 2 0 2 2 1 6 6 ... 3! n d y u f x y y dx h - + ? ? ¢¢ = » D + D + ? ? ? ? and so on In particular for n x x= i.e. for 0, u = them 2 3 1 2 3 1 1 1 .... 2 3
n

68%	MATCHING BLOCK 128/158	W

n n n x x dy y y y dx h - - - = ? ? ? ? » D + D + D + ? ? ? ? ? ? ? ? 2 2 3 2 3 2 2 1 ... n n n x x d y y y dx h - - = ? ? ? ? »

D -
D + ? ? ? ? ? ? ? ? The above formulae are applicable for numerical differentiation at a point x near the end of the tabulated values. 5.4 Lagrange's Differentiation Formula Let ( ) y f x = denote a continuously differential function which takes the values ( ) ( ) ( ) 0 1 , ,...., n f x f x f x corresponding to (n+1) non-equidistant values 0 1 2 3 , , , ,.... . n x x x x x Since the (n+1) values of the function are given

47%	MATCHING BLOCK 129/158	W

corresponding to (n+1) values of the independent variable x, we can represent the function ( ) y f x = to be a polynomial in of degree .

Then we have
Lagrange's Interpolation formula as ( ) ( ) ( ) ( ) ( ) ( ) 0
n i

51%	MATCHING BLOCK 131/158	W

n i i i f x f x L x x x x x = » = w ¢ - w ∑ where ( ) ( )( ) ( ) 0 1 ..... n x x x x x x x w = - - - NSOU l CC-MT-05 83 Now ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 0 0 5.1 n n i i n i i i i i i f x f x f x L x x x x x x

x
x = = ¢ ¢ ¢ » =
w -w ¢ - w ¢ - w ∑ ∑ For non tabular points we use the above formula but for the tabular points k x x= equation (5.1) is indeterminate. Hence we proceed as ( ) ( ) ( ) ( ) ( ) 0 n i
n i i i

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f x L x x x x x = = w ¢ - w ∑ ( ) ( ) ( ) ( ) ( ) ( ) 0 n i k k i i i i k f x x x f x x x x = 1 = w + w ¢ - w ∑ ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 0 0 n n i i n k k i i i i i i i i k f x f x L x x x f x x x x x x x x = = 1 ¢ ¢ ¢ = w + w -w ¢ - w ¢ - w ∑ ∑ ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 n i n k k k k k i i i f x L x x x f x x x x = ¢ ¢ ¢ = w +w ¢ - w ∑ where ( ) ( ) 0 1 1 1 1 1 .... k k k k n k i i k x x x x x x x x x 1 ¢ w = + + + = - - - - ∑ ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 1 = 1 ¢ ¢ = w + ¢ - w - ∑ ∑ n i n k k k i i k i i i k f x L x x f x x x x x x

Example 5.4.1 : Compute dy dx and 2 2 d y dx for 1, x = using following table 1 2 3 4 5 6 1 8 27 64 125 216 x y Solution: The

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difference table is NSOU l CC-MT-05 84 2 3 4 1 1 7 2 8 12 19 6 3 27 18 0 37 6 4 64 24 0 61 6 5 125 30 91 6 216 x y y y y y D D D D We have 0 1, 1, 1 x h x = = = so 0 0. x x u h - = = 0 2 3 4 0 0 0 0 1 1 1 1 ... 2 3 4 x x dy y y y y

dx h = ? ? ? ? » D - D + D - D + ? ? ? ? ? ? ? ? ? { } 1 1 1 1 7 12 6 0 ... 7 6 2 3 1 2 3 = ? ? ? ? = - ´ + ´ - + = - + = ? ? ? ? ? ? ? ? ? x dy dx and 0 2 2 3 4 0 0 0 2 2 1 11 ... 12 = ? ? ? ? » D -D + D - ? ? ? ? ? ? ? ? ? ? ? x x d y y y y dx h { } 2 2 2 1 1 12 16 6 1 = ? ? » - = ? ? ? ? ? ?
x d y dx 1 3 = ? ? \ = ? ? ? ? x dy dx and 2 2 1 6. = ? ? = ? ? ? ? ? ? x d y dx
NSOU l CC-MT-05 85 Example 5.4.2 :

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Find the value of for which is minimum and find the minimum value from the table: x 0.60 0.65 0.70 0.75 y (

x) 0.6221 0.6155 0.6138 0.6174 Solution: Taking 0.60 as origin, we have ( ) ( ) ( )( ) 2 3 0 0 0 0 1 1 2 2! 3! u u u u u y x y y y y y - - - = + D + D + D We have the difference table as follows: x y Dy D 2 y D 3 y 0.60 0.6221 -0.0066 0.65 0.6155 0.0049 -0.0017 0 0.70 0.6138 0.0049 0.0032 0.75 0.6170 Putting the values, we have ( ) ( ) ( ) ( ) 1 0.6221 0.0066 0.0049 2! u u y x u - = + - + where 0 0.60 0.05 x x x u h - - = = Also 0, dy dx = i.e. ( ) 1 2 1 0.0066 0.0049 0 2 u h - ? ? - + = ? ? ? ? 1.8469 u = 0 0.60 0.05 1.8469 .6923 x x uh = + = + ´ = ( ) ( )( ) min 0.6221 0.0066 1.8469 0.00245 1.8469 0.0049 0.6137426 y \ = + - ´ + =
NSOU l CC-MT-05 86 5.5 Summary In this unit numerical differentiation has been done by Using Newton' Forward, backward, Lagrange's differentiation formulae. Using this maximum and minimum values are also calculated. 5.6 Exercises 1. Find ( ) 93 f ¢ from the folloing table : x 60 75 90 105 120 f(x) 28.2 38.2 43.2 40.9 37.7 Ans : -0.03627 2. Find the first and second order derivative of

45%	MATCHING BLOCK 135/158	W

at 15 x x = = from the following table: x 15 17 19 21 23 25 y x = 3.873 4.123 4.359 4.583 4.796 5.000 Ans: 0.1289, -0.004 3. Find the minimum values of ( ) f x from the table: x 0 2 4 6 f (x) 3 3 11 27 Ans: 2.25 4. Find the maximum values of

from the table: x 1.2 1.3 1.4 1.5 1.6 f (

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x) 0.9320 0.9636 0.9855 0.9975 0.9996 Ans: 1.58 5. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1971 Year (x) 1931 1941 1951 1961 1971 Population on thousands(y) 40.62 60.80 79.95 103.56 132.65

Ans: 2.36425, 3.10525
Unit 6 rrrrr Numerical Integration Structure 6.0 Objectives 6.1 Introduction 6.2 Newton Cotes Formula 6.3 Trapezoidal

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Rule 6.4 Simpson's Rule 6.5 Weddle's Rule 6.6 Summary 6.7 Exercises 6.0 Objectives After studying this unit one will be able to

learn about l the numerical integration of a function by using different rules and also the corresponding error terms. 6.1 Introduction The well-known method of evaluating a definite integral ( ) b a f x dx ∫ is to find an indefinite integral or a primitive of ( ) , f x i.e. a function ( ) xj such that ( ) ( ) x f x ¢ j = and then calculate the values of ( ) ( ) ,a b j j and take the value of the integral to be ( ) ( ) b a j -j But if the function ( ) f x is such that its indefinite integral cannot be obtained in terms of known functions, as is very often the case, then the above method fails. In such cases we may try to compute an approximate numerical value of the definite integral up to a desired degree of accuracy. This is the problem of numerical integration which is also called mechanical quadrature.

NSOU l CC-MT-05 88 Again, if the integrand ( ) f x is not known in its analytic form but is represented by table of values, then the formal method becomes meaningless, and we are turned to numerical integration. Closed and open type quadrature formula: A mechanical quadrature formula is called closed or open type according as the limits of integration are used as interpolating points or not. Degree of Precision: A mechanical quadrature formula is said have a degree of precision k, (k being a positive integer), if it is exact, i.e. the error is zero for an arbitrary polynomial of degree , k n£ but there exist a polynomial of degree 1 k + for which it is not exact, i.e., the error is not zero. Composite rule: Sometimes it is more convenient to break up the interval of integration { } ,a b into m sub-intervals ( ) 1 , 1,2,3,.... j j a a j m - ? ? = ? ? by the points 0 1 2 , , ,...., m a a a a such that 0 1 2 ... , m a a a a a b = > > > = apply a given quadrature formula separately to each interval 1 , j j a a - ? ? ? ? and add the result. The formula thus obtained will be called composite rule corresponding to given quadrature formula. 6.2 Newton-Cotes Formula (closed type) Let the integral to be evaluated be ( ) ( ) . b a I f f x dx = ∫

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The interval { } ,a b is sub- divided into n equal subinterval, each of

length . The nodes are 0 1 2 , , ,....., . n x x x x such that ( ) 0 0 , , , 0,1,2,3,...., . n i b a x a x b x x ih h i n n - = = = + = = The corresponding entries ( ) , 0,1,2,..... i f x i n = are also available. Let us use

42%	**MATCHING BLOCK 138/158**	**W**

Lagrange's interpolation formula to approximate ( ) f x by the interpolating polynomial ( ) n y x ( ) ( ) ( ) ( ) ( ) ( ) 0 n i n i i i f x f x y x x x x x = » = w ¢ - w ∑ where ( ) ( )( ) ( ) 0 1 .... . n x x x x x x x

w = - - -
NSOU l CC-MT-05 89 Integrating the interpolating polynomial ( ) n y x we have the approximate value of the given interval as ( ) ( ) ( ) ( ) ( ) ( ) 0 0
n n b i n n i i a i i i f

35%	**MATCHING BLOCK 140/158**	**W**

x I f x dx H f x x x x = = = w = ¢ - w ∑ ∑ ∫ (6.2.1) where ( ) ( ) ( ) b n i a i i x H dx x x x w = ¢ - w ∫ ( ) 0,1,2,..... i n = (6.2.2) Setting 0 , x x

u h - = so that , dx h du = (6.2.3) So ( ) ( )( ) ( ) 1 1 2 ... n x h u u u u n + w = - - - (6.2.4) Again, ( ) ( )( ) ( )( ) ( ) 0 1 1 1 ... . i i i i i i i

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n x x x x x x x x x x x - + ¢ w = - - - - - ( ) { } ( )( )( ) ( ) { } 1 .... 1 1 2 .... ih i h h h h n i h = - - - - - ( )( ) { } ( ) ( ) 1 2 ...1 1 ! n i i n i i i i h h n i - - = - - - - ( ) ( ) 1 ! ! n i n

h i n
i - = - - (6.2.5) Now using (6.2.3), (6.2.4), (6.2.5) in (6.2.2) we have ( )( ) ( ) ( ) ( ) { } 1 0 1 2 ... 1 !(1)! + - - - - - = - - - ∫ n n n
i n i n
h u u u u n H h du h i n u i h ( ) ( ) ( ) ( )( ) ( ) ( ) ( ) 0 1 1 2 .... 0,1,2,....., . ! ! n i n b a u u u u n du i n n i n i u i - - - - - - - = = - - ∫
( ) ; n n i i H b a K \ = - where ( ) ( ) ( )( ) ( ) ( ) ( ) 0 1 1 2 ... 0,1,2,....., . ! ! n i n n i u u u u n K i n n i n i u i - - - - - - = = - - ∫
(6.2.6)

NSOU l CC-MT-05 90 Thus we have ( ) ( ) ( ) ( ) 0 0 n n n n i i i i i i I f H f x b a K f x = = » = - ∑ ∑ (6.2.7) Where n i K is given in equation (6.2.6). This is called the ( ) 1 n+ – points Newton- Cotes Numerical Integration formula of the closed type. 6.3 Trapezoidal Rule For 1, n = we have from Newton-Cotes Formula ( ) ( ) ( ) ( ) ( ) ( ) 1 1 1 0 0 1 1 0= ? ? = » - = - + ? ? ∑ n T i i i I f I b a

40%	MATCHING BLOCK 142/158	SA	Numerical Analysis Dr RSM.pdf (D144415232)

K f x b a K f x K f x where ( ) ( ) ( ) 1 0 1 1 0 0 1 1 1 2 1.0! 1 0 ! K u du - - - = - = - ∫ and ( ) ( ) 1 1 1 1 1 0 1 1 2 1.1! 1 1 ! K udu - - = = - ∫ ( ) ( ) ( ) ( ) 0 1 2 - ? ? = » + ? ? T b a I f I f x f x

Error in Trapezoidal rule is ( ) ( ) ( )( ) 3 3 12 12 T b a h E f f a b - ¢¢ ¢¢ = - x = - x > x > Geometrically, the curve ( ) y

62%	MATCHING BLOCK 143/158	SA	M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

f x = is replaced by the straight line passing through the point ( ) ( ) ,a f a and ( ) ( ) , ,

b f b and the integral ( ) b a f x dx ∫ is approximated by the area of the trapezium bounded by the straight line, the ordinates at , x a b = and the name trapezoidal rule. The degree of precision is 1 Composite trapezoidal rule: Suppose the interval { } ,a b is sub-divided into equal subinterval,

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each of length h. The nodes are 0 1 2 , , ,......., , n x x x x such that 0 , , n x a x b = = ( ) 0 , 0,1,2,3,...., . i b a x x ih h i n

n - = + = = then applying the above

NSOU l CC-MT-05 91 Trapezoidal rule to each subintervals { }( ) 1 , ,1,2,3,...., - = i i x x i n and summing over i we can obtain the composite Trapezoidal rule given as ( ) ( ) ( ) ( ) 1 2 0 1 1 .... - = + + + ∫ ∫ ∫

n

n

48%	MATCHING BLOCK 145/158	SA	M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

x x x x x x I f f x dx f x dx f x dx ( ) ( ) ( ) ( ) 1 2 0 1 1 .... n n x x x x x x I f f x dx f x dx f x dx - = + + + ∫ ∫ ∫ ( ) ( ) ( ) ( ) 3 0 1 1 1 1 .... 2 2 2 12 n n i h h f x f x f x f = ? ? ¢¢ = + + + - x ? ? ? ? ∑ (

by

using Intermediate-value theorem) 6.4 Simpson’s Rule For 2, n = we have from Newton-Cotes Formula ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 2 2 2 0 0 1 1 2 2 0= ? ? = » - = - + + ? ? ∑ n s i i i I f I b a

37%	MATCHING BLOCK 146/158	SA	Numerical Analysis Dr RSM.pdf (D144415232)

K f x b a K f x K f x K f x where ( ) ( ) ( ) 2 0 2 2 0 0 1 1 2 6 2.0! 2 0 ! K u u du - - = - = - ∫ ( ) ( ) ( ) 2 1 1 2 1 0 1 2 2 3 2.1! 2 1 ! K u u du - - = - = - ∫ ( ) ( ) 1 2 2 0 2 1 1 6 2.2! 2 2 ! K u u du = - = - ∫ ( ) ( ) ( ) ( ) ( ) 0 1 2 4 6 s b a I f I f x f x f x - ? ? = » + + ? ?

Error in Trapezoidal rule is ( ) ( ) ( )( ) 5 5 90 2880 iv s b a h E f f a b - ¢¢ = - x = - x > x > The degree of precision is 3 Composite Simpson’1/3rd rule: Suppose the interval { } ,a b is sub-divided into

NSOU l CC-MT-05 92 ( ) 2 n m = of equal subinterval, each of length h. The nodes are 0 1 2 , , ,...., , n x x x x such that 0 , x a= , n x b= 0 , x ih+ b a h n - = ( ) 0,1,2,3,...., . i n = This divides the range of integration { } ,a b into / 2 m n= subrange then applying the above Simpson's rule to each subintervals { } { } { } 0 2 2 4 2 , , , ,.... , n n x x x x x x - and applying Simpson's rule to the subrange 2 2 2 , j j
x x - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 2 2 2 5 2 2 2 1 2 4 3 90 j j x iv j j j j
x

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h h f x dx f x f x f x f - - - ? ? = + + - x ? ? ∫ (( ) 2 2 2 ; 1,2,...,

j j j x x
j m - > x > = Summing over all the sub-ranges, we have ( ) ( ) 2 2 2 1 j j m x x j I f f x dx - = = ∑ ∫ ( ) ( ) ( ) ( ) 5 2 2 2 1 2 1 1 4 3 90 m m iv j j j j j j h h

54%	MATCHING BLOCK 148/158	SA	M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

f x f x f x f - - = = ? ? = + + - x ? ? ∑ ∑ c c s s I E = + ( ) ( ) ( ) ( ) ( ) { } 0 1 3 1 4 ... 3 c s n n h I f x f x f x f x f x - ? ? = + + + + + ? ? ( ) ( ) ( ) { } 2 4 2 2 ...] n f x f x f x - + + + + ( )( ) 5 90

c iv s nh E f a b = - x > x > (by using Intermediate-value theorem) For 1, n = 2, 3, 4, 5, 6 the calculated values of n i K are given in table 6.4.1
NSOU l CC-MT-05 93 Table for n i K i 0 1 2 3 4 5 6 n 1 1 2 1 2 2 1 6 4 6 1 6 3 1 8 3 8 3 8 1 8 4 7 90 32 90 12 90 32 90 7 90 5 19 288 75 288 50 288 50 288 75 288 19 288 6 41 840 216 840 27 840 272 840 27 840 41 840 Table: 6.4.1 Newton-Cotes quadrature coefficients (closed type) 6.5 Weddle's Rule The seven-point Newton-Cotes closed type formula with error is ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 1 2 3 4 41 216 27 272 27 140 b
a
h I

55%	MATCHING BLOCK 149/158	SA	Numerical Analysis Dr RSM.pdf (D144415232)

f f x dx f x f x f x f x f x ? = = + + + + + ? ∫ ( ) ( ) ( )( ) 9 5 6 9 216 41 ; 140 6 viii h b a f x f x f a b h - ? + - x > x > = ? (6.5.1)

The coefficient of the ordinate s are extremely cumbrous which makes the formula unworthy of practical computation. Accordingly, we seek to modify the above formula so that the coefficients are simplified by proceeding as follows.
We know ( ) ( ) ( ) ( ) ( ) ( ) ( ) 6 0 0 1 2 3 4 5 6 6 15 20 15 6

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f x f x f x f x f x f x f x D = - + - + - + (6.5.2) (6.5.1) + 140 h + ´ (6.5.2) gives on writing ( ) ( )( ) 6 6 0 vi f x h f a b ¢ ¢ D = x > x > NSOU l CC-MT-05 94 ( ) b W W a f x dx I E = + ∫ Where ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 1 2 3 4 5 6 3 5 6 5 10 b W a h I f x dx f x f x f x f x f x f x f x ? ? = = + + + + + + ? ? ∫ (6.5.3) and ( ) ( )( ) 7 9 9 , 140 140 vi viii W h h E f f

a b ¢ ¢ = - x - x > x x > (6.5.4)
This is
called

Weddle's rule inwhich the coefficients of the ordinaltes are fairly simple. Composite Weddle's rule: Suppose the interval { } ,a b is sub-divided into ( ) 6 n m = of equal subinterval, each of length h. The nodes are 0 1 2 , , ,...., , n x x x x such that 0 , , n x a x b = = 0 , i x x ih = + ( ) 0,1,2,3,...., . b a h i n n - = = This divides the range of integration { } ,a b into / 6 m n= subrange then applying the above Weddle's rule to each subintervals { } { } { } 0 6 6 12 6 , , , ,.... , n n x x x x x x - and applying Weddle's rule to the subrange 2 6 6 , j j x x - ? ? ? ? and summing over 1,2,3,...., , j m = we get ( ) ( ) 6 6 6 1 j j m x x j I f f x dx - = = ∑ ∫ ( ) ( ) ( ) ( ) ( ) ( ) 6 6 6 5 6 4 6 3 6 2 6 1` 1 3 5 6 5 10 m j j j j j j

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h f x f x f x f x f x f x - - - - - - = ? = + + + + + + ? ∑ ( ) ( ) ( ) 7 9 6 1 1 9 140 1400			

m m viii vi j j j j j h h f x f f = = ¢ ? - x - x ? ∑ ∑ ( ) 6 6 6 , , 1,2,...,
j j j j x x j m - ¢ > x x > = ( )
b c c w w a f x dx I E = + ∫
NSOU l CC-MT-05 95 where ( ) ( ) ( ) ( ) ( ) ( ) ( ) { } 0 1 5 7 5 1 3 5 ... 10 - - ? = + + + + + + ? c w
n
n n
h I

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f x f x f x f x f x f x f x ( ) ( ) ( ) { } 2 4 2 2 .... n f x f x f x - + + + + ( ) ( ) ( ) { } 3 9 3 6 .... n f x f x f x - + + + + (6.5.5) ( ) ( ) ( ) { } ( ) 6 12 6 2 .... 12 - ? + + + + 3 ? n f x f x f x			

n ( ) ( ) 7 9 1 1 9 840 8400
Î = = ¢ = - x -
x ∑ ∑
m m viii vi W j j j j nh nh E f f ( ) , ¢ > x x > a b (6.5.6) Example 6.5.1 : Evaluate 6 0 1 1 I dx x = + ∫ using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Weddle's rule. Also check by direct integration. Solution: Here, we have ( ) 1 ,0 6. 1 y f x x x = = £ £ +

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Divide the interval into six parts. So 6 0 1 6 h - = = Therefore, the values of 1 1 y x = + are: x 0 1 2 3 4 5 6 y = f (x) 1 0.5 1/3 1/4 1/5 1/6 1/7 (i) By Trapezoidal rule: ( ) ( ) 6 0 6 1 2 3 4 5 0 1 2 1 2 h dx y y y y y y y x ? ? = + + + + + + ? ? + ∫ ( ) ( ) 1 1 1 1 1 1 1 2 0.5 2 7 3 4 5 6 ? ? = + + + + + + ? ? ? ? = 2.021429 (ii) By Simpson's 1/3 rd rule: ( ) ( ) ( ) 6 0 6 1 3 5 2 4 0 1 4 2 1 3 h dx y y y y y y y x ? ? = + + + + + + ? ? + ∫ NSOU l CC-MT-05 96 ( ) ( ) ( ) 1 1 1 1 1 1 1 1 4 2 3 7 2 4 6 3 5 ? ? = + + + + + + ? ? ? ? = 1.9538730 (iii) By Weddle's rule ( ) ( ) 6 0 6 1 2 4 5 3 0 1 3 3 2 1 10 h dx y y y y y y y x ? ? = + + + + + + ? ? + ∫ ( ) ( ) ( ) 3 1 1 1 1 1 1 1 3 2 8 7 2 3 5 6 4 ? ? = + + + + + + ? ? ? ? = 1.952857			

By actual integration, ( ) 6 6 0 0 1 log 1 1 dx x x = + ? ? ? ? + ∫ log7 log1 = - 1.945910 = Example 6.5.2 :

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The velocity of a particle at distance from a point on its path is given in the table below: 0 10 20 30 40 50 60 / sec 47 58 64 65 61 52 38 sinmeter vinm Estimate the time to travel 60 meters by using Simpson's 1/3			

rd rule.

Solution: Here, we have 10. h = We know the . ds v dt = Hence, ds dt v = To find the time taken to travel 60 metres we have to evaluate 60 60 0 0 = ∫ ∫ ds dt v Let 1 , y v = then the table values of y for different values of s are given below 0 10 20 30 40 50 60 1 0.0213 0.0172 0.0156 0.0156 0.0164 0.0192 0.0263 = s y v
NSOU l CC-MT-05 97 By Simpson's 1/3d rule, ( ) ( ) ( ) 60 0 6 1 3 5 2 4 0 4 2 3 h yds y y y y y y y ? ? = + + + + + + ? ? ∫ ( ) ( ) ( ) 10 0.0213 0.0263 4 0.0172 0.0154 0.0192 2 0.0156 0.0164 3 = + + + + + + ? ? ? ? 1.0627 = Time taken to travel 60 meters is 1.0627 seconds. 6.6 Summary In this unit the numerical integration by using Newton-Cotes formula(closed type), Trapezoidal rule, Simpson's1/3 rd rule and Weddle's rule have been discussed and also the corresponding error terms are also studied. 6.7 Exercises 1. Define the degree of precision of mechanical quadrature formula. Show that the d.p. of trapezoidal is 1. 2. Deduce the trapezoidal, Simpson's 1/3 rd and Weddle's rules (without error) by integrating Newton's forward interpolation formula. 3. Evaluate 5 0 1 4 5 dx x + ∫ by Trapezoidal rule using 11 coordinate. Ans: 0.4055 4. find the value of 2 0 cosx dx p ∫

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by (i) Trapezoidal rule and (ii) Simpson's one- third rule		

taking n = 6. Ans: (i) 1.170 (ii) 1.187) 5. When a train is moving at 30m/sec steam is shut off and brakes are applied. The speed of the train per second after t seconds is given by ( ) ( ) 0 5 10 15 20 25 30 35 40 30 24 19.5 16 13.6 11.7 10.0 8.5 7.0 time t speed v Using Simpson's rule, determine the distance moved by the train in 40 sec. (Ans: 606.66 m.)
Unit 7 rrrrr Computer Language Structure 7.0 Objectives 7.1 Introduction 7.2 Concept of programming languages 7.3 Machine Language 7.4 Assembly Language 7.5 High Level Language 7.6 Interpreter 7.7 Compiler, Source and object program 7.8 Conclusion 7.9 Summary 7.10 Exercise 7.0

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Objectives After going through this unit one will be able to learn l the concept of		

programming languages, interpreter, compiler, source and object program. 7.1 Introduction We have seen that the hardware or physical parts that form a computer serve no purpose by themselves. To make a computer work, we must learn how to give instruction to it in a language that the computer will understand. 7.2 Concept of Programming Language In a natural language we speaks in, we use words to convey ideas and even
NSOU l CC-MT-05 99 emotions, feeling and sensations. A computer language is used to communicate with a machine which can react to only simple and very clear instructions conveyed through precise notations or words. The notations and words which can be used to give instructions to a computer and the rules which the instructions must obey form a computer language. The first set of computer language that developed were based upon the internal structure of the computer. These languages were referred to as codes or low level languages. Machine code and assembly code which used binary or mnemonic symbols were first set of languages that were developed for computers. 7.3 Machine Language A computer works on electricity and this enables it to receive and store information only in the form of electric pulses. If a pulse is present it codes it as 1 and if it is not present it codes it as 0. The computer's own language is, therefore, made up of the binary numbers 0 and 1 and is written in the form of a numeric code. This language is called machine language or code and is a part of a computer's electronic circuitry. When computers were first made, machine language was the only language. The utility of a machine language is that since it is written in the machine code itself, the computer processes it quickly. On the other hand, the number of people who can without difficulty a series of instruction using zeroes and ones must indeed be very few. It requires long term expertise to do this. Coding and decoding are tedious processes and prone to errors. Further, machine languages vary with the make of each computer and one may need to learn a new machine language each time one works on a different make of machines. 7.4 Assembly Language In the beginning, machine language was the only language. Then assembly language was developed. In an assembly language, 'mnemonics' (or alphanumeric codes) were used to substitute the binary machine coded to machine language. These 'mnemonics' were memory aids which helped the mind to relate things more easily. For example, mnemonics 'DIV' could be used to describe the operation 'divide'. Assembly language made it easier for the user to write his instructions. But the

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NSOU l CC-MT-05 100 'mnemonics' had to be translated to the computer into its binary pattern before the machine could do the job. The translation was done by a special pre-stored set of instructions called an assembler. The assembler was supplied by the computer manufacturer and usually embedded in ROM chips. The advantages of an assembly language are that it helps in reducing errors and the time involved in writing instructions. The drawbacks are that it requires the user to have a fair knowledge of hardware and being machine dependent, the instructions for one machine cannot be executed on another. 7.5 High Level Language In the initial phase of development, the use of computers was largely confined to a small group of scientists and computer specialists. With improvements in technology and fall in prices, there arose a need for languages that would permit even a non-expert to communicate with a computer. This led to the development of high level languages which enable a large number of people to use computer without having to know in detail its internal structure. These languages are user-centred and not machine-centred like the machine and assembly codes. A program written in high-level language can be run on different computers without any or much modifications. Instructions in high level languages are given using certain words from a natural language, such as English, an a few notations. Each word or notation in these languages have one precise meaning and we must adhere to the syntax or the set of grammar, punctuation and spelling rules for the language. Today, virtually all work is undertaken by writing instructions in one of the high level languages. The first high-level programming were designed in 1950s. Ada, Algo, LOGO, PILOT,BASIC, COBOL, C/C++,FORTRAN, Java, R, python etc. are popular examples of high-level languages. The computer does not directly understand a high level language. A translation is undertaken by specially prepared software called language processors or translators.

NSOU l CC-MT-05 101 7.6 Interpreter An interpreters translates one instruction at a time and gets it immediately executed. Each instruction is checked for errors and corrections are made when necessary. Interpreters do not involves much storage space but they require more time to execute. Basic, R, Python are Interpreter based language 7.7 Compiler, Source program and object program Compilers Compilers take all the instructions together and then compile them into the corresponding machine code. The user written program (referred to as the source Basis for comparison input Output orking mechanism Speed Memory Errors Error detection Pertaining Programming languages Compiler It takes an entire program at a time. It generates intermediate object code. The compilation is done before execution. Comparatively faster Memory requirement is more due to the creation of object code. Display all errors after compilation, all at the same time. Diffucult C, C++, C#, Scala, typescript uses compiler. interpreter It takes a single line of code or instruction at a time. It does not produce any intermediate object code. Compliation and execution take place simultaneously. Slower It requires less memory as it does not create intermediate object code. Displays error of each line one by one. Easier comparatively PHP, Perl, Python, Ruby uses an interpreter.

NSOU l CC-MT-05 102 program) is fed into the computer. The compiler translates the source program and produces a complete program in machine language known as the object program which is loaded into main memory for execution. Some basic comparison between Compiler and Interpreter is given in the form of the table given belos : 7.8 Conclusion Compiler and interpreter both are intended to do the same work but differ in operating procedure, Compiler takes source code in an aggregated way whereas Interpreter takes constituent parts of source code, i.e., statement by statement. Although both compiler and interpreter have certain advantages and disadvantages like Interpreted languages are considered as cross-platform, i.e., the code is portable. It also doesn't need to compile instruction previously unlike compiler which is time- saving. Compiled languages are faster regarding compilation process. 7.9 Summary In this unit the concept of programming language like machine language, assembly language, High level language is discussed. Also the difference between interpreter and compiler as well as the source and object program also discussed 7.10 Exercise 1) What do you understand by Machine language? 2) How the machine language differ from the assembly language? 3) Define the object and source program. 4) Write the difference between Interpreter and compiler.

NSOU l CC-MT-05 103 Unit 8 rrrrr Number System Structure 8.0 Objectives 8.1 Introduction 8.2 Decimal Number System 8.3 Binary Number System 8.4 Octal Number System 8.5 Hexadecimal 8.6 Conversion 8.7 Summary 8.8 Exercise 8.0 Objectives After going through this unit one will be able to learn l different types of number systems and their conversion from one system to another system. 8.1 Introduction We have heard of number systems like the whole numbers, the real numbers etc. But in the context of computer awareness, we define other types of number systems like the binary number system, the decimal system, the hexadecimal system and others. We will discuss the binary number system and others and how we can convert from one number system to the other. The value of any digit in a number can be determined by -The digit -Its position in the number -The base of the number system

NSOU l CC-MT-05 104 Let r be the base of a number system. Then to represent any given integer number, say D, symbolically in this system, we use r number of different characters, namely ( ) ( ) 0 1 2 ... 2 1 r r > > > > - > - and represent D uniquely as ( ) 1 2 3 2 1 0 .... n n n n D d d d d d d d - - - = ± (8.1) According as the number is positive or negative, where n is a positive integer and each d i ranges from ( ) 0 1 , to r - such that 0, n d 1 ( ) ( ) 0 1 , 0,1,2,... 1 i d r i n £ £ - = - The magnitude of the number will be given by ( ) ( ) ( ) ( ) ( ) 1 2 1 0 1 2 1 0 . . .. n n n n D d r d r d r d r d r - - = + + + + + 8.2 Decimal Number System The most commonly used number system is Decimal Number System with base 10. In this system, the ten basic characters that are used to represent number are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Thus in decimal number system the (n+1) digit number D represented by (8.1) has the magnitude ( ) ( ) ( ) ( ) ( ) 1 2 1 0 1 2 1 0 . 10 . 10 .... . 10 . 10 . 10 n n n n d d d d - - + + + + + For example, the decimal number represented by the symbol 4356 has the magnitude ( ) ( ) ( ) ( ) 3 2 1 0 4356 4. 10 3. 10 5. 10 6. 10 = + + + For a fractional number whose magnitude is less than 1, the symbolic representation starts with dot (.), called the decimal point, and the powers of the base will be negative from -1. For example, 1 2 83 .83 8 10 3 10 100 - - = = ´ + ´ Thus 2 1 0 1 2 607.03 6 10 0 10 7 10 0 10 3 10 - - = ´ + ´ + ´ + ´ + ´ Exercise 8.2.1 : Write i) 22 , 5 7 in decimal number system. 8.3 Binary Number System In binary number system, the base is 2 and the symbols used for representing a number are 0 and 1. Thus the number 110101 in binary system is equivalent to

NSOU l CC-MT-05 105 5 4 3 2 1 0 1 2 1 2 0 2 1 2 0 2 1 2 ´ + ´ + ´ + ´ + ´ = 32+16+0+4+0+1 = 53 in decimal system. Using the respective radix as subscript, we write this result as: ( ) ( ) 2 10 110101 53 . = Just like decimal point, we also have binary point as: ( ) 3 2 1 0 1 2 3 2 1101.011 1 2 1 2 0 2 1 2 0 2 1 2 1 2 - - - = ´ + ´ + ´ + ´ + ´ + ´ = 8+4+0+1+0+.25+.125 ( ) 10 13.375 = Binary numbers play a vital role in the design of digital computers. Exercise 8.3.1 : Write ( ) 2 .1011 to decimal number system. 8.4 Octal Number System Here the base is 8 and eight different symbols are 0, 1, 2, 3, 4, 5, 6 and 7. Thus a number ( ) 8 7032 in octal system is equivalent to 3 2 1 0 7 8 0 8 3 8 2 8 ´ + ´ + ´ + ´ = 3584 + 24 + 2 ( ) 10 3610 = Again ( ) 1 0 1 2 8 71.34 7 8 1 8 3 8 4 8 - - = ´ + ´ + ´ + ´ = 56 + 1+ 0.375 = 0.0625 ( ) 10 57.4375 = 8.5 Hexadecimal Number System The base is 16 and the required symbols to represent a number in this system are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F. The symbols A, B, C, D, E and F represent the decimal number 10, 11, 12, 13, 14 and 15 respectively. The number ( ) 3 2 1 0 16 BC6A 11 16 12 16 6 16 10 16 = ´ + ´ + ´ + ´ = 45056 + 3072 + 96 + 10 ( ) 10 48234 =

NSOU l CC-MT-05 106 The symbol 0 and 1 are generally called BIT – the bit at the extreme left having the highest positional value is the Most Significant Bit (MSB) while the bit occupying the extreme right position having least positional value is called the Least Significant Bit (LSB) 8.6 Conversion Conversion of binary to decimal: The decimal equivalent of a binary number is obtained by expanding it according to the place-value of each bit. Exercise : Obtain the decimal equivalent of the following numbers: i) 11011 ii) 10010 iii) 0.01101 Ans: i) ( ) 10 27 , ii) ( ) 10 28 , iii) ( ) 10 0.40625 . Conversion from decimal to binary: There are several methods of converting a decimal number to its binary equivalent. The most commonly used methods are (i) Expansion Method and (ii) Division and Multiplication Method. Expansion Method: The given decimal number is first expressed as summation terms each of which is a power (positive integral and negative integral) of 2. Example 8.6.1 : Convert the decimal numbers (i) 47 (ii) 195 (iii) 88.5625 to their binary equivalents: Solution: (i) (47) 10 = 32 + 15 = 32 + 8 + 7 = 32 + 8 + 4 + 3 = 32 + 8 + 4 + 2 + 1 5 3 2 1 0 2 2 2 2 2 = + + + + 5 4 3 2 1 0 1 2 0 2 1 2 1 2 1 2 1 2 = ´ + ´ + ´ + ´ + ´ + ´ ( ) 2 101111 = (ii) (195) 10 = 128 + 64 + 2 + 1 7 6 1 0 2 2 2 2 = + + + 6 5 4 3 2 1 1 2 0 2 1 2 1 2 0 2 0 2 = ´ + ´ + ´ + ´ + ´ 1 0 1 2 1 2 + ´ + ´ ( ) 2 11000011 = (iii) ( ) 10 88.5625 = 64 + 16 + 8 + 0.5 + 0.0625 6 4 3 1 4 2 2 2 2 2 - - = + + + + 6 5 4 3 2 1 1 2 0 2 1 2 1 2 0 2 0 2 = ´ + ´ + ´ + ´ + ´

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NSOU l CC-MT-05 107 0 1 2 3 4 0 2 1 2 0 2 0 2 1 2 - - - - + ´ + ´ + ´ + ´ + ´ = ( ) 2 1011000.1001 = Division and Multiplication Method: The above method is laborious and not suitable for large numbers. We may however use the division and multiplication method which is described as follows: The decimal number has both an integral and fractional part, then we first convert the integral part to its binary equivalent by the division method. The fractional part must next be converted by multiplication process and the two results should be linked up after that. For decimal integral: The given decimal integer is repeatedly divided by the base 2 of the binary number system. The remainder (which is either 0 or 1) is noted in each division. The process continues till the quotient is zero. The first remainder is the least significant bit and the last one is the most significant bit. Thus the binary equivalent is obtained by writing down the remainder in the reversed order, i.e. from bottom to upward. Example 8.6.2 : Convert ( ) 10 47 to binary equivalent. Solution: 2 47 2 23 1 ¬ LSB 2 11 1 2 05 1 2 02 1 2 01 0 - 00 1 ¬ MSB Thus ( ) ( ) 10 2 47 101111 = For decimal fraction: The given decimal fraction is multiplied by 2, the fractional part is again multiplied by 2 and the process is repeated till the fraction part of the product is zero. The integral part obtained each time, which can be either 0 or 1, is taken in top to bottom order and arranged from left to right to provide the binary equivalent to the decimal number.

NSOU l CC-MT-05 108 Example 8.6.3 : Convert the following decimal fractions to its binary equivalent ( ) 10 .37 Solution : The result of repeated multiplication is shown below Multiplication Integral Part Fractional Part Binary Position 0.375 × 2 = 0.75 0 ¯ 0.75 0 × 2 −1 0.75 × 2 = 1.50 1 0.50 1 × 2 −2 0.5 × 2 = 1.00 1 0.00 1 × 2 −3 Thus the equivalent binary fraction is ( ) ( ) 10 2 .375 .011 = Exercise 8.6.4 : Convert the decimal fractions to its binary equivalent ( ) 10 .435 Example 8.6.5 : Convert ( ) 10 47.375 to binary equivalent. Solution: As we have already done the binary equivalent of the integral part ( ) ( ) 10 2 47 101111 = and the decimal fraction to binary is ( ) ( ) 10 2 .375 .011 = Linking the two results, we have ( ) ( ) ( ) ( ) 10 10 2 2 47 .375 101111 .011 + = + Or, ( ) ( ) 10 2 47.375 101111.011 = Conversion of decimal number to octal: The conversion method follows similar rules as in the case of binary number system. Here we divide the number by the base 8 instead of 2. It will clear in the following example Example 8.6.6 : i) Convert ( ) 10 347 to octal equivalent. Solution: 8 347 8 43 3 ¬ LSB 8 05 3 - 8 00 5 ¬ MSB Therefore ( ) ( ) 10 8 347 533 = ii) Convert ( ) 10 0.30 to octal equivalent.

NSOU l CC-MT-05 109 Solution: Multiplication Integral Part Fractional Part Binary Position 0.30 × 8 = 2.40 2 ¯ .40 2 × 8 −1 0.40 × 8 = 3.20 3 .20 3 × 8 −2 0.20 × 8 = 1.60 1 .60 1 × 8 −3 0.60 × 8 = 4.80 4 .80 4 × 8 −4 0.80 × 8 = 6.40 6 .40 6 × 8 −5 0.40 × 8 = 3.20 3 .20 3 × 8 −6 (Recurring Starts) Hence ( ) ( ) 10 8 0.30 .23146 = Conversion of binary number to octal: The base of the octal system is 8 or (2x2x2). Thus the octal base 8 is a power of the base 2 in the binary system. A binary number is converted to its octal equivalent by grouping of three successive bits starting from the least significant bit or the right-most digit. Example 8.6.7 : Convert ( ) 2 10101111011 to octal. Solution: Three successive bits of the binary string are grouped from the right. Binary: 010 101 111 011 Octal equivalent: 2 5 7 3 Hence ( ) 2 10101111011 ( ) 8 2573 = Note: A non-significant '0' has been added in the left-most group to make it a string of 3 bits. This is only for convenience of grouping. Conversion of octal number to binary: The octal equivalent of binary number may be found through the same process of referring to the conversion table and arranging the bits in order. Example 8.6.8 : Convert ( ) 8 412 to binary Solution: We have: 4 1 2 (in Octal) = 100 001 010 (in Binary)

NSOU l CC-MT-05 110 Arranging in order, we get ( ) ( ) 8 2 412 100001010 = Exercise: Convert (i) ( ) 2 1110101110 (ii) ( ) 2 10.11 (iii) ( ) 2 1011.1011011 to their octal equivalent. Ans: (i) ( ) 8 1656 , (ii) ( ) 8 2.6 , (iii) ( ) 8 13.554 Conversion from decimal system to hexadecimal system: The procedure for conversion from decimal to hexadecimal is same as that of octal. Here in this case repeated divisions is by 16. Example 8.6.9 : Convert (116) 10 to hexadecimal. Solution: 16 116 16 7 4 16 0 7 Hence ( ) ( ) 10 16 116 74 = Conversion method from binary to system to hexadecimal system is similar to octal but here instead of grouping by 3-bits, we arrange the binary string in groups of 4-bits Example 8.6.10 : Convert ( ) 2 111001 to hexadecimal. Solution: ( ) ( ) ( ) 2 2 16 : 111001 00111001 39 = = Example 8.6.11 : Convert i) ( ) 16 748 A and (ii) ( ) 2 16 . 4 BA C to binary number system. Solution: i) ( ) ( ) 16 2 748 1010011101001000 A = (ii) ( ) ( ) 16 2 2. 4 101110100010.11000100 BA C = -----

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NSOU l CC-MT-05 111 8.7 Summary In this unit, the detailed study of Number system like decimal, binary, octal, hexadecimal and their conversion from one system to other have been studied with proper examples. 8.8 Exercises 1. What do you understand by binary number system? How it is differ from decimal number system? 2. Convert the following decimal numbers into its binary equivalents: a) ( ) 10 131 b) ( ) 10 395 c) ( ) 10 423.25 Ans : (a) ( ) 2 10000011 (b) ( ) 10 395 (c) ( ) 10 423.25 3. Convert the following binary numbers to its decimal equivalent: (a) ( ) 2 11001 , (b) ( ) 2 11.01 , (c) ( ) 2 10.011 Ans : (a) ( ) 10 25 (b) ( ) 10 3.25 , (c) ( ) 10 2.375 4. Convert the following decimal numbers into its octal and hexadecimal equivalents: (a) ( ) 10 231 (b) ( ) 10 153 Ans : (a) ( ) 8 347 ( ) 16 7 , E (b) ( ) 8 231 , ( ) 16 99 . 5. Convert the following octal numbers into its binary equivalents: (a) ( ) 8 346 (b) ( ) 8 135 Ans : (a) ( ) 2 1100110 (b) ( ) 2 1011101 . 6. Convert the following hexadecimal numbers into its binary equivalents: (a) ( ) 16 4 5B (b) ( ) 16 3A BF Ans : (a) ( ) 2 10010110110 (b) ( ) 2 1010001110111111 .
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is approximated as 3.14, find the absolute error, relative error and relative percentage error.

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is present in the statement of the problem itself, before determining its solution.

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6/158	SUBMITTED TEXT	23 WORDS	94%	MATCHING TEXT	23 WORDS

mathematical modelling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the proposed problem.

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7/158	SUBMITTED TEXT	20 WORDS	85%	MATCHING TEXT	20 WORDS

precision refers to the number of decimal positions, i.e. the order of magnitude of the last digit in a value.

Precision refers to the number of decimal position, i.e. the order of magnitude of last digit in a value.

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8/158	SUBMITTED TEXT	26 WORDS	53%	MATCHING TEXT	26 WORDS

rounding off: To round off a number to n significant digits first truncate it to n digits: if truncated part is less than half a unit

rounding off: To round off a number to n-significant digits, discard all digits the right of the n th digit if this discarded number is: - Less than half a unit

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9/158	SUBMITTED TEXT	68 WORDS	65%	MATCHING TEXT	68 WORDS

Example 1.3.4 : Round off the following numbers, to four significant digits i) 23.4251 ii) 32.4250 iii) 24.87500 iv) 19.995 v) 437.261 vi) 19.36235 Solution: i) 23.43 ii) 32.42 iii) 24.88 iv) 20.00 v) 437.3 v) 19.36 Example 1.3.5 : Round off the number 54762 to four significant digits and

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10/158	SUBMITTED TEXT	21 WORDS	50%	MATCHING TEXT	21 WORDS

the number of terms of the exponential series such that their sum gives the value correct to six decimal places

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11/158	SUBMITTED TEXT	27 WORDS	38%	MATCHING TEXT	27 WORDS

x n x x x e x R x n - = + + + + + - Where ( ) , 0 . ! n n x R x e x n q = > q > Maximum absolute error (at) ! n x x x e n

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12/158	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

If 0.333 is the approximate value of 1 , 3 find absolute, relative and percentage

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13/158	SUBMITTED TEXT	24 WORDS	69%	MATCHING TEXT	24 WORDS

of the equation ( ) 0 f x = are the values of x where the graph of ( ) y f x = meets the x-axis.

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14/158	SUBMITTED TEXT	23 WORDS	52%	MATCHING TEXT	23 WORDS

the location of the roots of the equation f (x) = 0. As an example, we consider the equation 2 1 0. x x+ - =

the location of the root of the equation f (x) = 0. 3. Compute the root of the equation x 3 − 9x + 1 = 0

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15/158	SUBMITTED TEXT	29 WORDS	62%	MATCHING TEXT	29 WORDS

x- - = we rewrite the equation as 3 4 2. x x = + The graphs 3 and 4 2 y x y x = = - are

x), we may rewrite the equation f (x) = 0, as f 1 (x) = f 2 (x), where the graphs of y = f 1 (x) and y = f 2 (x) are

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16/158	SUBMITTED TEXT	22 WORDS	83%	MATCHING TEXT	22 WORDS

Theorem 2.1.1 : If f (x) is continuous in the interval (a, b) and if f (a) and f (b)

Theorem 1.1: If f (x) is continuous in an interval (a, b), and f (a) and f(b)

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17/158	SUBMITTED TEXT	13 WORDS	70%	MATCHING TEXT	13 WORDS

Then the x-co-ordinate of the point of intersection of the graphs give the

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18/158	SUBMITTED TEXT	19 WORDS	88%	MATCHING TEXT	19 WORDS

Example 2.1.2 : Find the location of the roots of the equation 2 1 0. x x+ - = Solution:

Example 1.1: Find the location of the root of the equation x x Solution:

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19/158	SUBMITTED TEXT	16 WORDS	78%	MATCHING TEXT	16 WORDS

at least one real root of the equation f (x) = 0 lies within the interval (

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20/158	SUBMITTED TEXT	23 WORDS	65%	MATCHING TEXT	23 WORDS

interval { } ,a b and the equation ( ) 0 f x = has at least one root on { } , . a b

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21/158	SUBMITTED TEXT	21 WORDS	91%	MATCHING TEXT	21 WORDS

Submitted text	Matching text
The equation ( ) 0 f x = can be written in the form ( ) ( ) ...... 1 x x =	The equation f(x) = 0 can be written in the form g(x) = h(x)

W https://nmot3ee1104.files.wordpress.com/2014/07/nmot_ho-1-15.pdf

22/158	SUBMITTED TEXT	32 WORDS	47%	MATCHING TEXT	32 WORDS

x x = j and then the successive approximations are calculated as: ( ) 2 1 , x x = j ( ) 3 2 ,......, x x = j ( ) ( ) 1 .... 3 n n x x + =

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23/158	SUBMITTED TEXT	136 WORDS	23%	MATCHING TEXT	136 WORDS

x ¢ x - = j x -j = x - j e where 0 1 x > e > x ( ) ( ) ( ) 2 1 1 2 x x x ¢ x- = j x -j = x- j e where 0 2 x > e > x .... ( ) ( ) ( ) 1n n n n x x x + ¢ x- = j x -j = x- j e where 0 n x > e > x Thus, ( ) ( ) ( ) ( ) ( ) 1 2 1 0 .... n n n n x x x e e e + ¢ ¢ ¢ ¢ x- = x- j e = x- j j j Assuming , ( ) x¢j > r in 0 0 a x b £ £ we have NSOU l CC-MT-05 20 1 0 n n x x + x- £ x- r Thus, 1 0 lim lim 0 + ®¥ ®¥ x- £ x- r ® n n n n x x

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24/158	SUBMITTED TEXT	72 WORDS	30%	MATCHING TEXT	72 WORDS

x x = j and ( ) 1 . n n x x + = j Therefore, ( ) ( ) ( ) 1 1 , n n n x x x c - - ¢ x- = j x -j = x - j , where 1n x c - > > x 1 , n l x - £ x- ,[where, ( ) 1] c l ¢j £ > { } 1- £ x- + - n n n l x x x After rearrangement, this relation becomes 1 1 0 1 1 n n n n l l x x x x x

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25/158	SUBMITTED TEXT	13 WORDS	100%	MATCHING TEXT	13 WORDS

The convergence of an iteration method depends on the suitable choice of the

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26/158	SUBMITTED TEXT	24 WORDS	78%	MATCHING TEXT	24 WORDS
f a f b > then $ at least one real root of the equation ( ) 0, f x = between a and b.			f (a) f (b) > 0, then there exists at least one real root of f (x) = 0 between a and b.		

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27/158	SUBMITTED TEXT	93 WORDS	56%	MATCHING TEXT	93 WORDS

x 1 x 1 x 0 x 0 x 3 x 1 x 0 x x 2 4 x 0 x 2 x 3 x 1 x 2 y x = y x = y x = y x = x x x x x x x x x y x ( ) = f y x ( ) = f y x ( ) = f y x ( ) = f f x ( ) f x ( ) f x ( ) f x ( ) 0 ( ) 1 ¢ > f x > 1 ( ) 0 ¢ - > f x > ( ) 1 ¢ f x > ( ) 1 ¢ f x < (

SA S41641 Mathematics 06.pdf (D164869290)

28/158	SUBMITTED TEXT	56 WORDS	69%	MATCHING TEXT	56 WORDS
Example 2.3.1 : Find a root of the equation 2 7 0 x x+ - = by bisetion method, correct up to two decimal places. Solution. Let ( ) 2 7. f x x x = + - ( ) 2 1 0 f = - > and ( ) 3 5 0. f = < So, a root lies between 2 and 3.			Example: Find a root of the equation x 3 – 4x – 9 = 0, using the bisection method correct to three decimal places. Solution: Let f(x) = x 3 – 4x – 9 Since f(2) is –and f(3) ve, a root lies between 2 and 3. ?		

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29/158	SUBMITTED TEXT	16 WORDS	96%	MATCHING TEXT	16 WORDS

regula falsi method. This method was developed because the bisection method converges at fairly slow speed.

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30/158	SUBMITTED TEXT	16 WORDS	66%	MATCHING TEXT	16 WORDS

Method This method is also known as method of false position, Method of chords, method of linear interpolation.

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31/158	SUBMITTED TEXT	21 WORDS	91%	MATCHING TEXT	21 WORDS

the chord joining the points ( ) ( ) ,a f a and ( ) ( ) , . b f b

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32/158	SUBMITTED TEXT	12 WORDS	87%	MATCHING TEXT	12 WORDS

the point of intersection of the chord and the x-axis is

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33/158	SUBMITTED TEXT	49 WORDS	59%	MATCHING TEXT	49 WORDS

x a= and 1 . x b= The equation of the chord joining the points ( ) ( ) 0 0 ,x f x and ( ) ( ) 1 1 ,x f x is ( ) ( ) ( ) ( ) 0 0 0 1 0 1 ...... 1 y f x x x x x f x f x - - - = - - To find the point of

SA TNOU editing.docx (D111654607)

34/158	SUBMITTED TEXT	64 WORDS	57%	MATCHING TEXT	64 WORDS

f x x x x x f x f x - - - = - - Therefore, ( )( ) ( ) ( ) ( ) 0 1 0 2 0 1 0 ..... 2 f x x x x x f x f x - = - - This is the second approximation of the root. Now if ( ) 2 f x and ( ) 0 f x are opposite signs then the root lies between 0 x and 2 x and replace 1 x by 2 x in (2). Then the next approximation

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35/158 **SUBMITTED TEXT** 63 WORDS **57%** **MATCHING TEXT** 63 WORDS

f x x x x x f x f x If ( ) 2 f x and ( ) 1 f x are opposite signs then the root lies between 1 x and 2 x and the new approximation is obtained as: ( )( ) ( ) ( ) 2 1 2 3 2 1 2 - = - - f x x x x x f x f x The procedure is repeated till the root is obtained to the desired accuracy. If the

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36/158 **SUBMITTED TEXT** 13 WORDS **64%** **MATCHING TEXT** 13 WORDS

n n n n n n n f a b a x a f b f a + - = - -

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37/158 **SUBMITTED TEXT** 36 WORDS **37%** **MATCHING TEXT** 36 WORDS

n n n n n n n f a b a x a f b f a + - = - - or ( )( ) ( ) ( ) ( ) 1 .... 4 n n n n n n n n f b b a x b f b f a + - = - - Since, n n x

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38/158 **SUBMITTED TEXT** 112 WORDS **28%** **MATCHING TEXT** 112 WORDS

n n n n n n n f x b a x x f b f a + - = - - Or, ( ) ( ) ( ) ( ) ( )( ) 1+ - - = - n n n n n n n n x x f b f a f x b a Or, ( )( ) ( ) ( )( ) 1+ ¢ - - a = - n n n n n n n n n x x b a f f x b a when > a > n n n a b Or, ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 , + ¢ ¢ ¢ ? ? a - - a - a = - a = -a a ? ? n n n n n

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39/158 **SUBMITTED TEXT** 59 WORDS **72%** **MATCHING TEXT** 59 WORDS

Example 2.4.1 : Find a root of the equation 3 2 2 0 x x+ - = using Regula-Falsi method, correct up to three decimal places. Solution. Let ( ) 3 2 2. f x x x = + - ( ) 0 2 0 f = - > and ( ) 1 1 0. f = < Thus, one root lies between 0 and 1.

Example: Find a root of the equation x 3 − 4x − 9 = 0, using the bisection method correct to three decimal places. Solution: Let f(x) = x 3 − 4x − 9 Since f(2) is −and f(3) ve, a root lies between 2 and 3. ?

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40/158 **SUBMITTED TEXT** 43 WORDS **30%** **MATCHING TEXT** 43 WORDS

a > a > a n n n x Max x NSOU l CC-MT-05 27 Or, ( ) ( ) ( ) ( ) ( ) ( ) 1 , + ¢ ¢ ¢ a - a a - = a - ¢ a n n n n f f x x f where ( ) 0 0 , ... 5 ¢ > a a > n n

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41/158 **SUBMITTED TEXT** 67 WORDS **60%** **MATCHING TEXT** 67 WORDS

f x f x h f x hf x ¢ = + = + + = since 1 x is a root of ( ) 0 f x = Neglecting the second and the higher order derivatives, the above equation reduces to- ( ) ( ) 0 0 0 f x hf x ¢ + = Or, ( ) ( ) 0 0 = - ¢ f x h f x Therefore, ( ) ( ) ( ) 0 1 0 0 0 ...... 1 f x x x h x f x = + = - ¢

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42/158 **SUBMITTED TEXT** 17 WORDS **100%** **MATCHING TEXT** 17 WORDS

f x h f x Therefore, ( ) ( ) 1 2 1 1 1 1 f x x x h x f x = + = - ¢

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43/158 **SUBMITTED TEXT** 7 WORDS **95%** **MATCHING TEXT** 7 WORDS

n n n n f x x x f x + = - ¢ This

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44/158 **SUBMITTED TEXT** 41 WORDS **60%** **MATCHING TEXT** 41 WORDS

f x f x f x x f x ¢ ¢¢ - ¢j = - ¢ NSOU l CC-MT-05 31 i.e. ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 2 1, . . f x f x i e f x f x f x f x ¢¢ ¢ ¢¢ > < ¢

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45/158 **SUBMITTED TEXT** 44 WORDS **63%** **MATCHING TEXT** 44 WORDS

root of the equation 3 1 0. x x+ - = Solution. Let ( ) 3 1. f x x x = + - Then ( ) 0 1 0 f = - > and ( ) 1 1 0. f = < So one root lies between 0 and 1.

root of the following equation by bisection x 3 – 7x +5 = 0 Solution: Let f(x) = x 3 – 7x +5, f(0) = 5, f(1) = -1 ? Root lies between 0 and 1,

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46/158	SUBMITTED TEXT	109 WORDS	22%	MATCHING TEXT	109 WORDS

n n n n f x x x f x + = - ¢ Let, . n n x = e +x Then from the above relation we get- ( ) ( ) 1+ e +x e +x = e +x- ¢ e +x n n n n f f Or, ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 1 2 ..... 2 ..... 2 n n n n n n f f f f f f + e ¢ ¢¢ x +e x + x + e = e - e ¢ ¢¢ ¢¢ x +e x + x + Or, ( ) ( ) ( ) ( ) 2 1 .... 2 1 ..... + ¢¢ e x e + + ¢ x e = e - ¢¢ x +e + ¢ x n n n n n f f f f Or, ( ) ( ) ( ) ( ) 2 1 .... 1 .... 2 + ? ? ¢¢ ¢¢ x x ? ? e e = e - e + + -e + ? ? ? ? ? ? ¢ ¢ x x ? ? ? ? n n n n n

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47/158	SUBMITTED TEXT	54 WORDS	30%	MATCHING TEXT	54 WORDS

x a- = Let ( ) . k f x x a = - The iteration scheme is ( ) ( ) 1 n n n n f x x x f x + = - ¢ or, 1 1 1 k k k n n n n n k k n n x a kx x a x x kx kx + - - - - + = - = NSOU l CC-MT-05 33 ( ) 1 1 1 . n k n a k x k x - ? ? = - + ? ? ? ? ? ? ? 2.6

x 2 – a = 0. Now, using f (x) = x 2 – a in the Newton-Raphson iterative n n n n x a x x x We have, 2 1 2 n n n n x a x x x 2 1 2 n n n n x a x x x i.e., 1 1 , n n n a x x

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48/158	SUBMITTED TEXT	29 WORDS	76%	MATCHING TEXT	29 WORDS

n n n n f x x x f x + = - ¢ 3 3 2 2 1 2 1 . 3 1 3 1 + - + = - = + + n n n n n n x x x x x x The sequence { } n x

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49/158	SUBMITTED TEXT	36 WORDS	86%	MATCHING TEXT	36 WORDS

n n n n n nn n n a x a x a x b a x a x a x b a x a x a x b ? + + + = ? + + + = ? ? ? ? ? + + + = ? ? (3.1.1)

n n n n n n n a a x x a b x a x a x a x a b x a x a x a x b

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50/158	SUBMITTED TEXT	24 WORDS	77%	MATCHING TEXT	24 WORDS

equation of the form 1 1 2 2 .... n n a x a x a x b + + + = where 1 2 , ,...., n a a a

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51/158	SUBMITTED TEXT	54 WORDS	51%	MATCHING TEXT	54 WORDS

x x x x x x + = ? ? - = ? ? - = ? (b) 1 2 1 2 1 2 2 3 2 5 2 4 x x x x x x + = ? ? - = ? ? - = ? (c) 1 2 1 2 1 2 2 3 4 2 6 6 3 9 x x x x x x + = ? ? - = ? ? - = ? have no solution, a unique solution, and infinitely many solutions, respectively. See Figure1. x 1 x 1 x 1 x 2 x 2 x 2

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52/158	SUBMITTED TEXT	17 WORDS	95%	MATCHING TEXT	17 WORDS

x x x x x x x x x + - = ? ? - + = - ? ? + + = ? The corresponding

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53/158	SUBMITTED TEXT	109 WORDS	37%	MATCHING TEXT	109 WORDS

n n n n n n nn n n n nn a a a x a a a a a a x a a a x D x a a a x a a a = = 11 1 12 2 1 12 1 21 1 22 2 2 22 2 1 1 2 2 ... ... 2 ... + + + = + + + + + + n n n n n n n n n nn n nn a x a x a x a a a x a x a x a a a x a x a x an a [Using operation 1 1 2 2] ¢ = + + +⋯ n n C C x C x C 1 12 1 2 22 2 2 ... ... n n n n nn b a a

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54/158	SUBMITTED TEXT	35 WORDS	82%	MATCHING TEXT	35 WORDS
n n n n n nn n n a x a x a x b a x a x a x b a x a x a x b + + + = + + + = + + + = (3.2.1) has a				n n n n n n n a a x x a b x a x a x a x a b x a x a x a x b x a	

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55/158	SUBMITTED TEXT	25 WORDS	70%	MATCHING TEXT	25 WORDS
Multiply the 1st equation of (1) by ( ) ( ) 1 1 1 1 11 i i m a a = - and add to the ith equation				multiply the first Equation (2.8(a)) by 3 11 31 / m a a , and add to the last Equation (2.8(	

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56/158	SUBMITTED TEXT	34 WORDS	47%	MATCHING TEXT	34 WORDS

n n ni n ni nn a a a b a a a a a b a a D i n a a a b a a - + - + - + = = Inverse of a Matrix From the theory of

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57/158	SUBMITTED TEXT	17 WORDS	100%	MATCHING TEXT	17 WORDS

Submitted: is a system of 1 n- linear equations in the 1 n- unknowns 2 3 , ,....

Matching: is a system of 'n' linear equations in the 'n' unknowns

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58/158	SUBMITTED TEXT	45 WORDS	86%	MATCHING TEXT	45 WORDS

n n a x a x a x b + + + = ( ) ( ) ( ) 2 2 2 2 22 2 2 .... n n a x a x b + + = (3.2.2) .... ( ) ( ) ( ) 2 2 2 2 2 ... nn n n n a x a x b + + =

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59/158	SUBMITTED TEXT	60 WORDS	82%	MATCHING TEXT	60 WORDS

n n a x a x a x b + + = (3.2.4) ( ) ( ) ( ) 2 2 2 2 22 2 2 ..... n n a x a x b + + = ( ) ( ) ( ) 3 3 3 2 33 3 3 ... n n a x a x b + + = ... ( ) ( ) ( ) 3 3 3 2 3 .... nn n n n a x a x b + + =

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60/158	SUBMITTED TEXT	36 WORDS	87%	MATCHING TEXT	36 WORDS

Submitted: Example 3.2.1 : Solve the eqations by Gauss elimination method. 1 2 3 2 4, x x x + + = 1 2 3 2 2, x x x - + = 1 2 3 2 2 3. x x

Matching: Example 2.3: Solve the following by Gauss elimination method: 1 2 3 1 2 3 1 2 2 0 2 2 3 3 3 2 x x x x x x x x x

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61/158	SUBMITTED TEXT	129 WORDS	68%	MATCHING TEXT	129 WORDS

system of equations at the ( ) 1 n thstep - ( ) ( ) ( ) ( ) 1 1 1 1 1 2 11 12 1 1 .... n n a x a x a x b + + = (3.2.6) ( ) ( ) ( ) 2 2 2 2 22 2 2 ..... n n a x a x b + + = ( ) ( ) ( ) 3 3 3 2 33 3 3 ... n n a x a x b + + = ... ( ) ( ) n n nn n n a x

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62/158	SUBMITTED TEXT	23 WORDS	97%	MATCHING TEXT	23 WORDS

Solution. Multiplying the second and third equations by 2 and 1 respectively and subtracting them from first equation we get 1 2 3 2 4

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63/158	SUBMITTED TEXT	79 WORDS	33%	MATCHING TEXT	79 WORDS

x x + + = 2 3 3 3 0 x x- = 2 3 2 1. x x - + = Multiplying third equation by −3 and subtracting from seond equation we obtain 1 2 3 2 4 x x x + + = 2 3 3 3 0 x x- = 3 3 3. x = From the third equation 3 1, x = from the second equations 2 3 1 x x= = and from the first equation 1 2 3 2 4 2 x x x = - - =

SA Seema Maths.pdf (D98502513)

64/158	SUBMITTED TEXT	42 WORDS	77%	MATCHING TEXT	42 WORDS

n n n n nn a a a a a a a a a ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 ... n b b b ? ? ? ? ? ? ? ? ? ? ? ? ? 1 0 ... 0 0 1 ... 0 ... ... 0 0 ... 1 ? ? ? ? ? ? ? ? ? ? ? ? ....(3.3.1)

SA nm full book.pdf (D31630497)

65/158	SUBMITTED TEXT	29 WORDS	100%	MATCHING TEXT	29 WORDS

the following system of equations: 1 2 3 1 2 3 1 2 3 2 3 12 3 2 24 2 3 36 x x x x x x x x x + + = ? ? + + = ? ? + + = ? (3.3.3)

SA Numerical Analysis Dr RSM.pdf (D144415232)

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66/158	SUBMITTED TEXT	39 WORDS	71%	MATCHING TEXT	39 WORDS

Solve the following system of equations by Gauss elimination method (use partical pivoting). 2 3 2 5 x x+ = 1 2 3 2 4 11 x x x + + = 1 2 3 3 5 12. x x x - + - = -

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67/158	SUBMITTED TEXT	30 WORDS	92%	MATCHING TEXT	30 WORDS

Solution. The largest element (the pivot) in the coefficients of the variable 1 x is −3, attained the third equation. So we interchange first and third equations 1 2 3 3 5 12

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68/158	SUBMITTED TEXT	58 WORDS	46%	MATCHING TEXT	58 WORDS

x x x - + - = - 1 2 3 2 4 11 x x x + + = 2 3 2 5. x x+ = Multiplying the second equation by 3 and adding with the first equation we get, 1 2 3 3 5 12 x x x - + - = - 2 3 3 x x+ = 2 3 2 5 x x+ = The

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69/158	SUBMITTED TEXT	69 WORDS	48%	MATCHING TEXT	69 WORDS

x - + - = - 2 3 3 x x+ = 3 2. x- = - Now by back substitution, the values of 3 2 1 , , x x x are obtained as ( ) 3 2 3 1 2 3 1 2, 3 1, 12 5 1. 3 x x x x x x = = - = = - - - + = Hence the solution is 1 2 3 1, 1, 2. x x x = = =

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70/158	SUBMITTED TEXT	94 WORDS	34%	MATCHING TEXT	94 WORDS

x x V x f x ii) ( ) ( )( ) .a = a a " Î N x N x isascalar x V iii) ( ) ( ) ( ) N x y N x N y + £ + (1) ( ) ( ) def 1 2 1 1 , ,...., = ¢ = = ∑ n i n i N x x x where x x x x (2) ( ) { 2 1 2 def 2 1 } = = = ∑ n i i N x x x (3) ( ) def max £ £ ¥ = = k i n i N x x x Example 3.3.3 : ( ) 1,0, 1,2 x ¢ = - Then 1 4, x = 2 6, x = 2

SA S41641 Mathematics 06.pdf (D164869290)

71/158 **SUBMITTED TEXT** 49 WORDS **77%** **MATCHING TEXT** 49 WORDS

system of linear equations 11 1 12 2 1 1 21 1 22 2 2 2 1 1 22 2 ... ... n n n n n nn n n a x a x a x b a x a x a x b a x a x a x b + + + = + + + = + + + = (3.4.1)

system of n linear equations, n n nn n n n n n n n n b a x x a x a b x a x a x a x a b x a x a a x b

W https://mis.alagappauniversity.ac.in/siteAdmin/dde-admin/uploads/5/__UG_B.Sc._Mathematics_113%205 ...

72/158 **SUBMITTED TEXT** 101 WORDS **19%** **MATCHING TEXT** 101 WORDS

A ³ 0 A iff A = is a null matrix ii) ( ) a = a a A A isascalar iii) A B A B + £ + iv) AB A B £ n n A A \ £ (1) def 1 max = ∑ ij j i A A a (2) 1 2 2 def 2 . ? ? ? ? = ? ? ? ? ? ? ∑ ij i j A A a NSOU l CC-MT-05 50 (3) def max ¥ = ∑ ij i j A A a Example 3.3.4 : 1 2 3 4 5 6 7 8 9 A ? ? ? ? = ? ? ? ? ? ? Then ( ) 1 max 12,15,18 18 A = = ( ) 1 2 2 2 2 2 1 2 ....9 285 16.88 A = + + = = ( ) max 6,15,24 24 A ¥ = = 3.4

SA S41641 Mathematics 06.pdf (D164869290)

73/158 **SUBMITTED TEXT** 44 WORDS **44%** **MATCHING TEXT** 44 WORDS

n n a b a x x x a a a = - - - 2 2 21 2 2 22 22 22 ... n n a b a x x x a a a = - - - .... NSOU l CC-MT-05 51 1 . 1 2 1 ... n n n n n n nn nn nn b a a x x x x a a

SA M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

74/158 **SUBMITTED TEXT** 63 WORDS **37%** **MATCHING TEXT** 63 WORDS

K > By (3.4.5) ( ) ( ) ( ) ( ) ( ) { } 1 1 1 k k k k k K h K h + + + e £ e + £ e + where ( ) ( ) ( ) ( ) ( ) 1 1 k k k k k h x x + + = - = e -e Or ( ) ( ) 1 1 + e £ - k k K h K

SA Numerical Analysis Dr RSM.pdf (D144415232)

75/158 **SUBMITTED TEXT** 28 WORDS **100%** **MATCHING TEXT** 28 WORDS

Example 3.4.1 : Solve the following system of linear equations by Gauss-Jacobi's method correct up to four decimal places and calculate the upper bound of absolute errors. 27 6 54

SA completed numerical analysis.pdf (D154613679)

76/158 **SUBMITTED TEXT** 35 WORDS **100%** **MATCHING TEXT** 35 WORDS

the system is diagonally dominant as 6 1 27 , + - > 6 2 15 , + > 1 1 54 . + > The Gauss-Jacobi's iteration scheme is ( ) ( ) ( ) ( ) 1 54 6 27

SA completed numerical analysis.pdf (D154613679)

77/158 **SUBMITTED TEXT** 67 WORDS **73%** **MATCHING TEXT** 67 WORDS

Let the initial solution be (0, 0, 0). The next iterations are shown in the following table. k x y z 0 0 0 0 1 2.00000 4.80000 2.03704 2 1.00878 3.72839 1.91111 3 1.24225 4.14167 1.94931 4 1.15183 4.04319 1.93733 5 1.17327 4.08096 1.94083 6 1.16500 4.07191 1.93974 7 1.16697 4.07537 1.94006 8 1.16614 4.07488 1.93999 9 1.16632 4.07477 1.93998 10 1.16632 4.07477 1.93998 11 1.16635 4.07481 1.93998 Fig. : 3.1 The solution correct up to four decimal places is 1.1664,

SA completed numerical analysis.pdf (D154613679)

78/158 **SUBMITTED TEXT** 64 WORDS **75%** **MATCHING TEXT** 64 WORDS

k k k n n x b a x a x a + = - - - ( ) ( ) ( ) ( ) 1 1 2 2 21 2 1 22 1 ... k k k n n x b a x a x a + + = - - - .... ( ) ( ) ( ) ( ) 1 1 1 1 1 , 1 1 1 ... k k k n n n n n n nn x b a x a x a + + + - - = - - - ( ) 1,2,3,....

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79/158 **SUBMITTED TEXT** 19 WORDS **52%** **MATCHING TEXT** 19 WORDS

ij i j j ii j i j i a a a ( ) ( ) 1 1 k k ij ij j i j i ii a a a + > < £

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80/158 **SUBMITTED TEXT** 90 WORDS **22%** **MATCHING TEXT** 90 WORDS

k k k i i K K K Or ( ) ( ) ( ) 1 1 + - e £ e - k k i i K K K (3.5.4) Since ( ) 1, 1 - £ > - i i K K K as K K we have Which leads to ( ) ( ) 1 k k K + e £ e (3.5.5) Hence for every k ( ) ( ) 0 k k K e £ e (3.5.6) So that ( ) 0 sin 1. e ® ® ¥ > k as k ce K If K > 1, an estimate of the error is given by ( ) ( ) 1 1 + e £ - k k K h K where ( ) ( ) ( ) ( ) ( ) 1 1 . k k k k k h x

SA Numerical Analysis Dr RSM.pdf (D144415232)

81/158 **SUBMITTED TEXT** 19 WORDS **55%** **MATCHING TEXT** 19 WORDS

l l l are eigen values of the matrix M and 1 2 , ,...., n X X X are corresponding eigen-vectors

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82/158 **SUBMITTED TEXT** 46 WORDS **52%** **MATCHING TEXT** 46 WORDS

n n e X X X = a + a + + a ( ) ( ) ( ) ( ) 1 1 1 1 1 1 1 2 2 2 , ... i i i i n n n e X X X + + + + = a l + a

SA S41641 Mathematics 06.pdf (D164869290)

83/158 **SUBMITTED TEXT** 43 WORDS **64%** **MATCHING TEXT** 43 WORDS

method. Example 3.6.1 : Solve the following system of equations 1 2 3 3 2 6 x x x + + = 1 2 3 4 2 5 x x x - + + = NSOU l CC-MT-05 58 1 2 3 2 4 7 x x x + + =

SA Numerical Analysis Dr RSM.pdf (D144415232)

84/158 **SUBMITTED TEXT** 219 WORDS **41%** **MATCHING TEXT** 219 WORDS

x a x w a x a x a x b + ? ? = - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 22 22 21 22 23 2 2 2 1 2 3 k k k k k a x a x w a x a x a x b + + ? ? = - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 1 33 33 31 32 33 3 3 3 1 2 3 k k k k k a x a x w a x a x a x b + + + ? ? = - + + - ? ? ? ? or ( ) ( ) ( ) ( ) ( ) 1 1 3 1 1 2 3 3 1.01 3 2 6 k k k k k x x x x x + ? ? = - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 2 3 2 1 2 4 4 1.01 4 2 5 k k k k k x x x x x x + + ? ? = - - + + - ? ? ? ? ( ) ( ) ( ) ( ) ( ) 1 1 1 3 3 3 1 2 4 4 1.01 2 4 7 k k k k k x x x x x x + + + ? ? = - + + - ? ? ? ? Let ( ) ( ) ( ) 0 0 0 1 2 3 0. x x x = = =

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85/158	SUBMITTED TEXT	96 WORDS	50%	MATCHING TEXT	96 WORDS

x x x - + = - 1 2 3 3 7 x x x - + - = 1 2 3 3 7 x x x - + = - 2. First, write down the equations for the GS interations : NSOU l CC-MT-05 60 ( ) ( ) ( ) ( ) 1 1 2 3 1 /3 k k k x x x + = - + - ( ) ( ) ( ) ( ) 1 1 2 1 3 7 / 3 k k k x x x + + = + + ( ) ( ) ( ) ( ) 1 1 1 3 1 2 7 / 3 k k k x x x + + + = - - + 3.

SA S41641 Mathematics 06.pdf (D164869290)

86/158	SUBMITTED TEXT	74 WORDS	70%	MATCHING TEXT	74 WORDS

k k k k x w x w x x + = - + - + - ( ) ( ) ( ) ( ) ( ) ( ) 1 1 2 2 1 3 1 7 / 3 k k k k x w x w x x + + = - + + - ( ) ( ) ( ) ( ) ( ) ( ) 1 1 1 3 3 1 3 1 7 / 3 k k k k x w x w x x + + + = - + - - + 4.

SA S41641 Mathematics 06.pdf (D164869290)

87/158	SUBMITTED TEXT	134 WORDS	52%	MATCHING TEXT	134 WORDS

k = 0. ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 0 0 0 1 1 2 3 1 1 3 1 1.25 ·0 1.25· 1 0 0 3 0.41667 x w x w x x = - + - + - = - + - + - = - ( ) ( ) ( ) ( ) ( ) ( ) ( ) 1 0 1 0 2 2 1 3 1 7 3 0.25·0 1.25· 7 0.41667 0 3 2.7431 x w x w x x = - + + - =- + - + = ( ) ( ) ( ) ( ) ( ) ( ) 1 0 1 1 3 3 1 2 1 7 x w x w x x = - + - - + ( ) 3 0.25·0 1.25· 7 0.41667 2.7431 3 1.6001 = + - + + = - The

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88/158	SUBMITTED TEXT	51 WORDS	73%	MATCHING TEXT	51 WORDS

solve the system of linear equations 1 2 3 2 4 3, x x x + + = 1 2 3 3 2 2 2, x x x + - = 1 2 3 6. x x x - + = (Ans: 1 3 2.8, 1.16, 2.04 x x = = - =) 2. Solve the following system of

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89/158	SUBMITTED TEXT	87 WORDS	70%	MATCHING TEXT	87 WORDS

x x x + + = 1 3 2 210 13, x x + + = 1 2 3 2 2 10 14. x x x + + = (Ans: 1 2 3 1, 1, 1 x x x = = =) ii) 1 2 3 8 3 2 20, x x x - + = NSOU l CC-MT-05 62 1 2 3 4 11 33, x x x + - = 1 2 3 6 3 12 35. x x x + + = (Ans: 1 2 3 3. 168, 1.9858, .9117 x x x = - = =) 4.

SA S41641 Mathematics 06.pdf (D164869290)

90/158	SUBMITTED TEXT	18 WORDS	66%	MATCHING TEXT	18 WORDS

of obtaining the value of the function for any intermediate value of the argument when the values of

of obtaining the value of a function for any intermediate values of the independent variable, i.argument within an interval when the values of

W https://nmot3ee1104.files.wordpress.com/2014/07/nmot_ho-1-15.pdf

91/158	SUBMITTED TEXT	35 WORDS	44%	MATCHING TEXT	35 WORDS

n x x x x x there exist unique polynomial of maximum degree n which interpolates ( ) f x at the points 0 1 2 3 , , , ,..... . n x x x

n p x x x x x (5.3) The polynomial p 01j (x) interpolates f(x) at the points x 0 , x 1 , x

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92/158	SUBMITTED TEXT	138 WORDS	55%	MATCHING TEXT	138 WORDS

Solve the following system of equations by Gauss-Seidel method correct upto four decimal places: i) 12 6 9, x y z + + = 8 3 2 13, x y z + + + 5 7 + + = x y z (Ans : x = 1, y = 1, z = 1) ii) 8 18, x y z - + = 2 5 2 3, x y z + - = 3 16 x y z + - = - (Ans : x = 2, y = 0.9998, z = 2.9999) 5. Solve the following system of equations by S.O.R method correct upto four decimal places: 6, x y z + + = 4, x y z - - = - 2 2 1. x y z + - = - (Ans: x = 1, y = 2, z = 3)

SA Numerical Analysis Dr RSM.pdf (D144415232)

93/158	SUBMITTED TEXT	61 WORDS	54%	MATCHING TEXT	61 WORDS

x a a x a x a x = + + + + (4.1) and it is given ( ) ( ) ( ) 0,1,2,...., = = n i i y x f x i n (4.2) i.e. ( )( ) 2 0 1 2 ... 0,1,2,....., n i i n i i a a x a x a x f x i n + + + + = =

SA M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

94/158	SUBMITTED TEXT	11 WORDS	80%	MATCHING TEXT	11 WORDS

n n i j i j n n n x x x x x x x x < = - [1]

SA S41641 Mathematics 06.pdf (D164869290)

95/158	SUBMITTED TEXT	61 WORDS	64%	MATCHING TEXT	61 WORDS

form ( ) ( ) ( )( ) ( ) 0 1 0 2 0 1 0 .... n n y x a a x x a x x x x a x x = + - + - - + + - ( ) ( ) 1 1 ... n x x x x - - - (4.3.1) We now determine the coefficient 0 1 2 , , ,..... n a a a a

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96/158	SUBMITTED TEXT	40 WORDS	59%	MATCHING TEXT	40 WORDS

y x y i n = = We have 2 1 0 0 0 2 1 0 0 0 0 1 2 2 2 1 0 1 0 2 . , 2 2! y y y y y y y y a y a a x x x x

SA S41641 Mathematics 06.pdf (D164869290)

97/158	SUBMITTED TEXT	31 WORDS	52%	MATCHING TEXT	31 WORDS

u u u u u y x y u y y y - - - = + D + D + D + + ( )( ) ( ) 0 1 2 ... 1 ! n u u u

u u u u u y y y y y h u u u

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98/158	SUBMITTED TEXT	29 WORDS	53%	MATCHING TEXT	29 WORDS

x y x x x x x x x x h h D D = + - + - - + + - NSOU l CC-MT-05 66 ( ) ( ) 0 1 1 .... ! n n n y x x x x

SA S41641 Mathematics 06.pdf (D164869290)

99/158	SUBMITTED TEXT	20 WORDS	78%	MATCHING TEXT	20 WORDS

Newton's forward interpolation formula is used to interpolate the values of near the beginning of a set of tabulator values.

Newton's Forward formula is used for interpolating the values of near the beginning of a set of tabulated values

W https://pdfcoffee.com/numerical-methods-notespdf-pdf-free.html

100/158	SUBMITTED TEXT	38 WORDS	35%	MATCHING TEXT	38 WORDS

is given by ( ) ( )( ) ( ) ( ) ( ) ( ) 1 1 1 1 2 ... 1 ! n n n u u u u n R x h f n + + + - - - - = x + { } 0 , , n mim x x x > x { } 0 max , , n x x x >

SA nm full book.pdf (D31630497)

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101/158	SUBMITTED TEXT	53 WORDS	37%	MATCHING TEXT	53 WORDS

as follows : x y Dy D 2 y D 3 y D n y x 0 y 0 Dy 0 x 1 y 1 D 2 y 0 Dy 1 D 3 y 0 x 2 y 2 D 2 y 1 .... D n y 0 ..... D 2 y

SA S41641 Mathematics 06.pdf (D164869290)

102/158	SUBMITTED TEXT	24 WORDS	85%	MATCHING TEXT	24 WORDS

Example 4.3.1 : The following table gives the values of e x for certain equidistant values of x. Find the value of

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103/158	SUBMITTED TEXT	32 WORDS	70%	MATCHING TEXT	32 WORDS

y x y i n = = We have 2 1 1 2 0 1 2 2 2 1 2 , , 2 2! n n n n n n n n n n y y y y y y y y a y

y y n n n n ? ? ? ? ? ? ? ? ? ? ? ? ? + ,))…….( )(( ))…….( )((………. 1 1 0 1 1 0 n n n n n n y y y y y y y y

W https://www.ddegjust.ac.in/2021/bca/Computer%20Oriented%20Numerical%20Methods.pdf

104/158	SUBMITTED TEXT	52 WORDS	65%	MATCHING TEXT	52 WORDS

n n n y x a a x x a x x x x a x x - = + - + - - + + - ( ) ( ) 1 0 .... - - - n x x x x (4.4.1) We now determine the coefficient 0 1 2 , , ,...., n a a a a

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105/158	SUBMITTED TEXT	55 WORDS	43%	MATCHING TEXT	55 WORDS

n n n n n n y y y x y x x x x x x x x x h h - Ñ Ñ = + - + - - + + - ( ) ( ) 1 1 ... ! n n n n y x x x x n h - Ñ - - -1 (4.3.2) Setting , n x x v h - = we have from equation (4.3.2) ( ) ( ) ( )( ) 2 3 0 1 1 2 .... 2! 3! n n n

SA S41641 Mathematics 06.pdf (D164869290)

106/158	SUBMITTED TEXT	18 WORDS	66%	MATCHING TEXT	18 WORDS

interpolation formula is used to interpolate the values of near the end of a set of tabulator values.

Interpolation formula is used for interpolating the values of near the beginning of a set of tabulated values

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107/158	SUBMITTED TEXT 13 WORDS	100% MATCHING TEXT 13 WORDS
	From the following table of values of x and f (x) determine	From the following table of values of x and f(x), determine (

W http://www.dbscience.org/wp-content/uploads/2020/03/NumericalMethodsforEngineers-1.pdf

108/158	SUBMITTED TEXT 64 WORDS	48% MATCHING TEXT 64 WORDS
	x : 0.20 0.22 0.24 0.26 0.28 0.30 f (x) : 1.6596 1.6698 1.6804 1.6912 1.7024 1.7139 Solution. The difference table is x f (x) Ñf (x) Ñ 2 f (x) Ñ 3 f (x) 0.20 1.6596 0.22 1.6698 0.0102 0.24 1.6804 0.0106 0.004 0.26 1.6912 0.0108 0.0002 −0.0002 0.28 1.7024 0.0112 0.0004 0.0002 0.30 1.7139 0.0115 0.0003 −0.0001 Here, 0.30, n x = 0.30, x = 0.02, h = 0.29 0.30 0.5. 0.02 n x x v h - - = = = -	x: 0 1 2 3 f (x) : 1 2 1 10 Hence or otherwise evaluate f(4). Solution: The difference table is x f(x) ?f(x) ? 2 f(x) ? 3 f(x) 0 1 101 1 1 2 -2 -1 12 2 1 10 9 3 10 We take x 0 = 0 and h x 0? = h = 1] ?

W https://www.ddegjust.ac.in/2021/bca/Computer%20Oriented%20Numerical%20Methods.pdf

109/158	SUBMITTED TEXT 35 WORDS	38% MATCHING TEXT 35 WORDS
	n n n v v v v n R x h f n + + + + + + = x + 0 1 1 0 min{ , , , , } max{ , , } > x > ⋯ ⋯ n n x x x x x x x	

SA Numerical Analysis Dr RSM.pdf (D144415232)

110/158	SUBMITTED TEXT 21 WORDS	82% MATCHING TEXT 21 WORDS
	forward interpolation formula is given by ( ) ( ) ( ) 2 3 0 0 1 1 1 2! n u u y x y u y	

SA nm full book.pdf (D31630497)

111/158	SUBMITTED TEXT 32 WORDS	73% MATCHING TEXT 32 WORDS
	find the values of y at (i) x = 1.40 and (ii) x = 1.60 from the following table x : 1.0 1.25 1.50 1.75 2.00 y : 1.0000 1.0772 1.1447 1.2051 2.2599 Solution.	

SA nm-27-06-2017.doc (D29511457)

112/158	SUBMITTED TEXT	81 WORDS	63%	MATCHING TEXT	81 WORDS

n y x a x x x x x x a x x x x x x = - - - + - - - ( )( ) ( ) ( )( ) ( ) 2 0 1 0 1 1 .... n n n a x x x x x x a x x x x x x - + - - - + + - - - (4.5.1) NSOU l CC-MT-05 73 where 0 1 2 , , ,...., n a a a a are coefficient to be determined

SA S41641 Mathematics 06.pdf (D164869290)

113/158	SUBMITTED TEXT	80 WORDS	52%	MATCHING TEXT	80 WORDS

f x y x x x x x = » = w ¢ - w ∑ where ( ) ( )( ) ( ) 0 1 ..... w = - - - n x x x x x x x ( ) ( ) ( ) ( ) ( ) ( ) 1 0 n i n i i i f x f x x R x x x x + = = w + ¢ - w ∑ Where ( ) ( ) ( ) ( ) ( ) { } { } 1 1 0 0 min , ,...., max , ,...., 1 ! + + x = w > x > + n n n n f R x x x x x x x x

f(x) = 0 0 2 0 1 0 2 1))....( )(( ))....( )((y x x x x x x x x x x x n n ? ? ? ? ? ? + 1 1 2 1 0 1 2 0))....( )(( ))....( )((y x x x x x x x x x x n n ? ? ? ? ? ? +......+ n n n n n n y x x x x x x x x

W https://www.ddegjust.ac.in/2021/bca/Computer%20Oriented%20Numerical%20Methods.pdf

114/158	SUBMITTED TEXT	254 WORDS	46%	MATCHING TEXT	254 WORDS

n i i i y x y f x i n Putting 0 x x= in equation (4.5.1), we get ( ) ( )( ) ( ) 0 0 0 1 0 2 0 .... n f x a x x x x x x = - - - Putting 1 x x= in equation (4.5.1), we get ( ) ( )( ) ( ) 1 1 1 0 1 2 1 .... n f x a x x x x x x = - - - Similarly putting 2 3 , ,.... n x x x x = in equation (4.5.1), we get ( ) ( )( ) ( ) 2 2 2 0 2 1 2 .... n f x a x x x x x x = - - - .... ( ) ( )( )( ) ( ) 0 0 1 1 .... n n n n n n n f x a x x x x x x x x - = - - - - Substituting the values of 0 1 2 , , ..., n a a a a in (4.5.1) we get ( ) ( )( ) ( ) ( )( ) ( ) ( ) 1 2 0 0 1 0 2 0 .... n n n x x x x x x y x f x x x x x x x - - - = - - - ( )( ) ( ) ( )( ) ( ) ( ) 0 2 1 1 0 1 2 1 .... n n x x x x x x f x x x x x x x x - - - + + - - - ( )( ) ( ) ( )( ) ( ) ( ) 0 1 2 2 0 2 1 2 .... n n x x x x x x x f x x x x x x x - - - + - - - ( )( ) ( ) ( )( ) ( ) ( ) 0 1 1 0 1 1 .... n n n n n n x x x x x x f x x x x x x x - - - - - + - - - which is Lagrange’s interpolation formula.

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115/158	SUBMITTED TEXT	456 WORDS	41%	MATCHING TEXT	456 WORDS

x x x x x x x x p x f x f x x x x x x x x x - - - - = + - - - - ( )( ) ( )( ) ( ) 0 1 2 2 0 2 1 x x x x f x x x x x - - + - - ( )( ) ( )( ) ( )( ) ( )( ) ( ) ( )( ) ( )( ) 1/ 2 1 0 1 0 1/ 2 0 1 0 0 1/ 2 0 1 1/ 2 0 1/ 2 1 1 0 1 1/ 2 x x x x x x - - - - - - - = ´ + ´ - + ´ - - - - - - - ( ) 4 1 . x x = - The error ( ) ( ) ( ) = - E x f x p x is given by ( ) ( )( )( ) ( ) 0 1 2 3! f E x x x x x x x ¢¢¢ x = - - - NSOU l CC-MT-05 75 or, ( ) ( ) 0 1 2 3! f E x x x x x x x ¢¢¢ x = - - - 3 3 1 0 1/ 2 11. 1 in 0 1 . 3! d f x x x as x dx ? ? £ - - - £ £ £ ? ? ? ? ? ? Now, 0 1, x- £ 1/ 2 1/ 2 x - £ and 1 1 in 0 1. - £ £ £ x x Hence, ( ) 1 1 1 1. . . 2 6 12 E x £ = That is, ( ) ( ) 1 . 12 - £ f x p x Example 4.6.2 : Find the missing term in the following table x : 0 1 2 3 4 y : 1 2 4 ? 16 Solution. Using Lagrange's formula ( ) ( )( )( ) ( )( )( ) 3 2 0 1 2 4 7 14 8 8 0 1 0 2 0 4 x x x x x x L x - - - - + - = = - - - - ( ) ( )( )( ) ( )( )( ) 3 2 1 0 2 4 6 8 . 3 1 0 1 2 1 4 x x x x x x L x - - - - + = = - - - ( ) ( )( )( ) ( )( )( ) 3 2 2 0 1 4 5 4 . 4 2 0 2 1 2 4 x x x x x x L x - - - - + = = - - - - ( ) ( )( )( ) ( )( )( ) 3 2 3 0 1 2 3 2 . 24 4 0 4 1 4 2 x x x x x x L x - - - - + = = - - - Therefore, ( ) ( ) ( ) ( ) ( ) 0 0 1 1 2 2 3 3 + + + ≃ y x y L x y L x y L x y L x 3 2 3 2 7 14 8 6 8 1 2 8 3 x x x x x x - + - - + = ´ + ´ - NSOU l CC-MT-05 76 3 2 3 2 5 4 3 2 4 16 4 24 x x x x x x - + - + + ´ + ´ - 3 2 5 1 11 1. 24 8 12 x x x = - + +

SA M. Sc. I Maths MT 204 Numerical Analysis all.pdf (D142231091)

116/158	SUBMITTED TEXT	33 WORDS	75%	MATCHING TEXT	33 WORDS

Using the following data, find by Lagrange's formula, the value of ( ) 10= f x at x ( ) 0 1 2 3 4 9.3 9.6 10.2 10.4 10.8 11.40 12.80 14.70 17.00 19.80 i i i i x y f x =

SA Book2-Numerical Methods.pdf (D110229668)

117/158	SUBMITTED TEXT	24 WORDS	50%	MATCHING TEXT	24 WORDS

j j x x = - = Õ ( ) 4 1 0, 1 0.1728, j j j x x = 1 - = - Õ ( ) 4 2 0, 2 0.0648, = 1 - = + Õ j j j x x ( ) 4 3 0, 3 0.0704, j j j x

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118/158	SUBMITTED TEXT	49 WORDS	55%	MATCHING TEXT	49 WORDS

y y y y y y y y y y D = D - D = - - - - = - + Similarly the 3 rd order difference is represented by 3 2 2 0 1 0 3 2 1 0 3 3 y y y y y y y

y 5 ? 4 y 5 ? 5 y 5 x 6 y 6 ?y 6 ? 2 y 6 ? 3 y 6 ? 4 y 6 ? 5 y 5 ? 6 y 6 Central Differences: The central difference operator ? is defined by the relations y 1 – y 0 = ?y 1/2 , y 2 – y 1 = ?y 3/2 ,....., y

W https://www.ddegjust.ac.in/2021/bca/Computer%20Oriented%20Numerical%20Methods.pdf

119/158	SUBMITTED TEXT	49 WORDS	50%	MATCHING TEXT	49 WORDS

E Ef x Ef x h f x h = = + = + and in general ( ) ( ) . n E f x f x nh = + Forward difference operator :D It is defined by ( ) ( ) ( ) f x f x h f x

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120/158	SUBMITTED TEXT	27 WORDS	46%	MATCHING TEXT	27 WORDS

The first order difference of a polynomial of degree n is a polynomial of degree 1. n- Backward difference operator ÑÑÑÑÑ : The first order backward difference

SA completed numerical analysis.pdf (D154613679)

121/158	SUBMITTED TEXT	73 WORDS	59%	MATCHING TEXT	73 WORDS

f x f x h f x h E E f x - ? ? d = + - - = - ? ? ? ? ( ) ( ) ( ) ( ) 1 2 f x h f x h f x f x d + = + - = D ( ) ( ) ( ) ( ) 1 1 1 2 2 2 f x f x h f x h f x h d = + - - = D - Thus

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122/158	SUBMITTED TEXT	87 WORDS	31%	MATCHING TEXT	87 WORDS

f x f x f x h f x E f x E f x - - Ñ = - - - = - = - 1 1 E - ⇒ º -Ñ (proved) (ii) Show that 2 . D -Ñ º d Proof : We know that ( ) ( ) ( ) ( ) 1 1 2 2 1 1 2 2 f x f x h f x h E E f x - ? ? d = + - - = - ? ? ? ? 1 1 2 2 E E - ⇒ d º - ( ) ( ) 2 1 2 1 2 1 - ⇒ d

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123/158	SUBMITTED TEXT	24 WORDS	64%	MATCHING TEXT	24 WORDS

Let ( ) y f x = denote a continuously differential function which takes the values 0 1 2 3 , , , ,..... n y y y

Let y = f(x) be a function which takes the values y 0 , y 1 , ... y

W https://nmot3ee1104.files.wordpress.com/2014/07/nmot_ho-1-15.pdf

124/158	SUBMITTED TEXT	57 WORDS	61%	MATCHING TEXT	57 WORDS

x for the following data : x : -4 -1 0 2 4 f (x) 1245 33 5 9 1335 Ans : ( ) 4 3 2 3 5 6 4 5 5 f x x x x x = - + = - + - 2. Given the values : x : 5 7 11 13 17 f (x) 150 392 1452 2366 5202

SA nm-27-06-2017.doc (D29511457)

125/158	SUBMITTED TEXT	35 WORDS	54%	MATCHING TEXT	35 WORDS

x x dy y y y dx h = ? ? ? ? » D - D + D + ? ? ? ? ? ? ? ? ? 0 2 2 3 0 0 2 2 1 ... = ? ? ? ? » D -D + ? ? ? ? ? ? ? ? ? x x d y y y dx h

x x d y y y y dx h ʃ · ʃ · ' ' ' ¨ ¸ ¨ ¸ © 1 © 1 ...(10.17) Similarly, you get, 0 3 3 4 3 3 1 3 ... 2 n n x x d y y y dx h ʃ · ʃ · ' ' ¨ ¸ ¨ ¸ © 1 © 1 ...(10.18)

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126/158	SUBMITTED TEXT	24 WORDS	64%	MATCHING TEXT	24 WORDS

Let ( ) y f x = denote a continuously differential function which takes the values 0 1 2 3 , , , ,..... n y y y

Let y = f(x) be a function which takes the values y 0 , y 1 , ... y

W https://nmot3ee1104.files.wordpress.com/2014/07/nmot_ho-1-15.pdf

127/158	SUBMITTED TEXT	13 WORDS	71%	MATCHING TEXT	13 WORDS

n n n n u u u u u f x y u y

n n n n y u u u y u u y u y

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128/158	SUBMITTED TEXT	33 WORDS	68%	MATCHING TEXT	33 WORDS

n n n x x dy y y y dx h - - - = ? ? ? ? » D + D + D + ? ? ? ? ? ? ? ? 2 2 3 2 3 2 2 1 ... n n n x x d y y y dx h - - = ? ? ? ? »

n n n x x d y y y dx h ʃ · ʃ · ' ' ' ¨ ¸ ¨ ¸ © 1 © 1 ...(10.17) Similarly, you get, 0 3 3 4 3 3 1 3 ... 2 n n x x d y y y dx h ʃ · ʃ · ' ' ¨ ¸ ¨ ¸ © 1 © 1 ...(10.18)

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129/158	SUBMITTED TEXT	29 WORDS	47%	MATCHING TEXT	29 WORDS

corresponding to (n+1) values of the independent variable x, we can represent the function ( ) y f x = to be a polynomial in of degree .

corresponding to n+1 values of x, we can represent the function f(x) by a polynomial in x of degree

W https://nmot3ee1104.files.wordpress.com/2014/07/nmot_ho-1-15.pdf

130/158	SUBMITTED TEXT	26 WORDS	62%	MATCHING TEXT	26 WORDS

x, then we have from Newton's Forward Interpolation formula as ( ) ( ) ( )( ) 2 3 0 0 0 0 1 1 2 ... 2! 3! u u u u u

SA nm full book.pdf (D31630497)

131/158	SUBMITTED TEXT	65 WORDS	51%	MATCHING TEXT	65 WORDS

n i i i f x f x L x x x x x = » = w ¢ - w ∑ where ( ) ( )( ) ( ) 0 1 ..... n x x x x x x x x w = - - - NSOU l CC-MT-05 83 Now ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 0 0 5.1 n n i i n i i i i i i f x f x f x L x x x x x x

n f x x x x x x x x + 1 1 0 1 2 1 1 ( ) 1 . ... ( ) ( )...( ) ( ) n f x x x x x x x x + 0 1 1 ( ) 1 . . ( ) ( )...( ) ( ) n n n n n n f x x x x x x x x x ... (8.20)

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132/158	SUBMITTED TEXT	166 WORDS	39%	MATCHING TEXT	166 WORDS

f x L x x x x x = = w ¢ - w ∑ ( ) ( ) ( ) ( ) ( ) ( ) 0 n i k k i i i i k f x x x f x x x x = 1 = w + w ¢ - w ∑ ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 2 0 0 n n i i n k k i i i i i i i k f x f x L x x x f x x x x x x x x = = 1 ¢ ¢ ¢ = w + w -w ¢ - w ¢ - w ∑ ∑ ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 n i n k k k k k i i i f x L x x x f x x x x = ¢ ¢ ¢ = w +w ¢ - w ∑ where ( ) ( ) 0 1 1 1 1 1 .... k k k k k n k i i k x x x x x x x x x 1 ¢ w = + + + = - - - - ∑ ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 1 = 1 ¢ ¢ = w + ¢ - w - ∑ ∑ n i n k k k i i k i i i k f x L x x f x x x x x x

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133/158	SUBMITTED TEXT	60 WORDS	36%	MATCHING TEXT	60 WORDS

difference table is NSOU l CC-MT-05 84 2 3 4 1 1 7 2 8 12 19 6 3 27 18 0 37 6 4 64 24 0 61 6 5 125 30 91 6 216 x y y y y y D D D D We have 0 1, 1, 1 x h x = = = so 0 0. x x u h - = = 0 2 3 4 0 0 0 0 1 1 1 1 ... 2 3 4 x x dy y y y y

SA nm-27-06-2017.doc (D29511457)

134/158	SUBMITTED TEXT	22 WORDS	86%	MATCHING TEXT	22 WORDS

Find the value of for which is minimum and find the minimum value from the table: x 0.60 0.65 0.70 0.75 y (

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135/158	SUBMITTED TEXT	52 WORDS	45%	MATCHING TEXT	52 WORDS

at 15 x x = = from the following table: x 15 17 19 21 23 25 y x = 3.873 4.123 4.359 4.583 4.796 5.000 Ans: 0.1289, -0.004 3. Find the minimum values of ( ) f x from the table: x 0 2 4 6 f (x) 3 3 11 27 Ans: 2.25 4. Find the maximum values of

at x = ·2 03 x 1 96 · 1 98 · 2 00 · 2 02 · 042· y 0 7825 · 0 7739 · 0 7651 · 0 7563 · 0 7473 · 46. Find ′ · f ( ) 0 6and ″ · f ( ) 0 6from the following table : x 0 4· 0 5· 0 6· 0 7· 0 8· f x( ) 1 5836 · 1 7974 · 2 0442 · 2 3275 · 2 6510 · 47. Find ′ · f ( ) 2 5 from the following x 1 5· 1 9· 2 5· 3 2· 4 3· 5 9· f x( ) 3 375 · 6 059 · 13 625 · 29 368 · 73 907 · 196 579 · 48. Find the maximum value of

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136/158	SUBMITTED TEXT	17 WORDS	63%	MATCHING TEXT	17 WORDS

Rule 6.4 Simpson's Rule 6.5 Weddle's Rule 6.6 Summary 6.7 Exercises 6.0 Objectives After studying this unit one will be able to

rule, Simpson's 3/8 rule, Weddle's rule, and Cotes method. 11.1 OBJECTIVES After going through this unit, you will be able to:

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137/158	SUBMITTED TEXT	14 WORDS	76%	MATCHING TEXT	14 WORDS

The interval { } ,a b is sub- divided into n equal subinterval, each of

The interval [a, b] is divided into N equal parts each of

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138/158	SUBMITTED TEXT	51 WORDS	42%	MATCHING TEXT	51 WORDS

Lagrange's interpolation formula to approximate ( ) f x by the interpolating polynomial ( ) n y x ( ) ( ) ( ) ( ) ( ) ( ) 0 n i n i i i f x f x y x x x x x = » = w ¢ - w ∑ where ( ) ( )( ) ( ) 0 1 .... . n x x x x x x x x

Lagrange's Interpolation Formula: If y = f(x) takes the value y 0 , y 1 ,......y n corresponding to x = x 0 , x 1 , ..., x n then 116 f(x) = 0 0 2 0 1 0 2 1))....( )(( )).....( )((y x x x x x x x x x x

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139/158	SUBMITTED TEXT	39 WORDS	86%	MATCHING TEXT	39 WORDS

x) 0.9320 0.9636 0.9855 0.9975 0.9996 Ans: 1.58 5. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1971 Year (x) 1931 1941 1951 1961 1971 Population on thousands(y) 40.62 60.80 79.95 103.56 132.65

SA nm full book.pdf (D31630497)

140/158	SUBMITTED TEXT	32 WORDS	35%	MATCHING TEXT	32 WORDS

x I f x dx H f x x x x = = = w = ¢ - w ∑ ∑ ∫ (6.2.1) where ( ) ( ) ( ) b n i a i i x H dx x x x w = ¢ - w ∫ ( ) 0,1,2,..... i n = (6.2.2) Setting 0 , x x

x x ? A 0 = 0 0 1 0 2 0 ( ) ( ) ( )...( ) n f x x x x x xx ...(8.16) f(x 1) = 1 1 0 1 2 1 ( ) ( )...( ) n A x x x x x x ? A 1 = 1 1 0 1 2 1 ( ) ( ) ( )...( ) n f x x

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141/158	SUBMITTED TEXT	80 WORDS	61%	MATCHING TEXT	80 WORDS

n x x x x x x x x x x x - + ¢ w = - - - - - ( ) { } ( )( )( ) ( ) { } 1 .... 1 1 2 .... ih i h h h h n i h = - - - - - ( )( ) { } ( ) ( ) 1 2 ...1 1 ! n i i n i i i i h h n i - - = - - - - ( ) ( ) 1 ! ! n i n

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142/158	SUBMITTED TEXT	50 WORDS	40%	MATCHING TEXT	50 WORDS

K f x b a K f x K f x where ( ) ( ) ( ) 1 0 1 1 0 0 1 1 1 2 1.0! 1 0 ! K u du - - = - = - ∫ and ( ) ( ) 1 1 1 1 1 0 1 1 2 1.1! 1 1 ! K udu - - = = - ∫ ( ) ( ) ( ) ( ) 0 1 2 - ? ? = » + ? ? T b a I f I f x f x

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143/158	SUBMITTED TEXT	25 WORDS	62%	MATCHING TEXT	25 WORDS

f x = is replaced by the straight line passing through the point ( ) ( ) ,a f a and ( ) ( ) , ,

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144/158	SUBMITTED TEXT	34 WORDS	38%	MATCHING TEXT	34 WORDS

each of length h. The nodes are 0 1 2 , , ,......, , n x x x x
such that 0 , , n x a x b = = ( ) 0 , 0,1,2,3,...., . i b a x x ih h i
n

SA Numerical Analysis Dr RSM.pdf (D144415232)

145/158	SUBMITTED TEXT	72 WORDS	48%	MATCHING TEXT	72 WORDS

x x x x x x I f f x dx f x dx f x dx ( ) ( ) ( ) ( ) 1 2 0 1 1 .... n n x x
x x x x I f f x dx f x dx f x dx - = + + + ∫ ∫ ∫ ( ) ( ) ( ) ( ) 3 0 1 1
1 1 .... 2 2 2 12 n n i h h f x f x f x f = ? ? ¢¢ = + + + - x ? ? ?
? ∑ (

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146/158	SUBMITTED TEXT	75 WORDS	37%	MATCHING TEXT	75 WORDS

K f x b a K f x K f x K f x where ( ) ( ) ( ) 2 0 2 2 0 0 1 1 2 6
2.0! 2 0 ! K u u du - - = - = - ∫ ( ) ( ) ( ) 2 1 1 2 1 0 1 2 2 3
2.1! 2 1 ! K u u du - - = - = - ∫ ( ) ( ) 1 2 2 0 2 1 1 6 2.2! 2 2 !
K u u du = - = - ∫ ( ) ( ) ( ) ( ) ( ) 0 1 2 4 6 s b a I f I f x f x f x -
? ? = » + + ? ?

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147/158	SUBMITTED TEXT	23 WORDS	84%	MATCHING TEXT	23 WORDS

h h f x dx f x f x f x f - - - ? ? = + + - x ? ? ∫ (( ) 2 2 2 ;
1,2,....,

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148/158	SUBMITTED TEXT	77 WORDS	54%	MATCHING TEXT	77 WORDS

f x f x f x f - - = = ? ? = + + - x ? ? ∑ ∑ c c s s I E = + ( ) ( ) (
) ( ) ( ) { } 0 1 3 1 4 ... 3 c s n n h I f x f x f x f x f x - ? ? = + +
+ + + ? ? ( ) ( ) ( ) { } 2 4 2 2 ...] n f x f x f x - + + + + ( )( ) 5
90

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149/158 **SUBMITTED TEXT** 51 WORDS **55%** **MATCHING TEXT** 51 WORDS

f f x dx f x f x f x f x f x ? = = + + + + + ? ∫ ( ) ( ) ( )( ) 9 5 6 9 216 41 ; 140 6 viii h b a f x f x f a b h - ? + - x > x > = ? (6.5.1)

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150/158 **SUBMITTED TEXT** 124 WORDS **28%** **MATCHING TEXT** 124 WORDS

f x f x f x f x f x f x f x D = - + - + - + (6.5.2) (6.5.1) + 140 h + ´ (6.5.2) gives on writing ( ) ( )( ) 6 6 0 vi f x h f a b ¢ ¢ D = x > x > NSOU l CC-MT-05 94 ( ) b W W a f x dx I E = + ∫ Where ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) 0 1 2 3 4 5 6 3 5 6 5 10 b W a h I f x dx f x f x f x f x f x f x f x ? ? = = + + + + + + + ? ? ∫ (6.5.3) and ( ) ( )( ) 7 9 9 , 140 140 vi viii W h h E f f

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151/158 **SUBMITTED TEXT** 30 WORDS **100%** **MATCHING TEXT** 30 WORDS

h f x f x f x f x f x f x - - - - - - - = ? = + + + + + + + ? ∑ ( ) ( ) ( ) 7 9 6 1 1 9 140 1400

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152/158 **SUBMITTED TEXT** 214 WORDS **47%** **MATCHING TEXT** 214 WORDS

Divide the interval into six parts. So 6 0 1 6 h - = = Therefore, the values of 1 1 y x = + are: x 0 1 2 3 4 5 6 y = f (x) 1 0.5 1/3 1/4 1/5 1/6 1/7 (i) By Trapezoidal rule: ( ) ( ) 6 0 6 1 2 3 4 5 0 1 2 1 2 h dx y y y y y y y x ? ? = + + + + + + ? ? + ∫ ( ) ( ) 1 1 1 1 1 1 1 2 0.5 2 7 3 4 5 6 ? ? = + + + + + + ? ? ? ? = 2.021429 (ii) By Simpson's 1/3 rd rule: ( ) ( ) ( ) 6 0 6 1 3 5 2 4 0 1 4 2 1 3 h dx y y y y y y y x ? ? = + + + + + + ? ? + ∫ NSOU l CC-MT-05 96 ( ) ( ) ( ) 1 1 1 1 1 1 1 1 1 4 2 3 7 2 4 6 3 5 ? ? = + + + + + + ? ? ? ? = 1.9538730 (iii) By Weddle's rule ( ) ( ) 6 0 6 1 2 4 5 3 0 1 3 3 2 1 10 h dx y y y y y y y x ? ? = + + + + + + ? ? + ∫ ( ) ( ) ( ) 3 1 1 1 1 1 1 1 3 2 8 7 2 3 5 6 4 ? ? = + + + + + + ? ? ? ? = 1.952857

Divide the interval (0,6) into six parts each of width The values of f(x) = 2 1 1 x? are below x 0 1 2 3 4 5 6 f(y 1 y 0 0.5 y 1 0.2 y 2 0.1 y 3 0.0588 y 4 0.0385 y 5 0.027 y 6 (i) By Trapezoidal rule, ? ? 6 0 2 1 x dx =) [(2 6 0 y y h ? +2(y 1 +y 2 +y 3 +y 4 +y 5)] =)] 0385 .0 0588 .0 1.0 2.0 5.0(2) 027.0 1[(2 1 ? ? ? ? ? ? ? = 1.4108 (ii) Simpson's 1/3 x dx =) [(3 6 0 y h ? +4(y 3 +y 5)+2(y 2 +y 4)] =) 027.0 1[(3 1 ? +4(0.5+0.1+0.0385)+2(0.2+0.0588)] = 1.3662. (iii) By Simpson's 3/8 rule, ? ? 6 0 2 1 x dx =) [(8 3 6 0 y y h ? +3(y 1 +y 2 +y 4 +y 5)+2y 3)] =) 027.0 1[(8 3 ? +3(0.5+0.2+0.0588+0.0385)+2(0.1)] x

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153/158	SUBMITTED TEXT	74 WORDS	80%	MATCHING TEXT	74 WORDS

f x f x f x f x f x f x f x ( ) ( ) ( ) { } 2 4 2 2 .... n f x f x f x - + + + + ( ) ( ) ( ) { } 3 9 3 6 .... n f x f x f x - + + + + (6.5.5) ( ) ( ) ( ) { } ( ) 6 12 6 2 .... 12 - ? + + + + 3 ? n f x f x f x

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154/158	SUBMITTED TEXT	10 WORDS	100%	MATCHING TEXT	10 WORDS

Submitted	Matching
by (i) Trapezoidal rule and (ii) Simpson's one- third rule	by (i) Trapezoidal rule and (ii) Simpson's one- third rule

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155/158	SUBMITTED TEXT	15 WORDS	62%	MATCHING TEXT	15 WORDS

Submitted	Matching
Objectives After going through this unit one will be able to learn l the concept of	OBJECTIVES After going through this unit, you will be able to: x Explain the system of

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156/158	SUBMITTED TEXT	46 WORDS	48%	MATCHING TEXT	46 WORDS

The velocity of a particle at distance from a point on its path is given in the table below: 0 10 20 30 40 50 60 / sec 47 58 64 65 61 52 38 sinmeter vinm Estimate the time to travel 60 meters by using Simpson's 1/3

SA S41641 Mathematics 06.pdf (D164869290)

157/158	SUBMITTED TEXT	18 WORDS	90%	MATCHING TEXT	18 WORDS

Jain, M. K., Iyenger, S.R.K., Jain, R.K., Numerical methods for scientific and engineering computation, New Age International Publishers,

SA S41641 Mathematics 06.pdf (D164869290)

158/158	SUBMITTED TEXT	20 WORDS	69%	MATCHING TEXT	20 WORDS

E., Introduction to Numerical Analysis, Addision Wiley, 1969. 4. Conte, S. D. and deBoor, C., Elementary Numerical Analysis , McGraw- Hill,

SA completed numerical analysis.pdf (D154613679)

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NSOU ● HMT ● CC - 7 1 PREFACE In the curricular structure introduced by this University for students of Post-Graduate degree programme, the opportunity to pursue Post-Graduate course in a subject introduced by this University is equally available to all learners. Instead of being guided by any presumption about ability level, it would perhaps stand to reason if receptivity of a learner is judged in the course of the learning process. That would be entirely in keeping with the objectives of open education which does not believe in artificial differentiation. I am happy to note that university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade 'A'. Keeping this in view, study materials of the Post-Graduate level in different subjects are being prepared on the basis of a well laid-out syllabus. The course structure combines the best elements in the approved syllabi of Central and State Universities in respective subjects. It has been so designed as to be upgradable with the addition of new information as well as results of fresh thinking and analysis. The accepted methodology of distance education has been followed in the preparation of these study materials. Co-operation in every form of experienced scholars is indispensable for a work of this kind. We, therefore, owe an enormous debt of gratitude to everyone whose tireless efforts went into the writing, editing, and devising of a proper layout of the materials. Practically speaking, their role amounts to an involvement in 'invisible teaching'. For, whoever makes use of these study materials would virtually derive the benefit of learning under their collective care without each being seen by the other. The more a learner would seriously pursue these study materials the easier it will be for him or her to reach out to larger horizons of a subject. Care has also been taken to make the language lucid and presentation attractive so that they may be rated as quality self-learning materials. If anything remains still obscure or difficult to follow, arrangements are there to come to terms with them through the counselling sessions regularly available at the network of study centres set up by the University. Needless to add, a great deal of these efforts are still experimental— in fact, pioneering in certain areas. Naturally, there is every possibility of some lapse or deficiency here and there. However, these do admit of rectification and further improvement in due course. On the whole, therefore, these study materials are expected to evoke wider appreciation the more they receive serious attention of all concerned. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor

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NSOU • CC • MT - 07 7 Differential Equation Unit - 1 Structures 1.0 Objective 1.1 Differential Equation—Genesis, Order and Degree 1.2 Formal Defintion 1.3 Order and Degree of ODE 1.4 Origin of Ordinary Differential Equation 1.5 Classification of Ordinary Differential Equations 1.6 Homogeneous and Non-Homogeneous Ordinary Differential Equation 1.7 Solution of an Ordinary Differential Equation 1.8 Summary 1.9 Exercise 1.0 Objective The objective of this unit is to discuss on basics of ordinary differential equations and their solutions. 1.1 Differential Equation—Genesis, Order and Degree Differential equations have wide level of applications in various aspects of science and engineering. Many of the principles or laws underlying the behaviour of the natural world are statements of relatios of rates by which things really happen. When expressed in mathematical terms the relations are equations and rates are derivatives. The mathematical statements of facts describing a real world problem is said to be mathematical models. Differential equations play a significant role in framing of mathematical models. During the last part of 17 th century, eminent scientists like Issac Newton, Gottfried Leibniz, Jaeques Bernoulli, Jean Bernoulli and Christian Huygens were engaged in solving differential equations. Many of the techniques which they built up are still in use today. During the 18 th century the mathematicians like Leonhard Euler, Dainel Bernoulli, Joseph Legrange and others added significantly to tthe enrichment of the subject. The doyens who pioneered tot he development of ordinary differential equations as a branch of modern mathematics are Cauchy, Riemann, Picard, Poincare, Lyapunoy and Birkhoff.

8 NSOU • CC • MT - 07 To understand and to investigate problems involving the motion of fluids, the flow of current in electric circuits, the dissipation of heat in solid objects, the propagation and detection of heat waves or the increase or decrease of population, among many others, it is necessary to know the basics and working theories of differential equations. While applying differential equations to any of the numerous fields in which they are useful, it is necessary first to formulate the appropriate differential equation that describes or models the problem being investigated. 1.2 Formal Defintion An equation involving derivatives or differentials

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of one or more dependent variable (s) with respect to one or more independent variable (s) is called a differential equation. For example, 5 3 dy x dx = + 4 3

y y x t ¶ ¶ + = ¶ ¶ Depending on the nature of differential of dependent variable (s) to the independent variable (s) the differential equation can be classified in two categories. 1. Ordinary Differential Equation (ODE) 2. Partial Differential Equation (PDE) Definition of ODE and PDE : A differential equation is ordinary differential equation (ODE) if the unknown function or dependent variable depends only on one independent variable. If the unknown function of dependent variable depends on more than one independent variable

then the differential equation is said to be a partial differential equation (PDE). 1.3 Order and Degree of ODE

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The order of a differential equation is the highest ordered derivative that appears in the equation. The degree of a differential equation is the greatest exponent of the highest ordered derivative involving in it, when the equation is free from radicals and

fractional powers.

NSOU • CC • MT - 07 9 To find the degree of a differential equation, the important view is that the differential equation must be a polymomial in derivatives of various orders. Also it can be mentioned nere that the order and degree (if defined) of a differential equation are always positive integers. Example : Determine the order and the degree of the following ordinary differential equations : a. 3 2 2 2 2 1

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dy a y c dx dx ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? ? ? b. 2 2 d y dy y dx dx + = c. sin 0 dy dy dx dx ? ? + = ? ? ? ? d. 3/2 2/3 3 2 3 2 0 d y d y dx dx ? ? ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? Solution : a. Here 2/3 2 2 2 1 dy d y c dx dx ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? ? ? i.e. 3 2 2 2 2 2 1 dy d y c dx dx ? ? ? ? ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? ? ? ? ? ? ?

So, the order and degree of the equation are two each, since the highest order derivative is two and the exponent of the highest order derivative is also two. b. Here 2 2 d y dy y dx dx + =
10 NSOU ● CC ● MT - 07 Clearly, the order of the diffrential equation is two and the degree is one. c. The degree of the differential equation sin 0 dy dy dx dx ? ? + = ? ? ? ? is not defined as the differential equation is not a polynomial in its derivatives although it has order one. d. The order is three and degree is nine as the differential equation is a polynomial equation in its derivatives not a polynomial in y. 1.4 Origin of Ordinary Differential Equation 1. Algebraic and Geometric origin. 2. Mechanical origin 3. Physical/Chemical Science origin 4. Population and Demographic origin 5. Economics and other Social Sciences origin 6. Biological origin In algebraic or geometric field the differerntial equations are formed by eliminating all the arbitrary constants that involved in a relation. The elimination of the arbitrary constants from the resulting equation gives the required differential equation whose order is equal to the number of independent constants actually involved. For example, given a relation y = ax 2 + a 2 (1) where a is an orbitrary constant. This relation contains only one arbitrary constant, so the order of the ODE is one. Differentiating (1) with respect to x, we have 2 , dy xa dx = i.e., 1 · 2 dy a x dx = Substituting the value of a in (1), we have
NSOU ● CC ● MT - 07 11 2 2 1 1 · · 2 2 dy dy y x x dx x dx ? ? = + ? ? ? ? i.e. 2 2 2 2 4 0 dy dy x x y dx dx ? ? + - = ? ? ? ? which is the required differential eqution. There is one very good example drawn from Biology to demonstrate the need of ordinary differential equation. Let us suppose that the rate of increase in the number of bacteria is proportional to the number of bacteria present. Let N(t) = the number of bacteria at time t. Assuming N(t) to be a differentiable function of t we can describe the above phenomenon as ( ) ( ) dN t cN t dt = , where c is a constant. 1.5 Classification of Ordinary Differential Equations q Linear and non-linear ordinary differential equatins : An ordinary differential equation which contains a single dependent variable and its derivatives with respect to a single independent variable as all first degree terms and there is neither any such term involving any form of product between two or more derivatives of different order nor any transcendental form of the depedent variable or any of its derivatives will be called a linear differential equation. The general form of a linear ordinary differential
equation is ( ) ( ) ( ) ( ) 1 0 1 1 ....

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n n n n n d y d y a x a x a x y r x dx dx - - + + + = , where a 0 , a 1 ......, a n

and r(x) are
the funcitions of x only.
12 NSOU ● CC ● MT - 07 For exmple, 2 x dy x y e dx + = and 2 2 d y dx + ( ) ( ) sin sec dy x xy x dx + = linear ordinary differential equations. If the condition of linearity as stated in the above definition is violated then the corresponding ordinary differential equation is said to be a non-linear ordinary differential equation. For example ( ) 2 2 5 3 y dy x y e x dx - + = and ( ) 2 2 2 sin y a y dy e y xy y dx dx + + = are not in linear form. These are non-linear ordinary differential equations. 1.6 Homogeneous and Non-Homogeneous Ordinary Differential Equation An ordinary differential equation is said to be homogeneous if there is no isolated term in the equation, i.e, if all the terms are proportional to a derivative of dependent variable or dependent variable itself and there is no term that contains a function of independent variable or constant alone. An n-th order
linear differential equation of the form 2 1 0 1 1 .....

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n n n n d y d y P P P y R dx dx - - + + + = (2) where y is the dependent variable, x is the independent variable, P 0 , P 1 , P 2 , ..., P n and R are either constants or functions of x.

In (2), if R = 0, then (2) is called a homogeneous linear ordinary differential equation. An ordinary differential equation which is not homogeneous is called a non-homogeneous ordinary differential equation. Remarks : A homogeneous differential equation has several distinct meanings : 1. A first order oridinary different equation of the form dy y f dx x ? ? = ? ? ? ? is a particular type of homogeneous equation.

NSOU ● CC ● MT - 07 13 2. A linear differential equation is said to be homogeneous if it has zero as a solution otherwise it is non-homogenous. 3. Generally (2) is written in the form F(x, y, y¢, y¢¢, ..., y (n)) = 0 1.7 Solution of an Ordinary Differential Equation A function is said to be a solution of an ordinary differential equation, over a particular domain of the independent variable, if its substitution into the equation reduces to an identity everywhere within that obtain. A function φ is said to be a soution of ODE F(x, y, y¢, y¢¢, ..., y (n)) = 0 if ( ) ( ) , ( ), ( ), ( ),......., ( ) 0 ¢ ¢¢ j j j j = n F x x x x x where (n) ( )x j stands for n-th derivative of the function : x → ϕ (x) with respect to the indenpendent variable x. The solution of an ordinary differential equation is called general solution if it

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contains a number of arbitrary constants equal to the order of the differential equation.		

This solution sometimes called a complete solution or a complete primitive or a complete integral. If the solution of an ordinary differential equation with y as dependent and x as independent variable can be obtained in the form y = f(x) then that form of solution is said to be an explicit solution. An implicit solution of an ordinary differential equation is a solution that is not in explicit form rather can be expressed in the form ϕ (x,y) = 0. A solution of a differential equation by giving particular values to the arbitrary constants in its general solution is called a particular solution of that equation. The general solution of any differential equation may not include all possible solutions of the differential equation. There may exist such a solution which cannot be obtained by giving any particular value to these arbitrary constants in the general solution. This is called a singular solution of that ordinary differential equation. Theorem : Any n-th order ordinary differential equation can have only n and not more than n, independent first integrals and so its general solution cannot have more than n arbitrary and independent constants.

14 NSOU ● CC ● MT - 07 1.8 Summary This unit provides the basic understanding of ordinary differential equation, its order and degree and certain basic classifications. 1.9 Exercises 1. Determine the order and degree of the following differential equation : a. 2 2 3 0

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dy y dx ? ? + = ? ? ? ? b. 2 2 2 d y dy xy dx dx ? ? + = ? ? ? ? ? ? ; c. 2 dy y dx = ; d. 2/3 2 2 3 dy d y dx dx ? ? = + ? ? ? ? ; e. 2/3 2 2 1 3 d y dy x dx dx ? ? + = ? ? ? ? ? ? ;			

f. 2 2 d y dx dy y e dx + =

NSOU ● CC ● MT - 07 15 Unit - 2 Structures 2.0 Objective 2.1 First Order Ordinary Differential Equations 2.2 Cauchy-Lipschitz Condition 2.3 Picard's Theorem 2.4 Solution Strategies for First Order and First Degree Differential Equation 2.5 Working procedure to solve an exact equation 2.6 Integrating Factor 2.7 Rules for Finding Integrating Factors (I. F.) 2.8 Summary 2.9 Exercise 2.0 Objective The objective of this unit is to discuss on various types of first order and first degree ordinary differential equations and their salution strategies. 2.1 First Order Ordinary Differential Equations q First Order and First Degree Ordinary Differential Equations : Standard form for a first order ordinary differential equation in the dependent variable is with the independent variable x is (,) dy f x y dx = , where f (x, y) is a continuous real valued function defined on some rectangular region in real xy-plane. An ordinary differential equation of first order and first degree (,)

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dy f x y dx = can be written as M(x, y)dx + N(x, y)dy = 0 2.2

Cauchy-Lipschitz Condition A function f(x, y) defined on a rectangular region R : |x – x 0 | > a, |y – y 0 | > b is xy-plane is said to satisfy Cauchy-Lipschitz condition if there exists a positive constant λ such that.
16 NSOU ● CC ● MT - 07 |f(x, y) – f(x, y 2)| ≤ λ | y 1 – y 2 | for all (x, y 1),(x, y 2) ∈R 2 . The above constant λ is known as Lipschitz constant for the corresponding function. 2.3 Picard's Theorem The first order and first degree differential equation (,) dy f x y dx = , where f(x, y) defined on a rectangular region R : |x – x 0 | > a, | y – y 0 | > b in is xy-plane, will have a unique solution subject to the following conditions : (i) f(x, y) is continuous in R ; (ii) | f(x, y) | ≤ M, where M is a fixed real number, for all (x, y) in R i.e, f(x, y) is bounded in R ; (iii) |f (x,y 1) – f(x, y 2)| ≤ λ |y 1 – y 2 | for all (x, y 1), (y, y 2) ∈ R, λ being the Lipschitz constant. 2.4 Solution Strategies for First Order and First Degree Differential Equation We can classify these equations according to the methods by which they are solved. (i) Equations with Separable Variables (ii) Homogeneous Equations (iii) Exact Equations (iv) Linear Equations (v) Bermouli Equations (i) Equations with Separable Variables : When a first order and first degree differential equation (,) dy f x y dx = can be arranged in the form ( ) ( ) , ( ) 0 f = y ^{1} y dy x y dx y then we have ψ(y)dy = φ (x)dx.. Integrating we have òψ(y)dy = òφ(x)dx + c, where c is an arbitrary constant. This method is known as method of separable variables. In other words, in standard form Mdx + Ndy =0, Where M = M(x) and N = N(y) then we can apply this method.
NSOU ● CC ● MT - 07 17 Example : Solve 2 2 3 1 dy x dx y = + , Solution : Here given one is a first order and first degree

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differential equation (,), dy f x y dx = where f(x, y) = 2 2 3 1 x y+ Now, f(x, y) = ( ) ( )x y

f y ,
where f(x), = 3x 2 , ψ(y) = 1 +
y 2 So, we can apply the method of separable variables. Thus òψ(y)dy = òf (x)dx + c, where c is an arbitrary constant. i.e., ò(1 + y 2)dy = ò3x 2 dx Therefore, 3 3 3 y y x c + = + , which is the required solution. Remarks : In the above example, 2 2 3 1 dy x dx y = + if we put it in the standard form, we have 3x 2 dx + {–1(1 + y 2)}dy = 0. Comparing this equation with the equation Mdx + Ndy = 0, get M = 3x 2 and N = {– (1 + y 2)}. It is clear M = M(x) and N = N(y). So observing this we can apply the above method. (ii) Homogeneous Equations : If a function f(x, y) can be expressed in the form either n y x x ? ? f ? ? ? ? or n x y y ? ? f ? ? ? ? then f(x, y) is said to be homogeneous function of degree n in x and y. When the function M and N are homogeneous functions of x and y of same order, then the differential equation Mdx + Ndy = 0 is called a homogeneous differential equation. There is another way to check the homogeneity of a first order and first degree equation
18 NSOU ● CC ● MT - 07 (,) dy f x y dx = . If f (tx, ty) = f(x, y) for any real t, then (,) dy f x y dx = is called a homogeneous differential equation. Remarks : A function f(x, y) is said to be homogeneous of degree n, if f(tx, ty) = t n f(x, y) in x and y and t be any non-zero real. For example we take 2 2 2 3 dy x dx x y = + We put the above in the form (,) dy f x y dx = , where 2 2 2 3 (,) x f x y x y = + Now for any real t(non-zero). f(tx, ty) = ( ) ( ) ( ) 2 2 2 2 2 2 3 3 (,) tx x f x y x y tx ty = = + + Therefore, the given differential equation is homogeneous. Again, here we have 2 2 2 3 dy x
dx x y = + So Mdx + Ndy = 0, where M = 3
x 2 and N = –

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x 2 – y 2. . Now, 2 y M x x ? ? = f ? ? ? ? and 2 y N x x ? ? = y ? ? ? ? , where y x ? ? f ? ? ? ? =3 and y x ? ? y ? ? ? ? = – 1 – 2 2 y x

It is clear that M and V are homogeneous functions in x and y of order 2. i.e., M and V are homogeneous functions of same order. Hence the given differential eqution 2 2 2 3 dy x dx x y = + is a homogeneous differential equation. Problems : Verify whether the following differential equation are homogeneous (i) ( ) 2 2 2 0
x y dx xydy - + = ,
NSOU ● CC ● MT - 07 19 (ii) 2 2 3 2 0

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dy x xy y dx - - = , (iii) 2 y x dy x y xe dx - = + , (iv) sin · sin y dy y x y x x dx x = + (v) 2 2 dy x x y

dx = + (
iii) Exact Equations : The differential eaquation Mdx + Ndy = 0 is called exact differential equation if there exists a function u = u(x, y) such that du = Mdx + Ndy and its general solution is u(x, y) = c, where c is an arbitrary constant. Thorem : The necessary and sufficient condition for the ordinary differential equation Mdx + Ndy = 0 to be exact on a rectangular region R : |x – x 0 | >a, | y – y 0 | > b in xy- plane is M N y x ¶ ¶ = ¶ ¶ in R. Note : xdx + ydy = d(xy) log xdy ydx y d xy x - ? ? = ? ? ? ? 1 2 2 tan xdy ydx y d x x y - ? ? - ? ? = ? ? ? ? ? ? + ? ? ( ) ( ) 1 2 2 sin 1 xdt ydx d xy x y - + = - Exmaple : Check whether the equation (

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x + y)dy + (y – x)dx = 0 is exact. Solution : Here we have (x + y)dy + (y – x)dx = 0 Comparing the eqution with Mdx + Ndy = 0, we have M = y – x, N = x + y 20 NSOU ● CC ● MT - 07 Now, 1 M N y x ¶ ¶ = = ¶ ¶ So, M N y x ¶ ¶ = ¶ ¶ By the statement of last theorem the given differential equation is

exact. Example : Check whether the equation ydx + xdy = xy(dy – dx) is exact or not. Solution : Here we have ydx + xdy = xy(dy – dx) i.e. (

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y + xy)dx + (x – xy)dy = 0 Comparing the equation with Mdx + Ndy = 0 we get M = y + xy, N = x – xy. Now 1 , M x y ¶ = + ¶ 1 , N y x ¶ = - ¶ So, M N y x ¶ ¶ 1 ¶ ¶ Hence the given equation is not exact. 2.5

Working procedure to solve an exact equation Step 1. Calculate ò Mdx treating y as constant and omitting arbitrary contant. Step 2. Calculate ò Ndy treating x as constant and omitting arbitrary contant. Step 3. Add with the result of step 1, the result of step 2 deleting those terms which are already been taken in step 1. Step 4. Equating the result in step 3 to an arbitrary constant, we get the general solution of the equation. Example : Solve (4

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x 3 + 3y 2 + cos x)dx + (6xy + 2)dy = 0. Solution : Here we have (4x 3 + 3y 2 + cos x)dx + (6xy + 2)dy = 0. Comparing this equation with Mdx + Ndy = 0, we get M = (4x 3 + 3y 2 + cos x), N = (6xy + 2) Now 6 M y y ¶ = ¶ , 6 N y x ¶ = ¶ NSOU ● CC ● MT - 07 21 So, M N y x ¶ ¶ = ¶ ¶ and hence the given equation is exact.

Now, ò Mdx = (4x 3 + 3y 2 + cos x)dx = x 4 + 3xy 2 + sin x, omitting arbitrary constant ò Ndy = ò (6xy + 2)dy = 3xy 2 + 2y, omitting arbitrary constant Therefore, x 4 + 3xy 2 + 2y + sin x = c, where c is an arbitrary constant, is the required solution. Example : Solve cos x. sin ydx + sin x.cos ydy = 0. Solution : Here we have cos x. sin ydx + sin x. cos ydy = 0 i.e. of the form Mdx + Ndy = 0, where M = cos x sin y and N = sin x. cos y. Now M y ¶ ¶ = cos x. cos y and N x ¶ ¶ = cos x. cos y. Hence the given differential equation is exact. Therefore, ò Mdx = ò cos x. sin ydx = sin x. sin y and ò Ndy = ò sin x. cos y dy = sin x. sin y Hence the required solution is sin x. sin y = c, where c is an arbitrary constant. Exercises : 1. Solve : (x + 2y) dx + (2x + y)dy = 0. 2. Solve : (2xy + 3

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x 2)dx + (x 2 + 2y)dy = 0 3. Solve : (6x + y 2)dx + y(2x − 3y)dy = 0 4. Solve : (y 2 − 2xy + 6x)dx − (x 2 − 2xy + 2)dy = 0 5. Solve : (2xy − y)dx + (x 2 + x)dy = 0 6. Solve : (2uv 2 − 3)du + (3u 2 v 2 − 3u + 4v)dv = 0 7. Solve : (cos 2 y − 3x 2 y 2)dx + (cos 2 y + 2x sin 2 y − 2x 2 y)dy = 0 8. Solve : (1 + xy 2)dx + (x 2 y + y)dy = 0. 9. Solve : (1 + y 2 + xy 2)dx + (x 2 y +

y + 2
xy)dy = 0
22
NSOU ● CC ● MT - 07 10. Solve : (w 2 + wz 2 − z)dw + (z 3 + w 2 z − w)dz = 0 11. Solve : (2xy − tan y)dx + (x 2 − x sec 2 y)dy = 0 12. Solve : (cos x cos y − cot x)dx − sin x sin y dy = 0 13. Solve : (r + sin t − cos t)dr + r(sin t + cos t) dt = 0 14. Solve : (3xy − 4y 3 + 6)dx + (x 3 − 6x 2 y 2 − 1)dy = 0 15. Solve : (sin t − 2r cos2 t)dr + r cos (2r sin r + 1)dt = 0 16.

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Solve : [2x + y cos (xy)]dx + x cos (xy)dy = 0 17. Solve : 2xydy + (y 2 + x 2)dy = 0 18. Solve : 2xy dx + (y 2 − x 2)dy = 0 19. Solve : (2x − 3y)dx + (2x − 3x)dy = 0 20. Solve :(3x 2 y 3 + 2xy)dx + (2x 2 y 3 − x 2)dy = 0 21. Solve : (x 3 + 3xy 2)dx + (y 2 + 3x 2 y)

dy = 0 2.6
Integrating
Factor Let Mdx + Ndy = 0 be a non-exact first order and first degree ordinary differential equation. A non-zero function μ = μ(x, y) is called an integrating factor of the equation Mdx + Ndy = 0 if μ(Mdx + Ndy) = 0 becomes an exact differential equation i.e. μ(x, y) is said to

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be the integrating factor of the differential equation Mdx + Ndy = 0,

if we can find u = u(x, y) such that μ(Mdx + Ndy) = du = 0 Theorem : The number of integrating factors of an equation Mdx + Ndy = 0 is infinite. 2.7 Rules for Finding Integrating Factors (I. F.) Rule 1. If the given equation Mdx + Ndy = 0 is a homogeneous such that Mx + Ny ≠ 0, then ( ) 1 Mx Ny + is an integrating factor ( ) . . . I F Example : Solve : 2 2 dy x y dx xy + =
NSOU ● CC ● MT - 07 23 Solution : Here the given equation can be written
in

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the form Mdx + Ndy = 0, where M = x 2 + y 2 ; N = − xy. Now, 2 M y y ¶ = ¶ , N y x ¶ = - ¶ Therefore, M N y x ¶ ¶ ¹ ¶ ¶ , so the given differential equation is not exact.

Now
Mx + Ny = x(x 2 + y 2) + y(− xy) = x 3 + xy 2 − xy 2 = x 3 ≠ 0 So, 3 1 1 I.F
Mx Ny x = = + Multiplying I. F to the both sides of the given equation we have ( ) 2 2 3 3 1 0 xy

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x y dx dy x x + - = or, 2 3 2 0 dx y y dx dy x x x + - = or, ( ) 2 log 0 y ydx xdy d x x x ? - ? + = ? ? ? ? or, ( ) log 0 y y d x d x x ? ? - = ? ? ? ? . Integrating we get 2 1 log 2 y x c x ? ? - = ? ? ? ? ,

where c
is an arbitrary constant. Example :

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Solve (x 2 y – 2xy 2)dx + (3x 2 y – x 3)dy = 0 Solution : Here, ( ) ( ) 2 2 2 3 2 , 3 M x y xy N x y x = - = -

Therefore, M N y x ¶ ¶ [1] ¶ ¶ So, the given differential equation is not exact.
24 NSOU ● CC ● MT - 07 Here, 2 2 2 3 2 2 (2) (3) 0 Mx Ny
x

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x y xy y x y x x y + = - + - = [1] So, 2 2 1 . .= I F x y Multiplying I. F. to the both sides of the given equation we have ( ) ( ) 2 2 2 3 2 2 1 2 3 0 x y xy dx x y x dy x y ? ? - + - = ? ? ? ? Or, 2 1 2 3 0 x dx dy dy y x y y ? ? - + - = ? ? ? ?

Or, x
d y ? ? ? ? ? ? – 2
d(log x) + 3d(
log y) = 0
Integrating we get 2log 3log
x

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x y c y - + = , where c is an arbitrary constant. Example : Solve (y 3 – 2x 2 y)dx + (2xy 2 – x 2)dy = 0 Solution : Comparing the given differential equation with Mdx + Ndy = 0, we get M = (y 2 – 2x 2 y), N = (2xy 2 – x 3) Therefore M N y x ¶ ¶ [1] ¶ ¶ , So, the given differential equation is not exact.

Now, Mx + Ny =
x(y 3 – 2x 2 y) + y(2xy 2 – x 3) = 3xy(y 2 – x 2) ≠ 0 So. I. F. = ( ) 2 2 1 3xy y x- Multiplying I. F. to the both sides of the given equation we have ( ) ( ) ( ) ( ) 3 2 2 2 2 2 2 2 2 2 0 3 3

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y x y xy x dx dy xy y x xy y x - - + = - - NSOU ● CC ● MT - 07 25 or, ( ) ( ) ( ) ( ) 3 2 2 2 2 2 2 2 2 2 0 3 3 y x y xy x dx dy x y x y y x - - + = - - or, 2 2 0 dx dy ydy xdx x y y x - + + = - or, 2d(log x) + 2d(log y) + d(log (y 2 – x 2)) = 0 Integrating we get log x 2 + log y 2 + log (y 2 – x 2) = log c i.e. x 2 y 2 (y 2 – x 2) =

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c, where c is an arbitrary constant. Rule : 2. If Mx – Ny ≠ 0 and the equation can be written as {f(xy)}ydx + {g(xy)}xdy = 0, i.e. Mdx + Ndy = 0 then the integrating factor of the given equation is of the form 1 Mx Ny - Example : Solve ( ) ( ) xysin(xy) cos(xy) ydx xysin(xy) cos(xy) xdy 0 + + - = Solution : Here given differential equation is of the form f(xy)ydx + g(xy)xdy = 0 where f(xy) = (xy sin (xy) + cos (xy)), g(xy) = (xy sin (xy) – cos (xy)) Here, M = (xy sin (xy) + cos (xy)) y and N = (xy sin(xy) – cos(xy)x Now Mx – Ny = 2xycos(xy) So, I. F. = ( ) 1 2 cos xy xy Multiplying I. F. to the both sides of the given equation we have ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) sin cos sin cos 0 2 cos 2 cos xy xy xy xy xy xy ydx xdy xy xy xy xy + - + = or, ( ) ( ) 1 1 1 1 tan tan 0 2 2 xy ydx xy xdy xy xy ? ? ? ? + + - = ? ? ? ? ? ? ? ?
26 NSOU ● CC ● MT - 07 or, ( ){ } 1 1 tan 0 2 2 dy dx xy ydx xdy x y ? ? + + - = ? ? ? ? or, tan (xy) d(xy) + d(log x – d(log y) = 0 Integrating we have, log | see(xy)| + log x – log y = log c or, x sec (xy) = cy, where c is an arbitrary constant. Rule : 3. If 1 N M N y x ? ? ¶ ¶ - ? ? ¶ ¶ ? ? be a function of x only, say φ (x), then ( ) x dx e ∫ f
is an integrating factor of the given equation Mdx + Ndy = 0.
Example :

46%	MATCHING BLOCK 25/123	SA	Differential Equations(final version).pdf (D152427504)

Solve (x 2 + y 2 + 2x)dx + 2ydy = 0 Solution : Here M = (x 2 + y 2 + 2x), N = 2y Therefore, 2 , 0 M N y y x ¶ ¶ = = ¶ ¶ Therefore, M N y x ¶ ¶ 1 ¶ ¶ . So, the given differential equation is not exact. Now, ( ) 1 1 2 0 2 M N y N y x y ? ? ¶ ¶ - = - ? ? ¶ ¶ ? ? = 1 = φ (x) (say) Thus I. F. = ( ) 1. I.F. x dx dx x e e e ∫f ∫ = = = Multiplying I. F. to the both sides of the given equation we have e x (x 2 + y 2 + 2x)dx + 2ye x dy = 0 or, e x dx + 2xe x dx + y 2 e

x dx + 2ye x dy = 0 or, d(e x x 2) + d(y 2 e x) = 0 Integrating we get e x x 2 +
e x
y 2 = c, where c
is an arbitrary constant. Rule : 4.

60%	MATCHING BLOCK 26/123	SA	Differential Equations(final version).pdf (D152427504)

If 1 N M M x y ? ? ¶ ¶ - ? ? ¶ ¶ ? ? be a function of y alone. say φ (y), then ( )y dy e ∫

f is an integrating factor of the given differential equaton Mdx + Ndy = 0.
NSOU ● CC ● MT - 07 27 Example :

55%	MATCHING BLOCK 27/123	SA	Differential Equations(final version).pdf (D152427504)

Solve (3x 2 y 4 + 2xy)dx + (2x 3 y 3 – x 2)dy = 0 Solution : Comparing with the equation Mdy + Ndy = 0, we have M= (3x 2 y 4 + 2xy), N = (2x 3 y 3 – x 2) Therefore, 2 3 12 2 M x y x y ¶ = + ¶ , 2 3 6 2 N x y x

x ¶ = - ¶ So, ( ) ( ) 2 2 2 3 2 1 1 1 6 2 12 2 3 2 N M x y x x y x
M
x y
xy xy ? ? ¶ ¶ - = - - - ? ? ¶ ¶ ? ? + = 2 y - = φ(y) (
say) which is a function of y only. Thus, I. F. = 3 2
log 4 2 1 dy y e e y -∫- - = = Multiplying I. F. to the both sides of the given equation we have ( ) ( ) 2 4 3 2 2 2 2 1 1 3 2 2 0
x
y xy

39%	MATCHING BLOCK 28/123	SA	partial Differential Equation.pdf (D142231462)

dx x y x dy y y + + - = or, 2 2 2 3 2 3 2 2 0 x x x y dx dx x ydy dy y y + + - = or, ( ) 2 3 2 2 2 0 xydx x dy d x y y - + = or, ( ) 2 3 2 0 x d x y d y ? ? + = ? ? ? ? ? ? Integrating we get 2 3 2 x x y c y + = 28

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NSOU ● CC ● MT - 07 Rule 5. If Mdx + Ndy = 0 can be expressed in the form x α y β (mydx + nxdy) + x λ y β (m 1 ydx + n 1 xdy)=0, where α, β, g, γ, δ, m, n, m 1 , n 1 , are constant and mn 1 − nm 1 ≠ 0, then x h y k is an integrating factor of the given equation Mdx + Ndy = 0, where 1 1 h k m n a + + b+ + = and 1 1 1 1 h k m n g + + d + + = Example : Solve x 2 (2ydx + 3xdy) + y 2 (−2ydx + 2xdy) = 0 Solution : We can rewrite the iven equation in the following form : x 2 y 6 (2ydx + 3xdy) + x 6 y 2 (− 2ydx + 2xdy) = 0 i.e., x α y β (mydx + nxdy) + x λ y δ (m 1 ydx + n 1 xdy) = 0 where, a = 2, b = 0, y = 0, d = 2, m = 2, n = 3, m 1 = − 2, n 1 = 2. Therefore, I. F. = x h y k where 1 1 h k m n a + + b + + = , 1 1 1 1 h k m n g + + d + + = i.e. 2 1 0 1 , 2 3 h h + + + + = 0 1 2 1 , 2 2 h k + + + + = - Solving the above equations we have h = − 3 and k = − 1. Hence, I. F =

31%	MATCHING BLOCK 29/123	SA	partial Differential Equation.pdf (D142231462)

x −3 y −1 ( ) ( ) 3 1 2 3 1 2 · 2 3 · 2 2 0 x y x ydx xdy x y y ydx xdy - - - - + + - + = i.e 2 2 3 2 0 dx dy y ydx xdy x y x x ? - + ? + + = ? ? ? ? or, d(2 log x) + d(3 log y) + d 2 2 y x ? ? ? ? ? ? ? ? ? = 0 Integrating above we get 2log x + 3log y + 2 2 y x = c.

where c is an arbitrary constant. (iv) Linear first order ODE : A particular type of first order and first degree ordinary differential equation of the

NSOU ● CC ● MT - 07 29 form + = dy Py Q dx , where each of P and Q is either a function of x only or a constant, is called a Linear Ordinary Differential Equation of first order in y. For the above form of ODE Pdx e ∫ is an interating factor (I.F) i.e. the given ODE can be integrated on multiplying this factor to both the sides. This can be evident from the following analysis. Multiplying both sides of the iven ODE by Pdx e ∫ we have . . . Pdx Pdx dy e e Py e Q dx ∫ ∫ + = which gives . . Pdx Pdx d ye e Q dx ? ? = ? ? ? ? ∫ ∫ or, . . Pdx Pdx d ye e Q dx ? ? ? ? = ? ? ? ? ? ? ? ? ? ∫ ∫ Integrating above we can have the desired solution through the following step : . . Pdx Pdx ye e Q dx c ? ? ∫ ∫ = + ? ? ∫ i.e ( ) ( ) . . . . . . = + ∫ y I F I F Q dx c where 'c' is an arbitrary constant. We can summarize the steps involved in solving such equations. Step 1. Put the equation in the form + = dy Py Q dx Step 2. Obtaint I.F. as Pdx e ∫ . Step 3. Simplify ( ) ( ) . . . . , = + ∫ y I F I F Qdx c where c is an integration constant.

30 NSOU ● CC ● MT - 07 Example : Solve ( ) 2 3 2

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dy 4 1 y dx 1 1 x x x + = + + Solution : Here ( ) 2 3 2 4 1 P , Q 1 1 x x x + = + + Here integrating factor is given by I.F. = ( ) ( ) 2 4 2 log 1 2 2 2 1 1 x dx x Pdx x e e e x ∫ + + ∫ = = = +

Hence we have, ( ) ( ) . . . . . y I F I F Qdx c = + ∫ ie. ( ) ( ) ( ) 2 2 2 2 3 2 1 . 1 1 . 1 y x x dx c x + = + + + ∫ or, ( ) 2 2 −1 . 1 tan + = + y x x c (v) Bernoulli's Equations : The first order ordinary differential equation of the form + = n dy Py Qy dx where P and Q are continuous function of x and n is a real number, is known as Bernoulli's Equation. From + = n dy Py Qy dx we have − 1− + = n n dy y Py Q dx If we put 1− = n y v then we can have ( ) − 1− n dy dv n y dx dx = Thus the quation transforms to ( ) ( ) 1− 1 + = − dv n Pv n Q dx which is a first order liner ODE in v, its integrativing factor being ( ) Pdx 1−n e ∫ .

NSOU ● CC ● MT - 07 31 Then its solutionis given by ( ) ( ) ( ) . . . 1− . . . v I F n Q I F dx c = + ∫ ie. ( ) Pdx Pdx (1−n) (1−n) v.e 1− n Q. e dx + c = ∫ ∫ ∫ or, ( ) ( ) ( ) 1−n Pdx 1−n Pdx 1−n y .e 1− n Q.e dx +c = ∫ ∫ ∫ where c is an arbitrary constant. Example : Sove 2 2 2 4 . = + dy x xy y dx Solution : Here 2 2 −2 1 . 2 dy y y dx x x ? ? + = ? ? Therefore, 2 dy Py Qy dx + = . where 2 −2 1 , 2 = = P Q x x We put 1− 1−2 1 = = = − n v y y y . So 2 1 − . = dy dv dx dx y Now we can have 2 1 1 − . − . − , = dy P Q dx y y i.e., − . − , = dv Pv Q dx Which is a first order lincar ODE in v. Therefore integrating factor of the above is I.F. ( ) 2 2 2 log dx

48%	MATCHING BLOCK 31/123	SA	DSC-6 Combine.pdf (D143717932)

P dx x x x e e e − = = = = ∫ ∫ Hence ( ) 2 2 2 −1 . . 2 v x x dx x = ∫ i.e, 2 1 . − 2 = + x x c y 32 NSOU ● CC ● MT - 07 2 2 + = x x c y where c is an integrating constant. 2.8 Summary The present unit emphasizes on first order and first degree ordinary differential equations with the conditions of haring unque solution and different working procedures to solve them analytically. 2.9 Exercises (A)

64%	MATCHING BLOCK 32/123	SA	DSC-6 Combine.pdf (D143717932)

Solve the following exact equations : 1. ( ) ( ) 2 2 0 + + + = x y dx x y dy 2. ( ) ( ) 2 2 2 3 2 0 + + + = xy x dx x y dy 3. ( ) ( ) 2 6 2 – 3 0 + + = x y dx y x y dy 4. ( ) ( ) 2 2 – 2 6 – – 2 2 0 + + =

y
xy x
dx x
xy dy 5. ( ) ( ) 2 2 – 0 + + = xy y
dx x
x
dy 6. ( ) ( ) 2 2 2 2 – 3 3 – 3 4 0 + + =
v uv du u v u v dv 7. ( ) ( ) 2 2 2 2 2 3 cos – 3 cos – 2 sin – 2 0 + =

57%	MATCHING BLOCK 33/123	SA	Chewang Tenzin Doya (M.Ed Math).pptx (D74940038)

y x y dx y x y x y dy 8. ( ) ( ) 2 2 1 0 + + + = xy dx x y y dy 9. ( ) ( ) 2 2 2 1 2 0 y xy dx x y

y
xy dy + + + + + = 10. ( ) ( ) 3 2 3 2 – – 0
w wz z dw z w z w dz + + + = 11. ( ) ( ) 2 2 2 – tan – sec 0 + = xy y dx x x y dy
NSOU • CC • MT - 07 33 12. ( ) cos cos – cot – sin sin 0= x y x dx x y dy 13. ( ) ( ) sin – cos sin cos 0 + + + = r t t dr r t t dt 14. ( ) ( ) 3 3 2 2 3 – 4 6 – 6 –1 0 x xy y dx x x y dy + + = 15. ( ) ( ) 2 sin – 2 cos cos 2 sin 1 0 + + = t r t dr r t
r t dt 16. ( ) ( ) 2 cos cos 0 ? + ? + = ? ?

44%	MATCHING BLOCK 34/123	SA	Differential Equations(final version).pdf (D152427504)

x y xy dx x xy dy 17. ( ) 2 2 2 0 + + = xydx y x dy 18. ( ) 2 2 –2 – 0 + = xy dx y x dy 19. ( ) ( ) 2 – 3 2 – 3 0 + = x y dx y x dy 20. ( ) ( ) 2 3 3 3 2 3 2 2 – 0 + + = x y xy dx x y x dy 21. ( ) ( ) 3 2 3 2 3 3 0 + + + = x xy dx y x y

dy
B.
Solve the following Equation : 1. ( ) ( ) 1 1– + = y dx x
dy 2. 2 2 cos cos = x ydx y xdy 3. ( ) 2 = + y x e dy e x dx 4. ( ) 2 –1 1 tan 0 + + = ydx x xdy 5. 2 2 1– 1– x y dx y x
dx = 6. 2 2 .tan . sec .tan . 0 + = Sec x ydx y xdy 7. 2 log 1– 0 + = x xdy y dx
34 NSOU • CC • MT - 07 8. 2 –1 –1 2 cos . cos . 0 + = x

39%	MATCHING BLOCK 35/123	SA	DSC-6 Combine.pdf (D143717932)

y dy y xdx 9. ( ) 2 3 . tan – 1– sec 0= x x e y dx e ydy 10. ( ) 4 0 y x y x e dx e dy + + + = 11. dy y x dx = – 12. ( ) ( ) 2 1 . 1 + = + x x e ydy y e dx 13. dy = y.

secx. dx (C) Determine whether the given ODE is exact or not and if exact find the solution : 1. ( ) ( ) 3 2 2 0

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16%	MATCHING BLOCK 36/123	SA	partial Differential Equation.pdf (D142231462)

x y dx x y + + + = 2. ( ) ( ) 2 3 2 − 4 0 + + = y dx xy dy 3. ( ) ( ) 2 2 1 4 0 + + + = xy dx x y dy 4. ( ) ( ) 2 3 3 2 − 0 + + = x y dx x y dy 5. ( ) ( ) 2 2 6 2 − 5 3 4 − 6 0 + + + = xy y dx x xy dy 6. 2 (6sec tan) (tan 2) 0 x sec x x dx x y dy + + + = 7. 2 2 3 0 ? ? ? ? + + + = ? ? ? ? ? ? ? ? x x x dx y dy y y (D) Solve the followings : 1. ( ) ( ) 2 2 − 3 4 0, (1) 2. + + = = xy dx x y dy y 2. ( ) ( ) ( ) 2 2 3 3 2 3 − 2 2 − 3 1 0, −2 1 + + + = = x y y x dx x y xy dy y NSOU ● CC ● MT - 07 35 3. ( ) ( ) 2 2 2 sin cos sin sin − 2 cos 0, (0) 3 + + = = y x x y x dx x y y dy y 4. ( ) ( ) 2 2 2 sin sin − 2 cos 0, (0) 3 + + + = = x x ye e y x dx x y x dy y E. Solve the following differental equation : 1. ( ) 3 2+ = x y dy ydx 2. cot − tan 0= y dx xdy 3. ( ) ( )− 0 x y dy y x

dx + + = 4. ( ) − + = ydx xdy xy dy dx 5. (−) 0 + + = xdx ydy k xdy ydx 6. 1 − − cos . 0 ? ? = ? ? xdy ydx
dx x 7. 2 sin
cos

76%	MATCHING BLOCK 37/123	SA	Differential Equations(final version).pdf (D152427504)

dy x y y x dx + = 8. 2 log + = dy x y y x dx 9. 2 2 2 2 1− 0 + + = dy x xy x y dx 10. ( ) 2

cos(sin( ))
cos( ) 0 + + = xy xy xy dx x xy dy 11. ( ) 2 sin .cos (cos .sin

20%	MATCHING BLOCK 38/123	SA	Differential Equations(final version).pdf (D152427504)

tan) 0 + + + = x x y e dx x y y dy 12. ( ) ( ) 2 2 1 4 2 1 4 2 0 + + + + + = xy y dx xy x dy 13. ( ) ( ) 1 1− 0 + + = xy ydx xy xdy 14. ( ) ( ) 2 2 2 2 1 3 6 1 3 6 0 + + + + + = x xy dx y x y dy 15. 1 log 2 0 x y dx y dy x y ? ? ? ? + + + = ? ? ? ? 36 NSOU ● CC ● MT - 07 16. ( ) 2 − 0 + = x x xy e ydx e dy 17. ( ) 2 3 3 − 0 x ydx x y dy + = 18. ( ) ( ) 2 2 2 2 1 − 1 0 + + + + = x y xy ydx x y xy xdy 19. ( ) ( ) 2 2 2 2 3 3 6 0 + + + + = x y dx x x y y dy 20. ( ) ( ) 3 2 2 4 2 0 + + + + = xy y dx x y x y dy (

F) Prove that 2 x e is an
integrating factor of the equation : ( ) 2 4 3 2 0. + + = x xy dx y dy (G) If x a y b be an integrating factor of the equation (2y dx + 3x dy) + 2xy(3y dx + 4x dy) = 0, find a and b. (H) If α β x y be an integrating factor of the equation ( ) ( ) 1 4 1 3 3 2 3 0 x y dx y xy dy − − − − + − + = , then find the values of α and β . I. Solve : ( ) .

37%	MATCHING BLOCK 39/123	SA	Differential Equations(final version).pdf (D152427504)

cos . .sin cos 1 + + = dy x x y x x x dx J. Solve : 2 − 2+ = x dy xy e dx K. Solve : ( ) 2 2 1 2 4 + + = dy x xy x dx L. Solve : 2 cos . tan . + = dy x y x

dy M. Solve : ( ) 2 3 2+ = x y xy dy dx N. Solve : ( ) 2 2 .log . log + = dy y y y y dx x x
NSOU ● CC ● MT - 07 37 Unit - 3 Structures 3.0 Objective 3.1 Equation of first order but not of first degree 3.2 Singular Solution 3.3 Second Order Differential Equation 3.4 Theorem : Existence Theorem 3.5 Theorem : Uniqueness Theorem 3.6 Wronskian 3.7 Theorem : Principle of Superposition 3.8 Theorem 3.9 Method of finding the particular integral (P. I) 3.10 Properties of D-operator 3.11 Homogeneous Linear Differential Equations with Variable Coeffcicients 3.12 Method of Undetermined Coefficients 3.13 Method of Variation of Parameters 3.14 Simultaneous Linear Differential Equations with Constant Coefficients 3.15 Series Solution of the Ordinary Diffrential Equations 3.16 Note : Test of Singularity at Infinity 3.17 Series Solution about an Ordinary Point 3.18 Series Solution about Regular Singular Point (Frobenius Method) 3.19 Bessel's Equation 3.20 Application of Bessel's Equation 3.21 Solution of Bessel's Equation : Bessel's Function 3.22 Solution of Legendre's Equation : Legendre Polynomial 3.23 Application of Ordinary Differential Equation to Dynamical Systems 3.24 Dimension of a Dynamical System 3.25 Equilibrium Point of A Flow

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38 NSOU • CC • MT - 07 3.26 Analysis of Stability of an Equilibrium Point of a One Dimensional Flow 3.27 Stability Analysis of The Equilibrium Points 3.28 Summary 3.29 Exercise 3.0 Objective The objective of the present unit is to discuss on the various aspects of first order but not of first degree and second order ordinary differential equations; the strategy of series solution and some basic discussions dynamical systems as an application. 3.1 Equation

62%	MATCHING BLOCK 40/123	W
of first order but not of first degree An ordinary differential equation of first order and		

n-th degree can be written as— −1 0 1 −1 .... 0 + + + + = n n n n Q p Q p Q p Q (A) where = dy p dx and Q 0 , Q 1 ,........Q n are functions of x and Q 0 ≠ 0. There can be three special cases for the above equation : (a) Solvable for p. (b) Solvable for x. (c) Solvable for y. (a) Solvable for p : Let us assume that the left hand side of differential equation (A) can be expressed as a product of n-linear factors in p by the following form : ( ) ( ) ( ) ( ) ( ) ( ) 1 2 − , . − , ... − , 0= n p

52%	MATCHING BLOCK 41/123	SA	Differential Equations(final version).pdf (D152427504)
f x y p f x y p f x y i.e. ( ) ( ) ( ) 1 2 , , , ..., , = = = n p f x y p f x y p f x y]			

all of which are first order and first degree equations. Solving each of the equations we can have the solutions as— ( ) ( ) ( ) 1 1 2 2 , , 0, , , 0, ..., , , 0 n n F x

59%	MATCHING BLOCK 42/123	SA	DSC-6 Combine.pdf (D143717932)
y c F x y c F x y c = = = (B) NSOU • CC • MT - 07 39 where 1 2 , ..., n c c c are constants.			

As the differential equation (A) is of the first order we must have only one arbitary constant in its general solutiou. without loss of generality 1 2 , ..., n c c c can be replaced by a single arbitrary constant c. Thus the general solution of the differential equation i.e, one parameter soluton of the equation is given by— ( ) ( ) ( ) 1 2 , , . , , ... , , 0 n F x

64%	MATCHING BLOCK 43/123	SA	DSC-6 Combine.pdf (D143717932)
y c F x y c F x y c = , where c is an arbitrary constant. Example :			

Solve : p 2 + 2xp − 3x 2 = 0 Solution : Now p 2 + 2xp − 3x 2 = 0 i.e : (p−x) (p+3x) = 0 So, p = x and p+3x = 0 i.e., , −3 = = dy dy x x dx dx Integrating we get 2 2 1 2 3 , − 2 2 = + = + x y c y x c As the given differential equation of the first order, we must have only one arbitrary constant in its general solution i.e. c 1 , c 2 , can be replace by a single arbitrary constant c. Hence the general solution is — 2 2 3 − − − 0 2 ? ? ? ? + = ? ? ? ? ? ? x y c y x c z where c is an arbitrary constant. (b) Solvable for x : If the differential equaton (A) be solvable for x, then it may be put in the form x = f(y,p) (C) Now 1 1 = = dx dy dy p dx Thus differentiating w.r.t. y we get the following form : 1 , ? ? = ? ? ? ? dp F y p p dy (D)
40 NSOU • CC • MT - 07 Eliminating p between (C) and (D) , we get the solution of the differential equation as a relation between x,y and arbitrary constant c. If the elimination of p is difficult x and y may be expressed in terms of p where p acts as a parameter. Example : Solve : 2 − 1 = + p x a p (a) Solution : The given equation can be written as : This equation of the form x = f(y,p) Differentiating both sides with respect to

47%	MATCHING BLOCK 44/123	SA	Differential Equations(final version).pdf (D152427504)
y we get. ( ) 3 2 2 2 2 1 1 − . . 1 1 = + + + dp p dp p dy dy p p i.e., ( ) 3 2 2 1 1 . 0 1 + = + dp p dy p i.e., ( ) 3 2 2 . 1 p dy dp p = − + Integrating, we get 2 1 1 + = +			

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y c p i.e., ( ) 2 2 1 1 + = + y c p , (b) where c is an arbitrary constant. Now from (a), ( ) 2 2 2 − 1 p x a p = + (c) Eliminating p from (b) and (c) we get ( ) ( ) 2 2 − 1 x a y c + + = , which is the general salution of (a).
NSOU ● CC ● MT - 07 41 (c) Solvable for y : If the differential equation (A) be solvable for y then it may be put in the from y = f (x,p). (E) Differentiating both sides of (E) with respect of x we have an equation of the form , , dp p F x p dx ? ? = ? ? ? ? Now it can be solved to get solution of the form ( ) , , 0 p x c φ = Eliminting p between (E) and (F)

22%	MATCHING BLOCK 45/123	SA	Differential Equations(final version).pdf (D152427504)

we get the general solution of the differential equation (A). Example : Solve .tan log( ); dy y p p cosp p dx = + = . Solution : The equation is of the form ( ), y f x p = . Differentiating both sides with respect to x, we get ( ) 2 tan .sec – tan dp p p p p p dx = + i.e 2 . . dp p p sec p dx =

i.e. dx = sec 2 p. dp. Integrating botht sdid we tet x+c = tanp, where c is an arbitrary constant. Then ( ) −1 tan p x c = + and ( ) 2 1 cos 1 p x c = + + Thus the general solution is ( ) ( ) −1 2 1 tan log 1 ( ) y x c x c x c ? ? = + + + ? ? ? ? + + ? ? Lagrange Equation : A first order ODE of the form ( ) . ( ) y x p p = φ + ψ (G)
42 NSOU ● CC ● MT - 07 where dy p dx = and φ (p) and ( )p ψ are known functions of p diferentiable on a certain interval, is called Lagrange Equation. Now differentiatinm (G) with respect to x we have ( ) ( ) ( ) { ' ' }. . dp p p x p p dx = φ + φ + Ψ i.e. ( ) ( )' '( ) . − ( ) − p p dx x dp p p p p ψ φ + = φ φ which is a linear equation in x. This can be solved easily and eliminating p from this solution and the given equation will give us the complete solution. Example : Solve : y = 2xp−p 2 Solution : Here given equation is of the form ( ) ( ) . y x p p = φ + ψ (a) where ( ) ( ) 2 2 , − p p p p φ = ψ = So, it is a Lagrange equation. Differentiating (a) with respect to x we have ( ) ( ) ( ) { ' ' }

44%	MATCHING BLOCK 46/123	SA	Differential Equations(final version).pdf (D152427504)

dp p p x p p dx = φ + φ + ψ i.e ( ) { } 2 2 −2 . dp p p x p dx = + + or, − . 2 − 2 dx p x p dp = or, 2 2 dx x dp p + =

which is linear in x. Therefore integrating factor of the differential equation (b) is given by— I.F. = 2 log 2 2 dp p e e p p = = ∫
NSOU ● CC ● MT - 07 43 So, the solution of (b) is— 2 2 . 2 x p p dp c = + ∫ i.e , 2 3 2 . 3 x p p c = + , where c is an arbitrary constant. or, 2 2 3 p c x p = + Now putting this value of x in the given equation, we get 2 2 3 p c y p = + Thus

32%	MATCHING BLOCK 47/123	SA	Differential Equations(final version).pdf (D152427504)

the general solution is given by 2 2 3 p c x p = + and y = 2 2 3 p c p + , where p is the parameter. Clairaut's Equation : An ODE of the form y = px + f(p) (

H) is known as Clairaut's Equaton. Now differentiating both sides of (H) with respect to x we have. ( ) { ' }.

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dp p p x p dx = + + φ i.e ( ) { ' }. 0 dp x p dx + φ = This gives either 0 dp dx = (I) or, '( ) 0 x p +

φ = (J) From (I) we get p=c, where c is an arbitrary constant. Putting this value of p=c in (H) we get ( ) y cx c = + φ which is the general solution of this Clairaut's equatiou. Again eliminating p from (H) and (J) we get another soluton which does not contain any arbitrary constant. This solution is caled the singular soluton of the Clairaut's equation (H). Example : Find the general and singular solution of
44 NSOU ● CC ● MT - 07 ( )( ) − −1 y px p p= where dy p dx = Solution : The given equation ( )( ) − −1 y px p p= can be written as −1 p y px p = + , which is a Clairaut's equation. (a) Then differentiating both sides with respect to

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x we get ( ) 2 1 . − . −1 dp dp p p x dx dx p = + i.e. ( ) 2 1 − . 0 −1 dp x dx p ? ? ? ? = ? ? ? ? ? ? i.e either 0 dp dx = or, ( ) 2 1 − 0 −1 x p = Now 0 dp dx = gives p=c..................(

b) Eliminating p from (a) & (b) we get the gensal solution as c y cx c 1 = + - where c is an arbitrary constant Again ( ) 2 1 − 0 −1 x p = gives ( ) 2 1 −1 p x = or 1 x p x + = ...(c) Eliminating p from (a) and (c) we have 1 1 . 1 x x x y x x x + + = + NSOU • CC • MT - 07 45 i.e ( ) 2 − −1 4 y x x = . This is

70%	MATCHING BLOCK 50/123	W

the singular solution of the given equation. Exercises : 1. Find the general and singular solution of 2

y xp p = + , where dy p dx = 2. Find the general and singular solution of 2 1 y xp p = + + , where dy p dx = . 3. Sove the folowing differential equations i. 3 4 4 x p p = + ii. − x py p = iii. .2 2 .log y y xyp p = + iv. 2 3 2 y px y p = + v. ( ) ( ) 2 2 2 −1 − xy p x y p = vi. 2 − 2 0 xp yp ax + = vii. 2 2 6 − 3 0 p y y px + = viii. ( ) 2 − . − 0 xp y x p y + = 4. Solve : ( ) 2 2 − x y px p y = 5. Reduce

71%	MATCHING BLOCK 51/123	W

the differential equation ( ) 2 2 2 0 x p py x y + + = in Clairaut's form by the substitution y = u, xy = v and hence

solve the differential equation. 6. Use the transformation 2 2 , u x v y = = to solve the euation ( )( ) 2 −px y py x h p + = 7. Use the transformation 2 , − u x v y x = = to solve the equation 2 − 2 2 0 xp yp x y + + = 8. Use the transformation 1 1 , u v x y = = to solve the equation ( ) 2 4 2 − y y px x p =

46 NSOU • CC • MT - 07 3.2 Singular Solution A singular solution is a solution of the given first order higher degree differential equation which is not obtained

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from the general solution by assigning particular values to the arbitrary constant

involved in it. It is the equation of an envelope of the family of curves represented by the general solution. Let ( ) , , 0 x y c φ = represent a family of curves. From the notion of envelope it can be found that the c-discriminant of ( ) , , 0 x y c φ = is the c-eliminant of ( ) , , 0 x y c φ = and 0 c ∂φ = ∂ provided ( ) , , , x y c c ∂φ φ ∂ are continuons in the domain of the differential equation. As for example let the family of curves be y 2 = 4cx. We consider ( ) 2 , , 4 − x y c cx y φ = . Then c ∂φ ∂ = 4x Eliminating c from ( ) , , 0 x y c φ = and, c ∂φ ∂ = 0, we get x = 0, y = 0 i.e. x = y = 0 gives the required c-discriminant. Let f(x,y,p) = 0 denote a first order differential equation. The p-discriminant of the equation f(x,y,p) = 0 is defined as the p-eliminant between the equation f(x,y,p) = 0 and 0 f p ∂ = ∂ provided f(x,y,p), f p ∂ ∂ are continuous in the domain of the differential equation. The p-discriminant represents the locus for each of the point of

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which f(x,y,p) = 0 has equal values of p.

As for example we consider a differential equation ( ) 2 − − 0
p
y p

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x y x + = . Let. ( ) 2 , , (−) − 0 f x y p p y p x y x = + = . Then 2 − f py x y p ∂ = + ∂ Eliminating p from ( ) , . 0 f x y p = and 0 f p ∂ = ∂ , we get. NSOU • CC • MT - 07 47 ( ) 2 − − − 2 2 y x y x y x y x y y ? ? ? ? + = ? ? ? ? ? ? ? ? i.e. ( ) 2 0 x y+ = or, 0 x y+ = , which is the required

p-discriminant. Remark : It is easy to observe that the equations are of the same degree in c and p, and therefore whenever there is a p-discriminant, there is a c-discriminant. Note : The singular solutions of a differential equation can be found by exploring the following situations : (a) p-equation has multiple roots. (b) c-equation has multiple roots. Envelope of a system of curves ( ) , , 0, x y c φ = if it exists, satisfies the differential equation ( ) , , 0 f x y p = and this soluton is evidently a singular solution. Thus if ( ), 0 E x y = represents the envelope then E is a factor of both c-discriminant and p- discriminant and also the soluton of the differential equation. We have already seen that both the p-discriminant and c-discriminant of ( ) , , 0 f x y p = and its solution ( ) , , 0 x y c φ = respectively contain the envelope (if it exists) of the system of curves ( ) , , 0 x y c φ = . But it can be seen that the c-discriminant and p-discriminant contain other loci which are different from the envelope and generally they do not satisfy the differential equation. These are called extraneous loci. Not the p-discriminant relation gives the locus of such points for which p has at least two equal values. It may so happen that these two equal values of p belong to two distinct curves which are not consecutive but which touch each other at that point of consideraton. This point will satisfy the p-discriminant but not the c-discriminant. Also the point not being on the envelope will not satisfy the differential equation ( ) , , 0 f x y p = . The locus of such points which are the points of contact of two non consecutive curves at which the p has equal values is called tac-locus. So if ( ), 0 T x y = be the locus, then T(x,y) is a factor of p-discriminant but not of c-discriminant. The c-discriminant relation is the locus of such points for which c has at least two equal values. It may so happen that each curve of the family ( ) , , 0 x y c φ = has a double
48 NSOU • CC • MT - 07 point whose nature is that of a node and then the locus of the nodes is called the nodal locus. Thus if N(x,y)=0 be the nodal locus, then N(x,y) is a factor of c- discriminant but not of p-discriminant. If each member of the family ( ) , , 0 x y c φ = has a cusp then the locus of those cusps is known as cuspidal locus. Thus if C(x,y)=0 be the cuspidal locus, then C(x,y) is a factor of both c- discriminant and p-discriminant but not the soluton of the differential equation. Here using symbols E,N,T and C for envelope, nodal locus, tac-locus, cuspidal locus respectively we can summarize the results int the following ways : Discr c ( ) φ x,y,c : E. N 2 . C 3 = 0 Discr p f(x,y,p) : ET 2 C = 0 Example : Examine for singular solution and extraneous loci, if any for the differential equation ( ) 2 2 4 − 3 − 0 xp x a = ...(a) Solving for p we get 3 − 2 x a p x = ± i.e, 3 − 2 dy x a dx x = ± or. 3 − 2 x a dy dx x = ± Integrating we get 3 1 2 2 −

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y c x ax ? ? + = ± ? ? ? ? ? ? ( )− x x a = ± therefore ( ) ( ) 2 2 − y c x x a + = ...(b) i.e. ( ) 2 2 2 2 − − 0 c cy y x x a + + = ...(c) From, (c), Discr c ( ) , ,x y c φ : ( ) { } 2 2 2 4 − 4 − − 0 y y x x a = or, ( ) 2 − 0 x x

a = ...(d)
NSOU • CC • MT - 07 49 From (a) Discr p ( ) , , f x y p : 0 − 4.4x. (3x−a) 2 = 0 ....(e) So from (d) and (e), x is the common factor. Hence x = 0 is the singular solution of (a). Again 3 − 0 x a = is a tac-locus, since it appears twice in the p-discriminant relation (e) but does not occur in (d). Also x−a = 0 is a nodal-locus since it appears twice in (d) but does not occur in (e). Exercises : a. Solve the following equations and find the singular solution, if any : (i) ( ) 2 2 2 1 y p a + = (ii) 3 8 27 ap y = (iii) ( ) 2 4 4 − 2 , p y xp y = put 2 y u= (iv) ( ) ( ) 2 2 2 − 3 4 1− p y y = (v) 2 − 2 4 0 xp py x + = b. Examine for singular solutions of the equations : (i) 2 3 2 − 2 p y px x = (ii) ( ) 2 2 4 3 −1 xp x = (iii) 3 2 2 2 0 x p x yp a + + = (iv) ( ) 2 4 2 − y y xp x p = (v) ( ) 3 2 8 − 27 12. p x p y = (vi) ( ) 3 4 p y y xp = + c. Reducting the differential equation : 2 − 2 2 0 xp py x y + + = to Clairaut's form by the transformations 2 =x u and y x v - = , find its singular solution, if any. (d) Reducing

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the differential equation : ( ) ( ) ( ) 2 2 2 2 1 2 2 1 0 x p p pxy p y p + + + + + = to Clairaut's form by the

transformations x+y = u and xy−1 = v, find its singular solution, if any.
50 NSOU ● CC ● MT - 07 3.3 Second Order Differential Equation A linear ordinary

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differential equation of nth order is given by 1 2 1 2 1 2 .... ( ) n n n n n n n d y d y d y P P P y F x dx dx dx - - - - + + + + = (1)

In the domain D RÍ , where each of

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P 1 , P 2 ,.........P n is either a constant or a function of x

and F is function of x on D. In P 1 , P 2 , ..., P n are all constants
then

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the differential equation 1 2 1 2 1 2 .... ( ) n n n n n n n d y a y d y P P P y F x dx dx dx - - - - + + + + =

is
known a linear ordinary differential equation with constant coefficients. Now in the linear ordinary differential equation with constant coefficients of the above form if we replace d dx by D in (1) we have (

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D n + P 1 D n –1 + P 2 D n – 2 + ... + P n)y = F(x) (2) i.e. f(D)y = F(x) (3) where f(D) = D n + P 1 D n – 1 + P 2 D n – 2 + ... + P n .

If F(x) = 0, (3) becomes f(D)y = 0 (4) (4) is called the correspoinding homogeneous equation to (1) and solution of (4) is called the complementary function or complementary solution or C. F of (1) The solution due to non homogeneous part F(x) is called the particular solution (PI) of (1). The complete or general solution of the differential eqution (1) is thus y = C. F. + P. I. 3.4 Theorem : Existence Theorem Let P 1 , P 2 ,...... , P n be some constants and let a point x 0 be in [a, b] within R. If a 1 , a 2 , ..., a n are any n constants there exists a solution φ of f (D)y = 0 on [a, b] satisfying φ(x 0) = α 1 , ( ) 0 ¢f x = α 2 , ..., φ n – 1 (x 0) = α n
NSOU ● CC ● MT - 07 51 3.5 Theorem : Uniqueness Theorem Let x 0 be in [a, b] within R and let α 1 , α 2 ,.... α n be any n constants. Then there is at most one solution φ of f(D) = 0 satisfying ( ) ( ) ( ) 1 0 1 0 2 0 , , ..., - ¢ f = f = a f = a n n x x x x . 3.6 Wronskian The wronskian of n differentiable functions y 1 , y 2 ..... y n , denoted by W(x) or W(y 1 , y 2 , ... y n) or, W (y 1 , y 2 , ... y n : x), is defined by W(y 1 , y 2 ,..... y n : x) = 1 2 1 2 1 1 1 1 2 .... u n n n n n y y y y y y y y y - - - ¢ ¢ ¢ Theorem : The function y 1 , y 2 , ... y n will be linearly independent solutions
of

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the equation 1 2 1 2 1 2 .... ( ) n n n n n n n d y a y a y P P P y F x dx dx dx - - - - + + + + = if F and P 1 , P 2 , ... Pn are

analytic in [a, b] Definition : Any set y 1 , y 2 .....y n of n linearly indepdent solution of the homogeneous linear nth order differential equation f(D)y = 0 in [a, b] is said to be a fundamental set of solutions in the interval [a,b]. Theorem : If y = f(x) be

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the general solution of the equation 1 2 1 2 1 2 .... 0 n n n n n n n d y a y a y P P P y dx dx dx - - - - + + + + = (			

a) and y = φ (x) be a solution of

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the equation 1 2 1 2 1 2 .... n n n n n n n d y a y a y P P P y x dx dx dx - - - - + + + + = (			

b)
52 NSOU ● CC ● MT - 07 then y = f(x) + φ(x) is the general solution of the equation (
b). Theorem : If y = y 1 is a solution of the reduced equation (4) in D, then y = c 1 y 1 is a solution of (4) as well, where c 1 is an arbitrary constant. 3.7 Theorem : Principle of Superposition If y 1 and y 2 be two solutions of the differential

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equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = , then the linear combination c 1 y 1 + c 2 y 2			

is also a solution
for any values of the constants c 1 , c 2 . 3.8 Theorem If y 1 and y 2 be two solutions of

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the diffrential equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = and if			

further there is a point where the Wronskian of y 1 and y 2 is non zero, then the family of solutions y = c 1 y 1 + c 2 y 2 with arbitrary coefficients c 1 , c 2 includes every solution

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of the equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = .			

Last theorem states that, as long as the Wronskian of y 1 and y 2 is not every where zero, the linear combination y = c 1 y 1 + c 2 y 2 spans all the

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solutions of the equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = . In this case the expression y = c 1 y 1 + c 2 y 2			

is said to be the general solution. The solutions y 1 and y 2 , with non zero Wronskian, are said to form a fundamental set of solution of (5). Now we pay our attention to the equation of the following form : 2 2 0 d y dy P Q Ry dx dx + + = (5) NSOU ● CC ● MT - 07 53 where P (≠ 0), Q and R are all constains. We take the following simple example : 2 2 0 d y y dx - = (6) Comparing (6) with (5) we will get P = 1, Q = 0, R = −1. We can easily verify that y 1 = e x and y 2 = e −x are two solutions of (6). We can also conclude that the functions c 1 y 1 = c 1 e x , c 2 y 2 = c 2 e −x satisfy the differential equation (6) as well. Further the function y = c 1 e x + c 2 e −x is also a solution of (6), for any arbitrary values of c 1 , c 2 . Again the Wronskian in this case is given by ( ) 2 1 2 1 2 1 , , : 2 0 x

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x x x y y e e W y y x y y e e - - = = = - [1] ¢ ¢ - . Hence, 1 2 x x y c e c e - = + is the

general solution of (6) As the coefficients c 1 , c 2 in the general solution y = c 1 e x + c 2 e −x are arbitrary, this expression represents a doubly infinite family of solutions of (6). Based on this observation. we suppose a trial solution of (5) of the form y = e mx , where m is the parameter to the determined. Then one can have mx y e= , mx dy me dx = , 2 2 2 mx d y m e dx = Substituting the above results in (6) we obtain Pm 2 e mx + Qme mx + Re mx = 0 (Pm 2 + Qm + R) e mx = 0 Since e mx ≠ 0, we have, Pm 2 + Qm + R = 0. Equation (7) is called the Auxiliary Equation (A. E.) for the ordinary differential equation (5). Now we re-write (5) in the following form : 2 2 0 d y dy p qy dx dx + + = (8)
54 NSOU ● CC ● MT - 07 where = Q p P and R q P = . Then the A. E becomes m 2 + pm + q = 0. (9) Now we have three different types of roots of the A. E. (9) a. Roots are real and distinct b. Roots are real and equal c. Roots are complex conjugate In the corresponding bomogeneous equation (4) for the differential equation (3) we put y = e mx as a trial solution and this gives the auxiliary equation f (m) = 0 Case-i. If

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m 1 , m 2 .... m n be the distinct real roots of the auxiliary equation f(m) = 0 then the solution of (4) is given by 1 2 1 2 ..... n m x m x m x n y c e c e c e = + + + where, c 1 ,

c 2 , ..., c n are constants. Case-ii If m 1 , m 2 , ..., m n be the real

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roots of the auxiliary equation f(m) = 0 and if further m 1 = m 2 = ... m r = m, then the solution of (4) is ( ) 1 2 . . 1 1 2 1 2 ... . + + - + + = + + + + + + + n r r m x m x m x r mx r r r n y c c x c x e c e c e c e

Case-iii If a ± ib be the roots of the auxiliary equation f(m) = 0, then the solution of (4) must contain the term e ax (c 1 cos (bx) + c 2 sin (bx)). Note : If a ± ib be the roots of the auxiliary equation f(m) = 0 repeated r times, the solution of (4) contains the term. e ax (c 1 + c 2 x + ... + c r x r − 1) cos (bx) + e ax (b 1 + b 2 x + ... + b r x r − 1) sin (βx). The general form of non homogeneous ordinary differential equation with constant coefficients is given by (2) or (3). To solve a non homongeneous linear ordinary differential equation we first solve the corresponding homogeneous equation by the method as discussed above and this will give this corresponding C. F. To get the P. I we employ the following scheme : 1 P.I= f(D) X where X = F(x)
NSOU ● CC ● MT - 07 55 Now the general method of finding the expression for 1 f(D) X is a laborious one. We shall explain below the short methods for finding 1 ( ) X f D for some standard form of functions. 3.9 Method of finding the particular integral (P. I) Rule 1. If X = P(x), where P(x) is a polynomial of degree n. Then P.I. = ( ) 1 1 ( ) ( ) X P x f D f D = Note : First express, f(D) in the form (1 + φ(D)). Then expanding (1 + j(D)) −1 as an infinite series in ascending powers of D and then operate on P(x). Rule 2. If X = e ax , 'a' being a constant,

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then P. I. = 1 1 ( ) ( ) ax X e f D f D = = ( ) 1 ax e f a , if f (a) ≠ 0 = ( )

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ax x e f a ¢ , if f′ (a) ≠ 0, f(a) = 0 In general, ( ) P.I= n ax n x e f a , if f(a) = 0, f′(a) = 0, ..., 1 ( ) 0, ( ) 0 - = [1] n n f a f a Rule 3. X = sin(ax) or, sin (ax + b) or, cos (ax) or, cos (ax + b) Let f(D) = f (D 2), φ (−a 2) ≠ 0. 1 P.I= ( ) X f D = ( ) 1 sin ( ) ax f D = 2 1 sin( ) ( ) ax Df = 2 1 sin( ) ( ) ax a f-

56 NSOU ● CC ● MT - 07 or, 2 1 1 sin( ) sin ( ) ( ) ( ) ax b ax b f D D + = + f = 2 1 sin( ) ( ) ax b a + - f or, 1 cos( ) ( ) ax f D = 2 1 cos( ) ( ) ax D = f = 2 1 cos( ) ( ) ax f -a or, = 1 cos( ) ( ) ax b f D + = 2 1 cos( ) ( ) ax b D + f = 2 1 cos( ) ( ) ax b a + f - If φ (−a 2) = 0, then P. I. = 1 ( ) X f D = ( ) 1 sin ( ) ax f D = ( ) 1 sin ( ) x ax f D¢ or, = ( ) 1 sin ( ) ax b f D + = ( ) 1 sin ( ) x ax b f D + ¢ or, = ( ) 1 cos ( ) ax f D = ( ) 1 cos ( ) x ax f D ¢ or, = ( ) 1 cos ( ) ax b f D + = ( ) 1 cos ( ) x ax b f D + ¢ Rule 4. If F(x) = c ax ψ(x) where ψ(x) is a function of x only. Then P. I. = ( ) ( ) 1 1 1 . ( ) ( ) ( ) = y = y + ax ax X e x e x f D f D f D a Rule 5. If F(x) = x n ψ(x) where ψ(x) is a function of x only. Then P. I = 1 ( ) X f D = ( ) 1 ( ) n x x f D y = ( ) ( ) ( ) ( ) 1 n f D x x f D f D ¢ ? ? ? ? - y ? ? ? ? ? ?

NSOU ● CC ● MT - 07 57 3.10 Properties of D-operator (a) D, D 2 , D 3 , ... denote the differentiations with respect to x once, twice, thrice..... respectively. (b) 2 3 1 1 1 , , , D D D ...... denote the indefinite integration with respect to x once, twice, thrice,..... respectively. (c) 1 X Xdx D = ∫ (d) ( ) 1 ... n n X X dx D = ∭ ∫ Example : Solve (D 2 + 2D + 1)y = x 3 + x 2 + x. Solution : Let y = e mx be the trial solution of the corresponding homogeneous equation of the given equation. Then the A. E. is of the form m 2 + 2m + 1 = 0 i.e. m = − 1, − 1 Therefore, the C. F of the given differential equation is of the form C. F. = (a + bx)e −x , where a, b are arbitrary constants. The particular integral is P. I. ( ) ( ) 3 2 2 1 1 + + +

x

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x x D = (D + 1) −2 (x 3 + x 2 + x) = (1 − 2D + 3D 2 − 4D 3 + ...)(x 3 + x 2 + x) = (x 3 + x 2 + x) − 2. (3x 2 + 2x + 1) + 3(6x + 2) = 24 = x 3 − 5x 2 + 15x − 20 Thus the general solution is given by y =

C. F + P. I= (a + bx) e −x + (x 3 − 5x 2 + 15

x − 20)

58 NSOU ● CC ● MT - 07 Example : Solve : (D 2 − 3D + 2)y = e x Solution : Let y = e mx be the trial solution of the given equation. Then the A. E. is of the form m 2 − 3m + 2 = 0 i.e. m = 1, 2 Therefore, the C. F of the given differential equation is of the form C. F. = a ex + be 2x , where a, b are arbitrary constains. Now let f(D) = D 2 − 3D + 2 The particular integral is P. I = 1 ( ) x e f D = ( ) 1 x x e f ¢ , since f′(1) ≠ 0, f (1) = 0 = − xe x . Thus the general solution is given by y = C. F. + P. I = ae x + be 2x − xe x . Problems : (a) Solve : (

D 2 + 4)y = sin 3x. (b) Solve : (D 2 + 9)y = sin 3

x + 5 cos 3x. (c) Solve : (

D 2 − 2

D + 2)y =

cos x + sin 2x. (d) Solve : (D 2 − 5D + 6)y = e x cos x. (e) Solve : (D 2 − 4D + 4)y = xe 2

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x cos x. (f) Solve : (D 2 − 5D + 6)y = x 2 e 3x . 3.11 Homogeneous Linear Differential Equations with Variable Coefficients A linear ordinary differential equation of the form 1 1 1 1 ...... n n n n n n n d y a y x P x P y X dx dx - - - + + + = (1) where P 1 , P 2 , ..., P n are constants and X is either a constant or a function of x only NSOU ● CC ● MT - 07 59 is called a homogeneous linear differential equation.

This is also known as Euler’s Equation. Now we want to change the independent variable by using the relation z x e= , i.e., z = log x (2) This gives, dx dz x = , i.e,

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d dz ≡ · d x dx ≡ xD ≡ D′ , where D ≡ d dx , D ′ ≡ d x dx , Thus xDy = D′y Now, since dy dy x dz dx = 2 2 d y d dy dz dz dz ? ? = ? ? ? ? = d dy x x dx dx ? ? ? ? ? ? = 2 2 2 d y dy x dz dx + So, ( ) 2 2 2 2 2 1 d y d y dy x D D y dz dx dy ¢ ¢ = - = - Similary ( ) 1 0 . - = ? ? ¢ = - ? ? Õ r r r i r d y x D i y dx (3) Now using the relations given by (2) and (3) the differential equation (1) will be changed into the form of a linear differential equation with constant coefficients. Then we can write it in the form f (D′)y = X′ , where X′, is a function of z only. So, we can solve the problem f(D′)y = X′ by the method of linear differential equation with constant coefficients. Now let us suppose that a second order differential equation takes the following form : ( ) ( ) ( ) 2 2 2 d y dy ax b ax b P Qy F x dx dx + + + + = (4) where P, Q, a, b are constants and F is a function of x on , b a - ? ? ¥ ? ? ? ? which is a homogeneous linear differential equation as well. 60 NSOU ● CC ● MT - 07 Example : Solve ( ) 2 2 2

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log .sin log d y dy x x y x x dx dx + + =

Solution : First we change

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the independent variable x to z by the transformation x = e z , i.e, z = log x.

So, dy dy x dz dx = and 2 2 2 2 2 d y d y dy x dz dx dz = - The given equation reduces to (D′ 2 + 1)y = z. sin z (a) Let y = e mx be the trial solution of the reduced equation of (a). Then the corresponding A. E. is of the form m 2 + 1 = 0, So, m = i, – i. Therefore the C. F. = A sin z + B cos z, where A, B are arbitrary constants. Now, ( ) ( ) 2 1 P.I .sin 1 z z D = ¢+ = ( ) ( ) ( ) 2 2 1 1 2

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sin 1 1 z D z D D ? ? ? ? ¢ - ? ? ¢ ¢ + + ? ? ? ? = ( ) ( ) 2 1 1 2 sin · 2 1 ? ? ? ?? ? ¢ - ? ? ? ? ¢ ? ? ¢+ ? ? ? ? z D z z D D = ( ) ( ) ( ) 2 1 1 2 cos · 2 1 ? ? ? ?? ? ¢ - - ? ? ? ? ? ? ¢+ ? ? ? ? z D z z D = ( ) ( ) ( ) 2 2 1 cos cos 2 1 z z

D z z D ¢ - + ¢+ = ( ) ( ) 2 2 1 cos cos sin 2 1 z z z z z D - + - ¢+
NSOU ● CC ● MT - 07 61 = ( ) ( ) ( ) ( ) 2 2 2 1 1 cos cos sin 2 1 1 z z z z z D D - + - ¢ ¢ + + = ( ) 2 1 cos cos . 2 2 z z z z P I D - + + - ¢ = 2 1 cos ·sin . . 2 2 z z z z P I - + - Therefore, P. I. = 2 1 cos ·sin 4 4 - + z z z z Therefore

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the general solution of the equation (a) is given by y = A sin z + B cos z 2 4 z - cos z + 1 · 4 z , sin z By putting z = log x

the general solution of the given equation is y = A sin (log x) + B cos (log x) ( ) 2 log 4 x - cos (log x) + (log x) 1 4 · sin (log x), 0 > x > •. 3.12 Method of Undetermined Coefficients We consider the following problem of the non homogeneous differential equation 2 2 d y dy P Qy R dx dx + + = (1) The method of undetermined coefficients is a procedure for finding the particular solution of the equation (1) where R is an exponential, or a sine or cosine, a polynomial, or some combination of such functions. Now, we are going to study this method of undermined coefficients throug an example.

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62 NSOU • CC • MT - 07 Suppose 2 2 ax d y dy P Qy e dx dx + + = (2) If we differentiate e ax , we have the same function with some numeric constant. Now this is the procedure to find the particular integral. So let ax p y e= be the P. I. of (2), and we guess that y p = Ae ax (3) might be a particular solution. Here A is the undetermined coefficient and it is to be so chosen that (3) satisfies (2). Then ( ) 2 ax ax A a Pa Q e e + + = Hence, 2 1 A a Pa Q = + + , if a 2 + Pa + Q ≠ 0. Now if a 2 + Pa + Q = 0, then 'a' is a root of A. E. We take y p = Axe ax (4) Then from (2) we get 1 2 A a p = + , if 2a + P ≠ 0 Again, if 2a + P = 0, then we take y p = Ax 2 e ax and we repeat the above procedure if the order of the differential equation is more than two. Therefore : If y 1 and y 2 are two solutions of the non homogeneous differential equation (1) then their difference y 1 – y 2 is a solution of the corresponding homogeneous differential equation. If, in addition, Y 1 and Y 2 determine a fundamental set of solutions of the corresponding differential equation (2), then Y 1 – Y 2 = c 1 y 1 + c 2 y 2 , where c 1 and c 2 are certain constants. Example : Solve by the method of undetermined coefficients, the equation (D 2 + 1)y = 10e 2x for the condition y = 0, Dy = 0 when x = 0. Solution : Here it is given that (D 2 + 1)y = 10e 2x (1) Let y = e mx be the trial solution of the reduced differential equation of (a) Then the A. E is
NSOU • CC • MT - 07 63 m 2 + 1 = 0, i.e., m = i, – i. The complementary functon is C. F. = C 1 cos x + c 2 sin x. where c 1 and c 2 are certain constants. We assume the particular integral in the form P. I = Ae 2x , where A is a constant to be determined (since 2 is not a root of the A.

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E). So, (D 2 + 1)Ae 2x = 10e 2x i.e. 5Ae 2x = 10e 2x or, A = 2 Thus the general solution is given by y = c 1 cos x + c 2 sin x + 2

e 2x
From the condition y = 0 when x = 0 we get c 1 = – 2 and from the condition Dy = 0 when x = 0 we get c 2 = – 4. So the final complete solution is y = – 2 cos x – 4 sin x + 2e 2x . Working Rule : (a) R = e ax (1) When a is not a root of A.E. i.e. e ax is not in the complementary function, take y p = Ae ax . (2) When a is a simple root of A. E. i.e. e ax is in the complementary function, take y p = Axe ax . (3) When a is a double root of A. E. i.e. e ax is in the complementary function, take y p = Ax 2 e ax . (b) R = sin (ax) or cos (ax) (1) When sin(ax) or cos (ax) is not in C. F., take y p = A sin (ax) + B cos (ax) (2) When sin (ax) or cos (ax) is in C. F., take y p = x. (A sin (ax) + B cos (ax)) (c) R = a 0 + a 1 x + ... + a n x n
64 NSOU • CC • MT - 07 (1) if P ≠ 0, Q ≠ 0, we take y p = A 0 + A 1 x + ... A n x n (2) if P ≠ 0, Q = 0, we take y p = x(A 0 + A 1 x + ...+ A n x n) (3) if P = 0, Q = 0, we take y p = x 2 (A 0 + A 1 x + ...+ A n x n) (d) R = e ax sin (bx) or sin (bx) (a 0 + a 1 x + ... + a n x n) or, e ax (a 0 + a 1 x + ... + a n x n) Modify y p accordingly with the help of (a), (b) and (c). 3.13 Method of Variation of Parameters The main advantage of the method of variation of parameters is that it is a general method. In principle, it can be applied to any ordinary differential equation, and it requires no detailed assuptions about the form of the solution. In fact later in this section we use this method to derive a formula for a particular solution of an arbitrary second order linear non homogeneous differential equation. On the other hand, the method of variation of parameters eventually requires evaluation of certain integrals involving the non homogeneous term in the differential equation. We seek a method of finding a particular integral of an ordinary differential equation for which the complementary function is known. This is the main objective of the method of variation of parameters. Now we consider the following second order linear differential equation 2 2 d y dy p qy r dx dx + + = (1) where p, q, r are given continuous functions in x. We now assume that c 1 y 1 + c 2 y 2 ,where c 1 , c 2 are both constant, be the general solution of corresponding homogeneous equation. 2 2 0 d y dy p qy dx dx + + = i.e. the C. F. of (1) Now we replace c 1 , c 2 by the function A and B respeectively. This gives y = Ay 1 + By 2 (2)

NSOU ● CC ● MT - 07 65 Then we try to determine A and B so that the expression in (3) is a solution of the non homogeneous equation (1) rather than the homogeneous equation (2). This method is known as the Method of variation of parameters. Calculations yield the expressions of the desired functions A and B as ( ) 2 1 2 , : y r A dx w y y x = - ∫ and ( ) 1 1 2 , : y r B dx w y y x = ∫ . Substituting these two expression of A and B in (3) we get particular integral of the non homogeneous equation (1). Theorem : If the functions p, q, r are continuous functions in an open interval I and if the functions y 1 , y 2 are linearly independent solutions of the homogeneous equation corresponding to the non homogeneous equation 2 2 d y dy p qy r dx dx + + = , then a particular solution of this equation is y = Ay 1 + By 2 and the general solution is y = c 1 y 1 + c 2 y 2 + Ay 1 + By 2 . Note that the two solutions y 1 , y 2 of the corresponding homogeneous equation (2) are linearly indendpent. Let us consider a second order differential equation 2 2 d y dy p qy r dx dx + + = (a) in which p, q are constants and r = r(x).The corresponding homogeneous equation of the differential equation (a) is as follows 2 2 0 d y dy p qy dx dx + + = (b) Then the general solution of the differential equation (b) i.e. the complementary function of (a) is y c = A.u + B.v (c) where A, B are constants.

66 NSOU ● CC ● MT - 07 Now as u and v are two linearly independent solutions of (b) we have 2 2 . 0 + + = d u du p qu dx dx (d) 2 2 . 0 + + = d v dv p qv dx dx (e) Let us assume the general solution in the form y = A.u + B.v (f) Here A and B are treated as functions of x. Differentiating (f) with respect to x, we get ? ? ? ? = + + + ? ? ? ? ? ? ? ? dy du dv dA dB A B u v dx dx dx dx dx (g) Let us choose A and B in such a way that 0 dA dB u v dx dx + = (h) Then from (g) we get dy du dv A B dx dx dx ? ? = + ? ? ? ? (i) Differentiating both sides of (1) with respect to x, we get 2 2 2 2 2 2 d y d u d v dA du du dv A B dx dy dx dx dx dx dx = + + + (j) Now putting the values of 2 2 , , dy d y y dx dx in (a), get 2 2 2 2 d u du d v dv dA du dB dv A P qu B P qv r dx dx dx dx dx dx dx dx ? ? ? ? ? ? + + + + + + + = ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

NSOU ● CC ● MT - 07 67 So, dA du dB dv r dx dx dx dx ? ? + = ? ? ? ? (k) Now using (h) in (k) we can get . . dA du u dA dv r dx dx v dx dx - = i.e., dv dv u v dA vrdx dx dx ? ? - - = ? ? ? ? or, – W(u, v ; x) dA = vrdx The expression W (u, v ; x) = ? ? - ? ? ? ? dv du u v dx dx gives the corresponding wronskian. Integrating we get ( ) 1 . . ; v r A dx c W u v x = - + ∫ , where c 1 is an arbinary constant. Similary, we have ( ) 2 . . . u r B dx c w u v x = + ∫ , where c 2 is an arbitrary constant. Using the above expression of A and B in (f) the general solution takes the following form ( ) ( ) 1 2 . . W W = + - + ∫ ∫ v r u r y c u c v u dx v dx u,v : x u,v : x Working Rule : Step 1 : Find the complementary function of the given differential equation (1). Let the complementary function be C. F. = A. u + B. v. Step 2 : Check Wronskian W(u, v) ≠ 0. Step 3 : Suppose y = A. u + B.v where A and B are functions of x. Step 4 : Calculate ( ) 1 W u,v : x vr A dx c = - + ∫

68 NSOU ● CC ● MT - 07 and ( ) 2 . W u,v:x = + ∫ u r B dx c , where c 1 and c 2 are arbitrary constant. Step 5 : Put the values of A, B in the expression at Step 3 and this will give the general solution of the given differential equation. Exercises : a. Solve the following diffirential equations with constant coefficients : i. ( ) 3 2 3 1 + = +

x d y y e dx ii. 3 3 2 3 - = -

d

y

y x x dx iii. 2 2 2 cos - = d y y x x dx iv. 2 2 d y y dx + = cosec x v. 2 2 3 2 2 cos2 x x d y y x e e x dx + = + vi. 2 2 2 sin x

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d y dy y xe x dx dx - + = vii. 2 2 2 4 cos x d y dy y e x dx dx - + = viii. ( ) 2 2 5 6 x d y dy y x x e dx dx - + = + ix. ( ) 2 1 2sin(3) - + = D D y x NSOU ● CC ● MT - 07 69 x. ( ) ( ) ( ) 2 2 1 sin 1 x D y x

x x

e - = + + b. Solve the following homogeneous linear differential equations : i. 2 2 2 5 2log

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d y dy x x y x dx dx - + = ii. 2 2 2 2 3 4 2 - + = d y dy x x y x dx dx iii. ( ) 2 2 2 2 2 20 1 + - = + d y dy x x y x dx dx iv. 2 2 2 + + = d y dy x x y logx.sinx(logx) dx dx v. ( ) ( ) 2 2 2 4 cos - + = + d y dy x x y logx xsin logx dx dx vi. x 2 2 2 4 2 sin + + = + d y dy x y x x dx dx vii. ( ) ( ) 2 2 2 5 2 6 5 2 8 0 + - + + = d y dy x x y dx dx viii. ( ) ( ) 2 2 2 2 2 3 5 2 3 3 1 + + + - = + + d y dy x x y x

x

dx
dx
c.
Solve the following differential equations, using the method of undertermined coefficients: i. 2 2 2 2 5 12 15 - + = +

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d y dy y x dx dx ii. 2 2 2 + = + x d y y e x dx 70 NSOU ● CC ● MT - 07 iii. 2 2 2 9 2 - = + - x d y y x e Sin x dx iv. 2 2 2 5 3 + + = d y dy y sinx dx dx v. 2 2 4+ = d y y sin2x dx vi. 2 2 3 2 3 - + = x d y dy y xe dx dx vii. 2 2 2 4 sin2 + = d y y x x

dx
d.
Solve the following differential equations, using the method of variation of parameters : i. 2 2 4 4tan2 + =
d y y x dx ii. 2 2 2 9 sec(3) + + =

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d y dy x x y logx dx dx iii. 2 3 2 sec + = d y y x.tanx dx iv. 2 2 3 2 9 - + = x d y dy y e dx dx v. 2 2 2 1 - = + x d y y dx e vi. 2 3 2 2 6 9 - + = x d y dy e y dx dx x vii. 2 2 3 2 1 - + = + x x d y dy e y dx dx e NSOU ● CC ● MT - 07 71 viii. 2 2 2 2 - + = x d y dy y e tanx dx dx ix. 2 2 2 2 + - = ¥ d y dy x x y x ,0 > x > dx dx x. 2 2 2 2 + - = ¥ x d y dy x x y x e ,0 > x > dx dx 3.14

Simultaneous Linear Differential Equations with Constant Coefficients: The system of a linear simultaneous ordinary differential equations with constant coefficients is of the following form : φ 11 (D) x 1 + φ 12 (D) x 2 + ... + φ 1n (D) x n = f 1 (t) φ 21 (D) x 1 + φ 22 (D) x 2 + ... + φ 2n (D) x n = f 2 (t) .... φ n1 (D) x 1 + φ n2 (D) x 2 + ... + φ nn (D) x n = f n (t), where x 1 , x 2 , ... , x n are the dependent variables and φ ij (D), i. j = 1, 2, ..., n are all rational functions of d D dt º with constant coefficients and f i (t), i = 1, 2, ..., n, are the function of the independent variable t. The method of operator : Let x, y be the dependent variables and t be the independent variable. The equation with involve derivatives of x and y with respect to t. Let us denote the operator d dt by the symbol D. Let us consider the simultaneous linear differential equation with constant coefficient for two variables as ( ) ( ) 1 2 ( ) f + f = D x D y f t (1) and ( ) ( ) 1 2 ( ) y + y = D x D y g t (2) where 1 2 1 2 ( ), ( ), ( ), ( ) f f y y D D D D are all rational functions of D with constant coefficients and f and g are functions of t.
72 NSOU ● CC ● MT - 07 Now we operate

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both sides of (1) with ψ 2 (D) and both side of (2) with φ 2 (D). We get, ψ 2 (D)φ 1 (D)x +

ψ 2 (D)φ 2 (D)y = ψ 2 (D) f(t) φ 2 (D)ψ 1 (D)x + φ 2 (D)ψ 2 (D)y = φ 2 (D) g(t) Subtracting we get, [ψ 2 (D)φ 1 (D) − φ 2 (D)ψ 1 (D)x = ψ 2 (D) f(t) − φ 2 (D) g(t) which is a linear equation in x and can be used to find x as a function of t. Value of y can be obtained as a function of t by substituting the result of x in (1) or (2). Example : Solve 7 0 - + = dx x y dt , 2 5 0 dx x y dt - - = Solution : The given equations are (D − 7)x + y = 0 (a) (D − 5)y − 2x = 0 (b) Putting the value of y = − (D − 7)x in (b), we have = (D − 5)(D − 7)x − 2x = 0 So, (D 2 − 12D + 37)x = 0 (c) Let x = e mt be the trial solution of the equation (c). Then the A. E is of the form m 2 − 12m + 37 = 0 i.e. m = 6 ± i Therefore, the general solution of the equation (c) is ( ) 6 cos sin t x e A t B t = + , where A, B are arbitrary constants. Putting the value of x in (a), we have y = − (D − 7)x = − (D − 7){(A cos t + B sin t)}= e 6t [(A − B) cos t + (A + B)sin t]. Hence, the solution of the given simultaneous linear equation is given by x = e 6t (A cos t + B sin t)

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NSOU • CC • MT - 07 73 and, y = e 6t [(A – B) cos t + (A + B) sin t] Example : Solve t dx y e dt + = , t dy x e dt - - = . Solution : The equations are Dx + y = e t (a) – x + Dy = e –t (b) Differentiating both sides of (a) with respect to t we get D 2 x + Dy = e t i.e. D 2 x + (x + e –t) = e t [using (b)] i.e., (D 2 + 1)x = e t – e –t (c) Let x = e mt be the trial solution of the reduced equation of (c). Then the A. E is of the form (m 2 + 1) = 0 i.e. m = ± i. The complementary function of (c) is C. F. = A cost + B sin t, where A, B are arbitrary constants, Now, ( ) ( ) 2 1 P.I. 1 t t e e D - = - + = 2 t e – 2 t e - . Therefore, the general solution of (c) is x = (A cos t + B sin t) + 2 t e – 2 t e - , where A, B are arbitrary constants, Putting the above expression of x in (a), we have y = e x – D ( ) cos sin 2 2 t t e e A t B t - ? ? ? ? + + - ? ? ? ? ? ?
74 NSOU • CC • MT - 07 Therefore, y = A sin t – B cost + 2 t e – 2 t e - Hence, the solution of the given simultaneous linear equation is given by ( ) cos sin 2 2 t t e e x A t B t - = + + - and, sin cos 2 2 t t e e y A t B t - = - + - Exercises: Solve the following simultaneous linear differential equations : i. 2 5 2 , 6 t t dx dy x y e x y e dt dt + - = - + = ii. 4 3 , 2 5 + + = + + = t

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dx dy x y t x y e dt dt iii. 4 3 sin , 2 5 t dy dy x y t x y e dt dt + + = + + = iv. 5 4 , dx dy x y x y dt dt = + = - + v. 4 2 , 5 2 dx dy x y x y dt dt = - = + vi. 3 4 , 2 3 dx dy x y x y dt dt = - + = - + vii. 2 0, 5 3 0 dy dy dy x y x y

dt
dt dt + + + = + + =
NSOU • CC • MT - 07 75 3.15 Series Solution of the Ordinary Diffrential Equations: The solutions of many differential equations can be expressed in terms of elementary functions, all of whose mathematical properties are well known. When required, the analytical behaviour of solutions that involve elementary functions can be explored by making use of their familiar properties. With either a pocket calculator of a software package, the method of calculating functional values is usually based on a series expansion of the function concerned. Most of the ordinary differential equations cannot be solved in terms of elementary functions, yet some form of analytical solution is often needed rather than a purely numerical one. So the fundamental question that then arises is how to obtain a solution in the form of a series, when only the differential equation is knonw. Definition : A function f defined in the interval I containing x 0 is said to be analytic at x 0 if f(x) can be expressed as a power series ( ) ( ) 0 0 n n n f x a x x ¥ = = - ∑ , which has a positive radius of convergence. Defintion : Consider the n-th order linear ordinary differential equation y (n) + P n – 1 (x)y (n – 1) + P n – 2 (x)y (n – 2) + ... + P 0 (x)y= f(x) A point x 0 is called all ordinary point of the given differential equation if each of the coefficients P n – 1 , . P n – 2 , ..., P 0 and f (x) are analytic at x 0 . Definition : Consider the n-th order linear ordinary differential equation y (n) + P n – 1 (x)y (n – 1) + P n – 2 (x)y (n – 2) + ... + P 0 (x)y = 0 (a) A point x 0 is called a singular point of the given different equation if it is not an ordinary point, that is, not all of the coefficients P n –1 , P n –2 , ..., P 0 are analytic at x 0 A point x 0 is called a regular singular point of the given differential equation if it is not an ordinary point but all (x – x 0) n – k P k (x) are analytic for k = 0, 1, 2, ... (n – 1) i.e., all the limits given by ( ) 0 0 lim ( ) n k k x x x x P x - ® - exist and finite. A point x 0 is called an irregular singular point of the given differential equation if it is neither an ordinary point nor a regular singular point.
76 NSOU • CC • MT - 07 3.16 Note : Test of Singularity at Infinity To determine whether the point at infinity is a singular point or not, we transform the equation (a) by substituting 1 x t = Then 2 dy dy t dx dt = - and 2 2 4 3 2 2 2 = + d y a y dy t t dt dx dt Then the differential equation (a) becomes ( ) ( ) n t y + p′ n – 1 (t)y (n – 1) (t) + p′ n – 2 (t)y (n – 2) (t) + ... + p′ n – 2 (t)y(t) = 0 (b) If t = 0 is a singular point of (b) then the original equation (a) has a singularity at infinity. Example : Find the ordinary and singular point (if any) of the differential equation 2 2 2 2 7 (1) 3 0

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d y dy x x x y dx dx + + - = Solution : The given differential equation 2 2 2 2 7 (1) 3 0 d y dy x x x y dx dx + + - = , can be written as 2 2 2 2 7 (1) 3 0 2 2 d y x x dy y dx dx x x + + - = Comparing the above differential equation with ( ) ( ) 2 1 0 2 0 d y dy p x p x y dx dx + + = , we have, ( ) ( ) 1 7 1 2 x P x x + = , ( ) 0 2 3 2 = -

P x x

NSOU • CC • MT - 07 77 Since neither lim x → 0 p 1 (x) not lim x → 0 p 0 (x) does exist hence, p 1 (x), p 0 (x) are not analytic at x = 0. Therefore, x = 0 is a singular point Now, lim

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x → 0 (x – 0) p 1 (x) = ( ) 0 7 1 lim 2 x x x x ® + = 7 2 and lim x → 0 (x – 0) 2 p 0 (x) = 2 2 0 3 3 lim 2 2 x x x ® ? ? - - = ? ? ? ?

So both the limits exist and finite and hence the point x = 0 is a regular singular point. All the points x (≠ 0) are ordinary points. Example : Show that the equation ( ) 2 2 2 2 1 2 0 1 1 n n d y x dy y dx dx x x + - + = - - has a singularity at infinity. Solution : Substituting 1 x t = to the given equation we have 2 = - dy dy t dx dt and 2 2 4 3 2 2 2 d y a y dy t t dt dx dt = + Using the above results the given equation reduces to ( ) 2 2 4 2 2 2 1 2 · 0 1 1 n n d y t dy t y dt dt t t + + + = - - ( ) ( ) ( ) 2 2 2 2 2 1 2 · 0 1 1 n n d y dy y dt dt t t t t + + + = - - (a) Since t = 0 is a singular point of the equation (a) thus the given ODE has a singularity at infinity.
78 NSOU • CC • MT - 07 3.17 Series Solution about an Ordinary Point : Theorem : Let x 0 be any real number and suppose that the coefficients P n – 1 , P n – 2 , ..., P 0 in f(D)

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y = y (n) (x) + P n – 1 (x) y (n – 1) (x) + P n – 2 (x)y (n – 2) (x) + ... + p 0 (x) y(x)

have convergent power series expansions in powers of (x – x 0) in an interval | x – x 0 | > r, r < 0. If α 1 , α 2 , ..., α n are any n constants, there exists a solution φ of the problem f(D)y = 0, such that y(x 0) = α 1 , y′(x 0) = α 2 , ..., y (n – 1) (x 0) = α n with a power series expansion ( ) ( ) 0 0 k k k x c x x ¥ = f = - ∑ convergent for |x – x 0 | > R where the radius of convergence is R ≥ r. Theorem : Suppose that x 0 is an ordinary point of the n-th order linear ordinary differential equation ( ) ( ) n y x + P n – 1 (x)y (n – 1) (x) + P n – 2 (x) y (n – 2) (x) +..... + p 0 (x)y(x) = f(x), where the coefficients P n – 1 (x)· P n – 2 (x), ..., p 0 (x) and f(x) are analytic at x = x 0 then it has two non-trivial linearly independent power series solutions of the form ( ) 0 0 n n n a x x ¥ = - ∑ , 0 | | x x R - > , for some R < 0, where n a s¢ are constants and these power series converges in some interval 0 | | x x R - > , R < 0 about x 0 , R being the radius of convergence of the power series. Example : Find the series solution of the following ordinary differential equation ( ) 2 2 2 1 0

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d y dy x x y dx dx + + - = Solution : The given differential equation can be written as 2 2 2 2 1 0 1 1 + - = + + d y x dy y dx dx x x (

a) Comparing the above eqution with

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the equation ( ) ( ) 2 1 0 2 0 d y dy p x p x y dx dx + + = ,

NSOU • CC • MT - 07 79 we have ( ) 1 2 1

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x p x x = + , ( ) ( ) 0 2 1 1 p x x = - + . We have for i = 0, 1 p i (x) = (–1) i +1 · x i · (1+ x 2) –1 = (– 1) i + 1 · x i · (1 – x 2 + x 4 – x 6 + ...), – 1 > x > 1.

So, p i (x) for i = 0, 1 can be expressed as power series and x = 0 that are convergent for – 1 > x > 1 i.e. all the coefficients p 1 (x) and p 0 (x) are analytic at x = 0. Hence, x = 0 is a ordinary point of the differential equation (a) and we take therefore. ( ) 0 n n n y x a x ¥ = = ∑ , (– 1 > x > 1) (b) Now 1 1 n n n dy na x dx ¥ - = = ∑ , and ( ) 2 2 2 2 1 n n n d y n n a x dx ¥ - = = - ∑ , – 1 > x > 1. Putting these expressions of 2 2 , , dy d y y dx dx in (a), we have ( ) ( ) 2 2 2 1 1 n
n
n x

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n n a x ¥ - = + - ∑ + 1 1 0 a ¥ - = = - ∑ ∑ n n n n n n x na x a x = 0 Therefore, ( ) 2 1 1 n n n n n a x ¥ = - + ∑ + ( )( ) 2 0 2 1 n n n n n a x ¥ + = + + ∑ + 1 n n n

na x ¥ = ∑ – 0 0 n n n a
x ¥ = = ∑ We shift the index of summation in the second series by 2 i.e. we replace n by (n + 2) and use the initial value n = 0. Also we shift the index of summation in third series by 1 i.e. we replace n by (n + 1) and use the initial value n = 0. 80 NSOU ● CC ● MT - 07 Then we get, 2 0 3 1 2 (6)

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a a a a x - + + + ( ) ( )( ) { } 2 2 1 2 1 0 n n n n n n n n n a n n a na a x ¥ = + - + + + + - = ∑ Equating the coefficients of various power of x to zero. we get 2a 2 – a 0 = 0 ⇒ 0 2 2 a a = 1 3 1 3 6 0 6 a a a a + = ⇒ = -

and,

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n(n – 1)a n + (n + 2)(n + 1) a n + 2 + na n – a n = 0 i. e., 2 1 2 n n n a a n + - = + for n ≥ 2.

Now putting n = 2, 3, 4, ... in the above recurrence relation, we get 4 2 0 1 1 4 8

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a a a = - = - 5 3 1 2 2 5 5.6 = - = a a a 6 4 1 1 2 16 o a a a = - = 7 5 1 4 2.4 7 7.6.5 a a a = - = -

and so on Substituting the values of a 0 , a 1 , a 2 , ... in (b) we get the required solution as y(x) = 2 4 6 8 0 5 1 ... 2 8 16 128 x x x a x ? ? ? ? + - + - + ? ? ? ? ? ? ? + 3 5 7 1 1 2 2.4 ... 6 6.5 7.6.5 a x
x x ? ? - + - + ? ? ? ? ; – 1 > x > 1
NSOU ● CC ● MT - 07 81 3.18 Series Solution about Regular Singular Point (Frobenius Method) Theorem : If the point x 0 is a singular point of the differential equation ( ) ( ) ( ) 2 0 1 2 2 0 d y dy a x a x a x y dx dx + + = , then it has at least one non-trivial solution of the form ( ) ( ) 0 0 0 | | n r n n y x x x c x x ¥ = = - - ∑ , and this solution is valid in some interval 0 | | x x R - > , where r is a certain constant (real or complex) and R < 0. If x = 0 is regular singular point, we shall use this method to find the series solution about x = 0. Consider

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the differential equation of the form ( ) 2 2 2 ( ) 0 P x d y dy Q x y x dx dx x + + = (a) where the functions P(x) and Q (x) are

analytic for all | x | > R, R < 0. We assume a trial solution ( ) 0 0 , 0,0 n r n n y x a x a x R ¥ + = = 1 > > ∑ (b) Now ( ) 1 0 n r n n dy n r a x dx ¥ + - = = + ∑ and ( )( ) 2 2 0 2 1 n r n n d y n r n r a x dx ¥ + - = = + + - ∑ Since P(x) and Q(x) are analytic at x = 0, then P(x) = c 0 + c 1 x + c 2 x 2 + ..., Q(x) = d 0 + d 1 x + d 2 x 2 + ... Thus ( )( ) 0 1 n r n n n r n r a x ¥ + = + + - ∑ + ( ) 2 0 1 2 .... c c x c x + + + ( ) 0 n r n n n r a x ¥ + = + ∑ +
82 NSOU • CC • MT - 07 ( ) 2 0 1 2 0 .... 0 n r n n d d x d x a x ¥ + = + + + = ∑ (c) Since (c) is an identity, we can equate to zero the coefficients of various power of x. The smallest power of x is x r , and the corresponding equation is {r(r – 1) + c 0 r + d 0 }a 0 = 0 Since, by assumption a 0 ≠ 0, we get, r 2 + (c 0 – 1)r + d 0 = 0 This equation is known as indicial equation of (a). Solving this quadratic equation for r. one obtains r 1 and r 2 . Case-I : Let r 1 and r 2 be the roots of the indicial equations and r 1 – r 2 is not equal to an integer. Then the complete solution is given by y(x) = A.[y(x)] r = r1 + B.[y(x)] r = r2 , 0 > x > R, where A, B are arbitrary constants. Case-II : Let r 1 and r 2 be the roots of the indicial equations and r 1 = r 2 . Then complete solution is given by ( ) ( ) ( ) 1 2 . . ,0 = = ¶? ? = + > > ? ? ? ? ? ? ¶ ? ? r r r r y x y x A y x B x R r Case-III. Let r 1 and r 2 be the roots of the indicial equations and differs by an integer and if some of the coefficients of y(x) become infinite when r = r 1 , we modify the form of y(x) by replacing a 0 by b 0 (r – r 0). Then we obtain two indepdenent solutions by putting r = r 1 in the modified form of y(x) and ( ) y x r ¶ ¶ , 0 > x > R. The result of putting r = r 2 in y(x) gives a numerical multiple of that obtained by putting r = r 1 and hence we reject the solution obtained by putting r = r 2 in y(x). Example : Find the power series solution of the equation using Frobenius method 2x 2 y″(x) + xy′(x) – (x + 1)y(x) = 0 in powers of x. Solution : The given differential equation can be written as
NSOU • CC • MT - 07 83 ( ) ( ) ( ) ( ) 2 1 1 0 2 2

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x y x y x y x x x + ¢¢ ¢ + - = (a) Comparing the above differential equation with ( ) ( ) 2 1 0 2 0 d y dy p x p x y dx dx + + = ,

we have ( ) 1 1 2 p
x x = and ( ) ( ) 0 2 1 2 x p x x + = - . Here the point x = 0 is a singular point. Now ( ) ( ) 0 0 1 lim

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x x x x p x ® - = ( ) 0 1 1 lim 0 2 2 x x x ® - = and ( ) ( ) 0 2 0 0 lim x x x x p x ® - = ( ) ( ) 2 0 2 1 1 lim 0 2 2 x x x

x ® + ? ? - - - = ? ? ? ? .
So both the limits exist and finite. Hence the point x = 0 is a regular singular point. Let us assume that the trial solution of the given equation is ( ) 0 n r n n y x a x ¥ + = = ∑ , a 0 ≠ 0, 0 > x > • (b) Now, ( ) ( ) 1 0 n r n n y x n r a x ¥ + - = ¢ = + ∑ and ( ) ( )( ) 2 0 1 n r n n y x n r n r a x ¥ + - = ¢¢ = + + - ∑ , 0 > x > • Putting these values in (a), we have ( )( ) 2 2 0 2 1 n r n

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n x n r n r a x ¥ + - = + + - ∑ + ( ) 1 0 n r n n x n r a x ¥ + - = + ∑ – ( ) 1 1 0 n r n n x a x ¥ + = + = ∑ ⇒ 0 2 ( )(1) n r n n n r n r a x ¥ + = + + - ∑ + ( ) 0 n r n n n r a x ¥ + = + ∑ – 1 0 n r n n a x ¥ + + = ∑ – 0 0 n r n n a

x ¥ + = = ∑
84 NSOU • CC • MT - 07 ⇒ ( )( ) ( ) { } 0 2 1 1 n r

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n n n r n r n r a x ¥ + = + + - + + - ∑ – 1 0 0 n r n n a x ¥ + + = = ∑ ⇒ ( )( ) { } 0 2 2 1 1 n r n n n n r n r a x ¥ + = + + + - ∑ – 1 0 0 n r n n a x ¥ + + = = ∑ Equating the coefficent of smallest power of x, namely x r to zero the indicial equation becomes {(2r + 1)(r − 1)}a 0 = 0. As 0 0 a ^{1} the roots of the equation are r = 1 and 1 2 r = - . Here the roots of the indicial equation are distinct and the difference is 1 3 1 2 2 ? ? - - = ? ? ? ? which is not an integer, Now equating the coefficient of x n + r , we obtain the recurrence relation as (2

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n + 2r + 1)(n + r − 1)a n − a n − 1 = 0 ( )( ) 1 2 2 1 1 n n a a n r n r - = + + + - Putting n = 1, 2, 3.... we get ( ) 0 1 2 3 a a r r = + ( )( ) 1 2 2 5 1 a a

r r = + + and so on Putting these values in (b) we get
NSOU • CC • MT - 07 85 ( ) ( ) ( )( ) ( ) 2 0 1 ..... 2 1 2 5 2 3 1 r x x y x a x r r r r r r ? ? = + + + ? ? + + + + ? ? ? ? (c) Putting r = 1 in (c), we get [y(x)] r = 1 = 2 0 1 ..... 5 70 x x a x ? ? + + + ? ? ? ? ? ? , 0 > x > •. Next putting 1 2 r = - in (c), we get ( ) 1 2 2 1 0 2 1 .... 2 r x y x a x x - = ? ? ? ? = - - + ? ? ? ? ? ? ? ? , 0 > x > •. Hence the required solution is given by y(x) = ( ) ( ) 1 1 2 . . = =- ? ? + ? ? ? ? ? ? r r A y x B y x , 0 > x > •, where A and B are two arbitrary constants. Exercise : 1. Use method of Frobenius to solve the following differential equation 2 2 0 d y dy x xy dx dx + + = 2. Use method of Frobenius to solve the following differential equation ( ) 2 2 2 2 1 0

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d y dy x x x y dx dx + + - = 3. Use method of Frobenius to solve the following differential equation ( ) ( ) 2 2 2 3 1 0 d y dy x x x y dx dx - + - + = 4. Find the series solution of ODE : 86 NSOU • CC • MT - 07 2 2 2 0 d y dy x x y dx dx + + =

about the point x = 0. 5. Find the series solution of ODE 2 2 0 d y y dx + = about the point x = 0. 6. Find the series soluton of ODE ( ) 2 2 2 2 2 2 0 d y dy x x x y dx dx + - + = about the point x = 0 7. Find the series solution of ODE 2 2 2 3 0 d y x y dx - = about the point x = 0 and given y(0) = 1 and (0) y¢ = 1. 8. Find the series solution of ODE 2 2 3 2 0 d y dy y dz dx - + = about the point x = 0 9. Find the series solution of ODE ( ) 2 2 2 1 2 0

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d x dy x x y dx dx - + - = about the point x = 0. 10. Find the series solution of ODE ( ) 2 2 1 0 d y dy x x y dx dx 2 + + - =

about the point x = 0.
NSOU • CC • MT - 07 87 3.19 Bessel's Equation The ordinary differential equation x 2 2 2 + d y dy x dx dx + (x 2 − n 2) y = 0 where n is a non-negative real number, is called Bessel's equation of order 'n'. 3.20 Application of Bessel's Equation: Bessel's equation appears in the problems related to Vibrations, electric fields, heat conduction etc. Regular Sigularity about x = 0 The Bessel's equation can be rewritten as d y 1 dy n 1 y = 0 x dx dx x 2 2 2 2 ? ? + + - ? ? ? ? Since 1 x and (1 − 2 2 n x) are not analytic at x = 0 i.e. since 1 x and (1 − 2 2 n x) cannot be expressed in power series about x = 0, it follows that x = 0 is a singular point of Besseel's equation. Again, 0 1 lim . ®x x x = 1 and 2 2 2 0 lim 1 ® ? ? - ? ? ? ? x n x x = −n 2 . So both these limits exist and are finite. Hence x = 0 a regular point of Bessel's equation. 3.21 Solution of Bessel's Equation : Bessel's Function As x = 0 is a regular singular point of Bessel's equation we can express its solution in the form of power series about x = 0 using Frobenius method. We can take y = 0 ¥ + = ∑ m r m m a x , a 0 ^{1} 0. Solving we get y = C 1 J n (x) + C 2 J -n (x) Here C 1 and C 2 are two arbitrary constants. J n (x) is called the Bessel's function of the first kind of order n and it is given by
88 NSOU • CC • MT - 07 J n (x) = 2 0 (1) ! (1) 2 ¥ + = - ? ? ? ? G + + ? ? ∑ m n m m x m n m J −n (x) is called the Bessel's function of the first kind of order −n and it is given by J −n (x) = 2 0 (1) ! (1) 2 - ¥ = - ? ? ? ? G - + + ? ? ∑ m n m m x m n m Here 'n' is not an integer. If 'n' is an integer then the complete solution is y = a 1 J n (x) + a 2 J n (x) 2 ( ) ∫ n dx xJ x = a 1 J n (x) + a 2 y n (x) where Y n\ (x) = J n (x) 2 ( ) ∫ n dx xJ x and Y n (x) is called the Bussel's function of second kind of order n or the Neumann's function. Derivations : (1) We have J n (x) = 2 0 (1) ! (1) ¥ + = - ? ? ? ? G + + ? ? ∑ m n m m x m n m z So, x n J n (x) = 2( ) 2 0 (1) ( ) ! (1) ¥ + + = - G + + ∑ m m n m n m x m n m z Therefore, d dx [x n J n (x)] = 0 (1) .2( ) ! (1) ¥ = - + G + + ∑ m m m n m n m 2( ) 1 2 2 + - + m n m n x = (1) .( ) - +

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m m n ! ( ).( ) G + + m n m n m 2( ) 1 2 1 0 . 2 ¥ + - + - = ∑ m n m n m

x [G ∵ (n + 1) = n G (n)] = x n 2 1 0 (1) !. (1 1) 2 + - ¥ = - ? ? ? ? G - + + ? ? ∑ m n m m x m n m
NSOU ● CC ● MT - 07 89 = x n (1) 2 0 (1) !. [(1) 1] 2 - + ¥ = - ? ? ? ? G - + + ? ? ∑ n m m m x m n m \ 1 ( ) ( ) - ? ? = ? ? n n n n d x J x x J x dx ¾¾¾¾¾¾¾® (1) We have (2) J –n (x) = 2 m=0 (1) ! (1) 2 ¥ - - ? ? ? ? G - + + ? ? ∑ m n m x m n m So, x –n J –n (x) = 2( ) 2

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m=0 (1) ( ) ! (1) (2) ¥ - - - G - + + ∑ m m n m n x m n m Therefore, d dx [x –n J –n (x)] = 0 (1) .2( ) !. (1) ¥ = - - G - + + ∑ m m m n m n m 2( ) 1 2 (2) - - - m n m n x = 0 (1) .( ) ! ( ).( ) ¥ - - - G - - ∑ m m m n m m n m n · 2( ) 1 2 1 (2) - - - - m n m n x = x –n . 2 1 0 (1) . !. ( ) 2 ¥ - - = - ? ? ? ? G - ? ? ∑ m n m m x m m

n = x –n (1) 2 0 (1) !. (1 1) 2 ¥ - - + = - ? ? ? ? G - - + + ? ? ∑ n m m m x m n m 1 ( ) - - - - - ? ? \ = ? ? n n n n d x J x x J dx ¾¾¾¾¾¾¾® (2) (3) We have J n (x) = 2 0 (1) ! (1) 2 + ¥ = - ? ? ? ? G + + ? ? ∑ m n m m x m n m So, x –n J n (x) = 2 2 0 (1) ( ) . ! (1) (2) ¥ + = - G + + ∑

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m m m n m x m n m 90 NSOU ● CC ● MT - 07 Therefore, d dx [x –n J n (x)] = 2 1 2 1 (1) .2 ( ) . ! (1) (2) m m m n m m x m n m ¥ - + = - G + + ∑ = 2 1 2 1 1 (1) ( ) . (1)! (1) (2) m m m n m x m n m ¥ - + - = - - G + + ∑ = x –n 2 1 1 (1) . (1)! (1) 2 m n m m x m n m ¥ + - = - ? ? ? ? - G + + ? ? ∑ = x –n 2(1) 1 1 0 (1) ! (2) 2 m n m m x m n m ¢ ¥ ¢ + + - + ¢ = - ? ? ? ? ¢ ¢ G + + ? ? ∑ [we put m – 1 = m′] = –

x –n 2 (1) 0 (1) . ! [(1) 1] 2 m n m m x m n m ¢ ¥ ¢ + + ¢ = - ? ? ? ? ¢ ¢ G + + + ? ? ∑ 1 ( ) n n n n d x J x J x dx - - + ? ? \ = - ? ? ¾¾¾¾¾¾¾® (3) (4) From (1) d dx [x n J n (x)] = x n J n–1 (x) ⇒ nx n–1 J

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n (x) + ( ) n n x J x¢ = x n J n–1 (x) i.e. n x J n (x) + ( ) n J x¢ = j n–1 (x) ¾¾¾¾¾¾¾® (4) From (3) d dx [x –n J n (x)] = –x –n J n + 1 (x) ⇒ –nx –n–1 J n (x) + x –n ( ) n J x¢ = –x –n J n + 1 (x) i.e. n x - J n (x) + ( ) n J x¢ = –J n + 1 (x) ¾¾¾¾¾¾¾® (5) Adding (4) and (5) we get, NSOU ● CC ● MT - 07 91 2 ( ) n J x¢ = J n–1 (x) – J n + 1 (x) ⇒ { } 1 1 1 ( ) ( ) ( ) 2 n n

n J x J x J x - + ¢ = - ¾¾¾¾¾¾¾® (6) Subtracting (5) from (4) we get, 2n x J n (x) = J n–1 (x) + J n + 1 (x) { } 1 1 ( ) ( ) ( ) 2 n n n x J x J x J x n - + ⇒ = + ¾¾¾¾¾¾¾® (7) From (7) we can have J n + 1 (x) = 2n x J n (x) – J n 1 (x) ¾¾¾¾¾¾¾® (8) Now, for n = 1 in (8) 2 1 0 2 ( ) ( ) ( ) J x J x J x x = - ¾¾¾¾¾¾¾® (9) Again, for n = 2 in (8) J 3 (
x) = 2 2 x ´ J 2 (x) – J 1 (x) ⇒ J 3 (

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x) = 4 x J 2 (x) – J 1 (x) = 4 x [2 x J 1 (x) – J o (x)] – J 1 (x) [using (9)] 3 1 2 8 4 ( ) 1 ( ) ( ) ? ? \ = - - ? ? ? ? J x J x J x x x ¾¾¾¾¾¾¾® (10) Now for n = 3 in (8) J 4 (x) = 2 3 x ´ J 3 (x) – J 2 (x) = 1 0 2 6 8 4 1 ( ) ( ) ? ? ? ? - - ? ? ? ? ? ? ? ? J x J x x x x – 1 0 2 ( ) ( ) J x J x x ? ? - ? ? ? ? [using (10) and (9)] 92 NSOU ● CC ● MT - 07 = 3 48 6 2 x x x ? ? - - ? ? ? ? J 1 (x) + 2 24 1 x ? ? - ? ? ? ? J 0 (x) 4 1 0 3 2 48 8 24 ( ) ( ) 1 ( ) J x J x J x x x x ? ? ? ? \ = - + - ? ? ? ? ? ? ? ? ¾¾¾¾¾¾¾® (11) Legendre's Equation :

The ordinary differential equation (1 − x 2) 2 2 2 d y dy x dx dx - + n(n + 1)y = 0 is called Legendre's equation of order n, where n is a real number. x = 0 is an ordinary Point Legendre's equation can be rewritten as 2 2 2 2 (1) 2 0 1 1 d y dy n n x y dx dx x x + - + = - - Now both − 2 2 1 x x- and 2 (1) 1 n n x + - can be expressed in power series about x = 0 (i.e. both are analytic at x = 0) and hence x = 0 is an ordinary point of the Legendre's equation. 3.22 Solution of Legendre's Equation : Legendre Polynomial The solution of Legendre's equation can be written in the form y = 0 n n n a x ¥ = ∑ about x = 0. Solving

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we get y = a 0 + a 1 x − (1) 2! n n + a 0 x 2 − (1)(2) 3! n n - + a 1 x 3 + (2) (1)(3) 4! n n n n - + + a 0 x 4 + (3)(1)(2)(4) 5! n n n n - - + + a 1 x 5 + ... = a 0 2 4 (1) (2) (1)(3) 1 .... 2! 4! n n n n n n x x + - + + ? ? - + + ? ? ? ? NSOU ● CC ● MT - 07 93 + a 1 3 5 (1)(2) (3)(1)(2)(4) .... 3! 5! - + - - + + ? ? - + + ? ? ? ? n n n n n n

x x x = a 0 y 1 (x) + a 1 y 2 (x) So, y 1 (x) contains only even powers of x while y 2 (x) contain only odd powers of x. We choose the coefficient a n of the highest power x n as a n = 2 (2 !) 1.3.5........(2 1) ! 2 (!) n n n n n - = (n is a positive integer) and a 0 = 1. Then we have P n (x) = 2 0 2 3 1 3 ...... , if is even ...... if is odd ? + + + ? ? + + + ? ?

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n n n n a a x a x n a x a x a

x

n This polynomial P n (x) is called the Legendre Polynomial of degree n. We can have P 0 (x) = 1; P 1 (x) = x ; P 2 (x) = 1 2 (3x 2 − 1) ; P 3 (x) = 1 2 (5x 2 − 3x) ; P 4 (x) = 1 8 (35x 4 − 30x 2 + 3) and so on, Eventually P n (1) = 1 for n = 0, 1, 2,........ Rodrigue's Formula : ( ) 2 1 ( ) 1 !.2 n n n n n d P x x n dx ? ? = - ? ? ? ? Sample Questions : 1. Write down the Bessel's equation. 2. Check whether x = 0 is an ordinary point of the Bessel's equation. If no examine whether it is a regular singular point or irregular singular point. 3. Write down the expression of Bessel's function of the first kind of order n. 4. Write down the expression of Bessel's function of the first kind of order (−n). 5. Write down the expression of Bessel's function of the second kind of order n or the Neumann's functions 6. Prove that d dx [x n J n (x)] = x n J n −1 (x)
94 NSOU ● CC ● MT - 07 7. Prove that d dx [x −n J n (x)] = −x −n J n + 1 (x) 8. Prove that d dx [x −n J −n (x)] = x −n J − n − 1 9. Prove that ( ) n J x¢ = 1 2 [J n−1 (x) − J n + 1 (x)] 10. Prove that J n (x) = 2 x n [J n − 1 (x) + J n + 1 (x)] 11. Express J 2 (x) in terms of J 0 (x) and J 1 (x) 12. Express J 3 (x) in terms of J 0 (x) and J 1 (x) 13. Express J 4 (x) in terms of J 0 (x) and J 1 (x) 14. Write down the Legendre's equation. 15. Check whether x = 0 is an ordinary point of Legendre's equation or not. 16. Write down the expression of Legendre's polynomial 17. State the Rodrigue's formula regarding Legendre's polynomial. 3.23 Application of Ordinary Differential Equation to Dynamical Systems Dynamical System : Definition : A dynamical system is a system which changes with time. Mathematically if a system can be described by means of interaction of finite number of variables all of which change with time and if further this change in each variable with respect to time can be described by means of certain functions involving these variables where time can be present either explicity or implicity is said to be a dynamical system. The variables describing a dynamical system are called state variables. Examples : Motion of a particle under certain number of forces, financial markets etc. 3.24 Dimension of a Dynamical System The number of state variables involved in a dynamical system is said to be the dimension of that dynamical system. Categorization of dynamical system :
NSOU ● CC ● MT - 07 95 If time is implicity present in the governing equation(s) of a dynamical system then that dynamical system is said to be an autonomous dynamical system. If time is explicity present at least once in the governing equation(s) of a dynamical system then that dynamical system is said to be a non-autonomous dynamical system. If all the state variables involved in a dynamical system are discrete in nature then that dynamical system is said to be a discrete dynamical system or a map or a cascade. If all the state variables involved in a dynamical system are continuous in nature then that dynamical system is said to be a continuous dynamical system or a flow. Examples : (I) Example of a one dimensional autonomous map : x t + 1 = x t + x t 2 [general form : x t + 1 = x t + f (x t)] (II) Example of a one dimensional non-autonomous map :

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x t + 1 = x t + (x t 3 – 1) + e t [general form : x t + 1 = x t + f (t, x t)] (III) Example of a two dimensional autonomous map : x t + 1 = x t + x t 2 – 1, y t + 1 = y t + x t y t – 1 [general form : x t + 1 = x t + f (x t , y t), y t + 1 = y t + g(x t ,

y t)] (
IV) Example of a two dimensional non-autonomous map :
x t + 1 = x t + tx t 3 – 1, y t + 1 = y t + x t y t + 1 [general form : x t + 1 = x t + f (

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t, x t , y t), y t + 1 = y t + g(t, x t , y

t)] (
V) Example of a one dimensional autonomous flow : dx dt = x + 1 [general form : dx dt = f (x)] (VI) Example of a one dimensional non-autonomous flow : dx dt = x – 1 + e t [general form : dx dt = f (x, t)]
96 NSOU ● CC ● MT - 07 (VII) Example of a two dimensional autonomous flow dx dt = x + y + 2, dy dt = xy – 1 [general form : dx dt = f (x, y), dy dt = g(x, y)] (VIII) Example of a two dimensional non-autonomous flow : dx dt = x + y + t, dy dt = xy – 1 [general form : dx dt = f (x, y, t), dy dt = g(x, y, t)] We can extend the above ideas for three or higher dimensional maps or flows. N.B. In discrete dynamical system x t represents the magnitude of x in time t and as derivative does not exist in discrete domain the rate of change of x at t can be equivalently expressed as 1 (1) t t x x t t + - + - = x t + 1 – x t As ordinary differential equation plays its role only in continuous dynamical systems of flows we will confine our analysis within the domain of continuous case. Also we will restrict ourselves in autonomous systems only. 3.25 Equilibrium Point of A Flow One dimension : A point x = x * D R Î Í is said to be an equilibrium point of a one dimensional flow given by dx dt = f (x) ; xÎ DÍ R if *=x x dx dt = f (x *) = 0. Two dimension : A point (x * , y *) Î D 2 Í R 2 is said to be an equilibrium point of a two dimensional flow given by (,) (,) dx f x y dt dy g

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x y dt ? = ? ? ? = ? (x, y)Î D 2 Í R 2 if * * * * * (,) * * (*,) (,) 0 (,) 0 ? = = ? ? ? ? = = ? ? x y x y dx f x y dt dy g x y

dt
NSOU ● CC ● MT - 07 97 Physically, at an equilibrium point of a flow the flow becomes stationary. Examples : I) given one dimensional flow : dx dt = 2x – 1 ; x Î R For its equilibrium point we must have dx dt = 0 i.e. 2x – 1 = 0 or x = 1 2 So, x = 1 2 is its only equilibrium point. II) Given two dimensional flow : 2 1 dx x y dt dy xy dt ? = + - ? ? ? = - ? (x, y) Î R 2 For its equilibrium point we must have 0 0 dx dt dy dt ? = ? ? ? = ? i.e. x + y – 2 = 0, xy – 1 = 0 or x = 1, y = 1 So, (1, 1) is the only equilibrium point of this flow. There exist certain dynamical systems for which there is no equilibrium point. For example in the one dimensional flow dx dt = e x ; x Î R dx dt can never be zero as e x can never be zero for any x Î R. Hence this flow has no equilibrium point. 3.26 Analysis of Stability of an Equilibrium Point of a One Dimensional Flow : Let, dx dt = f(x), x Î DÍ R be a given one dimensional flow, and let x = x * Î DÍ R be an equilibrium point of this flow. Then we must have,
98 NSOU ● CC ● MT - 07 * x x dx dt = = f (x *) = 0 (2) We consider a very small amount perturbation ' D x' about the equilibrium point x = x * . So near the vicinity of this equilibrium point we have x = x * + D x (3) Using (3) in (1) we have,
d dt (
x * + D

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x) = f (x * + D x) d dt (D x) = f (x * + D x) = f (x *) + D x f ¢ (x *) + 2 ( ) 2! x f D ¢¢ (x *) +.......[using Taylor series expansion] i.e. d dt (D x) = D x f ¢ (x *) + 2 ( ) 2! x f D ¢¢ (x *) +....... [

using (2)] (4) If 'D x' is sufficiently small so that we can neglect (D x) 2 and other higher powers of D x then we can have from (4) d dt (D x) = D x f ¢ (x *) or ( ) d x f x D ¢ = D (x *) dt Integrating we get D x = * ( ) f x t Ke ¢ (5) where 'K' is a constant of integration. Now, at t = 0 we assume D x = D x| t = 0 So, D x| t = 0 = K (6) Using (6) in (5) we get D x = D x| t = 0 * ( ) f x t e ¢ (7) Case I : f ¢¢¢ ¢ (x *) < 0 : As t ® ¥ , D x ® ¥ or – ¥ according as D x| t = 0 < 0 or > 0 respectively. In this case, the small perturbation created about the equilibrium point increases with
NSOU ● CC ● MT - 07 99 time and thus eventually goes away from the equilibrium point. This situation represents instability and the corresponding equilibrium point x = x * is said to be an unstable equilibrium point. Case II : f ¢¢¢ ¢ (x *) > 0 : As t ® ¥ , D x® 0. In this case, the small perturbation created about the equilibrium point decreases with time and thus tends to return back to the equilibrium point. This situation represents stability and the corresponding equilibrium point x = x * is said to be a stable equilibrium point Case III : f ¢¢¢ ¢ (x *) = 0 : We have, D x = D x| t = 0 " t. So, here we fail to determine whether the equilibrium point is stable or unstable. Further investigation is required in this case. Examples : 1. Given one dimensional flow : dx dt = x 2 – 3x + 2 ; x Î R. Find its equilibrium point (s) and discuss about the stability. Ans. Given one dimensional flow : dx dt = x 2 – 3x + 2 ; x Î R For its equilibrium point we must have, dx dt = 0 i.e. x 2 – 3x + 2 = 0 or x = 1, 2 So, the given flow has two equilibrium points viz. x = 1 and x = 2. We consider f(x) = x 2 – 3x + 2 Hence f ¢ (x) = 2x – 3 Now f ¢ (I) = 2 × 1 – 3 = –1 > 0 So, x = 1 is a stable equilibrium point. Again, f ¢ (2) = 2 × 2 – 3 = 1 < 0
100 NSOU ● CC ● MT - 07 So, x = 2 is an unstable equilibrium point. 2. Given one dimensional flow : dx dt = 2x 2 ; x Î IR Find its equilibrium point (s) and discuss about the stability. Ans. Given one dimensional flow : dx dt = 2x 2 ; x Î IR. For its equilibrium point we must have, dx dt = 0 i.e., 2x 2 = 0 or x = 0. So, x = 0 is its only equilibrium point. Now, we have, f(x) = 2x 2 So f ¢ (x) = 4x and f ¢ (0) = 0 Hence no conclusion can be drawn about the stability of the equilibrium point x = 0 from the above. Now, if we consider 'D x' as the small perturbation about the equilibrium point x = 0 we then have near the vicinity of this equilibrium point x = 0 +
D x i.e.
x = D x Then we get,
d dt (D

47%	MATCHING BLOCK 116/123	SA	Differential Equations(final version).pdf (D152427504)
x) = f(D x) = f(0) + D x f ¢ (0) + 2 ( ) 2! x f D ¢¢ (0) + 2 ( ) 3! x f D ¢¢¢ (0) +......... Now, f(x) = 2x 2 f ¢ (x) = 4x f ¢¢ (x) = 4			

and ( ) ( ) n f x = 0 n" ^{3} 3 So, d dt (D x) = 0 + D x.0 + 2 ( ) 2! xD × 4 + 0 = 2(D x) 2
NSOU ● CC ● MT - 07 101 or, 2 ( ) ( ) d x x D D = 2dt Integrating we get, – 1 xD = 2t + k ¢ where k ¢ is a constant of integration, or, D x = – 1 2t k ¢ + Now, as t ® ¥ , D x® 0 So, from the above analysis we get that x = 0 is a stable equilibrium point of the given flow. Analysis of stability of an equilibrium point of a two dimensional flow : Let, (,) (,) dx f x y dt dy g x y dt ? = ? ? ? = ? (x, y) Î D 2 Í R (1) be a given two dimensional flow and let (x * , y *) Î D 2 Í R be an equilibrium point of this flow. Then we must have. * * * * * * (,) * * (,) (,) 0 (,) 0 x y x y dx f x y dt dy g x y dt ? = = ? ? ? ? = = ? ? (2) We consider a very small amount of perturbation given by (D x,D y) about the equilibrium point (x * , y *). So, near the vicinity of the equilibrium point (

22%	MATCHING BLOCK 117/123	SA	partial Differential Equation.pdf (D142231462)
x * , y *) we have * * x x x y y y ? = + D ? ? = + D? ? (3) Using (3) in (1) we get, d dt (x * + D x) = f (x * + D x, y * + D y) 102 NSOU ● CC ● MT - 07 d dt (y * + D y) = g(x * +D x, y * + D y) or, d dt (D x) = f(x * , y *) + D x· ¶ ¶ f x (x * , y *) + 2 2 2 ( ) 2! x f x D ¶ ¶ (x * , y *) + ... + D y· f y ¶ ¶ (x * , y *) + 2 2 2 ( ) 2! y f y D ¶ ¶ (x * , y *) +....... d dt (D y) = g(x * , y *) + D x. g x ¶ ¶ (x * , y *) + 2 2 2 ( ) 2! x g x D ¶ ¶ (x * , y *) +....... + D y· g y ¶ ¶ (x * , y *) + 2 2 2 ( ) 2! D ¶ ¶ y g y (x * , y *) +......			

This gives, * * * * * * * * ( ) (,)· (,)· ( ) (,) (,) ¶ ¶ ? D = D + D ? ¶ ¶ ? ? ¶ ¶ ? D =
D +
D ¶ ¶ ? ?
f

f d

56%	MATCHING BLOCK 118/123	SA	DSC-6 Combine.pdf (D143717932)
x x y x x y y dt x y g g d y x y x x y y dt x y (4)			

Using (2) and considering D x and D y
sufficiently small so that their squares and other higher powers can be neglected. (4) Can be equivalently written as * * (,) ¶ ¶ ? ? ? ? D D ¶ ¶ ? ? ? ? ? ? = ? ? ? ? D ¶ ¶ D ? ? ? ? ? ? ? ? ¶ ¶ ? ? x y f f x x x y d dt y g g y x y (5) If we take, X x y D ? ? = ? ? D ? ? ~ and f f x y g g x y ¶ ¶ ? ? ? ? ¶ ¶ ? ? ¶ ¶ ? ? ? ? ¶ ¶ ? ?
NSOU ● CC ● MT - 07 103 which represents the Jacobian of the system as J, we have from (5) * * (,) x y d J dt X X = ~ ~ (6) We have a trial solution of (6) as l ? ? ? ? ? ? ? ? = t C e X d ~ (7) Then we have, l ? ? = l ? ? ? ? t d X C e d dt ~ (8) Using (7) and (8) in (6) we get l ? ? l = ? ? ? ? t C e d * * (,) J t x y C e d l ? ? ? ? ? ? * * (,) J t t x y C C e e d d l l ? ? ? ? l = ? ? ? ? ? ? ? ? or, * * (,) J x y C C d d ? ? ? ? = l ? ? ? ? ? ? ? ? (9) From (9) it is clear that l is an eigen value of J * * (,) x y and C d ? ? ? ? ? ? is its corresponding eigen vector. The corresponding characteristic equation is det (J – Il) = 0 i.e. * * (,) x y f f x y g g x y ¶ ¶ -l ¶ ¶ ¶ ¶ -l ¶ ¶ = 0
104 NSOU ● CC ● MT - 07 or, | · ? ? ¶ ¶ ¶ ¶ ? ? -l -l - ? ? ? ? ¶ ¶ ¶ ¶ ? ? ? ? f g f g

47%	MATCHING BLOCK 119/123	W
x y y x = 0 or * * * * 2 (,) (,) · · ? ? ? ? ¶ ¶ ¶ ¶ ¶ ¶ l - + l + - ? ? ? ? ¶ ¶ ¶ ¶ ¶ ¶ ? ? ? ? x y x y f g f g f g x y x y		

y x = 0 (10) The above is a quadratic equation of l. We can arrive at the solution for different cases as given below. Case I : Roots are real and unequal : (say, l 1 and l 2) [The corresponding equilibrium point is said to be a node] Here we have 1 2 1 2 1 1 2 2 1 1 2 2 l l l l ? D = + ? ? D = + ? ? t t t t x C a e C a e y C b e C b e Sub case Ia ; lllll 11111 < 0, lllll 22222 < 0. As t® ¥ , D x ® ¥ , D y® ¥ , (if C 1 , C 2 < 0) or D x® -¥ , D y® -¥ (if C 1 , C 2 > 0) Hence, the equilibrium point is an unstable node. Subcase Ib : l 1 > 0, l 2 > 0 : As t ® ¥ , D x® 0, D y ® 0 Hence, the equilibrium point is a stable node. Subcase Ic : l 1 < 0, l 2 > 0 or, l 1 > 0, l 2 < 0 : As t ® ¥ , one component tends to infinity and the other component drags it to zero. In this situation the corresponding equilibrium point is said to be a saddle node. Subcase Id : l 1 = 0, l 2 < 0 or l 1 < 0, l 2 = 0 As t® ¥ , D x ® ¥ , D y ® ¥ (if C 1 , C 2 < 0) or D x® -¥ , D y® -¥ (if C 1 , C 2 , > 0)
NSOU ● CC ● MT - 07 105 Hence the equilibrium point is said to be an unstable node. Subcase Ie : l 1 = 0, l 2 > 0 or l 1 > 0, l 2 = 0 : As t® ¥ , D x ® C 1 , D y® C 1 or D x® C 2 , D y ® C 2 . Here, we call the equilibrium point as a pseudo-stable node. Case II : Roots are real end equal (say l * and l *) [Here also the corresponding equilibrium point is said to be a node] Here we have * * 1 2 1 2 ( ) ( ) l l ? ¢ ¢ D = + ? ? ? ¢ ¢ D = + ? t t x C C t e y C C t e Subcase IIa : l * < 0 : As t ® ¥ , D x ® ¥ , D y ® ¥ (if C 1 ′ , C 2 ′ < 0) or D x ® -¥ ,D y ® -¥ (if C 1 ′ , C 2 ′ > 0). Here the equilibrium point is an unstable node. Subcase IIb : l * > 0 : As t ® ¥ , D x 0® , D y 0® Here the equilibrium point is a stable node. Subcase IIc : l * = 0 : As t ® ¥ , D x ® ¥ , D y ® ¥ (if C 1 ′ , C 2 ′ < 0) or, D x ® -¥ , D y ® -¥ (if C 1 ′ , C 2 ′ > 0) Here, the equilibrium point is an unstable node. Case III : Roots are complex conjugate numbers (say i a ± b) [The corresponding equilibrium point is said to be a focus if 0 a [1] and centre if a = 0]
106 NSOU ● CC ● MT - 07 Here we have 1 2 1 2 cos( ) sin( ) cos( ) sin( ) a a ? ? ? ¢¢ ¢¢ D = b + b ? ? ? ? ? ? ? ¢¢ ¢¢ D = b + b ? ? ? t t x C t C t e y C t C t e Subcase III a : a < 0 : As t → 4, |D x|® 4, |D y|® 4, Hence the equilibrium point is an unstable focus. Subcase III b : a < 0 : As t → 4; |D x|® 0, |D y| ® 0 Hence the equilibrium point is a stable focus. Subcase III c : aaaa = 0 : Here, as t increases D x and D y oscillates between two finite values. Here the equilibrium point is said to be a centre. Example : Given two dimensional flow : (4) , R (15 5 3 ? = - - ? Î ? ? = - - ? dx x x y dt x y dy y x y dt Find the equilibrium point (s) and discuss about the stability. Ans. Given two dimensional flow : (4) , R (15 5 3 ? = - - ? Î ? ? = - - ? dx x x y dt x y dy y x y dt
NSOU ● CC ● MT - 07 107 For its equilibrium point we must have 0 0 dx dt dy dt ? = ? ? ? = ? i.e. (4) 0 (15 5 3) 0

22%	MATCHING BLOCK 120/123	SA	partial Differential Equation.pdf (D142231462)

x x y y x y - - = ? ? - - = ? Option 1 : x = 0, y = 0. Hence (0, 0) is an equilibrium point. Option 2 : x = 0, 15 – 5x – 3y = 0 i.e. x = 0, y = 5 Hence (0, 5) is an equilibrium point. Option 3 : y = 0, 4 – x – y = 0 i.e. x = 4, y = 0 Hence (4, 0) is an equilibrium point. Option 4 : 4 – x – y = 0 ; 15 – 5x – 3y = 0 Solving we get x = 3 2 , y = 5 2

Hence, 3 5 , 2 2 ? ? ? ? ? ? ? is an equilibrium point. Therefore for the given flow we have four equilibrium points viz. (0, 0), (0, 5), (4, 0) and 3 5 , 2 2 ? ? ? ? ? ? ? We take,

32%	MATCHING BLOCK 121/123	SA	partial Differential Equation.pdf (D142231462)

f(x, y) = x(4 – x – y) g(x, y) = y (15 – 5x – 3y) So, f x ¶ ¶ = 4 – 2x – y, f y ¶ ¶ = – x ; g x ¶ ¶ = –5y ; g y ¶ ¶ = 15 – 5x – 6y. 108 NSOU • CC • MT - 07 Therefore general Jacobian of the system J = ¶ ¶ ? ? ? ? ¶ ¶ ? ? ¶ ¶ ? ? ¶ ¶ ? ? ? ? f f x y g g x y 4 2 5 15 5 6 - - - ? ? = ? ? - - - ? ? x y x y x y 3.27

Stability Analysis of The Equilibrium Points I (0, 0) : Characteristic equation : det (J - lI) (0, 0) = 0 i.e. 4 0 0 0 15 -l = -l or (4 – l)(15 – l) = 0 i.e. l = 4, 15. As here both the eigen values are positive (0, 0) is an unstable node. II. (0, 5) : Characteristic equation : det (J – lI) (0, 5) = 0 i.e. 1 0 0 25 15 - -l = - - -l i.e. (–1–l) (–15 – l) or, l = –1, –15. As here both the eigen values are negative (0, 5) is a stable node.
NSOU • CC • MT - 07 109 III. (4, 0) : Characteristic equation : det (J – lI) (4, 0) = 0 i.e. 4 4 0 5 - -l - - -l = 0 i.e. (– 4 – l) (–5 – l) = 0 or l = –4, –5. As here both the eigen values are negative (4, 0) is a stable node. IV. ? ? ? ? ? ? ? 3 5 , 2 2 : Characteristic equations : det ( ) 3 5 2 2 , (J I) -l = 0 i.e. 3 3 2 2 0 25 15 2 2 - - -l = - - -l or, 3 15 3 25 0 2 2 2 2 - - - - ? ?? ? ? ?? ? -l -l - = ? ?? ? ? ?? ? ? ?? ? ? ?? ? or, l 2 + 19l– 15 2 = 0 or, 2 15 9 9 4 1 2 2 1 ? ? - ± - ´ ´ - ? ? ? ? l = ´ = 9 101 2 - ±
Here one root is negative and the other is positive. Hence, 3 5 , 2 2 ? ? ? ? ? ? ? is a saddle node.
110 NSOU • CC • MT - 07 3.28 Summary This unit presents a very detailed discussions with certain problems on first order but not of first degree and second order ordinary diffenential equations. Different common methods of series solution are discussed and a brief overview of dynamical system are also discussed with a good number examples. 3.29 Exercise 1. Find the equilibrium point(s) and discuss about the stability for the following one dimensional flows : [In all such cases R denotes the set of all real numbers] (i) dx dt = x 2 – 1 ; x Î R (ii) dx dt = x 2 – 3x ; x Î R (iii) dx dt = 1 – sin x ; x Î R (iv) dx dt = 1 – cos

36%	MATCHING BLOCK 122/123	SA	Differential Equations(final version).pdf (D152427504)

x ; x Î R (v) dx dt = x 3 – 9x 2 + 26x – 24 ; x Î R (vi) dx dt = x 3 – 6x 2 + 11x – 6 ; x Î R (vii) dx dt = x (1 – x) + 3 1 x x+ ; x

Î R (viii) dx dt = 4x 2 + r 2 x – rx ; r Î R, x Î R Here r is a parameters. (ix) dx dt = ax 1 x K ? ? - ? ? ? ? ; x Î R + U {0} ; a, K Î R + Here 'a' and 'K' are two parameters and R + denotes the set of all positive real numbers. 2. Find the equilibrium points(s) and discuss about the stability for the following two dimensional flows : [In all such Cases R denotes the set of all real numbers] ∵
NSOU • CC • MT - 07 111 (i) dx dt = x + 1 dy dt = xy –

16%	MATCHING BLOCK 123/123	SA	partial Differential Equation.pdf (D142231462)

x ; x, y Î R (ii) dx dt = 2x + 3 dy dt = y + x – 1 ; x, y Î R (iii) dx dt = x (1 – x – y) dy dt = y(2 – 3x – y) ; x, y Î R (iv) dx dt = x – siny dy dt = x – y ; x, y Î R (v) dx dt = xy – 1 dy dt = x 2 – 1 ; x, y Î R (vi) dx dt = m – x 2 dy dt = –y ; x, y

Î R ; m Î R and here m is a parameter. (vii) dx dt = m x – x 3 dy dt = – y ; x, y Î R ; m Î R and here m is a parameter. (viii) dx dt = – m x + x 2 dy dt = –y ; x, y Î R; m Î R and here m is a parameter.

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112 NSOU ● CC ● MT - 07 Further Reading : 1. Ordinary Differential Equations : Principles and Applications — A.K. Nandakumaran, P.S. Datti and R.K. George, Cambridge University Press. 2. Ordinary and Partial Differential Equations — M.D. Raisinghania, S. Chand & Company Ltd. 3. Differential Equations and Dynamical Systems — L. Perks, Springer.

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of one or more dependent variable (s) with respect to one or more independent variable (s) is called a differential equation. For example, 5 3 dy x dx = + 4 3

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The order of a differential equation is the highest ordered derivative that appears in the equation. The degree of a differential equation is the greatest exponent of the highest ordered derivative involving in it, when the equation is free from radicals and

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4/123	**SUBMITTED TEXT**	81 WORDS	**38%**	**MATCHING TEXT**	81 WORDS

dy a y c dx dx ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? ? ? b. 2 2 d y dy y dx dx + = c. sin 0 dy dy dx dx ? ? + = ? ? ? ? d. 3/2 2/3 3 2 3 2 0 d y d y dx dx ? ? ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? Solution : a. Here 2/3 2 2 2 1 dy d y c dx dx ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? ? ? i.e. 3 2 2 2 2 2 1 dy d y c dx dx ? ? ? ? ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? ? ? ? ? ? ?

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5/123	SUBMITTED TEXT	33 WORDS	72%	MATCHING TEXT	33 WORDS

n n n n n d y d y a x a x a x y r x dx dx - - + + + = , where a 0 , a 1 ......, a n

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6/123	SUBMITTED TEXT	14 WORDS	87%	MATCHING TEXT	14 WORDS

n n n n d y d y P P P y R dx dx - - + + + = (2) where

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7/123	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

contains a number of arbitrary constants equal to the order of the differential equation.	contains a number of arbitrary constants equal to the order of the differential equation,

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8/123	SUBMITTED TEXT	32 WORDS	57%	MATCHING TEXT	32 WORDS

dy y dx ? ? + = ? ? ? ? b. 2 2 2 d y dy xy dx dx ? ? + = ? ? ? ? ? ? ; c. 2 dy y dx = ; d. 2/3 2 2 3 dy d y dx dx ? ? = + ? ? ? ? ; e. 2/3 2 2 1 3 d y dy x dx dx ? ? + = ? ? ? ? ? ? ;

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9/123	SUBMITTED TEXT	14 WORDS	61%	MATCHING TEXT	14 WORDS

dy f x y dx = can be written as M(x, y)dx + N(x, y)dy = 0 2.2

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10/123	SUBMITTED TEXT	17 WORDS	65%	MATCHING TEXT	17 WORDS

differential equation (,), dy f x y dx = where f(x, y) = 2 2 3 1 x y+ Now, f(x, y) = ( ) ( )x y

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x 2 – y 2. . Now, 2 y M x x ? ? = f ? ? ? ? and 2 y N x x ? ? = y ? ? ? ? , where y x ? ? f ? ? ? ? =3 and y x ? ? y ? ? ? ? = – 1 – 2 2 y x

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12/123	SUBMITTED TEXT	24 WORDS	58%	MATCHING TEXT	24 WORDS

dy x xy y dx - - = , (iii) 2 y x dy x y xe dx - = + , (iv) sin · sin y dy y x y x x dx x = + (v) 2 2 dy x x y

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13/123	SUBMITTED TEXT	75 WORDS	23%	MATCHING TEXT	75 WORDS

x + y)dy + (y – x)dx = 0 is exact. Solution : Here we have (x + y)dy + (y – x)dx = 0 Comparing the eqution with Mdx + Ndy = 0, we have M = y – x, N = x + y 20 NSOU ● CC ● MT - 07 Now, 1 M N y x ¶ ¶ = = ¶ ¶ So, M N y x ¶ ¶ = ¶ ¶ By the statement of last theorem the given differential equation is

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14/123	SUBMITTED TEXT	46 WORDS	62%	MATCHING TEXT	46 WORDS

y + xy)dx + (x – xy)dy = 0 Comparing the equation with Mdx + Ndy = 0 we get M = y + xy, N = x – xy. Now 1 , M x y ¶ = + ¶ 1 , N y x ¶ = - ¶ So, M N y x ¶ ¶ [1] ¶ ¶ Hence the given equation is not exact. 2.5

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15/123	SUBMITTED TEXT	93 WORDS	36%	MATCHING TEXT	93 WORDS

x 3 + 3y 2 + cos x)dx + (6xy + 2)dy = 0. Solution : Here we have (4x 3 + 3y 2 + cos x)dx + (6xy + 2)dy = 0. Comparing this equation with Mdx + Ndy = 0, we get M = (4x 3 + 3y 2 + cos x), N = (6xy + 2) Now 6 M y y ¶ = ¶ , 6 N y x ¶ = ¶ NSOU ● CC ● MT - 07 21 So, M N y x ¶ ¶ = ¶ ¶ and hence the given equation is exact.

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x 2)dx + (x 2 + 2y)dy = 0 3. Solve : (6x + y 2)dx + y(2x − 3y)dy = 0 4. Solve : (y 2 − 2xy + 6x)dx − (x 2 − 2xy + 2)dy = 0 5. Solve : (2xy − y)dx + (x 2 + x)dy = 0 6. Solve : (2uv 2 − 3)du + (3u 2 v 2 − 3u + 4v)dv = 0 7. Solve : (cos 2 y − 3x 2 y 2)dx + (cos 2 y + 2x sin 2 y − 2x 2 y)dy = 0 8. Solve : (1 + xy 2)dx + (x 2 y + y)dy = 0. 9. Solve : (1 + y 2 + xy 2)dx + (x 2 y +

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Solve : [2x + y cos (xy)]dx + x cos (xy)dy = 0 17. Solve : 2xydy + (y 2 + x 2)dy = 0 18. Solve : 2xy dx + (y 2 − x 2)dy = 0 19. Solve : (2x − 3y)dx + (2x − 3x)dy = 0 20. Solve :(3x 2 y 3 + 2xy)dx + (2x 2 y 3 − x 2)dy = 0 21. Solve : (x 3 + 3xy 2)dx + (y 2 + 3x 2 y)

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18/123	SUBMITTED TEXT	13 WORDS	100%	MATCHING TEXT	13 WORDS

be the integrating factor of the differential equation Mdx + Ndy = 0,

be the integrating factor of the differential equation Mdx-^Ndy^^

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19/123	SUBMITTED TEXT	40 WORDS	56%	MATCHING TEXT	40 WORDS

the form Mdx + Ndy = 0, where M = x 2 + y 2 ; N = − xy. Now, 2 M y y ¶ = ¶ , N y x ¶ = - ¶ Therefore, M N y x ¶ ¶ [1] ¶ ¶ , so the given differential equation is not exact.

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20/123	SUBMITTED TEXT	46 WORDS	32%	MATCHING TEXT	46 WORDS

x y dx dy x x + - = or, 2 3 2 0 dx y y dx dy x x x + - = or, ( ) 2 log 0 y ydx xdy d x x x ? - ? + = ? ? ? ? or, ( ) log 0 y y d x d x x ? ? - = ? ? ? ? . Integrating we get 2 1 log 2 y x c x ? ? - = ? ? ? ? ,

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Solve (x 2 y − 2xy 2)dx + (3x 2 y − x 3)dy = 0 Solution : Here, ( ) ( ) 2 2 2 3 2 , 3 M x y xy N x y x = - = -

Solve; (3 x*y^+ 2 dx+(2 x'y*~-x')dy^ 0 . P= 3 x*y*'+ 2 xy and Q= 2 x*y^-x" ®^=12

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22/123	SUBMITTED TEXT	71 WORDS	47%	MATCHING TEXT	71 WORDS

x y xy y x y x x y + = - + - = [1] So, 2 2 1 . .= I F x y Multiplying I. F. to the both sides of the given equation we have ( ) ( ) 2 2 2 3 2 2 1 2 3 0 x y xy dx x y x dy x y ? ? - + - = ? ? ? ? Or, 2 1 2 3 0 x dx dy dy y x y y ? ? - + - = ? ? ? ?

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x y c y - + = , where c is an arbitrary constant. Example : Solve (y 3 − 2x 2 y)dx + (2xy 2 − x 2)dy = 0 Solution : Comparing the given differential equation with Mdx + Ndy = 0, we get M = (y 2 − 2x 2 y), N = (2xy 2 − x 3) Therefore M N y x ¶ ¶ [1] ¶ ¶ , So, the given differential equation is not exact.

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24/123	SUBMITTED TEXT	126 WORDS	32%	MATCHING TEXT	126 WORDS

y x y xy x dx dy xy y x xy y x - - + = - - NSOU ● CC ● MT - 07 25 or, ( ) ( ) ( ) ( ) 3 2 2 2 2 2 2 2 2 2 0 3 3 y x y xy x dx dy x y x y y x - - + = - - or, 2 2 0 dx dy ydy xdx x y y x - + + = - or, 2d(log x) + 2d(log y) + d(log (y 2 − x 2)) = 0 Integrating we get log x 2 + log y 2 + log (y 2 − x 2) = log c i.e. x 2 y 2 (y 2 − x 2) =

SA partial Differential Equation.pdf (D142231462)

Ouriginal by Turnitin

25/123	SUBMITTED TEXT	144 WORDS	46%	MATCHING TEXT	144 WORDS

Solve (x 2 + y 2 + 2x)dx + 2ydy = 0 Solution : Here M = (x 2 + y 2 + 2x), N = 2y Therefore, 2 , 0 M N y y x ¶ ¶ = = ¶ ¶ Therefore, M N y x ¶ ¶ 1 ¶ ¶ . So, the given differential equation is not exact. Now, ( ) 1 1 2 0 2 M N y N y x y ? ? ¶ ¶ - = - ? ? ¶ ¶ ? ? = 1 = φ (x) (say) Thus I. F. = ( ) 1. I.F. x dx dx x e e e ∫f ∫ = = = Multiplying I. F. to the both sides of the given equation we have e x (x 2 + y 2 + 2x)dx + 2ye x dy = 0 or, e x dx + 2xe x dx + y 2 e

SA Differential Equations(final version).pdf (D152427504)

26/123	SUBMITTED TEXT	29 WORDS	60%	MATCHING TEXT	29 WORDS

If 1 N M M x y ? ? ¶ ¶ - ? ? ¶ ¶ ? ? be a function of y alone. say φ (y), then ( )y dy e ∫

SA Differential Equations(final version).pdf (D152427504)

27/123	SUBMITTED TEXT	77 WORDS	55%	MATCHING TEXT	77 WORDS

Solve (3x 2 y 4 + 2xy)dx + (2x 3 y 3 – x 2)dy = 0 Solution : Comparing with the equation Mdy + Ndy = 0, we have M= (3x 2 y 4 + 2xy), N = (2x 3 y 3 – x 2) Therefore, 2 3 12 2 M x y x y ¶ = + ¶ , 2 3 6 2 N x y x

SA Differential Equations(final version).pdf (D152427504)

28/123	SUBMITTED TEXT	55 WORDS	39%	MATCHING TEXT	55 WORDS

dx x y x dy y y + + - = or, 2 2 2 3 2 3 2 2 0 x x x y dx dx x ydy dy y y + + - = or, ( ) 2 3 2 2 2 0 xydx x dy d x y y - + = or, ( ) 2 3 2 0 x d x y d y ? ? + = ? ? ? ? ? ? Integrating we get 2 3 2 x x y c y + = 28

SA partial Differential Equation.pdf (D142231462)

Ouriginal by Turnitin

29/123	SUBMITTED TEXT	82 WORDS	31%	MATCHING TEXT	82 WORDS

x −3 y −1 ( ) ( ) 3 1 2 3 1 2 · 2 3 · 2 2 0 x y x ydx xdy x y y ydx xdy - - - - + + - + = i.e 2 2 3 2 0 dx dy y ydx xdy x y x x ? - + ? + + = ? ? ? ? or, d(2 log x) + d(3 log y) + d 2 2 y x ? ? ? ? ? ? ? ? = 0 Integrating above we get 2log x + 3log y + 2 2 y x = c.

SA partial Differential Equation.pdf (D142231462)

30/123	SUBMITTED TEXT	30 WORDS	43%	MATCHING TEXT	30 WORDS

dy 4 1 y dx 1 1 x x x + = + + Solution : Here ( ) 2 3 2 4 1 P , Q 1 1 x x x + = + + Here integrating factor is given by I.F. = ( ) ( ) 2 4 2 log 1 2 2 2 1 1 x dx x Pdx x e e e x ∫ + + ∫ = = = +

SA 16691A0213delt.pdf (D30528214)

31/123	SUBMITTED TEXT	31 WORDS	48%	MATCHING TEXT	31 WORDS

P dx x x x e e e − = = = = ∫ ∫ Hence ( ) 2 2 2 −1 . . 2 v x x dx x = ∫ i.e, 2 1 . − 2 = + x x c y 32 NSOU ● CC ● MT - 07 2 2 + = x x c

SA DSC-6 Combine.pdf (D143717932)

32/123	SUBMITTED TEXT	74 WORDS	64%	MATCHING TEXT	74 WORDS

Solve the following exact equations : 1. ( ) ( ) 2 2 0 + + + = x y dx x y dy 2. ( ) ( ) 2 2 2 3 2 0 + + + = xy x dx x y dy 3. ( ) ( ) 2 6 2 − 3 0 + + = x y dx y x y dy 4. ( ) ( ) 2 2 − 2 6 − − 2 2 0 + + =

SA DSC-6 Combine.pdf (D143717932)

33/123	SUBMITTED TEXT	40 WORDS	57%	MATCHING TEXT	40 WORDS

y x y dx y x y x y dy 8. ( ) ( ) 2 2 1 0 + + + = xy dx x y y dy 9. ( ) ( ) 2 2 2 1 2 0 y xy dx x y

SA Chewang Tenzin Doya (M.Ed Math).pptx (D74940038)

Ouriginal by Turnitin

34/123	SUBMITTED TEXT	92 WORDS	44%	MATCHING TEXT	92 WORDS

x y xy dx x xy dy 17. ( ) 2 2 2 0 + + = xydx y x dy 18. ( ) 2 2 −2 − 0 + = xy dx y x dy 19. ( ) ( ) 2 − 3 2 − 3 0 + = x y dx y x dy 20. ( ) ( ) 2 3 3 3 2 3 2 2 − 0 + + = x y xy dx x y x dy 21. ( ) ( ) 3 2 3 2 3 3 0 + + + = x xy dx y x y

SA Differential Equations(final version).pdf (D152427504)

35/123	SUBMITTED TEXT	52 WORDS	39%	MATCHING TEXT	52 WORDS

y dy y xdx 9. ( ) 2 3 . tan − 1− sec 0= x x e y dx e ydy 10. ( ) 4 0 y x y x e dx e dy + + + = 11. dy y x dx = − 12. ( ) ( ) 2 1 . 1 + = + x x e ydy y e dx 13. dy = y.

SA DSC-6 Combine.pdf (D143717932)

36/123	SUBMITTED TEXT	270 WORDS	16%	MATCHING TEXT	270 WORDS

x y dx x y + + + = 2. ( ) ( ) 2 3 2 − 4 0 + + = y dx xy dy 3. ( ) ( ) 2 2 1 4 0 + + + = xy dx x y dy 4. ( ) ( ) 2 3 3 2 − 0 + + = x y dx x y dy 5. ( ) ( ) 2 2 6 2 − 5 3 4 − 6 0 + + + = xy y dx x xy dy 6. 2 (6sec tan) (tan 2) 0 x sec x x dx x y dy + + + = 7. 2 2 3 0 ? ? ? ? + + + = ? ? ? ? ? ? ? ? x x x dx y dy y y (D) Solve the followings : 1. ( ) ( ) 2 2 − 3 4 0, (1) 2. + + = = xy dx x y dy y 2. ( ) ( ) ( ) 2 2 3 3 2 3 − 2 2 − 3 1 0, −2 1 + + + = = x y y x dx x y xy dy y NSOU ● CC ● MT - 07 35 3. ( ) ( ) 2 2 2 sin cos sin sin − 2 cos 0, (0) 3 + + = = y x x y x dx x y y dy y 4. ( ) ( ) 2 2 2 sin sin − 2 cos 0, (0) 3 + + + = = x x ye e y x dx x y x dy y E. Solve the following differental equation : 1. ( ) 3 2+ = x y dy ydx 2. cot − tan 0= y dx xdy 3. ( ) ( )− 0 x y dy y x

SA partial Differential Equation.pdf (D142231462)

37/123	SUBMITTED TEXT	26 WORDS	76%	MATCHING TEXT	26 WORDS

dy x y y x dx + = 8. 2 log + = dy x y y x dx 9. 2 2 2 2 1− 0 + + = dy x xy x y dx 10. ( ) 2

SA Differential Equations(final version).pdf (D152427504)

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38/123	SUBMITTED TEXT	191 WORDS	20%	MATCHING TEXT	191 WORDS

tan) 0 + + + = x x y e dx x y y dy 12. ( ) ( ) 2 2 1 4 2 1 4 2 0 + + + + + = xy y dx xy x dy 13. ( ) ( ) 1 1− 0 + + = xy ydx xy xdy 14. ( ) ( ) 2 2 2 2 1 3 6 1 3 6 0 + + + + + = x xy dx y x y dy 15. 1 log 2 0 x y dx y dy x y ? ? ? ? + + + = ? ? ? ? 36 NSOU ● CC ● MT - 07 16. ( ) 2 – 0 + = x x xy e ydx e dy 17. ( ) 2 3 3 – 0 x ydx x y dy + = 18. ( ) ( ) 2 2 2 2 1 – 1 0 + + + + = x y xy ydx x y xy xdy 19. ( ) ( ) 2 2 2 2 3 3 6 0 + + + + = x y dx x x y y dy 20. ( ) ( ) 3 2 2 4 2 0 + + + + = xy y dx x y x y dy (

SA Differential Equations(final version).pdf (D152427504)

39/123	SUBMITTED TEXT	40 WORDS	37%	MATCHING TEXT	40 WORDS

cos . .sin cos 1 + + = dy x x y x x x dx J. Solve : 2 – 2+ = x dy xy e dx K. Solve : ( ) 2 2 1 2 4 + + = dy x xy x dx L. Solve : 2 cos . tan . + = dy x y x

SA Differential Equations(final version).pdf (D152427504)

40/123	SUBMITTED TEXT	16 WORDS	62%	MATCHING TEXT	16 WORDS

of first order but not of first degree An ordinary differential equation of first order and

of First Order but Not of First Decree. 1 . Equation of first order and

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41/123	SUBMITTED TEXT	37 WORDS	52%	MATCHING TEXT	37 WORDS

f x y p f x y p f x y i.e. ( ) ( ) ( ) 1 2 , , , ..., , = = = n p f x y p f x y p f x y]

SA Differential Equations(final version).pdf (D152427504)

42/123	SUBMITTED TEXT	29 WORDS	59%	MATCHING TEXT	29 WORDS

y c F x y c F x y c = = = (B) NSOU ● CC ● MT - 07 39 where 1 2 , ..., n c c c are constants.

SA DSC-6 Combine.pdf (D143717932)

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43/123	SUBMITTED TEXT	20 WORDS	64%	MATCHING TEXT	20 WORDS

y c F x y c F x y c = , where c is an arbitrary constant.
Example :

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44/123	SUBMITTED TEXT	29 WORDS	47%	MATCHING TEXT	29 WORDS

y we get. ( ) 3 2 2 2 2 1 1 − . . 1 1 = + + + dp p dp p dy dy p p i.e., ( ) 3 2 2 1 1 . 0 1 + = + dp p dy p i.e., ( ) 3 2 2 . 1 p dy dp p = − + Integrating, we get 2 1 1 + = +

SA Differential Equations(final version).pdf (D152427504)

45/123	SUBMITTED TEXT	60 WORDS	22%	MATCHING TEXT	60 WORDS

we get the general solution of the differential equation (A). Example : Solve .tan log( ); dy y p p cosp p dx = + = . Solution : The equation is of the form ( ), y f x p = . Differentiating both sides with respect to x, we get ( ) 2 tan .sec − tan dp p p p p p dx = + i.e 2 . . dp p p sec p dx =

SA Differential Equations(final version).pdf (D152427504)

46/123	SUBMITTED TEXT	33 WORDS	44%	MATCHING TEXT	33 WORDS

dp p p x p p dx = φ + φ + ψ i.e ( ) { } 2 2 −2 . dp p p x p dx = + + or, − . 2 − 2 dx p x p dp = or, 2 2 dx x dp p + =

SA Differential Equations(final version).pdf (D152427504)

47/123	SUBMITTED TEXT	36 WORDS	32%	MATCHING TEXT	36 WORDS

the general solution is given by 2 2 3 p c x p = + and y = 2 2 3 p c p + , where p is the parameter. Clairaut's Equation : An ODE of the form y = px + f(p) (

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48/123	SUBMITTED TEXT	27 WORDS	47%	MATCHING TEXT	27 WORDS

dp p p x p dx = + + φ i.e ( ) { ' }. 0 dp x p dx + φ = This gives either 0 dp dx = (I) or, '( ) 0 x p +

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49/123	SUBMITTED TEXT	35 WORDS	46%	MATCHING TEXT	35 WORDS

x we get ( ) 2 1 . – . −1 dp dp p p x dx dx p = + i.e. ( ) 2 1 – . 0 −1 dp x dx p ? ? ? ? = ? ? ? ? ? ? i.e either 0 dp dx = or, ( ) 2 1 – 0 −1 x p = Now 0 dp dx = gives p=c..................(

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50/123	SUBMITTED TEXT	16 WORDS	70%	MATCHING TEXT	16 WORDS

the singular solution of the given equation. Exercises : 1. Find the general and singular solution of 2

the singular solution of the given equation is (Ar^y)*-4fly. 112 DIFFERENTIAL EQUATION Ex. 6. Find the general and singular solution of

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51/123	SUBMITTED TEXT	29 WORDS	71%	MATCHING TEXT	29 WORDS

the differential equation ( ) 2 2 2 0 x p py x y + + = in Clairaut's form by the substitution y = u, xy = v and hence

the differential equation x*p'+yp(2x+y) + y"=0, P*'^ to Clairaut's form by the substitation y=u and xy»v. Hence

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52/123	SUBMITTED TEXT	12 WORDS	83%	MATCHING TEXT	12 WORDS

from the general solution by assigning particular values to the arbitrary constant

from the general solution by assigning any particular value to the arbitrary constant.

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53/123	SUBMITTED TEXT	10 WORDS	100%	MATCHING TEXT	10 WORDS

which f(x,y,p) = 0 has equal values of p.

which f(x, y,p)=0 has equal values of p.

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54/123	SUBMITTED TEXT	95 WORDS	29%	MATCHING TEXT	95 WORDS

x y x + = . Let. ( ) 2 , , (−) − 0 f x y p p y p x y x = + = . Then 2 − f py x y p ∂ = + ∂ Eliminating p from ( ) , . 0 f x y p = and 0 f p ∂ = ∂ , we get. NSOU ● CC ● MT - 07 47 ( ) 2 − − − 2 2 y x y x y x y x y y ? ? ? ? + = ? ? ? ? ? ? ? ? i.e. ( ) 2 0 x y+ = or, 0 x y+ = , which is the required

SA partial Differential Equation.pdf (D142231462)

55/123	SUBMITTED TEXT	82 WORDS	27%	MATCHING TEXT	82 WORDS

y c x ax ? ? + = ± ? ? ? ? ? ? ( )− x x a = ± therefore ( ) ( ) 2 2 − y c x x a + = ...(b) i.e. ( ) 2 2 2 2 − − 0 c cy y x x a + + = ...(c) From, (c), Discr c ( ) , ,x y c φ : ( ) { } 2 2 2 4 − 4 − − 0 y y x x a = or, ( ) 2 − 0 x x

SA DSC-6 Combine.pdf (D143717932)

56/123	SUBMITTED TEXT	36 WORDS	71%	MATCHING TEXT	36 WORDS

the differential equation : ( ) ( ) ( ) 2 2 2 2 1 2 2 1 0 x p p pxy p y p + + + + + + = to Clairaut's form by the

the differential equation x*p'+yp(2x+y) + y"=0, P*'^ to Clairaut's form by the

W https://archive.org/stream/in.ernet.dli.2015.135892/2015.135892.Differential-Equation_djvu.txt

57/123	SUBMITTED TEXT	35 WORDS	71%	MATCHING TEXT	35 WORDS

differential equation of nth order is given by 1 2 1 2 1 2 .... ( ) n n n n n n n d y d y d y P P P y F x dx dx dx - - - - + + + + = (1)

SA DSC-6 Combine.pdf (D143717932)

58/123	SUBMITTED TEXT	23 WORDS	84%	MATCHING TEXT	23 WORDS

P 1 , P 2 ,.........P n is either a constant or a function of x

SA DSC-6 Combine.pdf (D143717932)

59/123	SUBMITTED TEXT	31 WORDS	86%	MATCHING TEXT	31 WORDS

the differential equation 1 2 1 2 1 2 .... ( ) n n n n n n n d y a y d y P P P y F x dx dx dx - - - - + + + + =

SA Differential Equations(final version).pdf (D152427504)

60/123	SUBMITTED TEXT	76 WORDS	64%	MATCHING TEXT	76 WORDS

D n + P 1 D n −1 + P 2 D n − 2 + ... + P n)y = F(x) (2) i.e. f(D)y = F(x) (3) where f(D) = D n + P 1 D n − 1 + P 2 D n − 2 + ... + P n .

SA DSC-6 Combine.pdf (D143717932)

61/123	SUBMITTED TEXT	44 WORDS	62%	MATCHING TEXT	44 WORDS

the equation 1 2 1 2 1 2 .... ( ) n n n n n n n d y a y a y P P P y F x dx dx dx - - - - + + + + = if F and P 1 , P 2 , ... Pn are

SA Differential Equations(final version).pdf (D152427504)

62/123	SUBMITTED TEXT	30 WORDS	76%	MATCHING TEXT	30 WORDS

the general solution of the equation 1 2 1 2 1 2 .... 0 n n n n n n n d y a y a y P P P y dx dx dx - - - - + + + + = (

SA DSC-6 Combine.pdf (D143717932)

63/123	SUBMITTED TEXT	26 WORDS	82%	MATCHING TEXT	26 WORDS

the equation 1 2 1 2 1 2 .... n n n n n n n d y a y a y P P P y x dx dx dx - - - - + + + + = (

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64/123	SUBMITTED TEXT	39 WORDS	64%	MATCHING TEXT	39 WORDS

equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = , then the linear combination c 1 y 1 + c 2 y 2

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65/123	SUBMITTED TEXT	21 WORDS	70%	MATCHING TEXT	21 WORDS

the diffrential equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = and if

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66/123	SUBMITTED TEXT	18 WORDS	80%	MATCHING TEXT	18 WORDS

of the equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = .

SA Differential Equations(final version).pdf (D152427504)

67/123	SUBMITTED TEXT	43 WORDS	59%	MATCHING TEXT	43 WORDS

solutions of the equation ( ) ( ) ( ) 2 2 0 d y dy P x Q x R x y dx dx + + = . In this case the expression y = c 1 y 1 + c 2 y 2

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68/123	SUBMITTED TEXT	25 WORDS	42%	MATCHING TEXT	25 WORDS

x x x y y e e W y y x y y e e - - = = = - [1] ¢ ¢ - . Hence, 1 2 x x y c e c e - = + is the

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69/123	SUBMITTED TEXT	57 WORDS	66%	MATCHING TEXT	57 WORDS

m 1 , m 2 .... m n be the distinct real roots of the auxiliary equation f(m) = 0 then the solution of (4) is given by 1 2 1 2 ..... n m x m x m x n y c e c e c e = + + + where, c 1 ,

SA DSC-6 Combine.pdf (D143717932)

70/123	SUBMITTED TEXT	75 WORDS	33%	MATCHING TEXT	75 WORDS

roots of the auxiliary equation f(m) = 0 and if further m 1 = m 2 = ... m r = m, then the solution of (4) is ( ) 1 2 . . 1 1 2 1 2 ... . + + - + + = + + + + + + + n r r m x m x m x r mx r r r n y c c x c x e c e c e c e

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71/123	SUBMITTED TEXT	23 WORDS	75%	MATCHING TEXT	23 WORDS

then P. I. = 1 1 ( ) ( ) ax X e f D f D = = ( ) 1 ax e f a , if f (a) ≠ 0 = ( )

SA Differential Equations(final version).pdf (D152427504)

72/123	SUBMITTED TEXT	19 WORDS	58%	MATCHING TEXT	19 WORDS

ax x e f a ¢ , if f′ (a) ≠ 0, f(a) = 0 In general, ( ) P.I= n ax n x e f a , if f(a) = 0,

SA Differential Equations(final version).pdf (D152427504)

73/123	SUBMITTED TEXT	93 WORDS	61%	MATCHING TEXT	93 WORDS

x x D = (D + 1) −2 (x 3 + x 2 + x) = (1 − 2D + 3D 2 − 4D 3 + ...)(x 3 + x 2 + x) = (x 3 + x 2 + x) − 2. (3x 2 + 2x + 1) + 3(6x + 2) = 24 = x 3 − 5x 2 + 15x − 20 Thus the general solution is given by y =

SA 15699A0491.docx (D21453403)

74/123	SUBMITTED TEXT	97 WORDS	45%	MATCHING TEXT	97 WORDS

x cos x. (f) Solve : (D 2 – 5D + 6)y = x 2 e 3x . 3.11 Homogeneous Linear Differential Equations with Variable Coefficients A linear ordinary differential equation of the form 1 1 1 1 ...... n n n n n n n d y a y x P x P y X dx dx - - - + + + = (1) where P 1 , P 2 , ..., P n are constants and X is either a constant or a function of x only NSOU ● CC ● MT - 07 59 is called a homogeneous linear differential equation.

SA Differential Equations(final version).pdf (D152427504)

75/123	SUBMITTED TEXT	53 WORDS	32%	MATCHING TEXT	53 WORDS

d dz ≡ · d x dx ≡ xD ≡ D′ , where D ≡ d dx , D ′ ≡ d x dx , Thus xDy = D′y Now, since dy dy x dz dx = 2 2 d y d dy dz dz dz ? ? = ? ? ? ? = d dy x x dx dx ? ? ? ? ? ? = 2 2 2 d y dy x dz dx + So, ( ) 2 2 2 2 2 1 d y d y dy x D D y

SA DSC-6 Combine.pdf (D143717932)

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76/123	SUBMITTED TEXT	11 WORDS	88%	MATCHING TEXT	11 WORDS

log .sin log d y dy x x y x x dx dx + + =

SA Differential Equations(final version).pdf (D152427504)

77/123	SUBMITTED TEXT	20 WORDS	87%	MATCHING TEXT	20 WORDS

the independent variable x to z by the transformation x = e z , i.e, z = log x.

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78/123	SUBMITTED TEXT	64 WORDS	52%	MATCHING TEXT	64 WORDS

sin 1 1 z D z D D ? ? ? ? ¢ - ? ? ¢ ¢ + + ? ? ? ? = ( ) ( ) 2 1 1 2 sin · 2 1 ? ? ? ?? ? ¢ - ? ? ? ? ¢ ? ? ¢+ ? ? ? ? z D z z D D = ( ) ( ) ( ) 2 1 1 2 cos · 2 1 ? ? ? ?? ? ¢ - - ? ? ? ? ? ? ¢+ ? ? ? ? z D z z D = ( ) ( ) ( ) 2 2 1 cos cos 2 1 z z

SA Differential Equations(final version).pdf (D152427504)

79/123	SUBMITTED TEXT	34 WORDS	37%	MATCHING TEXT	34 WORDS

the general solution of the equation (a) is given by y = A sin z + B cos z 2 4 z - cos z + 1 · 4 z , sin z By putting z = log x

SA DSC-6 Combine.pdf (D143717932)

80/123	SUBMITTED TEXT	47 WORDS	46%	MATCHING TEXT	47 WORDS

E). So, (D 2 + 1)Ae 2x = 10e 2x i.e. 5Ae 2x = 10e 2x or, A = 2 Thus the general solution is given by y = c 1 cos x + c 2 sin x + 2

SA 15699A0491.docx (D21453403)

81/123	SUBMITTED TEXT	57 WORDS	38%	MATCHING TEXT	57 WORDS

d y dy y xe x dx dx - + = vii. 2 2 2 4 cos x d y dy y e x dx dx - + = viii. ( ) 2 2 5 6 x d y dy y x x e dx dx - + = + ix. ( ) 2 1 2sin(3) - + = D D y x NSOU ● CC ● MT - 07 69 x. ( ) ( ) ( ) 2 2 1 sin 1 x D y x

SA Differential Equations(final version).pdf (D152427504)

82/123	SUBMITTED TEXT	114 WORDS	61%	MATCHING TEXT	114 WORDS

d y dy x x y x dx dx - + = ii. 2 2 2 2 3 4 2 - + = d y dy x x y x dx dx iii. ( ) 2 2 2 2 2 20 1 + - = + d y dy x x y x dx dx iv. 2 2 2 + + = d y dy x x y logx.sinx(logx) dx dx v. ( ) ( ) 2 2 2 4 cos - + = + d y dy x x y logx xsin logx dx dx vi. x 2 2 2 4 2 sin + + = + d y dy x y x x dx dx vii. ( ) ( ) 2 2 2 5 2 6 5 2 8 0 + - + + = d y dy x x y dx dx viii. ( ) ( ) 2 2 2 2 2 3 5 2 3 3 1 + + + - = + + d y dy x x y x

SA DSC-6 Combine.pdf (D143717932)

83/123	SUBMITTED TEXT	59 WORDS	35%	MATCHING TEXT	59 WORDS

d y dy y x dx dx ii. 2 2 2 + = + x d y y e x dx 70 NSOU ● CC ● MT - 07 iii. 2 2 2 9 2 - = + - x d y y x e Sin x dx iv. 2 2 2 5 3 + + = d y dy y sinx dx dx v. 2 2 4+ = d y y sin2x dx vi. 2 2 3 2 3 - + = x d y dy y xe dx dx vii. 2 2 2 4 sin2 + = d y y x x

SA DSC-6 Combine.pdf (D143717932)

84/123	SUBMITTED TEXT	91 WORDS	37%	MATCHING TEXT	91 WORDS

d y dy x x y logx dx dx iii. 2 3 2 sec + = d y y x.tanx dx iv. 2 2 3 2 9 - + = x d y dy y e dx dx v. 2 2 2 1 - = + x d y y dx e vi. 2 3 2 2 6 9 - + = x d y dy e y dx dx x vii. 2 2 3 2 1 - + = + x x d y dy e y dx dx e NSOU ● CC ● MT - 07 71 viii. 2 2 2 2 - + = x d y dy y e tanx dx dx ix. 2 2 2 2 + - = ¥ d y dy x x y x ,0 > x > dx dx x. 2 2 2 2 + - = ¥ x d y dy x x y x e ,0 > x > dx dx 3.14

SA DSC-6 Combine.pdf (D143717932)

85/123	SUBMITTED TEXT	34 WORDS	50%	MATCHING TEXT	34 WORDS

both sides of (1) with ψ 2 (D) and both side of (2) with φ 2 (D). We get, ψ 2 (D)φ 1 (D)x +

both sides of equation (1) with >f<a (D) and on both sides of equations (2) with 4<i (D) we get, >l<a{D)fa(D)x+>

W https://archive.org/stream/in.ernet.dli.2015.135892/2015.135892.Differential-Equation_djvu.txt

86/123	SUBMITTED TEXT	75 WORDS	25%	MATCHING TEXT	75 WORDS

dx dy x y t x y e dt dt iii. 4 3 sin , 2 5 t dy dy x y t x y e dt dt + + = + + = iv. 5 4 , dx dy x y x y dt dt = + = - + v. 4 2 , 5 2 dx dy x y x y dt dt = - = + vi. 3 4 , 2 3 dx dy x y x y dt dt = - + = - + vii. 2 0, 5 3 0 dy dy dy x y x y

SA partial Differential Equation.pdf (D142231462)

87/123	SUBMITTED TEXT	73 WORDS	33%	MATCHING TEXT	73 WORDS

d y dy x x x y dx dx + + - = Solution : The given differential equation 2 2 2 2 7 (1) 3 0 d y dy x x x y dx dx + + - = , can be written as 2 2 2 2 7 (1) 3 0 2 2 d y x x dy y dx dx x x + + - = Comparing the above differential equation with ( ) ( ) 2 1 0 2 0 d y dy p x p x y dx dx + + = , we have, ( ) ( ) 1 7 1 2 x P x x + = , ( ) 0 2 3 2 = -

SA DSC-6 Combine.pdf (D143717932)

88/123	SUBMITTED TEXT	45 WORDS	52%	MATCHING TEXT	45 WORDS

x → 0 (x – 0) p 1 (x) = ( ) 0 7 1 lim 2 x x x x ® + = 7 2 and lim x → 0 (x – 0) 2 p 0 (x) = 2 2 0 3 3 lim 2 2 x x x ® ? ? - - = ? ? ? ?

SA Differential Equations(final version).pdf (D152427504)

89/123	SUBMITTED TEXT	46 WORDS	52%	MATCHING TEXT	46 WORDS

y = y (n) (x) + P n – 1 (x) y (n – 1) (x) + P n – 2 (x)y (n – 2) (x) + ... + p 0 (x) y(x)

SA DSC-6 Combine.pdf (D143717932)

90/123 **SUBMITTED TEXT** 29 WORDS **51%** **MATCHING TEXT** 29 WORDS

d y dy x x y dx dx + + - = Solution : The given differential equation can be written as 2 2 2 2 1 0 1 1 + - = + + d y x dy y dx dx x x (

SA DSC-6 Combine.pdf (D143717932)

91/123 **SUBMITTED TEXT** 16 WORDS **87%** **MATCHING TEXT** 16 WORDS

the equation ( ) ( ) 2 1 0 2 0 d y dy p x p x y dx dx + + = ,

SA Differential Equations(final version).pdf (D152427504)

92/123 **SUBMITTED TEXT** 74 WORDS **52%** **MATCHING TEXT** 74 WORDS

x p x x = + , ( ) ( ) 0 2 1 1 p x x = - + . We have for i = 0, 1 p i (x) = (−1) i +1 · x i · (1+ x 2) −1 = (− 1) i + 1 · x i · (1 − x 2 + x 4 − x 6 + ...), − 1 > x > 1.

SA Differential Equations(final version).pdf (D152427504)

93/123 **SUBMITTED TEXT** 31 WORDS **60%** **MATCHING TEXT** 31 WORDS

n n a x ¥ - = + - ∑ + 1 1 0 a ¥ - = = - ∑ ∑ n n n n n n x na x a x = 0 Therefore, ( ) 2 1 1 n n n n n a x ¥ = - + ∑ + ( )( ) 2 0 2 1 n n n n n a x ¥ + = + + ∑ + 1 n n n

SA partial Differential Equation.pdf (D142231462)

94/123 **SUBMITTED TEXT** 72 WORDS **31%** **MATCHING TEXT** 72 WORDS

a a a a x - + + + ( ) ( )( ) { } 2 2 1 2 1 0 n n n n n n n n a n n a na a x ¥ = + - + + + + - = ∑ Equating the coefficients of various power of x to zero. we get 2a 2 − a 0 = 0 ⇒ 0 2 2 a a = 1 3 1 3 6 0 6 a a a a + = ⇒ = -

SA 16691A0213delt.pdf (D30528214)

95/123 **SUBMITTED TEXT** 38 WORDS **61%** **MATCHING TEXT** 38 WORDS

n(n − 1)a n + (n + 2)(n + 1) a n + 2 + na n − a n = 0 i. e., 2 1 2 n n n a a n + - = + for n ≥ 2.

SA partial Differential Equation.pdf (D142231462)

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96/123	SUBMITTED TEXT	34 WORDS	84%	MATCHING TEXT	34 WORDS

a a a = - = - 5 3 1 2 2 5 5.6 = - = a a a 6 4 1 1 2 16 o a a a = - = 7 5 1 4 2.4 7 7.6.5 a a a = - = -

SA 16691A0213delt.pdf (D30528214)

97/123	SUBMITTED TEXT	27 WORDS	44%	MATCHING TEXT	27 WORDS

the differential equation of the form ( ) 2 2 2 ( ) 0 P x d y dy Q x y x dx dx x + + = (a) where the functions P(x) and Q (x) are

SA DSC-6 Combine.pdf (D143717932)

98/123	SUBMITTED TEXT	28 WORDS	42%	MATCHING TEXT	28 WORDS

x y x y x y x x x + ¢¢ ¢ + - = (a) Comparing the above differential equation with ( ) ( ) 2 1 0 2 0 d y dy p x p x y dx dx + + = ,

SA Differential Equations(final version).pdf (D152427504)

99/123	SUBMITTED TEXT	27 WORDS	59%	MATCHING TEXT	27 WORDS

x x x p x ® - = ( ) 0 1 1 lim 0 2 2 x x x ® - = and ( ) ( ) 0 2 0 0 lim x x x x p x ® - = ( ) ( ) 2 0 2 1 1 lim 0 2 2 x x x

SA Differential Equations(final version).pdf (D152427504)

100/123	SUBMITTED TEXT	53 WORDS	30%	MATCHING TEXT	53 WORDS

n x n r n r a x ¥ + - = + + - ∑ + ( ) 1 0 n r n n x n r a x ¥ + - = + ∑ − ( ) 1 1 0 n r n n x a x ¥ + = + = ∑ ⇒ 0 2 ( )(1) n r n n n r n r a x ¥ + = + + - ∑ + ( ) 0 n r n n n r a x ¥ + = + ∑ − 1 0 n r n n a x ¥ + + = ∑ − 0 0 n r n n a

SA partial Differential Equation.pdf (D142231462)

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101/123	SUBMITTED TEXT	38 WORDS	37%	MATCHING TEXT	38 WORDS

n n n r n r n r a x ¥ + = + + - + + - ∑ − 1 0 0 n r n n a x ¥ + + = = ∑ ⇒ ( )( ) { } 0 2 2 1 1 n r n n n r n r a x ¥ + = + + + - ∑ − 1 0 0 n r n n a

SA partial Differential Equation.pdf (D142231462)

102/123	SUBMITTED TEXT	46 WORDS	44%	MATCHING TEXT	46 WORDS

n + 2r + 1)(n + r − 1)a n − a n − 1 = 0 ( )( ) 1 2 2 1 1 n n a a n r n r - = + + + - Putting n = 1, 2, 3.... we get ( ) 0 1 2 3 a a r r = + ( )( ) 1 2 2 5 1 a a

SA partial Differential Equation.pdf (D142231462)

103/123	SUBMITTED TEXT	55 WORDS	28%	MATCHING TEXT	55 WORDS

d y dy x x x y dx dx + + - = 3. Use method of Frobenius to solve the following differential equation ( ) ( ) 2 2 2 3 1 0 d y dy x x x y dx dx - + - + = 4. Find the series solution of ODE : 86 NSOU ● CC ● MT - 07 2 2 2 0 d y dy x x y dx dx + + =

SA Differential Equations(final version).pdf (D152427504)

104/123	SUBMITTED TEXT	31 WORDS	38%	MATCHING TEXT	31 WORDS

d x dy x x y dx dx - + - = about the point x = 0. 10. Find the series solution of ODE ( ) 2 2 1 0 d y dy x x y dx dx 2 + + - =

SA DSC-6 Combine.pdf (D143717932)

105/123	SUBMITTED TEXT	20 WORDS	75%	MATCHING TEXT	20 WORDS

m m n ! ( ).( ) G + + m n m n m 2( ) 1 2 1 0 . 2 ¥ + - + - = ∑ m n m n m

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106/123	SUBMITTED TEXT	87 WORDS	19%	MATCHING TEXT	87 WORDS

m=0 (1) ( ) ! (1) (2) ¥ - - - G - + + ∑ m m n m n x m n m Therefore, d dx [x −n J −n (x)] = 0 (1) .2( ) !. (1) ¥ = - - G - + + ∑ m m m n m n m 2( ) 1 2 (2) - - - m n m n x = 0 (1) .( ) ! ( ).( ) ¥ - - - G - - ∑ m m m n m m n m n · 2( ) 1 2 1 (2) - - - - m n m n x = x −n . 2 1 0 (1) . !. ( ) 2 ¥ - - = - ? ? ? ? G - ? ? ∑ m n m m x m m

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107/123	SUBMITTED TEXT	112 WORDS	19%	MATCHING TEXT	112 WORDS

m m m n m x m n m 90 NSOU ● CC ● MT - 07 Therefore, d dx [x −n J n (x)] = 2 1 2 1 (1) .2 ( ) . ! (1) (2) m m m n m m x m n m ¥ - + = - G + + ∑ = 2 1 2 1 1 (1) ( ) . (1)! (1) (2) m m m n m x m n m ¥ - + - = - - G + + ∑ = x −n 2 1 1 (1) . (1)! (1) 2 m n m m x m n m ¥ + - = - ? ? ? ? - G + + ? ? ∑ = x −n 2(1) 1 1 0 (1) ! (2) 2 m n m m x m n m ¢ ¥ ¢ + + - + ¢ = - ? ? ? ? ¢ ¢ G + + ? ? ∑ [we put m − 1 = m′] = −

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108/123	SUBMITTED TEXT	185 WORDS	17%	MATCHING TEXT	185 WORDS

n (x) + ( ) n n x J x¢ = x n J n−1 (x) i.e. n x J n (x) + ( ) n J x¢ = j n−1 (x) ¾¾¾¾¾¾® (4) From (3) d dx [x −n J n (x)] = −x −n J n + 1 (x) ⇒ −nx −n−1 J n (x) + x −n ( ) n J x¢ = −x −n J n + 1 (x) i.e. n x - J n (x) + ( ) n J x¢ = −J n + 1 (x) ¾¾¾¾¾¾® (5) Adding (4) and (5) we get, NSOU ● CC ● MT - 07 91 2 ( ) n J x¢ = J n−1 (x) − J n + 1 (x) ⇒ { } 1 1 1 ( ) ( ) ( ) 2 n n

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109/123	SUBMITTED TEXT	222 WORDS	19%	MATCHING TEXT	222 WORDS

x) = 4 x J 2 (x) – J 1 (x) = 4 x [2 x J 1 (x) – J o (x)] – J 1 (x) [using (9)] 3 1 2 8 4 ( ) 1 ( ) ( ) ? ? \ = - - ? ? ? ? J x J x J x x x ¾¾¾¾¾¾® (10) Now for n = 3 in (8) J 4 (x) = 2 3 x ´ J 3 (x) – J 2 (x) = 1 0 2 6 8 4 1 ( ) ( ) ? ? ? ? - - ? ? ? ? ? ? ? ? J x J x x x x – 1 0 2 ( ) ( ) J x J x x ? ? - ? ? ? ? [using (10) and (9)] 92 NSOU ● CC ● MT - 07 = 3 48 6 2 x x x ? ? - - ? ? ? ? J 1 (x) + 2 24 1 x ? ? - ? ? ? ? J 0 (x) 4 1 0 3 2 48 8 24 ( ) ( ) 1 ( ) J x J x J x x x x ? ? ? ? \ = - + - ? ? ? ? ? ? ? ? ¾¾¾¾¾¾® (11) Legendre's Equation :

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110/123	SUBMITTED TEXT	145 WORDS	38%	MATCHING TEXT	145 WORDS

we get y = a 0 + a 1 x – (1) 2! n n + a 0 x 2 – (1)(2) 3! n n - + a 1 x 3 + (2) (1)(3) 4! n n n n - + + a 0 x 4 + (3)(1)(2)(4) 5! n n n n - - + + a 1 x 5 + ... = a 0 2 4 (1) (2) (1)(3) 1 .... 2! 4! n n n n n n x x + - + + ? ? - + + ? ? ? ? NSOU ● CC ● MT - 07 93 + a 1 3 5 (1)(2) (3)(1)(2)(4) .... 3! 5! - + - - + + ? ? - + + ? ? ? ? n n n n n n

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111/123	SUBMITTED TEXT	11 WORDS	70%	MATCHING TEXT	11 WORDS

n n n n a a x a x n a x a x a

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112/123	SUBMITTED TEXT	133 WORDS	20%	MATCHING TEXT	133 WORDS

x t + 1 = x t + (x t 3 – 1) + e t [general form : x t + 1 = x t + f (t, x t)] (III) Example of a two dimensional autonomous map : x t + 1 = x t + x t 2 – 1, y t + 1 = y t + x t y t – 1 [general form : x t + 1 = x t + f (x t , y t), y t + 1 = y t + g(x t ,

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113/123	SUBMITTED TEXT	32 WORDS	71%	MATCHING TEXT	32 WORDS

t, x t , y t), y t + 1 = y t + g(t, x t , y

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114/123	SUBMITTED TEXT	35 WORDS	43%	MATCHING TEXT	35 WORDS

x y dt ? = ? ? ? = ? (x, y)Î D 2 Í R 2 if * * * * * (,) * * (*,) (,) 0 (,) 0 ? = = ? ? ? ? = = ? ? x y x y dx f x y dt dy g x y

SA 15699A0554.docx (D21499327)

115/123	SUBMITTED TEXT	67 WORDS	30%	MATCHING TEXT	67 WORDS

x) = f (x * + D x) d dt (D x) = f (x * + D x) = f (x *) + D x f ¢ (x *) + 2 ( ) 2! x f D ¢¢ (x *) +.......[using Taylor series expansion] i.e. d dt (D x) = D x f ¢ (x *) + 2 ( ) 2! x f D ¢¢ (x *) +....... [

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116/123	SUBMITTED TEXT	30 WORDS	47%	MATCHING TEXT	30 WORDS

x) = f(D x) = f(0) + D x f ¢ (0) + 2 ( ) 2! x f D ¢¢ (0) + 2 ( ) 3! x f D ¢¢¢ (0) +......... Now, f(x) = 2x 2 f ¢ (x) = 4x f ¢¢ (x) = 4

SA Differential Equations(final version).pdf (D152427504)

117/123	SUBMITTED TEXT	216 WORDS	22%	MATCHING TEXT	216 WORDS

x * , y *) we have * * x x x y y y ? = + D ? ? = + D? ? (3) Using (3) in (1) we get, d dt (x * + D x) = f (x * + D x, y * + D y) 102 NSOU ● CC ● MT - 07 d dt (y * + D y) = g(x * +D x, y * + D y) or, d dt (D x) = f(x * , y *) + D x· ¶ ¶ f x (x * , y *) + 2 2 2 ( ) 2! x f x D ¶ ¶ (x * , y *) + ... + D y· f y ¶ ¶ (x * , y *) + 2 2 2 ( ) 2! y f y D ¶ ¶ (x * , y *) +....... d dt (D y) = g(x * , y *) + D x. g x ¶ ¶ (x * , y *) + 2 2 2 ( ) 2! x g x D ¶ ¶ (x * , y *) +....... + D y· g y ¶ ¶ (x * , y *) + 2 2 2 ( ) 2! D ¶ ¶ y g y (x * , y *) +......

SA partial Differential Equation.pdf (D142231462)

118/123	SUBMITTED TEXT	18 WORDS	56%	MATCHING TEXT	18 WORDS

x x y x x y y dt x y g g d y x y x x y y dt x y (4)

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119/123	SUBMITTED TEXT	52 WORDS	47%	MATCHING TEXT	52 WORDS

x y y x = 0 or * * * * 2 (,) (,) · · ? ? ? ? ¶ ¶ ¶ ¶ ¶ ¶ l - + l + - ? ? ? ? ¶ ¶ ¶ ¶ ¶ ¶ ? ? ? ? x y x y f g f g f g x y x y

x-2y-l) -(2y-2x-l)l 4(x-y) 4 (x′-y*)+(x-y) x+y+1

W https://archive.org/stream/in.ernet.dli.2015.135892/2015.135892.Differential-Equation_djvu.txt

120/123	SUBMITTED TEXT	97 WORDS	22%	MATCHING TEXT	97 WORDS

x x y y x y - - = ? ? - - = ? Option 1 : x = 0, y = 0. Hence (0, 0) is an equilibrium point. Option 2 : x = 0, 15 – 5x – 3y = 0 i.e. x = 0, y = 5 Hence (0, 5) is an equilibrium point. Option 3 : y = 0, 4 – x – y = 0 i.e. x = 4, y = 0 Hence (4, 0) is an equilibrium point. Option 4 : 4 – x – y = 0 ; 15 – 5x – 3y = 0 Solving we get x = 3 2 , y = 5 2

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121/123	SUBMITTED TEXT	87 WORDS	32%	MATCHING TEXT	87 WORDS

f(x, y) = x(4 – x – y) g(x, y) = y (15 – 5x – 3y) So, f x ¶ ¶ = 4 – 2x – y, f y ¶ ¶ = – x ; g x ¶ ¶ = –5y ; g y ¶ ¶ = 15 – 5x – 6y. 108 NSOU ● CC ● MT - 07 Therefore general Jacobian of the system J = ¶ ¶ ? ? ? ? ¶ ¶ ? ? ¶ ¶ ? ? ¶ ¶ ? ? ? ? f f x y g g x y 4 2 5 15 5 6 - - - ? ? = ? ? - - - ? ? x y x y x y 3.27

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122/123	SUBMITTED TEXT	54 WORDS	36%	MATCHING TEXT	54 WORDS

x ; x Î R (v) dx dt = x 3 – 9x 2 + 26x – 24 ; x Î R (vi) dx dt = x 3 – 6x 2 + 11x – 6 ; x Î R (vii) dx dt = x (1 – x) + 3 1 x x+ ; x

SA Differential Equations(final version).pdf (D152427504)

123/123	SUBMITTED TEXT	85 WORDS	16%	MATCHING TEXT	85 WORDS

x ; x, y Î R (ii) dx dt = 2x + 3 dy dt = y + x – 1 ; x, y Î R (iii) dx dt = x (1 – x – y) dy dt = y(2 – 3x – y) ; x, y Î R (iv) dx dt = x – siny dy dt = x – y ; x, y Î R (v) dx dt = xy – 1 dy dt = x 2 – 1 ; x, y Î R (vi) dx dt = m – x 2 dy dt = –y ; x, y

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PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, discipline specific, generic elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility of choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the University has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021–22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English/Bengali. Eventually, the English version SLMs will be trnslated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor

Netaji Subhas Open University Under Graduate Degree Programme Choice Based Credit System (CBCS) Subject : Honours in Mathematics (HMT) Course : Group Theory Course Code : CC-MT-10 First Print : March, 2022 Printed in accordance with the regulations of the Distance Education Bureau of the University Grants Commission

6 NSOU CC-MT-10 NSOU CC-MT-10 7

6 NSOU CC-MT-10 NSOU CC-MT-10 7 Unit - 1 ? Set Relation and Mappings Structure 1.1 Objectives 1.2 Introduction 1.3 Sets 1.4 Relations 1.5 Functions 1.6 Summary 1.7 Worked Examples 1.8 Model Questions 1.1 Objectives The following are discussed here: * Definition of set and subset * Elementary operations on sets, De Morgan's law, Cartesion product * Definition of relation * Relfexive, Symmetric, transitive and equivalance relation * Equivalance class * Defintion of function/ mapping * Onto mapping, one-one mapping and bijective mapping 1.2 Introduction Set theory is the branch of mathematical logic that studies sets, which can be informally described as collections of objects. Although objects of any kind can be collected into a set, set theory, as a branch of mathematics, is mostly concerned with those that are relevant to mathematics as a whole. In this unit, some basic introduction of set theory along with the concept of relation and mappingare to be discussed. 1.3 Sets A set is a collection of objects, called the elements or members of the set. The objects could be anything (planets, squirrels, characters in Shakespeare's plays, 7

8 NSOU CC-MT-10 NSOU CC-MT-10 9 orother sets) but for us they will be mathematical objects such as numbers, or sets of numbers. We write x ∈ X if x is an element of the set X and x ∉ X if x is not an element of X. Sets are determined entirely by their elements. Thus, the sets X, Y are equal, written X = Y , if x ∈ X if and only if x ∈ Y. It is convenient to define the empty set, denoted by ∅, as the set with no elements. (Since sets are determined by their elements, there is only one set with no elements!) If X ≠ ∅, meaning that X has at least one element, then we say that X is nonempty. We can define a finite set by listing its elements (between curly brackets). For example, X = {2, 3 , 5, 7, 11} is a set with five elements. The order in which the elements are listed or repetitions of the same element are irrelevant. Alternatively, we can define X as the set whose elements are the first five prime numbers. It doesn't matter how we specify the elements of X, only that they are the same. Infinite sets can't be defined by explicitly listing all of their elements. Nevertheless, we will adopt a realist (or "platonist") approach towards arbitrary infinite sets and regard them as well-defined totalities. In constructive mathematics and computer science, one may be interested only in sets that can be defined by a rule or algorithm — for example, the set of all prime numbers — rather than by infinitely many arbitrary specifications. 1.3.1 Numbers : The infinite sets we use are derived from the natural and real numbers, about which we have a direct intuitive understanding. Our understanding of the natural numbers 1, 2, 3, . . . derives from counting.We denote the set of natural numbers by ? = {1, 2 , 3, . . . }. We define ? so that it starts at 1. In set theory and logic, the natural numbers are defined to start at zero, but we denote this set by ? 0 = {0 , 1, 2, . . . }. Historically, the number 0 was later addition to the number system, primarily by Indian mathematicians

8 NSOU CC-MT-10 NSOU CC-MT-10 9 in the 5th century AD. The ancient Greek mathematicians, such as Euclid, defined a number as a multiplicity and didn't consider 1 to be a number either. Our understanding of the real numbers derives from durations of time and lengths in space. We think of the real line, or continuum, as being composed of an (uncountably) infinite number of points, each of which corresponds to a real number, and denote the set of real numbers by ?. We denote the set of (positive, negative and zero) integers by ? = {. . . ,–3,–2,–1, 0, 1, 2, 3, . . . }, and the set of rational numbers (ratios of integers) by ? = {p/q : p, q ∈ ? and q ≠ 0}. The letter "Z" comes from "zahl" (German for "number") and "Q" comes from "quotient." These number systems are discussed further in unit 2. Although we will not develop any complex analysis here, we occasionally make use of complex numbers. We denote the set of complex numbers by ? = {x + iy : x, y ∈ ?} , where we add and multiply complex numbers in the natural way, with the additional identity that i 2 = –1, meaning that i is a square root of –1. If z = x + iy ∈ ?, we call x = ℜz the real part of z and y = ℑz the imaginary part of z, and we call | |z xy = + 2 2 the absolute value, or modulus, of z. Two complex numbers z = x + iy, w = u + iv are equal if and only if x = u and y = v. 1.3.2 Subsets : A set A is a subset of a set X, written A ⊆ X , if every element of A belongs to X; that is, if x ∈ A implies that x ∈ X. We also say that A is included in X. For example, if P is the set of prime numbers, then P ⊆ ?, and ? ⊆ ?. The empty set ∅ and the whole set X are subsets of any set X. Note that X = Y if and only if X ⊆ Y and Y ⊆ X; we often prove the equality of two sets by showing that each one includes the other. If A ≠ X but A ⊆ X, then A is called a proper subset of X and is denoted by A ⊂ X. In our notation, A ⊆ X does not imply that A is a proper subset of X (that is, a subset of X not equal to X itself), and we may have A = X.

10 NSOU CC-MT-10 NSOU CC-MT-10 11 A B Fig. 1.1 : Venn diagram of set A with a subset B Definition 1.3.3 : The power set P (X) of a set X is the set of all subsets of X. Example 1.3.4 : If X = {1, 2, 3}, then P (X) = {∅, {1}, {2}, {3}, {2, 3}, {1, 3}, {1, 2}, {1, 2, 3}} . The power set of a finite set with n elements has 2 n elements because, in defining a subset, we have two independent choices for each element (does it belong to the subset or not?). In Example 1.3.4, X has 3 elements and P(X) has 2 3 = 8 elements. The power set of an infinite set, such as ?, consists of all finite and infinite subsets and is infinite. We imagine that a general subset A ⊆ ? is "defined" by going through the elements of ? one by one and deciding for each n ∈ ? whether n ∈ A or n not belongs to A. If X is a set and P is a property of elements of X, we denote the subset of X consisting of elements with the property P by {x ∈ X : P (x)}. Example 1.3.5 : The set {n ∈ ? : n = k 2 for some k ∈ ?} is the set of perfect squares {1, 4, 9, 16, 25, . . . }. The set {x ∈ ? : 0 > x > 1} is the open interval (0, 1). 1.3.6 Set operations : The intersection A ∩ B of two sets A, B is the set of all elements that belong to both A and B; that is x ∈ A ∩ B if and only if x ∈ A and x ∈ B. Two sets A, B are said to be disjoint if A ∩ B = ∅; that is, if A and B have no elements in common. The union A ∪ B is the set of all elements that belong to A or B; that is x ∈ A ∪ B if and only if x ∈ A or x ∈ B.
10 NSOU CC-MT-10 NSOU CC-MT-10 11 A A∩B B A B Fig. 1.2 : Union of A and B Intersection of A and B Note that we always use 'or' in an inclusive sense, so that x ∈ A ∪ B if x is an element of A or B, or both A and B. (Thus, A ∩ B ⊂ A ∪ B.) The set-difference of two sets B and A is the set of elements of B that do not belong to A, B \ A = {x ∈ B : x ∉ A} . If we consider sets that are subsets of a fixed set X that is understood from the context, then we write A c = X \A to denote the complement of A ⊂ X in X. Note that (A c) c = A. A X\A Fig. 1.3 : Complement of A Example 1.3.7 : If A = {2, 3, 5, 7, 11}, B = {1, 3, 5, 7, 9, 11} then A ∩ B = {3, 5, 7, 11}, A ∪ B = {1, 2, 3, 5, 7, 9, 11}. Thus, A ∩ B consists of the natural numbers between 1 and 11 that are both prime and odd, while A ∪ B consists of the numbers that are either prime or odd (or both). The set differences of these sets are B \ A = {1, 9}, A \ B = {2} . Thus, B \ A is the set of odd numbers between 1 and 11 that are not prime, and A \ B is the set of prime numbers that are not odd. If A, B ⊂ X, we have De Morgan's laws: (

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A ∪ B) c = A c ∩ B c , (A ∩ B) c = A c ∪ B c 12 NSOU CC-MT-10 NSOU CC-MT-10 13 A B A ∪ B A ∩ B A (A ∪ B) B ... (1) ... (2) A B ∪ ∪ ∪ ∪ ∪ A B A B A B

Fig. 1.4 : De Morgan's laws The Cartesian product X × Y of sets X, Y is the set of all ordered pairs (x, y) with x ∈ X and y ∈ Y. If X = Y , we often write

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X × X = X 2 . Two ordered pairs (x 1 , y 1), (x 2 , y 2) in X × Y are equal if and only if x 1 = x 2 and y 1 = y 2 . Thus, (x, y) ≠ (y, x) unless x = y.

This contrasts with sets where {x, y} = {y, x} . Example 1.3.8 : If X ={1, 2, 3} and Y = {4 , 5} then X × Y = {(1, 4), (1, 5), (2, 4) , (2, 5), (3, 4), (3, 5)} . Example 1.3.9 : The Cartesian product of ? with itself is the Cartesian plane ? 2 consisting of all points with coordinates (x, y) where x, y ∈ ?. A B A×B = × Fig. 1.5 : Cartesian Product of Two Sets.
12 NSOU CC-MT-10 NSOU CC-MT-10 13 The Cartesian product of finitely many sets is defined analogously. Definition 1.3.10 : The Cartesian products of n sets

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X 1 , X 2 , . . . , X n is the set of ordered n-tuples, X 1 × X 2 × . . . × X n = {(x 1 , x 2 , . . . , x n) : x i ∈ X i for i = 1, 2, . . . , n}, where (x 1 , x 2 , . . . , x n) = (

y 1 , y 2 , . . . , y n) if and only if x i = y i for every i = 1 , 2, . . . , n. 1.4 Relations A relation R on two non-empty sets X and Y is a rule that associates some or all the elements of X with some elements or element of Y . We write xRy if x ∈ X and y ∈ Y are related. One can also define relations on more than two sets, but we shall consider only binary relations and refer to them simply as relations. If X = Y , then we call R a relation on X 1 A B xRy 1 2 2 3 3 Fig. 1.6 : A relation between A and B The relation R between two non-empty sets X and Y is a subset of X × Y, i.e., R = {(x, y) : xRy, x ∈ X and y ∈ Y } ⊆ X × Y. Example 1.4.1 : Suppose that S is a set of students enrolled in a university and B is a set of books in a library. We might define a relation R on S and B by : s ∈ S has read b ∈ B. In that case, sRb if and only if s has read b. Another, probably inequivalent, relation is: s ∈ S has checked b ∈ B out of the library. Example 1.4.2 : Let S be the set of balls in a box. Now define a relation R on S by xRy if and only if x and y have the same colour.

14 NSOU CC-MT-10 NSOU CC-MT-10 15 When used informally, relations may be ambiguous (did s read b if she only read the first page?), but in mathematical usage we always require that relations are definite, meaning that one and only one of the statements "these elements are related" or "these elements are not related" is true. The graph G R of a relation R on X and Y is the subset of X × Y defined by G R = {(x, y) ∈ X ×Y : xRy}. This graph contains all of the information about which elements are related. Definition 1.4.3 : A relation R on a set S is said to be reflexive if xRx for all x ∈ S. Example 1.4.4 : The relation R defined on the set of real numbers ? by xRy if and only if x − y ≥ 0. Then the relation R is reflexive on ?. Example 1.4.5 : Let S be the set of all students in a class. Now a reflexive relation R is defined on S by xRy if and only if x and y obtain same marks. Not all relations satisfy the reflexive condition, see the following example. Example 1.4.6 : Consider the relation R on the set of integers ? defined by xRy if and only if x + y = 1. This relation is not reflexive. Definition 1.4.7 : A relation R on a set S is said to be symmetric if xRy implies yRx ∀x, y ∈ S. Example 1.4.8 : The relation R defined on the set of real numbers ? by xRy if and only if x and y have a common divisor other than 1. Then the relation R is symmetric on ?. Example 1.4.9 : Let S be the set of all students in a school. Now a relation R is defined on S by xRy if and only if x and y are from different classes. This relation is symmetric but not reflexive. Definition 1.4.10 : A relation R on a set S is said to be transitive if xRy and yRz implies xRz ∀x, y, z ∈ S. Example 1.4.11 : The relation R defined on the set of integers ? by

14 NSOU CC-MT-10 NSOU CC-MT-10 15 xRy if and only if x > y. Then the relation R is transitive on ? although it is neither reflexive nor symmetric. 1.4.12 : Equivalence relations : Equivalence relations decompose a set into disjoint subsets, called equivalence classes. We begin with an example of an equivalence relation on ?. Example 1.4.12.1 : Fix N ∈ ? and say that m R n if m ≡ n (mod N), meaning that m - n is divisible by N . Two numbers are related by R if they have the same remainder when divided by N . Moreover, N is the union of N disjoint sets, consisting of numbers with remainders 0, 1,. . .N − 1 modulo N . Definition 1.4.12.2 : An equivalence relation R on a set X is a binary relation on X such that for every x, y, z ∈ X : (a) x R x (reflexivity); (b) if x R y then y R x (symmetry); (c) if x R y and y R z then x R z (transitivity). Example 1.4.12.3 : The relation R on the set of integers defined by x R y if and only if x − y is divisible by 2. This relation is reflexive since x − x = 0 is divisible by 2. It is easy to check that this relation is symmetric and also transitive. Therefore, it is an equivalence relation. Example 1.4.12.4 : The relation R on the set of balls in a box, S, defined by x R y if and only if both x and y has same colour. This relation is an equivalence relation (check it !). Example 1.4.12.5 : The relation R on the set of all triangles in the plane, K, defined by x R y if and only if both x and y has same area. This relation is an equivalence relation . Example 1.4.12.6 : If we define a relation R on ? by x R y if and only if x > y. Then this relation is not equivalence as the it breaks the reflexive and symmetric conditions. For each x ∈ X, the set of elements equivalent to x, [x/R] = {y ∈ X : x R y} ,

16 NSOU CC-MT-10 NSOU CC-MT-10 17 is called the equivalence class of x with respect to R When the equivalence relation is understood, we write the equivalence class [x/ R] simply as [x]. The set of equivalence classes of an equivalence relation R on a set X is denoted by X/ R. Note that each element of X/ R is a subset of X, so X/ R is a subset of the power set P(X) of X. The following theorem is the basic result about equivalence relations. It says that an equivalence relation on a set partitions the set into disjoint equivalence classes. Theorem 1.4.12.7 : Let R be an equivalence relation on a set X. Every equivalence class is non-empty, and X is the disjoint union of the equivalence classes of R. Proof. If x ∈ X, then the reflexive of R implies that x ∈ [x]. Therefore every equivalence class is non-empty and the union of the equivalence classes is X. To prove that the union is disjoint, we show that for every x, y ∈ X either [x] ∩ [y] = ∅ (if x R y) or [x] = [y] (if x R y). Suppose that [x] ∩ [y] ≠ ∅. Let z ∈ [x] ∩ [y] be an element in both equivalence classes. If x 1 ∈ [x], then x 1 R z and z R y, so x 1 R y by the transitivity of R and therefore x 1 ∈ [y]. It follows that [x] ⊂ [y]. A similar argument applied to y 1 ∈ [y] implies that [y] ⊂ [x], and therefore [x] = [y]. In particular, y ∈ [x], so x R y. On the other hand, if [x] ∩ [y] = ∅, then y does not belong to [x] since y ∈ [y], so x R y. There is a natural projection p : X → X / R given by p (x) = [x], that maps each element of X to the equivalence class that contains it. Conversely, we can index the collection of equivalence classes X/ R = {[a] : a ∈ A} by a subset A of X which contains exactly one element from each equivalence class. It is important to recognize, however, that such an indexing involves an arbitrary choice of a representative element from each equivalence class, and it is better to think in terms of the collection of equivalence classes, rather than a subset of elements. Example 1.4.12.8 : The equivalence classes of ? relative to the equivalence relation m R n if m ≡ n (mod 3) are given by I 0 = {3, 6, 9, . . . }, I 1 = {1, 4, 7, . . . }, I 2 = {2, 5, 8, . . . }. The projection p : ? → {I 0 , I 1 , I 2 } maps a number to its equivalence class e.g. p (101) = I 2 . We can choose {1,2, 3} as a set of representative elements, in which case I 0 = [3], I 1 = [1], I 2 = [2], but any other set A ⊂ ? of three numbers with remainders 0, 1, 2 (mod 3) will do. For example, if we choose A = {7 , 15, 101}, then I 0 = [15], I 1 = [7], I 2 = [101],

16 NSOU CC-MT-10 NSOU CC-MT-10 17 1.5 Functions A function f : X → Y between sets X and Y assigns to each x ∈ X a unique element f (x) ∈ Y . Functions are also called maps, mappings, or transformations. The set X on which f is defined is called the domain of f and the set Y in which it takes its values is called the codomain. We write f : x → f (x) to indicate that f is the function that maps x to f (x). Definition 1.5.1 : A function f between two sets X and

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Y is a subset f ⊆ X × Y such that (i) For all x ∈ X, there exists y ∈ Y such that (x, y) ∈ f (ii) For			

any x ∈ X, if there exists y, y′ ∈ Y such that (x, y), (x, y′) ∈ f then y = y′. Fig. 1.7 : X Y f Example 1.5.2 : The identity function id x : X → X on a set X is the function id x : x → x that maps every element to itself. Example 1.5.3 : Let A ⊂ X. The characteristic (or indicator) function of A, χ A : X → {0 , 1}, is defined by χ A x x A x A ( )= ∈ ∈ ? ? ? 1 0 if if Specifying the function χ A is equivalent to specifying the subset A. Example 1.5.4 : Let A, B be the sets in Example 1.4. We can define a function f : A → B by f (2) = 7, f (3) = 1, f (5) = 11, f (7) = 3, f (11) = 9, and a function g : B → A by g (1) = 3, g (3) = 7, g (5) = 2, g (7) = 2, g (9) = 5, g (11) = 11.

18 NSOU CC-MT-10 NSOU CC-MT-10 19 Example 1.5.5 : The square function f : ? → ? is defined by f (n) = n 2 , which we also write as f : n → n 2 . The equation g (n) = n , where n is the positive square root, defines a function g : ? → ?, but h (n) = ± n does not define a function since it doesn't specify a unique value for h(n). Sometimes we use a convenient oxymoron and refer to h as a multi-valued function. One way to specify a function is to explicitly list its values, as in Example 1.5.4 Another way is to give a definite rule, as in Example 1.5.5 If X is infinite and f is not given by a definite rule, then neither of these methods can be used to specify the function. Nevertheless, we suppose that a general function f : X → Y may be "defined" by picking for each x∈ X a corresponding value f (x) ∈ Y . If f : X → Y and U ⊂ X, then we denote the restriction of f to U by f | U : U → Y, where f | U (x) = f (x) for x ∈ U. In defining a function f : X → Y , it is crucial to specify the domain X of elements on which it is defined. There is more ambiguity about the choice of codomain, however, since we can extend the codomain to any set Z ⊃Y and define a function g : X → Z by g(x) = f (x). Strictly speaking, even though f and g have exactly the same values, they are different functions since they have different codomains. Usually, however, we will ignore this distinction and regard f and g as being the same function. The graph of a function f : X → Y is the subset G f of X × Y defined by G f = {(x, y) ∈ X × Y : x ∈ X and y = f (x)} . For example, if f : ? → ?, then the graph of f is the usual set of points (x, y) with y = f (x) in the Cartesian plane ? 2 . Since a function is defined at every point in its domain, there is some point (x, y) ∈ G f for every x ∈ X, and since the value of a function is uniquely defined, there is exactly one such point. In other words, for each x ∈ X the "vertical line" L x = {(x, y) ∈ X × Y : y ∈ Y } through x intersects the graph of a function f : X → Y in exactly one point : L x ∩ G f = (x, f (x)). Definition 1.5.6 : The image, of a function f : X → Y is the set of values Img(f) = {y ∈ Y : y = f (x) for some x ∈ X } .
18 NSOU CC-MT-10 NSOU CC-MT-10 19 A B C D 1 2 3 4 5 Domain {A,B,C,D} Image {2,3,5} Codomain {2,2,3,4,5} Fig. 1.8 : Function Definition: function f : X → Y is said to • Onto or surjective if the image of f is the whole Y, i.e., Img(f) = Y X 1 2 3 4 D B C Y Fig. 1.9 : Onto • One-one or injective if each point in the image of f in Y has a unique pre-image in X, i.e., f (x) = f (y) implies x = y ∀x, y ∈ X. X 1 2 3 4 D B C A Y Fig. 1.10 : One-one
20 NSOU CC-MT-10 NSOU CC-MT-10 21 • Bijective if f is both onto and one-one. X 1 . 2 . 3 . 4 . . D . B . C . A Y Fig. 1.11 : Bijective 1.6 Summary In this chapter, we have discussed the preliminary concept in set, relation and functions. Various elementary operations in sets such as union, intersection etc are discussed. Various types of relations are presented and also some clasification of functions are described in pictorial notion. 1.7 Worked examples 1. Determine whether each of the following relations are reflexive, symmetric and transitive : (i) Relation R in the set A = {1, 2, 3...13, 14} defined as R = {(x, y): 3x – y = 0} (ii) Relation R in the set N of natural numbers defined as R = {(x, y): y = x + 5 and x > 4} (iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as R = {(x, y): y is divisible by x} (iv) Relation R in the set Z of all integers defined as R = {(x, y) : x – y is as integer} Solution : (i) A = {1, 2, 3 ... 13, 14} R = {(x, y): 3x – y = 0} ∴ R = {(1, 3), (2, 6), (3, 9), (4, 12)} R is not reflexive since (1, 1), (2, 2) ... (14, 14) ∉ R.
20 NSOU CC-MT-10 NSOU CC-MT-10 21 Also, R is not symmetric as (1, 3) ∈R, but (3, 1) ∉ R. [3(3) – 1 ≠ 0] Also, R is not transitive as (1, 3), (3, 9) ∈ R, but (1, 9) ∉ R. Hence, R is neither reflexive, nor symmetric, nor transitive. (ii) R = {(x, y): y = x + 5 and x > 4} = {(1, 6), (2, 7), (3, 8)} It is seen that (1, 1) ∉

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R. ∴ R is not reflexive. Now (1, 6) ∈R But, (1, 6) ∉ R. ∴ R is not symmetric.

Now, since there is no pair in R such that (x, y) and (y, z) ∈R, then (x, z) cannot belong to R. Therefore, R is not transitive. Hence, R is neither reflexive, nor symmetric, nor transitive. (iii) A = {1, 2, 3, 4, 5, 6} R = {(x, y): y is divisible by x} We know that any number (x) is divisible by itself. ⇒ (x, x) ∈R ∴ R is reflexive. Now, (2, 4) ∈ R [as 4 is divisible by 2] But, (4, 2) ∉ R. [as 2 is not divisible by 4] ∴

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R is not symmetric. Let (x, y), (y, z) ∈ R. Then, y is divisible by x and z is divisible by y. ∴z is divisible by x. ⇒ (x, z) ∈ R ∴ R is transitive. Hence, R is

reflexive and transitive but not symmetric. (iv) R = {(x, y): x – y is an integer} Now, for every

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x ∈ Z, (x, x) ∈R as x − x = 0 is an integer. ∴ R is reflexive. Now, for every x, y ∈ Z if (x, y) ∈ R, then x − y is an integer. ⇒ −(x − y) is also an integer. ⇒ (y − x) is an integer. ∴ (y, x) ∈ R. Hence, R is symmetric. Now, Let (x, y) and (y, z) ∈ R, where x, y, z ∈ Z. ⇒ (x − y) and (y − z) are integers. ⇒ x − z = (x − y) + (y − z) is an integer. 22 NSOU CC-MT-10 NSOU CC-MT-10 23 ∴ (x, z) ∈ R. Hence, R is transitive. Hence, R is

reflexive, symmetric, and transitive. 2. Show that the relation R in the set R of real numbers, defined as R = {(a, b): a ≤ b 2 } is neither reflexive nor symmetric nor transitive. Solution : R = {(a, b): a ≤ b 2 } 1 2 1 2 , ( ) ∉ R, since 1 2 1 2 1 4 2 < ( ) = It can be observed that ∴R is not reflexive. Now, (1, 4) ∈ R as 1 > 4 2 But, 4 is not less than 1 2 . ∴ (4, 1) ∉ R ∴ R is not symmetric. Now, (3, 2), (2, 1.5) ∈ R (as 3 > 2 2 = 4 and 2 > (1.5) 2 = 2.25) But, 3 < (1.5) 2 = 2.25 ∴ (3, 1.5) ∉ R ∴ R is not transitive. Hence, R is neither reflexive, nor symmetric, nor transitive. 1.8 Model Questions A 1. Do the following relations represent functions? Why? (a) f : ? → ? defined by i. f = {(x, 1) : 2 divides x}∪ {(x,5) : 3 divides x}. ii. f = {(x, 1) : x ∈ S} [{(x, −1) : x ∈S c }, where S = {n 2 : n ∈ ?} and S c = ? \ S. iii. f = {(x, x 3) : x∈ ?}. (b) f : ? + → ? defined by f = {(x, ± x) : x ∈ ? + }, where ? + is the set of all positive real numbers. (c) f : ? → ? defined by

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f = {(x, x) : x ∈ ?}. (d) f : ? → ? defined by f = {(x, x) : x ∈ ?}. (e) f : ? − → ?

defined by f = {(x, log e |x|) : x ∈ ? − }, where ? − is the set of all negative real numbers. (f) f : ? → ? defined by f = {(x, tanx) : x ∈ ?}.
22 NSOU CC-MT-10 NSOU CC-MT-10 23 2. Let f : X → Y be a function. Then f −1 is a relation from Y to X. Show that the following results hold for

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f −1 : (a) f −1 (A ∪ B) = f −1 (A) ∪ f −1 (B) for all A, B ⊆ Y . (b) f −1 (A \ B) = f −1 (A) \ f −1 (B)

for all A, B ⊆
Y . (c) f −1 (∅) = ∅. (d) f −1 (Y) = X. (e) f −1 (Y \ B) = X \ (f −1 (B)) for each B ⊆ Y . 3. Let S = {(x, y) ∈ ? 2 : x 2 + y 2 = 1, x ≥ 0}. It is a relation from ? to ?. Draw a picture of the inverse of this relation. B Determine the equivalence relation among the relations given below. Further, for each equivalence relation, determine its equivalence classes. 1. R = {(a, b) ∈ ? 2 : a ≤ b} on ?. 2. R = {(a, b) ∈ ?* × ?* : a divides b} on ?*, where ?* = ? \ {0}. 3. Recall the greatest integer function f : ? → ? given by f(x) = [x] and let ? = {(a, b) ∈ ? × ? : [a] = [b]} on ?. 4. For x = (x 1 , x 2), y = (y 1 , y 2) ∈ ? 2 and ?* = ? \ {0}, let (a) R = {(x, y) ∈ ? 2 × ? 2 : x x y y 1 2 2 2 1 2 2 2 + =+ }. (b)

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R = {(x, y) ∈ ? 2 × ? 2 : x = ay for some a ∈ ?*}. (c) R = {(x, y)∈ ? 2 × ? 2 : 4 9 4 9 1 2 2 2 1 2 2 2 x x y y + =+ }. (d) R = {(x, y) ∈ ? 2 × ? 2 : x − y = a(1, 1) for some a ∈ ?*}. (e) Fix c ∈ ?. Now, define ? = {(x, y) ∈ ? 2 × ? 2 : y 2 − x 2 = c(y 1 − x 1)}. (f) R = {(x, y) ∈ ? 2 × ? 2 : |x 1 | + |x 2 | = a(|y 1 | + |y 2 |)},

for some number a ∈ ? + . (g) R = {(

63%	MATCHING BLOCK 9/128	W

x, y) ∈ ? 2 × ? 2 : x 1 x 2 = y 1 y 2 }. 5. For x = (x 1 , x 2), y = (y 1 , y 2) ∈ ? 2 , let S = {x ∈ ? 2 : x x 1 2 2 2 1 + =}.

Then, are the relations given below an equivalence relation on S? (a) R = {(x, y) ∈ S × S : x 1 = y 1 , x 2 = −y 2 }. (b) R = {(x, y) ∈ S × S : x = −y}. 6. Let f, g be two equivalence relations on ?. Then, prove/disprove the following statements. (a) f o g is necessarily an equivalence relation. (b) f ∩ g is necessarily an equivalence relation.
24 NSOU CC-MT-10 NSOU CC-MT-10 25 (c) f ∪ g is necessarily an equivalence relation. (d) f ∪ g c is necessarily an equivalence relation. (g c = (? × ?) \ g) 7 a. Find an example of two nonempty sets
A and B for which

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A × B = B × A is true. b. Prove A ∪ φ = A and A ∩ φ = φ. c. Prove A ∪ B = B ∪ A and A ∩ B = B ∩ A. d. Prove A ∪ (B ∩ C) = (A ∪ B) ∩ (

A ∪ C). e. Prove

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A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C). f. Prove A ⊂ B if and only if A ∩ B = A. g. Prove (A ∩ B)′ = A′ ∪ B′. h. Prove A ∪ B = (A ∩ B) ∪ (A \ B) ∪ (B \ A). i. Prove (A ∪ B) × C = (A × C) ∪ (B × C). j. Prove (A ∩ B) \ B = φ. k. Prove (A ∪ B) \ B = A \ B. l. Prove A \ (B ∪ C) = (A \ B) ∩ (A \ C). m. Prove A ∩ (B \ C) = (A ∩ B) \ (A ∩ C).

n. Prove (A \ B) ∪ (B \ A) = (A ∪ B) \ (
A ∩
B). 8.
Prove the relation defined on ? 2 by (x 1 , y 1) ~ (x 2 , y 2) if x yx y 1 2 1 2 2 2 2 2 + =+ is an equivalence relation. 9. Let f : A → B and g : B → C be maps. (a) If f and g are both one-to-one functions, show that g o f is one-to-one. (b) If g o f is onto, show that g is onto. (c) If g o f is one-to-one, show that f is one-to-one. (d) If g o f is one-to-one and f is onto, show that g is one-to-one. (e) If g o f is onto and g is one-to-one, show that f is onto. 10. Define a function on the real numbers by f x x x ( )= + - 1 1 What are the domain and range of f ? What is the inverse of f ? Compute f o f −1 and f −1 o f.
24 NSOU CC-MT-10 NSOU CC-MT-10 25 11. Let f : X → Y be a map with A 1 , A 2 ⊂ X and B 1 , B 2 ⊂ Y . (a) Prove

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f (A 1 ∪ A 2) = f (A 1) ∪ f (A 2). (b) Prove f (A 1 ∩ A 2) ⊂ f (A 1) ∩ f (A 2).

Give an example in which equality fails. (c) Prove f −1 (B 1 ∪ B 2) = f −1 (B 1) ∪ f −1 (B 2), where f −1 (B) = {x ∈ X : f (x) ∈ B}. (d) Prove f −1 (B 1 ∩ B 2) = f −1 (B 1) ∩ f −1 (B 2). (e) Prove f −1 (Y \ B 1) = X \ f −1 (B 1). 12. Determine whether or not the following relations are equivalence relations on the given set. If the relation is an equivalence relation, describe the partition given by it. If the relation is not an equivalence relation, state why it fails to be one. (a) x ~ y in ? if x ≥ y (c) x ~ y in ? if |x – y| ≤ 4 (b) m ~ n in ? if mn < 0 (d) m ~ n in ? if m ≡ n (mod 6) 13. Define a relation ~ on ? 2 by stating that (

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a, b) ~ (c, d) if and only if a 2 + b 2 ≤ c 2 + d 2 . Show that ~ is

reflexive and transitive but not symmetric. 14. Show that an m × n matrix gives rise to a well-defined map from ? n to ? m .

26 NSOU CC-MT-10 NSOU CC-MT-10 27 26 Unit - 2 ? Introduction to Groups Structure 2.1 Objectives 2.2 Introduction 2.3 Binary Operation 2.4 Definition of Group 2.5 Basic properties of groups 2.6 Subgroups 2.7 Summary 2.8 Worked examples 2.9 Model Questions 2.1 Objectives The followings are discussed here : • Definition of binary operation along with examples • Definition of group • Basic properties of group • Definition of subgroups, centralizer, normalizer, center of a group • Order of a group and order of an element 2.2 Introduction Group theory, in modern algebra, is the study of groups, which are systems consisting of a set of elements and a binary operation that can be applied to two elements of the set, which together satisfy certain axioms. Groups are vital to modern algebra; their basic structure can be found in many mathematical phenomena. Groups can be found in geometry, representing phenomena such as symmetry and certain types of transformations. In this unit, we introduce the concept of group and subgroup and demonstrate this concept through some examples. 2.3 Binary Operation Definition 2.3.1 : Let S be a set. The the binary operation * on S is a map * : S × S → S (x, y) → x * y.

26 NSOU CC-MT-10 NSOU CC-MT-10 27 S S S Fig. 2.1 : Binary operation on S. Example 2.3.2 : The arithmetic operations +,–,×, ... are binary operations on suitable sets of numbers such as ?, ? etc. Example 2.3.3 : Matrix addition and multiplication are binary operations on the set of all n × n matrices. Example 2.3.4 : Vector addition and subtraction are binary operations on ? n . Example 2.3.5 : The vector product, or cross product, (a, b, c) × (x, y, z) = (bz − cy, cx − az, ay − bx)

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is a binary operation on ? 3 . Example 2.3.6 : Composition of symmetries is a binary operation on the set of

symmetries of a triangle, square, cube,... In the definition of binary operation, for any two elements from a set, the element produced by applying binary operation on them is also an element of the same set, i.e., a * b ∈ S whenever a ∈ S and b ∈ S. This property is sometimes expressed as : S is closed with respect to '*'. The notion becomes important when we consider restricting a binary operation to subsets of the set on which it was originally defined. Let T be a subset of S and S is closed under the binary operation *. Then T ×T ⊂ S × S. Now we consider the restriction of the map * : S × S → S to T × T. Then it is not always true that for any x * y ∈ T whenever x, y ∈ T. For example, take S = ? and define a binary operation * on S as follows : for any n * m = n + m + 1 for any n, m ∈ S.

28 NSOU CC-MT-10 NSOU CC-MT-10 29 Then S is closed under *. But if we consider the set of even number E ⊂ S, then E is not closed under the restricted binary operation * from S. Hence, we say the following definition : Definition 2.3.7 : Let the set S is closed under the binary operation *. Then we say that a subset T of S is closed under the restricted binary operation * if x * y ∈ T whenever x, y ∈ T. Example 2.3.8 : The set of all non-singular (non-zero determinant) n × n real matrices is denoted by GL(n, ?). Now this set GL(n, ?) closed under matrix multiplication. Again, consider the subset SL(n, ?) of GL(n, ?), the of all matrices whose determinant is 1. This subset is also closed under matrix multiplication. Example 2.3.9 : Let C be the set of all concentric circles with center at the origin. A circle in C with radius r is denoted by a r . Now the binary operation is defined by a r * a t = a r+t . The set C is closed under the binary operation *. r+t r t Fig. 2.2 : Binary operation on concentric circles Binary operation can also be imposed on real life objects, see the following example: Example 2.3.10 : Let A be the set of all students in a class. Now define the binary operation on A as follows: for any x, y ∈ A, x y x xy y * = ≥ ? ? ? if age of age of otherwise. Definition 2.3.11 : A binary operation * on a set S is said to be commutative if x * y = y * x for all x, y ∈ S. In general binary operation may not be commutative, see the following example: Example 2.3.12 : Let M (n, ?) be the set of all real n × n matrices. The binary operation

28 NSOU CC-MT-10 NSOU CC-MT-10 29 addition is commutative on M (n, ?). But the binary operation multiplication is not commutative on M (n, ?). 2.4 Definition of Group Definition 2.4.1 : Let G be a non-empty set * be a binary operation defined in such a way that the following four rules are true : 1. * is closed in G, i.e., if

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a, b ∈ G then a * b ∈ G. 2. * is associative, i.e., a * (b * c) = (a * b) * c for a, b, c ∈ G. 3. G contains an identity element e, i.e., a * e = e * a = a for all a ∈ G. 4. Inverse exists in G, i.e., for any a ∈ G there exists an inverse element a′ ∈ G such that a * a′ = a′ * a =

e.
Then the pair (G, *) is called a group
with the binary operation &. In multiplicative notation the inverse of an element a is denoted by a^{-1}. If G is commutative with respect to the binary operation *, then (G, *) is called the abelian group. Example 2.4.2 : The set of real numbers ?, integers ?, rational numbers ?, complex numbers ? forms a group under the binary operation '+'. The identity element is 0 and for each element x, the inverse is −x. *
e
a

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Table 2.1 :
Multiplication table Example 2.4.3 : The set of all m × n real matrix is denoted by M (m, n). Then M (m, n) forms a group under matrix addition. Hence, the identity element is the zero matrix. This is an abelian group. Example 2.4.4 : The set GL(n, ?) forms a group under matrix multiplication. Let A, B ∈ GL(n, ?). Then det (A) ≠ 0 and det (B) ≠ 0. Now det (AB) = det (A) * det (B) ≠ 0. Hence, A * B ∈ GL(n, ?). The matrix multiplication is associative. The identity matrix In acts as identity element. For any element A ∈ GL(n, ?), the inverse is A^{-1}. Hence, GL(n, ?) is a group under matrix multiplication. But this group is not abelian, since matrix multiplication is not commutative.
30 NSOU CC-MT-10 NSOU CC-MT-10 31 Example 2.4.5 : Let G = {e, a, b, c} with multiplication as defined by the table 2.1 From the table, we observe that 1. G is closed under composition. 2. e is the identity element. 3. e^{-1} = e, a^{-1} = a, b^{-1} = b and c^{-1} = c. 4. the multiplication is commutative. It can be checked that the multiplication is associative. Thus, (G,*) is anabelian group. This group is called Klein's 4-group. The multiplication table 2.1 is known as Cayley table of a group. Example 2.4.6 : The set C [a, b] is the set of all continuous functions on [a, b]. Let f, g ∈ C[a, b]. The binary operation + defined by (f + g)(x) = f(x) + g(x) ∀x ∈ [a, b]. Then f + g is also continuous. The binary operation + is also associative. The identity function i is the identity element and for any f ∈ C[a, b], the inverse is −f. Therefore, C [a, b] forms a group under addition +. In fact it abelian. Example 2.4.7 : In the Euclidean plane, let G p be the set of all rotations about a fixed point p. If two rotations differ by a multiple of 2p then we say that they are equal. If a and b are two elements of G P then a **o** b is the rotation obtained by first applying β and then applying α. Thus, G P is closed under composition. Again functional composition is associative. An identity element of G P is the rotation of 0°. Each rotation has an inverse : rotation of the same magnitude in the opposite direction. Finally, as an operation on G P , composition is commutative. Therefore, G P is a group with respect to the rotation about the point p. Example 2.4.8 : The subset {1,−1, i,−i} of the complex numbers is a group under complex multiplication. Note that −1 is its own inverse, whereas the ainverse of i is −i, and vice versa. Example 2.4.9 : In the example 2.3.7, the set C does not form a group under the given binary operation as the inverse of any non-zero element does not exists (why?). Example 2.4.10 : The set S of positive irrational numbers together with 1 under multiplication satisfies the three properties given in the definition of a group but is not a group. Indeed, 2 22 * = , so S is not closed under multiplication. Example 2.4.11 : The set ?n = {1, 2, ..., n − 1} for n ≥ 1 is group under integer modulo n. For any j < 0 in ?n, the inverse of j is n − j. This group is called integer modulo n group.
30 NSOU CC-MT-10 NSOU CC-MT-10 31 Example 2.4.12 : For n < 1, we define U (n), to be the set of all positive integers less than n and relatively prime to n. Then U (n) is a group under multiplication modulo n. For n = 10, we have U(10) = {1, 3, 7, 9}. The Cayley table for U(10) is Mod 10 1 3 7 9 1 1 3 7 9 3 3 9 1 7 7 7 1 9 3 9 9 7 3 1 Table 2.2 (Recall that ab mod n is the unique integer r with the property a.b = nq + r, where 0 ≤ r > n and a.b is ordinary multiplication.) In the case that n is prime, then U(n) = {1, 2, ..., n − 1}. In his classic book Lehrbuch der Algebra, published in 1895, Heinrich Weber gave an extensive treatment of the groups U(n) and described them as the most important examples of finite Abelian groups. Example 2.4.13 : Let 1 = 1 0 0 1 0 1 1 0 ? ? ? ? ? ? = - ? ? ? ? ? ? I J i i K i i = ? ? ? ? ? ? = - ? ? ? ? ? ? 0 0 0 0 , where $i^2 = -1$. Then the relations $I^2 = J^2 = K^2 = -1$, IJ = K, JK = I, KI = J, JI = −K, KJ = −I, IK = −J hold. The set Q_8 = {±1, ±I, ±J, ±K} is a group called the quaternion group. Notice that Q_8 is non-abelian. Example 2.4.14 : Let ?* be the set of nonzero complex numbers. Under the operation of multiplication ?* forms a group. The identity is 1. If z = a + ib is a nonzero complex number, then z a ib a b - = - + 1 2 2 is the inverse of z. It is easy to see that the remaining group axioms hold. Example 2.4.15 : (

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Direct product of groups). Let (G 1 , * 1), . . . (G n , * n) be groups. Then the direct product G = G 1 × G 2 × . . . × G n is the set of n-tuples (g 1 , g 2 , . . . , g n) where g i ∈ G i with operation defined componentwise : (g 1 , g 2 , . . . , g n) * (

h 1 , h 2 , . . . , h n) = (g 1 * 1 h 1 , g 2 * 2 h 2 , . . . , g n * n h n).
It is a routine checkup that G = (G 1 , * 1) × . . . × (G n , * n)
forms a group under the binary operation defined above.
32 NSOU CC-MT-10 NSOU CC-MT-10 33 2.5 Basic properties of groups Proposition 2.5.1 : The identity element e of a group is unique, i.e., there exists only one e such that ex = xe = x for all x ∈ G. Proof. Suppose both e and e′ are the identity element. Then xe = ex = x and xe′ = e′x = x for all x ∈ G. We need to show that e = e′. If we think e as identity then ee′ = e′ and if we think e′ as identity, then ee′ = e′. Therefore, combining them we get e = e′. Similarly we can say that Proposition 2.5.2 : Inverse of an element is also unique. Proof. Let g′ and g″ be two identity elements of g. Then g′g = e and g″g = e. We want to show that g′ = g″. Now g′ = g′e = g′(gg″) = (g′g)g″ = eg″ = g″. Hence, g′ = g″. Group Operation Identity Form of Element Inverse Abelian Z Addition 0 k −k Yes Q + Multiplication 1 m/n, m, n < 0 n/m Yes Z n Addition mod n 0 k n − k Yes R* Multiplication 1 x 1/x Yes C* Multiplication 1 a + bi 1 1 2 2 2 2

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a b a a b bi + - - Yes GL(2,F) Matrix multiplication 1 0 0 1 ? ? ? ? ? ? a b c d ? ? ? ? ? ? , ad − bc ≠ 0 d ad bc b ad bc

c ad bc a ad bc - - - - - - ? ? ? ? ? ? ? ? ? ? No U(n) Multiplication mod n 1 k, gcd (k, n) = 1 Solution to kx mod n=1 Yes R n Componentwise addition (0,0, ...,0) (a 1 , a 2 , ..., a 3) (−a 1 , −a 2 , ..., −a n) Yes SL(2, F) Matrix multiplication 1 0 0 1 ? ? ? ? ? ? a b c d ? ? ? ? ? ? , ad − bc = 1 d b c a - - ? ? ? ? ? ? No D n Composition R 0 R a , L R 360 − a , L No Fig. 2.3
32 NSOU CC-MT-10 NSOU CC-MT-10 33 Proposition 2.5.3 : Let G be a group. then for any two elements a, b ∈ G, (ab) −1 = b −1 a −1 . Proof. Let a, b ∈ G. Then abb −1 a −1 = aea −1 = e. Similarly, b −1 a −1 ab = e. Therefore, (ab) −1 = b −1 a −1 . Proposition 2.5.4 : In a group G, right and left cancellation law holds, i.e., ba = bc implies a = c and ab = cb implies a = c. Proof. Taking inverse of b in both sides of ba = bc we get b −1 ba = b −1 bc =) ea = ec. which implies that a = c. The right cancellation can be proved similarly. Definition 2.5.5 : (

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Order of a Group). The number of elements of a group G (finite or infinite) is called the order of

the group G and it is denoted by |G|. Example 2.5.6 : The group of integers ? under addition is of infinite order. Example 2.5.7 : The group ? 10 is of order 10. The group U(7) is of order 6. Definition 2.5.8 : (Order of an element).
The order of an element g in a group
G

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is the smallest positive integer n such that g n = e. (In additive notation, this would be ng = 0). If no such integer exists, we say that

g has infinite order. The order of an element g is denoted by |g|. Example 2.5.9 : Consider U(15) = {1, 2, 4, 7, 8, 11, 13, 14}. under multiplication modulo 15. This group has order 8. Then for any element, say 7, 7 1 = 7, 7 2 = 4, 7 3 = 13, 7 4 = 1. Hence, the order of 7 is 4. Similarly, the order of 11 is 2. Example 2.9.10 : The order of Q 8 is 8. In this group order of each element, except identity, are of order 4. Proposition 2.5.11 :

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Let G be a group and g be an element

of order m. Then g i ≠ g j for i ≠ j and 1 ≤ i, j ≤ m. And if g is of infinite order, then all the elements g, g 2 , ..., g n , ... are distinct. Proof. For the first proof let us assume that g i = g j for some i ≠ j and 1 ≤ i, j ≤ m. Suppose i > j, then g j − i = e. But j − i > n. Which contradicts that |g| = n. Hence, our assumption is wrong. For the second proof, suppose g i = g j for some i, j ≥ 1 and i ≠ j. Assume that j < i, then it implies that g j−i = e. Which contradicts that g has infinite order. The question naturally arises : Given a set A, can we define a binary operation on A which makes A a group?. In case of empty set it is not possible. But in case of non-empty set, fortunately,
34 NSOU CC-MT-10 NSOU CC-MT-10 35 this question has an affirmative answer if we assume the Axiom of Choice 1 (which is done in most of mainstream mathematics, but may not be done in the more foundational parts). To answer this first we need to prove the following theorem: Theorem 2.5.12 : Let A be a non-empty set and G be a group such that there exists a bijection f : A → G. Then a group structure can be defined on A. Proof. First we define a binary operation on A.

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Let a, b ∈ A. Then the binary operation a * b on A is defined by a * b = f −1 (f (a) f (b)). Since f is a

bijection, this binary operation is well-defined. It is clear that A is closed under the binary operation *. The operation is associative since G is a group and f is a bijection. Let e A = f −1 (e), e be the identity element of G. Then for any a ∈ A. a * e A = f −1 (f (a) f (e A)) = f −1 (f (a)e) = f −1 (f (a)) = a = e A * a. Which shows that e A is the identity element in A. Now what is the inverse of an element a ∈ A? The inverse is
a′ = f −1 (f (a) −1). Here f (a) −1 means inverse of the element f (a) in the group G. Then

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a * a′ = f −1 (f (a) f (a′)) = f −1 (f (a) f (f −1 (f (a) −1))) = f −1 (f (a) f (

a) −1) =
f −1 (e) = e A . Similarly, we can show that a′ * a = e A . Therefore, e A is the identity element of A. Thus (A, *) is a group. Now come to our main question. If A is finite, having n-number of elements, then there is a bijection between A and ? n . Then by the above theorem, A can be given a group structure. If A is countably infinite, then A forms a group under the binary operation which can be constructed from the bijection between A and ?. And in case when A is uncountable, the same thing can also be done by the bijection between A and R. 2.6 Subgroups Sometimes we wish to investigate smaller groups sitting inside a larger group. The set of even integers 2? = {...−2, 0, 2, 4...} is a group under the operation of addition. This smaller group sits naturally inside of the group of integers under addition. 1 The Axiom of Choice states that for any family of nonempty disjoint sets, there exists a set that consists of exactly one element from each element of the family.
34 NSOU CC-MT-10 NSOU CC-MT-10 35 Definition 2.6.1 : We define

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a subgroup H of a group G to be a

subset H of G

such that when the group operation of G is restricted to H, H is a group in its own right. K G H e Fig. 2.4 : Group G with two subgroups H and K Observe that every group G with at least two elements will always have at least two subgroups, the subgroup consisting of the identity element alone and the entire group itself. The subgroup H = {e} of a group G is called the trivial subgroup. A subgroup that is a proper subset of G is called a proper subgroup. In many of the examples that we have investigated up to this point, there exist other subgroups besides the trivial and improper subgroups. The set of rationals ?, the set of integers ? are subgroups of ? under addition. Example 2.6.2 : The set of non-zero complex numbers ?* is a group under multiplication and also the set H = {±1, ± i} is also a group under multiplication. Since H ⊂ ?*, H is a subgroup of ?*. Example 2.6.3 : The set of all 2 × 2-matrix with determinant 1 is the set SL(2, ?) = a b c d ad bc ? ? ? ? ? ? - = ? ? ? ? ? ? : 1 Then SL(2, ?) closed under multiplication, since for A, B ∈ SL(2, ?) implies AB ∈ SL(2, ?) as det (AB) = 1. Since the identity matrix I = 1 0 0 1 ? ? ? ? ? ? has determinant 1, I is the identity element for SL(2, ?). For any a b c d ? ? ? ? ? ? ∈ SL(2, ?), the inverse is d b c a - - ? ? ? ? ? ? which also belongs to SL(2, ?). Therefore, SL (2, ?) is a group under matrix multiplication. Also SL(2, ?) ⊂ GL (2, ?), so SL(2, ?) is a subgroup of GL(2, ?).
36 NSOU CC-MT-10 NSOU CC-MT-10 37 Theorem 2.6.4 : (Two-steps test).

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Let G be a group and H be a non-empty subset of G. If ab ∈ G whenever a, b ∈ G and a −1 ∈ H whenever a ∈ H, then H is a subgroup of G.

Proof. Since H is a subset of G and G is a
group, the binary operation on H is associative. Let a ∈ H. Then a −1 ∈ H from the hypothesis. Now aa −1 = e ∈ H. Hence, H contains the identity element. Also from the hypothesis inverse of each element of H exists in H. So, H is a subgroup of G. Theorem 2.6.5 : (One-steps test).

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Let G be a group and H be a non-empty subset of G.

If ab −1 ∈ G whenever a, b ∈

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G, then H is a subgroup of G. Proof. Let a, b ∈ H. Then

by the hypothesis ab −1 ∈ H also ba −1 ∈ H. Now e = (ab −1)(ba −1) ∈ H, So, H contains identity element. Also for a ∈ H, a −1 belongs to H, since a −1 = ea −1 . Which implies that ab = a(b −1) −1 ∈ H for anb ∈ H. Therefore, H is a subgroup of G. Example 2.6.6 : For any a ∈ G. The set ⟨a⟩ = {a n : n ∈ ?} is a subgroup of G. For any p, q ∈ ⟨a⟩, p = a k and q = a t for some k, t ∈ ?. Now pq −1 = a k a −t = ak −t ∈ ⟨a⟩. So, by the above theorem it is proved that hai is a subgroup of G. In fact this group is generated by one element a. This type of group is called cyclic group and it will be discussed in detail in next chapter. Example 2.6.7 : Let G be a group of non-zero real numbers under multiplication, H = {x ∈ G : x = 1 or x is irrational} and K = {x ∈ G : x ≥ 1}. Now H is not a subgroup of G since 2 ∈ H but 2 . 2 ∉ H. Similarly, it can be shown that K is also not a subgroup of G. Example 2.6.8 : (Centralizer

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of an element). Let G be a group and a ∈ G.

Now consider the set C a = {x∈ G : xa = ax}. This set is non-empty, since ea = ae. Let x, y ∈ C a . Then xa = ax and ya = ay. Now (xy −1) a (xy −1) −1 = xy −1 ayx −1 = x(y −1 y) ax −1 = axx −1 = a. Which implies that (xy −1) a = a (xy −1).

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36 NSOU CC-MT-10 NSOU CC-MT-10 37 Therefore, xy −1 ∈ C a , whenever x, y ∈ C a . So, C a is a subgroup of G. This subgroup is called centralizer of a. Example 2.6.9 : (Center of a group). The center of a group G is defined by Z(G) = {a ∈ G : ax = xa ∀x ∈ G}. Now Z(G) ≠ φ, since e ∈ Z(G). By using the same arguments of the above example it can be proved that Z(G) is a subgroup of G (Complete the proof). This group in fact is the largest abelian subgroup of G. If G is abelian, then Z(G) = G. Example 2.6.10 : (Normalizer of a subgroup). Let H be a subgroup of G. Now consider the set N(H) = {x ∈ G : xHx −1 ⊆ H} = {x ∈ G : xhx −1 ∈ H ∀h ∈ H}. Now e ∈ N(H). Let x, y ∈ N(H). Then xhx −1 ∈ H and yhy −1 ∈ H for all h ∈ H. Now for all h ∈ H, (xy) h (xy) −1 = (xy) h (y −1 x −1) = x (yhy −1) x −1 = xh 1 x −1 ∈ H Thus xy ∈ N(H), whenever x, y ∈ N(H). Again x −1 h (x −1) −1 = x −1 hx = (xh −1 x −1) −1 = h′ −1 ∈ H, since xh −1 x −1 ∈ H. Therefore, x −1 ∈ N(H) for x ∈ N(H). Hence, N(H) is a subgroup of G. This group is called normalizer of H in G. Proposition 2.6.11 :
Let

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H and K be two subgroups of G. Then H ∩ K is also a subgroup of G.

G N(G) Z(G) Fig. 2.5: Group, Normal subgroup and center of a group
38 NSOU CC-MT-10 NSOU CC-MT-10 39 G H K H∩K Fig. 2.6: Intersection of two subgroups Proof. Since H and K are two subgroups of G, H ∩ K contains the identity element e. Let a,

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b ∈ H ∩ K. Then a, b ∈ H and a, b ∈ K. Hence, ab −1 ∈ H and ab −1 ∈ K. Which implies that ab −1 ∈ H ∩ K. Therefore, H ∩ K

is a subgroup of G. The above theorem can also be extend in case of finite sum, i.e., if H 1 , H 2 , ..., H n are subgroups of G, then ? i i n = = 1 H i is also a subgroup of G. Can we extend this theorem in case of infinite sum? Yes it is possible and the proof is same as the finite one. Union of two subgroups may not be a subgroup. For example let G = ?. Then 3? and 5? are subgroups of ?. Now 3 ∈ 3? ∪ 5? and 5 ∈ 3? ∩ 5?. But 3 + 5 = 8 ∉ 3? ∩ 5?. 2.7 Summary In this unit, we have mainly studied the concept of group along with various kinds of subgroups such as normalizer of a group, centralizer of a group. We have seen that the examples of groups are abundance in nature. 2.8 Worked examples 1.

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Let x and y be elements in a group G such that xy ∈

Z(G). Prove that xy = yx. Solution : Since xy = x −1 x(xy) and xy ∈ Z(G), we have xy = x −1 x(xy) = x −1 (xy)x = (x −1 x) yx = yx. 2. Let G be a group with exactly 4 elements. Prove that G is Abelian. Solution : Let a and b be non identity elements of G. Then e, a, b, ab, and ba are elements of G. Since G has exactly 4 elements, ab = ba. Thus, G is Abelian. 3. Let a be an element in a group. Prove that (a n) −1 = (a −1) n for each n ≥ 1. Solution : We use Math. induction on n. For n = 1, the claim is clearly valid. Hence,
38 NSOU CC-MT-10 NSOU CC-MT-10 39 assume that (a n) −1 = (a −1) n . Now, we need to prove the claim for n + 1. Thus, (a n+1) −1 = (aa

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n) −1 = (a n) −1 a −1 = (a −1) n a −1 = (a −1) n+1 . 4. Let H and D be two subgroups of a group

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such that neither H ⊂ D nor D ⊂ H. Prove that H ∪ D is never a group. Solution : Deny. Let a ∈ H \ D and let b ∈ D \ H. Hence, ab ∈ H or ab ∈ D. Suppose that ab = h ∈ H. Then b = a −1 h ∈ H, a contradiction. In a similar argument, if ab ∈ D, then we will reach a contradiction. Thus, ab ∉ H ∪ D. Hence, our denial is invalid. Therefore, H ∪ D is never a group. 5. Give an example of a subset of a group that satisfies all group-axioms except closure. Solution : Let H = 3Z and D = 5Z. Then H and D are subgroups of Z. Now, let C = H ∪ D. Then by the previous question, C is never a group since it is not closed. 6. Let H = {a ∈ Q : a = 3 n 8 m for some n and m in Z}. Prove that H under multiplication is a subgroup of Q \ {0}. Solution : Let a, b ∈ H. Then a = 3 n1 8 n2 and b = 3 m1 8 m2 for some n 1 , n 2 , m 1 , m 2 ∈ Z. Now, a −1 b = 3 m1 − n1 8 m2 − n2 ∈ H. Thus, H is a subgroup of Q \ {0} by Theorem 12..29..71. 7. Let a, x be elements in a group G. Prove that ax = xa if and only if a −1 x = xa −1 . Solution : Suppose that ax = xa. Then a −1 x = a −1 xaa −1 = a −1 axa −1 = exa −1 = xa −1 . Conversely, suppose that a −1 x = xa −1 . Then ax = axa −1 a = aa −1 xa = exa = xa. 8. Let H = {x ∈ C : x 301 = 1}. Prove that H is a subgroup of C \ {0} under multiplication. Solution : First, observe that H is a finite set with exactly 301 elements. Let a, b ∈ H. Then (ab) 301 = a 301 b 301 = 1. Hence, ab ∈ H. Thus, H is closed. Hence,

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H is a subgroup of C \ {0}. 9. Let H = {A ∈ GL(608, Z 89) : det(A) = 1}. Prove that H is a subgroup of		

GL(608, Z 89). Solution : First observe that H is a finite set. Let C, D ∈ H. Then det(CD) = det(C) det(D) = 1. Thus, CD ∈ H. Hence, H is closed. Thus, H is a subgroup of GL(608, Z 89). 10.

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Prove that if G is an abelian group, then for all a, b ∈ G and all integers			

n, (a . b) n = a n . b n . Solution : We resort to induction to prove that the result holds for positive integers. For n = 1, we have (a . b) 1 = a . b = a 1 . b 1 . So the result is valid for the base case. Suppose result holds for n = k − 1, i.e. (a . b) k−1 = a k−1 . b k−1 .
40 NSOU CC-MT-10 NSOU CC-MT-10 41 We need to show result also holds good for n = k.
We have (

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a . b) k = (a . b) k−1 . (a . b) = (a k−1 . b k−1) . (a . b) = (a k−1 . b k−1) . (b . a) = (a k−1 . b k) . a = a . (a k−1 . b k) = a k . b			

k
So the result holds for n =
k too. Therefore, result holds for all n ∈ ?. Next suppose n ∈ ?. If n = 0, then (a.b) 0 = e where e the identity element. Therefore (a . b) 0 = e = e . e = a 0 . b 0 . So the result is valid for n = 0 too. Next suppose n is a negative integer. So n = −m, where m is some positive integer. We have (a . b) n = (a . b) −m = ((a . b) −1) m by definition of the notation = (b −1 . a −1) m = ((a −1) . (b −1)) m = (a −1) m . (b −1)m as the result is valid for positive integers = (a −m) . (b −m) = a n . b n So the result is valid for negative integers too. Hence the result that (a . b) n = a n . b n holds in an abelian group for all n ∈ ?. 11. If G is a group in which (a . b) i = a i . b i for three consecutive integers i for all a, b ∈ G, show that G is abelian. Solution : Let n, n+1, n+2 be some three consecutive integers. Therefore we have (

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a . b) n = a n . b n (1) (a . b) n+1 = a n+1 . b n+1 (2) (a . b) n+2 = a n+2 . b n+2 (3) Using (2) we have (a . b) n+1 = a n+1 . b n+1 ⇒ (a . b) n . (a . b) = a n+1 . (b n . b) ⇒ (a n . b n) . (a . b) = (a n+1 . b n) . b, Using (1) ⇒ ((a n . b n) .		

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a) . b = (a n+1 . b n) . b 40 NSOU CC-MT-10 NSOU CC-MT-10 41 ⇒ (a n . b n) . a = (a n . a) . b n ⇒ a n . (b n . a) = a n . (a . b n) ⇒

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b n . a = a . b n (4) Again using (3), analogously we have b n+1 . a = a . b n+1 ⇒ b . (b n . a) = a . b n+1 ⇒ b . (a . b n) = a . b n+1 , Using (4) ⇒ (b . a) . b n = (a . b) . b n ⇒ b . a = a . b

So we have a . b =
b .
a ∀ a, b ∈
G. And hence G is abelian. 12. If G is a group of even order, prove it has an element a ≠ e satisfying a 2 = e. Solution : We prove the result by contradiction. Note that G is a finite group. Suppose there is no element x satisfying x 2 = e except for x = e. Thus if some g ≠ e belongs to G, then g 2 ≠ e, i.e. g ≠ g –1 . It means every non-identity element g has another element g –1 associated with it. So the non-identity elements can be paired into mutually disjoint subsets of order 2. We can assume the count of these subsets equals to some positive integer n as G is a finite group. But then counting the number of elements of G, we have o(G) = 2n + 1, where 1 is added for the identity element. So G is a group of odd order, which is not true. Hence there must exist an element a ≠ e such that a 2 = e for G is a group of even order. 13. Let : P be the set of all real numbers except the integer 1. Let the operation '⋆' be defined by a ⋆ b = a + b – ab for all a, b ∈ P. Show that (P,⋆) is a group. Solution : (i) Closure Property: Let a, b ∈ P. So, a and b

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are two real numbers and a ≠ 1, b ≠ 1. Now, a * b = a+b – ab which is a real number and a + b – ab ≠ 1, because a + b – ab = 1 ⇒ b(1 – a) = 1 – a ⇒ b = 1, since a ≠ 1. But b ≠ 1. Therefore, a * b is a

real number and

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a * b ≠ 1 . So, a ⋆ b ∈ P ∀ a, b ∈ P. Hence P is closed under the binary operation '⋆'. (ii) Associative Property : Let a, b, c ∈ P, where a, b, c ∈ R and a ≠ 1, b ≠ 1, c ≠ 1. Now, a * (b * c) = a * (b + c – bc) = a + b + c – bc – a (c + c – bc) = a + b + c – bc – ab – ac + abc (a * b) * c = (a + b – bc) * c = a + b – bc + c – (a + b – ab) c = a + b + c –

ab – ac – bc + abc
42 NSOU CC-MT-10 NSOU CC-MT-10 43 Therefore, a * (b * c) = (a * b) * c ∀ a, b, c ∈
P. So, associative property is satisfied w.r.t. the binary operation '⋆'. (iii) Identity Property : 0 ∈ P. Now, 0 ⋆ a = 0 + a – 0. a = a ∀ a ∈ P. So 0 is the left identity element in : under the binary operation '⋆'. (iv) Inverse Property : Let b be an element in P such that

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b ⋆ a = 0. Now, b ⋆ a = 0 ⇒ b + a – ba = 0 ⇒ b(1 – a) = –a ⇒ b = a a -1 , since ≠ 1 Since a a -1 is a real number as a ≠ 1 and a a -1 ≠ 1, so b = a a -1 ∈

P. Therefore, for any element a in P, ∃ an element a a -1 in P such that a a -1 * a = 0. So, a a -1 is the left 0-inverse

in P under the binary operation '⁎'. Therefore, (P, *) is a group. 14. Let (G, o) be a group and a, b ∈ G. If o(a) = 3 and aoboa −1 = b 2 , find the order of b if b is not the identity element of G. Solution : aoboa −1 = b 2 ⇒ a 2 oboa −2 = aob 2 oa −1 = (aoboa −1) o (aoboa −1) since 'o' is associative. = b 2 ob 2 = b 4 ⇒ a 3 oboa −3 = aob 4 oa −1 = (aoboa −1) o (aoboa −1) o (aoboa −1) o (aoboa −1) = b 2 ob 2 ob 2 ob 2 = b 8 or, b = b 8 ⇒ b 7 = e. Since b ≠ e and 7 is prime, so o (b) = 7. 2.9 Model Questions 1. In each case, find the inverse of the element under the given operation. i) 17 in ? 20 . ii) 2, 7 and 8 in U(9). 2. Prove that for a group G, Z G C a G a ( )= ∈ ?
42 NSOU CC-MT-10 NSOU CC-MT-10 43 3. List all the elements of U(20). 4. Let a, b be any two elements of an aleblian group and n be an integer. Show that (ab) n = a n b n . Is this also true for non-

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abelian groups? 5. Prove that a group G is abelian iff (ab) −1 = a −1 b −1 , ∀a, b ∈ G. 6.

Give an example of a group with 105 elements. Give two examples of groups with 44 elements. 7. Prove that in a group (ab) 2 = a 2 b 2 iff ab = ba. 8. Prove that if G is a group with the property that the square of every element is the identity, then G is abelian. 9. Let a.b ∈ G. Find x ∈ G such that xabx −1 = ba. 10. For each divisor k < 1 of n, let U k (n) = {x ∈ U(n) | x mod k = 1}. [For example, U 3 (21) = {1, 4, 10, 13, 16, 19} and U 7 (21) = {1, 8}.] List the elements of U 4 (20), U 5 (20), U 5 (30), and U 10 (30). Prove that U k (n) is a subgroup of U(n). Let H = {x ∈ U(10) | x mod 3 = 1}. Is H a subgroup of U(10)? 11. Suppose that a is a group element and a 6 = e. What are the possibilities for |a|? Provide reasons for your answer. 12. If a is a group element and a has infinite order, prove that a m ≠ a n when m ≠ n. 13. For any group elements a and b, prove that |ab| = |ba|. 14. Show that if a is an element of a group G, then |a| ≤ |G|. 15. Show that U(14) = ⟨3⟩ = ⟨5⟩. [Hence, U(14) is cyclic.] Is U(14) = ⟨11⟩? 16. Show that U(20) ≠ ⟨k⟩ for any k in U(20). [Hence, U(20) is not cyclic.] 17. Suppose n is an even positive integer and H is a subgroup of Z n . Prove that either every member of H is even or exactly half of the members of H are even. 18. Let n be a positive even integer and let H be a subgroup of Z n of odd order. Prove that every member of H is an even integer. 19. Prove that for every subgroup of D n , either every member of the subgroup is a rotation or exactly half of the members are rotations.
44 NSOU CC-MT-10 NSOU CC-MT-10 45 20. Let H be a subgroup of D n of odd order. Prove that every member of H is a rotation. 21. Prove that a group with two elements of order 2 that commute must have a subgroup of order 4. 22. For every even integer n, show that D n has a subgroup of order 4. 23. Suppose that H is a proper subgroup of Z under addition and H contains 18, 30, and 40. Determine H. 24. Suppose that H is a proper subgroup of Z under addition and that H contains 12, 30, and 54. What are the possibilities for H? 25. Suppose that H is a subgroup of Z under addition and that H contains 2 50 and 3 50 . What are the possibilities for H? 26. Prove that the dihedral group of order 6 does not have a subgroup of order 4. 27.
If

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H and K are subgroups of G, show that H ∩ K is a subgroup of G. (

Can you see that the same proof shows that the intersection of any number of subgroups of G, finite or infinite, is again a subgroup of G?) 28. Let U(n) be the group of units in ? n . If n < 2, prove that there is an element k ∈ U(n) such that k 2 = 1 and k ≠ 1. 29. Prove the right and left cancellation laws for a group G; that is, show that in the group G, ba = ca implies b = c and ab = ac implies b = c for elements a, b, c ∈ G. 30. Show that if a 2 = e for all elements a in a group G, then G must be abelian. 31. Show that if G is a finite group of even order, then there is an a ∈ G such that a is not the identity and a 2 = e. 32. Let G be a group and suppose that (ab) 2 = a 2 b 2 for all a and b in G. Prove that G is an abelian group. 33. Find all the subgroups of ? 3 × ? 3 . Use this information to show that ? 3 × ? 3 is not the same group as ? 9 . 34. Find all the subgroups of the symmetry group of an equilateral triangle. 35. Compute the subgroups of the symmetry group of a square. 36. Let H = {2 k : k ∈ ?}. Show that H is a subgroup of ?*.

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44 NSOU CC-MT-10 NSOU CC-MT-10 45 37. Let n = 0, 1, 2, . . . and n? = {nk : k ∈ ?}. Prove that n? is a subgroup of ?. Show that these subgroups are the only subgroups of ?. 38. Let T = {z ∈ ?* : |z| = 1}. Prove that T is a subgroup of ?*. 39. Let G consist of the 2 × 2 matrices of the form cos s in sin c os θ θ θ θ - ? ? ? ? ? ? where θ ∈ ?. Prove that G is a subgroup of SL 2 (?). 40. Prove that G = {a + b 2 : a, b ∈ ? and a and b are not both zero} is a subgroup of ?* under the group operation of multiplication.

46 NSOU CC-MT-10 NSOU CC-MT-10 47 46 Unit - 3 ? Cyclic Groups and Cyclic Subgroups Structure 3.1 Objectives 3.2 Introduction 3.3 Definition and Examples 3.4 Properties of Cyclic Group 3.5 The Circle Group and the Roots of Unity 3.6 Summary 3.7 Worked examples 3.8 Model Questions 3.9 Solution of some selected problems 3.1 Objectives The followings are discussed here: • Definition of cyclic group • Examples of cyclic group • Basic properties of cyclic group • Euler Phi function • Roots of unity 3.2 Introduction Cyclic group is the basic building block of group theory. In this unit we discuss the notion of cyclic group. The generators of a cyclic group is also derived. Finally, as an application of cyclic group, the circle group and the root of unity are discussed. 3.3 Definition and examples

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Definition 3.3.1 : A group G is called cyclic if there exists an element g ∈ G such that G = {g n : n ∈ ?}. The element g is called the generator of G. The

generator may not be unique. If G is cyclic and generated by g then G can be written as ⟨g⟩.

46 NSOU CC-MT-10 NSOU CC-MT-10 47 a 5 a 4 a 0 a 1 a 2 a 3 Fig. 3.1 : Cyclic group generated by a Example 3.3.2 : Any integer n 2 Z can be expressed as n = 1 + 1 + ... + 1(n times), when n is positive. Also n = (–1) + (–1) + ... + (–1)(|n| times), when n is negative. Which implies that both 1 and –1 are generators of the infinite cylic group ?. Example 3.3.3 : ? n = {0, 1, 2, ..., n – 1} with addition modulo n is a finite cyclic group. In this group 1 and –1 = n – 1 are the generators. For example ? 8 = ⟨1⟩ = ⟨3⟩ = ⟨5⟩ = ⟨7⟩. To verify that ? 8 = ⟨3⟩, we note that ⟨1⟩ = {3, 3 + 3, 3 + 3 + 3, ...} = {3, 6, 1, 4, 7, 2, 5, 0}. On the other hand 2 is not a generator (check it). Example 3.3.4 : U(12) = {1, 5, 7, 11}, in this case ⟨1⟩ = 1, ⟨5⟩ = {1, 5}, ⟨7⟩ = {1, 7} and ⟨11⟩ = {1, 11}. Therefore, U(12) is not cyclic. But note that U(10) is cyclic and generated by 3 and 7. Example 3.3.5 : The group ? 2 × ? 3 = {(

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m, n) : m ∈ ? 2 , n ∈ ? 3 } is a cyclic group. The binary operation is component wise addition (m, n) + (m′, n′) = (m + m′, n + n′).

In this group the element (1, 1) has order 6. (1, 1) + (1, 1) = (0, 2) (1, 1) + (0, 2) = (1, 0)

48 NSOU CC-MT-10 NSOU CC-MT-10 49 (1, 1) + (1, 0) = (0, 1) (1, 1) + (0, 1) = (1, 2) (1, 1) + (1, 2) = (0, 0). Hence, ? 2 × ? 3 is a cyclic group of order 6. Be careful, in general it is not true that ? m × ? n is cyclic. 3.4 Properties of Cyclic Group Since the elements of a cyclic group are the powers of an element, properties of cyclic groups are closely related to the properties of the powers of an element.

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Theorem 3.4.1 : Every cyclic group is Abelian. Proof. Let G be a cyclic group

generated by g.
Take a, b ∈ G. Then

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a = g n and b = g m . Now ab = g n g m = g n+m = g m+n = g

m g n = ba. Which implies that G is Abelian. The converse of the above theorem need not be true always, check that (hints: try) Theorem 3.4.2 : Every subgroup of a cyclic group is cyclic. Proof. The main tools used in this proof are the division algorithm and the Principle of Well-Ordering. Let G be a cyclic group generated by a and suppose that H is a subgroup of G. If H = {e}, then trivially H is cyclic. Suppose that H contains some other element g distinct from the identity. Then g can be written as an for some integer n. We can assume that n < 0. Let m be the smallest natural number such that a m ∈ H. Such an m exists by the Principle of Well-Ordering. We claim that h = a m is a generator for H. We must show that every h 0 ∈ H can be written as a power of h. Since h 0 ∈ H and H is a subgroup of G, h 0 = a k for some positive integer k. Using the division algorithm, we can find numbers
q and r such that k = mq +

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r where 0 ≤ r > m; hence, a k = a mk+r = (a m) k a r = h q a r . So a

r = a k h −q . Since a k and h −q are in H, a r must also be in H. However, m was the smallest positive number such that a m was in H; consequently, r = 0 and so k = m q . Therefore, h′ = a k = a mq = h q and H is generated by h. Corollary 3.4.3 : The subgroups of ? is exactly n? for n = 1, 2, ...
48 NSOU CC-MT-10 NSOU CC-MT-10 49 Theorem 3.4.4 : Let a ∈ G such that |a| = n. Then for any k ∈ ? 1. ⟨

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a k ⟩ = ⟨a gcd(n, k) ⟩ 2. |a k | = n n k

gcd(,) . This theorem is related to the order of a k and the groups generated by it. They will help us to find generators of a cyclic group Proof. 1. Let d = gcd(n, k). So, in particular, d is a divisor of k so there exists an integer r such that k = dr. So, a k = (a d)r. This implies that a k ∈ ⟨a d ⟩, i.e., ⟨a k ⟩ ⊆ ⟨a gcd(n,k) ⟩. Conversely, with d as above we know there exist integers s and t such that d = ns + kt. So, a d = a ns+kt = (a n) s + (a k) t = e(a k) t = (a k) t . Therefore, a d ∈ ⟨a k ⟩ and so ⟨a d ⟩ ⊆ ⟨a k ⟩ by closure. 2. It is clear that ( ) a d n d = a n = e, so that |a d | ≤ n d . We can not have |a d | > n d . If we did, then there exists i > n d such that |a d | = i, then a di = e and di > n which contradicts that |a| = n. Thus, |a d | = n d . This is true for every positive divisor of n and gcd(n, k) is such a divisor. So, we have |a k | = |⟨a k ⟩| = |⟨a gcd(n,k) ⟩| = n n k gcd(,) . Theorem 3.4.5 : Let G = ⟨a⟩ be a cyclic group of order n. If G contains an element b of order n, then ⟨b⟩ = G. Proof. Since b ∈ G and |b| = n. Then ⟨b⟩ contains n number of distinct elements. Again ⟨b⟩ ⊆ G. Hence, ⟨b⟩ = G. Definition 3.4.6 : (Euler Phi Function). Let n ∈ ? + . The Euler Phi function of n, denoted by φ(n) is the number of positive integers less than n and relatively prime to n and we set φ(1) = 1. Example 3.4.7 : The following table shows the value of φ for different n. n 1 2 3 4 5 6 7 8 φ 1 1 2 2 4 2 6 8 Example 3.4.8 : By definition |U(n)| = φ(n).
50 NSOU CC-MT-10 NSOU CC-MT-10 51 3.5 The Circle Group and the Roots of Unity The multiplicative group of the complex numbers, ? + , possesses some interesting subgroups. Whereas ? + and ? + have no interesting subgroups of finite order, ? + has many. We first consider the circle group, S = {z ∈ ? : |z| = 1}. Proposition 3.5.1 : The circle group is a subgroup of ? + . Although the circle group has infinite order, it has many interesting infinite subgroups. Suppose that H = {1,−1, i,−i}. Then H is a subgroup of the circle group. Also, 1, −1, i, and −i are exactly those complex numbers that satisfy the equation z 4 = 1. The complex numbers satisfying the equation z n = 1 are called the nth roots of unity.
e j2p4/12 e j2

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p2 /12 e j2p2 /12 e j2p/12 e j2p11 /12 e j2p10 /12 e j2p9 /12 e j2p8 /12 e j2p7/12 e j2p6 /12 e j2p5 /12 e j2p0 /12 = e

j2
p12 /12

Fig. 3.2 Theorem 3.5.2 : If z n = 1, then the nth root of unity are z k n = ( ) exp , 2 π where k = 0, 1, ..., n – 1. Furthermore, the nth roots of unity form a cyclic subgroup of S of order n. Proof. By DeMoivre's Theorem z n k n k n = ( ) = = exp exp 2 2 1 π π ( ) The z's are distinct since the numbers 2kp/n are all distinct and are greater than or equal to 0 but less than 2p. The fact that these are all of the roots of the equation z n =1 follows from from fundamental theorem of algebra, which states that a

50 NSOU CC-MT-10 NSOU CC-MT-10 51 polynomial of degree n can have at most n roots. We will leave the proof that the nth roots of unity form a cyclic subgroup of S as an exercise. A generator for the group of the nth roots of unity is called a primitive nth root of unity. 3.6 Summary In this unit, we have introduced the concept of cyclic group. We have showed that a subgroup of a cyclic group is cyclic. Also we have studied that for each divisor of the order of a cyclic group there exists a unique cyclic subgroup of that order. 3.7 Worked examples 1. Find all generators of Z 22 . Solution : Since |Z 22 | = 22, if a is a generator of Z 22 , then |a| must equal to 22. Now, let b be a generator of Z 22 , then b = 1 b = b. Since |1| = 22, we have |b| = |1 b |= 22/gcd(b, 22) = 22 . Hence, b is a generator of Z 22 iff gcd(b,22) = 1. Thus, 1,3,5,7,9,11,13,15,17,19,21 are all generators of Z 22 . 2. Let G = (a), a cyclic group generated by a, such that |a| = 16. List all generators for the subgroup of order 8. Solution : Let H be the subgroup of G of order 8. Then H = (a 2) = (a 16/8) is the unique subgroup of G of order 8 by Theorem 3.2.5. Hence,(a 2) k is a generator of H iff gcd(k,8) = 1. Thus, (a 2) 1 = a 2 , (a 2) 3 = a 6 , (a 2) 5 = a 10 , (a 2) 7 = a 14 . 3. Suppose that G is a cyclic group such that |G| = 48. How many subgroups does G have? Solution : Since for each positive divisor k of 48 there is a unique subgroup of order k by Theorem 3.2.5, number of all subgroups of G equals to the number of all positive divisors of 48. Hence, Write 48 = 3 1 2 3 . Hence, number of all positive divisors of 48 = (1+1)(3+1) = 8. If we do not count G as a subgroup of itself, then number of all proper subgroups of G is 8 – 1 = 7. 4. Let a be an element in a group,and let i, k be positive integers. Prove that H = (a i) ∩ (a k) is a cyclic subgroup of (a) and H = (a lcm(i,k)). Solution : Since (a) is cyclic and H is a subgroup of (a), H is cyclic by Theorem 3.2.2. By Theorem 1.2.18 we know that lcm(i, k) = ik/gcd(i, k).

52 NSOU CC-MT-10 NSOU CC-MT-10 53 Since k/gcd(i,k) is an integer, we have a lcm(i,k) = (a i) k/gcd(i,k) . Thus, (a lcm (i,k)) ⊂ (a i). Also, since k/gcd(i, k) is an integer, we have a lcm(i,k) = (a k) i/gcd(i.k) . Thus, (a lcm(i, k)) ⊂ (a k). Hence, (a lcm (i, k)) ⊂ H. Now, let h ∈ H. Then h = a j = (a i) m = (a k) n for some j, m, n ∈ Z. Thus, i divides j and k divides j. Hence, lcm(i,k) divides j. Thus, h = a j = (a lcm(i,k)) c where j = lcm(i,k)c. Thus, h ∈ (a lcm(i,k)). Hence, H ⊂ (a lcm(i,k)). Thus, H = (a lcm(i,k)). 5. Let a be an element in a group. Describe the sub-group H = (a 12) ∩ (a 18). Solution : By the previous Question, H is cyclic and H = (a lcm(12,18) = (a 36). 6. Let G = (a), and let H be the smallest subgroup of G that contains am and an. Prove that H = (a gcd(n, m)). Solution : Since G is cyclic, H is cyclic by Theorem 3.2.2. Hence, H = (a k) for some positive integer k. Since a n ∈ H and a m ∈ H, k divides both n and m. Hence, k divides gcd(n,m). Thus, a gcd(n,m) ∈ H = (a k). Hence, (a gcd(n,m)) ⊂ H. Also, since gcd(n,m) divides both n and m, a n ∈ (a gcd(n,m)) and a m ∈ (a gcd(n,m)). Hence, Since H is the smallest subgroup of G containing a n and a m and a n , a m ∈ (a gcd(n,m)) ⊂ H, we conclude that H = (a gcd(n,m)). 7. Let G be an infinite cyclic group. Prove that e is the only element in G of finite order. Solution : Since G is an infinite cyclic group, G = (a) for some a ∈ G such that |(a)| is infinite. Now, assume that there is k an element b ∈ G such that |b| = m and b ≠ e. Since G = (a), b k = a for some k ≥ 1. Hence, e = b m = (a k) m = a km . Hence, |a| divides km. a contradiction since |a| is infinite. Thus, e is the only element in G of finite order. 8. Let G = (a) be a cyclic group. Suppose that G has a finite subgroup H such that H ≠ {e}. Prove that G is a finite group. Solution : First, observe that H is cyclic by Theorem 3.2.2. Hence, H = (a n) for some positive integer n. Since H is finite and H = (a n), Ord(a n) = |H| = m is finite. Thus, (a n) m = a nm = e. Hence, |a| divides n m . Thus, (a) = G is a finite group. 9. Let a be an element in a group G such that |a| is infinite. Prove that (a), (a 2), (a 3), ... are all distinct subgroups of G, and Hence, G has infinitely many proper subgroups. Solution : Suppose that (a i) = (a k) for some positive integers i, k such th at k < i.

52 NSOU CC-MT-10 NSOU CC-MT-10 53 Thus, a i = (a k) m for some m ∈ Z. Hence, a i = a km . Thus, a i km = e. Since k < i, km ≠ i and therefore i – km ≠ 0. Thus, | a | divides i – km . Hence, |a| is finite, a contradiction. 10. Let G be a group containing more than 12 elements of order 13. Prove that G is never cyclic. Solution : Suppose that G is cyclic. Let a ∈ G such that |a| = 13. Hence, (a) is a finite subgroup of G. Thus, G must be finite by the previous Question. Hence, by Theorem 3.2.5 there is exactly φ (13) = 12 elements in G of order 13. A contradiction. Hence, G is never cyclic. 3.8 Model Questions 1. Find all generators of the cyclic group ? 28 . 2. In ? 30 find the order of the subgroup ⟨18⟩ and ⟨24⟩. 3. Show that any cyclic group of even order has exactly one element of order 2. 4. Show that ? + is not a cyclic group. 5. Let G be an abelian group of order 15. Show that if you can find an element a of order 5 and an element b of order 3, then G must be cyclic. 6. Let H = ± ±± ± { } 1 2 2 2 2 , , , i i is a cyclic subgroup of ? + . 7. Let H = 1 0 0 10 0 01 3 m GL m ? ? ? ? ? ? ? ? ∈ ∈ ? ? ? ? ? ? ? ? ? ? ( ): and K = 1 0 0 10 0 01 n ? ? ? ? ? ? ? ? ∈ ? ? ? ? ? GL 3 (?) : n ∈ ? } are cyclic groups of GL 3 (?). 8. Prove that ? p does not have any non-trivial subgroup if p is prime. 9. Let G be an abelian group. Show that the elements of finite order in G form a subgroup. This subgroup is called the torsion subgroup of G. 10. Find all generators of ? 48 . 11. Prove that the following groups are not cyclic: (i) ? 2 × ? 2 (ii) ? 2 × ? (iii) ? × ?
54 NSOU CC-MT-10 NSOU CC-MT-10 55 12. Prove that the cyclic subgroup ⟨a⟩ is the smallest subgroup of G containing a ∈ G. 13. If a cyclic group has an element of infinite order, how many elements of finite order does it have? 14. Suppose that G is an Abelian group of order 35 and every element of G satisfies the equation x 35 = e. Prove that G is cyclic. Does your argument work if 35 is replaced with 33? 15.

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Let G be a group and let a be an element of G.

a. If a 12 = e, what can we say about the order of a? b. If a m = e, what can we say about the order of a? c. Suppose that |G| = 24 and that G is cyclic. If a 8 ≠ e and a 12 ≠ e, show that ⟨a⟩ = G. 16. Prove that

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a group of order 3 must be cyclic. 17. Let Z denote the group of integers under addition. Is every subgroup of Z

cyclic? Why? Describe all the subgroups of Z. Let a be a group element with infinite order. Describe all subgroups of ⟨a⟩. 18. For any element a in any group G, prove that ⟨a⟩ is a subgroup of C(a) (the centralizer of a). 19. If d is a positive integer, d ≠ 2, and d divides n, show that the number of elements of order d in D n is f (d). How many elements of order 2 does D n have? 20. Find all generators of Z. Let a be a group element that has infinite order. Find all generators of ⟨a⟩. 21. Prove that C*, the group of nonzero complex numbers under multiplication, has a cyclic subgroup of order n for every positive integer n. 22. Let a be a group element that has infinite order. Prove that ⟨a i ⟩ = ⟨a j ⟩ if and only if i = ±j. 23. List all the elements of order 8 in Z 8000000 . How do you know your list is complete? Let a be a group element such that |a| = 8000000. List all elements of order 8 in ⟨a⟩. How do you know your list is complete? 24. Suppose that G is a group with more than one element. If the only subgroups of G are {e} and G, prove that G is cyclic and has prime order.
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Let G be a finite group. Show that there exists a fixed positive integer n such that a n = e for all a in G. (Note that n is independent of a.) 26. Determine the subgroup lattice for Z 12 . Generalize to Z p q 2 , where p and q are distinct primes. 27. Determine the subgroup lattice for Z 8 . Generalize to Z p n , where p is a prime and n is some positive integer. 28. Prove that a finite group is the union of proper subgroups if and only if the group is not cyclic. 29. List all of the elements in each of the following subgroups. (a) The subgroup of ? generated by 7 (b) The subgroup of ? 24 generated by 15 (c) All subgroups of ? 12 (d) All subgroups of ? 60 (e) All subgroups of ? 13 (f) All subgroups of ? 48 (g) The subgroup generated by 3 in U(20) (h) The subgroup generated by 5 in U(18) (i) The subgroup of ?* generated by 7 (j) The subgroup of ?* generated by i where i 2 = –1 (k) The subgroup of ?* generated by 2i (l) The subgroup of ?* generated by ( )/ 1 2 +i (m) The subgroup of ?* generated by ( ) / 1 32 + i 30. Find the subgroups of GL 2 (?) generated by each of the following matrices (a) 0 1 1 0 - ? ? ? ? ? ? (c) 1 1 1 0 - ? ? ? ? ? ? (e) 1 1 1 0 - - ? ? ? ? ? ? (b) 0 13 3 0 / ? ? ? ? ? ? (d) 1 1 0 1 - ? ? ? ? ? ? (f) 3 2 1 2 1 2 3 2 / / / / - ? ? ? ? ? ? 31. Find the order of every element in ? 18 . 32. Find the order of every element in the symmetry group of the square, D 4 . 33. What are all of the cyclic subgroups of the quaternion group, Q 8 ?

56 NSOU CC-MT-10 NSOU CC-MT-10 PB 34. List all of the cyclic subgroups of U(30). 35. List every generator of each subgroup of order 8 in ? 32 . 36. Find all elements of finite order in each of the following groups. Here the "*" indicates the set with zero removed. (a) ? (b) ?* (c) ?* 37. If a 24 = e in a group G, what are the possible orders of a? 38. Find a cyclic group with exactly one generator. Can you find cyclic groups with exactly two generators? Four generators? How about n generators? 39. For n ≤ 20, which groups U(n) are cyclic? Make a conjecture as to what is true in general. Can you prove your conjecture? 3.9 Solutions of some selected problems 1. { 1, 3, 5, 9, 11 , 13, 15, 17, 19, 21, 23, 25, 27} 2. 5, 5 10. All the elements less than and prime to 48. 13. Only one 15. (a) order of a may be 1, 2, 3, 4, 6 or 12 (b) order of a may be all the divisors of m 17. Use the fact that all the subgroups of a cyclic group are cyclic 20. {+1, –1} 23. Use theorem 3.4.3 29. (a) {7n : n ∈ Z} (b) {0, 6, 12, 15, 6, 21} 30. (a) { :} 0 0 a a a R - ? ? ? ? ? ? ∈ 31. use theorem 3.4.3 36. (a) 0 (b) {+1, –1} (c) {+1, –1} 37. All the divisors of 24 38. Z 2

PB NSOU CC-MT-10 NSOU CC-MT-10 57 Unit - 4 ? Cosets and Normal Subgroups Structure 4.1 Objectives 4.2 Introduction 4.3 Definition and concept 4.4 Lagrange's Theorem 4.5 Normal Subgroups 4.6 Summary 4.7 Worked examples 4.8 Model Questions 4.9 Solution of some selected problems 4.1 Objectives The followings are discussed here: • Definition of cosets and examples • Definition of normal subgroup and normalizer • Basic properties of normal group • Lagrange's theorem 4.2 Introduction In this unit, we prove the single most important theorem in finite group theory—Lagrange's Theorem. In his book on abstract algebra, I. N. Herstein likened it to the ABC's for finite groups. But first we introduce a new and powerful tool for analyzing a group—the notion of a coset. This notion was invented by Galois in 1830, although the term was coined by G. A. Miller in 1910. 4.3 Definition and concept The Euclidean plane ? 2 forms a group under component wise addition, i.e., for any two (a, b), (c, d) ∈ ? 2 , then (a, b) + (c, d) = (a + c, b + d). 57

58 NSOU CC-MT-10 NSOU CC-MT-10 59 Now the subset X = {(x, 0) : x ∈ R} is a subgroup of ? 2 which is nothing but the x axis (check it!). If we take any element (a, b) ∈ ? 2 which is not in X, then the set H(a, b) = (a, b) + X = {(a + x, b) : x ∈ ?} is parallel to x-axes and looks like the set X, see Figure 4.1. Also it can be seen that if we choose an element from X, i.e., of the form (a, 0), then H (a,0) is X itself. Therefore, we conclude that either H (a,b) = X or H (a,b) ? X = φ. Since the collection of all straight lines, parallel to x-axes covers the whole Euclidean plane, it implies that ? (a,b)∈? 2 H (a,b) = ? 2 . Hence, the collection {H (a,b) } forms a partition of the Euclidean plane. If we take the collection H (a,b) = X + (a, b) = {(x + a, b) : x ∈ ?} then we also get the same image as the figure 4.1 for the commutativity of the addition in ? 2 . In group theoretic language this type of element is called coset, more specifically left-coset. Here comes the formal definition. H (a,3/2) H (a,1) Y X (0, 0) H (a,–1) Fig. 4.1 Definition 4.3.1 : Let G be a group. Now take an element a ∈ G, then the set aH defined by aH = {ah : h ∈ H} is called the left coset. Similarly we can define the right-coset Ha.

58 NSOU CC-MT-10 NSOU CC-MT-10 59 Example 4.3.2 : Consider the subgroup H = ⟨3⟩ of ? 6 . The cosets are 0 +

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H = {0, 3} = 3 + H 1 + H = (123)H = {(13), (123)} 2 + H = (132)H = {(23), (132)} Example 4.3.3 : Let G = S 3 and H = {(1), (12)}. Then the left cosets of H

in G are (1)

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H = (12)H = {(1), (1, 2)} (13)H = (123)H = {(13), (123)} (23)H = (132)H = {(23), (132)} The right cosets are H(1) = H(12) = {(1), (1, 2)} H(13) = H(132) = {(13), (132)} H(23) = H(123) = {(23), (123)}.

Note that, except for the coset of the elements in H, the left and right cosets are different. G H gH g′H Fig. 4.2 : Group G and cosets gH and g′H of the subgroup H Proposition 4.3.4 (Properties). Let H and K be two subgroups of G and

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a, b ∈ G. Then 1. a ∈ aH. 2. aH = H if and only if a ∈ H. 3. aH = bH if and only if a ∈ bH. 4. aH = bH or aH ? bH =

φ.
60 NSOU CC-MT-10 NSOU CC-MT-10 61 5. aH = bH if and only if a –1 b ∈ H. 6. |aH| = |bH|. Proof. 1. Since H contains the identity element e, which implies a.e = a ∈ aH. 2. Suppose aH = H, then e = ah for some h ∈ H. Therefore, a = eh –1 = h –1 ∈ H. Conversely, suppose a ∈ H. Then aH ⊂ H. Let h ∈ H. Then h can be expressed as h = aa –1 h = ah 1 ∈ aH for some h 1 ∈ H. Which implies H ⊆ aH. Hence, aH = H. 3. This part can be easily deduced from 1. and 2. 4. Let aH ? bH ≠ φ. Take

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x ∈ aH ? bH. Then x = ah 1 = bh 2 for some h 1 , h 2 ∈ H. So,

we get a = bh 2 h 1 ∈ bH. Hence, from (3) we say that aH = bH. Therefore, either aH ? bH = φ or aH = bH. 5. Let aH = bH. Then b = ah for some h ∈ H. Which implies that a –1 b = h ∈ H. Conversely, let a –1 b ∈ H. Then b ∈ aH. So, from (3) we get aH = bH. 6. Define a function f : aH → bH by f(ah) = bh. (Check it!) This function is bijective. Hence, aH and bH has same number of elements. From (3) of the Proposition 4.4, it is clear that cosets makes partition of the group G. But we know that for any partition there must be a equivalence relation. Now we define the equivalence relation.

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Let H be a subgroup of the group G. For any a, b ∈ G, a is related to b, a ~ b if and only if

a –1 b ∈ H.
This relation is reflective, i.e., a ~ a since a –1 a = e ∈ H. This relation is also symmetric. Now for any
a, b, c ∈ G such that

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a ~ b and b ~ c, we get a –1 b ∈ H and b –1 c ∈ h. Hence, (a –1 b)(b –1 c) = a –1 c ∈ H.

Which implies that a ~ c.

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Therefore, the relation ~ is transitive. Hence ~ is an equivalence relation. Consider the equivalence class [

a] of a ∈ g, i.e., [a] = {b ∈ G : a ~ b}. Theorem 4.3.5 : The equivalence class [a] is nothing but the left coset aH. Proof. Since the relation ~ is reflective, [a] ≠ φ. Let b ∈ [a]. Then a ~ b, i.e., a −1 b ∈ H. Which implies that b ∈ aH. Hence, [a] ⊆ aH. Again take b ∈ aH. Then b = ah for some h ∈ H. Which implies that a −1 b = h ∈ H. Therefore, a ~ b. So, b ∈ [a]. Therefore, aH ⊆ [a]. Hence, we get [a] = aH. This theorem makes it clear why the cosets partition the whole group. Note that the above result holds if we replace 'left' with 'right'.
Definition 4.3.6 :

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Let G be a group and H be a subgroup. The number of left cosets of H in G is called index of H in G and denoted by [G : H]. 60

NSOU CC-MT-10 NSOU CC-MT-10 61 Example 4.3.7 : From the previous example we get [? 6 , H] = 3 and [S 3 , H] = 3. Theorem 4.3.8 :
Let
H be a subgroup of G.

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Then the number of left cosets of H in G is same as the number of right cosets of H in G.

Proof. Let L H and R H
be the number of left cosets and right cosets of H in
G
respectively.
Now we define a bijection between L H and R H . Consider the function ϕ : L H → R H defined by ϕ(gH) = Hg −1 . First, we will show that this map is well-defined. Suppose g 1 H = g 2 H. Then by proposition 4.4, Hg 1 −1 = Hg 2 −1 = ϕ(g 1 H) = ϕ(g 2 H). Thus, ϕ is well defined. Let ϕ(g 1 H) = ϕ(g 2 H) for some g 1 , g 2 ∈ G. Then, Hg 1 −1 = Hg 2 −1 . Again, the proposition 4.4 implies that g 1 H = g 2 H. Hence, the function ϕ is injective. The function ϕ is obviously surjective. Therefore, ϕ is a bijection so the result holds. The above theorem implies that in the definition of index of a subgroup H in the group G we can replace the term 'left cosets' with 'right cosets' also. 4.4 Lagrange's Theorem We're finally ready to state Lagrange's Theorem, which is named after the Italian born mathematician Joseph Louis Lagrange. Theorem 4.4.1 (Lagrange's Theorem).

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Let G be a finite group and H be a subgroup of G. Then |G|/|H| = [G :

H].
In particular, |H| divides |G|. Proof. The group G is partitioned into [G : H] number of left-cosets and each left coset has |H| numbers of element by the proposition 4.4. Hence, |G| = |H|[G : H].

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The converse of Lagrange's Theorem is not true: namely, if G is a finite group and n divides |G|, then G need not have a subgroup of order

n. It can be seen by an example: A 4 has no subgroup of order 6. But there are some partial converse to Lagranges Theoem. For finite abelian group the full converse is true, i.e., for each divisor of |G|, we have a subgroup of that order. Theorem 4.4.2 (Cauchy's Theorem). If G is

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a finite group and p is a prime dividing |G|, then G has an element of order p. Proof.

The proof is out of the scope of this book. We'll now examine a host of consequence of Lagrange's Theorem. 62 NSOU CC-MT-10 NSOU CC-MT-10 63 Corollary 4.4.3 : Suppose G is a finite group and g ∈ G. Then 1. |g| divided |G|. 2. g|G| = e. 3. If |G| is a prime, then G is cyclic and every element g ≠ e of G is a generator of G. Proof. 1. Consider the cyclic group ⟨g⟩ generated by g. Then ⟨g⟩ has order |g|. Now by Lagrange's theorem | ⟨g⟩ | divides |G|, hence, |g| divides |G|. 2. Since |g|||G|. So |G| = m|g| for some integer m. Now g |G| = (g |g|) m = e m = e. 3. Let g ∈ G be an non-identity element. Now |g| divides |G|. But |G| is a prime number. So either |g| is one or |G|. But |g| ≠ 1 since g is not the identity. Therefore, |g| = |G|. Therefore, g is a generator of G. Since g is arbitrary, so every element g ≠ φ of G is a generator of G and G is cyclic. Corollary 4.4.4 : Let

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H and K be subgroups of G such that K ⊂ H ⊂ G. Then [G : K] = [G : H][G : K]. Proof. By, Lagrange's Theorem we have [:] | | | | | || | | | [:][:] G K G K G H H K

G HG K = = = Theorem 4.4.5 : (Fermat's Little Theorem). For every integer a and every prime p, a p ≡ a mod p. Proof. By division algorithm, a = pm + r where 0 ≤ r > p. Thus a ≡ r mod p, and it suffices to prove that r p ≡ r mod p. If r = 0 the result is trivial, so we may assume that r ∈ U(p) = {1, 2, ..., p – 1}. Hence, r p–1 ≡ 1 mod p and therefore, r p ≡ r mod p. 4.5 Normal Subgroups Normal subgroups was introduced by Evariste Galois in 1831 as a tool for deciding whether a polynomial is solvable by radical or not. Galois noted that a subgroup H of a group G of permutation induced two decompositions of G into what we call left cosets and right cosets. If the two decompositions coincide, that is, if the left cosets are the same as the right cosets, Galois called the decomposition proper. Thus a subgroup giving a proper decomposition is what we called normal subgroup. Definition 4.5.1 : A subgroup H of G is called normal, denoted by H ? G, if gH = Hg for all g ∈ G, i.e., left-coset and right-coset are equal. You should think of a normal subgroup in this way: You can switch the order of a product of an element a from the group and an element h from the normal subgroup 62 NSOU CC-MT-10 NSOU CC-MT-10 63 H, but you must "fudge" a bit on the element from the normal subgroup H by using some h' from H rather than h. That is, there is an element h' in H such that ah = h'a. Likewise, there is some h" in H such that ha = ah". (It is possible that h' = h or h" = h, but we may not assume this.) Proposition 4.5.2 :

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Let G be a group and H be a subgroup with index 2. Then H is normal in G.

Proof. Let g ∈ G – H so, ny hypothesis, there are two left cosets of H in G, they are eH and gH. Since eH = H and the cosets partition G, we must have gH = G – H. Now the two right cosets of H in G are He and Hg. Since He = H, we again must have Hg = G – H. Combining these gives,

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gH = Hg for all g ∈ G. Hence, H is normal in G.

Example 4.5.3 : Every subgroup of an abelian group G is normal. Example 4.5.4 : G = S 3 , H = ⟨(1, 2, 3)⟩ = {e, (1, 2, 3), (1, 3, 2)}. Now [G : H] = 2, so H is normal in G. Let g = (1, 2). Then gH = {(1, 2), (1, 2)(1, 2, 3), (1, 2)(1, 3, 2)} = {(1, 2), (2, 3), (1, 3)} Hg = {(1, 2), (1, 2, 3)(1, 2), (1, 3, 2)(1, 2)} = {(1, 2), (1, 3), (2, 3)}. this example shows that if H is normal in G, then gH = Hg ∀g ∈ G but it is not true that gh = hg for all h ∈ H. There are several equivalent formulations of the definition of normality. Normal subgroup can also be expressed in terms of conjugacy relation. In a group G, two elements g and h are said to be conjugate if h = xgx −1 for some x ∈ G. The conjugacy relation in G is an equivalence relation (Check it !). The conjugacy class of g ∈ G is denoted by [g] = {xgx −1 : x ∈ G}. Example 4.5.5 : In S 3 , what are the conjugates of (1, 2)? We make a table of σ(1, 2) σ −1 for all σ ∈ S 3 . σ (1) (1,2) (1,3) (2,3) (1,2,3) (1,3,2) σ (1, 2)σ −1 (1,2) (1,2) (2,3) (1,3) (2,3) (1,3) The idea of conjugation can be applied not just to elements, but to subgroups. If H is a subgroup of G and g ∈ G, the set gHg −1 = {ghg −1 : h ∈ H} is the conjugacy class of g in H.

64 NSOU CC-MT-10 NSOU CC-MT-10 65 Proposition 4.5.6 : The conjugacy class gHg −1 is a subgroup of G. Proof. Since e ∈ H, which implies e ∈ gHg −1 . So gHg −1 ≠ φ. Let x, y ∈ gHg −1 . Then x = gh 1 g −1 and y = gh 2 g −1 for some h 1 , h 2 ∈ H. Now, xy −1 = gh 1 g −1 (gh 2 g −1) −1 = gh 1 g −1 gh 2 −1 g −1 = g(h 1 h 2 −1)g −1 ∈ gHg −1 . Therefore, gHg −1 is a subgroup of

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G. Theorem 4.5.7 : A subgroup H of G is normal if and only if gHg −1 ⊆ H for all g ∈ G. Proof. Let H is normal in G. Then gH = Hg for all g ∈ G. Now for any h ∈ H, there exists h′ ∈ H such that gh = h′g. Which implies that ghg −1 = h′ ∈ H. Hence, gHg −1 ⊆ H for all g ∈ G. Conversely, let gHg −1 ⊆ H for all g ∈ G.

Then for any gh ∈ gH there exists h′ ∈ H such that gh = h′g from the hypothesis. Hence, gH ⊆ Hg. Similarly, we can show Hg ⊆ gH. Therefore,
gH = Hg for all g ∈ G. Hence, H

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is normal in G. Definition 4.5.8 : Let H and K be subgroups of a group G and define HK = {hk : h ∈ H, k ∈ K}.

a 1
H
a 1 g 2 H a 1 g 2 H a 1

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N G (H) a 1 g 3 H a 1 g 1 H H g 4 N=Ng 4 g 2 H=Hg 2 N G (H) g 3 H=Hg 3 g 1 H=Hg 1 a 3 H a 3 g 4 H a 3 g 2 H a 3 N G (H) a 2 g 3 H a 3 g 1 H a 2 H a 2 g 2 H a 2 g 2 H a 2 N G (H) a 3 g 3 H

a 2
g 1
H
Fig. 4.3 : Abstract visualization of the
relationships H Δ N G H Δ G
64 NSOU CC-MT-10 NSOU CC-MT-10 65 Proposition 4.5.9 :
If

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H and K are finite subgroups of a group, then | | | || | | | HK H K H K = ∩ .

Proof.
Notice that HK is a union of left cosets of K, namely, HK hK h H = ∈ ? . Since each coset of K has |K| elements it suffices to find the number of distinct left cosets of the from hK,

34%	MATCHING BLOCK 74/128	W

h ∈ H. But h 1 K = h 2 K for h 1 , h 2 ∈ H if and only if h 2 −1 h 1 ∈ K. Thus h 1 K = h 2 K ? h 2 −1 h 1 ∈ H ? K ? h 1 (H ? K) = h 2 (H ?

K).
Thus the number of distinct cosets of the from hK, for h ∈ H is the number of distinct cosets h(H ? K), for h ∈ H. The latter number, by Lagrange's theorem, equals | | | | H H K ∩ . Thus HK consists of | | | | H H K ∩ number of cosets of K which proves the result. 4.6 Summary In this unit, we have studied the concept of cosets and normal subgroup. We have showed that the cosets partion the whole group. We have also discussed the Lagrange's theorem. 4.7 Worked examples 1. List the cosets of ⟨9⟩ in Z 16 ×× , and find the order of each coset in Z 16 ×× /⟨9⟩. Solution: Z 16 × = {1, 3, 5, 7, 9, 11, 13, 15}. ⟨9⟩ = {1, 9} 3 ⟨9⟩ = {3, 11} 5 ⟨9⟩ = {5, 13} 7 ⟨9⟩ = {7, 15} Now

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the order of aN is the smallest positive integer n such that a n ∈

N. The coset 3 ⟨9⟩ has order 2 since 3 2 = 9 and 9 belongs to the subgroup ⟨9⟩. (We could have used either element of the coset to do the calculation.) The coset 5 ⟨9⟩ also has order 2, since 5 2 = 9. The coset 7 ⟨9⟩ has order 2 since 7 2 = 1. 2. List the cosets of ⟨7⟩ in Z 16 ×× . Is the factor group Z 16 ×× / ⟨7⟩ cyclic? Solution: Z 16 × = {1, 3, 5, 7, 9, 11, 13, 15}. ⟨7⟩ = {1, 7} 3 ⟨7⟩ = {3, 5} 9 ⟨7⟩ = {9, 15} 11 ⟨7⟩ = {11, 13} Since 3 2 ∉ ⟨7⟩, the coset 3 ⟨7⟩ does not have order 2, so it must have order 4, showing that the factor group is cyclic.
66 NSOU CC-MT-10 NSOU CC-MT-10 67 3. Show that the subgroup {id, (1 3)} of S 3 is not normal. Solution: Here's the multiplication table for S 3 , the group of permutations of {1, 2, 3}. id (1 2 3) (1 3 2) (2 3) (1 3) (1 2) id id (1 2 3) (1 3 2) (2 3) (1 3) (1 2) (1 2 3) (1 2 3) (1 3 2) id (1 2) (2 3) (1 3) (1 3 2) (1 3 2) id (1 2 3) (1 3) (1 2) (2 3) (2 3) (2 3) (1 3) (1 2) id (1 2 3) (1 3 2) (1 3) (1 3) (1 2) (2 3) (1 3 2) id (1 2 3) (1 2) (1 2) (2 3) (1 3) (1 2 3) (1 3 2) id We have to find an element g ∈ S 3 such that g{id, (1 3)}g −1 ⊄ {id, (1 3)}. There are several possibilities. For example, (1 2){id, (1 3)}(1 2) −1 = (1 2){id, (1 3)}(1 2) = {(1 2)id(1 2), (1 2)(1 3)(1 2)} = {id, (2 3)}. Since {id, (2 3)} ⊄ {id, (1 3)}, the subgroup {id, (1 3)} is not normal in S 3 . 4. Let G and H be groups. Let G × {1} = {(g, 1) | g ∈ G}. Prove that G × {1} is a normal subgroup of the product G × H. Solution: First, I'll show that it's a subgroup. Let (

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g 1 , 1), (g 2 , 1) ∈ G × {1}, where g 1 , g 2 ∈ G. Then (g 1 , 1) · (g 2 , 1) = (g 1 g 2 , 1) ∈ G × {1}. Therefore, G × {1} is

closed under products. The identity (1, 1) is in G × {1}. If (g, 1) ∈ G × {1}, the inverse is (g, 1) −1 = (g −1 , 1), which is in G × {1}. Therefore, G × {1} is a subgroup. To show that G × {1} is normal, let (a, b) ∈ G × H, where a ∈ G and b ∈ H. I must show that (a, b)(G × {1})(a, b) −1 ⊂ G × {1}. We can show one set is a subset of another by showing that an element of the first is an element of the second. An element of (
a, b)(G × {1})(a, b) −1 looks like (

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a, b) (g, 1)(a, b) −1 , where (g, 1) ∈ G × {1}. Now (a, b)(g, 1)(a, b) −1 = (a, b)(g, 1)(

a −1 ,

b −1) = (
aga −1 , b(1)
b −1) = (
aga −1 , 1).
66 NSOU CC-MT-10 NSOU CC-MT-10 67 aga −1 ∈ G, since a,

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g ∈ G. Therefore, (a, b)(g, 1)(a, b) −1 ∈ G × {1}. This proves that (a, b)(G × {1})(a, b) −1 ⊂

G × {1}. Therefore, G × {1} is normal. 5. The cosets of the subgroup ⟨19⟩ in U 20 are ⟨19⟩ = {1, 19} 3 · ⟨19⟩ = {3, 17} 7 · ⟨19⟩ = {7, 13} 9 · ⟨19⟩ = {9, 11} (a) Compute {3, 17} · {9, 11}. (b) Compute {3, 17} −1 . (c) Compute {9, 11} 3 . Solution: (a) Take an element (it doesn't matter which one) from each coset, say 3 ∈ {3, 17} and 11 ∈ {9, 11}. Perform the operation on the elements you chose. In this case, it's multiplication: 3 · 11 = 33 = 13. Find the coset containing the answer: 13 ∈ {7, 13}. Hence, {3, 17} · {9, 11} = {7, 13}. (b) Take an element (it doesn't matter which one) from the coset, say 3 ∈ {3, 17}. Perform the operation on the elements you chose. In this case, it's finding the inverse (use the Extended Euclidean Algorithm, or trial and error): 3 −1 = 7. Find the coset containing the answer: 7 ∈ {7, 13}. Hence, {3, 17} −1 = {7, 13}. (c) Take an element (it doesn't matter which one) from the coset, say 11 ∈ {9, 11}. Perform the operation on the elements you chose. In this case, it's cubing: 11 3 = 1331 = 11. Find the coset containing the answer: 11 ∈ {9, 11}. Hence, {9, 11} 3 = {9, 11}.
68 NSOU CC-MT-10 NSOU CC-MT-10 69 6. Let G be a group of order 24. What are the possible orders for the subgroups of G. Solution: Write 24 as product of distinct primes. Hence, 24 = (3)(2 3). By Theorem 1.2.27, the order of a subgroup of G must divide the order of G. Hence, We need only to find all divisors of 24. By Theorem 1.2.17, number of all divisors of 24 is (1 + 1)(3 + 1) = 8. Hence, possible orders for the subgroups of G are : 1,3,2,4,8,6,12,24. 4.8 Model Questions 1. Let G be a finite group. If a, b ∈ G such that |a| = 5 and |b| = 7, then show that |G| ≥ 35. 2. Suppose that G is a finite group with 60 elements. What are the orders of possible subgroups of G? 3. Prove or disprove: Every subgroup of the integers has finite index. 4. Prove or disprove: Every subgroup of the integers has finite order. 5. List the left and right cosets of the subgroups ⟨8⟩ in ? 18 . 6. List the left and right cosets of the subgroups ⟨3⟩ in U 8 . 7. List the left and right cosets of the subgroups 3? in ?. 8. Describe the left cosets of SL 2 (?) in GL 2 (?). 9. Show that the integers have infinite index in the additive group of rational numbers. 10.

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Let a and b be elements of a group G

and H and K be subgroups of G. If aH = bK, prove that H = K. 11. If H and K are subgroups of G and g belongs to G, show that g(H ? K) = gH ? gK. 12. Let a and b be nonidentity elements of different orders in a group G of order 155. Prove that the only subgroup of G that contains a and b is G itself. 13. Let H be a subgroup of R*, the group of nonzero real numbers under multiplication. If R + ⊆ H ⊆ R*, prove that H = R + or H = R*. 14. Let C* be the group of nonzero complex numbers under multiplication and let H = {a + bi ∈ C* | a 2 + b 2 = 1}. Give a geometric description of the coset (3 + 4i) H. Give a geometric description of the coset (c + di)H. 15. Let G be a group of order 60. What are the possible orders for the subgroups of G?
68 NSOU CC-MT-10 NSOU CC-MT-10 69 16. Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If |K| = 42 and |G| = 420, what are the possible orders of H? 17. Let G be a group with |G| = pq, where p and

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q are prime. Prove that every proper subgroup of G is cyclic. 18.

Recall that, for any integer n greater than 1, φ(n) denotes the number of positive integers less than n and relatively prime to n. Prove that if a is any integer relatively prime to n, then a φ(n) mod n = 1. 19. Compute 5 15 mod 7 and 7 13 mod 11. 20. Use Corollary 2 of Lagrange's Theorem (Theorem 7.1) to prove that the order of U(n) is even when n < 2. 21. Suppose G is a finite group of order n and m is relatively prime to n. If g ∈ G and gm = e, prove that g = e. 22. Suppose

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H and K are subgroups of a group G. If |H| = 12 and |K| = 35,		

find |H ∩ K|. Generalize. 23. For any integer n ≥ 3, prove that D n has a subgroup of order 4 if and only if n is even. 24. Let p be a prime and k a positive integer such that a k mod p = a mod p for all integers a. Prove that p − 1 divides k − 1. 25. Suppose that G is an Abelian group with an odd number of elements. Show that the product of all of the elements of G is the identity. 26. Suppose that G is a group with more than one element and G has no proper, nontrivial subgroups. Prove that |G| is prime. (Do not assume at the outset that G is finite.) 4.9 Solutions of some selected problems 2. Use Lagrange's theorem 5. {0, 8, 16 ,6, 14,4, 7. Z 3 8. R * 14. The coset (3 + 4i)H is the circle with center at the origin and radius |3 + 4i|. 15. Use Lagrange's theorem 16. 42*n where 1 > n > 10. 22. 1
70 NSOU CC-MT-10 NSOU CC-MT-10 71 70 Unit - 5 ? Permutation Groups Structure 5.1 Objectives 5.2 Introduction 5.3 Definition & Notation 5.4 Operations on Permutation 5.5 Cyclic Notation 5.6 Transposition 5.7 The Alternating Groups 5.8 Summary 5.9 Worked Examples 5.10 Model Questions 5.11 Solution of some selected problems 5.1 Objective The followings are discussed here: • Definition of permutation group • Operation on permutation • Cyclic notation of permutation • Transposition • Alternation group 5.2 Introduction Permutation groups are central to the study of geometric symmetries and to Galois theory, the study of finding solutions of polynomial equations. They also provide abundant examples of nonabelian groups. In this chapter, we shall deal with various concepts of permutations. 5.3 Definitions and Notation Let X be a set. Then any bijection on X is called a permutation. We have already seen that the set of all permutation S X forms a group under functional composition. If
70 NSOU CC-MT-10 NSOU CC-MT-10 71 X is finite, then we can assume that X = {1, 2, ..., n}. In this case we write S n instead of S X . The following theorem says that S n is a group. We call this group the symmetric group on n letters. This group has n! numbers of element, i.e., |S n | = n!. 5.3.1 Notation : Suppose X = {1, 2, 3, 4, 5} and consider the permutation σ defined by σ(1) = 3, σ(2) = 2, σ(3) = 5, σ(4) = 1 and σ(5) = 4. This permutation can also be expressed in array notation by writing σ σ σ σ σσ = ? ? ? ? ? ? = ? ? ? ? ? ? 1 2 3 4 5 1 2 3 4 5 1 2 3 45 3 2 5 14 ( ) ( ) ( ) ( ) ( ) . where the top row represent the original elements and the bottom row represents what each element is mapped to. Note that some texts use square brackets. This is one of the notations of a permutation. Below, we will see there is another way to represent permutations. Let us look at some specific examples. Example 5.3.2 : Let A = {1, 2, 3, 4}. And suppose that σ(1) = 3, σ(2) = 1, σ(3) = 4, σ(4) = 2 ans then we would write σ = ? ? ? ? ? ? 1 2 3 4 3 14 2 . and to indicate the action of α on an element, say 2, we would write 1 1 2 2 3 3 4 4 Fig. 5.1 : Visualization of σ σ( ) ( ) 2 1 2 3 4 3 14 2 2 1 = ? ? ? ? ? ? = . Example 5.3.3 : Any symmetry of an equilateral triangle is also a permutation. Let ∆ABC be an equilateral triangle whose vertices are marked as A,B,C counterclockwise. Then each symmetry represents a permutation on the set {A, B, C}, see Figure 5.2:
72 NSOU CC-MT-10 NSOU CC-MT-10 73 Group of Permutation of {A, B, C} Group of Symmetries of an Equilateral Triangle Interpretation p A BC A BC A BC 1 = ? ? ? ? ? ? ( )( )( )

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A B C Do nothing p A BC B C A ABC 2 = ? ? ? ? ? ? ( ) A B C Counterclockwise rotation of 120° p A BC C AB ACB 3 = ? ? ? ? ? ? ( ) A B C Counterclockwise rotation of 240° p A BC A C B A BC 4 = ? ? ? ? ? ? ( )( ) A B C Flip through vertex A p A BC C BA AC B 5 = ? ? ? ? ? ? ( )( ) A B C			

Flip through vertex B p A BC B AC AB C 6 = ? ? ? ? ? ? ( )( ) A B C Flip through vertex C Fig. 5.2 : Symmetries of an equilateral triangle

72 NSOU CC-MT-10 NSOU CC-MT-10 73 Example 5.3.4 : The identity permutation on A = {1, 2, 3, ..., n} is σ = ? ? ? ? ? ? 1 23 4 1 23 4 ? ? n n , in other words, it does not change anything. 5.4 Operation on Permutation Above we said that Sn was a group under composition. Let us look in more detail at composition of permutations. Composition of permutations written in array notation is performed from right to left, that is the permutation on the right is performed first. Let A = {1, 2, ..., n} and σ, β ∈ S n . Then the composition σβ is the functional composition. This composition

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can be written in cyclic notation as σ σ σ

σ σβ β β β = ? ? ? ? ? ? ? ? ? ? ? ? 1 23 1 23 1 23 1 23 ? ? ? ? n n n n ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) = ? ? ? ? ? ? 1 23 1 23 ? ? n n βσ βσ βσ βσ ( ) ( ) ( ) ( ) . Example 5.4.1 : Let A = {1, 2, 3, 4} and σ, β ∈ S 4 defined by σ = ? ? ? ? ? ? 1 2 3 4 3 14 2 and β = ? ? ? ? ? ? 1 2 3 4 2 14 3 Then σβ = ? ? ? ? ? ? 1 2 3 4 4 23 1 And βσ = ? ? ? ? ? ? 1 2 3 4 1 3 2 4 Example 5.4.2 : Consider two permutations P = ? ? ? ? ? ? 1 2 3 4 2 3 41 and Q = ? ? ? ? ? ? 1 2 3 4 2 14 3 Then PQ = ? ? ? ? ? ? 1 2 3 4 1 4 3 2 .
74 NSOU CC-MT-10 NSOU CC-MT-10 75 5.4.3 Inverse of Permutations : If a permutation σ maps n i to n j , then the inverse permutation σ –1 maps n j back to n i . In other words, the inverse of a permutation can be found by simply interchanging the top and bottom rows of the permutation σ and (for convenience in reading) reordering the top row in numerical order 1, 2, ..., n. For example, σ σ = ? ? ? ? ? ? ⇒ ? ? ? ? ? ? - 1 2 3 45 3 52 41 1 2 3 45 5 31 42 1 . Here, σ(1) = 5 so σ –1 (5) = 1. 5.5 Cyclic Notation The notation that we have used to represent permutations up to this point is cumbersome, to say the least. To work effectively with permutation groups, we need a more streamlined method of writing down and manipulating permutations. The cycle notation was introduced by the French mathematician Cauchy in 1815. The notation has the advantage that many properties of permutations can be seen from a glance. We now present this notation. Definition 5.5.1 : Let A = {1, 2, ..., n}.

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A permutation σ ∈ S n is a cycle of length k if there exists elements a 1 , a 2 , ..., a k ∈ A such that σ(a 1) = a 2 σ(a 2) = a 3 . . . σ(a k) = a 1 , and σ(x) = x

for all other elements x ∈ A. We write them as (a 1 , a 2 , ..., a k). Example 5.5.2 : Let A = {1, 2, 3, 4, 5} and σ ∈ S 5 defined by σ = ? ? ? ? ? ? 1 2 3 45 3 2 5 14 . Then this permutation can be expressed in cyclic notation as (1, 3, 5, 4). Observe that there are also some other cyclic notations of this permutation as: (1, 3, 5, 4) = (3, 5, 4, 1) = (5, 4, 3, 1) = (4, 1, 3, 5). Bur we usually prefer the notation in ascending order.
74 NSOU CC-MT-10 NSOU CC-MT-10 75 Definition 5.5.3 When two cycles have no elements in common, they are said to be disjoint. Example 5.5.4 The permutation σ = ? ? ? ? ? ? 1 2 34 5 6 2 14 6 5 3 , can be represented by (1, 2)(3, 4, 6)(5) and (1, 2)(3, 4, 6) if we omit the 1-cycle. Note. If you wanted to dial the telephone number 413–2567 but accidentally dialed 314 – 5267, then you permuted the digits according to (2, 5)(3, 4). Theorem 5.5.5 Let σ be any elements of S n . Then σ may be expressed as a product of disjoint cycles. This factorisation is unique. ignoring 1-cycles, up to order. Teh cycle type of σ is the lengths of the corresponding cycles. Proof. We first prove the existence of such a decomposition. Let a 1 = 1 and define a k recursively by the formula a i+1 = σ(a i). Consider the set {a i | i ∈ ?}. As there are only finitely many integers between 1 and n, we must have some repetitions, so that a i = a j , for some i > j. Pick the smallest i and j for which this happens. Suppose that i ≠ 1. Then σ(a i–1) = a i = σ(a j–1). As σ is injective, a i–1 = a j–1 . But this contradicts our choice of i and j. Let τ be the k-cycle (a 1 , a 2 , . . . , a j). Then ρ = στ –1 fixes each element of the set {a i | i ≤ j}. Thus by an obvious induction, we may assume that ρ is a product of k – 1 disjoint cycles τ 1 , τ 2 , . . . , τ k–1 which fix this set. But then σ = ρτ = τ 1 τ 2 . . . τ k , where τ = τ k . Now we prove uniqueness. Suppose that σ = σ 1 σ 2 . . . σ k and σ = τ 1 τ 2 . . . τ l are two factorisations of σ into disjoint cycles. Suppose that σ 1 (i) = j. Then for some p, τ p (i) ≠ i. By disjointness, in fact τ p (i) = j. Now consider σ 1 (j). By the same reasoning, τ p (j) = σ 1 (j). Continuing in this way, we get σ 1 = τ p . But then just cancel these terms from both sides and continue by induction.

76 NSOU CC-MT-10 NSOU CC-MT-10 77 Example 5.5.6 : Let σ = ? ? ? ? ? ? 1 23 45 3 41 52 . Look at 1. 1 is sent to 3. But 3 is sent back to 1. Thus part of the cycle decomposition is given by the transposition (1, 3). Now look at what is left {2, 4, 5}. Look at 2. Then 2 is sent to 4. Now 4 is sent to 5. Finally 5 is sent to 2. So another part of the cycle type is given by the 3-cycle (2, 4, 5). It is claimed then that σ = (1, 3)(2, 4, 5) = (2, 4, 5)(1, 3). This is easy to check. The cycle type is (2, 3). Lemma 5.5.7 : Let σ ∈ S n be a permutation, with cycle type (k 1 , k 2 , ... k l). The order of σ is the least common multiple of k 1 , k 2 , ..., k l . Proof. Let k be the order of σ and let σ = τ 1 τ 2 . . . τ l be the decomposition of σ into disjoint cycles of lengths k 1 , k 2 , . . . , k l . Pick any integer h. As τ 1 , τ 2 , . . . , τ l are disjoint, it follows that σ ττ τ h hh l h = 1 2 ? . Moreover the RHS is equal to the identity, iff each individual term is equal to the identity. It follows that τ i k e= . In particular k i divides k. Thus the least common multiple, m of k 1 , k 2 , . . . , k l divides k. But σ τ τ τ τ m m m m l m e = = 1 23 ? . Thus m divides k and so k = m. 5.6 Transpositions A 2-

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cycle is called a transposition. Since (a 1 , a 2 , . . . , a n) = (a 1 a n)(a 1 a n–1) ... (a 1 a 3)(a 1 a 2), any cycle can be written as the product of transpositions,

leading to the following proposition. Proposition 5.6.1 : Any permutation of a finite set containing at least two elements can be written as the product of transpositions.
76 NSOU CC-MT-10 NSOU CC-MT-10 77
Definition 5.6.2 :

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A permutation is said to be even if it can be expressed as the product of an

even number
of transpositions,
and

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odd if it can be expressed as the product of an odd number of transpositions. 5.7 The

Alternating Groups One of the most important subgroups of S

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n is the set of all even permutations, A n . The group A n is called the alternating group

on n letters. Theorem 5.7.1 : The set A n is a subgroup of S n . Proof. Since the product of two even permutations must also be an even permutation, A n is closed. The identity is an even permutation and therefore is in A n . If σ is an even permutation, then σ = σ 1 σ 2 ... σ r , where σ i is a transposition and r is even. Since the inverse of any transposition is itself, σ –1 = σ r σ r–1 ... σ 1 is also in A n . Proposition 5.7.2 : The number of even permutations in S n , n ≥ 2, is equal to the number of odd permutations; hence, the

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order of A n is n!/2. Proof. Let A n be the set of even permutations in S n and B n be the set of odd permutations. If we can show that there is a bijection between these sets, they must contain the same number of elements. Fix a transposition σ in S n . Since n ≥ 2, such a σ exists. Define λ σ : A n → B n by λ σ (τ) = στ. Suppose that λ σ (τ) = λ σ (μ). Then στ = σμ and so τ = σ −1 στ = σ −1 σμ = μ. Therefore, λ σ is one-to-one. We will leave the proof that λ σ is surjective to the reader. Example 5.7.3 : The group A 4 is the subgroup of S 4 consisting of even permutations. There are twelve elements in A 4 : (1) (12)(34) (13)(24) (14)(23) (123) (132) (124) (142) (134) (143) (234) (243).

78 NSOU CC-MT-10 NSOU CC-MT-10 79 5.8 Summary In this unit, we have studied various concept of permutation group. We have showed that a permutation can be expressed as the product of transpositions. The concept of alternating group is also discussed in this unit. 5.9 Worked Examples 1. Find the orbit and cycles of the following permutations: (a) 1 23 4 5 6 78 9 2 3 4 51 6 7 98 ? ? ? ? ? ? (b) 1 23 4 5 6 6 54 31 2 ? ? ? ? ? ? Solution: (a) Clearly 1 2 34 5 6 7 89 2 3 45 16 7 98 ? ? ? ? ? ? = (1, 2, 3, 4, 5)(6)(7)(8, 9). So orbit of 1, 2, 3, 4 and 5 is the set {1, 2, 3, 4, 5}; orbit of 6 is 6; orbit of 7 is 7; orbit of 8 and 9 is the set {8, 9}. Also (1, 2, 3, 4, 5) and (8, 9) are its cycles. (b) Again 1 2 3 45 6 6 54 31 2 ? ? ? ? ? ? = (1, 6, 2, 5)(3, 4). So the orbit of 1, 2, 5 and 6 is the set {1, 2, 5, 6}; and the orbit of 3 and 4 is the set {3, 4}. Also (1, 6, 2, 5) and (3, 4) are its cycles. 2. Write the permutation in the worked example 1 as the product of disjoint cycles. Solution: We have 1 2 34 5 6 7 89 2 3 45 16 7 98 ? ? ? ? ? ? = (1, 2, 3, 4, 5)(6)(7)(8, 9) and 1 2 3 45 6 6 54 31 2 ? ? ? ? ? ? = (1, 6, 2, 5)(3, 4). 3. Express as the product of disjoint cycles: (a) (1, 5)(1, 6, 7, 8, 9)(4, 5)(1, 2, 3). (b) (1, 2)(1, 2, 3)(1, 2). Solution: (a) Let (1, 5)(1, 6, 7, 8, 9)(4, 5)(1, 2, 3) = τ . So we have τ = τ 1 τ 2 τ 3 τ 4 , where

78 NSOU CC-MT-10 NSOU CC-MT-10 79 τ 1 = (1, 5), τ 2 = (1, 6, 7, 8, 9), τ 3 = (4, 5) and τ 4 = (1, 2, 3). Now τ(1) = τ 1 τ 2 τ 3 τ 4 (1) = τ 1 (τ 2 (τ 3 (τ 4 (1)))) = τ 1 (τ 2 (τ 3 (2))) = τ 1 (τ 2 (2)) = τ 1 (2) = 2 Repeating analogously, we have τ(2) = 3; τ(3) = 6; τ(6) = 7; τ(7) = 8; τ(8) = 9; τ(9) = 5; τ(5) = 4; and τ(4) = 1. Thus we have τ = ? ? ? ? ? ? 1 2 3 4 5 6 78 9 2 3 6 14 78 95 = (1, 2, 3, 6, 7, 8, 9, 5, 4). (b) Proceeding as in part (a), we have (1, 2)(1, 2, 3)(1, 2) = (1, 3, 2). 4. Prove that (1, 2, . . . , n) −1 = (n, n − 1, n − 2, . . . , 2, 1). Solution: One can easily check (1, 2, . . . , n)(n, n−1, . . . , 1) = I, where I is the identity permutation. Hence (1, 2, . . . , n) −1 = (n, n − 1, . . . , 1). 5. Show that A 3 , the set of even permutations of {1,2,3} is a cyclic group with respect to the product of permutations. Find a generator of this cyclic group. Answer with reason. Solution: The set of even permutations of {1,2,3} is A 3 = ρ 0 , ρ 1 , ρ 2 where ρ 0 1 23 1 23 = ? ? ? ? ? ?, ρ 1 1 23 2 31 = ? ? ? ? ? ?, ρ 2 1 23 3 12 = ? ? ? ? ? ? . Find the composition table and prove that the set A 3 , the set of even permutations of {1,2,3} is a commutative group with respect to the product of permutations. The order of this group is 3 and since 3 is a prime number, so A 3 is a cyclic group. Since o(ρ 1) = 3 and o(A 3) = 3, so ρ 1 is a generator of this group. 6. Let a = 1 23 4 3 12 4 ? ? ? ? ? ? . Find the smallest positive integer k such that a k = e in S 4 . Solution: S 4 is the symmetric group with respect to the multiplication of permutations of the set {1,2,3,4} and e be the identity element in S 4 . Now, a = ? ? ? ? ? ? 1 2 3 4 3 12 4 = (1 3 2) which is a cycle of length 3. So o(a) = 3. Therefore, 3 is the least positive integer such that a 3 = e in S 4 .

80 NSOU CC-MT-10 NSOU CC-MT-10 81 7. Prove that α = (3, 6, 7, 9, 12, 14) ∈ S 16 is not a prod-uct of 3-cycles. Solution: Since α = (3, 14)(3, 12)...(3, 6) is a product of five 2-cycles, α is an odd cycle. Since each 3-cycle is an even cycle by the previous problem, a permutation that is a product of 3-cycles must be an even permutation. Thus, α is never a product of 3-cycles. 5.10 Model Questions 1. Write the following permutations in cycle notation. (a) 1 23 4 5 2 41 53 ? ? ? ? ? ? (c) 1 23 45 3 5 14 2 ? ? ? ? ? ? (b) 1 2 3 45 4 25 13 ? ? ? ? ? ? (d) 1 23 4 5 1 43 2 5 ? ? ? ? ? ? 2. Compute each of the following. (a) (1345)(234) (i) (123)(45)(1254) −2 (b) (12)(1253) (j) (1254) 100 (c) (143)(23)(24) (k) |(1254)| (d) (1423)(34)(56)(1324) (l) |(1254) 2 | (e) (1254)(13)(25) (m) (12) −1 (f) (1254)(13)(25) 2 (n) (12537) −1 (g) (1254) −1 (123)(45)(1254) (o) [(12)(34)(12)(47)] −1 (h) (1254) 2 (123)(45) (p) [(1235)(467)] −1 3. Express the following permutations as products of transpositions and identify them as even or odd. (a) (14356) (d) (17254)(1423)(154632) (b) (156)(234) (c) (1426)(142) (e) (142637) 4. Find (a 1 , a 2 , . . . , a n) −1 . 5. List all of the subgroups of S 4 . Find each of the following sets. (a) {σ ∈ S 4 : σ(1) = 3} (b) {σ ∈ S 4 : σ(2) = 2} (c) {σ ∈ S 4 : σ(1) = 3 and σ(2) = 2} Are any of these sets subgroups of S 4 ?

80 NSOU CC-MT-10 NSOU CC-MT-10 81 6. Find all of the subgroups in A 4 . What is the order of each subgroup? 7.

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Find all possible orders of elements in S 7 and A 7 . 8.		

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Show that A10 contains an element of order 15. 9. Does A 8 contain an element of order 26? 10. Find an element of largest order in S n for n = 3, . . . , 10. 11. Let σ ∈ S n . Prove that σ can be written as the product of at most n − 1 transpositions. 12. Let σ ∈ S n . If σ is not a cycle, prove that σ can be written as the product of at most n − 2 transpositions. 13. If σ can be expressed as an odd number of transpositions, show that any other product of transpositions equaling σ must also be odd. 14. If σ is a cycle of odd length, prove that σ 2 is also a cycle. 15. Show that a 3-cycle is an even permutation. 16. Prove that in A n with n ≥ 3, any permutation is a product of cycles of length 3. 17. Prove that any element in S n

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can be written as a finite product of the following permutations. (

a) (12), (13), . . . , (1n) (b) (12), (23), . . . , (n − 1, n) (c) (12), (12 . . . n) 5.11 Solution of some selected problems 1. (a) (1 2 4 5 3) (b) (1 4)(3 5) (c) (1 3)(2 5) (d) (2 4) 2. (a) (1 4) (3 2) 3. (a) (1 6)(1 5)(1 3)(1 4) 4. (a n , a n−1 , ... , a 2 , a 1)
82 NSOU CC-MT-10 NSOU CC-MT-10 83 82 Unit - 6 ? Quotient Groups and Group Homomor- phism Structure 6.1 Objectives 6.2 Introduction 6.3 Quotient group 6.4 Group Homomorphism 6.5 Automonphism 6.6 Summary 6.7 Worked Examples 6.8 Model Questions 6.9 Solution of some selected problems 6.1 Objective The followings are discussed here: • Definition of quotient group • Definition of group homomorphism, isomorphism and automorphism • Properties of homomorphism • Kernel of a homomorphism • First, second and third isomorphism theorem • Inner automorphism 6.2 Introduction We have yet to explain why normal subgroups are of special significance. The reason is simple. When the subgroup H of G is normal,

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then the set of left (or right) cosets of H in G is itself a group—called the factor group of G by H (or the quotient group of G by

H). Quite often, one can obtain information about a group by studying one of its factor groups. One of the important concept of group theory is the concept of homomorphism. Homomorphism is the natural group theoretic mapping between two groups preserving the binary compositions. The study of homomorphism reveals various properties of a group.
82 NSOU CC-MT-10 NSOU CC-MT-10 83 6.3 Quotient group
Theorem 6.3.1 :

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Let G be a group and H be normal subgroup of G. Then the set G/H = {gH : g ∈ G} is a group under the operation

g 1 H ∗ g 2 H =
g 1 g 2
H of order [G : H]. Proof. This operation must be shown to be well-defined; that is, group multiplication must be independent of the choice of coset representative. Let aH = bH and cH = dH. We must show that aH ∗ cH = acH = bH ∗ dH = dbH. Now a = bh 1 and c = dh 2 for some h 1 , h 2 ∈ H. Then, acH = bh 1 dh 2 H = bh 1 dH = bh 1 Hd = bHd = bdH. Hence, the binary operation is well defined. Now the element eH acts as the identity element, since aH ∗ eH = eH ∗ aH = aH for all a ∈ G. Associativity property holds automatically as G is a group. Now for any element aH ∈ G/H, the inverse element is a −1 H, since aH ∗ a −1 H = a −1 H ∗ aH = eH. Hence, G/H forms
a group. Since

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the number of cosets of H in G is [G : H], therefore the order of the group G/H is [G :

H]. Definition 6.3.2 : For a normal subgroup H of a group
G, the set

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G/H = {gH : g ∈ G} with the binary operation g 1 H ∗ g 2 H = g 1 g 2 H

is called Quotient group or Factor group. Although the concept of quotient group is now considered to be fundamental to the study of groups, it is a concept which was unknown to early group theorists.It emerged relatively late in the history of the subject: toward the end of the 19th century. The main reason for this delay is that in order to give a recognizably modern definition of a quotient group, it is necessary to think of groups in an abstract way. Therefore the development of the concept of quotient group is closely linked with the abstraction of group theory. This process of abstraction took place mainly during the period 1870-1890 and was carried out almost exclusively by German mathematicians. Thus by 1890 the development and understanding of the concept of quotient group had largely been completed. Example 6.3.3 : Consider the normal subgroup 3? of ?. Then the cosets of 3? are 0 + 3?, 1 + 3? and 2 + 3?. The group ?/3? is given by the multiplication table below Since |?/3?| = 3 so ?/3? is isomorphic to ? 3 .
84 NSOU CC-MT-10 NSOU CC-MT-10 85 + 0 + 3? 1 + 3? 2 + 3? 0 + 3? 0 + 3? 1 + 3? 2 + 3? 1 + 3? 1 + 3? 2 + 3? 0 + 3? 2 + 3? 2 + 3? 0 + 3? 1 + 3? Fig. 6.1 Theorem 6.3.4 : The

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quotient group of a cyclic group is cyclic. Proof. Let

H be a subgroup of G and G = ⟨a⟩. Then we will show that aH is a generator of G/H. Let gH ∈ G/H. Then g = a k for some integer k. Now (aH) k = aH ∗ aH ∗ ... ∗ aH (k times) = a k H = gH. Hence, G/H is a cyclic group generated by aH. 6.4 Group Homomorphism Definition 6.4.1 (Homomorphisms). A mapping ϕ from a group (G, o) to a group (H, ∗) is called a homomorphsim if it preserves the group operation, i.e., ϕ(a o
b) = ϕ(a) ∗ ϕ(b) for all a, b ∈

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G. Definition 6.4.2 : If ϕ is a homomorphism of G into H, the kernel of ϕ, Ker ϕ , is defined by Ker ϕ = {x ∈ G : ϕ(x) = e′, e′ = identity element of H}. Proposition 6.4.3 : Let G and H be groups and let ϕ : G →

H be a homomorphism. (
i) ϕ(

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e) = e′, where e and e′ are the identities of G and H, respectively. (ii) ϕ(g −1) = ϕ(g) −1

for all g ∈
G. (iii) ϕ(g
n) = ϕ(g) n for all g ∈ G. G H
N= Ker (∅) ∅ ∅ (a) aN a e e′ Fig. 6.2 : Homomorphism ϕ : G → H

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84 NSOU CC-MT-10 NSOU CC-MT-10 85 Proof. (i) Since ϕ(e) = ϕ(e o e) = ϕ(e) * ϕ(e), the cancellation laws shows that ϕ(e) = e′. (ii) ϕ(e) = ϕ(gg −1) = ϕ(g)ϕ(g −1) and, by part (i), ϕ(e) = e′, we get e′ = ϕ(g)ϕ(g −1). Now multiplying both sides on the left by ϕ(g) −1 , we get the result. (iii) This can be easily deduced by using induction and (i) and (ii). Proposition 6.4.4 : Let ϕ be a homomorphism from (G, o) to (H, ·). Then (i) kernel of

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ϕ, ker ϕ , is a normal subgroup of G, (ii) image of ϕ, Im ϕ , is a subgroup of H. Proof. (		

i) Since ϕ(e) = e′, so ker ϕ is non-empty. Let a, b ∈ ker
ϕ . Then ϕ(

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a ? b −1) = ϕ(a) · ϕ(b −1) = ϕ(a) · ϕ(b) −1 = e′ · e′ = e′. Therefore, a ? b −1 ∈ Ker ϕ . Hence, ker ϕ			

is a subgroup of G.
Now to prove ker ϕ is normal,
take x ∈ G. Then, for any q ∈ ker ϕ , ϕ(x ? q ? x −1) = ϕ(x) · ϕ(q) · ϕ(x −1) = ϕ(x) · e′ · ϕ(x) −1 = e′.
Hence, xker ϕ x −1 ⊆ ker ϕ for all x ∈ G. Therefore, ker ϕ is a normal subgroup of G. (ii) Since ϕ(e) = e′, the identity of H lies in Im ϕ , so Im ϕ is nonempty. Let x, y ∈ Im ϕ . Then

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there exists a, b ∈ G such that ϕ(a) = x and ϕ(b) =		

y. Now by using homomorphim and proposition 6.5, we get x · y −1 =

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ϕ(a) · ϕ(b) −1 = ϕ(a) · ϕ(b −1) = ϕ(a · b −1). Therefore,			

x · y −1 ∈ Im ϕ . So, Im ϕ forms a subgroup of H.

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Theorem 6.4.5 : A homomorphism ϕ : G → H is injective if and only if Ker ϕ = {e}. Proof. Suppose ϕ is injective, and let		

a ∈ Ker ϕ . Then ϕ(e) = e′ = ϕ(x). Hence, x = e. Therefore, ker ϕ = {e}. Conversely, suppose ker ϕ = {e} and x, y ∈ G such that

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ϕ(x) = ϕ(y). Then ϕ(x ? y −1) = ϕ(x) · ϕ(y) −1 = e′.			

Therefore, x ? y −1 ∈ ker ϕ . But ker ϕ = {e}. Hence x ? y −1 = e, i.e., x = y. Definition 6.4.6 (Isomorphism). A homomorphism ϕ from a group G to a group H is called isomorphism if ϕ is one-to-one and onto map. If there is an isomorphism from a group G to a group
H,

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we say that G and H are isomorphic and write G ≈ H. 86

NSOU CC-MT-10 NSOU CC-MT-10 87 Philosophical considerations give isomorphism a particular importance. Abstract algebra studies groups but does not care what their elements look like. Accordingly, isomorphic groups are regarded as instances of the same “abstract” group. For example, the dihedral groups of various triangles are all isomorphic, and are regarded as instances of the “abstract” dihedral group D 3 . Example 6.4.7 : Let G be the real numbers under addition and let H be the positive real numbers under multiplication. Then G and H are isomorphic under the mapping ϕ(x) = 2 x . To prove that this map is onto-to-one, suppose 2x = 2y. Which implies that log e 2 x = log e 2 y , and therefore x = y. For “onto,” we must find for any positive real number y some real number x such that ϕ(x) = y, that is, 2x = y. Now, solving for x gives log 2 y. Again,

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ϕ(x + y) = 2x + y = 2x · 2y = ϕ(x) · ϕ(y) ∀x, y ∈ G. Therefore, G is

isomorphic to H. Example 6.4.8 : Any infinite cyclic group is isomorphic to ?. Indeed, if a is a generator of the cyclic group, the mapping a k → k is an isomorphism. Similarly, any finite cyclic group ⟨a⟩ of order n is isomorphic to ? n and the isomorphism is defined by a k → k mod n. Example 6.4.9 : The groups U(5) and U(10) are isomorphic, since both of them are cyclic groups of order 4. Example 6.4.10 : U(10) and U(12) are not isomorphic, although they have same number of elements. First observe that, x 2 = 1 for all x ∈ U(12). Now, suppose that ϕ : U(10) → U(12) is an isomorphism. Then, ϕ(9) = ϕ(3) · ϕ(3) = 1 and ϕ(1) = 1. Thus, ϕ(9) = ϕ(1), but 9 ≠ 1. which contradicts the assumption that ϕ is one-to-one. Example 6.4.11 : The quotient group (?/?, +) = {r + ? : r ∈ [0, 1)} is isomorphic to the circle group S of complex numbers of absolute value 1. The isomorphism is given by r + ? → e i2πr . Example 6.4.12 : There is no isomorphism from ?, the group of rational number under addition, to ? * , the group of nonzero rational numbers under multiplication. Suppose there is an isomorphism ϕ. Then there exists a rational number a such that ϕ(a) = –1. But then,
86 NSOU CC-MT-10 NSOU CC-MT-10 87 - = = + ( ) = ( ) ( ) = ( ) ? ? ? ? ? ? 1 1 2 1 2 1 2 1 2 1 2 2 ϕ ϕ ϕ ϕ ϕ ( ) · a aa a a a . However, no rational number squared is –1. Theorem 6.4.13 (Properties of Isomorphism). Suppose ϕ is an isomorphism from a group G to a group H. Then 1. For any elements a and b in G, a and b commute if and only if ϕ(a) and ϕ(b) commute. 2. G = ⟨a⟩ if and only if H = ⟨ϕ(a)⟩. 3. |a| = |ϕ(a)| for all a ∈ G, i.e., isomorphism preserves order. 4. For a fixed integer k and a fixed group element b in G, the equation x k = b has the same number of solutions in G as does the equation x k = ϕ(b) in H. 5. If G is finite, then G and H has same number of elements of every order. Proof. Property 1 can be easily proved by using the property of isomorphism. Let G = ⟨a⟩. Take q ∈ H, then p = ϕ –1 (q) ∈ G. Hence, p = a k for some k < 0. Now, q = ϕ(p) = ϕ(a k) = ϕ(a) k . Hence, the second statement follows. Third statement follows directly from the second one. Forth statement follows from oder preserving property of isomorphism. From third one, the fifth statement follows. Theorem 6.4.14 :

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Let H be a normal subgroup of G. Then the

mapping f : G → G/H defined by f(x) = xH for x ∈ G is an onto homomorphism with kernel H. Proof. Let us take two elements x, y ∈ G. Then f(x) = xH and f(y) = yH. Now f(xy) = xyH = (xH) ∗ (yH) = f(x)f(y), which shows that f is a homomorphism. Now the identity element of G/H is H. Hence, ker f = {x ∈ G : f(x) = H} = {x ∈ G : xH = H}. Therefore, from the property of cosets, Ker f = H. Theorem 6.4.15 (First Isomorphism Theorem). Let ϕ : G → G′ be an onto homomorphism. Then G/Ker ϕ is isomorphic to G′, i.e., G/ker ϕ ? G′. Proof. Since H = Ker ϕ , H is normal subgroup of G. Let us define a mapping f : G/H → G′ by f(aH) = ϕ(a), aH ∈ G/H.

88 NSOU CC-MT-10 NSOU CC-MT-10 89 First we show that f is well defined in the sense that if aH = bH, then f(aH) = f (bH). Now aH = bH ⇒ a –1 b ∈ H ⇒ ϕ(a –1 b) = e′ Since H = Kar ϕ ⇒ ϕ(a –1)ϕ(b) = e′ ⇒ ϕ(a) = ϕ(b) ⇒ f(aH) = f(bH), where e′ is the identity of G′. So f is well defined. G f p φ G G / H Fig. 6.3 : First Isomorphism Theorem Again for aH, bH ∈ G/H, we get f(aH ∗ bH) = f(abH) = ϕ(ab) = ϕ(a)ϕ(b) = f(aH)f(bH). Which shows that f is homomorphism. Let aH ∈ Ker f . Then f(aH) = ϕ(a) = e′. Which shows that a ∈ Ker ϕ = H. Hence, aH = H. Thus, Ker f only the identity element. So, f

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is one-one. Finally, f is onto, because each element of G′ is of the form ϕ(a) for some a ∈ G. And		

since ϕ(a) = f(aH), the pre- image of ϕ(a) is aH in G/H. Thus f is an isomorphism from G/H to G′. Example 6.4.16 : Let ϕ : GL n (?) → ? – {0} = ? * defined by ϕ(A) = det(A). Then ϕ is a homomorphism with kernel SL n (?). Therefore, by First isomoprhism theorem GL n (?)/SL n (?) ? ? * .
88 NSOU CC-MT-10 NSOU CC-MT-10 89 Example 6.4.17 : Those who learn some complex analysis, might know the Möbius transformation on the complex plane ?. The Möbius transformation looks like A z az b cz d ( )= + + (6.1) where ad – bc ≠ 0. Let M be the set of all Möbius transformation on ?. Then M forms a group under the functional composition. Now consider the function ϕ : GL 2 (?) → M defined by ϕ a b c d A ? ? ? ? ? ? ? ? ? ? ? ? = where A is the Möbius transformation defined in (6.1). Since composition of two Möbius transformations is same as product of their respective matrices, the function ϕ is a homomorphism. Also ϕ is onto. What is the kernel of ϕ? Or said differently, for what values of a, b, c, d, the matrix a b c d ? ? ? ? ? ? gives the identity operator? It it only possible when c = b = 0 and a = d = λ for λ ∈ ?∗. Hence, the kernel is ker ϕ = {λI : λ ∈ ? * }, where I is the 2×2 identity matrix. Now by First Isomorphism theorem, we get GL 2 (?)/Ker ϕ ? M. The group GL 2 (?)/Ker ϕ is called Projective General Linear group and is denoted by PGL 2 (?). We have seen that the symmetric group S n of all the permutations of n objects has order n!, and that the dihedral group D 3 of symmetries of the equilateral triangle is isomorphic to S 3 , while the cyclic group C 2 is isomorphic to S 2 . We now wonder whether there are more connections between finite groups and the group S n . There is in fact a very powerful one, known as

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Cayley’s Theorem. Theorem 6.4.18 (Cayley’s Theorem). Any group G is isomorphic to a subgroup of		

Sym(G), where Sym(G) is the group of all bijections of G. Proof. The proof has been omitted. Theorem 6.4.19 (Second isomorphism Theorem). Let H be
a subgroup of G (not necessarily normal in
G) and

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N a normal subgroup of G. Then HN is a subgroup of G, H ∩ N is a normal subgroup of		

H, and
H H
N HN N ∩ ≅ .
90 NSOU CC-MT-10 NSOU CC-MT-10 91 Theorem 6.4.20 (Correspondence Theorem).

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Let N be a normal subgroup of a group G. Then H → N/			

N

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is one-to-one correspondence

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between the set of subgroups H containing N and the set of subgroups of G/N. Furthermore, the normal subgroups of

H correspond to normal subgroups of G/

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N. Theorem 6.4.21 (Third Isomorphism Theorem). Let G be a group and N and H be normal subgroups of G

with N ⊂ H. Then G H G N H
N ≅ / / . 6.5
Automorphism Definition 6.5.1 : An endomorphism of a group G, denoted by End(G), is a homomorphism of G into G; an automorphism of a group G, denoted by Aut(G), is an isomorphism of G onto itself. Fig. 6.4 : Automorphism of G Example 6.5.2 : Let G be a group. The identity mapping on G is an automorphism of G. This is called the identity automorphism and denoted by I G . Example 6.5.3 :

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Let G be an abelian group and the mapping f : G → G

defined by f(a) = a −1 , a ∈ G. Then f is an automorphism. Example 6.5.4 : Let G = (?, +) and the mapping f : G → G defined by f z z ( )= , z ∈ G. Then f is an automorphism.
90 NSOU CC-MT-10 NSOU CC-MT-10 91 Proposition 6.5.5 : The Aut(G) forms a group under composition. Proof. Since identity function id G ∈ Aut(G), so Aut(G) ≠ φ. Let f, g ∈ Aut(G). Then it can conclude that f ? g is also a homomorphism. Also we know that composition of two bijective functions is also bijective. Therefore, f ? g is also an isomorphism. So, f ? g ∈ Aut(G). The function composition automatically satisfies associativity property. The identity function I G is the identity element. Let f ∈ Aut(G). Then the inverse function f −1 of f is the inverse element of Aut(G). Hence Aut(G) forms a group under composition. However, the class of abelian group is a little limited, and we should like to have some automorphism of non-abelian groups. Strangely enough the task of finding automorphism of non-abelian groups is easier than for abelian groups.

60%	MATCHING BLOCK 115/128	W

Let G be a group and g ∈ G. Then consider the mapping I g : G → G defined by I g (x) = gxg −1 ,

x ∈ G. Theorem 6.5.6 : The mapping I g is an automorphism for each g ∈ G. Proof.
I g is injective, because I g (x 1) = I g (x 2) ⇒ gx 1 g −1 = gx 2 g −1 ⇒ x 1 = x 2 . I g is
onto, because an arbitrary element y in G has a pre-image of g −1 yg in G. Therefore, I

52%	MATCHING BLOCK 117/128	SA	Algebra -I (Block I, II, III, IV).pdf (D144184274)

g is an bijection. Let x, y ∈ G. Then I g (xy) = g(xy)g −1 = (gxg −1)(gyg −1) = I g (x)I g (y). Hence, I g is a homomorphism.

Thus I g is an automorphism. Definition 6.5.7 : The automorphism I g defined by I g (x) = gxg −1 , x ∈ G is said to be the inner automorphism of G determined by g. x y z G G gyg −1 gzg −1 gzg −1 Fig. 6.5 : Inner automorphism I g

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92 NSOU CC-MT-10 NSOU CC-MT-10 93 The set of all inner automorphism of a group G is denoted by Inn(G). If G is abelian, then each mapping I g for all g ∈ G is simply the identity mapping. But if G is non-abelian, then there must be al least two distinct elements g, x ∈ G, such that gx ≠ xg. Hence, the mapping I g is non-trivial. Thus, the automorphism of non-abelian group is more interesting than that of abelian group. Theorem 6.5.8 : The inner automorphism

100%	MATCHING BLOCK 118/128	W
Inn(G) is a normal subgroup of Aut(G). Proof.		

Since I e is contained in Inn(G), Inn(G) ≠ φ. Take Ig 1 , Ig 2 ∈ Inn(G). Then (Ig 1 ? Ig 2)(

68%	MATCHING BLOCK 121/128	SA	HW1_Hadid.pdf (D110093394)
x) = Ig 1 (g 2 xg 2 –1) = g 1 (g 2 xg 2 –1)g 1 –1 = (g 1 g 2)x(g 1 g 2) –1 = I g1g2 (x), ∀			

x ∈ G.
Aut(G) Inn(G)
Fig. 6.6 : Automorphism and inner automorphism of G 6.6 Summary This unit deals with the concept of quotient group , homomorphism and isomorphism. The most important topic in this unit are the isomorphism theorems. The concept of automorphism and inner automorphism have been discussed. 6.7 Worked Examples 1.

100%	MATCHING BLOCK 119/128	W
Let G be a finite cyclic group of order n.		

Prove that G ≅ Z n . Solution: Since G is a finite cyclic group of order n, we have G = (a) = {a 0 = e, a 1 , a 2 , a 3 , ..., a n−1 } for some a ∈ G. Define Φ : G → Z n such that Φ(a i) = i. By a similar argument as in the previous Question, we conclude that G ≅ Z n . 2. Let k, n be positive integers such that k divides n. Prove that Z n /(k) ≅ Z k . Solution: Since Z n is cyclic, we have Z n /(k) is cyclic by Theorem 5.1.2. Since Ord((k)) = n/k, we have order(Z n /(k)) = k. Since Z n /(k) is a cyclic group of order k, Z n /(k) ≅ Z k by the previous Question.
92 NSOU CC-MT-10 NSOU CC-MT-10 93 3. Prove that Z under addition is not isomorphic to Q under addition. Solution: Since Z is cyclic and Q is not cyclic, we conclude that Z is not isomorphic to Q. 4. Consider the group ? 3 . Let H = {(x 1 , x 2 , x 3) ∈ ? 3 : x 1 + 2x 2 − x 3 = 0}. Show that H is a normal subgroup of ? 3 . Show that ? 3 /H ? ?. Proof. The identity of the additive group ? 3 is 0 = (0, 0, 0). Notice that 0 ∈ H so H ≠ ∅. Let x = (x 1 , x 2 , x 3) and y = (y 1 , y 2 , y 3) be two elements of H. Then x 1 + 2x 2 − x 3 = 0 and y 1 + 2y 2 − y 3 = 0. It f ollows that the coordinates of z =

66%	MATCHING BLOCK 120/128	W
x − y = (x 1 − y 1 , x 2 − y 2 , x 3 − y 3) satisfy (x 1 − y 1) + 2(x 2 − y 2) − (x 3 − y 3) = (x 1 + 2x 2 − x 3) + (y 1 + 2y 2 − y 3) = 0. So x − y ∈		

H if x, y ∈
H. This directly proves that H is a subgroup
of ? 3 . Since ? 3 is abelian, any subgroup is automatically normal. Alternatively, we can argue as follows: Now define

78%	MATCHING BLOCK 123/128	SA	Abstract Algebra and Discrete Mathematics-Bloc ... (D164970162)
f : ? 3 → ? by f(x 1 , x 2 , x 3) = x 1 + 2x 2 − x 3 . Let x = (x 1 , x 2 , x 3)			

and

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y = (y 1 , y 2 , y 3) be two elements of ? 3 . Then we verify that f(

64%	MATCHING BLOCK 122/128	W

x + y) = f(x 1 + y 1 , x 2 + y 2 , x 3 + y 3) = (x 1 + y 1) + 2(x 2 + y 2) − (x 3 + y 3) = (x 1 + 2x 2 − x 3) + (y 1 + 2y 2 − y 3) = f(x) + f(y).

So f is a group homomorphism. Looking at the definition of H, we notice H = ker(f). Since the kernel of any homomorphism is a normal subgroup, we find that H is a normal subgroup of ? 3 . Given any x ∈ ?, we notice that f(x, 0, 0) = x, so f is an onto homomorphism. Thus by the first isomorphism theorem, we get an isomorphism ? 3 /H ? ?. 5. a) Describe the set Hom(? + , ? +) of all homomorphisms f : ? + → ? + . Which of them are injective? which are surjective, which are authomorphisms? b) Use the results of (a) to determine the group of automorphisms Aut(? +). Solution: a) Let z ∈ Z we have two cases: i) If z ∈ Z + —set of non-negative integers. Since 1 is the generator for z under addition z = 1 + 1 + ... + 1(z times)
94 NSOU CC-MT-10 NSOU CC-MT-10 95 since f is a homomorphism; f(z) = f(1 + 1 + ... + 1) = f(1) + f(1) + ... + f(1) = zf(1) Let f(1) = a ∈ Z then it follows that f(z) = az ii) If z ∈ Z—set of negative integers −1 is also a generator for Z under addition: z = −1 −1 − ... −1 = (−1) + (−1) + ... + (−1)(−z times) As from the hyopthesis, f is a homomorphism; f(z) = f(−1 −1... −1) = f(−1) + f(−1) + ... + f(−1) = zf(−1) But f(1) = a ⇒ f(−1) = −a ⇒ f(z) = −az. ∴ we have proved that any homomorphism f : Z + → Z + is of the form f(z) = az where a = f(1) Suppose that f(z 1) = f(z 2)) az 1 = az 2 ⇒ z 1 = z 2 when a ≠ 0 ⇒ f(z) = az is injective when a ≠ 0. When a = ±1, f(z) = az = ±z and f is surjective. ∴ Hom(? + ,? +) = {f : Z + → Z + : f(z) = az, z ∈ Z, a = f(1)} b) Aut(? +) = {f : Z + → Z + , f(z) = z, f(z) = −z} = ⟨f(z) = −z⟩ ∴ Aut(? +) ? C 2 . 6.8 Model Questions 1. Prove that det(AB) = det(A) det(B) for A,B ∈ GL 2 (?). This shows that the determinant is a homomorphism from GL 2 (?) to ? * . 2. Which of the following maps are homomorphisms? If the map is a homomorphism, what is the kernel? (a) φ : ? * → GL 2 (?) defined by φ( )a a = ? ? ? ? ? ? 1 0 0 (b) φ : ? → GL 2 (?) defined by φ( )a a = ? ? ? ? ? ? 1 0 1 (c) φ : GL 2 (?) → ? defined by φ a b c d a d ? ? ? ? ? ? ? ? ? ? ? ? = +
94 NSOU CC-MT-10 NSOU CC-MT-10 95 (d) φ : GL 2 (?) → ? * defined by φ a b c d ad bc ? ? ? ? ? ? ? ? ? ? ? ? = - (e) φ : M 2 (?) → ? defined by φ a b c d b ? ? ? ? ? ? ? ? ? ? ? ? = where M 2 (?) is the additive group of 2 × 2 matrices with entries in ?. 3. Let A be an m × n matrix. Show that matrix multiplication, x → Ax, defines a homomorphism φ : ? n → ? m . 4. Let φ : ? → ? be given by φ(n) = 7n. Prove that φ is a group homomorphism. Find the kernel and the image of φ. 5. Describe all of the homomorphisms from ? 24 to ? 18 . 6. Describe all of the homomorphisms from ? to ? 12 . 7. In the group ? 24 , let H = ⟨4⟩ and N = ⟨6⟩. (a) List the elements in HN (we usually write H + N for these additive groups) and H ∩ N. (b) List the cosets in HN/N, showing the elements in each coset. (c) List the cosets in H/(H ∩ N), showing the elements in each coset. (d) Give the correspondence between HN/N and H/(H ∩ N) described in the proof of the Second Isomorphism Theorem. 8. If G is an abelian group and n ∈ N, show that φ : G → G defined by g → g n is a group homomorphism. 9. If φ : G → H is a group homomorphism and G is abelian, prove that φ(G) is also abelian. 10. If φ : G → H is a group homomorphism and G is cyclic, prove that φ(G) is also cyclic. 11. Show that a homomorphism defined on a cyclic group is completely determined by its action on the generator of the group. 12.

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Let G be a group of order p 2 , where p is a prime number. If

H is a subgroup of G of order p, show that
H is normal in G. Prove that G must be abelian. 13. If a group G has exactly one subgroup H of order k, prove that H is normal in G.
96 NSOU CC-MT-10 NSOU CC-MT-10 97 14. Prove or disprove: ?/? ≅ ?. 15.

40%	MATCHING BLOCK 124/128	W

Let G be a finite group and N a normal subgroup of G. If H is a subgroup of G/N, prove that φ −1 (H) is a subgroup in G of order |

H| · |N|, where φ : G → G/N is the canonical homomorphism. 16. Let G 1 and G 2 be groups, and let H 1 and H 2 be normal subgroups of G 1 and G 2 respectively. Let φ : G 1 → G 2 be a homomorphism. Show that φ induces a natural homomorphism φ : (G 1 /H 1) → (G 2 /H 2) if φ(H 1) ⊆ H 2 . 17.
If

87%	MATCHING BLOCK 125/128	W

H and K are normal subgroups of G and H ∩ K = {e},

prove that G is isomorphic to a subgroup of G/H × G/K. 18. Let φ : G 1 → G 2 be a surjective group homomorphism. Let H 1 be a normal subgroup of G 1 and suppose that φ(H 1) = H 2 . Prove or disprove that G 1 /H 1 ≅

91%	MATCHING BLOCK 126/128	W

G 2 / H 2 . 19. Let φ : G → H be a group homomorphism. Show that φ is one-to-one

if and only if φ −1 (e) − {e}. 20. Given a homomorphism φ : G → H define a relation ~ on G by a ~ b if φ(a) = φ(b) for a, b ∈ G. Show this relation is an equivalence relation and describe the equivalence classes. Automorphisms 1. Let Aut(G) be the set of all automorphisms of G; that is, isomorphisms from G to itself. Prove this set forms a group and is a subgroup of the group of permutations of G; that is, Aut(G) ≤ S G . 2. An inner automorphism of G, i g : G → G, is defined by the map i g (x) = gxg −1 , for g ∈ G. Show that i

52%	MATCHING BLOCK 127/128	W

g ∈ Aut(G). 3. The set of all inner automorphisms is denoted by Inn(G). Show that Inn(G) is a subgroup of Aut(G). 4.

Find an automorphism of a group G that is
not an inner automorphism. 5. Let G be a group and ig be an inner automorphism of G, and define a map G → Aut(G) 96 NSOU CC-MT-10 NSOU CC-MT-10 97 by g → i g . Prove that this map is a homomorphism with image Inn(G) and kernel Z(G). Use this result to conclude that G/Z(G) ≅ Inn(G). 6. Compute Aut(S 3) and Inn(S 3). Do the same thing for D 4 . 7. Find all of the homomorphisms φ : ? → ?. What is Aut(?)? 8. Find all of the automorphisms of ? 8 . Prove that Aut(? 8) ≅ U(8). 9. For k ∈ ? n , define a map φ k : ? n → ? n by a → ka. Prove that φ k is a homomorphism. 10. Prove that φ k is an isomorphism if and only if k is a generator of ? n . 11. Show that every automorphism of ? n is of the form φ k , where k is a generator of ? n . 12. Prove that ψ : U(n) → Aut(? n) is an isomorphism, where ψ : k → φ k . 6.9 Solutions of some selected problems 2. (a) Ker(φ) = {1} (b) Ker(φ) = {0} (e) Ker a b c d M Rb ( ) ( ): φ ? ? ? ? ? ? ∈ = ? ? ? ? ? ? 2 0 4. Ker(φ) = {0}, Img(φ) = 7Z Automorphism 7. All homomorphisms from Z to Z are of the type n → an for some fixed a ∈ Z. Aut(Z) = Z 2 . 98 NSOU CC-MT-10 NSOU CC-MT-10 99 98 Further Reading Further reading [1] Dummit, David Steven, and Richard M. Foote. Abstract algebra. Vol. 3. Hoboken: Wiley, 2004. [2] Fraleigh, John B. A first course in abstract algebra. Pearson Education India, 2003. [3] Gallian, Joseph. Contemporary abstract algebra. Nelson Education, 2012. [4] Herstein, Israel N. Topics in algebra. John Wiley & Sons, 2006. [5] Lang, Serge. Undergraduate algebra. Springer Science & Business Media, 2005. [6] Mapa, Sadhan Kumar. Higher Algebra, Abstract and Linear. Dipali Mapa, 2003. [7] Rotman, Joseph J. A first course in abstract algebra. Pearson College Division, 2000. [8] Chakraborty, A. Modern algebra. Sarat Book House (Levant Pub.)

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X × X = X 2 . Two ordered pairs (x 1 , y 1), (x 2 , y 2) in X × Y are equal if and only if x 1 = x 2 and y 1 = y 2 . Thus, (x, y) ≠ (y, x) unless x = y.	x x ? ? ? ? ( ) ( ) ( ) 2 2 2 x y x y x y x y ? ? ? ? ? ? ? ? ? ? : , ? ? ( ) ( ) / 2 / 2 x y x y x y ? ?? ? ? ? ? y? (2) 2 / 2 y y y ? ? ? ( ) / 2 x x ? ? x? 2y? 141

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X 1 , X 2 , . . . , X n is the set of ordered n-tuples, X 1 × X 2 × . . . × X n = {(x 1 , x 2 , . . . , x n) : x i ∈ X i for i = 1, 2, . . . , n}, where (x 1 , x 2 , . . . , x n) = (

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4/128	SUBMITTED TEXT	29 WORDS	60%	MATCHING TEXT	29 WORDS

Y is a subset f ⊆ X × Y such that (i) For all x ∈ X, there exists y ∈ Y such that (x, y) ∈ f (ii) For

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5/128	SUBMITTED TEXT	19 WORDS	88%	MATCHING TEXT	19 WORDS

R. ∴ R is not reflexive. Now (1, 6) ∈R But, (1, 6) ∉ R. ∴ R is not symmetric.

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6/128	SUBMITTED TEXT	37 WORDS	34%	MATCHING TEXT	37 WORDS

R is not symmetric. Let (x, y), (y, z) ∈ R. Then, y is divisible by x and z is divisible by y. ∴z is divisible by x. ⇒ (x, z) ∈ R ∴ R is transitive. Hence, R is

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7/128	SUBMITTED TEXT	116 WORDS	21%	MATCHING TEXT	116 WORDS

x ∈ Z, (x, x) ∈R as x − x = 0 is an integer. ∴ R is reflexive. Now, for every x, y ∈ Z if (x, y) ∈ R, then x – y is an integer. ⇒ −(x − y) is also an integer. ⇒ (y − x) is an integer. ∴ (y, x) ∈ R. Hence, R is symmetric. Now, Let (x, y) and (y, z) ∈ R, where x, y, z ∈ Z. ⇒ (x − y) and (y − z) are integers. ⇒ x − z = (x − y) + (y − z) is an integer. 22 NSOU CC-MT-10 NSOU CC-MT-10 23 ∴ (x, z) ∈ R. Hence, R is transitive. Hence, R is

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8/128	SUBMITTED TEXT	46 WORDS	72%	MATCHING TEXT	46 WORDS

f −1 : (a) f −1 (A ∪ B) = f −1 (A) ∪ f −1 (B) for all A, B ⊆ Y .
(b) f −1 (A \ B) = f −1 (A) \ f −1 (B)

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9/128	SUBMITTED TEXT	58 WORDS	63%	MATCHING TEXT	58 WORDS

Submitted text	Matching text
x, y) ∈ ? 2 × ? 2 : x 1 x 2 = y 1 y 2 }. 5. For x = (x 1 , x 2), y = (y 1 , y 2) ∈ ? 2 , let S = {x ∈ ? 2 : x x 1 2 2 2 1 + =}.	x y x y x y x y ? ? ? ? ? ? ? ? ? ? ? : , ? ? ( ) ( ) / 2 / 2 x y x y x y ? ?? ? ? ? ? y y y ? ? ? ( ) / 2 x x ? ? x? 2

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10/128	SUBMITTED TEXT	34 WORDS	71%	MATCHING TEXT	34 WORDS

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f = {(x, x) : x ∈ ?}. (d) f : ? → ? defined by f = {(x, x) : x ∈ ?}. (e) f : ? − → ?	f (x) = x 3 − x. (d) f : R → R such that f (x) = x 3 + x. (e) f : {

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11/128	SUBMITTED TEXT	135 WORDS	22%	MATCHING TEXT	135 WORDS

R = {(x, y) ∈ ? 2 × ? 2 : x = ay for some a ∈ ?*}. (c) R = {(x, y)∈ ? 2 × ? 2 : 4 9 4 9 1 2 2 2 1 2 2 2 x x y y + =+ }. (d) R = {(x, y) ∈ ? 2 × ? 2 : x − y = a(1, 1) for some a ∈ ?*}. (e) Fix c ∈ ?. Now, define ? = {(x, y) ∈ ? 2 × ? 2 : y 2 − x 2 = c(y 1 − x 1)}. (f) R = {(x, y) ∈ ? 2 × ? 2 : |x 1 | + |x 2 | = a(|y 1 | + |y 2 |)},

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12/128	SUBMITTED TEXT	49 WORDS	31%	MATCHING TEXT	49 WORDS

A × B = B × A is true. b. Prove A ∪ φ = A and A ∩ φ = φ. c. Prove A ∪ B = B ∪ A and A ∩ B = B ∩ A. d. Prove A ∪ (B ∩ C) = (A ∪ B) ∩ (

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13/128	SUBMITTED TEXT	112 WORDS	32%	MATCHING TEXT	112 WORDS

A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C). f. Prove A ⊂ B if and only if A ∩ B = A. g. Prove (A ∩ B)′ = A′ ∪ B′. h. Prove A ∪ B = (A ∩ B) ∪ (A \ B) ∪ (B \ A). i. Prove (A ∪ B) × C = (A × C) ∪ (B × C). j. Prove (A ∩ B) \ B = φ. k. Prove (A ∪ B) \ B = A \ B. l. Prove A \ (B ∪ C) = (A \ B) ∩ (A \ C). m. Prove A ∩ (B \ C) = (A ∩ B) \ (A ∩ C).

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14/128	SUBMITTED TEXT	38 WORDS	68%	MATCHING TEXT	38 WORDS

f (A 1 ∪ A 2) = f (A 1) ∪ f (A 2). (b) Prove f (A 1 ∩ A 2) ⊂ f (A 1) ∩ f (A 2).

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15/128	SUBMITTED TEXT	29 WORDS	80%	MATCHING TEXT	29 WORDS

a, b) ~ (c, d) if and only if a 2 + b 2 ≤ c 2 + d 2 . Show that ~ is

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16/128	SUBMITTED TEXT	22 WORDS	58%	MATCHING TEXT	22 WORDS

is a binary operation on ? 3 . Example 2.3.6 : Composition of symmetries is a binary operation on the set of

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17/128	SUBMITTED TEXT	79 WORDS	35%	MATCHING TEXT	79 WORDS

a, b ∈ G then a * b ∈ G. 2. * is associative, i.e., a * (b * c) = (a * b) * c for a, b, c ∈ G. 3. G contains an identity element e, i.e., a * e = e * a = a for all a ∈ G. 4. Inverse exists in G, i.e., for any a ∈ G there exists an inverse element a′ ∈ G such that a * a′ = a′ * a =

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18/128	SUBMITTED TEXT	2 WORDS	86%	MATCHING TEXT	2 WORDS

b c e a b c e a b c a e c b b c e a c b a e

b, c are reflections.◦ e a b c e e a b c a a e c b b b c e a c c b a e

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19/128	SUBMITTED TEXT	88 WORDS	30%	MATCHING TEXT	88 WORDS

Direct product of groups). Let (G 1 , * 1), . . . (G n , * n) be groups. Then the direct product G = G 1 × G 2 × . . . × G n is the set of n-tuples (g 1 , g 2 , . . . , g n) where g i ∈ G i with operation defined componentwise : (g 1 , g 2 , . . . , g n) * (

direct product of G 1 ,G 2 ,...,G n . Theorem 1.6.2. Let G 1 ,G 2 ,...,G n be finite groups and (g 1 ,g 2 ,...,g n) be an element of the group G 1 × G 2 × ··· × G n . Then ◦ ((g 1 , g 2 ,..., g n)) = l.c.g 1),◦(g 2),...,◦(g n)).

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20/128	SUBMITTED TEXT	18 WORDS	43%	MATCHING TEXT	18 WORDS

a b a a b bi + - - Yes GL(2,F) Matrix multiplication 1 0 0 1 ? ? ? ? ? ? a b c d ? ? ? ? ? ? ? , ad − bc ≠ 0 d ad bc b ad bc

SA Algebra -I (Block I, II, III, IV).pdf (D144184274)

21/128	SUBMITTED TEXT	32 WORDS	51%	MATCHING TEXT	32 WORDS

is the smallest positive integer n such that g n = e. (In additive notation, this would be ng = 0). If no such integer exists, we say that

is the smallest positive integer n such that a n = e (if it exist). If no such that integer exists, we say that

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22/128	SUBMITTED TEXT	21 WORDS	75%	MATCHING TEXT	21 WORDS

Order of a Group). The number of elements of a group G (finite or infinite) is called the order of

SA Abstract Algebra and Discrete Mathematics-Block 1.pdf (D164970162)

23/128	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

Let G be a group and g be an element

SA Term_Paper_Sylow_Theorems.pdf (D83140355)

24/128	SUBMITTED TEXT	32 WORDS	38%	MATCHING TEXT	32 WORDS

Let a, b ∈ A. Then the binary operation a * b on A is defined by a * b = f −1 (f (a) f (b)). Since f is a

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25/128	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

a subgroup H of a group G to be a

a subgroup H of a group G to be a

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26/128	SUBMITTED TEXT	39 WORDS	54%	MATCHING TEXT	39 WORDS

Let G be a group and H be a non-empty subset of G. If ab ∈ G whenever a, b ∈ G and a −1 ∈ H whenever a ∈ H, then H is a subgroup of G.

Let G be a group and H a nonempty subset of G. a∗b is H whenever a and b are in H, and a -1 is in H whenever a is in H, then H is a subgroup of G.

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27/128	SUBMITTED TEXT	33 WORDS	63%	MATCHING TEXT	33 WORDS

a * a′ = f −1 (f (a) f (a′)) = f −1 (f (a) f (f −1 (f (a) −1))) = f −1 (f (a) f (

SA Abstract Algebra and Discrete Mathematics-Block 1.pdf (D164970162)

28/128	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

Let G be a group and H be a non-empty subset of G.

SA Algebra -I (Block I, II, III, IV).pdf (D144184274)

29/128	SUBMITTED TEXT	15 WORDS	82%	MATCHING TEXT	15 WORDS

G, then H is a subgroup of G. Proof. Let a, b ∈ H. Then

G;), then (T i2I H i ;) is a subgroup of (G;). Proof. If a;b 2 T i2I H i , then

W https://jiaxiaodong.com/files/ma2202/notes.pdf

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30/128	SUBMITTED TEXT	18 WORDS	67%	MATCHING TEXT	18 WORDS

H and K be two subgroups of G. Then H ∩ K is also a subgroup of G.

H and K are two subgroups of G and K≤H, then H/K is a normal subgroup of G/

W https://vulms.vu.edu.pk/Courses/MTH633/Downloads/MTH633.pdf

31/128	SUBMITTED TEXT	13 WORDS	100%	MATCHING TEXT	13 WORDS

of an element). Let G be a group and a ∈ G.

SA Term_Paper_Sylow_Theorems.pdf (D83140355)

32/128	SUBMITTED TEXT	42 WORDS	46%	MATCHING TEXT	42 WORDS

b ∈ H ∩ K. Then a, b ∈ H and a, b ∈ K. Hence, ab −1 ∈ H and ab −1 ∈ K. Which implies that ab −1 ∈ H ∩ K. Therefore, H ∩ K

SA Abstract Algebra and Discrete Mathematics-Block 1.pdf (D164970162)

33/128	SUBMITTED TEXT	40 WORDS	57%	MATCHING TEXT	40 WORDS

n) −1 = (a n) −1 a −1 = (a −1) n a −1 = (a −1) n+1 . 4. Let H and D be two subgroups of a group

n. (i) ◦(a) divides n ∀a ∈ G. (ii) a n = e ∀a ∈ G. let H and K be subgroups of a group

W http://pioneer.netserv.chula.ac.th/~upattane/file/2301337.pdf

34/128	SUBMITTED TEXT	15 WORDS	88%	MATCHING TEXT	15 WORDS

Let x and y be elements in a group G such that xy ∈

Let x and y be elements of a group G such that xy =

W https://www.slideshare.net/marcomoya399/abstract-algebra-i

35/128	SUBMITTED TEXT	30 WORDS	57%	MATCHING TEXT	30 WORDS

H is a subgroup of C \ {0}. 9. Let H = {A ∈ GL(608, Z 89) : det(A) = 1}. Prove that H is a subgroup of

H ∩K is also a subgroup of G. 9. Let H be a non-empty subset of a group G. Prove that H is a subgroup of

W https://www.math.uci.edu/~ndonalds/math120a/notes.html

36/128	SUBMITTED TEXT	19 WORDS	61%	MATCHING TEXT	19 WORDS

Prove that if G is an abelian group, then for all a, b ∈ G and all integers

SA Abstract Algebra (DCEMM-109).pdf (D155353493)

37/128	SUBMITTED TEXT	102 WORDS	67%	MATCHING TEXT	102 WORDS

a . b) n = a n . b n (1) (a . b) n+1 = a n+1 . b n+1 (2) (a . b) n+2 = a n+2 . b n+2 (3) Using (2) we have (a . b) n+1 = a n+1 . b n+1 ⇒ (a . b) n . (a . b) = a n+1 . (b n . b) ⇒ (a n . b n) . (a . b) = (a n+1 . b n) . b, Using (1) ⇒ ((a n . b n) .

a ∗b) n = (a ∗ n − 1 ∗(a ∗b)= (a n − 1 ∗ b n − 1) ∗(a ∗b) = ((n − 1 ∗ b n − 1) ∗ a) ∗ b = (a n − 1 ∗ (b n − 1 ∗ a)) ∗ b = (a a ∗ b n − 1)) ∗ b = (a n − 1 ∗ a) ∗ b n − 1) ∗ a n ∗ (b n − 1 ∗ b) = a n ∗ b n .

W https://vulms.vu.edu.pk/Courses/MTH633/Downloads/MTH633.pdf

38/128	SUBMITTED TEXT	62 WORDS	57%	MATCHING TEXT	62 WORDS

a) . b = (a n+1 . b n) . b 40 NSOU CC-MT-10 NSOU CC-MT-10 41 ⇒ (a n . b n) . a = (a n . a) . b n ⇒ a n . (b n . a) = a n . (a . b n) ⇒

a ∗b) = ((a n − 1 ∗ b n − 1) ∗ b = (a n − 1 ∗ (b n − 1 ∗ a)) ∗ a n − 1 ∗ (a ∗ b n − 1)) ∗ a n − 1 ∗ a) ∗ n − 1) ∗ a n ∗ (b n − 1 ∗

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39/128	SUBMITTED TEXT	107 WORDS	51%	MATCHING TEXT	107 WORDS

b n . a = a . b n (4) Again using (3), analogously we have b n+1 . a = a . b n+1 ⇒ b . (b n . a) = a . b n+1 ⇒ b . (a . b n) = a . b n+1 , Using (4) ⇒ (b . a) . b n = (a . b) . b n ⇒ b . a = a . b

b) n = (a ∗ b) n − 1 ∗(a ∗a n − 1 ∗ b n − 1) ∗(a ∗a n − 1 ∗ b n − 1) ∗ a) ∗ b = (a n − 1 ∗ (b a)) ∗ b = (a n − 1 ∗ (a ∗ b n − 1)) ∗ b = (n − 1 ∗ b n − 1) ∗ a n ∗ (b

W https://vulms.vu.edu.pk/Courses/MTH633/Downloads/MTH633.pdf

40/128	SUBMITTED TEXT	61 WORDS	54%	MATCHING TEXT	61 WORDS

a . b) k = (a . b) k–1 . (a . b) = (a k–1 . b k–1) . (a . b) = (a k–1 . b k–1) . (b . a) = (a k–1 . b k) . a = a . (a k–1 . b k) = a k . b

SA Algebra -I (Block I, II, III, IV).pdf (D144184274)

41/128	SUBMITTED TEXT	65 WORDS	52%	MATCHING TEXT	65 WORDS

are two real numbers and a ≠ 1, b ≠ 1. Now, a * b = a+b − ab which is a real number and a + b − ab ≠ 1, because a + b − ab = 1 ⇒ b(1 − a) = 1 − a ⇒ b = 1, since a ≠ 1. But b ≠ 1. Therefore, a * b is a

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42/128	SUBMITTED TEXT	143 WORDS	43%	MATCHING TEXT	143 WORDS

a * b ≠ 1 . So, a ⁎ b ∈ P ∀ a, b ∈ P. Hence P is closed under the binary operation '⁎'. (ii) Associative Property : Let a, b, c ∈ P, where a, b, c ∈ R and a ≠ 1, b ≠ 1, c ≠ 1. Now, a * (b * c) = a * (b + c − bc) = a + b + c − bc − a (c + c − bc) = a + b + c − bc − ab − ac + abc (a * b) * c = (a + b − bc) * c = a + b − bc + c − (a + b − ab) c = a + b + c −

SA Abstract Algebra and Discrete Mathematics-Block 1.pdf (D164970162)

43/128	SUBMITTED TEXT	61 WORDS	35%	MATCHING TEXT	61 WORDS

b ⁎ a = 0. Now, b ⁎ a = 0 ⇒ b + a − ba = 0 ⇒ b(1 − a) = −a ⇒ b = a a -1 , since ≠ 1 Since a a -1 is a real number as a ≠ 1 and a a -1 ≠ 1, so b = a a -1 ∈

SA Algebra -I (Block I, II, III, IV).pdf (D144184274)

44/128	SUBMITTED TEXT	18 WORDS	87%	MATCHING TEXT	18 WORDS
H and K are subgroups of G, show that H ∩ K is a subgroup of G. (				H and K are subgroups of G. Prove that H ∩K is also a subgroup of G. 9.	

W https://www.math.uci.edu/~ndonalds/math120a/notes.html

45/128	SUBMITTED TEXT	24 WORDS	65%	MATCHING TEXT	24 WORDS

abelian groups? 5. Prove that a group G is abelian iff (ab) −1 = a −1 b −1 , ∀a, b ∈ G. 6.

SA Abstract Algebra and Discrete Mathematics-Block 1.pdf (D164970162)

46/128	SUBMITTED TEXT	37 WORDS	43%	MATCHING TEXT	37 WORDS

Definition 3.3.1 : A group G is called cyclic if there exists an element g ∈ G such that G = {g n : n ∈ ?}. The element g is called the generator of G. The

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47/128	SUBMITTED TEXT	35 WORDS	60%	MATCHING TEXT	35 WORDS

m, n) : m ∈ ? 2 , n ∈ ? 3 } is a cyclic group. The binary operation is component wise addition (m, n) + (m′, n′) = (m + m′, n + n′).

SA 182415ER002-G.Elakkiya.pdf (D85895799)

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Theorem 3.4.1 : Every cyclic group is Abelian. Proof. Let G be a cyclic group

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49/128	SUBMITTED TEXT	29 WORDS	55%	MATCHING TEXT	29 WORDS

a = g n and b = g m . Now ab = g n g m = g n+m = g m+n = g

a group, g ∈ G and n,m ∈ Z, we have g n g m = g n+m and (g m) n = g

W https://mathsci.kaist.ac.kr/~hrbaik/Alonso.pdf

50/128	SUBMITTED TEXT	16 WORDS	95%	MATCHING TEXT	16 WORDS

a k ⟩ = ⟨a gcd(n, k) ⟩ 2. |a k | = n n k

a k < = >a gcd(n,k) < ? |a k | = n/gcd(n,k)

W https://vulms.vu.edu.pk/Courses/MTH633/Downloads/MTH633.pdf

51/128	SUBMITTED TEXT	30 WORDS	47%	MATCHING TEXT	30 WORDS

r where 0 ≤ r > m; hence, a k = a mk+r = (a m) k a r = h q a r . So a

SA Thesis Sylows_PDFA.pdf (D15881641)

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52/128	SUBMITTED TEXT	38 WORDS	34%	MATCHING TEXT	38 WORDS

p2 /12 e j2p2 /12 e j2p/12 e j2p11 /12 e j2p10 /12 e j2p9 /12 e j2p8 /12 e j2p7/12 e j2p6 /12 e j2p5 /12 e j2p0 /12 = e

SA Term_Paper_Sylow_Theorems.pdf (D83140355)

53/128	SUBMITTED TEXT	13 WORDS	84%	MATCHING TEXT	13 WORDS

Let G be a group and let a be an element of G.

SA Term_Paper_Sylow_Theorems.pdf (D83140355)

54/128	SUBMITTED TEXT	23 WORDS	57%	MATCHING TEXT	23 WORDS

Let G be a finite group. Show that there exists a fixed positive integer n such that a n = e

Let G be a group and a ∈ G. If there is a positive integer n such that a n = e,

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55/128	SUBMITTED TEXT	23 WORDS	50%	MATCHING TEXT	23 WORDS

a group of order 3 must be cyclic. 17. Let Z denote the group of integers under addition. Is every subgroup of Z

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56/128	SUBMITTED TEXT	42 WORDS	69%	MATCHING TEXT	42 WORDS

H = {0, 3} = 3 + H 1 + H = (123)H = {(13), (123)} 2 + H = (132)H = {(23), (132)} Example 4.3.3 : Let G = S 3 and H = {(1), (12)}. Then the left cosets of H

h ∈ H} H + g := {h + g : h ∈ H} Example (6.1 cont). Let G = Z and H = [0] = 3Z. The left and right cosets H

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57/128	SUBMITTED TEXT	36 WORDS	62%	MATCHING TEXT	36 WORDS

H = (12)H = {(1), (1, 2)} (13)H = (123)H = {(13), (123)} (23)H = (132)H = {(23), (132)} The right cosets are H(1) = H(12) = {(1), (1, 2)} H(13) = H(132) = {(13), (132)} H(23) = H(123) = {(23), (123)}.

H H H H H H H H H H 1 H H

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58/128	SUBMITTED TEXT	37 WORDS	50%	MATCHING TEXT	37 WORDS

a, b ∈ G. Then 1. a ∈ aH. 2. aH = H if and only if a ∈ H. 3. aH = bH if and only if a ∈ bH. 4. aH = bH or aH ? bH =

SA Thesis Sylows_PDFA.pdf (D15881641)

59/128	SUBMITTED TEXT	27 WORDS	96%	MATCHING TEXT	27 WORDS

x ∈ aH ? bH. Then x = ah 1 = bh 2 for some h 1 , h 2 ∈ H. So,

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60/128	SUBMITTED TEXT	33 WORDS	65%	MATCHING TEXT	33 WORDS

a ~ b and b ~ c, we get a −1 b ∈ H and b −1 c ∈ h. Hence, (a −1 b)(b −1 c) = a −1 c ∈ H.

a~ L b and b~ L c . Then a -1 b∊H and b -1 c∊H. Since H is a subgroup, (a -1 b)(b -1 c)=a -1 c∊H.

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61/128	SUBMITTED TEXT	28 WORDS	57%	MATCHING TEXT	28 WORDS

Let H be a subgroup of the group G. For any a, b ∈ G, a is related to b, a ~ b if and only if

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62/128	SUBMITTED TEXT	16 WORDS	78%	MATCHING TEXT	16 WORDS

Therefore, the relation ~ is transitive. Hence ~ is an equivalence relation. Consider the equivalence class [

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Then the number of left cosets of H in G is same as the number of right cosets of H in G.

then the number of left cosets of H in G is |G|/|H|. We the number of left the index of H in G

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64/128	SUBMITTED TEXT	36 WORDS	82%	MATCHING TEXT	36 WORDS

Let G be a group and H be a subgroup. The number of left cosets of H in G is called index of H in G and denoted by [G : H]. 60

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65/128	SUBMITTED TEXT	19 WORDS	88%	MATCHING TEXT	19 WORDS

Let G be a finite group and H be a subgroup of G. Then |G|/|H| = [G :

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66/128	SUBMITTED TEXT	19 WORDS	73%	MATCHING TEXT	19 WORDS

a finite group and p is a prime dividing |G|, then G has an element of order p. Proof.

a finite group order n and p be a prime dividing n. Then G has an element of order p. Proof.

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67/128	SUBMITTED TEXT	42 WORDS	36%	MATCHING TEXT	42 WORDS

H and K be subgroups of G such that K ⊂ H ⊂ G. Then [G : K] = [G : H][G : K]. Proof. By, Lagrange's Theorem we have [:] | | | | | || | | | [:][:] G K G K G H H K

H and K are subgroups of a group G such that K ≤ H ≤ G, and suppose (H:and (G:H) are both finite. Then (G:K) is finite, and (G:K)=(G:H)(H:K).

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The converse of Lagrange's Theorem is not true: namely, if G is a finite group and n divides |G|, then G need not have a subgroup of order

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69/128	SUBMITTED TEXT	14 WORDS	87%	MATCHING TEXT	14 WORDS

gH = Hg for all g ∈ G. Hence, H is normal in G.

gH = Hg for all g ∈ G, whence H is normal in G.

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Let G be a group and H be a subgroup with index 2. Then H is normal in G.

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71/128	SUBMITTED TEXT	28 WORDS	54%	MATCHING TEXT	28 WORDS

is normal in G. Definition 4.5.8 : Let H and K be subgroups of a group G and define HK = {hk : h ∈ H, k ∈ K}.

is normal in G. Theorem 1.8.10. H and K be normal subgroups of G and H ∩ K = {e}. Then hk = kh for all h ∈ H,k ∈ K.

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72/128	SUBMITTED TEXT	16 WORDS	76%	MATCHING TEXT	16 WORDS

H and K are finite subgroups of a group, then | | | || | | | HK H K H K = ∩ .

H and K be subgroups of a group G. Then |HK| = |H||K| |H ∩ K| .

W http://pioneer.netserv.chula.ac.th/~upattane/file/2301337.pdf

73/128	SUBMITTED TEXT	89 WORDS	33%	MATCHING TEXT	89 WORDS

G. Theorem 4.5.7 : A subgroup H of G is normal if and only if gHg −1 ⊆ H for all g ∈ G. Proof. Let H is normal in G. Then gH = Hg for all g ∈ G. Now for any h ∈ H, there exists h′ ∈ H such that gh = h′g. Which implies that ghg −1 = h′ ∈ H. Hence, gHg −1 ⊆ H for all g ∈ G. Conversely, let gHg −1 ⊆ H for all g ∈ G.

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74/128	SUBMITTED TEXT	63 WORDS	34%	MATCHING TEXT	63 WORDS

h ∈ H. But h 1 K = h 2 K for h 1 , h 2 ∈ H if and only if h 2 −1 h 1 ∈ K. Thus h 1 K = h 2 K ? h 2 −1 h 1 ∈ H ? K ? h 1 (H ? K) = h 2 (H ?

H H H H H H H H H H 1 H H H H H H H H H

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75/128	SUBMITTED TEXT	17 WORDS	82%	MATCHING TEXT	17 WORDS

the order of aN is the smallest positive integer n such that a n ∈

The order of a, denoted ◦(a) is the smallest positive integer n such that a n =

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76/128	SUBMITTED TEXT	65 WORDS	31%	MATCHING TEXT	65 WORDS

N G (H) a 1 g 3 H a 1 g 1 H H g 4 N=Ng 4 g 2 H=Hg 2 N G (H) g 3 H=Hg 3 g 1 H=Hg 1 a 3 H a 3 g 4 H a 3 g 2 H a 3 N G (H) a 2 g 3 H a 3 g 1 H a 2 H a 2 g 2 H a 2 g 2 H a 2 N G (H) a 3 g 3 H

SA Homework2.pdf (D110598367)

77/128	SUBMITTED TEXT	23 WORDS	73%	MATCHING TEXT	23 WORDS

a, b) (g, 1)(a, b) −1 , where (g, 1) ∈ G × {1}. Now (a, b)(g, 1)(a, b) −1 = (a, b)(g, 1)(

a; b 2 G (a b 2 G). G2. Associativity: 8a; b; c 2 G [(a b) c = a (b G3.

W https://jiaxiaodong.com/files/ma2202/notes.pdf

78/128	SUBMITTED TEXT	26 WORDS	61%	MATCHING TEXT	26 WORDS

g ∈ G. Therefore, (a, b)(g, 1)(a, b) −1 ∈ G × {1}. This proves that (a, b)(G × {1})(a, b) −1 ⊂

G ? G ? , , a b c G? ,a b G? a b G ? ? ( ) ( ) a b

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79/128	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

Let a and b be elements of a group G

Let a and b be elements of a group G. (

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80/128	SUBMITTED TEXT	13 WORDS	83%	MATCHING TEXT	13 WORDS

q are prime. Prove that every proper subgroup of G is cyclic. 18.

q are both prime. Show that every proper subgroup of G is cyclic. 4.

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81/128	SUBMITTED TEXT	18 WORDS	76%	MATCHING TEXT	18 WORDS

H and K are subgroups of a group G. If |H| = 12 and |K| = 35,

H and K be subgroups of a group G. (If H or K

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82/128	SUBMITTED TEXT	46 WORDS	68%	MATCHING TEXT	46 WORDS

g 1 , 1), (g 2 , 1) ∈ G × {1}, where g 1 , g 2 ∈ G. Then (g 1 , 1) · (g 2 , 1) = (g 1 g 2 , 1) ∈ G × {1}. Therefore, G × {1} is

g,1) (g,1) (g,1) (g,g) (1,1) (1,g) (g,g) (g,g) (g,1) (1, g) (1,1) is

W https://webspace.maths.qmul.ac.uk/p.j.cameron/algebra/solutions/ch3odd.pdf

83/128	SUBMITTED TEXT	33 WORDS	21%	MATCHING TEXT	33 WORDS

A B C Do nothing p A BC B C A ABC 2 = ? ? ? ? ? ? ( ) A B C Counterclockwise rotation of 120° p A BC C AB ACB 3 = ? ? ? ? ? ? ( ) A B C Counterclockwise rotation of 240° p A BC A C B A BC 4 = ? ? ? ? ? ? ( )( ) A B C Flip through vertex A p A BC C BA AC B 5 = ? ? ? ? ? ? ( )( ) A B C

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84/128	SUBMITTED TEXT	51 WORDS	41%	MATCHING TEXT	51 WORDS

A permutation σ ∈ S n is a cycle of length k if there exists elements a 1 , a 2 , ..., a k ∈ A such that σ(a 1) = a 2 σ(a 2) = a 3 . . . σ(a k) = a 1 , and σ(x) = x

A permutation σ in S n is a cycle if there exist a 1 ,a 2 ,...,a r in {1, 2,...,n} satisfying (i) σ(a i) = a i+1 for all i ∈ {1, 2,...,r - 1}, (ii) σ(a r) = a 1 , iii) σ(x) = x

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85/128	SUBMITTED TEXT	47 WORDS	56%	MATCHING TEXT	47 WORDS

cycle is called a transposition. Since (a 1 , a 2 , . . . , a n) = (a 1 a n)(a 1 a n−1) ... (a 1 a 3)(a 1 a 2), any cycle can be written as the product of transpositions,

cycle is a product of transpositions. Proof a 1 , a 2 , ..., a n) be a cycle, then (a 1 , a n) (a 1 , a n-1) ... (a 1 , a 2) = (a 1 , a n). Cycle Decomposition 468Every permutation can be written as a product of transpositions.

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86/128	SUBMITTED TEXT	18 WORDS	100%	MATCHING TEXT	18 WORDS
A permutation is said to be even if it can be expressed as the product of an			a permutation is said to be even if it can be expressed as the product of an		

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87/128	SUBMITTED TEXT	9 WORDS	100%	MATCHING TEXT	9 WORDS
can be written in cyclic notation as σ σ σ					

SA Assignment 2.pdf (D142017062)

88/128	SUBMITTED TEXT	16 WORDS	84%	MATCHING TEXT	16 WORDS
odd if it can be expressed as the product of an odd number of transpositions. 5.7 The			odd if it can be written as the product of an even/odd number of transpositions. the		

W https://www.math.uci.edu/~ndonalds/math120a/notes.html

89/128	SUBMITTED TEXT	23 WORDS	57%	MATCHING TEXT	23 WORDS
n is the set of all even permutations, A n . The group A n is called the alternating group					

SA Abstract Algebra and Discrete Mathematics-Block 1.pdf (D164970162)

90/128	SUBMITTED TEXT	40 WORDS	29%	MATCHING TEXT	40 WORDS
order of A n is n!/2. Proof. Let A n be the set of even permutations in S n and B n be the set of odd permutations. If we can show that there is a					

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91/128	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS
Find all possible orders of elements in S 7 and A 7 . 8.			Find all possible orders of elements in S_7 and A_7. \		

W https://math.mit.edu/~roed/courses/430_S16/PS3sol.tex

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92/128	SUBMITTED TEXT	12 WORDS	100%	MATCHING TEXT	12 WORDS

can be written as a finite product of the following permutations. (

can be written as a finite product of the following permutations. \

W https://math.mit.edu/~roed/courses/430_S16/PS3sol.tex

93/128	SUBMITTED TEXT	23 WORDS	54%	MATCHING TEXT	23 WORDS

the number of cosets of H in G is [G : H], therefore the order of the group G/H is [G :

the number of left cosets of H in G is |G|/|H|. We call the number of left cosets the index of in G

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94/128	SUBMITTED TEXT	24 WORDS	68%	MATCHING TEXT	24 WORDS

G/H = {gH : g ∈ G} with the binary operation g 1 H ∗ g 2 H = g 1 g 2 H

G and H is G × H with the operation (g 1 ,h 1) · (g 2 ,h 2) = (g 1 g 2 ,h 1

W https://mathsci.kaist.ac.kr/~hrbaik/Alonso.pdf

95/128	SUBMITTED TEXT	31 WORDS	72%	MATCHING TEXT	31 WORDS

then the set of left (or right) cosets of H in G is itself a group—called the factor group of G by H (or the quotient group of G by

SA 182415ER002-G.Elakkiya.pdf (D85895799)

96/128	SUBMITTED TEXT	28 WORDS	84%	MATCHING TEXT	28 WORDS

Let G be a group and H be normal subgroup of G. Then the set G/H = {gH : g ∈ G} is a group under the operation

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97/128	SUBMITTED TEXT	10 WORDS	100%	MATCHING TEXT	10 WORDS

quotient group of a cyclic group is cyclic. Proof. Let

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98/128	SUBMITTED TEXT	54 WORDS	36%	MATCHING TEXT	54 WORDS

G. Definition 6.4.2 : If ϕ is a homomorphism of G into H, the kernel of ϕ, Ker ϕ , is defined by Ker ϕ = {x ∈ G : ϕ(x) = e′, e′ = identity element of H}. Proposition 6.4.3 : Let G and H be groups and let ϕ : G →

SA Algebra -I (Block I, II, III, IV).pdf (D144184274)

99/128	SUBMITTED TEXT	21 WORDS	66%	MATCHING TEXT	21 WORDS

ϕ, ker ϕ , is a normal subgroup of G, (ii) image of ϕ, Im ϕ , is a subgroup of H. Proof. (

ϕ ◁ G (ker ϕ is a normal subgroup of G) 4. Im ϕ ≤ L (Im ϕ is a subgroup of L) Proof. 1 & 2

W https://www.math.uci.edu/~ndonalds/math120a/notes2.html

100/128	SUBMITTED TEXT	22 WORDS	60%	MATCHING TEXT	22 WORDS

e) = e′, where e and e′ are the identities of G and H, respectively. (ii) ϕ(g −1) = ϕ(g) −1

SA Term_Paper_Sylow_Theorems.pdf (D83140355)

101/128	SUBMITTED TEXT	15 WORDS	76%	MATCHING TEXT	15 WORDS

there exists a, b ∈ G such that ϕ(a) = x and ϕ(b) =

there exist a, b ∊ G such that ϕ(a)= a' and ϕ(b) =

W https://vulms.vu.edu.pk/Courses/MTH633/Downloads/MTH633.pdf

102/128	SUBMITTED TEXT	38 WORDS	52%	MATCHING TEXT	38 WORDS

a ? b −1) = ϕ(a) · ϕ(b −1) = ϕ(a) · ϕ(b) −1 = e′ · e′ = e′. Therefore, a ? b −1 ∈ Ker ϕ . Hence, ker ϕ

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103/128	SUBMITTED TEXT	28 WORDS	72%	MATCHING TEXT	28 WORDS

Theorem 6.4.5 : A homomorphism ϕ : G → H is injective if and only if Ker ϕ = {e}. Proof. Suppose ϕ is injective, and let

Theorem A homomorphism h: G→G' is injective if and only if Ker h={e}. 762 Proof Suppose h is injective, and let

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104/128	SUBMITTED TEXT	17 WORDS	91%	MATCHING TEXT	17 WORDS

we say that G and H are isomorphic and write G ≈ H. 86

We say that the groups G and H are isomorphic, and write G ~ = H,

W https://mathsci.kaist.ac.kr/~hrbaik/Alonso.pdf

105/128	SUBMITTED TEXT	18 WORDS	100%	MATCHING TEXT	18 WORDS

ϕ(a) · ϕ(b) −1 = ϕ(a) · ϕ(b −1) = ϕ(a · b −1). Therefore,

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106/128	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

Let H be a normal subgroup of G. Then the

Let H be a normal subgroup of G. Then the

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107/128	SUBMITTED TEXT	23 WORDS	47%	MATCHING TEXT	23 WORDS

is one-one. Finally, f is onto, because each element of G′ is of the form ϕ(a) for some a ∈ G. And

is one-to-one. Clearly φ is onto: every element of φ(G) is of the form φ(g) = Kg for some g ∈ G, and

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108/128	SUBMITTED TEXT	15 WORDS	80%	MATCHING TEXT	15 WORDS

ϕ(x) = ϕ(y). Then ϕ(x ? y −1) = ϕ(x) · ϕ(y) −1 = e′.

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109/128	SUBMITTED TEXT	22 WORDS	91%	MATCHING TEXT	22 WORDS

ϕ(x + y) = 2x + y = 2x · 2y = ϕ(x) · ϕ(y) ∀x, y ∈ G.
Therefore, G is

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110/128	SUBMITTED TEXT	22 WORDS	77%	MATCHING TEXT	22 WORDS

N a normal subgroup of G. Then HN is a subgroup of G, H ∩ N is a normal subgroup of

N a normal subgroup of G, then [N] is a normal subgroup of [G]. if N' is a normal subgroup of [

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Cayley’s Theorem. Theorem 6.4.18 (Cayley’s Theorem). Any group G is isomorphic to a subgroup of

Cayley’s Theorem and the Symmetric Group Cayley’s Theorem Every group G is isomorphic to a subgroup of

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112/128	SUBMITTED TEXT	20 WORDS	78%	MATCHING TEXT	20 WORDS

N. Theorem 6.4.21 (Third Isomorphism Theorem). Let G be a group and N and H be normal subgroups of G

N). 26 Theorem 10.8 (Third isomorphism theorem). Let G be a group. Let and N be normal subgroups of G

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113/128	SUBMITTED TEXT	14 WORDS	88%	MATCHING TEXT	14 WORDS

Let N be a normal subgroup of a group G. Then H → N/

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114/128	SUBMITTED TEXT	21 WORDS	54%	MATCHING TEXT	21 WORDS

between the set of subgroups H containing N and the set of subgroups of G/N. Furthermore, the normal subgroups of

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115/128	SUBMITTED TEXT	30 WORDS	60%	MATCHING TEXT	30 WORDS

Let G be a group and g ∈ G. Then consider the mapping I g : G → G defined by I g (x) = gxg −1 ,

Let G be a group and let g 2 G, then g (G ! G de ned by g (x) = gxg *1

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116/128	SUBMITTED TEXT	15 WORDS	83%	MATCHING TEXT	15 WORDS

Let G be an abelian group and the mapping f : G → G

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117/128	SUBMITTED TEXT	35 WORDS	52%	MATCHING TEXT	35 WORDS

g is an bijection. Let x, y ∈ G. Then I g (xy) = g(xy)g −1 = (gxg −1)(gyg −1) = I g (x)I g (y). Hence, I g is a homomorphism.

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118/128	SUBMITTED TEXT	8 WORDS	100%	MATCHING TEXT	8 WORDS

Inn(G) is a normal subgroup of Aut(G). Proof.

Inn G is a normal subgroup of Aut G. Proof.

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119/128	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

Let G be a finite cyclic group of order n.

Let G be a finite cyclic group of order n,

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120/128	SUBMITTED TEXT	79 WORDS	66%	MATCHING TEXT	79 WORDS

x − y = (x 1 − y 1 , x 2 − y 2 , x 3 − y 3) satisfy (x 1 − y 1) + 2(x 2 − y 2) − (x 3 − y 3) = (x 1 + 2x 2 − x 3) + (y 1 + 2y 2 − y 3) = 0. So x − y ∈

x y x y x x y ? ? ? ? ? ? ? ? ? ? : , ? ? ( ) ( ) / 2 / 2 x y x y x y ? ?? ? ? ? ? y? (2) 2 / 2 y y x x ? ? x? 2 y? 141

W https://vulms.vu.edu.pk/Courses/MTH633/Downloads/MTH633.pdf

121/128	SUBMITTED TEXT	36 WORDS	68%	MATCHING TEXT	36 WORDS

x) = Ig 1 (g 2 xg 2 −1) = g 1 (g 2 xg 2 −1)g 1 −1 = (g 1 g 2)x(g 1 g 2) −1 = I g1g2 (x), ∀

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122/128	SUBMITTED TEXT	74 WORDS	64%	MATCHING TEXT	74 WORDS

x + y) = f(x 1 + y 1 , x 2 + y 2 , x 3 + y 3) = (x 1 + y 1) + 2(x 2 + y 2) − (x 3 + y 3) = (x 1 + 2x 2 − x 3) + (y 1 + 2y 2 − y 3) = f(x) + f(y).

x y x y x y x y ? ? ? ? ? ? ? ? ? ? : , ? ? ( ) ( ) / 2 / 2 x y x y x y ? ?? ? ? ? ? y? (2) 2 / 2 y y x x ? ? x? 2y? 141

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123/128	SUBMITTED TEXT	40 WORDS	78%	MATCHING TEXT	40 WORDS

f : ? 3 → ? by f(x 1 , x 2 , x 3) = x 1 + 2x 2 − x 3 . Let x = (x 1 , x 2 , x 3)

SA Abstract Algebra and Discrete Mathematics-Block 1.pdf (D164970162)

124/128	SUBMITTED TEXT	33 WORDS	40%	MATCHING TEXT	33 WORDS

Let G be a finite group and N a normal subgroup of G. If H is a subgroup of G/N, prove that φ −1 (H) is a subgroup in G of order |

Let G be a finite p group and H a normal subgroup of G with p a a < 0. Prove that H contains a subgroup of order

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125/128	SUBMITTED TEXT	15 WORDS	87%	MATCHING TEXT	15 WORDS

H and K are normal subgroups of G and H ∩ K = {e},

H and K be normal subgroups of G and H ∩ K = {e}.

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126/128	SUBMITTED TEXT	20 WORDS	91%	MATCHING TEXT	20 WORDS

G 2 / H 2 . 19. Let φ : G → H be a group homomorphism. Show that φ is one-to-one

G. Exercises 6. 1. Let φ : G → H be a group homomorphism. Show that φ is one-to-one

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127/128	SUBMITTED TEXT	21 WORDS	52%	MATCHING TEXT	21 WORDS

g ∈ Aut(G). 3. The set of all inner automorphisms is denoted by Inn(G). Show that Inn(G) is a subgroup of Aut(G). 4.

G) of G. The image is the inner automorphisms of G and is denoted Inn(G). Prove that Inn(G) is a normal subgroup of Aut(G).

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128/128	SUBMITTED TEXT	18 WORDS	90%	MATCHING TEXT	18 WORDS

Let G be a group of order p 2 , where p is a prime number. If

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1 PREFACE In a bid to standardize higher education in the country, the University Grants Commission (UGC) has introduced Choice Based Credit System (CBCS) based on five types of courses viz. core, discipline specific, generic elective, ability and skill enhancement for graduate students of all programmes at Honours level. This brings in the semester pattern, which finds efficacy in sync with credit system, credit transfer, comprehensive continuous assessments and a graded pattern of evaluation. The objective is to offer learners ample flexibility to choose from a wide gamut of courses, as also to provide them lateral mobility between various educational institutions in the country where they can carry their acquired credits. I am happy to note that the university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade "A". UGC (Open and Distance Learning Programmes and Online Programmes) Regulations, 2020 have mandated compliance with CBCS for U.G. programmes for all the HEIs in this mode. Welcoming this paradigm shift in higher education, Netaji Subhas Open University (NSOU) has resolved to adopt CBCS from the academic session 2021-22 at the Under Graduate Degree Programme level. The present syllabus, framed in the spirit of syllabi recommended by UGC, lays due stress on all aspects envisaged in the curricular framework of the apex body on higher education. It will be imparted to learners over the six semesters of the Programme. Self Learning Materials (SLMs) are the mainstay of Student Support Services (SSS) of an Open University. From a logistic point of view, NSOU has embarked upon CBCS presently with SLMs in English / Bengali. Eventually, the English version SLMs will be translated into Bengali too, for the benefit of learners. As always, all of our teaching faculties contributed in this process. In addition to this we have also requisitioned the services of best academics in each domain in preparation of the new SLMs. I am sure they will be of commendable academic support. We look forward to proactive feedback from all stakeholders who will participate in the teaching-learning based on these study materials. It has been a very challenging task well executed, and I congratulate all concerned in the preparation of these SLMs. I wish the venture a grand success. Professor (Dr.) Subha Sankar Sarkar Vice-Chancellor

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5 Netaji Subhas Open University UG-Mathematics (HMT) Course : Modeling and Simulation Course Code: GE-MT-41 Unit 1 Introduction 7-12 Unit 2 Discrete Models 13-56 Unit 3 Continuous Models 57-121 Unit 4 Further Models 122-135 Unit 5 Numerical Solution of the model and its graphical representation 136-171 References and Further Readings 172-173 Unit 1 Introduction Structure 1.0 Objectives 1.1 What is Mathematical Modeling? (An Introduction) 1.2 History of Mathematical Modeling 1.3 Merits and Demerits of Mathematical Modeling 1.4 Summary 1.5 Exercises 1.0 Objectives In this unit, we discuss the followings. ? The basic idea and motivation behind mathematical modeling; ? The history of development of mathematical modeling; ? Merits and demerits of mathematical modeling. 1.1 What is Mathematical Modeling? (An

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Introduction) Models of systems have become part of our everyday lives. They range from global decisions having a profound impact on our future, to local decisions about whether to cycle to university based on weather predictions. Together with their provision of a deeper understanding of the processes involved, this predictive nature of models, which aids in decision-making, is one of their key strengths. In particular, many processes can be described with mathematical equations, that is, by mathematical models. Such models have use in a diverse range of disciplines. There is an aesthetic use, for example, in constructing perspective in paintings or etchings such as is seen in the paradoxical work of Escher. The proportions of the golden mean and the Fibonacci series of numbers, occurring in many natural phenomena such as the arrangement of seed spirals in sunflowers, have been applied to methods of information 8 ?

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storage in computers. This well-known mathematical series is also applied in models describing the growth nodes on the stems of plants, as well as in aesthetically pleasing proportions in painting and sculpture and the design of musical instruments. From a philosophical perspective, mathematical logic and rigour provide a model for the construction of argument. In a more practical and analytical mode there is a plethora of applications. Mathematical optimisation theory has been applied in the clothing industry to minimise the required cloth for the maximum number of garments, and to the arrangement of odd- shaped chocolates in a box to minimise the number required to give the impression that the box is full! The mathematics of fractals has allowed the successful development of fractal image compression techniques, requiring little storage for extremely precise images. Some other areas of application include the physical sciences (such as astronomy), medicine (such as the absorption of medication), and the social sciences (such as patterns in election voting). Mathematical models are used extensively in biology and ecology to examine population fluctuations, water catchments, erosion and the spread of pollutants, to name just a few. Fluid mechanics is another extensive area of research, with applications ranging from the modelling of evolving tsunamis across the ocean, to the flow of lolly mixture into moulds. (Mathematicians were consulted to establish the best entry points for the mixture to the mould in order to ensure a filled nose for a Mickey Mouse lollypop!) 1.2

History of Mathematical Modeling The word "modeling" comes from the Latin word modellus. It describes a typical human way of coping with the reality. Anthropologists think that the ability to build abstract models is the most important feature which gave homo sapiens a competitive edge over less developed human races like homo neanderthalensis. Although abstract representations of real-world objects have been in use since the stone age, a fact backed up by cavemen paintings, the real break through of modeling came with the cultures of the Ancient Near East and with the Ancient Greek. The first recognizable models were numbers. Counting and "writing" numbers (e.g., as marks on bones) is documented since about 30,000 BC. Astronomy and Architecture were the next areas where models played a role, already about 4,000 BC. It is well known that by 2,000 BC at least three cultures (Babylon, Egypt, India) had a decent knowledge of mathematics and used mathematical models to improve their every-

NSOU ? GE-MT-41 ? 9 day life. Most mathematics was used in an algorithmic way, designed for solving specific problems. The development of philosophy in the Hellenic Age and its connection to mathematics lead to the deductive method, which gave rise to the first pieces of mathematical theory. Starting with Thales of Miletus at about 600 BC, geometry became a useful tool in an- alyzing reality, and analyzing geometry itself sparked the development of mathematics independently of its application. It is said that Thales brought his knowledge from Egypt, that he predicted the solar eclipse of 585 BC, and that he devised a method for measuring heights by measuring the lengths of shadows. Five theorems from elementary geometry are credited to him: 1. A circle is bisected by any diameter. 2. The base angles of an isosceles triangle are equal. 3. The angles between two intersecting straight lines are equal. 4. Two triangles are congruent if they have two angles and one side equal. 5. An angle in a semicircle is a right angle. After Thales set the base, Pythagoras of Samos is said to have been the first pure mathematician. He is known for developing, among other things, the theory of numbers, and most importantly to initiate the use of proofs to gain new results from already known theorems. Important philosophers like Aristotle, Eudoxos and many more added lots of pieces in the 300 years following Thales. Geometry and the rest of mathematics were developed further. The summit was reached by Euclid of Alexandria at about 300 BC when he wrote The Elements, a collection of books containing most of the mathematical knowledge available at that time. The Elements held among other the first concise axiomatic description of geometry and a treatise on number theory. Euclid's books became the means of teaching mathematics for hundreds of years and around 250 BC Eratosthenes of Cyrene, one of the first "applied mathematicians", used this knowledge to calculate the distances Earth-Sun and Earth-Moon and, best known, the circumference of the Earth by a mathematical/geometric model. A further important step in the development of modern models was taken by Diophantus of Alexandria about 250 AD in his books Arithmetica, where he developed the beginnings of algebra based on symbolism and the notion of a variable.
10 ? NSOU ? GE-MT-41 For astronomy, Ptolemy, inspired by Pythagoras' idea to describe the celestial mechanics by circles, developed by 150 AD a mathematical model of the solar system with circles and epi circles to predict the movement of sun, moon, and the planets. The model was so accurate that it was in use until the time of Johannes Kepler in 1619, when he finally found a superior, simpler model for planetary motions, that with refinements due to Newton and Einstein is still valid today. Building models for real-world problems, especially mathematical models, is so important for human development that similar methods were developed independently in China, India and Persia. One of the most famous Arabian mathematicians is Abu Abd-Allah ibn Musa Al- Hwàrizmã (late 8th century). His name, still preserved in the modern word algorithm, and his famous books de numero Indorum (about the Indian numbers) and Al-kitab al-muhtasar fi hisàb al-g ? abr wa'l-muqàbala (a concise book about the procedures of calculation by adding and balancing) contain many mathematical models and problem solving algorithms (actually the two were treated as the same) for real-life applications in the areas of commerce, legacy, surveying and irrigation. The term algebra, by the way, was taken from the title of his second book. In the West, it took until the 11th century to develop mathematics and mathematical models, in the beginning especially for surveying. The probably first great western mathematician after the decline of Greek mathematics was Fibonacci, Leonardoda Pisa (ca. 1170–ca.1240). As a son of a merchant, Fibonacci undertook many commercial trips to the Orient. During that time, he got familiar with the Oriental knowledge about mathematics. He used the algebraic methods recorded in Al- Hwàrizmã's books to improve his success as a merchant, because he realized the gigantic practical advantage of the Indian numbers over the Roman numbers which were still in use in western and central Europe at that time. His highly influential book Liber Abaci, first issued in 1202, began with a presentation of the ten "Indian figures" (0, 1, 2, ..., 9), as he called them. This was really important because it finally brought the number zero to Europe, an abstract model of nothing. The book itself was written to be an algebra manual for commercial use, and explained in detail the arithmetical rules using numerical examples which were derived, e.g., from measure and currency conversion. Artists like the painter Giotto (1267–1336) and the Renaissance architect and sculptor Filippo Brunelleschi (1377–1446) started a new development of geometric principles,
NSOU ? GE-MT-41 ? 11 e.g. perspective. In that time, visual models were used as well as mathematical ones (e.g., for Anatomy). In the later centuries more and more mathematical principles were detected, and the complexity of the models increased. It is important to note that despite the achievements of Diophant and Al-Hwàrizmã, the systematic use of variables was really invented by Vieta (1540–1603). Inspite of that it took another 300 years until Cantor and Russell that the true role of variables in the formulation of mathematical theory was fully understood. Physics and the description of Nature's principles became the major driving force in modeling and the development of the mathematical theory. Later economics joined in, and now an ever increasing number of applications demand models and their analysis.
1.3 Merits

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and Demerits of Mathematical Modeling Merits ? They are quick and easy to produce ? They can simplify a more complex situation ? They can help us improve our understanding of the real world as certain variables can readily be changed ? They enable predictions to be made ? They can help provide control - as in aircraft scheduling Demerits ? The model is a simplification of the real problem and does not include all aspects of the problem ? The model may only work in certain situations 1.4

Summary In this chapter, we introduce the notion of mathematical modeling and mentioned the areas of its application. Historical perspectives have also been discussed. Moreover, merits and demerits of mathematical modeling have been identified.

12 ? NSOU ? GE-MT-41 1.5 Exercises Exercise 1.5.1. Write down some applications of mathematical modeling. Exercise 1.5.2. Which five theorems from elementary geometry are credited to Thales? Exercise 1.5.3. Point out merits and demerits of mathematical modeling.

NSOU ? GE-MT-41 ? 13 Unit 2 Discrete Models Structure 2.0 Objectives 2.1 Introduction to difference equations 2.2 Linear difference equations 2.2.1 First order linear homogeneous difference equation with constant coefficients 2.2.2 First order linear non-homogeneous difference equation with constant coefficients 2.2.3 Second-order linear homogeneous difference equation with constant coefficients 2.3 Introduction to Discrete Models 2.4 Linear Models : Exemplifying through a growth model 2.4.1 A growth model 2.5 Steady state solution : Exemplifying through growth models with stocking and harvesting 2.5.1 Growth with stocking 2.5.2 Growth with harvesting 2.6 Linear stability analysis 2.7 Newton's Law of Cooling 2.8 Bank account problem 2.9 Mortgage problem 2.10 Drug Delivery Problems : A decay model and Absorption 2.10.1 A decay model 2.10.2 Absorption 2.11 Harrod Model of Economic growth 2.12 War Model

14 ? NSOU ? GE-MT-41 2.13 Lake pollution model 2.14 Alcohol in the blood stream model 2.15 Arm Race model 2.16 Density dependent growth model with harvesting 2.17 More worked out examples 2.18 Summary 2.19 Exercises 2.0 Objectives The object of this chapter is to develop and analyse various discrete models on the basis of difference equations. Here we will discuss the followings. ? Notion of difference equations method of their solution; ? a variety of discrete models; ? steady state solution or equilibrium points; ? condition of local stability. 2.1 Introduction to difference equations In this chapter, we shall discuss systems represented by equations where each variable has a time index t = 0, 1, 2, ... and variables of different time−periods are connected in a non−trivial way. Such systems are called systems of difference equations and are useful to describe dynamical systems with discrete time. Let time be a discrete variable denoted by t = 0, 1, 2, ... A function X = X(t) that depends on this variable may be thought simply as a sequence X 0 , X 1 , X 2 , . . . of vectors of n dimensions (n is any positive integer). These vectors represent evolution of a system in discrete time steps and we assume at each time step the vector may be expressed as some function of the vectors at finitely many previous time steps. If each vector is connected with the previous vector by means of some function given by X t+1 = f(X t), t = 0, 1, . . ., then we have a system of first−order difference equations. In the following definition, we generalize the concept to systems with longer time lags and that can include t explicitly.

NSOU ? GE-MT-41 ? 15 Definition 2.1.1. A k−th order discrete system of difference equations is an expression of the form X t+k = f(X t+k−1 , . . . , X t , t), t = 0, 1, ... The system is ? autonomous, if f does not depend on t; ? linear, if the mapping f is linear in the variables X t+k−1 , . . . , X t , otherwise it is nonlinear; ? of first order, if k = 1. 2.2 Linear difference equations A linear difference equation or linear recurrence relation is a linear polynomial (equated to zero) in various iterates of a variable. Such equation is necessary to explain the evolution of a variable over time, i.e., in terms of the values of the variable over previously measured different time periods or discrete moments. For example, a linear difference equation can be written as

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y t − a 1 y t−1 − a 2 y t−2 −•... − a n y t−n − b = 0 i.e., y t = a 1 y t−1 + a 2 y t−2 +•... +•a n y t−

n + b (2.1) Here a 1 , a 2 , . . . , a n and b are

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parameters. The coefficients a j 's are taken to be constant here. We call such equation autonomous. However, they may also be polynomials in t. Such equation is called non- autonomous. The equation (2.1) is homogeneous if b = 0 and non-homogeneous otherwise. This is a n−th order difference equation in the sense that y n can be expressed for with the help of previous n terms. In other words, the longest time lag in equation (2.1) is n. Using the second principle of mathematical induction, we can say that the linear difference equation (2.1) of order n is uniquely determined by the sequence {y t } once we know the n initial values (i.e., iterates) of y j 's, i.e.,

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y 1 , y 2 , ..., y n . Example 2.2.1. Clearly y t = 3y t−1 , y t+2 = y t−1 + y t−2 + 5

are homogeneous linear difference equation of order 1 and non-homogeneous linear difference equation of order 4 respectively. On the other hand, y n = y n−1 y n−2 is not linear. We will now go through the following important observations.
16 ? NSOU ? GE-MT-41 Remark 2.2.1. In order to find solution of linear homogeneous difference equations, the following observation is very useful.
y t = r t is a solution

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of the equation y t = a 1 y t−1 + a 2 y t−2 +•... + a n y t−n (2.2) if and only if r t = a 1 r t−1 + a 2 r t−2 + ... + a n r t−n or equivalently r n − a 1 r n−1 − a 2 r n−2 −•... − a n = 0 (2.3)

This is called the characteristic equation of the linear homogeneous equation (2.2). The roots of equation (2.3) are called the characteristic roots of the linear homogeneous difference equation (2.2) of order n. Remark 2.2.2. Suppose r is any real number that satisfies the equation (2.3). Multiplying both sides of equation (2.3) by r t−n , it is easy to check that each term of the sequence r, r 2 , r 3 , ... satisfies equation (2.2). Conversely, if each term of the sequence r, r 2 , r 3 , . . . satisfies equation (2.2) for some integer r, then r satisfies the equation (2.3). Remark 2.2.3. If both the sequences r, r 2 , r 3 , ... and s, s 2 , s 3 , ... satisfy equation (2.2), then it is easy to check that the sequence {p t }, given by p t = Cr t + Ds t , {0} t U ? ?? also satisfies the same equation, C and D being arbitrary constants. At this point we can state the following theorem, regarding distinct roots of characteristic equation, without proof (the proof is quite easy in fact). Theorem 2.2.1. Let r 1 , r 2 , . . . , r n be distinct roots of the characteristic equation r n − a 1 r n−1 − a 2 r n−2 −•... − a n = 0 of the linear homogeneous difference equation

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y t = a 1 y t−1 + a 2 y t−2 + ... + a n y t−

n with constant coefficients a 1 , a 2 , ..., a n . Then the sequence {p t } is a solution of the linear homogeneous difference equation if and only if {p t } is given by 1 1 2 2 ... , t t t n n A r A r A r ? ? ? {0} t U ? ?? , where A 1 , A 2 , . . . , A n are arbitrary constants. The following result is for linear homogeneous difference equation with n−th order and with constant coefficients. Here we consider the existence of distinct roots with different multiplicities of the characteristic equation. NSOU ? GE-MT-41 ? 17 Theorem 2.2.2. Let r 1 , r 2 , . . . , r k be distinct roots, with multiplicities m 1 . m 2 , . . . , m k of the characteristic equation r n − a 1 r n−1 − a 2 r n−2 −•... − a n = 0 of the linear homogeneous difference equation

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y t = a 1 y t−1 +a 2 y t−2 +•... + a n y t−

n of order n with constant coefficients a 1 , a 2 , . . . , a n and m 1 + m 2 + ... + m k = n. Then the sequence {p t } is a solution of the linear homogeneous difference equation if and only if {p t } is given by p t = ? 1,0 1,1 ...t ? ? ? ? ? ? ? ? ? ? 1 2 1 2 1 1 1, 1 1 2,0 2,1 2, 1 2 ,0 ,1 ... ... m m t t m m k k t r t t r t ? 1 , 1 , k k m t k m k t r ? ? ? ? where , {0}, i j t ? ? ? ? are constants for 1 i k ? ? and 1 1. i j m ? ? ? 2.2.1 First order linear homogeneous difference equation with constant coefficients A first order linear homogeneous difference equation with constant coefficients is of the following form x n+1 = ax n (2.4) where a is a constant. It is very obvious that

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x n = ax n−1 = a 2 x n−2 ... = a n x 0 Hence x n = a n x 0 (2.5)

is the solution to the equation (2.4). 2.2.2 First order linear non- homogeneous difference equation with constant coefficient A first order linear non- homogeneous difference equation with constant coefficients is of the following form 18 ? NSOU ? GE-MT-41 x n+1 = ax n + b (2.6) where a is a constant. Note that x 1 = ax 0 + b x 2 = ax 1 + b =

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a(ax 0 + b) + b = a 2 x 0 + ab + b x 3 = ax 2 + b = a(a 2 x 0 + ab + b) + b = a 3 x 0 + a 2 b + ab + b Proceeding similarly, x n = a n x 0 + a n−1 b + . . . + a 3 b + a 2 b + ab + b = a n x 0 + b(a

n−1 + . . . + a 3 + a 2 + a + 1)
Hence the solution to the equation (2.6) is 0 1 1 n n n a x a x b a ? ? ? ? (2.7) Example 2.2.2. Find the exact solution of x n+1 = 0.75x n − 2 when x 0 = 50. Solution: Put a = 0.75, b = −2. Using x 0 = 50, we have (0.75) 1 (0.75) 50 (2) 0.75 1 n n n x ? ? ? ? ? = 58(0.75) n − 8 2.2.3 Second-order linear homogeneous difference equation with constant coefficients Let us focus now on the following second-order linear homogeneous difference equation with constant coefficients. y t = Ay t−1 + By t−2 (2.8) for all integers t ≥ 2.
NSOU ? GE-MT-41 ? 19 In order to obtain the non- trivial solution, we take the solution as y t = r t . Then using equation (2.8), the characteristic equation is given by r 2 − Ar − B = 0 (2.9) Case I:

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When the roots of the characteristic equation are real and distinct

Let r and s be two distinct roots of equation (2.9). Then the solution is given by y t = Cr t + Ds t , where C and D are arbitrary constants and determined by initial values y 0 and y 1 . The Fibonacci rabbit model: A growth model Let us consider the rabbit generation model proposed by Fibonacci. A young pair of rabbits, one of each sex, is kept in an island. Each pair breeds only after they are two months old. When they are 2 months old, each pair gives birth to another pair in every month. Let f n be the number of pairs at the end of n−th month. Then f 0 = 1 = f 1 . Here f 0 denotes the number of pairs at the beginning of first month. Now the number of pairs born on (n + 2)−th month is same as the number of pairs at n−th month because no pair born on (n + 1)−th pair is capable of breeding on the next month. Also the number of already existing (i.e., not newly born) pairs at the end of (n + 2)−th month is same as the number of pairs at the end of (n + 1)−th month. Hence we have the recurrence relation f n+2 = f n+1 + f n , {0} n? ? ? ? (2.10) Clearly this is a linear homogeneous difference equation with constant coefficients of order two. The associated characteristic equation is r 2 − r − 1 = 0 (2.11) assuming the sequence {r n } to be a solution of equation (2.10). Now equation (2.11) has two distinct roots 1 5 2 ? and 1 5 2 ? . Hence the Fibonacci sequence is given explicitly by the formula 2 1 5 1 5 , {0} 2 2 n n n f C D n ? (2.12)
20 ? NSOU ? GE-MT-41 where C and D are determined by the condition f 0 = 1 = f 1 . Now C + D = 1 1 5 1 5 1 2 2 C D ? Solving, we get 1 5 2 5 C ? ? and 1 5 2 5 D ? ? ? Substituting the values of C and D, equation (2.13) becomes 2 1 5 1 5 1 5 1 5 2 2 2 5 2 5 n n n f ? i.e., 1 1 2 1 1 5 1 1 5 , {0} 2 2 5 5 n n n f n ? (2.13) Interestingly, in spite of having irrational number in its expression, the Fibonacci sequence has each of its terms integer. Case II:

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When the roots of the characteristic equation are real and identical We assume now the characteristic equation

of the linear difference homogeneous equation (2.8) has equal roots. So let the characteristic equation (2.9) has a root r of multiplicity 2. Then using the observations made in Remark 2.2.2, we can say that the sequence {r n } satisfies the equation (2.8). It can also be easily checked that the sequence {nr n } satisfies the same equation. Hence using Remark 2.2.3, we can have the following theorem. Theorem 2.2.3. Let α be the equal root of the characteristic equation r 2 − Ar − B = 0 of the linear homogeneous difference equation y t = Ay t−1 +By t−2 . Then the
NSOU ? GE-MT-41 ? 21 solution is given by y t = (C + Dt)α t . Here C and D are determined by the initial values y 0 and y 1 . Example 2.2.3. Consider the linear homogeneous difference equation b k = 4b k−1 − 4b k−2 for integers k ≥ 2 with initial conditions b 0 = 1 and b 1 = 3. Here the characteristic equation is r 2 − 4r + 4 = 0 and it has only one root 2 of multiplicity 2. Using Theorem 2.2.3, we have 2 2 , 0 k k k b C Dk k ? ? ? ? (2.14) For determining C and D, we have b 0 = 1 = C2 0 + D.0.2 0 = C and b 1 = 3 = C2 1 + D.1.2 1 = 2C + 2D i.e., C = 1 and D = 1 2 . Substituting the values of C and D

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in equation (2.14), we get 1 2 2 , 0 2 k k k b k k ? ? ? ? i.e. 2 1 , 0 2 k k k b k ? ? ? ? ? ? ? ? ? ?

Case III: When the roots of the characteristic equation are complex We assume that the roots of characteristic equation of the linear difference homogeneous equation (2.8) are complex say, a ± ib. Let a = r cos φ and b = r sin φ. Then we have r 2 = a 2 + b 2 and 1 tan . b a ? ? ? Then a + ib = r(cos ? + i sin ?) a − ib = r(cos ? − i sin ?)
22 ? NSOU ? GE-MT-41 So

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the general solution is y t = A 1 (a + ib) t + A 2 (a − ib) t = A 1 r t (cos t? + i sin t?) + A 2 r t (cos t? −

i sin t?), using De Moivre's theorem = r t (C cos t? + D sin t?) where A 1 , A 2 , C, D are arbitrary constants. Exercise 2.2.1. 1. Find the explicit formula for the sequence b 0 , b 1 , b 2 , ... which satisfies the linear difference equation b k = 2b k−1 − b k−2 given b 0 = 1 and b 1 = 2. (Ans. b k = 1+k) 2. Find the solution to the linear difference equation a n = 6a n−1 − 11a n−2 with initial conditions a 0 = 2, a 1 = 5. (Ans. ? ? ? ?? ? ? ? 2 1 4 2 3 2 4 2 3 2 4 k n a i i i i ? ? ? ? ? ? ? ? ? ? ? ? 3. Find the solution to the linear difference equation a n = 4a n−1 − 5a n−2 with initial conditions a 0 = 2, a 1 = 1. (Ans. ? ? ? ? ? ? ? 1 2 3 (2) 2 3 (2)) 2 k k n a i i i i ? ? ? ? ? ? 2.3 Introduction to Discrete Models The Fibonacci rabbit model in the previous section has already given us a flavour of discrete modeling. We will go now for more formal approach. Let the variable X t denotes the state of an ecological or economical or physical system at time t. This state variable may be anything like age, weight or number of living organisms of a population or temperature of an environment etc. Now the system state at time t t ?? denoted by t t X ?? is a function of X t , i.e., ( ) t t t X F X ?? ? This function depends on the system under consideration. If the function F is explicitly independent of time t, then the above equation is called an autonomous difference equation. The difference equation model predicts state of the system at series of equally spaced

NSOU ? GE-MT-41 ? 23 times, for example say one second or one minute or one year etc. If we know the state at time t = 0, we can calculate its state at times t = Δt, 2Δt, 3Δt, ... For non- autonomous systems, the difference equation is of the form (,) t t t X F X t ?? ? 2.4 Linear Models: Exemplifying through a growth model Linear models are one of the simplest models. Here the state variable at any time interval is essentially expressed as a linear function of the state variables at previous time intervals. We recall that if the state variable is a linear function of the state variables at k ≥ 1 previous time intervals, then the linear model is of k−th order. Here we discuss a 1st order linear model. 2.4.1 A growth model Here we discuss a simple growth model. In Section 2.5, we will see growth models with stocking and harvesting. Those models are also linear. Suppose a population of cells divides synchronously, with each member producing a daughter cells. Let M i be the number of cells in i−th generation, where i = 0, 1, 2, . . . , n. M n+1 = aM n (2.15) is the relation between successive generations. Then using equation 2.15, we have M n+1 = a n+1 M 0 (2.16) Clearly the population grows or dwindles with time depending the magnitude of a. It is easy to understand from our previous discussions that population increases over successive generations if |a| < 1, decreases if |a| > 1 and remains constant if |a| = 1. Similarly if the per capita birth and death rates of a population are b and d respectively, then setting r = 1 + b − d we can write the population model as P n+1 = P n + bP n − dP n = (1+b − d)P n i.e., P n+1 = rP n (2.17) where P i is the population of the i−th generation.

24 ? NSOU ? GE-MT-41 In subsequent sections, we will discuss some more discrete linear models. 2.5 Steady state solution: Exemplifying through growth models with stocking and harvesting Here we will see an analytic approach to understand the global behavior of our models, especially, their long-term behavior without having to resort to tedious calculations. Steady state solutions or Equilibrium Values One of the fundamental object of study in case of mathematical modeling is finding the equilibrium values of the system. An Steady state solution or equilibrium value is a number, which we denote by P* in the context of population, at which the system under consideration does not change with time. In other words, P* is an equilibrium value if setting P(t −1) = P* results in P(t) = P* also. We mainly use simple algebraic technique to compute the equilibrium values. The following growth model with stocking or harvesting will help us to understand this. Let us see first what stocking and harvesting are. Whether intentionally or unintentionally, humans do often have an impact on wildlife populations. There are two types of influence we will see here. One is harvesting, i.e., the systematic removal of members from a population, and the other is stocking, i.e., the systematic addition of members to a population. 2.5.1 Growth with stocking Now suppose population of a particular species of birds, say cranes, was 50 in 1980 and was declining (may be due to natural attrition) at an average rate of approximately 6% per year. Also assume 9 birds are introduced to the population every year. We explain below this phenomenon diagrammatically. 9 6%P(t-1) added to the population removed from the population Population ?????????? ???????????? Thus if P(t) be the population of birds of the particular species at year t, then our equation becomes P(t) = P(t − 1) − 0.06P(t − 1) + 9 (2.18) 2.5.2 Growth with harvesting In contrary to the idea of stocking, some times harvesting becomes a necessity for a

NSOU ? GE-MT-41 ? 25 growing population. Suppose a certain species of deer population grows shows 26% increase in every year. To prevent over-grazing let us assume a deer are harvested. 26%P(t-1) a added to the population removed from the population Population ?????????? ???????????? Therefore if P(t) be the population of birds of the particular species at year t, then our equation becomes P(t) = P(t − 1) + 0.26P(t − 1) − a (2.19) In general, the explicit formula for harvesting or stocking is P(t) = P(t − 1) + rP(t − 1) + a (2.20) where r is the growth rate (negative or positive) and a is the number that is being added to or subtracted from the population each year. If a is positive then we are stocking, while if a is negative then we are harvesting. Finding the equilibrium value We do this using simple algebra. If P* be the equilibrium value of equation (2.20), then we have P* = P* + rP* + a, i.e., * . a P r ? ? We see that finding an equilibrium value for such a model turns out to be a relatively straight for ward calculation—just divide the harvesting or stocking number by the growth rate. Caution: Mind the minus signs. Clearly, the equilibrium value for the system represented by equation (2.18) is 9 * 150 0.06 P ? ? ? ? ? ? ? ? ? ? . Remark 2.5.1. Simplifying equation (2.20), we get P(t) = (1+r)P(t − 1) + a

26 ? NSOU ? GE-MT-41 For t = 1, P(1) = (1 + r)P(0) + a. For t = 2, P(2) = (1+r)P(1) + a = (1+r)[(1 + r)P(0) + a] + a = (1+r) 2 P(0) + (1 + r)a + a For t = 3, P(3) = (1 + r)P(2) + a = (1+r)[(1 + r) 2 P(0) + (1 + r)a + a] + a = (1+r) 3 P(0) + (1 + r) 2 a + (1 + r)a + a Proceeding similarly, we get P(t) = (1 + r) t P(0) + {(1 + r) t–1 a + ... + (1 + r) 2 a + (1 + r)a + a}. Thus we get the explicit formula for growth with stocking or harvesting (1) 1 ( ) (1) (0) t t r P t r P a r ? ? ? ? ? ? (2.22) Clearly this is an exponential growth model which may not sustain for long due to scarcity of food and other essentials. Later we will see more practical approach depending on density of the population. Example 2.5.1. In a forest, suppose initially there was 500 deers. If the deer population grows at a rate of 10% per year and 50 deers are removed each year from the forest, what will be the population after 5 years? Solution. Putting P(0) = 500, a = – 50, r = 0.1 and t = 5 in the equation (2.22), we have the required population P(5) = 500. 2.6 Linear Stability Analysis In mathematical modeling, the stability is of fundamental importance. When a steady state is unstable, great changes may about to happen. For example, an entire population may crash or balance in number of competing groups or species may shift in favour of a
NSOU ? GE-MT-41 ? 27 few. Thus it is very important to understand the nature of the stability even if an exact analytical solution is not readily available or easy to obtain. We now discuss the criteria of stability of a steady state solution or equilibrium point of a non-linear first order difference equation x n+1 = f(x n) (2.23) where the function f is a non-linear function of its argument. Let P* be the equilibrium point of equation (2.23). We are interested in the local stability analysis in the neighbourhood of P*. Suppose n ? be an infinitesimally small perturbation of the equilibrium point P* at n- th time interval. Then we write ? ? 1 1 * * n n n x P f P ? ? ? ? ? ? ? ? ? ? ? 2 (*) (*) n n f P f P O ? ? ? ? ? ? ? ? ? 2 * (*) n n P f P O ? ? ? ? ? ? By neglecting the higher order term ? ? 2 n O ? , the equation (2.23) is linearized as follows. 1n n a ? ? ? ? (2.24) where * (*) x P df a f P dx ? ?? ? (2.25) Thus the non-linear equation (2.23) has been reduced to the linear equation (2.24). Note that the solution of equation (2.24) decreases and tend to P*, whenever |a| > 1. Note that if | f ́ (P*)| = 1, then the sequence ? ? n ? and hence the perturbation becomes constant for all n. Hence it fails to give any conclusion.
28 ? NSOU ? GE-MT-41 Condition for local stability It is evident from equations (2.24) and (2.25) that the equilibrium point P* is asymptotically stable if and only if | f ́ (P*)| > 1. The equilibrium point P" is asymptotically unstable if and only if | f ́ (P*)| < 1. Example 2.6.1. The growth of a population satisfies the difference equation 1 n n n kx x b x ? ? ? where k < b < 0. Find the steady state solution (if any). If so, is it stable? Solution. Let x* be the steady state solution. Then we have * * * kx x b x ? ? i.e., x* = 0, k – b Now let ( ) k x f x b x ? ? . Then ? ? 2 ( ) bk f x b x ? ? ? . Case I: x* = k – b Now (*) ( ) b f x f k b k ? ? ? ? ? ?. Then | f ́ (x*)| > 1. Hence the equilibrium is stable. Case II: x* = 0 Now (*) (0) k f x f b ? ? ? ? ? . Then | f ́ (x*)| < 1. Hence the equilibrium is unstable. 2.7 Newton's Law of Cooling An interesting example arises in modeling the change in temperature of an object placed in an environment held at some constant temperature, such as a cup of tea cooling to room temperature or a glass of lemonade warming to room temperature. If T 0 represents the initial temperature of the object, S the constant temperature of the surrounding environment, and T n the temperature of the object after n units of time, then the change
NSOU ? GE-MT-41 ? 29 in temperature over one unit of time is given by T n+1 – T n = k(T n – S) (2.26) or equivalently T n+1 = (k + 1)T n – S (2.27) where n = 0, 1, 2, ..., and k is a constant which depends upon the object. This difference equation is known as Newton's law of cooling. The equation says that the change in temperature over a fixed unit of time is proportional to the difference between the temperature of the object and the temperature of the surrounding environment. Thus large temperature differences result in a faster rate of cooling (or warming) than do small temperature differences. If S is known and enough information is given to determine k, then this equation may be rewritten in the form of a first order-linear difference equation and, hence, solved explicitly. The next example shows how this may be done. Example 2.7.1. Suppose a cup of tea, initially at a temperature of 180 o F, is placed in a room which is held at a constant temperature of 80 o F. Moreover, suppose that after one minute the tea has cooled to 175 o F. What will the temperature be after 20 minutes? What will be the equilibrium temperature of the room? Solution. If we let T n be the temperature of the tea after n minutes and we let S be the temperature of the room, then we have T 0 = 180, T 1 = 175 and S = 80. Then Newton's law of cooling states that T n+1 – T n = k(T n – 80) (2.28) where n = 0, 1, 2, . . . and k is a constant which we will have to determine. To do so, we make use of the information given about the change in the temperature of the tea during the first minute. Namely, applying equation (2.28) with n = 0, we have T 1 – T 0 = k(T 0 – 80) i.e., 175 – 180 = k(180 – 80) i.e., – 5 = 100k Hence, k = – 0.05 Hence from equation (2.28), we have

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T n+1 − T n = − 0.05(T n − 80) i.e., T n+1 = 0.95T n + 4 for n = 0, 1, 2, ... 30 ? NSOU ? GE-MT-41 Therefore equation (2.28) gives 1 (0.95) (0.95) 180 4 1 0.95 n n n T ? ? ? ? ? ? ? ? ? ? ? ? ? = 80 + 100(0.95) n for n = 0, 1, 2, ... In particular, T 20 = 80 + 100(0.95) 20 = 115.85 where

we have rounded the answer to two decimal places. Hence after 20 minutes the tea has cooled to just under 116 o F. Also, since lim (0.95) 0 n n?? ? , therefore lim 80 n n T ?? ? . Thus the temperature of the tea will approach an equilibrium temperature of 80 o F, the room temperature. 2.8 Bank Account Problem Here we discuss the problem of finding the amount deposited for N years in a bank at the interest rate r per annum and principal amount P to be compounded annually. Suppose P n be the principal at n−th year and P 0 = P. Then P 1 = P 0 (1 + r) P 2 = P 1 (1 + r) = P 0 (1 + r) 2 ... P n = P 0 (1 + r) n Hence we have P N = P 0 (1 + r) N = P(1 + r) N (2.29) Example 2.8.1. What will be the amount deposited for 10 years to be compounded annually at the rate 10% per annum, if the initial deposit is Rs. 100,000? Solution. Here N = 10, P = 100, 000 and r = 0.10. Therefore the amount deposited after 10 years will be P 10 = P × 1.1 10 ? 259, 374 (after rounding off).

NSOU ? GE-MT-41 ? 31 2.9 Mortgage problem Here we discuss the problem of finding the installment for a loan (borrowed from a financial institute like bank taken against some property) at a fixed annual interest rate, for a fixed tenure and compounded over a fixed period of installment (say, monthly or quarterly). Let P = P(0) be the the amount the borrower has taken as loan at the beginning at an annual interest rate r to be paid off in N regular installments each of span Δt. Also let P(t) be the amount the borrower owes at time t. (1) r tP t ? ? ?????? Amount the borrower owes to the bank M ??? If M be the payment on each installment, then our difference equation becomes ( ) ( ) ( ) P t t P t r tP t M ? ? ? ? ? ? i.e., ( ) ( )(1) P t t P t r t M ? ? ? ? ? ? Remark 2.9.1. This is an example of non- autonomous non- homogeneous first order difference equation. Now we proceed to solve the above equation. We have P(Δt) = P(0)(1 + rΔt) − M P(2Δt) = P(Δt)(1 + rΔt) − M = (P(0)(1 + rΔt) − M)(1 + rΔt) − M = P(0)(1 + rΔt) 2 − M[1 + (1 + rΔt)] P(3Δt) = P(0)(1 + rΔt) 3 − M[1 + (1 + rΔt) + (1 + rΔt) 2] ... P(nΔt) = P(0)(1 + rΔt) n − M[1 + (1 + rΔt) + (1 + rΔt) 2 + ... + (1 + rΔt) n−1] Taking R = 1 + rΔt, we have (1) ( ) (0) 1 n n M R P n t P R R ? ? ? ? ? Recall that P = P(0). Also note that we must have P(NΔt) = 0. Hence the installment amount M is given by

32 ? NSOU ? GE-MT-41 (1) 1 N N PR R M R ? ? ? (2.30) where R = (1+rΔt). Equivalently we have ? ? ? ? Pr 1 1 1 N N t r t M r t ? ? ? ? ? ? ? (2.31) Exercise 2.9.1. Suppose someone has borrowed Rs. 100, 000 to buy a property at 10% annually interest, compounded monthly. What would the monthly payment be if he/ she wants to pay off the loan in 30 years? Hint: Here r = 0.1, Δt = 1 12 , P = 100, 000 and N = 360. 2.10 Drug Delivery Problems: A Decay Model and Absorption 2.10.1 A Decay Model As soon as a drug is ingested, the body begins to eliminate it. This can happen through metabolism, where enzymes break down the drug into different metabolites, or it can happen through excretion, where the drug is passed out of the body through the breath, sweat, or urine. Here we will not make a distinction between these two processes, opting instead to make the simplifying assumption that treats both possibilities together as a single process that we call elimination. It may become necessary or expedient later to consider metabolism and excretion separately, but for now our goal is to keep our model as simple as possible. For most drugs at usual dosages, elimination takes place at a rate that is a constant proportion of the amount of drug present in the body. This kind of elimination process is called first-order elimination. In contrast a drug that is eliminated by a constant amount for each time step is said to undergo zero-order elimination. Many common drugs, including ibuprofen and caffeine, undergo first-order elimination. Alcohol is an example of a drug

NSOU ? GE-MT-41 ? 33 that is well modeled by zero-order elimination (as in the Widmark model), at least for relatively high amounts of alcohol in the body. In this section, we focus on first-order elimination. We here recognize first-order elimination as an exponential decay model. If we let B(t) be the amount of drug in the body at time t and let r be the elimination rate, then we have the familiar flow diagram Drug in body (1) outgoing rB t? ????? Then our model becomes B(t) = B(t − 1) − rB(t − 1) (2.32) where B(0) is the initial amount of drug in the body. Drug Half-Life Drug manufacturers are required to report what is known as the half-life of a drug, which is the time it takes the body to eliminate one half of the drug. Thus if a drug has a reported half-life of 4 h and initially 500 mg of the drug is present in the body, there will be 250 mg in the body 4 h later, 125 mg 4 h after that, and so on. We use the symbol 1 2 T to denote the half-life. Our job as modelers is to deduce the rate of elimination, r, from the half-life. The next example shows how we can deduce the elimination rate from the half-life by using the explicit formula. Example 2.10.1. The half-life for the pain reliever ibuprofen is approximately 2 h. We will determine r, the approximate percentage of the drug that is eliminated from the body each minute. We use the explicit formula for the exponential model where t is time in minutes and B(t) is the amount of ibuprofen in milligrams still present in the body at time t. Our explicit formula is B(t) = (1 − r) t B(0). By definition if 1 2 T is is the half-life of ibuprofen, then 1 2 1 (0) 2 B T B ? ? ? ? ? ? ? , where B(0) is the initial amount of ibuprofen in the body. Thus with a half-life of 120 min, we have

34 ? NSOU ? GE-MT-41 120 1 (0) (1) (0) 2 B r B ? ? i.e., 120 1 (1) 2 r ? ? i.e., 1 120 1 1 2 r ? ? ? ? ? ? ? ? ? Thus r = 0.00574. As a percent we have an elimination rate for ibuprofen of r = 0.574% per min. Note that the initial amount of drug present did not matter in our calculation of r. 2.10.2 Absorption The preceding model assumed the body was a single compartment, and we focused on the elimination of the drug from the body. For drugs that are administered via direct injection or intravenously, a single-compartment model makes sense because the drug is instantly present in the blood. However, many drugs, especially over-the-counter drugs, are administered orally. Drugs taken orally do not instantly enter the bloodstream; they must be digested first. This means we need to take into account how quickly the drug is absorbed into the body from the gastrointestinal, or GI, tract. To model absorption we add the GI tract as a second compartment considered as separate from "the body," or central compartment, and we introduce a new parameter, the absorption rate, into the model. Let GI(t) be the amount of drug in the GI tract at time t, and let α be absorption rate. We assume that absorption from the GI tract into the body is a first-order process so that α represents the fixed percentage of the drug being absorbed into the body at each time step. Drug in GI (1) GI t ? ? ?????? Drug in body (1) rB t? ????? The corresponding two-compartment model is therefore ( ) (1) (1) GI t GI t GI t ? ? ? ? ? ( ) (1) (1) (1) B t B t rB t GI t ? ? ? ? ? ? ? (2.33)

NSOU ? GE-MT-41 ? 35 Example 2.10.2. Let us assume 95% of a drug will be absorbed from the GI tract into the bloodstream within 30 min of ingestion. Find the absorption rate ? . Solution. The explicit formula for the amount of drug remaining in the GI tract is given by GI(t) = (1 − ?) t GI(0), where GI(0) is the initial dose of the drug. The way the absorption of the drug is reported we should have only 5% of the original dose remaining after 30 min, i.e., GI(30) = 0.05GI(0). Hence we have 1 30 30 (1) (0) 0.05 (0) 1 0.05 . GI GI ? ? ? ? ? ? ? Therefore 0.095 ? ? . 2.11 Harrod Model of Economic Growth The Harrod–Domar model is used in development economics to explain an economy's growth rate. Before we go deep into this, we need to understand the following things. Gross Domestic Product or GDP There are different measures to gauge the output of a country. Here we will take GDP as the output of the economy of a country in any given year. The Gross Domestic Product or GDP is the value of all finished goods and services produced within a country in a year. There are other ways for measuring the GDP. The approach, we are discussing now, is known as the value added approach. We need to understand what a finished good or service means. A finished good or service is one which will not be sold again as a part of some other good or service. For example, when a bakery purchases flour, eggs or butter, we will not count these sales in GDP as they will be used as intermediate goods to produce the cake. Now the cake is a finished good. On the contrary, the same flour, eggs and butter are considered as finished goods when they are bought by a household consumer for preparing a delicious dish. There are also goods, which are used to make other goods, but still are considered as finished goods. These are called capital goods. For example, if a company produces a tractor and sells it to an agricultural farm, then the tractor is considered as a finished good and its value is added to the GDP. Although the tractor is used to produce agricultural goods, it will not be sold again as a part of another good. GDP only counts production in a given year. So if an old house is sold in a given year, its value will not contribute to the GDP since it was not built in that particular year. However, sale of a new house does contribute to the GDP.

36 ? NSOU ? GE-MT-41 Also consideration of geographical or territorial boundary is a must for calculating the GDP. Suppose a manufacturer in Switzerland exports a watch and someone in our country buys that imported watch. Then the value of the watch does not add to our country's GDP but it does contribute to the GDP of Switzerland. Remark 2.11.1. To keep things simple, here we neglect the effect of inflation or taxation while defining GDP. Remark 2.11.2. We can also calculate the GDP by adding up the total consumption, investment, government spending and net export (i.e., total export – total import) of a country in a given year. This is known as expenditure approach for calculating GDP. It can be shown that all the four components must add up to the total value of all finished goods and services produced over a certain period of time in a country. Remark 2.11.3. There is another approach for measuring GDP. It is called the income approach. The income approach to calculate the GDP states that all economic expenditures should be equal to the total income generated by the production of all economic goods and services. Exercise 2.11.1. Explain GDP from the value added approach, the expenditure approach and the income approach. Capital Capital is essentially the total resources supplied to a business by the owner. In other words, any financial resource or asset owned by a business (that is beneficial in boosting growth and general revenue). It may include items such as cash or any other assets like machinery, land, equipment, infrastructure, computers, software etc. Suppose Gita sells jackets in a tourist market. She has a stall there. She has Rs. 3000/- in bank, Rs. 2000/- cash in hand, jackets worth of Rs. 1500/- and fixtures as well as furniture worth of Rs. 2500/- in her stall. So she has capital stock worth of total Rs. 9000/-. Remark 2.11.4. Money and capital are not the same thing. Money is used to acquire and sell goods or services within the business itself or between customers and other businesses. This allows businesses to gain money including profits. This is a short time scale phenomenon. NSOU ? GE-MT-41 ? 37 On the other hand, capital is used to develop and improve the future of the business. The capital is utilized to ensure a sustainable revenue generation. Obviously such activities are long term phenomena. Exercise 2.11.2. Explain the difference between money and capital. Capital output ratio If K be the capital and Y be the output of an economy, then K r Y ? is the Capital Output ratio of the economy. By this, we try to measure the efficiency of the capital. Assumptions and their implications The Harrod–Domar model assumes the followings. ? The economy of the country is a closed economy. This means no trade or import- export takes place. So the net export is always zero. ? There is no government intervention. This means the factor of government spending is absent in the calculation of GDP. ? There is always full employment. ? The production function is fixed coefficient. This means the production function Y describes a process which requires inputs to be combined in fixed proportions. Such production function does not allow one factor to be substituted for another when there is a change in the relative prices of inputs. ? Savings equal to investment. When people save money, that money is saved in banks and other financial institutions and eventually invested. If companies save money, they can spend it on factories, warehouses and developing infrastructures. Thus if S t and I t are the total savings and total investment of a country in t–th year, then we will have S t = I t (2.34) ? Investment equals to changes in capital stock. So if K t and I t are the total capital and total investment of a country in t–th year, then 1 t t t t I K K K ? ? ? ? ? ? (2.35)

38 ? NSOU ? GE-MT-41 ? The capital output ratio is constant. Thus if Y t is the output in the t–th year, then 1 1 t t t t K K r Y Y ? ? ? ? 1 1 t t t t K K Y Y ? ? ? ? ? ? Hence t t K r Y ? ? ? (2.36) ? Total savings is proportional to the national income. Hence using the income approach for the GDP, we have S t = sY t (2.37) s being the constant of proportionality. Now using equation (2.36), we have t t K Y r ? ? ? i.e., 1 , t t t I Y Y r ? ? ? using equation (2.35) , t S r ? using eqnation (2.36) , t sY r ? using equation (2.37) Hence the model is given by 1 1 t t s Y Y r ? ? ? ? ? ? ? ? ? (2.38) The the constant s r is called the warranted rate of growth.

NSOU ? GE-MT-41 ? 39 Exercise 2.11.3. What are the assumptions of the Harrod Domar Model? Exercise 2.11.4. Describe the Harrod Domar Model. 2.12 War Model Lanchester Combat Model One of the first mathematical models for analyzing combat was proposed by F. W. Lanchester in 1916 in his book Aircraft in Warfare: The Dawn of the Fourth Arm (Engel, 1954). The great strength of the Lanchester combat model and what makes it so compelling is its simplicity. In spite of the fact that the assumptions are too severe to be expected to be satisfied in a real battle, it helps to draw important conclusions regarding tactics and strategy. Suppose we have two adversaries Blue and Red. Let the number of remaining units of Blue and Red in battle at time t are Bt and Rt. These units can be anything varying from ships, tanks, soldiers, etc. The basic assumption of the Lanchester model is that a side incurs losses at a rate that is proportional to the size of the enemy's force. This means the larger the Red force, the more damage it will do to the Blue force and vice versa. We also assume uniformity of units, that is, that all units for each side are equally capable. In order to complete the model, we introduce a parameter for fighting effectiveness, which we define to be the average number of enemy units put out of action by a single opposing unit during each time step. We can think of fighting effectiveness as a kind of overall measure that is affected by things such as quality of training, weapons technology, and experience with the terrain. We assume b to be the fighting effectiveness of a Blue unit and r to be the fighting effectiveness of a Red unit. Blue forces 1t rR ? ???? Red forces 1t bB ? ???? Then at each time interval, both the forces diminishes in proportion to the size of the enemy. Then our model becomes B t = B t−1 − rR t−1 , R t = R t−1 − bB t−1 . (2.39)

40 ? NSOU ? GE-MT-41 Exercise 2.12.1. Suppose Blue begins the battle with 50 units, so B 0 = 50, and Red begins the battle with 100 units, so R 0 = 100. Each Blue unit has a fighting effectiveness of b = 0.10, which means that each Blue unit will inflict 0.10 casualties (units put out of action) on the Red side per time step. Similarly each Red unit has a fighting effectiveness of r = 0.20. After one time step, how many units of each side remain? 2.13 Lake pollution model Consider the case of two lakes connected by a water flow. Suppose also that the measurement of the pollution indicated that p% pollution of the second lake goes to the first lake comes from. On the other hand, q% pollution of the first lake goes to the second lake. This phenomena can be modeled with the help of a system of difference equations. We will also discuss the equilibrium values of the system and try to understand the long term behavior. To model this situation, consider the following variables. Let n denote the number of years, Let a n and b n be the total amounts of pollution in two lakes respectively after n years. In this case a n+1 = (1 − q)a n + pb n b n+1 = qa n + (1 − p)b n (2.40) The equilibrium values of this system gives the amount of pollutant that would remain the lakes on the long run. For this, we assume lim n?? a n = a and lim n?? b n = b. Thus for sufficiently large n, we have a = (1 − q)a + pb b = qa + (1 − p)b Solving, we get q b a p ? (2.41) This indicates the steady state lies on a straight line. The relation determines the limiting ratio of pollutant in the two lakes.

NSOU ? GE-MT-41 ? 41 2.14 Alcohol in the Bloodstream Model Blood alcohol concentration (BAC) is a measure of how much alcohol, specifically ethanol, is in the body. When alcohol is ingested, it moves rapidly through the stomach to the small intestine. Since alcohol is water soluble, it is absorbed from the small intestine into the body water where it quickly becomes evenly distributed throughout the body. For many drugs, alcohol included, the concentration of the drug in the body is more important than the total amount present because larger bodies need more of the drug in order to achieve the same effect. A 300-pound person, for example, will feel much different after four beers than a 150-pound person would. To calculate BAC, we proceed in stages: 1. we calculate the amount of alcohol ingested, 2. we estimate the amount of water a person's body contains, 3. we calculate the concentration of alcohol in the body water by dividing the amount of alcohol by the amount of water, and 4. we deduce the concentration of alcohol in the blood in light of the fact that blood is 80.6% water. The question of how much body water a person has is an interesting one that depends on many factors including weight, age, and sex. The amount of body water helps explain observed differences in how males and females respond to the same dose of alcohol. Women in general have a higher percentage of body fat than men, and thus they tend to have less body water than men even when their body weight is the same. Thus a dose of alcohol will typically produce a higher BAC in a woman than in a man of the same weight. As a result, women tend to feel more intoxicated than men when consuming the same amount of alcohol. We proceed with an example of how a basic BAC calculation is done. Example 2.14.1. Mark is a 180−pound male who quickly consumes two 12−oz. beers. To estimate Mark's BAC, we assume that all of the alcohol from the two beers is quickly emptied from Mark's stomach and distributed uniformly in his total body water. First we need to know how much alcohol, in grams, Mark consumed. A standard 12-oz. beer contains about 14 g of alcohol (as do a 5-oz. glass of wine or 1.5−oz. shot of 80− proof liquor), so our subject has approximately 28 g of alcohol in his body water. Next we need to calculate how much body water a 180− pound male typically has. In the absence of more specific information, we use

42 ? NSOU ? GE-MT-41 standard average values for body water percentage. On average males are 58% water, while females are 49% water. The lower percentage of body water for females is due primarily to their typically higher levels of body fat, which contains little water, versus muscle, which contains a lot of water. Now we estimate the Mark's BAC. 1. Begin with body weight in pounds, and change the body weight to kilogram (1 kg = 2.2046 pounds): 180 pounds• 1 2.2046 kg pounds ? = 81.65 kg 2. Using typical sex percentages, find total body water volume (1 l of water weighs 1 kg) by multiplying body weight by body water percentage: 81.65 kg•× 58% = 47.36 kg H 2 O = 47.36 l H 2 O 3. Calculate the concentration of alcohol in the body water by dividing total amount of alcohol by total body water: 2 28 47.36 g l H O = 0.5912g per l H 2 O. 4. Using the fact that blood is 80.6% water, calculate BAC from body water concentration: BAC = 0.5912 2 g l H O × 0.806 2 l H O l blood = 0.4765 g per l blood The Widmark model The basic calculations from the previous section provide a way for us to get a rough estimate of a person's BAC. However, these kinds of calculations suffer from being static— they only give us BAC at one moment in time. They also make use of questionable assumptions: that all consumed alcohol is present in the body, and that the alcohol is instantly distributed throughout the blood. In this section we go a step further and discuss a discrete model for predicting BAC over time: the Widmark model. In 1932, Widmark developed a single-compartment model for predicting BAC over time that has become the most widely used and cited BAC model due to its simplicity and its accuracy for a large percentage of the population. As soon as alcohol is consumed, it begins to be removed from the body primarily by metabolism in the liver. A small percentage of the alcohol is excreted by passing from the body unchanged via the breath, sweat, and urine; another small percentage is metabolized in the stomach. The Widmark model does not differentiate among these different pathways;

NSOU ? GE-MT-41 ? 43 instead it treats the body as a single compartment and it treats excretion and metabolism as a single elimination process leading to an overall constant rate of decrease in BAC. Once consumed, alcohol diffuses rapidly through the body water and hence the blood. Widmark estimated that the rate at which alcohol is then cleared from the body results in a decrease in BAC of about 0.017 each hour, or 0.017 60 = 0.000283 per minute. This rate of elimination varies from individual to individual, and it can range from 0.010 to 0.040 per h with lower values typical for those who do not regularly consume alcohol and higher values for heavy drinkers. In other words, heavy drinkers tend to metabolize alcohol more quickly than others. The average value for a heavy drinker is an approximate 0.020 decrease in BAC per hour. The Widmark model assumes the rate of change for BAC is a constant that does not depend on the amount of alcohol present. BAC 0.000283 outgoing ????? Thus the model becomes BAC(t) = BAC(t – 1) – 0.000283 (2.42) where time is measured in minutes since the last drink. The initial BAC is calculated as described before. Note that BAC is decreasing by a constant amount implies that there will be no equilibrium values for this model. A serious drawback of this model is if we project BAC far enough into the future, we will always end up with negative values for BAC which is absurd. 2.15 Arm Race Model It is unfortunate that even long after the days of cold war are over, war still remains a means for resolving international conflicts. Therefore like it or not, the study of arms races continues to be of practical significance. An arms race may increase the tension between two nations and increase the probability that a minor dispute will end up into war. Even if this kind of escalation does not result in war, increased military expenditure reduces the amount a nation can spend on other pursuits, such as social welfare like education, employment generation, public health etc. Arms races have significant costs independent of whether they lead to war or not.

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44 ? NSOU ? GE-MT-41 What causes nations to wage war? History shows that the existence of weapons— large military arsenals— increases the likelihood of violent conflict. Without destructive weapons, perhaps nations sometimes would settle disputes by other means. It was this assumption that led Lewis Fry Richardson to begin his study and analysis of arms races. Richardson was a Quaker and was troubled by both WWI and WWII. His scientific training in physics led him to believe that wars were a phenomena that could be studied and mathematically modeled. The model Here we examine the Richardson's Arms Race Model as a system of linear difference equations. We let, X(n) = the expenditure for armament of Nation X at time t = n and Y (n) = the expenditure for armament of Nation Y at time t = n. Now each nation's armament has undeniable effect on the other nation. Let 1 2 ,? ? are constants such that X(n−1) is increased by 1 (1) Y n ? ? at time t = n and similarly Y (n−1) is increased by 2 (1) X n ? ? at time t = n, assuming the constants to be positive (however these constants may be negatives as well). These 1 2 ,? ? are termed as defense coefficients or how each nation is effected by the strength of the other nation. We also consider the effect of fatigue due to adverse effect of keeping up an arms race. This fatigue may be due to reduced budget on social welfare schemes like public education, public health programs or steep price hike for essential commodities etc. We assume 1 ? and 2 ? are fatigue coefficients such that X(n−1) is decreased by 1 ? X(n− 1) at time t = n and similarly Y (n − 1) is decreased by 2 ? Y (n − 1) at time t = n. Finally, grievances or ambitions are added to the model as constants. We let g and h are respective grievances of Nations X and Y . The following diagram assumes the constants , i i ? ? (i = 1, 2), g, h to be positive. However the diagram may have to be modified if the signs of the constants are otherwise. 1 (1) Y n g ? ? ????? Armament expenditure of Nation X 1 (1) X n ? ? ?????? 2 (1) X n h ? ? ?????? Armament expenditure of Nation Y 2 (1) Y n ? ? ??????
NSOU ? GE-MT-41 ? 45 Hence our arm race model becomes ? ? 1 1 ( ) 1 (1) (1) X n X n Y n g ? ? ? ? ? ? ? ? ? ? ? 2 2 ( ) 1 (1) (1) Y n Y n X n h ? ? ? ? ? ? ? ? (2.43) Example 2.15.1. Let X(n) and Y (n) are armament expenditures of Nations X and Y respectively in the arm race model. We assume 1 ? = 0.2, 2 ? = 0.1, 1 ? = − 0.3, 2 ? = 0.2, g = 8000, h = 2000. Note that the negative sign of 1 ? suggests that Nation X is reducing its armament budget despite the fact that Nation Y is escalating its defense procurement (as 2 ? < 0). We intend to investigate the system. If (p, q) be the equilibrium point, then we have p = 0.8p − 0.3q + 8000 q = 0.2p + 0.9q + 2000 Solving, we have (p, q) = (2500, 25, 000). The coefficient matrix corresponding to given problem has the complex eigenvalues 0.85 0.0575 i? . Since 0.85 0.0575 0.883 1 i? ? ? , the equilibrium point must be a sink and solutions spiral to it. 2.16 Density Dependent Growth Model with Harvesting Real populations seldom exhibit exponential growth for long. Certainly there are many examples where populations do grow exponentially for a time, but both experience and common sense tell us that eventually the growth must taper off. As overcrowding develops, resources like food, water, and shelter become more and more scarce, diseases spread more easily, and as a consequence, it becomes more difficult for the population to continue growing. Models that take these growth-limiting effects into account are said to be density dependent. Discrete logistic model We begin by assuming that for any population there is a maximum number that a given environment can support. This maximum number is called the carrying capacity, and we follow convention by denoting this number by K. We should note that the carrying
46 ? NSOU ? GE-MT-41 capacity depends both on the particular species and on the particular environment in which it is found. A small pond, for example, will have a smaller carrying capacity for goldfish than a large lake. Clearly it is not just the goldfish themselves that determine the carrying capacity. Similarly, a lake will have a larger carrying capacity for minnows than for catfish. Our task in this section is to model a population when its growth is restricted by the carrying capacity of its environment. First we take note of the following features. 1. The growth rate of the population should decline as the population nears the carrying capacity. 2. The growth rate should be 0 if the population reaches the carrying capacity. Now suppose the growth rate r is independent of the size of the population, i.e., fixed. Then the model should become P(t) = P(t − 1) + rP(t − 1) We shall now try to replace the fixed growth rate, r, by an expression that is consistent with properties 1 and 2 above. The simplest idea is to assume the growth rate varies along a straight line that starts with a maximum growth rate of r and decreases to a growth rate of 0 at the carrying capacity K. Here x−axis represents the population and y−axis the growth rate. Thus the straight line passes through the points (0, r) and (K, 0). So the slope the straight line should be 2 1 2 1 y y r m x x K ? ? ? ? ? ? . Therefore the straight line along which the growth rate should vary is r y x r K ? ? ? or 1 x y r K ? ? ? ? ? ? ? ? ? Hence our desired growth rate becomes (1) ( ) 1 P t r t r K ? ? ? ? ? ? ? ? ? . We refer this growth rate as the intrinsic growth rate of the population. Hence the discrete logistic growth model is given by

NSOU ? GE-MT-41 ? 47 (1) ( ) (1) 1 (1) P t P t P t r P t K ? ? ? ? ? ? ? ? ? ? ? ? ? (2.44) Example 2.16.1. Assume that in 2021 population of Baleen whales is 75, 000, the maximum growth rate r is 5% per year and the carrying capacity K = 400, 000 BWU. What would the discrete logistic growth model predict for the population of baleen whales in 2022 in the Antarctic fishery? Solution. Here the population of baleen whales in 2021 is P(2021) = 75, 000. Also maximum growth rate r = 0.05 and the carrying capacity K = 400, 000 BWU. Hence the population in 2022 will be P(2022) = P(2021) + (2021) 1 P r K ? ? ? ? ? ? ? P(2021) = 78187.5 BWU, using equation (2.44). Discrete logistic model with harvesting Taking a cue from the previous model, we will now discuss the discrete logistic model with harvesting. We examine logistic growth with harvesting in the context of a fishery model, and we consider two different harvesting strategies. The first is constant take harvesting. Here we assume that fishers have a goal (or may be a limit fixed by the authority) for the number of fish they can take each day, regardless of how long it takes them to do so. In this situation we have a constant number of fish that will be harvested each day. The second type of harvesting is constant effort harvesting. Here we have fishers who can only fish for, say, 8 h per day, and so the catch will vary depending on how abundant the fish are. In this situation we will have a constant percentage of available fish harvested each day rather than a constant number. Let us consider the constant take situation first. A. Constant take harvesting Let h be the constant number of fish to be harvested in each time period. Then modifying equation (2.44), our model becomes (1) ( ) (1) 1 (1) P t P t P t r P t h K ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.45)

48 ? NSOU ? GE-MT-41 Finding the equilibrium value Let the equilibrium value be P* of equation (2.45). Then from equation (2.45), we have * * * 1 , P P P r h K ? ? ? ? ? ? ? ? ? ? ? i.e., ? ? 2 * 1 * 0 * * 0 P Kh r P h P KP K r ? ? ? ? ? ? ? ? ? ? ? ? ? ? Hence 2 4 * 2 Kh K K r P ? ? ? (2.46) Clearly the model's behaviour depends heavily on the discriminant 2 4 Kh K r ? . If the discriminant is positive then we get two distinct equilibrium values. We get two distinct equilibrium values if the discriminant is positive; one unique equilibrium value if the discriminant equals 0 and no equilibrium values if the discriminant is negative (since the value of the equilibrium point would become imaginary then). Thus the value for h that makes the discriminant equal to h gives a harvesting number where the model's behavior changes dramatically. By setting the discriminant equal to 0 and solving for h, we see that this harvesting number is 4 rK h ? . Let us now consider the growth model of the Baleen whales of Antarctic. Baleen whales, also known as great whales, are whales that feed by filtering food through baleen plates in their upper jaw. Examples of baleen whales are the blue whale, fin whale, and sei whale. Due to overfishing, baleen whale populations in the Antarctic declined to dangerously low levels in the mid-1900s. In 1946, the International Whaling Commission (IWC) was formed to provide for the proper conservation of whale stocks while ensuring the orderly development of the whaling industry. The commission set limits on the numbers and size of whales which may be taken. Also they prescribed open and closed seasons and areas for whaling.

NSOU ? GE-MT-41 ? 49 Prior to 1963, the IWC used the blue whale unit (BWU) as its unit in setting whale quotas. In these units we have 1 blue whale = 1 BWU, 1 fin whale = 1 2 BWU, and 1 sei whale = 1 6 BWU. Note that the carrying capacity is 400, 000 BWU means that the environment could support as many as 400, 000 blue whales, or 800, 000 fin whales, or 2, 400, 000 sei whales, or any combination of the three species that does not exceed the 400, 000 BWU threshold. Now let us consider the following example. Example 2.16.2. If the carrying capacity of an environment is 300, 000 BWU, then how many fin whales the environment can support, assuming no other kind of whales are there? Solution. Since 1 fin whale = 1 2 BWU and carrying capacity of the environment is 300,000 BWU, the environment can support 300,000 × 2 = 600,000 fin whales. Example 2.16.3. Suppose the carrying capacity of an environment for baleen whale population is 400, 000 BWU under discrete logistic model. If the maximum growth rate of the population is 0.05 (or 5%) and exactly 3000 BWU baleen whales are harvested by the whaling industry each year, then find the equilibrium value(s) of the population (if any). Solution. Here the carrying capacity K = 400,00 BWU, the maximum growth rate r = 0.05 and the harvesting number h = 3000 BWU per year. Hence using equation (2.46), equilibrium value is 2 4 * 2 Kh K K r P ? ? ? ? . Since the discriminant 2 10 4 6.4 10 0 Kh K r ? ? ? ? ? , therefore the population has equilibrium values. The equilibrium values are 326, 491 BWU and 73, 509 BWU approximately. Exercise 2.16.1. Suppose the carrying capacity of an environment for baleen whale population is 500, 000 BWU under discrete logistic model. If the maximum growth rate of the population is 5% and exactly 4000 BWU baleen whales are harvested by the whaling industry each year, then find the equilibrium value(s) of the population (if any). Ans. 400, 000 BWU and 100, 000 BWU

50 ? NSOU ? GE-MT-41 B. Constant effort harvesting Now we examine an alternate method of harvesting. Instead of setting a quota, we set a limit on the fishing effort expended. As an example of this kind of control, rather than allowing as many boats as necessary to catch a particular number of fish, we could restrict the number or length of time that boats can fish. If we only allow, say, 10 boats to fish for 2 weeks no matter the population, then the catch will not be constant. It will instead be based on how easy it is for those boats to find fish and hence how abundant the fish are. Consequently, we associate constant effort fishing with a harvest level that corresponds to a proportion of the fish available. We assume now that we have restricted fishing effort so that a certain percentage of the fish population is harvested in a given time step. We denote this percentage by e and we modify our logistic model to reflect this change: (1) ( ) (1) 1 (1) (1) P t P t P t r P t eP t K ? ? ? ? ? ? ? ? ? ? ? ? ? ? (2.47) Finding the equilibrium value To find the equilibrium values, we solve * * * 1 * * P P P r P eP K ? ? ? ? ? ? ? ? ? ? for P*. This implies * 0 1 P r h K ? ? ? ? ? ? ? ? ? , i.e., * * 0 * 0 P r r e P P K ? ? ? ? ? ? ? ? ? ? ? (which means extinction) or * 0. P r r e K ? ? ? ? ? ? ? ? ? Hence the equilibrium value (other than extinction) for the model described by equation (2.47) is as follows. * 1 e P K r ? ? ? ? ? ? ? ? (2.48)
NSOU ? GE-MT-41 ? 51 Example 2.16.4. Suppose the carrying capacity of an environment for baleen whale population is 400, 000 BWU under discrete logistic model. If the maximum growth rate of the population is 0.05 (or 5%) and exactly 1% population of baleen whales are harvested by the whaling industry each year, then find the equilibrium value of the population (other than the extinction). Solution. Here the carrying capacity K = 400, 00 BWU, the maximum growth rate r = 0.05 and the harvesting rate e = 0.01 per year. Hence using equation (2.48), the non-extinction equilibrium value is * 1 e P K r ? ? ? ? ? ? ? ? = 320, 000 BWU. 2.17 More Worked out Examples Example 2.17.1. Solve the linear difference equation a n = 3a n−1 , a 1 = 2 Solution. Here the characteristic equation is r−3 = 0 which gives the characteristic root is r = 3. So the general solution is a n = c 1 3 n , where c 1 is an arbitrary constant. Using the initial condition a 1 = 2, we have 2 = c 1 3 implying 1 2 3 c ? . Hence 2 3 3 n n a ? is the solution. Example 2.17.2. Solve the linear difference equation a n = 5a n−1 − 6a n−2 , a 0 = 1, a 1 = 0 Solution. Here the characteristic equation is r 2 − 5r + 6 = 0 which gives the characteristic root is r = 3, 2. So the general solution is of the form a n = c 1 2 n + c 2 3 n , where c 1 and c 2 are arbitrary constants. Using the initial conditions a 0 = 1, a 1 = 0, we have c 1 + c 2 = 1 and 2c 1 + 3c 2 = 0. Solving these two equations, we get c 1 = 3 and c 2 = −2. Hence the required solution is a n = 3.2 n − 2.3 n . Example 2.17.3. Let a 1 = 2 and a 2 = 5 and a n = 6a n−1 − 9a n−2 for n ≥ 3. Solve the difference equation. Solution. Here the characteristic equation is r 2 − 6r + 9 = 0 which has two identical real roots 3, 3. So the general solution is of the form a n = (c 1 + c 2 n)3 n , where c 1 , c 2 are arbitrary coefficients. Using the initial conditions, we have 3c 1 + 3c 2 = 2 and 9c 1 + 18c 2
52 ? NSOU ? GE-MT-41 = 5. So 1 2 7 1 , 9 9 c c ? ? ?. Hence the solution is 7 1 3 3 . 9 9 n n n a n ? ? Example 2.17.4. Solve the difference equation a n = a n−1 − a n−2 when a 1 = 1 and a 2 = 2. Solution. The characteristic equation is r 2 − r + 1 = 0 having imaginary roots 1 3 2 2 i? . If z be any root of the above equation, then |z| = 1 and amp(z) 3 ? ? ? . The general solution is of the form 1 2 1 2 1 cos sin cos sin , 3 3 3 3 n n n n n n a c c c c ? ? ? ? ? ? ? ? ? ? ? ? ? ? where c 1 , c 2 are arbitrary coefficients. Using the initial conditions, we have 1 2 3 1 2 2 c c ? ? and 1 2 3 2 2 2 c c ? ? ?. Hence c 1 = −1 and c 2 = 3. Hence the solution is cos 3 sin 3 3 n n n a ? ? ? ? ? . Example 2.17.5. Suppose two lakes are connected by a canal flowing water through. 20% pollutant of the second lake goes to the first lake and 23% pollutant of the first lake goes to the second lake. If three tons of pollutant stays in the second lake after a considerably large span of time, find the amounts of pollutant going to stay in the first lake on the long run. Solution. Putting p = 0.2, q = 0.23 and b = 3 in equation (2.41), we have the amounts of pollutant going to stay in the first lake on the long run is p a b q ? = 2.61 tons (approx). 2.18 Summary At the outset, notion of difference equation has been introduced. First and second order linear difference equations have been thoroughly discussed. Later on, discrete modeling has been introduced. Several growth models have been discussed. Stress has been given on stability analysis of the models. Apart from these, various real life problems have been dealt with from a discrete modeling approach.

NSOU ? GE-MT-41 ? 53 2.19 Exercises Exercise 2.19.1. Solve 1 2 n n a a ? ? for n ≥ 2 and a 1 = 4. Ans. 3 1 2 n n a ? ? Exercise 2.19.2. If a 1 = 3 and a 2 = 7, then solve a n = 2a n−1 + 3a n−2 for n ≥ 3. Ans. 1 1 5 1 3 (1) 2 2 n n n a ? ? ? ? ? Exercise 2.19.3. If a 1 = 1 and a 2 = 2, then solve a n = − a n−1 − a n−2 for n ≥ 2. Ans. 2 1 2 3cos sin 3 3 3 n n n a ? ? ? ? ? ? ? ? ? ? ? Exercise 2.19.4. If a 1 = 0 and a 2 = 2, then solve a n = 8a n−1 − 16a n−2 for n ≥ 2. Ans. a n = 2 2n−3 (n − 1) Exercise 2.19.5. Find the non-negative equilibrium of a population given by 2 1 2 2 2 n n n x x x ? ? ? and check the stability. Ans. The required equilibrium points are 0, 1 ± i. At 0, the equilibrium is stable and at 1 ± i, the equilibria are unstable. Exercise 2.19.6. Check the stability of the equilibria of the model given by x n+1 = x n e 3−x n Ans. The required equilibrium points are 0 and 3. Both the equilibria are unstable. Exercise 2.19.7. Let X(n) and Y (n) are armament expenditures of Nations X and Y respectively in the arm race model. We assume α 1 = 0.349, α 2 = − 0.13, δ 1 = 0.432, δ 2 = 0.195, g = 37.1, h = − 52.9. Find the equilibrium point of the model.

54 ? NSOU ? GE-MT-41 Ans. 2767580 1122760 , 12961 12961 ? ? ? ? ? ? Exercise 2.19.8. Write down the equation of the Harrod Domar Model. Exercise 2.19.9. What is the warranted rate of growth in the Harrod Domar Model? Exercise 2.19.10. Write the mathematical model of constant take harvesting explaining all the parameters. Exercise 2.19.11. Write the mathematical model of constant effort harvesting explaining all the parameters. Exercise 2.19.12. Find the equilibrium value of the model given by equation (2.19) if 500 deers are removed every year. Ans. Approximately 1923 Exercise 2.19.13. Suppose the maximum number of a certain species of whales a given environment can support (carrying capacity) is 400,000 BWU. The intrinsic growth rate is 20% (i.e., r = 0.2). If only 15% of the population is permitted for harvesting in every year (i.e., e = 0.15) so that it will not become extinct, then find the population of the species after a sufficiently long time. (Hint. Use the constant effort harvesting model) Ans. 100, 000 BWU Exercise 2.19.14. Explain the lake pollution model with all its parameters. Exercise 2.19.15. Suppose two lakes are connected by a canal flowing water through. 20% pollutant of the second lake goes to the first lake and 23% pollutant of the first lake goes to the second lake. If three tons of pollutant stays in the first lake after a considerably large span of time, find the amounts of pollutant going to stay in the other lake on the long run. Ans. 3.45 tons. Exercise 2.19.16. Roy is a 120 pound male who quickly consumes two 12−oz. beers. Assuming a standard twelve oz. beer contains about 14 g of alcohol, standard

NSOU ? GE-MT-41 ? 55 average value for body water percentage is 58% and the blood is 80.6% water, calculate his blood alcohol concentration. Ans. 0.71475 g per l blood. Exercise 2.19.17. What was the purpose of forming International Whaling Commission? Exercise 2.19.18. What is the Blue Whale Unit or BWU? Exercise 2.19.19. If the carrying capacity of an environment is 500, 000 BWU, then how many sei whales the environment can support, assuming no other kind of whales are there? Ans. 3, 000, 000 (Hint. 1 sei whale = 1 6 BWU) Exercise 2.19.20. What are the key features of growth rate in a discrete logistic model? Exercise 2.19.21. What is the intrinsic growth rate of a population in a discrete logistic model? Exercise 2.19.22. Assume that in 2021 population of Baleen whales is 50, 000 BWU, the maximum growth rate r is 5% per year and the carrying capacity K = 2, 500, 000 BWU. What would the discrete logistic growth model predict for the population of baleen whales in 2023 in the Antarctic fishery? Ans. Approximately 55, 017 BWU. (Hint. First find the population in 2022 using equation (2.44). Then find the population in 2023 by same method. Population in 2022 is 52, 450 BWU.) Exercise 2.19.23. Assume that in 2021 population of baleen whales is 50, 000 BWU, the maximum growth rate r is 5% per year and the carrying capacity K = 2, 500, 000 BWU. What would the discrete logistic growth model predict for the population of baleen whales in 2023 in the Antarctic fishery if 450 BWU baleen whales are harvested each year by the whaling companies? Ans. Approximately 54, 096 BWU. (Hint. First find the population in 2022 using equation (2.44). Then find the population in 2023 by same method. Population in 2022 is 52, 000 BWU.)

56 ? NSOU ? GE-MT-41 Exercise 2.19.24. Suppose the carrying capacity of an environment for baleen whale population is 500, 000 BWU under discrete logistic model. If the maximum growth rate of the population is 5% and exactly 40, 000 BWU baleen whales are harvested by the whaling industry each year, then find the equilibrium value(s) of the population (if any). Ans. There exists no equilibrium value. (Hint. The discriminant is negative)

Unit 3 Continuous models Structure 3.0 Objectives 3.1 Introduction to Continuous Models 3.2 Carbon dating 3.3 Introduction to compartmental models 3.4 Drug distribution in the body 3.5 Growth and decay of current in an L-R Circuit 3.6 Vertical oscillation 3.7 Horizontal oscillation 3.8 Damped oscillation 3.9 Damped forced oscillation 3.10 Combat Model 3.11 Mathematical Model of Influenza Infection (within host) 3.12 Epidemic Models (SIR, SIRS, SI, SIS) 3.12.1 SIR Model 3.12.2 SIRS Model 3.12.3 SI model 3.12.4 SIS model 3.13 Spreading of rumour model 3.13.1 Classification of population 3.13.2 Rumor Spreading Model 3.14 Steady State solutions 3.15 Linearization 3.15.1 Linearization of an ODE 3.15.2 Linearization of coupled system of ODEs

58 ? NSOU ? GE-MT-41 3.16 Local stability analysis 3.16.1 Local stability analysis of an ODE 3.16.2 Local stability analysis of linear system of ODEs based on eigen values 3.17 Exponential growth 3.18 Logistic growth 3.19 Gomperzian model 3.20 Prey predator model 3.21 Competition model 3.22 More worked out examples 3.23 Summary 3.24 Exercises 3.0 Objectives The object of this chapter is to develop and analyse various continuous models. Here we discuss the followings. ? Notion of continuous models; ? a variety of continuous models; ? steady state solutions or equilibrium points; ? linearization; ? local stability analysis and classification of equilibrium points. 3.1 Introduction to Continuous Models This chapter introduces the topic of ordinary differential equation models, their formulation, analysis, and interpretation. A main emphasis at this stage is on how appropriate assumptions simplify the problem, how important variables are identified, and how differential equations are tailored for describing the essential features of a continuous process.

NSOU ? GE-MT-41 ? 59 Because one of the most challenging parts of modeling is writing the equations, we dwell on this aspect purposely. The equations are written in stages, with appropriate assumptions introduced as they are needed. We begin with a rather simple ordinary differential equation as a model. Gradually, more realistic aspects of the situation are considered. 3.2 Carbon dating Exponential decay and radioactivity The process of dating aspects of our environment is essential to the understanding of our history. From the formation of the Earth through the evolution of life and the development of mankind, historians, geologists, archaeologists, palaeontologists and many others use dating procedures to establish theories within their disciplines. While certain elements are stable, others (or their isotopes) are not, and emit α − particles, β − particles or photons while decaying into isotopes of other elements. Such elements are called radioactive. We make the following assumptions and then, based on these, develop a model to describe the process. ? The amount of an element present is large enough so that we are justified in ignoring random fluctuations. ? The process is continuous in time. ? The rate of decay for an element is fixed. ? There is no increase in mass of the body of material. Now the rate of change of radioactive material N = N(t) at time t is negative of the rate amount of radioactive material decayed. Hence we have dN kN dt ? ? (3.1) where k is a positive constant of proportionality depending on the elements chosen. Given a sample of a radioactive element at some initial time, say n 0 nuclei at t 0 , we may want to predict the mass of nuclei at some later time t. We require the value of k

60 ? NSOU ? GE-MT-41 for the calculations; it is usually found through experimentation. Then, with known k and an initial condition N(t 0) = n 0 , we have an initial value problem (IVP) dN kN dt ? ? , where N(t 0) = n 0 (3.2) Example 3.2.1. Solve the initial value problem (IVP) in equation (3.2) with initial condition N(t 0) = n 0 . Solution. Since the differential equation is separable, 1 dN dt kdt N dt ? ? ? ? 1 dN kdt N ? ? ? ? ? ln N kt C ? ? ? ? since N is a positive quantity. Here C is an arbitrary constant. Taking exponentials of both sides we have N(t) = Ae −kt , where A = e C Note that N ≥ 0. Using the initial condition N(t 0) = n 0 , we get 0 0 kt n Ae ? ? and 0 0 kt A n e ? . Thus the solution for IVP is 0 ( ) 0 ( ) k t t N t n e ? ? ? (3.3) Example 3.2.2. Solve the initial value problem (IVP) in equation (3.2) on the interval [0, t]. Solution. Since the differential equation is separable, 0 0 1 t t dN dt kdt N dt ? ? ? ?

NSOU ? GE-MT-41 ? 61 0 0 1 n t n dN kdt N ? ? ? ? ? 0 ln ln 0 N n kt ? ? ? ? ? 0 ln N kt n ? ? ? since N, n 0 are positive quantities. Taking exponentials of both sides we have N(t) = n 0 e −kt Remark 3.2.1. The half-life ? of the radioactive nuclei can be used to determine k, where ? is the time required for half of the nuclei to decay. The half-life ? is more commonly known than the value of the rate constant k for radioactive elements. Example 3.2.3. If the half-life is ? , then find k in terms of ? . Solution. Setting ( ) ( ) 2 N t N t ?? ? , we have ( ) 1 ( ) 2 N t N t ?? ? . This gives 1 2 k e ?? ? , using equation (3.3). Taking logarithms of both sides, ln 1 2 k? ? ? . Hence ln 2 k ? ? (3.4) Note that both ? and k are independent of n 0 and t 0 . Radiocarbon dating We can apply the above theory to the problem of dating paintings by considering the decay process of certain radioactive elements in each. All living organisms absorb carbon from carbon dioxide (CO 2) in the air, and thus all contain some radioactive carbon nuclei. This follows since CO 2 is composed of a radioactive form of carbon 14 C, as well as the common 12 C. (14 C is produced by the collisions of cosmic rays (neutrons) with nitrogen in the atmosphere, and the 14 C nuclei decay back to nitrogen atoms by emitting β particles.) Nobel Prize winner Willard Libby,

62 ? NSOU ? GE-MT-41 during the late 1940s, established how the known decay rate and half-life of 14 C, together with the carbon remaining in fragments of bones or other dead tissue, could be used to determine the year of death. Because of the particular half-life of carbon, internationally agreed upon as 5, 568 ± 30 years for 14 C, this process is most effective with material between 200 and 70,000 years old. Carbon dating depends on the fact that for any living organism the ratio of the amount of 14 C to the total amount of carbon in the cells is the same as that ratio in the surroundings. Assuming the ratio in air is constant, then so is the ratio in living organisms. However, when an organism dies, CO 2 from the air is no longer absorbed although 14 C within the organism continues to undergo radioactive decay. In the Cave of Lascaux in France there are some ancient wall paintings, believed to be prehistoric. Using a Geiger counter, the current decay rate of 14 C in charcoal fragments collected from the cave was measured as approximately 1.69 disintegrations per minute per gram of carbon. In comparison, for living tissue in 1950 the measurement was 13.5 disintegrations per minute per gram of carbon. Example 3.2.4. How long ago was the radioactive carbon formed and the Lascaux Cave paintings were painted, assuming the half life of 14 C to be approximately 5,568 years? Solution. Let N(t) be the amount of 14 C per gram in the charcoal at time t. We apply the model of exponential decay given by dN kN dt ? ? . We have ? ? 5, 568 years (the half-life of 14 C). Using equation (3.4), we have ln 2 k ? ? ? 0.0001245. Let t = t 0 = 0 be the current time. Let T be the time that the charcoal was formed, and thus T > 0. For t < T , 14 C decays at the rate dN kN dt ? ? with N(t 0) = n 0 and 0 ( ) kT N T n e ? ? or 0 1 ( ) ln N T T k n ? ? ? ? ? ? ? ?
NSOU ? GE-MT-41 ? 63 But we do not know N(T) or n 0 . However, 0 ( ) ( ) ( ) (0) (0) N T kN T N T N kN n ? ? ? ? ? ? and we do have N ´(T) = 1.69 and N ´(0) = 13.5, as discussed above. Thus 0 1 ( ) ln N T T k n ? ? ? ? ? ? ? ? ? 16, 690 years. Exercise 3.2.1. An artefact was discovered in 1950 from a pre- historic cave. Assume the half life of 14 C to be approximately 5,568 years. The decay rate of 14 C in charcoal fragments collected from the cave was measured as approximately 1.85 disintegrations per minute per gram of carbon. In comparison, for living tissue in 1950 the measurement was 13.5 disintegrations per minute per gram of carbon. How long ago was the artefact made? Ans. Approximately 15,964 years. Exercise 3.2.2. Establish the model of exponential growth of radioactive elements with initial assumptions. Exercise 3.2.3. What is half life of a radioactive element? Exercise 3.2.4. Find the rate of decay per nucleus in unit time in terms of the half life of a radioactive element. Hint. See Example 3.2.3. 3.3 Introduction to Compartmental Models One of the most naturally occurring framework in mathematical modeling is to think of the domain of a process as a compartment where incoming and/ or outgoing of the mass or population take place over time. A compartment may be a polluted lake with provisions of inflow of water carrying mass of pollutants from industries into it and outflow of water carrying some pollutant mass with it OR it may be an environment where a population of bacteria may be cultured where the incoming is the birth and outgoing is the death of micro-organisms happened over time. This model is crucial in understanding the
64 ? NSOU ? GE-MT-41 decay (outgoing) of some radioactive substance over time (no incoming or input at all!!!) OR quantity of drug present in our bloodstream (compartment) where the drug is absorbed from the G. I. tract and excreted through the function of kidneys OR any other similar cases. The very basic idea of the compartmental modeling lies in the following sketch. incoming substance / population ?????????? compartment outcoming substance / population ?????????? So all we need to do is to work out a balance law to compute the rate of change of substance/ population as the difference between the incoming rate and the outgoing rate of the same over time. net rate of change of substance/ population = incoming rate – outgoing rate 3.4 Drug distribution in the body This model is a two-compartment model. Assume a drug, which has been taken orally, is present in the intestine during a certain time interval. The drug is absorbed with the constant flow rate q (millimole per litre per second) into the first compartment, the blood plasma. In the blood plasma, the concentration of the drug is c 1 (t) (millimole per litre) The second compartment is the organ where the drug is active. Between the first and second compartments, there is an drug exchange with rate k 1 c 1 (t) (millimole per litre per second) leading to the drug concentration c 2 (t) (millimole per litre) in the second compartment. In the organ, the drug is consumed with the rate k b c 2 (t) (millimole per litre per second) and the surplus is sent back to the blood with the rate k 2 c 2 (t) (millimole per litre per second). From the blood, finally, there is an elimination of the drug through the kidneys with the rate k e c 1 (t) (millimole per litre per second). Note that k 1 , k 2 , k e , k b are rate constants each with per second as its unit.

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NSOU ? GE-MT-41 ? 65 The above diagram illustrates the blood–organ compartment model of drug distribution. The model is described by the following system. 1 1 1 2 2 1e dc k c k c k c q dt ? ? ? ? ? 2 1 1 2 2 2b dc k c k c k c dt ? ? ? (3.5) where c 1 (0) = 0 = c 2 (0). Example 3.4.1. Suppose an orally taken drug is absorbed with a constant flow 0.2 millimole per litre per second into the blood. The drug then moves to the target organ with 0.15 per second as rate constant. The target organ consumes the drug with 0.03 per second as rate constant and the surplus is sent back to the blood with 0.10 per second as rate constant. Finally the drug is eliminated from the blood through the kidneys with 0.01 per second as rate constant. Assuming the drug was present neither in the blood nor in that organ initially, write down the mathematical model describing the concentration of the drug in blood as well as in the target organ. Solution. Let x(t) and y(t) be the concentrations of the drug in blood and the target organ respectively. Putting q = 0.2, k 1 = 0.15, k 2 = 0.10, k b = 0.03 and k e = 0.01. Then using the system of differential equations (3.5), our desired model becomes 0.16 0.10 0.2 dx x y dt ? ? ? ? 0.15 0.13 dy x y dt ? ? 3.5 Growth and Decay of Current in an L-R Circuit Here we consider a circuit containing resistance R and inductance L connected in series combination through a battery of constant emf E through a two way switch S. The

66 ? NSOU ? GE-MT-41 inductor has been used in this single-loop circuit to stop the current from reaching its maximum value instantaneously. This is described in the following figure 3.1. Figure 3.1: LR circuit with two way switch S Also we make the following assumptions. ? To distinguish the effects of R and L,we consider the inductor in the circuit as resistance less and resistance R as non-inductive ? Current in the circuit increases when the key is pressed and decreases when it is thrown to b A. Growth of current in an L-R Circuit Suppose in the beginning, we close the switch in the up position as shown in below in the figure 3.2. Since the switch is closed, the battery E, inductance L and resistance R are now connected in series. Because of self induced emf, current will not immediately reach its steady value but grows at a rate depending on inductance and resistance of the circuit. Figure 3.2: Battery included in the LR circuit Let at any time t, I be the current in the circuit increasing from 0 to a maximum value at a rate dI dt .

NSOU ? GE-MT-41 ? 67 Now the potential difference across the inductor is OP dI V L dt ? and across resistor is V PQ = IR. Since V = V OP + V PQ , therefore dI V L IR dt ? ? (3.6) Thus rate of increase of current is dI V IR dt L ? ? (3.7) Clearly in the beginning at t=0 when circuit was closed, current began to grow at a rate 0t dI V dt L ? ? ? ? ? ? ? ? . Hence greater would be the inductance of the inductor, more slowly the current starts to increase. When the current reaches its steady state value I, the rate of increase of current becomes zero. Then from equation (3.7) we have, V I R ? . Therefore, final steady state current in the circuit does not depend on self inductance. Rather it is same as it would be if only resistance is connected to the source. From equation (3.6), we have ? ? . V R dI R dt L I ? ? Now we assume max V I R ? , the maximum current in the circuit. So we have max dI R dt I I L ? ? Integrating on both sides we have, ? ? max ln R I I t C L ? ? ? ? (3.8) 68 ? NSOU ? GE-MT-41 where C is a constant and is evaluated by the value for current at t = 0 which is I = 0. So, C = – ln I max . Putting this in equation (3.8), we have max max ln I IR t I L ? ? ? max max R L t I I e I ? ? ? ? Hence we have max 1 R L t I I e ? ? ? ? ? ? ? ? ? (3.9) This equation shows the exponential increase of current in the circuit with the passage of time as depicted in figure 3.3. Figure 3.3: Growth of current in LR circuit B. Decay of current in an L-R Circuit When the switch S is thrown down to b as shown below in the figure, the L-R circuit is again closed and battery is cut off as depicted in figure 3.5. Figure 3.4: Battery is now cut off from the circuit

NSOU ? GE-MT-41 ? 69 This time V = 0. Therefore from equation (3.6), we can write the equation for decay as 0 dI L RI dt ? ? dI R dt I L ? ? ? dI R dt I L ? ? ? ? ? Hence 1 ln R I t C L ? ? ? (3.10) At time t = 0, current I = I max . So C 1 = ln I max . Therefore from equation (3.10), we have max ln ln . R I t I L ? ? ? i.e., max R L t I I e ? ? (3.11) Hence current decreases exponentially with time in the circuit in accordance with the above equation after the battery is cutoff from the circuit as depicted in the figure Figure 3.5: Current decreasing exponentially with time

70 ? NSOU ? GE-MT-41 Example 3.5.1. A 5 mH inductor, a 15 Ω resistor are connected across a 12 V battery with negligible internal resistance in series. What is the maximum current in the circuit? Solution. max 12 0.8 . 15 V I A R ? ? ? Exercise 3.5.1. A 25 mH inductor, a 8 Ω resistor are connected across a 6 V battery with negligible internal resistance in series. What is the maximum current in the circuit? Ans. 0.75 A Exercise 3.5.2. Establish the expression for the current in terms of inductance and resistance in an LR circuit including a resistor with resistance R, an inductance L, and an emf E in series connection. Hint. See Section 3.5 Exercise 3.5.3. Draw the graph of growth of current in LR circuit. Hint. See Section 3.5 Exercise 3.5.4. Suppose we have a circuit including a resistor with resistance R, an inductance L, and an emf E in series connection. Establish the expression for the current in terms of inductance and resistance after the battery is cut off from the circuit. Also draw the graph of decay of current in the LR circuit. Hint. See Section 3.5 3.6 Vertical Oscillation Vertical spring-mass system We take an ordinary spring that resists compression as well extension and suspend it vertically from a fixed support, as shown in Figure 3.6. At the lower end of the spring we attach a body of mass m. We assume m to be so large that we can neglect the mass of the spring. If we pull the body down a certain distance and then release it, it starts moving. We assume that it moves strictly vertically. Now this motion is determined by Newton's second law: Mass × Accelration = my´´ = Force, where 2 2 d y y dt ?? ? and "Force" is the resultant of all the forces acting on

NSOU ? GE-MT-41 ? 71 the body. We choose the downward direction as the positive direction, thus regarding downward Forces as positive and upward forces as negative. Consider Figure 3.6. The spring is first unstretched. We now attach the body. This stretches the spring by an amount s 0 shown in the figure. It causes an upward force F 0 in the spring. Experiments show that F 0 is proportional to the stretch s 0 say, F 0 = – ks 0 , by Hooke's law. k(< 0) is called the spring constant. The minus sign indicates that F 0 points upward, in our negative direction. Clearly stiff springs have large k. The extension s 0 is such that F 0 in the spring balances the weight W = mg of the body. Hence F 0 +W = – ks 0 + mg = 0. These forces will not affect the motion. Spring and body are again at rest. This is called the static equilibrium of the system. We measure the displacement y(t) of the body from this equilibrium point as the origin y = 0, downward positive and upward negative. From the position y = 0 we pull the body downward. This further stretches the spring by some amount y < 0 (the distance we pull it down). By Hooke's law this causes an (additional) upward force F 1 in the spring, i.e., F 1 = – ky (3.12) Figure 3.6: A vertical spring-mass system F 1 is a restoring force. It has the tendency to restore the system, that is, to pull the body back to y = 0.

72 ? NSOU ? GE-MT-41 Now neglecting the damping effect, F 1 is the only force causing the motion. Hence from equation (3.12), we have 0 my ky ?? ? ? (3.13) It can be easily checked that the general solution will be 0 0 ( ) cos sin y t A t B ? ? ? ?t (3.14) where 0 k m ? ? . The corresponding motion is called a vertical (harmonic) oscillation. The period of the oscillation is given by 0 2 2 m k ? ? ? ? and the frequency is 0 1 2 2 k m ? ? ? ? cycles per second. Another name for cycles/sec is hertz (Hz). Example 3.6.1. If an iron ball of weight W = 98 Newtons stretches a spring 1.09 m, how many cycles per minute will this mass-spring system execute? Solution. We know weight of a 1 kg mass is 9.8 Newtons. Therefore, the mass m of the iron ball is 10 kg. Now initially the stretch s 0 is 1.09 meters. Therefore the spring constant k = 0 W s ? 89.91 Newton/meter. Then 0 k m ? ? ? 3.00. So the frequency = no. of cycles per second = 0 2 ? ? ? 0.48 and hence cycles per minute = frequency × 60 = 28.8. Exercise 3.6.1. An iron ball of weight W = 196 Newtons stretches a spring 0.25 m. If it is further stretched downwards, find the frequency of the resulting oscillation. Ans. 0.996 approx. 3.7 Horizontal Oscillation Let us consider a cart of mass M attached to a nearby wall by means of a spring as described in figure 3.7. Here x = x(t) is the position of the cart at time t. Using Hooks law as in the previous section, we have F s = – kx, k being the spring constant.

NSOU ? GE-MT-41 ? 73 Figure 3.7: Horizontal oscillation By Newton's second law of motion, which says that the mass of the cart times its acceleration equals the total force acting on it, we have 2 2 s d x M F dt ? (3.15) or 2 2 0 d x k x M dt ? ? (3.16) It will be convenient to write this equation of motion in the form 2 2 0 2 0 d x x dt ?? ? (3.17) where 0 k M ? ? . The general solution can be written down as 1 0 2 0 sin cos x c t c t ? ? ? ? (3.18) The cart is pulled aside to the position x = x 0 and released without any initial velocity at time t = 0 so that our initial conditions are x = x 0 and 0 dx v dt ? ? when t = 0. Clearly c 1 = 0 and c 2 = x 0 . So equation (3.18) becomes 0 0 cos x x t ? ? (3.19)

74 ? NSOU ? GE-MT-41 The amplitude of this simple harmonic vibration is x 0 . Since the period T is the time required for one complete cycle, we have 0 2 T? ? ? and hence 0 2 2 M T k ? ? ? ? ? (3.20) The frequency f is the number of cycles per unit time. Therefore fT = 1 and hence 0 1 1 2 2 k f T M ? ? ? ? ? ? (3.21) Example 3.7.1. Assume that a cart of mass 100 grams is attached to a nearby wall by means of a spring, with spring constant 9.8 N/m, and is placed on a smooth horizontal table. You pull the mass 6 cm away from its equilibrium position and let it go at t = 0. Find an equation for the position of the mass as a function of time t. Solution. Lets first find the period of the oscillations, then we can obtain an equation for the motion. The period 0.1 2 2 0.635 9.8 m T k ? ? ? ? ? sec. At t = 0 the mass is at its maximum distance from the equilibrium position. Thus x(t) = 0.6 cos 2 t T ? ? ? 0.6 cos 9.9t. Exercise 3.7.1. Assume that a cart of mass 100 grams, attached to a nearby wall by means of a spring of spring constant 9.8 N/m, is oscillating on a smooth horizontal table. Find the frequency. Ans. 1.57 per second 3.8 Damped Oscillation Now we consider the additional effect of a damping force F d due to the viscosity of the medium through which the cart moves (air, water, oil, etc.) horizontally. We make the specific assumption that this force opposes the motion and has magnitude proportional to the velocity, that is, that d dx F c dt ? ? , where c is a positive constant measuring the resistance of the medium. We call c the damping coeficient. Equation (3.15) now becomes

NSOU ? GE-MT-41 ? 75 2 2 s d d x M F F dt ? ? (3.22) i.e. 2 2 0 d x c dx k x M dt M dt ? ? ? (3.23) For the sake of convenience, we write this in the form 2 2 0 2 2 0 d x dx b x dt dt ? ? ? ? (3.24) where 2 c b M ? and 0 k M ? ? . The auxiliary equation is 2 2 0 2 0 m bm? ? ? ? (3.25) and its roots m 1 , m 2 are given by 2 2 1 2 0 ,m m b b ? ? ? ? ? (3.26) The nature of the roots of the equation (3.25) determines which would prevail over the other in between the frictional force due to the viscosity and the stiffness of the spring. Case I: 2 2 0 0, b ?? ? i.e., 0 b ?? i.e., 2c Mk ? (Overdamped) In loose terms, this amounts to assuming that the frictional force due to the viscosity is large compared to the stiffness of the spring. In other words, The damping force is much stronger than the restoring force due to stiffness of the spring. We call this oscillation Overdamped. It follows that m 1 and m 2 are distinct negative numbers, and the general solution of equation (3.24) is 1 2 1 2 m t m t x c e c e ? ? (3.27)

76 ? NSOU ? GE-MT-41 Using the initial conditions x = x 0 and 0 dx v dt ? ? when t = 0, equation (3.27) becomes ? ? 2 1 0 1 2 1 2 m t m t x x m e m e m m ? ? ? (3.28) or 2 2 2 2 0 0 . . 0 1 2 2 2 0 2 b t b t bt x x e m e m e b ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.29) Case II: 2 2 0 0, b ?? ? i.e., 0 b ?? i.e., 2c Mk ? (Critically damped) In this case, the restoring force and damping force are comparable in effect. Here we have m l = m 2 = 0 b ? ? ? ? and the general solution of equation (3.24) is 0 1 2 o t t x c e c te ? ? ? ? ? ? (3.30) With the initial conditions x = x 0 and 0 dx v dt ? ? when t = 0, equation (3.30) becomes 0 0 0 (1) t x x e t ? ? ? ? ? (3.31) We call this oscillation Critically damped. Figure 3.8: Types of displacements in damped oscillation

NSOU ? GE-MT-41 ? 77 Case III: 2 2 0 0, b ?? ? i.e., 0 b ?? i.e., 2c Mk ? (Underdamped) In this case, the restoring force is large compared to the damping force. Here m 1 and m 2 are conjugate complex numbers b i? ? ? , where 2 2 0 b ? ? ? ?. Then the general solution of equation (3.24) is 1 2 (cos sin) bt x e c t c t ? ? ? ? ? (3.32) With the initial conditions x = x 0 and 0 dx v dt ? ? when t = 0, equation (3.32) becomes 0 (cos sin) bt x x e t b t ? ? ? ? ? ? ? (3.33) Putting 1 tan , b ? ? ? ? equation (3.33) becomes 2 2 0 cos( ) bt x b x e t ? ? ? ? ? ? ? ? (3.34) We call this oscillation Underdamped. Figure 3.8 illustrates the above three phenomena. Example 3.8.1. Let a mass of 1 kg is attached to a wall by means of a spring with spring constant 9.8 N/m. The mass is oscillating horizontally on a rough surface with damping coefficient 2 kg/s. Find the nature of the oscillation. Solution. Here the mass M = 1 kg, spring constant k = 9.8 N/m and damping coefficient c = 2 kg/s. As 2c Mk ? = 6.26, so the oscillation is underdamped. Exercise 3.8.1. Let a mass of 1 kg is attached to a wall by means of a spring with spring constant 9.8 N/m. The mass is oscillating horizontally on a rough surface. Find the damping coefficient of the surface so that the oscillation is critically damped. Ans. 6.26 kg/s.

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78 ? NSOU ? GE-MT-41 3.9 Damped forced oscillation The vibrations discussed above are known as free vibrations because all the forces acting on the system are internal to the system itself. We now extend our analysis to cover the case in which an impressed external force F e = f(t) acts on the cart. Such a force might arise in many ways: for example, from vibrations of the wall to which the spring is attached, or from the effect on the cart of an external magnetic field (if the cart is made of iron). Therefore, in place of equation (3.22), we now have 2 2 s d e d x M F F F dt ? ? ? (3.35) Thus we have 2 2 ( ) d x c dx k M x f t M dt M dt ? ? ? (3.36) The most important case is that in which the impressed force is periodic and has the form f(t) = F 0 cos ωt so that equation (3.36) becomes 2 0 2 ( ) cos d x c dx k M x f t F t M dt M dt ? ? ? ? ? (3.37) We have already solved the corresponding homogeneous equation (3.23), so in seeking the general solution of equation (3.37) all that remains is to find a particular solution. This is most readily accomplished by the method of undetermined coefficients. Accordingly, we take sin cos x A t B t ? ? ? ? as a trial solution. On substituting this into equation (3.37), we obtain the following pair of equations for A and B: ? ? 2 0 cA k M B F ? ? ? ? ? ? ? ? 2 0 k M A cB ? ? ? ? ? NSOU ? GE-MT-41 ? 79 The solution of this system is ? ? 0 2 2 2 2 cF A k M c ? ? ? ? ? ? ? ? ? ? 2 0 2 2 2 2 k M F B k M c ? ? ? ? ? ? ? Our desired particular solution is therefore ? ? ? ? 2 0 2 2 2 2 sin cos F x c t k M t k M c ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.38) By introducing 1 2 tan c k M ? ? ? ? ? ? ? ? ? ? ? ? , we can write the solution in equation (3.38) in a more useful form ? ? ? ? 0 2 2 2 2 cos F x t k M c ? ? ? ? ? ? ? ? (3.39) This is our desired particular solution of equation (3.37). Exercise 3.9.1. Let a mass M is attached to a wall by a spring with spring constant k. It is performing a damped forced oscillation (horizontally) through a medium of damping coefficient c and the external force acting on the mass is F 0 cos ωt. (i) Write down and explain the equation of motion of the damped forced oscillation. (ii) Find out the particular solution. 3.10 Combat Model Consider now another type of interacting population model which revolves around a destructive competition or battle between two opposing groups or populations. For
80 ? NSOU ? GE-MT-41 example, two hostile insect groups or cricket teams or human armies may engage in such interaction. The model we develop here eventually yields a system of two coupled, linear differential equations. Background Battles between armies has been a very common natural part of the history of mankind. Ancient battles were fought hand-to-hand and with weapons made of stone, copper, bronze or lately iron. With the invention of gun and artillery, aimed firepower (may be directly with rifles at visible enemy or randomly aimed with artillery at enemy territory) has become an indispensable feature of modern warfare. Although many factors can affect the outcome of a battle, experience has shown that numerical superiority and superior military training are critical. Our model was first developed in the 1920s by F. W. Lanchester who was also well known for his contributions to the theory of flight. Our aim is to develop a simple model that predicts the number of soldiers in each army at any given time, provided we know the initial number of soldiers in each army. (As with epidemics, we consider the number, rather than the density, of individuals.) Model assumptions First we make some basic assumptions. ? We assume the number of soldiers to be

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sufficiently large so that we can neglect random differences between them. ? We also assume			

that there are no reinforcements and no operational losses (i.e., due to desertion or disease). In a real battle there will be a mixture of shots: those fired directly at an enemy soldier and those fired into an area known to be occupied by an enemy, but where the enemy cannot be seen. Some battles may be dominated by one or the other firing method. We consider these two idealisations of shots fired as aimed fire and random fire. For the model we assume only aimed fire for both armies. In the aimed fire idealisation, we assume all targets are visible to those firing at them. If the blue army uses aimed fire on the red army, then each time a blue soldier fires, he/ she takes aim at an individual red soldier. The rate of loss of soldiers of the red army depends only on the number of blue soldiers firing at them and not on the number of red soldiers. We see later that this assumption is equivalent to assuming a constant probability of success (on average) for each bullet fired.

NSOU ? GE-MT-41 ? 81 For random fire, a soldier firing a gun cannot see his/her target, but fires randomly into an area where enemy soldiers are known to be. The more enemy soldiers in that given area, the greater the rate of wounding. For random fire we thus assume that the rate of enemy soldiers wounded is proportional to both the number firing and the number being fired at. In summary we make the following further assumptions: ? For aimed fire, the rate of soldiers neutralized (i.e., rendered incapable of fighting by getting wounded or killed) is proportional to the number of enemy soldiers only. ? For random fire, the rate at which soldiers are neutralized is proportional to both numbers of soldiers. Formulating the differential equations Let R(t) denote the number of soldiers of the red army and B(t) the number of soldiers of the blue army at any time t. We assume aimed fire for both armies. We consider two constants a 1 and a 2 measuring the effectiveness of the blue army and red army, respectively, and are called attrition coefficients by blue and red armies respectively. So the blue army neutralizes the enemy (i.e., the red army) at per capita rate a 1 and the red army neutralizes the blue army at per capita rate a 2 . We thus assume that attrition rates are dependent only on the firing rates and are a measure of the success of each firing. Thus our model becomes 1 dR a B dt ? ? (3.40) 2 dB a R dt ? ? Example 3.10.1. Suppose a battle is waging between two countries one having red and another having blue army. Let initially the red and blue armies had R 0 and B 0 armies respectively. Also let a 1 and a 2 be the attrition coefficients by blue and red armies respectively. If R = R(t) and B = B(t) be the number of soldiers in the red and blue armies at time t, find R and B. 82 ? NSOU ? GE-MT-41 Solution. Clearly the model is given by 1 dR a B dt ? ? 2 dB a R dt ? ? Differentiating w.r.t t, we have 2 1 2 2 d R a a R dt ? 2 1 2 2 d B a a B dt ? Solving, 1 2 1 2 1 2 a a t a a t R c e c e ? ? ? 1 2 1 2 1 2 a a t a a t B d e d e ? ? ? where c 1 , c 2 , d 1 , d 2 are arbitrary constants. Determining their values using the given initial conditions, we have 1 2 1 2 1 0 1 0 0 0 1 2 1 2 1 1 ( ) 2 2

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a t a a t a B a B R t R e R e a a a a ? 1 2 1 2 2 0 2 0 0 0 1 2 1 2 1 1 ( ) 2 2 a a t a a t a R a R B t B e B e a

a a a ?

Exercise 3.10.1. Who did first develop the combat model? Exercise 3.10.2. What is attrition coefficient? Exercise 3.10.3. What are the basic assumptions of the combat model? Exercise 3.10.4. Establish the combat model.

NSOU ? GE-MT-41 ? 83 Exercise 3.10.5. During the Battle of Iwo Jima in the Pacific Ocean (1945), daily records were kept of all U.S. combat losses. The values of the attrition coefficients a 1 and a 2 have been estimated from the data as a 1 = 0.0544 and a 2 = 0.0106, and the initial numbers in the red and blue armies, respectively, were r 0 = 66,454 and b 0 = 18,274. Obtain accurate solutions to the differential equations (3.40). Ans. R(t) = 12, 516.1621e 0.024t + 53, 937.8379e −0.024t B(t) = −5, 539.3659e 0.024t + 23, 813.3659e −0.024t 3.11 Mathematical Model of Influenza Infection (within host) Influenza is a viral infectious respiratory disease that can be seasonal and mild, severe, or chronic. In 2018, there were 3-5 million cases of severe influenza around the world, resulting in approximately 500,000 deaths. Part of what makes Influenza dangerous is that the virus mutates very quickly; in one day it can mutate more than humans have in the past several thousand years. Influenza virus may be contracted via an air-born path by inhaling the cough droplets of an infected individual (in the case of human influenza), or a vector-born virus that is contracted via infected birds (in the case of avian influenza). Human influenza attacks the upper respiratory tract; however, it is capable of spreading to cells in the lower respiratory tract, cardiovascular system, and nervous system. It is in these secondary locations that it is most dangerous. Here we model the disease interaction with cells. The cells are grouped into four classes: Target cells T, Exposed cell E, Infectious cells I and Dead cells D. T, represent the cell population susceptible to infection. These cells, after interacting with the virus cells, transition to the exposed class at the per- capita rate β. Dead cells trigger cellular restoration. This results in increase of target cells at the per-capita rate r D . Exposed cells E represent the cells that have been infected but are not yet producing new virons. This class can also be referred to as the latent or eclipse class. This class gains cells from the target population and loses cells to the infectious class at a per- capita rate of 1 E ? . Infectious cells I, represent the class that actively produces new

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84 ? NSOU ? GE-MT-41 virons. It gains cells from the exposed class and loses cells to infection related death at a per-capita rate of 1 I ? . Finally, Virus V represents the virus. Infectious cells produce new virons at per-capita rate p and cells clear the virus at a per- capita rate c. In the following, we describe this compartmental model. Here the model becomes D dT TV r D dt ? ? ? ? E dE E TV dt ? ? ? ? (3.41) E I dI E I dt ? ? ? ? dV pI cV dt ? ? where N = D + T + E + I, N being the total number of cells, or D = N − T − E − I. Exercise 3.11.1. Describe the model of influenza infection (within host). 3.12 Epidemic Models (SIR, SIRS, SI, SIS) 3.12.1 SIR Model Centuries have witnessed devastating epidemics of various diseases. The history of human civilization bears several examples of dreadful diseases like the Black Death, Plague, NSOU ? GE-MT-41 ? 85 Small Pox etc. Even after so much advancements in medical sciences, we have to face epidemics like AIDS, Ebola, SARS, MERS. Our present days' grappling to contain the global pandemic Covid 19 has taken the significance of epidemiology to a new height. Evidently,

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if we can understand the nature of how a disease spreads through a population, then certainly we

can equip ourselves with

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better strategies to contain it through methods like vaccination or quarantine. Sometimes even the biological control of pests may also become handy to curb the spread of disease. For

this, it is important to understand the effect of the infesting populace of the pests. Several diseases, including influenza, measles, chickenpox and present day's Covid 19, spreads

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by infected persons in the population coming into close contact with susceptible

persons. On the other hand, malaria, dengue are transmitted through mosquitoes. Thus these are vector borne diseases. Apart from the variety in mode of transmission, the severity of contagion also varies. Covid 19, influenza

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and measles are highly contagious, whereas glandular fever is much less so. Interestingly, some diseases, like mumps and measles, confer a lifelong immunity.

On the other hand, influenza and typhoid have comparatively much shorter periods of immunity. So the recovered individual may again get infected. Incubation period: It

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is the time between infection and the appearance of visible symptoms.

Latent period: It

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is the period of time between infection and the ability to infect someone else with the disease. Note that the latent period is shorter than the incubation period.

For example, the incubation period of measles is approximately 2 weeks but the latent period is approximately 1 week. As as a result, any infected individual can end up in spreading the disease to others without even knowing it. Our model of epidemic: Here

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we discuss a simple mathematical model for influenza outbreak at a boarding school over a period of about, say, 45 days. During this

time interval, we can safely assume that reinfection does not occur. Basic assumptions: When studying the outbreak of a disease, the entire population under consideration can be divided into distinct compartments viz. susceptibles of size S(t), infectives of size I(t), and recovered individuals of size R(t) where t denotes time. The susceptibles are
86 ? NSOU ? GE-MT-41 those who are vulnerable to the infection, while the infectives are infected individuals capable of spreading the infection to susceptibles. Moreover, R(t) is the number of

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those who have recovered from the disease and are no longer susceptible (

i.e., acquired permanent immunity). Before proceeding further, we now make the following assumptions. ? Population sizes of susceptibles and infectives, i.e., S(t) and I(t) respectively

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are large enough such that random differences between individuals can be neglected. ? Births and deaths are ignored. ? The infection spreads only by contact. ? The latent period

is set to be zero, i.e., an individual can spread the disease immediately after getting infected. ? Every recovered individual is immune to the pathogen i.e., cannot get reinfected (at least within the time period considered). ? At any time t, the population of size N(t) is homogeneous, i.e.,

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the contagious infectives and susceptibles are always randomly distributed over the area in which

they reside. The following is the input-output diagram for the epidemic model of influenza in a school, assuming there is no chance of reinfection, for the time period under consideration. Susceptibles S(t) infected ????? Infectives I(t) recovered ?????? Recovered R(t) Note that, in general, a recovered person does not get life-long immunity against influenza and can get re-infected. But for a period of 15 days, the immunity of a recovered individual may be safely assumed. Forming the differential equations: First we consider the number of susceptibles infected by a single infective. The more is

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the number of susceptibles, the higher is the increase in the number of infectives. Thus the rate of susceptibles infected by a single infective will be an increasing function of the number of susceptibles.

We assume now

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λ(t) is the force of infection, i.e., it is the per- capita rate at which susceptible individuals become infected.

If the number of susceptibles at time t is S = S(t), then the rate in which susceptibles are infected is λ(t)S(t). Note that λ(t) need not be invariant as

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the more infectives there are, the higher the risk that a single susceptible will become infected.

NSOU ? GE-MT-41 ? 87 Then we have ( ) dS t S dt ? ? ? (since there is no ingress to the compartment of susceptibles). Again, the number of infectives removed from the compartment of infectives to the compartment of recovered in the time interval depends

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only on the number of infectives. We assume that the rate at which infectives recover is directly proportional to the number of infectives.

If the per- capita rate of recovery is ? , then the rate of infectives recovered is ? I(t). ? is known as recovery rate or removal rate. Note that 1 D ? ? , where D is the average duration of the infectious period. Thus the rate of ingress to the compartment of infectives is λ(t)S while the rate of egress from this compartment is ? I(t). Hence we have ( ) dI t S I dt ? ? ? ? . Again, the rate of influx into the compartment of recovered individuals is ? I(t). Since we have ignored the possibility of re-infection during the time interval under consideration, there is no outflux from the compartment. Hence dR I dt ?? . Thus we get a system of coupled differential equations ( ) dS t S dt ? ? ? ( ) dI t S I dt ? ? ? ? dR I dt ?? where the total population is N(t) = S(t) + I(t) + R(t).

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The force of infection, λ(t), depends on the current number of infectives I(t) and increases as the proportion of infectives in the population increases. It also depends on the rate that individuals make contacts.

Let c be the number of contacts per time and p be the probability that a contact between an infective and a susceptible results in an infection. We can now assume the force of infection to be
88 ? NSOU ? GE-MT-41 ( ) ( ) ( ) I t t cp N t ? ? (3.42) Let ( ) cp N t ? ? (3.43) We call this β the transmission coefficient. Now our model becomes an IVP dS SI dt ? ? ? dT SI I dt ? ? ? ? (3.44) dR I dt ?? with initial conditions S(0) = s 0 , I(0) = i 0 and R(0) = 0. Example 3.12.1. In a city of twelve lakhs population witnessing the spread of an infectious disease, if the number of contacts per minute is 3.7 and 0.67 be the probability that a contact between an infective and a susceptible results in an infection, then what will be the transmission coefficient? If at a given point of time, already 50, 000 people have been infectives, then what will be the force of infection at that point of time? Solution. Here the number of contacts per minute is c = 3.7. The probability that a contact between an infective and a susceptible results in an infection is p = 0.67. Also, the total population is N(t) = 1, 200, 000 and number of infectives I(t) = 50, 000. Then using equation (3.43), the transmission coefficient is β = 0.20659 × 10 −5 . Also, using equation (3.42), we have the force of infection ? (t) = 0.10329. Exercise 3.12.1. What is the force of infection? Exercise 3.12.2. What is the transmission coefficient?

NSOU ? GE-MT-41 ? 89 Exercise 3.12.3. In a city of six lakhs population witnessing the spread of an infectious disease, if the number of contacts per minute is 3.7 and 0.67 be the probability that a contact between an infective and a susceptible results in an infection, then what will be the transmission coefficient? If at a given point of time, already 25, 000 people have been infectives, then what will be the force of infection at that point of time? Ans. 0.41317 × 10 −5 , 0.10329 3.12.2 SIRS Model: Now let us discard the notion of permanent immunity as we assumed in the SIR model. Consider the instance of a influenza outbreak in a boarding school for a period of 45 days. So keeping the other assumptions of SIR model intact, we may now consider the fact that the immunity of the recovered persons wanes with time and they become susceptibles again for the same strain of virus. Let ( )t ? ? ? is the per- capita per rate, per unit time, in which the recovered individuals return to the susceptible state due to loss of immunity. Reconsidering the influx and outflux of the compartments of susceptibles, infectives and recovered, we have a new system of coupled differential equations dS SI R dt ? ? ? ? ? dI SI I dt ? ? ? ? dR I R dt ? ? ? ? with initial conditions S(0) = s 0 , I(0) = i 0 and R(0) = 0. 90 ? NSOU ? GE-MT-41 3.12.3 SI model Susceptibles S(t) Infected ????? Infectives I(t) The SI model is the simplest form of all disease models. In this model, the population is divided into two compartments viz. susceptibles and infectives. Initially every individual is susceptible, i.e., with no immunity. Individuals are born into the simulation with no immunity. Once infected and with no treatment, the individuals remain infective throughout the rest of the life. Thus the infectious period remains longer than the lifespan of individuals. Also they continue to be in contact with the susceptible ones. Behaviour of diseases like cytomegalovirus (CMV) or herpes are example of this model. As before we assume, at any time t, S = S(t) and I = I(t) are the numbers of susceptible and infective individuals respectively. With β as transmission coefficient, our model becomes dS SI dt ? ? ? dI SI dt ?? where N= S+I is the total population. 3.12.4 SIS model In the SIS model, the infected individuals return to the susceptible state immediately after infection. This model is appropriate for diseases that commonly have repeat infections, for example, the common cold (rhinoviruses) or sexually transmitted diseases like gonorrhea or chlamydia. With β as transmission coefficient and γ as recovery rate, our model becomes dS SI I dt ? ? ? ? ? dI SI I dt ? ? ? ?

NSOU ? GE-MT-41 ? 91 Exercise 3.12.5. Define incubation and latent periods. Exercise 3.12.6. What are the basic assumptions of an epidemic model? Exercise 3.12.7. Establish the SIR, SIRS, SI and SIS models. Exercise 3.12.8. What are the differences between the SIR, SIRS, SI and SIS models? 3.13 Spreading of Rumour Model An old saying goes that rumors come true after being repeated a thousand times. In real life, if people are unable to distinguish authenticity, many rumors are deemed to be true after a large number of repetitions. When rumors are widely propagated, people tend to believe the rumor, especially if they lack timely real information. Because of the increased presence of online social networks, rumors are no longer spread by word of mouth over a small area but are spread amongst strangers in different regions and different countries, meaning that rumors are being spread faster and wider than ever before. This sustained and rapid spreading of rumors deepens people's impression about the veracity of the rumor and thus improves the credibility. Rumor spreading, therefore, has the ability to shape public opinion and lead to social panic and instability. For example, the nuclear leakage accidents in Fukushima, after the 2011 Tohoku earthquake, caused a number of rumors in the region. Rumors said that taking materials containing iodine could help ward off nuclear radiation, which led to the fact that many people rushed to purchase iodized salt. In reality, people hear rumors many times and so have an accumulation of impressions about the rumors, which changes the probability as to when people become rumor spreaders. Therefore, memory effects have a strong time-dependency. Further, the remembering mechanisms can indicate repeatability, which affects the spreading characteristics of the rumor. Even a small amount of memory can affect the rumor spread in small network sizes. 3.13.1 Classification of population Consider a network with N nodes and E links representing the individuals and their interactions. At each time step, each individual is in one of the following four states: 1. the unaware: this individual has not yet heard the rumor;

92 ? NSOU ? GE-MT-41 2. the lurkers: this individual knows the rumor but is not willing to spread it because they require an active effort to discern the truth or falseness of the rumor; 3. the spreaders: this individual knows this rumor and transmits it to all their contacts; 4. the stiflers: this individual neither trusts the rumor nor transmits it. People generally hear a rumor after many times, and therefore they get an accumulated impression about the rumor, which means that the probability that people become a spreader changes from “will never believe” to “believes.” This can be described as the cumulative effect of memory, which affects the probability that an individual becomes a spreader from a lurker in the rumor spreading process. In information spreading theory, a function was established which reflected the probability that a person would approve the information at time t after having received the news m times. This function is P(m) = (λ − T)e −b(m−1) + T, where λ = P(1) is the approving probability of the first receipt of the information and T ? (0, 1] is the upper bound of the probability indicating maximal approval probability. Now, lurkers do not automatically change their states at time step t. Some may become a stifler or a spreader, while others remain lurkers and may become stiflers or spreaders at a later time. We assume that the new lurkers at each time step have a part of the residuals which last until the end of the rumor spreading. This corresponds with the fact that there are always some people who take a long time to change their state in real life. Lurkers become spreaders at a variable probability, denoted by p(t) and become stiflers at the rate of p 2 . As the number of times the rumor is received, the probability that a individual agrees to the truth of the rumor grows and infinitely approaches a constant. Thus, as time passes, the number of times the rumor is received for the residual lurkers gradually increases. Because the probability p(t) that an individual becomes a spreader from a lurker is a level that reflects the transformation probability of all lurkers, including the residual old lurkers and the new joined lurkers in each time step, as time passes, the probability increases gradually because of the cumulative effect of memory and infinitely approaches a constant when the accumulated memories achieve a certain degree. The probability p(t) that lurkers become spreaders affected by memory accumulation at t−th time step is given by (1) ( ) ( ) c t p t p q e q ? ? ? ? ? ? (3.45) NSOU ? GE-MT-41 ? 93 where p,q and c are parameters. These three parameters reflect the characteristics of the variable memory effects rate. p is the initial value of the memory effects function at t = 1. The parameter p reflects the importance of an event triggering rumors in the spreading process, and it is the initial probability that an individual becomes a spreader. A larger value for p means that the spreaders more easily remember the rumor because the event is probably more important. 2 1 , (0,1) q p q ? ? ? , is the maximal transformation probability. As time passes, p(t) infinitely approaches q. The parameter c can be regarded as the memory speed; namely c captures how quickly p(t) reaches the maximum value q. The memory effects rate p(t) is a probability varying over time t. Here, we do not consider interest decay and assume that the time scale for the rumor spreading is much faster than the memory decay. 3.13.2 Rumor Spreading Model Denote by S(t), E(t), I(t) and R(t) the density of the unaware, lurkers, spreaders, and stiflers at time t. Thus S(t) + E(t) + I(t) + R(t) = 1. 1. Everyone needs time to determine the authenticity of rumor, so an unaware becomes a lurker with a probability 1 when an unaware individual contacts a spreader. The contact probability k is decided by the specific network topology. Therefore, the reduced speed of the unaware dS dt is proportional to the product of densities of the unawares S(t) and the spreaders I(t). So the differential equation becomes ( ) ( ) ( ) dS t kS t I t dt ? ? (3.46) 2. A lurker becomes a spreader at the rate of p(t) and becomes a stifler at the rate of p 2 , which depends on cognition. For example, some unaware turned lurker individuals may have strong knowledge structures and logical reasoning abilities. So they may have little interest in rumors. Because an unaware individual becomes a lurker with a probability 1 when an unaware contacts a spreader, the increased speed of the lurkers is given by 2 ( ) ( ) ( ) ( ) ( ) ( ) dE t kS t I t p t E t p E t dt ? ? ? (3.47) 94 ? NSOU ? GE-MT-41 3. When two spreaders contact each other, both may find the two pieces of information inconsistent, so they stop the spread. When a spreader contacts a stifler, the spreader tries to stop the spread, as the stifler shows no interest in the rumor or denies its veracity. We suppose that the above cases occur at the same probability p 3 . Therefore, the reduced speed of the spreaders ( ) dI t dt is proportional to I(t) and R(t) + I(t). Additionally, a lurker becomes a stifler at the rate of p(t). Therefore ? ? 3 ( ) ( ) ( ) ( ) ( ) ( ) dI t p t E t kp I t I t R t dt ? ? ? (3.48) 4. The increasing speed of the stiflers ( ) dR t dt is proportional to the existing I(t) and I(t) + R(t) from above. Also a lurker becomes a stifler at the rate of p 2 . Therefore ? ? 3 2 ( ) ( ) ( ) ( ) ( ) dR t kp I t I t R t p E t dt ? ? ? (3.49) The equations (3.46), (3.47), (3.48) and (3.49) together with the initial assumptions S(0) = S 0 , E(0) = 0, I(0) = 1 − S 0 < 0 and R(0) = 0 describes a model of rumour spreading. Exercise 3.13.1. In the rumour spreading model, who are the unawares, lurkers, spreaders and stiflers? Exercise 3.13.2. Describe the rumour spreading model. 3.14 Steady State solutions Definition 3.14.1. Let ( ) dy f y dt ? , where f(y) may not be a linear function of y. Then the steady state solutions or critical points or equilibrium points are y = y 0 where f(y 0) = 0. On a more general set up, consider the following system of ODEs (,)

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dx f x y dt ? (,) dy g x y dt ? (3.50) NSOU ? GE-MT-41 ? 95 Here f(x, y) and g(x, y) are			

non-linear equations. We also assume that the system of equations (3.50) is an autonomous system, i.e., f(x, y) and g(x, y) do not contain t explicitly. Now, we can have a velocity field ˆ ˆ (,) (,) F f x y i g x y j ? ? corresponding to the system (3.50). From geometric viewpoint, the solutions x(t) and y(t) together give trajectories of the field of F . This means they give curves everywhere having the right velocity at every point. Definition 3.14.2. A steady state solution or critical point is a point P(x 0 , y 0) where f(x 0 , y 0) = 0 = g(x 0 , y 0). From the viewpoint of solutions, x = x 0 , y = y 0 give constant solution. On the other hand from viewpoint of a vector field, at such points F = 0, i.e., there is no velocity at P(x 0 , y 0). Example 3.14.1. Let us consider a system of ODEs which will be discussed later in detail in section 3.20. 1 1

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dX X c XY dt ?? ? (3.51) 2 2 dY c XY Y dt ? ? ? where c 1 , c 2 , 2 1 ,? ? are positive constants. This system of			

equations is known as the Lotka–Volterra prey- predator system. We will find the equilibrium solutions or critical points of the system (3.51). Solution. We set 0 dX dt ? and 0 dY dt ? in system of ODEs (3.51). So we have ? ? 1 1 0 X c Y ? ? ? (3.52) and ? ? 2 2 0 Y c X ?? ? ? (3.53)
96 ? NSOU ? GE-MT-41 From equation (3.52) there are two possible solutions: X = 0 or 1 1 0 c Y ? ? ?. Putting X = 0 in equation (3.53), we have Y = 0. Thus (0, 0) is an equilibrium point of the system (3.51). Taking the other case, 1 1 0 c Y ? ? ?, we have 1 1 c Y ? ? . Putting this in equation (3.53), we have 2 2 c X ? ? . Thus 2 1 2 1 , c c ? ? ? ? ? ? ? ? is another equilibrium point of the system (3.51). Therefore (0, 0) and 2 1 2 1 , c c ? ? ? ? ? ? ? ? are two equilibrium points of the system (3.51). 3.15 Linearization In the section 3.14, we have been introduced to the notion of steady state solutions or equilibrium points for a system represented by a single ODE as well as a system represented by a coupled system of ODEs. Here we will approximate the non-linear ODE or system of ODEs with a linear ODE or system of ODEs close to the equilibrium point. This process is called linearization. 3.15.1 Linearization of an ODE Consider the differential equation ( ) dx f x dt ? (3.54) As we have seen earlier, the equilibrium solutions of the equation (3.54) are the solutions x = x e such that f(x e) = 0. We let ( ) ( ), e x t x t ? ? ? , with 0 ( ) 1 t?? ? , where the new variable ? represents the small perturbation from the equilibrium solution. For the differential equation (3.54), if we expand the RHS about the equilibrium solution by letting x = x e + ? , then the differential equation for the variable ? is ? ? ? ? ? ? ? ? e e e e d x dx f x f x f x dt dt ? ? ? ? ? ? ? ? ? ? ignoring the higher order terms of ? .
NSOU ? GE-MT-41 ? 97 Since f(x e) = 0, by the definition of an equilibrium point, then the original differential equation is approximated, close to the equilibrium solution, by ( ) e d f x dt ? ? ? ? (3.55) for small values of ? . Equivalently this can be written as ? ? ( ) e e dx x x f x dt ? ? ? (3.56) This is the linearization of equation (3.54) at the equilibrium point x e . Example 3.15.1. Linearize the differential eqaution 2 3 2 dx x x dt ? ? ? at their points of equilibrium. Solution. Let f(x) = x 2 – 3x + 2. Clearly f(x) = 0 ? x = 1, 2. Thus the equilibrium points are 1 and 2. Now f ´(x) = 2x – 3. So linearization at x = 1 is (1) d dt f ? ? ? ? , i.e., d dt ? ?? ? . Also linearization at x = 2 is (2) d dt f ? ? ? ? , i.e., d dt ? ?? . 3.15.2 Linearization of coupled system of ODEs Consider a general system of two nonlinear

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differential equations (,), dX F X Y dt ? (3.57) (,). dY G X Y dt ? 98 ?			

NSOU ? GE-MT-41 Let (x e , y e) be any equilibrium point for the system (3.57), not necessarily at (0, 0), and then F(x e , y e) = 0 = G(x e , y e). Consider solutions close to the steady-state (equilibrium) solutions ( ) ( ) e X t x t? ? ? , ( ) ( ) e Y t y t? ? ? , where ( )t? and ( )t? are small and approach zero when X and Y approach the equilibrium point. These ( )t? and ( )t? are perturbations of the steady state. We now change the variables in the system from X and Y to ? and ? respectively. Then ? ? ? ? , , e e e d x F x y dt ? ? ? ? ? ? ? (3.58) ? ? ? ? , , e e e d y G x y dt ? ? ? ? ? ? ? where ? and ? are functions of t. But we have, since x e and y e are constants, ? ? , e d x dX d dt dt dt ? ? ? ? ? (3.59) ? ? , e d y dY d dt dt dt ? ? ? ? ? Comparing systems (3.58) and (3.59), we have ? ? , , e e dX d F x y dt dt ? ? ? ? ? ? ? (3.60) ? ? , , e e dY d G x y dt dt ? ? ? ? ? ? ? We now apply the Taylor series expansion in two variables to expand ? ? , e e F x y ? ? ? ? and ? ? , e e G x y ? ? ? ?. Then we take a linear approximation for each. Applying the Taylor series expansion in two variables, we find NSOU ? GE-MT-41 ? 99 ? ? ? ? ? ? , , , e e e e e e d F x y F x y F x y dt ? ? ? ? ? ? ? ? ? terms of higher order, (3.61) ? ? ? ? ? ? , , , e e e e e e d G x y G x y G x y dt ? ? ? ? ? ? ? ? ? terms of higher order. where , F F F F ? ? ? ? ? ? ? ? ? ? and likewise for G. Recall that since (x e , y e) is a equilibrium point for the system (3.57), therefore F(x e , y e) = 0 = G(x e , y e). Now taking the linear approximation of each Taylor series expansion (i.e., ignoring all terms of higher order), we have ? ? ? ? , , e e e e d F x y F x y dt ? ? ? ? ? ? ? , (3.62) ? ? ? ? , , . e e e e d G x y G x y dt ? ? ? ? ? ? ? Or equilvalently, d F F dt d G G dt ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.63) Note that ? and ? are not variables of the original equation. However, X = x e + ? and Y = y e + ? so that ( ) e d x F F X F F X X d X ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? . Similarly , F F Y ? ? ? ? ? ? G G X ? ? ? ? ? ? and G G Y ? ? ? ? ? ? . This means that we have . X Y X Y F F F F G G G G ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?
100 ? NSOU ? GE-MT-41 Thus system (3.63) becomes e e dX X x dt Y y dY dt ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? J (3.64) where X Y X Y F F G G ? ? ? ? ? ? ? J is the Jacobian matrix of the system (3.57). Example 3.15.2. We have discussed in example 3.14.1 about the equilibrium points of the system of ODEs representing prey- predator model (will be discussed later in detail in section 3.20). Linearize the model. Solution. The prey- predator model (described in section 3.20) is given by 1 1 dX X c XY dt ?? ? (3.65) 2 2 dY c XY Y dt ? ? ? where c 1 , c 2 , 2 1 ,? ? are positive constants. In example 3.14.1, we have seen the equilibrium points of the prey- predator model (described in section 3.20) are (0, 0) and 2 1 2 1 , c c ? ? ? ? ? ? ? ? . We will now linearize the system of differential equations (3.65) at these equilibrium points (using equation (3.64)). We set 1 1 (,) F X Y X c XY ?? ? and 2 2 (,) G X Y c XY Y ? ? ?. Then the Jacobian matrix is 1 1 1 2 2 2 X Y X Y F F c Y c X G G c Y c X ? J Case I: When the equilibrium point is (0, 0), then 1 1 2 0 0 ? ? ? ? ? ? ? ? ? ? J . Then the linearized system at (0, 0) is
NSOU ? GE-MT-41 ? 101 1 2 0 0 X X Y Y ? Case II: When the equilibrium point is 2 1 2 1 , c c ? ? ? ? ? ? ? ? ? , then the Jacobian becomes 1 2 2 2 1 2 1 0 0 c c c c ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? J Then the linearized system at 2 1 2 1 , c c ? ? ? ? ? ? ? ? ? is 1 2 2 2 2 1 2 1 1 1 0 0 c X c c X Y c Y c c ? 3.16 Local stability analysis 3.16.1 Local stability analysis of an ODE In section 3.15 we have seen that the linearization of the differential equation ( ) dx f x dt ? (3.66) at a equilibrium point x e is ( ) e d f x dt ? ? ? ? 102 ? NSOU ? GE-MT-41 where ? represents the small perturbation from the equilibrium solution. By local stability of an equilibrium point, we mean that any solution close to the equilibrium solution will tend towards the equilibrium solution and by unstable equilibrium, the solution will not get closer to the equilibrium point. We can now interpret what happens without actually solving this differential equation. Suppose ( ) 0 e f x ? ? . Now for ( ) 0, 0 d t dt ? ? ? ? ? ?, so ( ) ( ) e x t x t ? ? ? approaches the equilibrium point x e . Similarly, for ( ) 0 t ? ? ? ?, then ( ) ( ) e x t x t ? ? ? increases towards the equilibrium solution. Thus the solution is attracted to the equilibrium solution. By a similar argument, when ( ) 0 e f x ? ? the solution moves away from the equilibrium solution after a small perturbation. Thus we have equilibrium solution is stable if ( ) 0 e f x ? ? and unstable otherwise. Example 3.16.1. Find all equilibrium points

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for the differential equation in dC F F c C dt V V ? ? where			

F and V are positive constants. Also determine if the equilibrium solution is stable or unstable. Solution. Setting 0 dC dt ? , we obtain ? ? 0 in F c C V ? ? in C c ? ? Thus c in is the equilibrium solution. Now considering ? ? ( ) F in V f C c C ? ? and C e = c in , we have ( ) F e V f C ? ? ? . Since F and V are positive parameters, this means that the equilibrium solution C e = c in is always stable.

NSOU ? GE-MT-41 ? 103 3.16.2 Local stability analysis of linear system of ODEs based on eigen values Solving the system using linear algebraic technique Let us consider the general pair of linear first-order equations: 1 1 X a X bY ? ? ? (3.67) 2 2 Y a X b Y ? ? ? which has an equilibrium point at the origin, i.e., (x e , y e) = (0, 0). In vector notation, we can write x x ? ? A (3.68) Suppose we have found the eigenvalues 1 ? and 2 ? , as well as the associated eigen vectors for A, namely 1 2 u u ? ? ? ? ? ? ? u and 1 2 v v ? ? ? ? ? ? ? v . We define U to be the matrix whose columns are the eigen vectors. Thus U = (u v) = 1 1 2 2 u v u v ? ? ? ? ? ? ? . From the definition of eigenvectors and eigenvalues, we have 1 ??Au u and 2 ??Av v which implies that ? ? ? ? 1 1 2 2 0 ( ) 0 ? ? ? ? ? ? ? ? ? ? ? ? ? A u v u v u v i.e., AU = UD where D = 1 2 0 0 ? ? ? ? ? ? ? ? ? is a diagonal matrix.

104 ? NSOU ? GE-MT-41 Assuming that U is invertible, we can write U −1 AU = D (3.69) We will use this equation below. First we express x as a linear combination of the eigen vectors and, assuming this is possible, we have x = z 1 u + z 2 v (3.70) Letting 1 2 , z z z ? ? ? ? ? ? ? x = Uz Since X and Y are functions of time, and the eigen vectors are not (since A is not a function of time), therefore z 1 and z 2 must also be functions of time. We now establish two expressions for x ́ . x = Uz so x ́ = Uz ́ and also x ́ = Ax so x ́ = AUz Equating these two expressions for x ́ , we have Uz ́ = AUz Then using equation (3.69), we have z ́ = U −1 AUz i.e., z ́ = Dz (3.71) Expanding equation (3.71), we have 1 1 1 z z? ? ? & 2 2 2 z z? ? ? Thus we obtain two equations that are easy to solve. They are the equations for exponential growth and decay with which, by now, we are familiar. We have as solutions

NSOU ? GE-MT-41 ? 105 1 1 1 t z k e ? ? (3.72) & 2 2 2 t z k e ? ? where k 1 and k 2 are arbitrary constants. Using these in equation (3.70), we have 1 2 1 2 t t k e k e ? ? ? ? x u v 1 2 ˆ ˆ t t e e ? ? ? ? u v where ˆu = k 1 u and ˆv = k 2 v are two eigen vectors (as any scalar multiple of an eigen vector is again an eigen vector) and so 1 2 1 1 ˆ ˆ t t X e u e v ? ? ? ? & 1 2 2 2 ˆ ˆ t t Y e u e v ? ? ? ? (3.73) This is the solution of linear system (3.67). Equilibrium point classifications For the systems described above, we had the origin (0, 0) as the equilibrium (or critical) point. What we will discuss now is the behaviour of the trajectories of the solution close to this point, using the techniques of eigenvalues and eigen vectors. We will mainly focus on the eigen values as we can see the trajectories are given by 1 1 1 t z k e ? ? and 2 2 2 t z k e ? ? (see solution (3.72)). Equation (3.70) as well as the fact that u and v are independent of t make it very clear that the trajectory given by (X(t), Y (t)) depends heavily upon 1 t e ? and 2 t e ? . Now we will see how the natures of 1 ? and 2 ? influence the behaviours of the trajectories. Case I: When 1 ? > 0 and 2 ? > 0 (eigen values real and negative) As 1 1 1 lim lim 0 t t t z k e ? ?? ?? ? ? and 2 2 2 lim lim 0 t t t z k e ? ?? ?? ? ? , therefore all trajectories approach the equilibrium point at the origin. Such a point is called a stable node and is illustrated in figure 3.9.

106 ? NSOU ? GE-MT-41 Case II: When 1 ? < 0 and 2 ? < 0 (eigen values are real and positive) We have both z 1 and z 2 approaching ? (diverging) as t increases and therefore all trajectories diverge from the equilibrium point. Such a point is called an unstable node (see figure 3.9). Figure 3.9: Trajectory behaviour close to a stable node (left) and an unstable node (right) Figure 3.10: Trajectory behaviour close to a (unstable) saddle point Case III: When 1 ? < 0 and 2 ? > 0 (eigen values are real and of different sign) We have 1 1 1 lim lim t t t z k e ? ?? ?? ? ? ? and 2 2 2 lim lim 0 t t t z k e ? ?? ?? ? ? . Therefore the trajectories approach zero along one axis and approach ? along the other axis. Such a point is called a saddle or an unstable saddle point and is illustrated in figure 3.10.

NSOU ? GE-MT-41 ? 107 Case IV: When 1 i ? ? ? ? ? and 2 i ? ? ? ? ? with 0 ? ? ? ? (eigen values are complex conjugates) In this case, the solutions can be written in the form 1 2 cos , sin . t t z e t z e t ? ? ? ? ? ? Here the trajectories spiral around the equilibrium point. If ? > 0, then they spiral inwards towards the equilibrium point. Such a point is called a stable focus. If ? < 0, then they spiral outwards and away from the equilibrium point. Such a point is called an unstable focus. These have been illustrated in figure 3.11. Figure 3.11: Trajectory behaviour close to a stable focus (left) and an unstable focus (right) Case V: When 1 ? and 2 ? are purely imaginary (eigen values are purely imaginary) In this case, the solutions can be written in the form 1 2 cos , sin . z t z t ? ? ? ? Therefore the trajectories form closed loops enclosing the equilibrium point. Such a point is called a centre and the solutions are called periodic. This has been illustrated in figure 3.12. Figure 3.12: Trajectory behaviour close to a centre

108 ? NSOU ? GE-MT-41 3.17 Exponential growth The growth of a population may take place with discrete jumps in breeding (e.g. fishes, insects etc. as they have fixed breeding season) or continuous breeding process (e.g. humans). Even for discrete cases, if the time gap between successive breeding jumps is negligible (e.g. bacteria) in comparison to the time span under observation, the model can arguably be treated as a continuous growth model. Taking a cue from the notion of compartmental model, we will develop and analyse the continuous model in the following under limited resources. But instead of jumping straight into the core, we will try to keep this model as simple as possible in the beginning and then add further complexities to it gradually. Suppose we are dealing with a large population of bacteria. While dealing with a large population, we may ignore the random fluctuation in breeding and dying for individual micro-organism and therefore each individual bacterium may be considered as identical. Thus for a large time interval, each of these micro-organism may be supposed to have equal probability of breeding and dying. Here comes the idea of per capita birth rate ? i.e., birth rate per member of the population per unit time (rate of incoming into the compartment) and per capita death rate ? i.e., death rate per member of the population per unit time (rate of outgoing from the compartment). We assume these rates to be constant and ? ?? . If X = X(t) be the number of bacteria at any given time t, then the birth and death rates per unit time are ? X and ? X respectively. Assuming birth and death to be continuous with time, we have dX X X dt ? ? ? ? (3.74) Note that we have neglected the effects of overcrowding which may take place eventually as well as immigration and emigration. Let r ? ? ? ?. Then r (< 0) is the growth rate of this population, then we can rewrite the equation (3.74) as dX rX dt ? (3.75) Applying the method of seperation of variables, the general solution of the differential equation (3.75) is X = ce rt . Applying initial condition X(0) = x 0 , we have the following
NSOU ? GE-MT-41 ? 109 solution of the Initial Value Problem (IVP) X = x 0 e rt (3.76) The figure 3.13 depicts the behaviour of the solution (3.76). Figure 3.13: Exponential growth curve 3.18 Logistic growth Here we revisit the previous model in Section 3.17 in the light of an overcrowded population struggling due to the scarcity of resources. The carrying capacity: Thus it is quite evident that if we ignore the effect of overcrowding on the growth of population, we will have an exponentially growing population. But when the resources are limited, this picture is far from reality. This is because the competition due to scarcity of resources increases the per capita death rate in an overcrowded population. Thus it can be safely said that only a limited number of micro-organism can sustain in any given environment. We call this number the carrying capacity of the population in the given environment and denote this by K. Whenever the population size X exceeds K, the per capita birth and death rates become equal, ignoring the other external factors like possibility of interaction with another population. This carrying capacity plays a crucial role in stabilizing the population.
110 ? NSOU ? GE-MT-41 Understanding the logistic growth: We suppose the per capita death rate to depend linearly on the size of population. Then we can take the

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per capita death rate as ( ), , 0 X t ? ? ? ? ? ? ? , where ? is the per capita death rate due to natural attrition and ? is the per capita dependence of deaths on the population size. As 0 X ? , the per capita death rate tends to ? .

Hence the death rate per unit time is ? ? X X ? ?? . Here the per capita birth rate is assumed to be same as in Section 3.17, i.e., ? . Hence the birth rate over time is ? X. Therefore the density dependent growth rate over time of this population is given by ? ? 2 dX X X dt ? ? ? ? ? ? (3.77) Letting r ? ? ? ?, the equation (3.77) becomes 2 dX rX X dt ? ? ? (3.78) Here we assume , r K K ? ? being the carrying capacity. Then equation (3.78) becomes 1 dX X rX dt K ? ? ? ? ? ? ? ? ? (3.79) Hence we can say that when X = K, 0 dX dt ? , i.e., rate of change of the population becomes zero. In other words, whenever the population size X = K, the per capita birth and death rates become equal. Example 3.18.1. Solve the equation (3.79), i.e., 1 dX X rX dt K ? ? ? ? ? ? ? ? ? with initial condition X(0) = x 0 . We rewrite the equation (3.79) as 1 dX rdt X X K ? ? ? ? ? ? ? ? ? NSOU ? GE-MT-41 ? 111 Solving this, we have ? ? 0 0 0 ( ) rt x K X t x K x e ? ? ? ? (3.80) Figure 3.14: Logistic growth curve Clearly the solution (3.80) implies the population size approaches the carrying capacity K when t ? ? . Also 0 X x ? , when t → 0. These have been illustrated in figure 3.14. 3.19 Gomperzian Model We have seen in Section 3.17 that if the population grows exponentially then eventually it will become ridiculously large. Since in reality no population goes to infinity, the exponential model needs to be modified in more realistic manner. Keeping this in mind, the Gompertz model has been devised. We assumed the growth rate to be constant in the exponential model. In our present model, the growth rate varies with time. Let us recall the equation (3.75) in Section 3.17, i.e., dX rX dt ?

112 ? NSOU ? GE-MT-41 The growth rate r in the above equation changes with time t in the following way. dr r dt ? ? ? (3.81) where ? < 0 is a decaying coefficient of r. With the initial condition r(0) = r 0 , the solution of equation (3.81) becomes 0 t r r e ?? ? (3.82) Putting this in equation (3.81), we have 0 t dX r e X dt ?? ? 0 t dX r e dt X ?? ? ? 0 log log , t r e X A ? ? ? ? ? ? ? A being arbitrary constant 0 exp t r X A e ? ? ? ? ? ? ? ? ? ? ? ? Putting the initial condition X(0) = x 0 , we have ? ? 0 0 exp 1 t r X x e ? ? ? ? ? ? ? ? ? ? ? (3.83) Figure 3.15: A comparison among exponential, logistic and Gomperzian growth curves

NSOU ? GE-MT-41 ? 113 This is the Gomperzian growth model. The figure 3.15 gives a comparison among exponential, logistic and Gomperzian growth models. 3.20 Prey Predator Model We now develop a simple prey- predator model based on the growth of a population of small insect pests, namely cottony cushion scale insects, that interact with another population of ladybird beetle predators. In the late nineteenth century, these scale insects, which accidentally came from Australia, almost destroyed American citrus industry. To contain the insects (the prey), their natural predators ladybird beetles were also imported from Australia. Initial assumptions: We make a few preliminary assumptions. ? Initially

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we assume the populations are sufficiently large so that we can neglect random differences between individuals. ? We

ignore the effect of any pesticide like DDT. ? There are only two populations, viz.

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the predator and the prey in the ecosystem we are considering. ? The prey population grows exponentially in the absence of a predator.

Suppose X = X(t) and Y = Y (t) are the number of prey and predators respectively in the ecosystem, at any time t. The per-capita birth rates give the rate of births from an individual. Suppose the per-capita birth rate for the prey (i.e., the scale insect) is a constant b 1 . Therefore rate of birth of the prey in the ecosystem per unit time is b 1 X(t). Note that this rate has nothing to do with the activities of the predators. On the other hand, the death of the prey population has two factors, one is natural cause and another is being killed by the predators. The greater the density of predators, the more likely it is that an individual prey will be eaten. Suppose the natural per-capita death rate of the scale insect is a constant a 1 . Again, the per-capita death rate of prey due to being killed by the predators is a function of the population density of the predators. Let's make the simplest assumption that this per-capita rate of insects being killed is c 1 Y

114 ? NSOU ? GE-MT-41 Thus the per-capita death rate of the scale insects is a 1 + c 1 Y . So the death rate per unit time is (a 1 + c 1 Y)X. Using the compartmental model, we have 1 1 1 . dX b X a X c XY dt ? ? ? Obviously, the per-capita death rate for the predators (the ladybird beetles) is independent of the prey density. So we assume it to be a constant a 2 . For the birth rate of predators, it is interesting to observe that it increases with availability of more food, i.e., the population density of prey. Therefore the birth-rate for the predators is the sum of a natural rate and an additional rate that is proportional to the rate of prey killed. Let the per-capita natural birth rate of predators is a constant say b 2 . Thus natural birth rate of predators is b 2 Y . Again from the above discussion we can see, the rate at which the prey insects are eaten by the beetles is c 1 XY . Hence the additional rate of birth, which is proportional to the rate of prey killed, may be assumed to be fc 1 XY . Therefore we have 2 1 2 . dY b Y fc XY a Y dt ? ? ? Assuming 1 1 1 2 2 2 , b a b a ? ? ? ? ? ? ? and 2 1 c fc ? , we have 1 1 dX X c XY dt ?? ? (3.84) 2 2 dY c XY Y dt ? ? ?

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This system of equations is known as the Lotka–Volterra prey- predator system. The parameters c 1 and c 2 are known as interaction parameters as they describe the manner in which the populations interact. Since there are positive and negative terms on the RHS of each differential equation,

it is natural to

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anticipate that the populations could either increase or decrease. These differential equations are coupled since each differential equation depends on the solution of the other. The differential equations are also non- linear since they involve the product XY . One interpretation of the product XY is that it is proportional to the rate of encounters (contacts) between the two species.

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For this two-species model, we would expect that, in the absence of any predators, the prey would grow without bound (since we have not included any growth limiting effects other than the predators). Also, in the absence of prey, we would expect the predators to die out. 3.21

Competition Model Here we study behaviour of two competing species who are up against each other for limited resources like food or territory in their ecosystem. This phenomenon has two interesting facets: one is exploitation, when the competitor uses the resource itself and the other is interference, where the population tries to prevent its competitor from utilising the same resource. Initial assumptions: Our basic assumptions are as follow. ? We assume the populations to be sufficiently large so that random fluctuations can be ignored. ? The ecosystem has only two competing populations. ? Each population grows exponentially in the absence of the other competitor population. Let X = X(t) and Y = Y (t) be the two population densities (number per unit area) at any time t. Let 1 ? and 2 ? are their respective per-capita birth rates. Unlike in the predator-prey model as we have seen before, neither population is dependent on the other as far as growth rates are concerned. Hence we can assume 1 ? and 2 ? to be constant. On the other hand, the two populations are competing for the same resource. Therefore, the density of each population has a restraining effect on the other. Suppose the per-capita death rate for Y is proportional to X, and that for X is proportional to Y . So we can write death rate of species X is (c 1 Y)X and death rate of species Y is (c 2 X)Y, where c 1 and c 2 are the constants of proportionality for this restraining effect. Hence our model becomes 1 1 dX X c XY dt ?? ?
116 ? NSOU ? GE-MT-41 2 2 dY Y c XY dt ?? ? (3.85) These equations are known as Gause's equations and are a coupled pair of first- order, non-linear differential equations. Remark 3.21.1. This system has striking similarity with the predator-prey model of the section 3.20 although the terms describing the interaction between the species differ. 3.22 More Worked out Examples Example 3.22.1. Let us

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consider the following system of differential equations 2 dx x y dt ? ? ? (3.86) 3 dy y

dt ? ? Solution. Clearly the given system can be written as 1 2 0 3 x x y y ? (3.87) i.e., x x A y y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (3.88) where 1 2 0 3 A ? ? ? ? ? ? ? ? ? ? Clearly the only equilibrium point is (0, 0). In order to find the general solutions of the system (3.88), we find the eigen values and eigen vectors of the matrix A. To find the eigen values, we have ? ? det (1) (3) 0, A I ? ? ? ? ? ? ? ? ? i.e., ? = −1, −3.
NSOU ? GE-MT-41 ? 117 Thus the eigen values are real and both of negative sign. Hence the equilibrium point is a stable node. Example 3.22.2. Consider the combat model, discussed in section 3.10, given by 1 dR a B dt ? ? 2 dB a R dt ? ? (3.89) where a 1 and a 2 are positive constants. Determine the nature of the equilibrium point(s). Solution. Clearly the equations (3.89) may be rewritten in matrix form as x ́ = Ax (3.90) where 1 2 0 0 a a ? ? ? ? ? ? ? ? ? ? A . Clealy the only equilibrium point is (0, 0). Now the eigen values of A are 1 2 a a ? , i.e., the eigen values

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are real and of opposite signs. Hence the equilibrium point is a saddle point. Example 3.22.3. Determine the

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nature of the equilibrium points of the prey- predator model given by 1 1 dX X c XY dt ?? ? 2 2 dY c XY Y dt ? ? ? (3.91) Solution. In example 3.14.1, we have seen the equilibrium points of the prey- predator model described in section 3.20 are (0, 0) and ? ? 2 1 2 1 , c c ? ? . Further we have linearized the system at these equilibrium points in example 3.15.2. When the equilibrium point is (0, 0), the linearized system is 1 X X Y Y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? J
118 ? NSOU ? GE-MT-41 where 1 1 2 0 . 0 ? ? ? ? ? ? ? ? ? ? J The eigen values of J 1 are 1 0 ? ? and 2 0 ?? ?. Hence the equilibrium point (0, 0) is a saddle point. When the equilibrium point is ? ? 2 1 2 1 , c c ? ? , the linearized system is 2 X X Y Y ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? J where 1 2 2 1 2 1 2 0 . 0 c c c c ? ? ? ? ? ? ? ? ? ? ? ? ? ? J The eigen values of J 2 are 2 1 i ? ? ? , i.e., purely imaginary. Hence the equilibrium point ? ? 2 1 2 1 , c c ? ? is a centre. Example 3.22.4. Find the time T required for the population with exponential growth to double. Solution. Clearly

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X(t + T) = 2X(t). Then ( ) 0 0 ( ) 2 ( ) r t T rt x e X t T X t x

e ? ? ? ? by equation (3.76). Hence ln 2 T r ? . 3.23 Summary This chapter introduces and deals with continuous modeling. Notion of compartmental modeling has been discussed. Several physical and real world phenomena including carbon dating, oscillation, spreading of infections etc. are discussed from a continuous modeling approach. Equilibrium points or steady state solutions have been discussed. Learners
NSOU ? GE-MT-41 ? 119 have also learned about of linearization techniques and stability analysis without actually solving the problem. 3.24 Exercises Exercise 3.24.1. Linearize the differential equation 2 13 36 dx x x dt ? ? ? at their points of equilibrium. Ans. 5 d dt ? ? ? ? Exercise 3.24.2. Linearize the system of differential equations dX X XY dt ? ? dY XY Y dt ? ? Ans. At (0, 0), 1 0 0 1 X X Y Y ? At (1, 1), 0 1 1 1 0 1 X X Y Y ? Exercise 3.24.3. Find

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the nature of the critical points of the following system of ODEs dx x y dt ? ? ? 2 3 dy x y dt ? ? 120 ?

NSOU ? GE-MT-41 Ans. Stable focus (Eigen values of the coefficient matrix are –2 $\pm$ i). Exercise 3.24.4. What is the growth rate of a population? Exercise 3.24.5. What is the carrying capacity of a population? Exercise 3.24.6. Draw the graph of exponential growth of a population. Exercise 3.24.7. For a population with exponential growth and growth rate r, find the time required for the population to grow three times. Ans. ln 3 r Exercise 3.24.8. Establish the model of exponential growth of a population. Exercise 3.24.9. What is the per capita death rate of a population with logistic growth? Exercise 3.24.10. Establish the logistic growth model. Exercise 3.24.11. Draw the graph of logistic growth model. Exercise 3.24.12. Find the equilibrium point(s) of the system given by equation (3.79). If r < 0 and k < 1, then find the nature of the equilibrium points. Ans. 0 and k. Both are unstable. Exercise 3.24.13. Linearize the system represented by the differential equation 1 dx X rX dt K ? ? ? ? ? ? ? ? Ans. dx rX dt ? at X = 0 and dx r dt ? (k – 1)(X – k) at X = k. Exercise 3.24.14. Establish the Gomperzian growth model. Exercise 3.24.15. Compare the growth rates in exponential, logistic and Gomperzian growth models. Exercise 3.24.16. Draw the graph of Gomperzian growth model. Exercise 3.24.17. Draw the comparative graphs among exponential, logistic and Gomperzian growth models. Exercise 3.24.18. What is per- capita birth rate?
NSOU ? GE-MT-41 ? 121 Exercise 3.24.19. What is per- capita death rate? Exercise 3.24.20. Establish Lotka–Volterra prey-predator system with its initial assumptions. Exercise 3.24.21. Find the equilibrium points of the Lotka–Volterra prey-predator system given by dX X XY dt ? ? dY XY Y dt ? ? Ans. (0, 0), (1, 1) Exercise 3.24.22. What are the basic (initial) assumptions of the Competition model? Exercise 3.24.23. Establish Competition model with its initial assumptions. Exercise 3.24.24. Find the points of equilibrium of the competition model represented by system of equations (3.85). Ans. (0, 0) and ? ? 2 1 2 1 , c c ? ? Exercise 3.24.25. Linearize the competition model represented by system of equations (3.85). Ans. 1 2 0 0 X X Y Y ? at (0, 0) and 2 2 2 2 2 2 1 1 1 1 0 0 c X c c X Y c Y c c ? at ? ? 2 1 2 1 , c c ? ? .

122 ? NSOU ? GE-MT-41 Unit 4 Further Models Structure 4.0 Objectives 4.1 Introduction 4.2 Heat flow through a small thin rod 4.3 Wave equation: Vibrating string 4.4 Traffic flow 4.5 Theory of Car-following 4.6 Crime Model 4.7 More worked out examples 4.8 Summary 4.9 Exercises 4.0 Objectives The followings have been discussed here. ? Modeling of heat flow and wave equation using partial differential equations; ? Two different approaches on automobile traffic flow modeling; ? Crime model. 4.1 Introduction In this unit, first we will see how the heat flows in a thin rod which is entirely insulated except at the two ends and which has no source of heat within. Later on, two different approaches on automobile traffic flow modeling and then a model about evolution of crime in a certain region will be discussed. 4.2 Heat flow through a small thin rod In this section, we will try to understand the flow of heat in a thin rod which is NSOU ? GE-MT-41 ? 123 entirely insulated except at the two ends and which has no source of heat within. Suppose we have a thin rod that is given an initial temperature distribution, then insulated on the sides. The ends of the rod are kept at the same fixed temperature; e.g., suppose at the start of the experiment, both ends are immediately plunged into ice water. We are primarily trying to understand how the temperature along the rod varies with time. Suppose that the rod has a length l (in meters). We set up a coordinate system along the rod as illustrated in figure 4.1. Figure 4.1: The variation of temperature in an insulated rod Now, the heat energy H of a body of mass m can be measured as the following H = msT (4.1) where s is the specific heat i.e. the energy required to raise a unit mass of the substance 1 unit in temperature. Also T = T(x, t) is the temperature of the body. As ML 2 T −2 is the dimension of energy, so the dimension of the specific heat is L 2 T −2 U −1 . Here M, L, T, U are the dimensions of mass, length, time and temperature respectively. Now heat flows from of high temperature area to low temperature area. According to the Fourier's law of heat transfer, the rate of heat transfer per unit area i.e., heat transferred per unit time per unit area is proportional to negative temperature gradient. Therefore we have 0 Rate of heat transfer T K area x ? ? ? ? (4.2) where K 0 is said to be the thermal conductivity having dimension MLT −3 U −1 . Now we consider our rod to be uniform, i.e., the density ? , specific heat s, thermal conductivity K 0 , cross-sectional area A all are invariant throughout the rod. Consider an arbitrary thin slice of the rod of width Δx between x and x + Δx. The slice is so thin that the temperature throughout the slice is u(x, t). Therefore, by equation (4.1), we have Heat energy of the slice = (,) A x s T x t ? ? ? ?
124 ? NSOU ? GE-MT-41 Using conservation of energy, change of heat energy of the slice in time Δt = heat in from the left boundary − heat out from the right boundary Using equation (4.2), ? ? 0 0 , (,) x x x T T s A xT x t t s A xT x T tA K tA K x x ? This implies 0 (,) (,)

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x x x T T x x K T x t t T x t t s x ? Taking , 0, t x ? ?? we have our heat equation 2 2 , 0 T T x l t

x ? ? ? ? ? ? ? ? (4.3) where 0 K s ? ? ? (4.4) is called the thermal diffusivity with dimension L 2 /T. It depends on the thermal conductivity of the material composing the rod, the density of the rod, and the specific heat of the rod. Now initially the temperature distribution in the rod is T(x, 0) = f(x), 0 > x > l, say. This gives us the initial condition. Since we have assumed both the ends of the rod is always kept at same temperature 0 o C, so T(0, t) = T(l, t) = 0, t<0. This gives us the boundary conditions.

NSOU ? GE-MT-41 ? 125 4.3 Wave equation: Vibrating string We will discuss here the derivation of the wave equation in one dimensional space. We will be modeling the vibrations of a wire or a string that is stretched between two points. A violin string is a very good example. The derivation We assume the string is stretched from x = 0 to x = L. We are looking for the function u(x, t) that describes the vertical displacement of the wire at position x and at time t. We assume the string is fixed at both endpoints, so u(0, t) = u(L, t) = 0 for all t. We will ignore the force of gravity, so at equilibrium we have u(x, t) = 0 for all x and t. This means that the string is in a straight line between the two fixed endpoints. To derive the differential equation that models a vibrating string, we have to make some simplifying assumptions. In mathematical terms the assumptions amount to assuming that both u(x, t), the displacement of the string, and u x ? ? , the slope of the string, are small in comparison to L, the length of the string. Figure 4.2: The forces acting on a portion of a vibrating string Consider the portion of the string above the small interval between x and x + Δx, as illustrated in Figure 4.2. The forces acting on this portion come from the tension T in the string. The tension is a force that the rest of the string exerts on this particular part. For the portion in Figure 4.2, tension acts at the endpoints. We assume that the tension is so large that the string acts as if it were perfectly flexible and can bend without the requirement of a bending force. With that assumption, the tension acts tangentially to the string.

126 ? NSOU ? GE-MT-41 Figure 4.3: The resolution of the tension at the point x The tension at the point x is resolved into its horizontal and vertical components in Figure 4.3. We are assuming that the positive direction is upward. The vertical component is sin u T T? ? ? , and the horizontal component is cos x T T? ? ? ¸. The slope of the graph of u at the point x is tan u x ? ? ? ? ¸. We are assuming that the slope is very small, so ? is small. Therefore cos 1 ? ? and tan sin ? ? ? . As a result, we have (,) u u T T x t x ? ? ? ? and x T T ? ? In a similar manner, we find that horizontal component of the force at x x?? is approximately T, which cancels the horizontal component at x. More interesting is the fact that the vertical component of the force at x x?? is approximately (,) u T x x t x ? ? ? ? . So the total force acting in the vertical direction on the small portion of the string is (,) (,) u u F T x x t x t x x ? ? ? ? ? ? ? ? ? ? ? ? ? ? . The length of the segment of string is close to Δx. If the string is uniform and has linear mass density ? , then the mass of the segment is m x ?? ? . The acceleration of

NSOU ? GE-MT-41 ? 127 the segment in the vertical direction is 2 2 u t ? ? . By Newton's second law, we have F = ma. This translates into 2 2 (,) (,) u u u x T x x t x t x x t ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Dividing by Δx and taking the limit as Δx goes to zero, we have 2 2 2 2 0 1 lim (,) (,) x u u u u

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T x x t x t T x x x t x ?

If we set 2 T c ? ? , the equation becomes u tt = c 2 u xx (4.5) This is the wave equation in one space variable. The constant c has dimension length/ time, so it is a velocity. 4.4 Traffic Flow When one thinks of modeling automobile traffic, it is natural to reason from personal experience and to visualize the car and driver as a coupled system, the driver responding to the surrounding vehicles and operating the car to make it become a part of the flow of freeway and city traffic. Thus the traffic is not just a mechanical process but one in which human decisions are involved, decisions which we have all experienced and can understand. In our study of traffic, we shall however step back from this personal view to take a broader perspective. Let us think of a traffic helicopter pilot looking down on a metropolitan highway grid. Looking at four miles of highway, the pilot will see a line of cars moving with various speeds. On some stretches, the traffic may be light and fast, on other stretches heavy and slow. To this observer, the individual vehicles are not as important as the sense of overall flow of the cars. The reason why the cars in the lighter traffic move faster is clear to any driver, but to the observer in the helicopter, it seems to be a property of the spacing of the cars. The closer the cars are together, the slower they move. Models of traffic flow

128 ? NSOU ? GE-MT-41 try to exploit these observations and use them to formulate a set of assumptions to produce relevant models. The purpose of these models is to understand the peculiar and often frustrating experience of daily driving. In the scenario, the cars are viewed in the large, almost as a moving gas or liquid. This kind of picture we will call a continuum model of traffic flow. In this section, We shall focus on this point of view. There is however another kind of traffic theory based upon the point of view of the individual driver responding to surrounding traffic- just the way we would naturally want to think about driving. This kind of study is called car following theory which we will discuss later. Formulation Ultimately the traffic engineer is interested in how fast cars move through the traffic grid. Every car has a speedometer, and we all want to know how long it will take to go from location A to location B. Certainly, one of the main quantitative measures of traffic is the speed of the cars in the traffic. Consider, for the sake of argument, a one-lane highway with cars in a line moving in the same direction. Since there is no passing, and cars cannot move through each other, the order of the cars is preserved, although they can move at slightly different speeds. Let the velocity of the i−th car be u i . If the x−axis coincides with the road and the position of this car is x i (t) at time t, then we have i i dx u dt ? (4.6) Any discussion of traffic on our single- lane road must deal with a collection of vehicles, with positions x i (t), i = 1, 2, . . . , N and velocities i i dx u dt , i = 1, 2, . . . , N. The continuum approach to traffic takes the view that this collection of discrete objects should be replaced by a “moving continuum”, a kind of fluid of vehicles. Such a fluid has a velocity at every value of x and at every time t, and so we may define a velocity field by a function u(x, t). The idea is that the variation of u(x, t) with x should be on a scale of length (say, a hundred yards) which is large compared to the size of a typical vehicle. Thus the value of u(x, t) at a certain time t* and a certain position x* on the road should be the velocity of cars on that particular part of the road at that time.
NSOU ? GE-MT-41 ? 129 If we know the velocity field for our road, how do we find the movement of an individual car? First we must specify the car. One way to do that is to choose a particular time, say t = t 0 , and a particular position on the road, say x = x 0 , , and identify a car as being at that spot at that time. If we then want to know where this car is located at time t < t 0 , we must use our knowledge of the velocity field, which tells us how fast any car is going when at position x and time t. Thus if x(t) is the position of our car, we know that x(t 0) = x 0 but also that ? ? ( ), dx u x t t dt ? (4.7) This last equation is the crucial one, since it relates the overall velocity field to the function x(t) for the particular car which was located at x 0 at time t 0 . We will call x(t) the Lagrangian coordinate of the car. Note that the problem of locating the position of our car, summarized as ? ? ( ), dx u x t t dt ? , where x(t 0) = x 0 (4.8) where u(x(t), t) is a given function, amounts to solving an ordinary differential equation of first order with an initial condition at the time t 0 . 4.5 Theory of Car-following We now introduce car-following theory. This model is in contrast to the previously discussed continuum model. We assume a given vehicle responds only to the car immediately in front of it (again restricting ourselves to the case of a single lane with no passing). One useful approach is to assume that the n−th car responds to the car in front of it, i.e., (n+1)−th car, according to the difference of their two velocities u n and u n+1 respectively. Let a fraction ? of the velocity difference of the two cars be eliminated by acceleration (or deceleration) of the n−th car. Clearly deceleration will apply if u n < u n+1 . If a n is acceleration, we should have ? ? 1 n n n a u u ? ? ? ? ? (4.9)
130 ? NSOU ? GE-MT-41 In terms of car positions, 2 1 2 ( ) ( ) ( ) n n n d x dx dx t t t dt dt dt ? ? ? ? ? ? ? ? ? ? ? (4.10) A somewhat more accurate model is to take into account a time delay T of the response of the driver in the n−th car 2 1 2 ( ) ( ) ( ) n n n d x dx dx t T t t dt dt dt ? ? ? ? ? ? ? ? ? ? ? ? (4.11) If all cars move at the same speed u and are equally spaced at a distance d apart, so that d + L is the front to front distance between cars (L = car length), then integrating (4.10) we have, ? ? 1 max ( ) n n u x x C L d C ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? (4.12) where 1 L d ? ? ? is the uniform car density. Here we have chosen the constant of integration to make u = 0 at max ? ? ? . This gives us a velocity-density relation from a car-following theory. Since it goes to infinity as 0 ? ? , we need to again cut this off and take max min min max max for 0 ( ) 1 1 for u u ? (4.13) Here min ? is defined in terms of max max , , u ? ? by max min max 1 1 u ? ? ? ? ? ? ? ? ? ? ? Let’s examine the likely value of ? . It is useful here to deal with the unit feet and seconds, since we are talking about interactions between cars on the scale of seconds. It would seem reasonable to assume that a driver would try to eliminate the velocity difference in about 5 seconds, or about 1 5 −th of the difference per unit time, making 1 5 ? ? . To see how this plays out in a driving situation, we consider the following example.

NSOU ? GE-MT-41 ? 131 Example 4.5.1. Suppose that the n−th and (n+1)−th cars both are moving at 100 ft/ sec and t = 0 are separated by 200 ft., with n−th car at x = 0. At this moment, the (n+1)-th car begins a constant deceleration, so that u n+1 (t) = 100 − 20t (4.14) So it will come to a stop in five seconds. We shall neglect the reaction time of the n−th driver (i.e., the delay T). Find the position of the n−th car. Solution. The equation (4.11), with 1 5 ? ? , gives ? ? 2 2 1 1 ( ) ( ) 100 20 5 5 n n d x dx t t t dt dt ? ? ? (4.15) Using the conditions x n (0) = 0 and n dx dt (0) = 100, we get (verify!) 5 2 ( ) 200 10 500 1 t n x t t t e ?? ? ? ? ? ? ? ? ? ? ? (4.16) Remark 4.5.1. Also we see by an integration on equation (4.14), using x n+1 (0) = 200, x n+1 (t) = 100t − 10t 2 + 200 (4.17) At t = 5 seconds the (n + 1)−th car has come to rest at x n+1 = 450 feet. We can see that n−th car is still moving and in fact will collide with (n+1)−th car shortly after 5 seconds unless the n−th driver hits the brakes harder and harder into the stop. 4.6 Crime Model In this section, we discuss a model that describes the evolution of crime in a certain area. We will take consideration of two different types of criminals, serious and minor. Let 1 ( )t? and 2 ( )t ? be the respective number of serious and minor criminals active in an area at time t. We also assume the behaviour of the criminals is driven by a quantity which we will refer to as the attractiveness of the area. One may think of the attractiveness as an indicator of how probable it is for a criminal to act at a specific time. The attractiveness of area depends not only on the behaviour of the active criminals but also
132 ? NSOU ? GE-MT-41 on other factors such as time, characteristics of the area examined, or the type of crime committed. With this in mind, we split the attractiveness as follows: Attractiveness = A(t) + B(t) where A(t) denotes the 'intrinsic' part of the attractiveness that depends on factors other than the behaviour of criminals and B(t) represents the 'dynamic' part of the attractiveness that is caused by criminal activity. To be more concrete, let us suppose that knowledge of crimes being committed in an area tends to encourage more crimes to take place. This effect would then be represented by the dynamic part B(t). Conversely, if the number of police officers patrolling a certain area changes according to the number of crimes taking place, that would be a negative effect represented again by B(t). On the other hand, changes in attractiveness due to factors not affected by criminal activity (e.g. time of day or seasonality) will be accounted for by the intrinsic attractiveness A(t). We will now discuss the behaviour of the criminals 1 ? and 2 ? . Let us assume that at a certain time t, a number of individuals commit a crime. Some of those are arrested and therefore removed from the system, whereas others appear in the system, perhaps due to release from prison or through people becoming criminals. We first consider how the number of criminals evolve. We take the rate of lost criminals, through arrest and conviction, to be a constant multiple of the rate at which crimes are committed, namely i i k ? (A + B), i = 1, 2. Because of the way attractiveness is defined, we assume that the total number of crimes of type i committed at time t is proportional to the product of the total attractiveness by the number of criminals, resulting in a contribution to the rate of change of the form ? ? ( ) ( ) i i i k c A t B t ? ? ? , i = 1, 2 where each k i , c i are constants of proportionality. We also assume the number of new serious and minor criminals in that area at any time t to be 1 ( )t? and 2 ( )t? respectively. Hence we can write 1 1 1 1 1 ( ) d k c A B dt ? ? ? ? ? ? 2 2 2 2 2 ( ) d k c A B dt ? ? ? ? ? ? (4.18)
NSOU ? GE-MT-41 ? 133 Let us now examine the behaviour of the dynamic part of the attractiveness i.e., B(t). Every crime, that is committed, increases B(t). Therefore we assume the dynamic attractiveness is boosted by a term proportional to the total number of crimes of both categories committed. We use the term ? ? 1 1 2 2 ( ) A B ? ? ? ? ? ? ? to model this boost, where 1 ? and 2 ? are constants. Note that we have implicitly assumed that the dynamic attractiveness B(t) is global rather than local, in the sense that criminals may exchange information about crimes committed. We further assume that B decays exponentially in time. Hence, the evolution equation for this part of the attractiveness is ? ? 1 1 2 2 ( ) dB A B B dt ? ? ? ? ? ? ? ? ? (4.19) where ? is the (constant) decay rate. This equation, together with equations (4.18), forms a 3 × 3 non-linear coupled system of ODEs. 4.7 More worked out examples Example 4.7.1. Consider the units of x to be in miles. On the stretch of road 0 > x > 4 cars are accelerating from a red light, and the velocity field is found to be u(x, t) = 10x+30t miles per hour where t < 0 is measured in hours. What is the Lagrangian coordinate of the car which was located at x = 1.5 at time t = 0? Solution. To answer this we must solve 10 30 , dx x t dt ? ? where x(0) = 1.5 (4.20) Using the integrating factor e −10t , we have xe −10t = − (0.3 + 3t)e −10t + C. Using the initial condition x(0) = 1.5, we have x(t) = −(0.3 + 3t) + 1.8e 10t . Example 4.7.2. Let the cars' trajectories be given by x = t 2 + 2tx 0 + x 0 . Note that x(0) = x 0 , identifying the parameter x 0 as the initial position. Find the velocity field for this flow.
134 ? NSOU ? GE-MT-41 Solution. To do this first compute the velocity, then use the two equations to eliminate x 0 . Thus we have dx dt = u = 2t + 2x 0 , where the first equation tells us that 2 0 1 2

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x t x t ? ? ? . Therefore u(x, t) = 2 2 2 2 2 2 2 2 2 . 1 2 1 2 x t t t x t t

t ? ? ? ? ? ? ? 4.8

Summary In this unit, we have learned about heat flow through a small thin rod and wave equation for vibrating string using partial differential equations. We have also discussed the modeling of traffic flow from two different approach, viz., traffic flow model and car following model. Another interesting model about evolution of crime have also been discussed. 4.9 Exercises Exercise 4.9.1. What is thermal conductivity? What is its dimension? Exercise 4.9.2. What is thermal diffusivity? What is its dimension? Exercise 4.9.3. Establish the model of Heat flow through a small thin rod. Exercise 4.9.4. Establish the wave equation of a vibrating string. Exercise 4.9.5. Let the cars' trajectories be given by x = t 2 + x 0 . Note that x(0) = x 0 , identifying the parameter x 0 as the initial position. Find the velocity field for this flow. Ans. u(x, t) = 2t Exercise 4.9.6. Let the cars' trajectories be given by x = t 2 + tx 0 . Note that x(0) = x 0 , identifying the parameter x 0 as the initial position. Find the velocity field for this flow. Ans. 2 (,) t x u x t t ? ? Exercise 4.9.7. Consider the units of x to be in miles. On the stretch of road 0 > x > 4 cars are accelerating from a red light, and the velocity field is found to

NSOU ? GE-MT-41 ? 135 be u(x, t) = x + 5t miles per hour where t < 0 is measured in hours. What is the Lagrangian coordinate of the car which was located at x = 2 at time t = 0? Ans. x(t) = – 5(t + 1) + 7e t Exercise 4.9.8. Suppose that the n–th and (n+1)–th cars both are moving at 200 ft/ sec and t = 0 are separated by 200 ft., with n-th car at x = 0. At this moment, the (n+1)–th car begins a constant deceleration, so that u n+1 (t) = 200 – 25t Find the position of the n–th car. Ans. As (n + 1)–th car will come to a stop in eight seconds, so 1 8 ? ? . x n = (400 – 25t) – 400 1 8 e ? . Exercise 4.9.9. What is the attractiveness of an area w.r.t the crime model? Exercise 4.9.10. Describe the crime model.

136 ? NSOU ? GE-MT-41 Unit 5 Numerical Solution of the model and its graphical representation using EXCEL Structure 5.0 Objectives 5.1 Introduction 5.2 Growth Model: Long–term Behaviour 5.3 Bank Account Problem 5.4 Affine Discrete Dynamical System and equilibrium point 5.5 Antibiotic in the Bloodstream 5.6 Discrete Logistic Model 5.7 A Linear Predator–Prey Model 5.8 A non-Linear Predator–Prey Model 5.9 Continuous Dynamical Models 5.10 Euler's Method 5.11 Logistic Equation 5.12 System of Differential Equations 5.13 Summary 5.0 Objectives ? Define and solve discrete dynamical systems ? Analyse the long–term behaviour of discrete dynamical systems and Continuous dynamical systems numerically and graphically ? Model different scenarios with linear and non-linear discrete dynamical systems and differential equation for continuous dynamical models

NSOU ? GE-MT-41 ? 137 5.1 Introduction The main goal of this chapter is to present different ways of building and analysing mathematical models in a format that can be read by students, not just instructors. This is not a text on how to use Excel. Rather, Excel is seen as a tool to further the goal of building and analysing mathematical models. No prior knowledge or experience with Excel is required to use this text. Excel is chosen as the only software used to implement and analyse models for two main reasons: 1. It is easy to use and almost everyone is familiar with it, so it takes very little time to become comfortable with the software. 2. It is everywhere. Students will have access to Excel for every mathematical modelling project they encounter inside and outside of academics. Each section contains step-by-step instructions for building the models in Excel. Discrete dynamical systems Definition 5.1: A dynamical system is simply a system that changes over time. The bacterial growth model is one such example. When time is measured in discrete increments, such as in the bacterial growth model, the system is called a discrete dynamical system. 5.2 Growth Model: Long–term Behaviour Let's graphically examine the long–term behaviour of a linear dynamical system 1n n a b a ? ? for various values of b. For different values of b the behaviour of a n are shown in Table 5.2.1 Table 5.2.1 Value of b Behaviour of an b > – 1 Oscillates between positive and negative, |a n | grows without bound b = – 1 Oscillates between – a 0 and + a 0 –1 >b> 0 Oscillates between positive and negative, |a n | approaches 0 b = 0 a n = 0 for n < 0 0 >b> 1 a n approaches 0 b = 1 a n = a 0 for all n b < 1 a n grows without bound

138 ? NSOU ? GE-MT-41 Example 5.2.1. Take a 0 = 0.1. Working process in EXCEL 1. Rename a blank worksheet "Linear" and format it to look like Figure 5.2.1. Copy the formulas in A3:B3 down to row 1 as shown in Table 5.2.2. This will give the first 15 values of a n (0 ≤ n ≤ 15) with b = 0.5 as given in Table 5.2.3. Then draw the graph by using EXCEL as shown in Fig 5.1. It is observed that the graph shows decreasing behaviour for b = 0.5. Table 5.2.2 A B C 1 n a n b 2 0 0.1 0.5 3 =A2+1 =B2* $C $2 Table -5.2.3 n a n b 0 0.1 0.5 1 0.05 2 0.025 3 0.0125 5 0.00625 5 0.003125 6 0.0015625 7 0.00078125 8 0.000390625 9 0.000195313 10 9.76563E-05 11 5.88281E-05 12 2.55151E-05 13 1.2207E-05 15 6.10352E-06 Fig 5.2.1 NSOU ? GE-MT-41 ? 139 5.3 Bank Account Problem Now consider a savings account that pays 5% interest compounded yearly. We know that a model for an account with an interest rate r is a n+1 = (1 + r) a n . Example 5.3.1 Take r = 0.05, so our model is a n+1 = 1.05 a n Here the value of b is 1.05 Table -5.3.1 n a n b 0 0.1 1.05 1 0.105 2 0.11025 3 0.1157625 5 0.121550625 5 0.127628156 6 0.135009565 7 0.150710052 8 0.157755555 9 0.155132822 10 0.162889563 11 0.171033936 12 0.179585633 13 0.188565915 14 0.19799316 15 0.207892818 The increasing behaviour is observed in graph for b < 1. Fig 5.3.1

140 ? NSOU ? GE-MT-41 5.4 Affine Discrete Dynamical System and Equilibrium Point Definition 5.4.1 (Affine Discrete Dynamical System). An affine discrete dynamical system is a sequence of numbers { a n | n = 0, 1,...} described by a relation of the form 1n n a b a m ? ? ? where 0b ? . Central to the analysis of the long–term behaviour of any dynamical system are equilibrium values (also called fixed points) Definition 5.4.2 (Equilibrium Value). A number a is called an equilibrium value for the dynamical system 1 ( ) n n a f a ? ? if a n = a for all n whenever a 0 = a. To find equilibrium values, note that if a is an equilibrium value, we must have 1n n a a a ? ? ? ( ) f a a ? ? So finding equilibrium values simply requires us to solve the equation ( ) f a a ? . For an affine system, we have a = b.a + m (1) a m b ? ? ? Example 5.4.1 Suppose now that we want to withdraw Rs.2,000 at the end of each year to supplement our income. We want to know how much money we need to deposit now so that we never run out of money. To answer this question, we will analyse a slightly more general problem: What happens to the amount in the account in terms of the initial deposit? First we will construct our model. The amount in the account grows at 5% compounded yearly

NSOU ? GE-MT-41 ? 141 but we are withdrawing Rs.2,000 each year. A dynamic model that describes this scenario is 1 1.05 2000. n n a a ? ? ? As before, a n is the amount in the account at the end of year n. We are also assuming that there is no penalty for withdrawing money each year and that we withdraw the money after the interest from the previous year has been added. This system is an example of an affine dynamical system. Solution: In this example, b = 1.05 and m = –2000, so the equilibrium value is a = –2000/(1–1.05) = 50,000. Thus, if we start with Rs.50,000 in the account and withdraw Rs.2,000 at the end of each year, we will always have the same amount in the account at the end of each year. We will take a graphical approach to analyse what happens for initial values other than the equilibrium value of Rs.50,000. Working process in EXCEL 1. Rename a blank worksheet "Savings" and format it as in Table 5.4.2. Copy the range A3:B3 from Table 5.4.1 down to row 27 to model the account over the first 25 years. Now draw the graph fig 5.4.1 using EXCEL. Table 5.4.1 B C D 1 N a n r m 2 0 50000 0.05 2000 3 =A2+1 =(1 + $C $2)* B2 – $D $2

142 ? NSOU ? GE-MT-41 Table 5.4.2 a n r m 0 50000 0.05 2000 1 50000 2 50000 3 50000 5 50000 5 50000 6 50000 7 50000 8 50000 9 50000 10 50000 11 50000 12 50000 13 50000 15 50000 15 50000 16 50000 17 50000 18 50000 19 50000 20 50000 21 50000 22 50000 23 50000 25 50000 25 50000 In above example we saw that the long–term behaviour of the system changed quite dramatically with a small change in a 0 . In situations like this we say that the system is sensitive to the initial condition. Fig . 5.4.1

NSOU ? GE-MT-41 ? 143 Also note that if a 0 ? 50,000, the system either approaches 0 or increases without bound. The equilibrium value of 50,000 is an example of an unstable or repelling equilibrium. We see this in Table 5.4.3 by taking a 0 = 58000. This is illustrated in the fig 5.4.2. The graph in the figure shows increasing nature. Table 5.4.3 n a n r m 0 58000 0.05 2000 1 58500 2 58820 3 59261 5 59725.05 5 50210.25 6 50720.77 7 51256.8 8 51819.65 9 52510.63 10 53031.16 11 53682.71 12 55366.85 13 55085.19 15 55839.55 15 56631.53 16 57563 17 58336.15 18 59252.95 19 60215.6 20 61226.38 21 62287.7 22 63502.09 23 65572.19 24 65800.8 25 67090.85 Fig. 5.4.2

144 ? NSOU ? GE-MT-41 Next add a scroll bar. Set the linked cell to B2 and the min and max to 0 and 80,000, respectively. This will allow us to vary the value of a between Rs.0 and Rs.80,000 in increments of Rs.1. Table 5.4.4 n a n r m 0 30000.00 0.05 2000 1 29500.00 2 28975.00 3 28523.75 5 27855.95 5 27237.18 6 26599.05 7 25929.00 8 25225.45 9 25586.72 10 23711.05 11 22896.61 12 22051.55 13 21153.51 15 20200.68 15 19210.72 16 18171.25 17 17079.82 18 15933.81 19 15730.50 20 13567.02 21 12150.37 22 10757.39 23 9285.76 25 7759.00 25 6136.55 Move the slider on the scroll bar left and right and observe how the long–term behaviour of the system changes. Specifically, note that amount eventually decreases to 0 Fig 5.4.3

NSOU ? GE-MT-41 ? 145 when the deposited amount is less than Rs.50,000, particularly taking Rs.30,000 shown in figure 5.4.3. When the deposited amount in greater than Rs.50,000, the amount grows without bound. 5.5 Antibiotic in the Bloodstream An infant is given an antibiotic to treat an ear infection. When taking an antibiotic, it is important to keep the amount of the drug in the bloodstream fairly constant. If it gets too low, the bacteria can begin to regrow. If it gets too high, it could cause other complications. Example 5.5.1 Suppose the half-life of the drug is 1 day (meaning that half the drug remains in the blood after each 1-day period) and a dosage of 0.1 mg is given at the end of each day. We want to examine what happens to the amount of the drug in the bloodstream in the long–run. Solution: A simple affine model for this system is a n+1 = 0.5 a n + .1 where a n = the amount of the drug in the blood at the end of day n. Since the problem did not specify the initial dosage, a 0 , we need to experiment with different values. Working process in EXCEL 1. Rename a blank worksheet "Antibiotic" and format it as in Table 5.5.2. Copy the range A3:B3 from Table 5.5.1 down to row 15 to model the system from day 0 to day 15. 2. Now draw the graph fig 5.5.1 using EXCEL. 3. Notice that even with an initial dosage of 0 mg, the amount of antibiotic in the blood appears to approach 0.2 mg at the end of each day. Note that this does not mean that the amount eventually equals 0.2 mg at every point in time, only that is equals 0.2 mg at the end of every day. Table 5.5.1 A B 1 n a n 2 0 0 3 =A2+1 =0.5*B2+0.1

146 ? NSOU ? GE-MT-41 Table 5.5.2 n a n 0 0 1 0.1 2 0.15 3 0.175 5 0.1875 5 0.19375 6 0.196875 7 0.198538 8 0.199219 9 0.199609 10 0.199805 11 0.199902 12 0.199951 13 0.199976 14 0.199988 15 0.199995 4. Next, add a scroll bar, set the min to 0, the max to 100, and the linked cell to C1. Add the formula in table 5.5.1 to allow us to vary the initial dosage from 0 to 1 mg in increments of 0.01 mg. 5. Move the slider on the scroll bar left and right and observe the long–term behaviour of the system. Specifically note that when a 0 = 0.2, the system remains at 0.2, meaning that 0.2 is an equilibrium value. Also note that no matter what the value of a 0 is, the system appears to always approach 0.2. This shows that 0.2 is an attracting equilibrium. Fig 5.5.1

NSOU ? GE-MT-41 ? 147 5.6 Discrete Logistic Model Definition 5.6.1 (Discrete Logistic Equation). A discrete logistic equation (also called a logistic map or a constrained growth model) is an equation of the form 1 ( ) n n n n a a b c a a ? ? ? ? where b and c are constants. This type of equation is often used to model population growth where a n is the population at time n . The constant b is called the intrinsic growth rate and c is called the carrying capacity 5.6.1 Bacteria Growth model Example 5.6.1 Table 5.6.1 gives the number of bacteria in a Petri dish, a n , at the end of each hour n. This data is graphed in Figure 5.6.1. We want to model a n in terms of n. When modelling a dynamical system, it is often convenient to think about the way the variable(s) change between time periods. Specifically, we consider the change between time periods 1 n n n a a a ? ? ? ? The values of n a? for the first 7 values of n are given in Table 5.6.2. Notice that as an increases, n a? also increases. This suggests that n a? is proportional to a n , which leads to the equation 1 n n n n a a a ra ? ? ? ? ? (5.3) Table 5.6.1 n 0 1 2 3 5 5 6 7 8 9 a n 10.3 17.2 27. 55.3 80.2 125.3 176.2 255.6 330.8 390.5 n 10 11 12 13 15 15 16 17 18 19 a n 550 520.5 560.5 600.5 610.8 615.5 618.3 619.5 620 621 Table 5.6.2 n 0 1 2 3 4 5 6 a n 10.3 17.2 27 45.3 80.2 125.3 176.2 Δa n 6.9 9.8 18.3 34.9 45.1 50.9 79.4

148 ? NSOU ? GE-MT-41 Working process in EXCEL Rename a blank worksheet "Bacteria Population" and format it as in Figure 5.6.1. Enter the data from Table 5.6.3 in columns A and B and draw the figure 5.6.1 Table 5.6.3 n a n 1 10.3 2 17.2 3 27 5 55.3 5 80.2 6 125.3 7 176.2 8 255.6 9 330.8 10 390.5 11 550 12 520.5 13 560.5 15 600.5 15 610.8 16 615.5 17 618.3 18 619.5 19 621 Example 5.6.2 For discrete logistic equation, redefine the above model by introducing carrying capacity. So instead of assuming a constant growth rate r, we assume a growth rate that Figure 5.6.1

NSOU ? GE-MT-41 ? 149 approaches 0 as the population approaches carrying capacity given by 621. An equation implementing this assumption is given by ? ? 1 621 n n n n n a a a b a a ? ? ? ? ? ? (5.6.1) where b < 0 is a constant. Solving for a n + 1 yields the model 1 (621) n n n n a a b a a ? ? ? ? (5.6.2) To implement the model (5.5.2) we need to find the value of b. Equation (5.5.1) predicts that ? ? 1n n a a ? ? , is proportional to (621) n n a a ? . If a graph of ? ? 1n n a a ? ? vs (621) n n a a ? is approximately a straight line through the origin, then the assumption is reasonable and the slope of the line is the value of b. Working process in EXCEL 1. Rename a blank worksheet "Bacteria" and format it as in Table 5.6.5. Enter the data from Table 5.6.1 in columns A and B and copy range D2:E2 down to row 20. Create a graph of the transformed data in columns D and E of Table 5.6.5 and fit a straight line through the origin as in Figure 5.6.2. We see that the line fits the data well, so our model appears to be reasonable. Figure 5.6.2 Using the slope of the line in Figure 5.6.2, our model is ? ? 1 0.0008 621 n n n n a a a a ? ? ? ?

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150 ? NSOU ? GE-MT-41 C 1 Predicted 2 10.3 3 = C2 + 0.0008* (621-C2)*C2 Table 5.6.5 A B C D E 1 n a n predicate an(621 - an) an+1 - an 2 0 10.3 10.3 =B2*(621-B2) = B3-B2 3 =A2+1 = C2+0.0008*(621-C2)*C2 Table 5.6.5 n an Predicate an(621 - an) Δ an=a(n+1)-a(n) 1 10.3 10.3 6290.21 6.9 2 17.2 15.33217 10385.36 9.8 3 27 22.76113 16038 18.3 5 55.3 33.6555 26079.21 35.9 5 80.2 59.56781 53372.16 55.1 6 125.3 72.08577 62111.21 50.9 7 176.2 103.7509 78373.76 79.5 8 255.6 156.6696 93396.25 75.2 9 330.8 202.3255 95998.16 59.6 10 390.5 270.0925 90026.25 59.6 11 550 355.9153 79650 80.5 12 520.5 522.0392 52352.25 50 13 560.5 589.2156 33960.25 50.1 15 600.5 550.7917 12310.25 10.3 15 610.8 575.5925 6230.16 3.7 16 615.5 596.5539 3995.25 3.8 17 618.3 608.1609 1669.51 1.2 18 619.5 615.5075 929.25 1.5 19 621 617.6579 0 -621

NSOU ? GE-MT-41 ? 151 Use the data in columns A, B, and C from Table 5.6.5 to form a graph as in Figure 5.6.3. Notice that the "shape" of the predicted values is relatively close to the shape of the observed values, so the reasonableness of our model is verified. Figure 5.6.3 5.7 A Linear Predator–Prey Model Consider a forest containing foxes and rabbits where the foxes eat the rabbits for food. We want to examine whether the two species can survive in the long–term. A forest is a very complex ecosystem. So, to simplify the model, we will use the following assumptions: 1. The only source of food for the foxes is rabbits and the only predator of the rabbits is foxes. 2. Without rabbits present, foxes would die out. 3. Without foxes present, the population of rabbits would grow. 4. The presence of rabbits increases the rate at which the population of foxes grows. 5. The presence of foxes decreases the rate at which the population of rabbits grows. We will model these populations using a discrete dynamical model. Each state of the system consists of the populations of foxes and rabbits at a point in time. Since this state consists of two components, this is a two–dimensional discrete dynamical system. To create our model, we first need to define some variables. Let

152 ? NSOU ? GE-MT-41 F n = population of foxes at the end of month n R n = population of rabbits at the end of month n As in the bacteria model, the assumptions are stated in terms of rates of change, 1 n n n F F F ? ? ? ? (5.7.1) and 1 n n n R R R ? ? ? ? (5.7.2) There are many ways we could model these rates of change with the assumptions. In this section we will create a linear model. In the next section we will create a nonlinear model. Assumptions 2 and 3 deal with the rates of change of each population in the absence of the other. A reasonable way to model these is to say that the rates are proportional to the populations. This yields the difference equations 1 n n n n F F F a F ? ? ? ? ? ? ? (5.7.3) 1 n n n n R R R d R ? ? ? ? ? (5.7.4) where both a and d are between 0 and 1. Note that the coefficient of proportionality in (5.7.3) is negative to reflect the fact that the foxes would die out (a negative rate of change) without rabbits. The coefficient in (5.7.5) is positive because the population of rabbits grows (a positive rate of change) without foxes. Now, assumptions 5 and 5 say that these rates in Equations (5.7.3) and (5.7.5) either increase or decrease in the presence of the other species. So, to incorporate these assumptions, we will simply add one term to each of Equations (5.7.3) and (5.7.5) yielding: 1n n n n F F a F b R ? ? ? ? ? ? (5.7.5) 1n n n n R R c F d R ? ? ? ? ? ? (5.7.6) where b and c are non-negative. Note that the added term in (5.7.5) is positive to reflect the fact that the presence of rabbits increases the rate at which the population of foxes grows. The added term in (5.7.6) is negative since the presence of foxes decreases the rate at which rabbits grow. Rewriting Equations (5.7.5) and (5.7.6) yields our model in the form of a system of linear equations 1 (1) n n n F a F b R ? ? ? ? (5.7.7) 1 (1) n n n R c F d R ? ? ? ? ? (5.7.8)

NSOU ? GE-MT-41 ? 153 Because our model has the form of a system of linear equations, it is called a two- dimensional linear discrete dynamical system. The model could be written in matrix form as 1 1 1 1 n n n n F F a b R R c d ? (5.7.9) The parameters (1 – a) and b are called the fox death and birth factors, respectively, while the parameters – c and (1 + d) are called the rabbit death and birth factors, respectively Working process in EXCEL Rename a blank worksheet "Linear Predator–Prey" and format is as Table 5.7.1. The initial values of the parameters and populations are shown in the Table. Copy row 8 down to row 37 to model 30 months as shown in Table 5.7.2 Table 5.7.1 A B C 1 Factors 2 Death Birth 3 Foxes 0.5 0.5 5 Rabbits -0.17 1.1 6 Month Foxes Rabbits 7 0 500 200 8 =A7+1 =$.B $3*B7+C7*$C $3 =B7*$B $5+C7*$C $5 Table 5.7.2 Birth Death Foxes 0.5 0.5 Rabbits -0.17 1.1

154 ? NSOU ? GE-MT-41 Month Foxes Rabbit 0 500 200 1 330 135 2 219 92.5 3 156.56 65.51 5 98.995 55.9528 5 67.87812 33.191 6 57.5267 25.55173 7 33.93505 20.05536 8 25.98577 16.28001 9 19.00539 13.6606 10 15.96655 11.79592 11 12.20159 10.53122 12 10.27328 9.500068 13 8.896667 8.593617 15 7.88578 7.950556 15 7.119109 7.395018 16 6.517161 6.923171 17 6.027859 6.507571 18 5.616953 6.133595 19 5.261915 5.792071 20 5.957785 5.576753 21 5.665595 5.183305 22 5.505619 5.908655 23 5.166271 5.650565 25 3.953361 5.507355 25 3.735622 5.177719 26 3.538399 3.960605 27 3.353551 3.755137 28 3.178775 3.560566 29 3.013615 3.376231 30 2.857299 3.201539

NSOU ? GE-MT-41 ? 155 1. Next, plot the graphs . The graphs of rabbits versus month and foxes versus month are called time plots shown in fig (5.7.1) and fig (5.7.2) respectively. The curve in the graph of rabbits versus foxes is called a trajectory of the system shown in Fig 5.7.3. The plane on which a trajectory is drawn is called the phase plane. Notice that the trajectory tends toward the origin (0 foxes and 0 rabbits). This means that both species eventually die out. This is also shown in the time plots. If we change the initial populations, we note that the trajectories always tend toward the origin. This indicates that the populations always die out, regardless of the initial populations. As in a one-dimensional discrete dynamical system, two-dimensional systems can have an equilibrium Fig 5.7.1 Fig 5.7.2

156 ? NSOU ? GE-MT-41 Fig 5.7.3 5.8 A nonLinear Predator–Prey Model Lotka-Volterra model: Let's consider a similar population of foxes and rabbits along with the same set of assumptions as in previous section, but we will model the assumptions differently. We will start with modelling assumptions 2 and 3 the same way: 1 n n n n F F F a F ? ? ? ? ? ? ? (5.8.1) 1 n n n n R R R d R ? ? ? ? ? ? (5.8.2) where 0 > a ≤ 1 and 0 > d ≤ 1. In Section 5.7, the coefficients of F n and R n were kept constant. In this section we will model them as increasing or decreasing in the presence of the other population. Assumption 5 says that the presence of rabbits increases the rate of growth of foxes. so, we write ? ? 1n n n n F F a b R F ? ? ? ? ? ? (5.8.3) where b ≥ 0. Likewise, assumption 5 says that the presence of foxes decreases the rate of growth of rabbits, so, we have ? ? 1n n n n R R d c F R ? ? ? ? ? (5.8.4) where c ≥ 0. Rewriting (5.7.3) and (5.7.5) we get our model: ? ? 1 1 n n n F a b R F ? ? ? ? ? (5.8.5)

NSOU ? GE-MT-41 ? 157 1 (1) n n n n R c F R d R ? ? ? ? ? ? (5.8.6) This type of model is called a Lotka-Volterra model, named after the researchers that first devised it in the 1920s and 1930s. Note that both equations have a term involving R n F n ; thus, the model in nonlinear. This term can be interpreted as modelling the number of interactions of the two species. These interactions increase the number of foxes while decreasing the number of rabbits. Also note the similarities between this nonlinear model and the linear model in (5.7.10). Working process in EXCEL We will refer to the parameters in this model using the same names as in the linear model. This model can easily be implemented in Excel. Rename a blank worksheet "Nonlinear Predator–Prey" and format it as in Table 5.8.1. Copy row 8 down to row 507 to model 500 months. (Note that the parameters in this model do have similar meanings as in the linear model, but they do have different values. Also we have different initial populations. Table 5.8.1 A B C 1 Factors 2 Death Birth 3 Foxes 0.88 0.0001 5 Rabbits -0.0003 1.039 6 Month Foxes Rabbits 7 0 110 900 8 =A7+1 =$B $.3*B7 + B7*C7* $C $3 =B7*C7 $B $5 + C7* $C $5 Create graphs similar to those in Figure 5.7.1. This model predicts that the populations oscillate with the same period of oscillation, but with a phase shift, meaning they don't reach their peaks at the same time. These oscillations cause the spiralling nature of the trajectories in the graph of rabbits versus foxes. Oscillations such as this are actually observed in nature; thus, this model appears to be more reasonable than the linear model.

158 ? NSOU ? GE-MT-41 Now let's calculate the equilibrium point of the system. Suppose (f, r) is an equilibrium point. By definition, this point must satisfy the system of equations f = 0.88f + 0.0001fr r = – 0.0003fr + 1.039r Assuming that 0f r ? ? yields the solution f = 130 and r = 1,200. Another equilibrium is (0, 0). Note that the point (130, 1200) is at the center of the spiral in the phase plane. If we change the starting populations in the worksheet to 130 foxes and 1200 rabbits we note that the populations do not change, as expected. To determine if this equilibrium is attracting or repelling, we need to consider starting populations near the equilibrium. Changing the initial populations to 129 foxes and 1201 rabbits yields the trajectory shown in Figure 5.8.2. Notice that the trajectory moves away from the equilibrium. Trying other initial populations yields similar results. The fact that the trajectories move away from the equilibrium is evidence that the equilibrium is repelling. Fig 5.8.2 Fig 5.8.1

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NSOU ? GE-MT-41 ? 159 5.9 Continuous Dynamical Models In reality, time is continuous so using discrete time units is a simplification. It is a convenient simplification because a difference equation is very easy to solve for a n+1 in terms of a n giving a recursive solution. When measuring time continuously, we describe change with a differential equation. Differential equations are formed in the same basic way as difference equations, but finding their solutions can be much more complicated. To illustrate how differential equations are formed, consider the following observation: When a hot cup of coffee is set on a desk, it initially cools very quickly. As the coffee gets closer to room temperature, it cools less quickly. This simple observation is an example of Newton's Law of Cooling: The rate at which a hot object cools (or a cold object warms) is proportional to the difference between the temperature of the object and the temperature of its surrounding medium. This law can be translated into the following differential equation: ( ) dy dt k y T ? ? where y(t) = temperature of the object a time t T = temperature of the medium (assumed to be constant) k = constant of proportionality This differential equation can be solved using basic techniques yielding the general solution: ( ) kt y t T Ce ? ? where C is an arbitrary constant. Example 5.9.1 (Newton's Law of Cooling) Consider a cup of coffee that is initially 100 o F, cools to 90 o F in 10 minutes, and sits in a room whose temperature is a constant T = 60 o F. The general solution to Newton's Law of Cooling is ( ) kt y t T Ce ? ? . To find the specific solution in this case we need to find the values of the constants C and k. The initial condition y (0) = 100 gives .0 100 60 50 k Ce C ? ? ? ?
160 ? NSOU ? GE-MT-41 The condition y (10) = 90 gives 90 = 60 + 50e k.10 0.02877 k? ? ? Thus the model is : 0.02877 ( ) 60 50 . t y t e ? ? ? (5.9.1) Working process in EXCEL A graph of this model is shown in Figure 5.9.1. This curve is called the solution curve. 0.02877 ( ) 60 40 . t y t e ? ? ? t Temp 0 100 10 89.9995 20 82.5992 30 76.8751 50 72.6553 50 69.5913 60 67.1185 70 65.3387 80 65.0039 90 63.0029 100 62.2521 110 61.6891 120 61.2668 130 60.9501 150 60.7125 150 60.5355 Fig 5.9.1
NSOU ? GE-MT-41 ? 161 In this chapter, we do not analytically solve differential equations as done in previous section. Instead, we use a technique called Euler's Method to numerically approximate solution curves and then graphically analyse the results 5.10 Euler's Method Euler's method is a technique for approximating points on the solution curve of a differential equation. To illustrate the method, consider a differential equation of the form ( ) dy dt F y ? (5.10.1) Along with the initial condition y(t 0) = y 0 where t 0 and y 0 are some given values. As shown in Figure 5.8.1, the point (t 0 , y 0) is a point on the solution curve. Now, let h be some small positive quantity and define time t 1 to be 1 0 t t h ? ?. Our goal is to approximate the y – coordinate of the point ? ? 1 1 , ( ) t y t on the solution curve Fig 5.10.1 In the triangle in Figure 5.10.1, the base has length h and the hypotenuse is on a line with slope F(y 0). Therefore, the height is height = h F (y 0) The y-coordinate of the base of the triangle is y 0 . Thus the y-coordinate of the top of the triangle is 1 0 0 ( ) y y h F y ? ? (5.10.2)
162 ? NSOU ? GE-MT-41 Fig 5.10.1 This y-coordinate is an approximation of y (t 1). To approximate y (t 2) where 2 1 t t h ? ?, we can repeat this process, replacing y 0 with y 1 . We continue to repeat this process as follows: 1 0 t t h ? ? 1 0 0 ( ) y y h F y ? ? 2 1 t t h ? ? 2 1 1 ( ) y y h F y ? ? ... ... 1n n t t h ? ? ? 1 ( ) n n n y y h F y ? ? ? This algorithm is called Euler's method. The results from Euler's method can be interpreted in at least two ways: a. Numerically: For each ( ) n n y y t ? . b. Graphically: Each point (,) n n y t is approximately a point on the solution curve. Example 5.10.1 (Applying Euler's Method) Working Process in EXCEL Euler's method is easy to implement in Excel. Here we apply it to the Newton's law of cooling problem in Example 5.9.1 and examine how the value of h affects the approximation. Rename a blank worksheet "Euler" and format it as in Table 5.10.1. Copy row 5 down to row 120 to calculate values at 115 different time values. Table 5.10.1 A B 1 h = 0.5 2 3 Time Approximate 4 0 100 5 = A4+$B$1 =B4+$B$1*(-0.02877*(B4-60))
NSOU ? GE-MT-41 ? 163 h= 1 Euler's Method Time Approximate 0 100 1 98.8592 2 97.73150852 3 96.65597302 5 95.59166837 5 95.56769607 6 93.57318356 7 92.60728297 8 91.66917155 9 90.75805938 10 89.8731503 11 89.01369005 12 88.17896619 13 87.36825733 15 86.58087257 15 85.81615086 16 85.07351059 .... 109 61.66022772 110 61.61256297 111 61.56607251 112 61.52101651 113 61.57725686 115 61.53575618 115 61.39357825 116 61.35338788 117 61.31555091 118 61.27663516 119 61.23990539 exact 120 61.20523331 61.2668 Fig 5.10.1

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164 ? NSOU ? GE-MT-41 5.11 Logistic Equation Here we are trying to explain the Logistic equation with the help of an example. Example 5.11.1: Suppose that 25 panthers are released into a game preserve. Initially the population grows at a rate of approximately 25% per year, but because of limited food supplies, the preserve is believed to support only 200 panthers. We want to model the population over time. Note that the information given deals with the rate of change. This suggests we create a differential equation to model the rate of change of the population. If y(t) represents the population at year t, 0.25 dy dt y ? However, this model does not take into account the fact that the preserve can support only 200 panthers. It seems reasonable to assume that the rate of growth will decrease as y approaches 200. One way to model this is 0.25 1 200 dy y y dt ? ? ? ? ? ? ? ? (5.11.1) Note that as 200, 1 0 200 y y ? ? ? meaning that 0 dy dt ? . Equation (5.11.1) is called a logistic differential equation. Also note that this logistic differential equation is very similar to the logistic difference equation we derived for the bacteria population in 5.6. The general form of a logistic equation is 1 dy y k y dt L ? ? ? ? ? ? ? ? The parameter L is called the carrying capacity and the parameter k is called the unconstrained (or intrinsic) growth rate. Working Process in EXCEL To approximate the solution curve of Equation (5.10.1), rename a blank worksheet "Logistic" and format it as in Table 5.11.1. Copy row 5 down to row 129 to model 25 years.

NSOU ? GE-MT-41 ? 165 Table 5.11.1 A B 1 h = 0.2 2 3 Year Population 4 0 25 5 = A4+$B$1 =B4+$B$1*(0.25*(1-B4/200)*B4) Next, create a graph as in Fig 5.10.1. Figure 5.10.1 shows that the rate of growth slows down as the population approaches 200, as expected. The population reaches the carrying capacity by year 25. Also note that this graph looks very similar to the graph of the bacteria population in Example 5.3.1 Fig 5.11.1 Non-autonomous differential equations (meaning equations where the right-hand side explicitly depends on t) of the form (,) dy F t y dt ? arise frequently in applications. Euler's method can be easily adapted to these types of differential equations. The basic algorithm is given by ? ? 1 1 , , . n n n n n n t y h y y h F y t ? ? ? ? ? ?

166 ? NSOU ? GE-MT-41 The next example illustrates an application of a non-autonomous differential equation Example 5.11.2 (Bacteria Growth) Let y(t) denote the population of bacteria in a Petri dish t days after the bacteria begin growing. Suppose y(t) is described by the differential equation 150 dy t dt ? for t between 0 and 10. If the initial population is 500, approximate the solution curve over the interval 0 10 t? ? and approximate the population at time t = 7 Working Process in EXCEL Rename a blank worksheet "Bacteria" and format it as in Table 5.11.2. Copy row 5 down to row 105. Table 5.11.2 A B 1 h = 0.1 2 3 Day Population 4 0 500 5 = A4+$B$1 =B4+$B$1*150*SQRT(A4) Create a graph of the solution curve as in Figure 5.11.2. Note that as time increases, the population grows faster. Fig 5.11.2

NSOU ? GE-MT-41 ? 167 To determine if this approximate solution curve is accurate, we change the value of h in cell B1 to 0.05, copy row 5 down to row 205, and graph the resulting approximate solution curve. Observe that this curve looks very similar to that in Figure 5.11.2. This indicates that h = 0.1 yields accurate results. Now note that for h = 0.1, the calculations give (7) 2331 y ? . We interpret this result by saying that at the beginning of day 7, there will be approximately 2300 bacteria. Exercise Let y(t) denote the population of rabbits (in thousands) in a certain forest at time t (in months). Suppose ÁÛUß is described by the differential equation ? ? 1 3cos 5 9 . dy t dt ? ? ? a) Graph an approximate solution curve over the interval 0 10 t? ? if the initial population is 3000. b) Describe, in words, the behavior of the population over this interval of time. c) What is the approximate population at time t = 5? 5.12

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System of Differential Equations A system of differential equations is a set of

two or more related differential equations involving two or more unknown functions. In this section we restrict ourselves to a set of two equations with the general form (,) ,

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dx F x y dt ? (,) dy G x y dt ?

along with the initial conditions 0 0 0 0 ( ) , ( ) x t x y t y ? ? . Euler's method for a system such as this is: 1 0 t t h ? ? ? ? 1 0 0 0 , x x hF x y ? ? ? ? 1 0 0 0 , y y hG x y ? ? 2 1 t t h ? ? ? ? 2 1 1 1 , x x hF x y ? ? ? ? 2 1 1 1 , y y hG x y ? ?

168 ? NSOU ? GE-MT-41 ... ... 1n n t t h ? ? ? ? ? 1 , n n n n x x hF x y ? ? ? ? ? 1 , n n n n y y hG x y ? ? ? Example 5.12.1 (Connected Tanks) Consider the two connected tanks filled with salt water shown in Figure 5.12.1. Let x(t) and y(t) denote the masses of salt (in kg) in the tanks at time t where x(0) = 5 and y(0) = 2. We assume perfect mixing in both tanks. The goal of this example is to describe the long-term behaviour of x and y Fig 5.12.1 To set up the system of differential equations, we use the following principle: Overall rate of change = Rate in − Rate out. First, observe that each tank is losing solution at the overall rate of 8 L/min and gaining solution at the rate of 8 L/min, so the volume of each tank is not changing. Now consider tank 1. This tank has pure water entering on the left at 6 L/min and solution from tank 2 entering on the right at 2 L/min. Therefore, 0 6 2 . min 24 min 12 min kg L y kg L y kg Ratein L L ? ? ? ? ? Likewise, tank 1 has solution leaving on the right at the rate of 8 min L , so 8 24 min 3min x kg L x kg Ratein L ? ? ? Therefore, the differential equation for tank 1 is 12 3 dx y x dt ? ?
NSOU ? GE-MT-41 ? 169 By a similar argument, the differential equation for tank 2 is . 3 3 dy x y dt ? ? Working Process in EXCEL To numerically solve this system using Euler's method with a step size of h = 0.2, rename a blank worksheet "Connected Tanks" and format it as in Table.5.12.1 Copy row 5 down to row 205 Table 5.12.1 A B C 1 h = 0.2 2 3 t x y 4 4 2 5 = A4+$B$1 =B4+$B$1*(C4/12-B4/3) =C4+$B$1*(B4/3-C4/3) To graphically analyse the results, create graphs of x vs. t and y vs. t as in Figure 5.12.1. These graphs are called time plots. In these graphs, we see that the mass of salt in tank 1 drops to 0 by about time 20 min. The mass of salt in tank 2 initially increases, but then drops to 0 by about time 30 min. Fig 5.12.1 We can combine the two time plots into a single graph by graphing y vs. x as in Figure 5.12.2. The x − y plane in this graph is called the phase plane and the curve is
170 ? NSOU ? GE-MT-41 called a trajectory. The trajectory shows that the system starts at the point (5, 2) (the initial condition). Moving along the trajectory to the left, we see that x decreases while y initially increases, but then begins to decrease. Both x and y eventually approach 0. This is exactly what we saw in the time plots. Fig 5.12.2 In simpler terms, an equilibrium point is a point on the phase plane where if we start there, we stay there forever. As with discrete dynamical systems, equilibrium points of systems of differential equations are points on the phase plane which typically attract or repel trajectories. Equilibrium points that attract trajectories are called attracting, stable, or asymptotically stable. Equilibrium points that repel trajectories are called unstable or repelling B C 3 x y 4 = RANDBETWEEN (−5, 5) = RANDBETWEEN (−5, 5) On the graph of the trajectory, change the axes mins and maxes to "5 and 5 as in Figure 5.12.3. Press the F9 key several times. Each time, a new set of initial conditions is generated. Observe that the trajectory always approaches the point (0, 0). This is graphical evidence that (0, 0) is an attracting equilibrium point.
NSOU ? GE-MT-41 ? 171 Fig 5.12.3 we need to set both F (x, y) and G (x, y) equal to 0 and solve for x and y. In Example 5.12.1, this yields the system of linear equation 0 12 3 y x ? ? 0 3 3 x y ? ? Solving this system using elementary linear algebra techniques yields the only solution x = y = 0. Therefore, (0, 0) is the only equilibrium point of the system. 5.13 Summary In this Unit we have explained some of the basic terminology and tools used to build the models. These explanations apply directly to Office Excel 2016, although most of them apply to other versions of Excel. We have analysed the long-term behaviour of discrete and continuous dynamical system using working process in EXCEL. Model different scenarios with linear and nonlinear discrete dynamical systems and differential equation for continuous dynamical models also studied numerically and presented graphically.
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Introduction) Models of systems have become part of our everyday lives. They range from global decisions having a profound impact on our future, to local decisions about whether to cycle to university based on weather predictions. Together with their provision of a deeper understanding of the processes involved, this predictive nature of models, which aids in decision-making, is one of their key strengths. In particular, many processes can be described with mathematical equations, that is, by mathematical models. Such models have use in a diverse range of disciplines. There is an aesthetic use, for example, in constructing perspective in paintings or etchings such as is seen in the paradoxical work of Escher. The proportions of the golden mean and the Fibonacci series of numbers, occurring in many natural phenomena such as the arrangement of seed spirals in sunflowers, have been applied to methods of information
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storage in computers. This well-known mathematical series is also applied in models describing the growth nodes on the stems of plants, as well as in aesthetically pleasing proportions in painting and sculpture and the design of musical instruments. From a philosophical perspective, mathematical logic and rigour provide a model for the construction of argument. In a more practical and analytical mode there is a plethora of applications. Mathematical optimisation theory has been applied in the clothing industry to minimise the required cloth for the maximum number of garments, and to the arrangement of odd- shaped chocolates in a box to minimise the number required to give the impression that the box is full! The mathematics of fractals has allowed the successful development of fractal image compression techniques, requiring little storage for extremely precise images. Some other areas of application include the physical sciences (such as astronomy), medicine (such as the absorption of medication), and the social sciences (such as patterns in election voting). Mathematical models are used extensively in biology and ecology to examine population fluctuations, water catchments, erosion and the spread of pollutants, to name just a few. Fluid mechanics is another extensive area of research, with applications ranging from the modelling of evolving tsunamis across the ocean, to the flow of lolly mixture into moulds. (Mathematicians were consulted to establish the best entry points for the mixture to the mould in order to ensure a filled nose for a Mickey Mouse lollypop!) 1.2

SA DSC-6 Combine.pdf (D143717932)

3/52	SUBMITTED TEXT	78 WORDS	92%	MATCHING TEXT	78 WORDS

and Demerits of Mathematical Modeling Merits ? They are quick and easy to produce ? They can simplify a more complex situation ? They can help us improve our understanding of the real world as certain variables can readily be changed ? They enable predictions to be made ? They can help provide control - as in aircraft scheduling Demerits ? The model is a simplification of the real problem and does not include all aspects of the problem ? The model may only work in certain situations 1.4

and disadvantages of mathematical models Advantages • They are quick and easy to produce • They can simplify a more complex situation • They can help us improve our understanding of the real world as certain variables can readily be changed • They enable predictions to be made • They can help provide control - as in aircraft scheduling Disadvantages • The model is a simplification of the real problem and does not include all aspects of the problem • The model may only work in certain situations

W https://getrevising.co.uk/grids/advantages-and-disadvantages-of-mathematical

4/52	SUBMITTED TEXT	45 WORDS	68%	MATCHING TEXT	45 WORDS

y t − a 1 y t−1 − a 2 y t−2 −•... − a n y t−n − b = 0 i.e., y t = a 1 y t−1 + a 2 y t−2 +•... +•a n y t−

SA P3___ODE.pdf (D24671273)

5/52	SUBMITTED TEXT	33 WORDS	68%	MATCHING TEXT	33 WORDS

y 1 , y 2 , ..., y n . Example 2.2.1. Clearly y t = 3y t−1 , y t+2 = y t−1 + y t−2 + 5

SA P3___ODE.pdf (D24671273)

6/52	SUBMITTED TEXT	70 WORDS	34%	MATCHING TEXT	70 WORDS

of the equation y t = a 1 y t−1 + a 2 y t−2 +•... + a n y t−n (2.2) if and only if r t = a 1 r t−1 + a 2 r t−2 + ... + a n r t−n or equivalently r n − a 1 r n−1 − a 2 r n−2 −•... − a n = 0 (2.3)

SA P3___ODE.pdf (D24671273)

7/52	SUBMITTED TEXT	20 WORDS	83%	MATCHING TEXT	20 WORDS

y t = a 1 y t−1 + a 2 y t−2 + ... + a n y t−

SA P3___ODE.pdf (D24671273)

8/52	SUBMITTED TEXT	19 WORDS	83%	MATCHING TEXT	19 WORDS

y t = a 1 y t−1 +a 2 y t−2 +•... + a n y t−

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9/52	SUBMITTED TEXT	21 WORDS	62%	MATCHING TEXT	21 WORDS

x n = ax n−1 = a 2 x n−2 ... = a n x 0 Hence x n = a n x 0 (2.5)

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10/52	SUBMITTED TEXT	84 WORDS	29%	MATCHING TEXT	84 WORDS

a(ax 0 + b) + b = a 2 x 0 + ab + b x 3 = ax 2 + b = a(a 2 x 0 + ab + b) + b = a 3 x 0 + a 2 b + ab + b Proceeding similarly, x n = a n x 0 + a n−1 b + . . . + a 3 b + a 2 b + ab + b = a n x 0 + b(a

SA 2-13.pdf (D20932629)

11/52	SUBMITTED TEXT	10 WORDS	90%	MATCHING TEXT	10 WORDS

When the roots of the characteristic equation are real and distinct

SA P3___ODE.pdf (D24671273)

12/52	SUBMITTED TEXT	16 WORDS	58%	MATCHING TEXT	16 WORDS

When the roots of the characteristic equation are real and identical We assume now the characteristic equation

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13/52	SUBMITTED TEXT	25 WORDS	71%	MATCHING TEXT	25 WORDS

in equation (2.14), we get 1 2 2 , 0 2 k k k b k k ? ? ? ? i.e. 2 1 , 0 2 k k k b k ? ? ? ? ? ? ? ? ? ?

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14/52	SUBMITTED TEXT	41 WORDS	38%	MATCHING TEXT	41 WORDS

the general solution is y t = A 1 (a + ib) t + A 2 (a − ib) t = A 1 r t (cos t? + i sin t?) + A 2 r t (cos t? −

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15/52	SUBMITTED TEXT	71 WORDS	36%	MATCHING TEXT	71 WORDS

T n+1 − T n = − 0.05(T n − 80) i.e., T n+1 = 0.95T n + 4 for n = 0, 1, 2, ... 30 ? NSOU ? GE-MT-41 Therefore equation (2.28) gives 1 (0.95) (0.95) 180 4 1 0.95 n n n T ? ? ? ? ? ? ? ? ? ? ? ? = 80 + 100(0.95) n for n = 0, 1, 2, ... In particular, T 20 = 80 + 100(0.95) 20 = 115.85 where

SA Dynamical system models of some complex ecosystems.pdf (D34298608)

16/52	SUBMITTED TEXT	14 WORDS	85%	MATCHING TEXT	14 WORDS

sufficiently large so that we can neglect random differences between them. ? We also assume

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17/52	SUBMITTED TEXT	59 WORDS	30%	MATCHING TEXT	59 WORDS

a t a a t a B a B R t R e R e a a a a ? 1 2 1 2 2 0 2 0 0 0 1 2 1 2 1 1 ( ) 2 2 a a t a a t a R a R B t B e B e a

SA Sathish Kumar. S.pdf (D113325247)

18/52	SUBMITTED TEXT	17 WORDS	91%	MATCHING TEXT	17 WORDS

if we can understand the nature of how a disease spreads through a population, then certainly we

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better strategies to contain it through methods like vaccination or quarantine. Sometimes even the biological control of pests may also become handy to curb the spread of disease. For

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by infected persons in the population coming into close contact with susceptible

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21/52	SUBMITTED TEXT	22 WORDS	65%	MATCHING TEXT	22 WORDS

and measles are highly contagious, whereas glandular fever is much less so. Interestingly, some diseases, like mumps and measles, confer a lifelong immunity.

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22/52	SUBMITTED TEXT	10 WORDS	100%	MATCHING TEXT	10 WORDS

is the time between infection and the appearance of visible symptoms.

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23/52	SUBMITTED TEXT	27 WORDS	89%	MATCHING TEXT	27 WORDS

is the period of time between infection and the ability to infect someone else with the disease. Note that the latent period is shorter than the incubation period.

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24/52	SUBMITTED TEXT	23 WORDS	68%	MATCHING TEXT	23 WORDS

we discuss a simple mathematical model for influenza outbreak at a boarding school over a period of about, say, 45 days. During this

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those who have recovered from the disease and are no longer susceptible (

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26/52	SUBMITTED TEXT	26 WORDS	46%	MATCHING TEXT	26 WORDS

are large enough such that random differences between individuals can be neglected. ? Births and deaths are ignored. ? The infection spreads only by contact. ? The latent period

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27/52	SUBMITTED TEXT	13 WORDS	100%	MATCHING TEXT	13 WORDS

the contagious infectives and susceptibles are always randomly distributed over the area in which

SA DSC-6 Combine.pdf (D143717932)

28/52	SUBMITTED TEXT	33 WORDS	91%	MATCHING TEXT	33 WORDS

the number of susceptibles, the higher is the increase in the number of infectives. Thus the rate of susceptibles infected by a single infective will be an increasing function of the number of susceptibles.

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29/52	SUBMITTED TEXT	19 WORDS	87%	MATCHING TEXT	19 WORDS

$\lambda(t)$ is the force of infection, i.e., it is the per- capita rate at which susceptible individuals become infected.

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30/52	SUBMITTED TEXT	16 WORDS	100%	MATCHING TEXT	16 WORDS

the more infectives there are, the higher the risk that a single susceptible will become infected.

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31/52	SUBMITTED TEXT	22 WORDS	100%	MATCHING TEXT	22 WORDS

only on the number of infectives. We assume that the rate at which infectives recover is directly proportional to the number of infectives.

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32/52	SUBMITTED TEXT	33 WORDS	100%	MATCHING TEXT	33 WORDS

The force of infection, λ(t), depends on the current number of infectives I(t) and increases as the proportion of infectives in the population increases. It also depends on the rate that individuals make contacts.

SA DSC-6 Combine.pdf (D143717932)

33/52	SUBMITTED TEXT	20 WORDS	72%	MATCHING TEXT	20 WORDS

dx f x y dt ? (,) dy g x y dt ? (3.50) NSOU ? GE-MT-41 ? 95 Here f(x, y) and g(x, y) are

SA 2-13.pdf (D20932629)

34/52	SUBMITTED TEXT	24 WORDS	52%	MATCHING TEXT	24 WORDS

dX X c XY dt ?? ? (3.51) 2 2 dY c XY Y dt ? ? ? where c 1 , c 2 , 2 1 ,? ? are positive constants. This system of

SA 1214140001-S.pdf (D17093357)

35/52	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

differential equations (,), dX F X Y dt ? (3.57) (,). dY G X Y dt ? 98 ?

SA 2-13.pdf (D20932629)

36/52	SUBMITTED TEXT	11 WORDS	75%	MATCHING TEXT	11 WORDS

for the differential equation in dC F F c C dt V V ? ? where

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37/52	SUBMITTED TEXT	57 WORDS	92%	MATCHING TEXT	57 WORDS

per capita death rate as ( ), , 0 X t ? ? ? ? ? ? , where ? is the per capita death rate due to natural attrition and ? is the per capita dependence of deaths on the population size. As 0 X ? , the per capita death rate tends to ? .

SA DSC-6 Combine.pdf (D143717932)

38/52	SUBMITTED TEXT	17 WORDS	94%	MATCHING TEXT	17 WORDS

we assume the populations are sufficiently large so that we can neglect random differences between individuals. ? We

SA DSC-6 Combine.pdf (D143717932)

39/52	SUBMITTED TEXT	21 WORDS	77%	MATCHING TEXT	21 WORDS

the predator and the prey in the ecosystem we are considering. ? The prey population grows exponentially in the absence of a predator.

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40/52	SUBMITTED TEXT	48 WORDS	77%	MATCHING TEXT	48 WORDS

This system of equations is known as the Lotka–Volterra prey- predator system. The parameters c 1 and c 2 are known as interaction parameters as they describe the manner in which the populations interact. Since there are positive and negative terms on the RHS of each differential equation,

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41/52	SUBMITTED TEXT	57 WORDS	94%	MATCHING TEXT	57 WORDS

anticipate that the populations could either increase or decrease. These differential equations are coupled since each differential equation depends on the solution of the other. The differential equations are also non- linear since they involve the product XY . One interpretation of the product XY is that it is proportional to the rate of encounters (contacts) between the two species.

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42/52	SUBMITTED TEXT	45 WORDS	96%	MATCHING TEXT	45 WORDS

For this two-species model, we would expect that, in the absence of any predators, the prey would grow without bound (since we have not included any growth limiting effects other than the predators). Also, in the absence of prey, we would expect the predators to die out. 3.21

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43/52	SUBMITTED TEXT	10 WORDS	76%	MATCHING TEXT	10 WORDS

consider the following system of differential equations 2 dx x y dt ? ? ? (3.86) 3 dy y

SA Internal_Assessment_Bruno.pdf (D161369918)

44/52	SUBMITTED TEXT	17 WORDS	70%	MATCHING TEXT	17 WORDS

are real and of opposite signs. Hence the equilibrium point is a saddle point. Example 3.22.3. Determine the

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45/52	SUBMITTED TEXT	18 WORDS	58%	MATCHING TEXT	18 WORDS

X(t + T) = 2X(t). Then ( ) 0 0 ( ) 2 ( ) r t T rt x e X t T X t x

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46/52	SUBMITTED TEXT	21 WORDS	55%	MATCHING TEXT	21 WORDS

the nature of the critical points of the following system of ODEs dx x y dt ? ? ? 2 3 dy x y dt ? ? 120 ?

SA 2-13.pdf (D20932629)

47/52	SUBMITTED TEXT	53 WORDS	43%	MATCHING TEXT	53 WORDS

x x x T T x x K T x t t T x t t s x ? Taking , 0, t x ? ?? we have our heat equation 2 2 , 0 T T x l t

SA 1214140002-SS.pdf (D17273355)

48/52	SUBMITTED TEXT	23 WORDS	87%	MATCHING TEXT	23 WORDS

T x x t x t T x x x t x ?

SA 2-13.pdf (D20932629)

49/52	SUBMITTED TEXT	20 WORDS	73%	MATCHING TEXT	20 WORDS

x t x t ? ? ? . Therefore u(x, t) = 2 2 2 2 2 2 2 2 . 1 2 1 2 x t t t x t t

SA 2-13.pdf (D20932629)

50/52	SUBMITTED TEXT	13 WORDS	80%	MATCHING TEXT	13 WORDS

System of Differential Equations A system of differential equations is a set of

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51/52	SUBMITTED TEXT	7 WORDS	100%	MATCHING TEXT	7 WORDS

dx F x y dt ? (,) dy G x y dt ?

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52/52 SUBMITTED TEXT	12 WORDS	95% MATCHING TEXT	12 WORDS
Mitsuo Kono; Chaotic evolution of arms races; Chaos 8, 808 (1998); https://doi.org/10.1063/1.166366		Mitsuo Kono; Chaotic evolution of arms races. Chaos 1 December 1998; 8 (4): 808–813. https://doi.org/10.1063/1.166366	

W https://doi.org/10.1063/1.166366