NETAJI SUBHAS OPEN UNIVERSITY

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B. Ed. Spl. Ed. (M.R./H.I./V.I.)-ODL

PEDAGOGY OF TEACHING MATHEMATICS

A-4 (Part-II)

B. Ed. Spl. Ed. (M. R. / H. I. / V. I)-
ODL Programme

AREA - A

A - 4 (PART-II) : PEDAGOGY OF TEACHING MATHEMATICS

A COLLABORATIVE PROGRAMME OF
NETAJI SUBHAS OPEN UNIVERSITY
AND
REHABILITATION COUNCIL OF INDIA

AREA - A
PEDAGOGY OF TEACHING MATHEMATICS
COURSE CODE - A4 (PART-II)
PEDAGOGY OF TEACHING MATHEMATICS

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Sri Mohan Kumar Chattopadhyay
Registrar

Netaji Subhas Open University

From the Vice-Chancellor's Desk

Dear Students, from this Academic Session (2015-17) the Curriculum and Course Structure of B. Ed.- Special Education have been thoroughly revised as per the stipulations which featured in the Memorandum of Understanding (MoU) between the Rehabilitation Council of India (RCI) and the National Council for Teacher Education (NCTE). The newly designed course structure and syllabus is comprehensive and futuristic has, therefore, been contextualized and adopted by NSOU from the present academic session, following the directives of the aforesaid national statutory authorities.

Consequent upon the introduction of new syllabus the revision of Self Instructional Material (SIM) becomes imperative. The new syllabus was circulated by RCI for introduction in the month of June, 2015 while the new session begins in the month of July. So the difficulties of preparing the SIMs within such a short time can easily be understood. However, the School of Education of NSOU took up the challenge and put the best minds together in preparing SIM without compromising the standard and quality of such an academic package. It required many rigorous steps before printing and circulation of the entire academic package to our dear learners. Every intervening step was meticulously and methodically followed for ensuring quality in such a time bound manner.

The SIMs are prepared by eminent subject experts and edited by the senior members of the faculty specializing in the discipline concerned. Printing of the SIMs has been done with utmost care and attention. Students are the primary beneficiaries of these materials so developed. Therefore, you must go through the contents seriously and take your queries, if any, to the Counselors during Personal Contact Programs (PCPs) for clarifications. In comparison to F2F mode, the onus is on the learners in the ODL mode. So please change your mind accordingly and shrug off your old mindset of teacher dependence and spoon feeding habits immediately.

I would further urge you to go for other Open Educational Resources (OERs) - available on websites, for better understanding and gaining comprehensive mastery over the subject. From this year NSOU is also providing ICT enabled support services to the students enrolled under this University. So, in addition to the printed SIMs, the e-contents are also provided to the students to facilitate the usage and ensure more flexibility at the user end. The other ICT based support systems will be there for the benefit of the learners.

So please make the most of it and do your best in the examinations. However, any suggestion or constructive criticism regarding the SIMs and its improvement is welcome. 1 must acknowledge the contribution of all the content writers, editors and background minds at the SoE, NSOU for their respective efforts, expertise and hard work in producing the SIMs within a very short time.

Professor (Dr.) Subha Sankar Sarkar
Vice-Chancellor, NSOU

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AREA - A

A-4 (PART-II) : PEDAGOGY OF TEACHING MATHEMATICS

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Netaji Subhas Open University

AREA - A
A-4 : PEDAGOGY OF TEACHING MATHEMATICS

A-4 ❐ Pedagogy of Teaching Mathematics

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Unit I ❑ Nature of Mathematics

1.1. Introduction

1.2. Objectives

1.3 Meaning, Nature, Importance and value of Mathematics

1.3.1 Meaning (Desination from Greek word; English Dictionary; Indian language, Saying of Maxwell, Bartrand Russell, A.N.Whitehead etc)

1.3.2 Nature (As a science of reasoning; a scinece of symbols; an abstract science; Science of numbers; a language; survival of mankind; Interpreter fo relalife phenomenon; duductive reasoning etc)

1.3.3 Importance (in school currimulum; in matheamatical aepect; in civilizations and culture; in other subjects).

1.3.4 Value

1.1. Introduction

Longback, Comte observes as follows, "All science education which does not commence with Mathematics is, of necessity, defective at its foundation". Roger Bacon observs "Mathematics is the gate and key of the science. Neglect of Mathematics causes injury to all knowledge. Science he who is ignorant of it cannot know the other sciencs or the things of the world. What is more, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy."

Mathematics plays an important role for acquring knowledge in any branch of science and scientific theories are also based on mahematical knoledge. Kothari Commission (1964 – 66) mentioned "we cannot over stress the importance of mathematical in relation to science, education and research. This has always been so, but at no time has the significance of mathematical been greater than today. It is important that deliberate effort is made to place India onthe world map of mathematics, within the next two decades or so.'' National Curruculum Frame work– 2005 has emphsaised "Developing children's abilities for mathematicsation is the main goal of mathematics education. The narrow aim of school mathematics is to

develop useful capabilities, particularly those relating to numeracy, numbers, number operations, measurement, decimals and percentages. The higher aims is to develop the child's resources to think and reason mathematically, to purpose assumptions to logical conclusions and to handle abstraction. It includes a way of doing things and the ability and attitude to formulate and solve problems."

So the role of mathematics, is many folded. Mathematics develops the skills of visualisation and representation of a person about physical fenomena. C. A. Coulson says "Our world is becoming increasingly mathematical"

In this persspective we will discuss the nature, importance and value of mathematics.

1.2. Objectives :

At the end of this subunit, you will be able to

- acquire a clear perspedtive of meaning and nature of mathematics.
- explain the importance of mathematics in multi-dimentional fields.
- appreciate different values of mathematics as a school subject.

1.3.1. Meaning of Mathematics :

Etymologically the term 'Mathematics' is derived from two Greek words, 'Manthanein' means 'learning' and 'Techne' means 'an art or technique'. So, mathematics means the art of learning related to faculties.

In many Indian languages the vernacular word for mathematics is 'Ganita' which means the science of space and quantity which helps us in solving many problems of life using numeration and calculation.

In Sanskrit, it is said, 'Ganita Shastra' which means science of counting and calculation for mankind.

The dictinary meaning of Mathematics is, "it is either the science of number and space or the science of measurement, quantity and magnitude." So mathematics is the science of quantity, measurement and spatial relationss. It is a systematised and well organised branch of science expressed symbolically. Mathematics is difined in different ways by diffient schools.

- "Mathematics is the indispensable intrument of all physical resource" Kant.
- "Mathematics is engaged in fact in the profound study of art and expression of beauty"—I. B. Shaw
- "Mathematics is the art of saying the same thing in many different ways"—Makwell
- "Mathematics may be difined as "The subject in which we never know what we are talking about, nor whether what we are saying is true"—Bertrand Russel
- "Mathematics is the queen of science and arithmetic is the queen of all mathematics"–Gaoss
- "Mathematics is a science of order and measure"—Descrutes
- "Mathematics is a way to settle in the mind a habit of reasoning"—Loyce.

  Mathematical culture shaves with the humanistic culture some characteristics like beauty, elegance, depth, emotional involvement, artistic nature, crativity, persuit of truth etc.

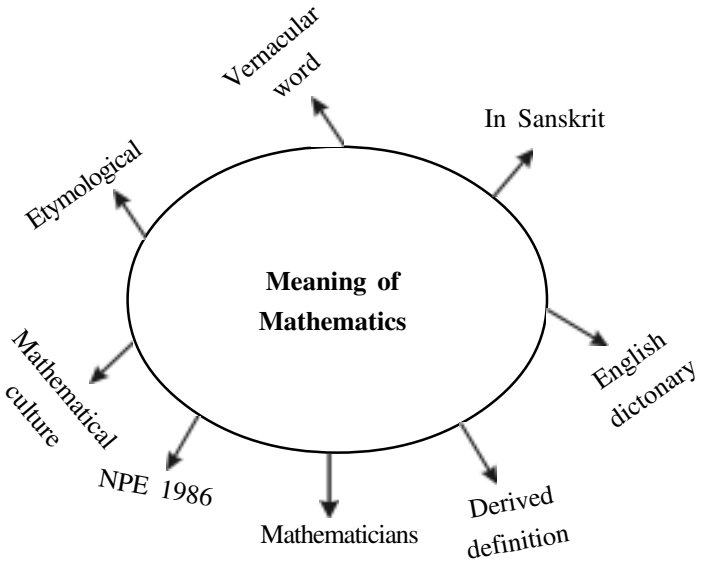
The Natioanl Policy on Education 1986, mentions

- "Mathematics should be visualised as the vehicle to train a child to think, reason, analyse and to articulate logically.
- Mathematics is the science of patters and forms in numbers and space. The thrill of mathematics lies in the discovrey of these patterns.
- Mathematics is both an expaimental and deductive science. While proofs are important, the discovery of pattern, is even more important and is certainly more exciting.

Derived definition :

So mathematics can be definied as a syptematised, organised branch of science which deals with quantitative facts forming generalisation, establishing relationship and developing logical thinking and reasoning.

So the word Mathematics denotes two different senses namely one as a method used to solve the problem of physical natue (tool) and second to generalise truths (logical frameworks)



1.3.2 Nature of Mathematics : On the basis of various thoughts of the greatest mathematical scienctists of all times the following aspects on nature of mathematics are emerged.

1.3.2.1 As a science of reasoning :

According to Locke 'Mathematics is a way to settle in mind a hibite of reasonging'. Mathematics is based on logical reasoning. This logic helps us to come to a conclusion or to proof a statement. The reasoning may be of two types (a) inductive reasoning–based on observeation and experience of a individual cases to arrive of a generalised conclusion (b) indeductive reasoning–where generalization is made on the basis of predetermind aioms or postualates. Laws of deductive logic are the basis of valid reasoning. These laws are

(a) Law of identity

(b) Law of excluded middle and

(c) Law of contradiction.

So mathematics is regarded as a highly disciplined model of thinking.

1.3.2.2. Mathematics as a science of symbols.

Symbols are plenty in mathematics. Each symbol has a definite meaning. These symbol are accepted as universal. The main characteristics of mathematical languages are simplicity, accuracy and very precisious.

The other features of mathematical language are the following.

(i) Like other languages, the mathematics language also has it own grammer. A definition must be stated cclearly and precisely without confusion.

(ii) Mathematical languages, clearly distinguishes between number and numereal, fraction and fractional, angle and angular etc.

(iii) Some common words are also used in mathematic in different contexts like 'Cone' 'angle', 'root', is used as a roof of an equation, as squre or cube root and main source etc.

(iv) To arive at a mathematical solution, we have to follow a algorithm that depends on sequence of steps.

(v) Verbal statement, can be precisely expressed using mathematical symbol i.e. language. A is greater than B. B is greater than C can be state $A > B > C$. The commulative law of addition and multiplication of real nmber system is expressed in verbal language as 'addition and multiplication of two real numbers is independetn of order.'

In mathematical language is it can be stated as $a + b = b + a$, $a \times b = b \times a$, $a, b_{+} \in R$

Important mathematical statements relations and operations are $+$, $-$, $\times$, $\div$, $<$, $>$, $\leq$, $\%$, $\surd$, Σ, etc.

(vi) Sometimes, the same mathematical operation can be expressed in different mathematical language. The adition, can be stated as 'find the sum', 'find the value', find the whole', 'find the total', 'how many is all', the summation of, Σ, etc, Mathematical symbol may be reffered as 'tool of communication'.

1.3.2.3. Mathamtics as an Abstract scicne.

Scientific theories, laws etc are expressed also in mathematical language which beocme abstract.

$E = mc^2$, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ etc.

Some mathematical concepts cannot be physically verified, they are abstract. For example Euclid's lines are assumed to have no width and points are no size similarly,

the concept of infinity, negative numbers limit etc. Can not be learned through practical experiences. Hence they are abstract.

1.3.2.4. Mathamatics as a study of structure.

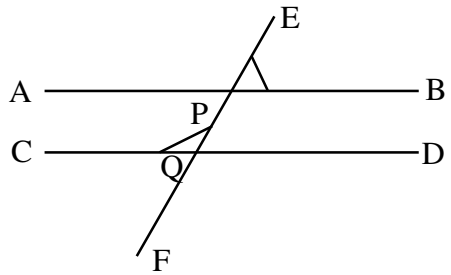
Mathamtical structure is a mathematical system obtained by some sort of arrangement, formation or putting togethere of parts. For this we apply the properties like commutative, associative as dribute operations. Plane analytic geometry is considered as a superstructure based on the structure known as real number system. Similarly, number system, group, field, ring, vector etc, are example of meathamtical structures.

1.3.2.5. Mathematics as a science of precision and accuracy.

Mathematical results are exact and precised. It may be either right or wrong, accpted or rejected. Thue is no in between the might and wrong. Sometimes we have to give emphasis on approximation in mathematical results but that also depends on degree of accuracy.

1.3.2.6 Mathamatics as a deductive science.

In deductive reasining we proceed in the following way : if something is true which is considered as fundamnemtal assumption, then the subsequent deduction based on accepted assumptions must be true. 'If a line intersects two parallel lines, the alternative angles are equeal.'



AB ∥ CD; E F, intersects at P and Q points. ∠EPB = ∠EQC etc.

Hence the validity of the deduced conclusion depends on the consistency of the assumption i.e., premises Air contains O_2, N_2, CO_2 etc in the following percentages, of a mixture of O_2, N_2, CO_2 etc contains such percentages, then if may be treated

as air. This type of deduction is made by applying logical reasoning. So the phases of mathematical deduction are (a) Premises (basis consideration or axioms) (b) The proces of reasoning and (c) Conclusion (accepted definition or theorem)

1.3.2.7 Mathematics as an inductive science.

Inductive reasoning is based on the principle that if a relationship holds good for few particular cases and even for any similar case then the relationship can be generalised into a rule or formula. If $(a + b)(a + b) = a^2 + 2ab + b^2$, $(m + n)(m + n) = m^2 + 2mn + n^2$ etc then $(p + q)^2 = p^2 + 2pq + q^2$.

Mathematical definitions and rules are formed through induction based on particulaar facts, examples. The child may use measurement, manipulation as some consturctive activities to arrive at a relationship using mathematical symbols. This symbolic form based on particular instances is called as law or rule. The phase are :

Observation and direct experiences $\rightarrow$ Generalised form/ relationship $\rightarrow$ Use of symbols through reasoning (law/rule is formed)

1.3.2.8. Mathematics as a science of number.

Before the invention of proper numbers, ancient people uses the notations of 'one' and 'many'. But necessity of man compels them to write symbols for numbers which we use today–1, 2, 3, 4, 5, 6, 7, 8 and 9. Those counting numbers are invented with in a centuries of development. Scinetific information was expressed by using numbers. Thus mathematica is considered as a science of numbers. For example,

(a) 1, 2, 3, 4.....are called numberd. These numbers are used in counting. Hence then unmbers are called counting numbers.

(b) On the same way 0, 1, 2, 3, 4,.....are called whole numbers. Those are used in meaning. Hence these numbers are called measuring numbers.

(c) .....–4, –3, –2, –1, 2, 3, 4.....are called integers. Those numbers are used to indicate direction. So there are called directional numbers.

By using the numbers, we can explain the natural phenomenon, scientific discoveries, economic growth, environmental information etc.

1.3.3. Importance of Mathamatics.

In the words of Young "Mathematical is the only subject that encourages and develops logical thinking. If enables the student to discriminate between essential and non essential. If helps them to shift facts, to draw conclusions without ambiguity and that is the subject by which they may learn what is meant by rigid reasoning."

The importance of mathematics is multidimenssional and widespread. So being numberate is becoming more important than being litrate. The importance of mathematica can be judged from three aspects namely (a) Socical aspect, (b) Application aspect and (c) Mathematical aspect.

1.3.3.1 Importance of Mathematical from social aspect.

- Daily life activities of man demand mastery of number and mathematical vocabulary like commission, discount, profit and loss, percent and like.
- The study of mathematics helps to develop logical thinking and reasoning, creative imagination.
- Mathematics enhances the ability to apply mathematical idesa in other branches of knowledge and even in daily life situation.
- Many vocations require mathematical skills.
- Mathematics helps man to discover the mysteries of universe and to overcome superstitions.

1.3.3.2 Importance of Mathematics from application aspect.

- Mathematics provides sufficient skills to meet the demand of daily life.
- It provides a clear undrstanding of laws of nature.
- Various cultural art like painting, drawing and sculpture are based on mathematical knowledge.
- Music utilizes mathematics.
- Mathematical Quizes, puzzles, magic squares, tools are both entertaining and challenging to develop power reasoning.
- Mathematics has a positiver correlation with other frames of science as well as with social sciences and also with languages-mathematics in used have as tool of other subjects.

- Lofly A Zadeh has dicovered two types of fuzzy set namely Intuitionistic and Neutrosophic fuzzy set which has revolutionied the application areas of mathematics.

1.3.3.3 Importance of Mathematics from mathematical aspect.

- Mathematical contents are gradually increasing and other related subjects are also developed by mathematics.
- Mathematics improves the ability to perform calculation with speed and accuracey. It also develops the concepts of symmetry, similarity and the skill to use instruments with precission.
- Mathematics improves the power of

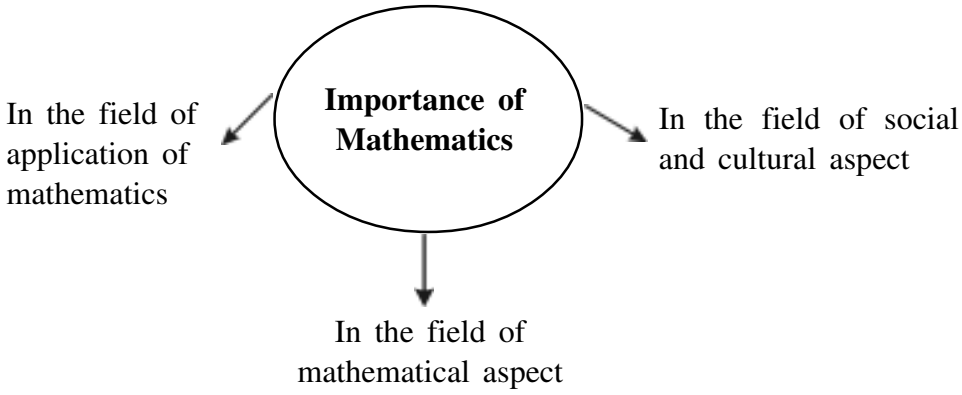
(a) estimation and approximation of results more accuality.

(b) being more systematic in finding relaionship, drawing conclusion.

(c) interpretion of numerical data using graphs etc.

(d) taking independent decisions in administrative and social, issues.

(e) thniking alternative methods of solving problems



1.3.4. Value of Mathematics.

In the language of Courant and Robin "Mathematics is an expression of the human mind which reflects the active will, the contemplative reason and the desire for aesthetic perfection. Its basic elements are logic and intuition, eralysis and constructions generality and individuality."

Mathematics does not contain only asbtract information, some process laws and

formula. It must contain avenues of developing a good number of values.

According to Encyclopida "Value means relative worth or importance". We know 'value can not be taught, it must be caught.' Buy by teaching or learning a subject one must obtain such qualities which one essential for a person to live in society and which is desirable for a person.

Teaching of mathematics will also foster the following values.

1. **Utilitarian value**
2. **Intellectual value**
3. **Aesthetic value**
4. **Moral value**
5. **Recreational value.**

1.3.4.1 Utilitarian value :

In the words of I. G. Kemeny "Whether man's travel carries him into space or into theoretical science, his passport must be stamped with the mathematician's seal or approval.'

The utilitarian values of mathematics are many and varried consider the applications of mathematics in our daily life. We start our works, complete those following the time schedule and planning. We face comercial transaction, fill payment, construction expenses, use of electronic gudget, tour & travels etc in our daily life. All these activities are directly influenced mathematics.

- **Other frames**

We have identified the relationship between mathematics and other branches of science and humanities and have realised the utilitarian value of mathematics starting from scientists, educationists, administrations businessmen teachinicuans and even the foal utilise mathematics knowingly or unknowingly. So mathematics for all.

- Mathematics is business and industry—The development of a country depends also on the expression of its business of industry. Both utilise the application on the mathematics.

1.3.4.2 Intellectual value :

Intellect is the mental capacity with the help of which we can think and acquire

knowledge. It is the mental faculty by which we understand the concrete and abstract life experiences. If regulates our thinking, imagination and responses to any stimulus. All sorts of creation, manipulation and structuring of knowledge are the output of intellectual vaules, mathematics regulates those processes standign from thinking, analysing, arriving at a conclusion and verifying the result.

According to Duffon "Mathematics furnishes the power of deliberate thought and accurate statement and to speck trugh."

So students must love and enjoy mathematics. To inclucate such mental faculty, the teachers may8 arrnage the following activities.

- To allow the students to be exposed in different branches of mathematics.
- To arrange mathematics Quitz, Quest, seminer.
- To acquaint the student with mathematical logical for solving different problems.
- To include the biographies and contributies of national and international mathematics.
- To enourage the students to participate in teaching learning process and in mathematics laboratoraies.
- To organise mathematical exhibition, fair, fun for mathematics etc.
- To conduct group mathematical project for exchange of views.

1.3.4.3 Aesthetic value :

In the words of Thorndike "Education as a whole should foster the higher impersonal pleasure" Mathematics also fulfills that function. Mathematics is alos sources of delight and joy. Geometrical drawings, artistic diagrams, engineering figures etc, are based on the principles of mathematics. Proper shope and appropriate size brings beauty to the objects in nature.

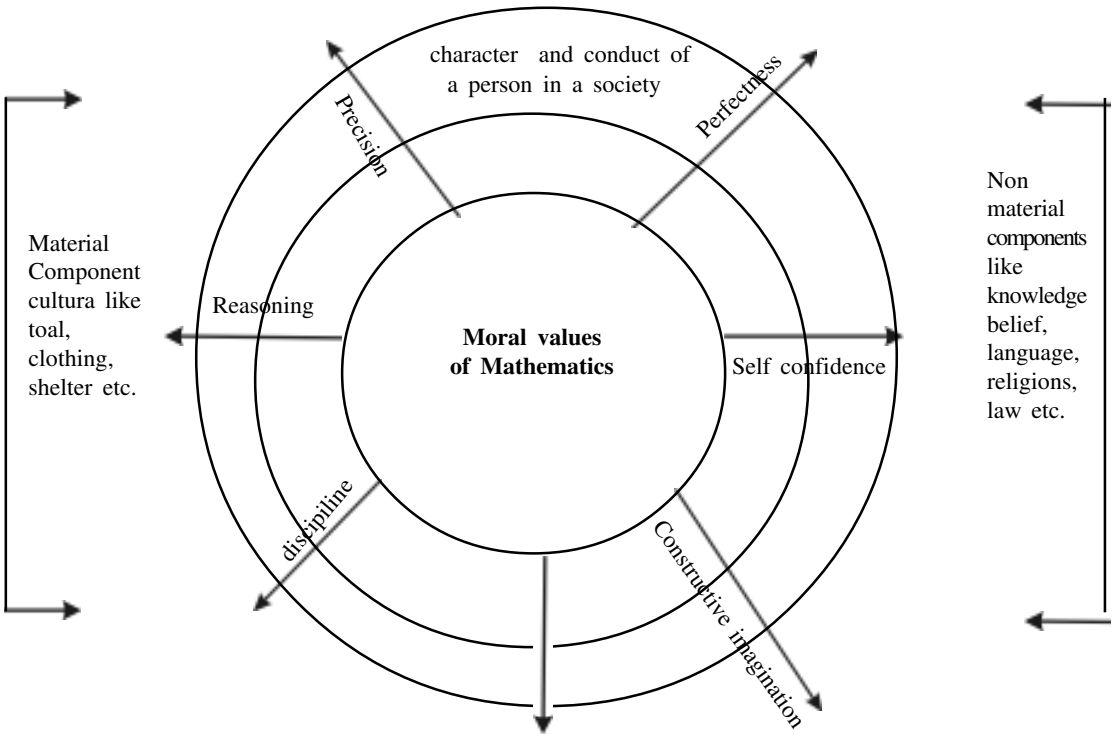
Symmetry is the corner stone of beautyful creation of the universe. For example two parts (belt and right) of a human body, loves of a tree, beehibes, nests of birds, spider net has beautiful mathematical structure. The feeling of Aesthetic measure (M) cembe calculated using the formula.

$M = \frac{O}{C}$ where O = The property of harmony, as symmetry of orders.

C = Complexity of the objects.

1.3.4.4 Moral values :

Morality means the charancteristics and conduct of a person with respect to his/her behaviour is the society. The human character and conduct is determind by the culture of the person. The culture includes knowledge, belief, morals, custom, law and habits etc. The culture is influenced both by material components like clothing, machine tools etc as well as non-material components like language, literature, out religion, rule of the government. Moreover, both the material and nonmaterial component of culture are controlled direct by or indirectly by mathematics. Hence mathematics has a strong moral value. The moral value of mathematics are exihibited in multidimentional powers like "discipline', reasoning capacity, concentration of mind, precission in delaing, perfectness in work, neatness in all kinds objects, self confidence in arriving at a conclusion, constructive imagination for solving a problem, capacity of judging the problamatic sitituation.



Thus moral values of mathematics develop in the learner good character, desirable cultura, honesty, attitude of oepn-mindnees, deductive and inductive reasoning etc.

1.3.4.5. Recreation value :

Galileo has mentioned 'Mathematics is the language in which God has written th Universe."

For recreation or enjoyment we also explore the language of the universe. For recreation we even undergo such activities which requrire power of reasoning symbolisation and critical thinking etc. Recreational activities give the person fresh energy for solving problems, performing creative activities. While solving mathemcatical problem one can enjoy a lot by facing challegning sitiuation and ultimatley solving the problem. The learnier is delighted by discovering such new knowledge.

Mathematical puzzle, induction and finding alternate ways to solve problem are the sources of recreation value of mathematics.

1.2. Axioms, Postulates, assumption and Hypothesis in Mathematics (Mathematical assertions)

The word axiom comes from the Greek word 'axioms' means 'That which is thought worthy or fit' or 'that which commends itself as evident'. In mathematics, it is a statement that is so evident as well-establishment, that it is accepted without controversy or questions. It can be used as the premise or starting point for further reasoning or arguments.

Axion is, a rule as a statment that is a accpted as true without proof.

In mathematics, three are two types of axiomx namely

(i) Logical axioms and (ii) non-logical axioms.

Logical axioms are unally statements that are considered to be true within the system of logic they define. For example A and B may represent two points A and B when as AB may be a live, live segment etc.

Non-logical axioms is not a self-evident trugh, used in deduction to build a mathematical theory.

Example : $a + b = b + a$ is true in arithmetic

❑ **Characteristics :**

(1) It is any mathematical statement from which other statements may be logically derrived.

(2) An axiom in one system may be a theorem in another and vice versa.

(3) An axiom is a self evident assumption which is common to many branches of science.

Example : when can equal amount is taken from equals, an equal amount results. In mathematics, a clear distinction can be made between logical and non logical.

Logical axioms are certain formulas in a formad language that are universally valid. They are satisfied by every assignment of values.

On the other hand, non-logical axioms are formulas that are theory specific assumptions. For examples the natural numbers and integers involve the same logical axioms. The non-logical axioms aim to satisfy what is special about a particular structure like groups. These are reffered as axioms in mathematical discourse. Every mathematical theory starts from a given set of non-logical axioms and ultimately formalised to logical formulas.

Example : For natural numbers.

Relevent terms '0' (Zero); "number"; "successor" Axiomx AI '0' (Zero) is a number A2. The successro of a number is a number. A3. No two numbers have the same successor A4. 0 is not the successor of any number a5. If p is a property such that (a) q/ has the pro;erly p and (b) whenever nhas the property P, the successor of n has the property P, then every number has property P

The 5th axiom is termed as 'The principle of mathematical induction'.

❑ **Postulate :**

The meaning of the word is to 'demand'.

Euclide demands that 'any two points can be joined by a straight line.' All the foundation of the famous science depend certain additional hypothesis which were accepted without proof. Such a hypothesis were known as postulate.

❑ **Characteristics :**

(1) The postulates of each particular science were different.

(2) The validity of postulate is determined by means of real-world experience.

(3) These are very basis, self evident assertions.

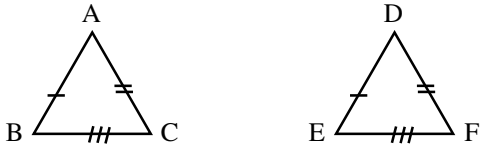
(4) These are non logical axioms.

Fromk Euclid's Elements, the following fine postulates are obtained.

1. Things which are equal to the same thing are also equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equals are subtreated from equals, the reaminders are equal.
4. Things which concide with one another are equeal to one another.
5. The whole is greated than the part.

Postulate is a true statement which does not require to be proved.

Example : We use pootulates SSS.



General assumptions :

1. Things which are equal to the same thing are also equal to one another.
2. If equal are added to equals, the wholes are equal.
3. If equals are subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to are another.
5. The whole is greater than the part.

However, modern mathematics develops theories and axioms like field theory, group theory, topology, vector spaces, hyperbolic geometry without any particular application in mind. Mathematicians now consider that axioms should be regarded as purely forms statements and not as facts based on experience. Modern mathematics strengthens its foundation to such an extent that mathematical therories can be regarded as mathematical objects and mathematics itself can be regarded as a branch of logic.

❑ **Assumption :**

Something taken for granted or accepted astrue withoit proof, as supposition.

It is a statement that is used as the premise of a particular argument but may not be otherwise accepted. Latin 'assumptionem' meaning a taking as receiving."

So Assumption is a supposition on the current situation or a presupposition on futrue course of events, assumed to be true is the absense of positive proof but necessary to established a rule/law.

❑ **Characteristics :**

1. A statements that is assumed to be true and from which a conclusion can be chuwn

   a straight line– which will never meet, but on the surface of earch which has a curvature, it is assumed.

2. It is hypothesis that is takne for granted.
3. Form the fundamental assumption something is developmed or exploined.
4. It is the act of assuming or taking for granted.
5. An assumption may be taken as postulate as axiom.
6. It may be taken as presumption for establishing a theory.
7. It is accpted cause and effect relationship.

❑ **Hypothesis :**

Hypothesis is the part of a condition statement. It is statement or idea which gives an explantion to a sciences of observation.

Example : If the three sides of a triangle measure.

The same, the trinalge is equilateral.

The hypothesis is 'all three sides of a triangle measure the same'.

(1) A statement that might be true, which can then be tested.

(2) Sometimes the hypothesis would not be tested.

(3) Steps underlying the formation of hypothesis are :

(to make some observations)

↓

Collect some data based on the observations

↓

Make a conclusion (Hypothesis)

↓

To test the hypothesis taking more evidence/date

↓ ↓

If the evidences is in agreement If the data/evidences contradict

↓ ↓

The hypothesis is correct It must be rejected as amended for verification

(4) Mathematics is based on deductive reasoning so a good hypothesis is essential for formation/explanations of a mathe.

1.3. Historical Development of Notations and Number System

1.3.1 Development of Mathematical Notations.

Mathematical notations comprises the symbol used to write mathematical equations and formulas. It implies a set of well defind representations of quantities and symbols operators. The history includes Hindu-Arabic numbers, betters from. The Roman, Greek, Hebrew and German alphabets.

The development of mathematical notation can expressed in phases.

1st The rhetorical stage. At this stage calculations are performed by words only.

2nd phase The syncopated stage

Operation and quatities are represent symbol

The numerical symbos consisted of stroks or notches cut in wood or stone.

The symbolic system was in use by medieval Indian mathematicians and in Europe since the middle of the 17th century. The Egyptian mathematics had a symbol for one, ten, one hundred, one thousand, ten thousand, one hundred-thousand and one million.

Some common Mathematical symbols with history.

= (the equal sign) means "is the sameas' first introduced in the 1557 (book 'The Whetstone of Witte by Robert Recorde.)

$<$ (the less than sign) means "is striclty lessthen",

$>$ (greater than sign) means "is striclty greater than sign". First appeared in Artis Analytica Praxip ad Awquations Abgebraicas Regolverdas. ("The Analytical Arts Applied to Solving Algebraic Equation" by Thoms Harriot (1631).

$\leq$ ("less then or equal") and $\geq$ ("greater than or equals" by Pieree Bouguer in 1734

$\therefore$ (three dots) means "therefore" ("Teach yourself Algebra" (1659) by Joharn Rahn

$\ni$ (the such that sign) means "under the condition that". Commonly to abbraviate 'such that".

$\Rightarrow$ (the implies sign) means "logically implies that".

$\Leftrightarrow$ means "if and only if".

$\forall$ (the universal quantities symbol means "for all" Gerhard Gentzen (1935)

Investigation an Logical Reasoning"

$\exists$ (The existential quantities) means "there exists" first used in 1897 book "For mulaire de mathematiqures by Giuseppe Peano

Set threory notation :

$\subset$ (Proper subset) means "This set is a subset of " and $\supset$ (the includes sign) measns "this set has a subset". Those are used in 1890 book Vorlesungen iiber dia Algebra der Logik ("Lectures on the Algebra of the Logic") by Ernst Schroder.

$\in$ (Element of) means "in an element of" and first appeared in the 1895 book Formulaire de mathematiques by Giusepp Peano.

$\cup$ (The union sign) means "take the elements that are in either set", and $\cap$ (The intersection sign) means "take the elements that he two stes hore is common." They use introduced in the 1888 book "Geometric Calculus based upon The teachings of H. Grassman. Prededed by the operations of deductive logic" by Giuseppe Peano.

Φ (The mult set or empty set symbol) measn "the set without any elements is it", first used in 1939 book "Elements de mathematique" by N. Bourbaki.

$\propto$ (infinity) denotes "a number of cubitrarily large magnitude "First appeared in print in the 1655 book "On Conic Section" by Gohn Wallis.

π (The ratio of the circumference to the diameter of a circle) denotes the number 3.141592653589.... and was first used by William Gones in his 1706 book

Synopsis Plama morum mathematics ("A New Introduction to the Mathematics") Many people speculate that gones chose the letter π because it is the first letter in Greek word perimetron, which roughly means 'aroung'.

1.3.2. Development of Number system.

The historical development of number system can be grouped into following phases.

1.3.2.1. Egyptians (3000 – 1000 B.C.) Ciphered number system.

Important features are.

- Two numeration system..
- Important tally system 'Hieroglyphics'.
- Those system when based on groupong of 10.
- Thy used their numeration system for measruesment.

1.3.2.2. Babylonians (2000 – 200 B.C.)

Important features are.

- Their number system based on goruping of 60.
- Have the position system.
- Writing ws as clay tablets.
- Problem was spacing between position.
- Finally they used dot to seprate those numbers.

1.3.2.3. Maya and Romans (300 B. C.)

Important features

- Those were similar to Babyglonians.
- Problems related to spacing were removed.
- Grouping of numbers was based on 20.
- Based on odd use of 18.

Romans' system was.

- Similar to the Egyptian system.
- They wrote larger numbers by putting a bar over.
- Addition and subtraction can be made.

1.3.2.4. Regarding Place Value of Zero :

- Babyglonians started the place value using theri dot.
- In 600 A.D. Hindus started to use 10 place value system
- Hindu recognized zero as a number
- In 9th centrury, Arabs started to use the Hindu system
- Indian word sunyo means absense of quantity
- Mahavira proved that number multiplied by zero will become zero.
- Bhaskare proved a number divided by zero will renult infinite quantity.

1.3.2.5. Place Value of Zero :

- Babylonians started place value using their dot.
- In 9th century Arabs accupted Hindus system.
- In 18th century zero occupy a place in algebraic equation.

1.3.2.5. Fractions :

- Egyptians first use fractions as "parts".
- Babylonians started base sixty system to include fraction.
- Russian had a unit-fraction method.
- In their nine chapters on mathematical art, Chinese mathematicism thought about fraction similar to our.
- Chinese avoided using improper fractions.

1.3.2.6. Nagetive Numbers :

- In 7th century, Brahmagupta recognised that negative number can be treated as debt.

- In 17th century, negative numbers were accpted.
- Descartes recognised negative roots as 'false roots'.
- Euler recognised negative numbers as debts and treated that product of two negative numbers is a positive number.

1.3.2.7. Complex Number :

- In early times if the quadratic formula was used to square root of a negative number then there was no solution.
- Rafael Bombelli formed out a new language to treat these negative radicols.
- Bombelli showed that sometimes the square roots of a negative number can be used to find real solution.
- Euler used complex nunbers a lot, but could not explain about their nature.
- Argund represented imaginary numbers geometrically on a plane.
- Gauss showed how complex number could be used in Mathematics.

Number can be calssified into sets, called $\frac{a}{b}$ number system.

main number system of Roman numbers are the following :

N	Natural	0, 1, 2, 3, 4,....or 1, 2, 3, 4,....
Z	Integer	...,–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5
Q	Rational	where a and a are integer a and b is not 0
R	Read	The limit of a contergent sequence of rational numbers
C	Complex	a+ib where a and b are real number and i is the square rot of –1

Premitive and prehistoric number system are numbers systems that have been developed when then has been need to express mognitudes. Thus the "bundle-of-sticks" method is used to represent magnitude. It can be used to express anything but quantities. Following Table shows the number words used amont the

Bushmen in South Africa

Number	Word or Combination
1	Xa
2	t'oa
3	'quo
4	t'oa – t'oa
5	t'oa – t'oa ta
6	t'oa – t'oa – t-oa

1.4. Contribution of Great Mathematics

1.4.1. Srinivasa Ramanujam FRS

He was an Indian mathematician. Though he had atmost no formal training in pure mathematics, he made extrordinary contribution to mathematical analysis, number theory, infinite service and continued fractions.

During his short life (22 December 1887 – 26 April 1920). Ramanujam idependently complied neraly 3.900 result (mostly identities and equations)

$\sqrt{1+2\sqrt{1+3\sqrt{1+\ldots\ldots}}}$

Ramanujum, with the help of Ramanujum Aiyer, had his work pulished inthe Journal of the Indian Mathematial society. He posed the problem in the Journal as follws.

He waited for a solution over six months. At the end, Ramanujan published the solution himself. On page 105 of his first notebook, he formulated an equation that could be used to solve the infinitity rested radical problem

$$x + n + a = \sqrt{ax + (n+a)^2 + x\sqrt{a(x+n) + (n+a)^2 + (x+n)\sqrt{\ldots\ldots\ldots}}}$$

using this equation the problems can be solved.

Ramanujum has written a good number of creative article. Prof Handly recommended the following seven articles as most significent.

They are —

i. Modular Equation and Approximatians to π—Quarterly journal of mathematics. Vol. 45.

ii. Highly composition Numbers– Proceeding London Mathematical Society. Vol. 14

iii. On Certain Arithmetical Functions Trase Continental Philosophical Society.

iv. On Certain Trignometirc sums and Their Applications to the Theory of Nunbers. — Transe Continental Philosophical Society.

v. Some Properties of P(n), the Number of Partitions on 'n'. Transe Continent Philisophical Society.

vi. Proof of Certin Identities in Combinatory Analysis.

vii. Congruance Properties of Partitions—Math. Thematia Zeitschmift. Vo. 9

Besides his published work, Ramanujum left behind several notebooks, which have been the object of research. The Enlish mathematician G. N. Watson wrote a series of papers about them. At present, the American mathematician C. Berndt has written a multi-volume study of the notebook in 1997.

Ramanujam worked out the Riemann series, the illiptic integrads, hypergeomtric series, the functional equations of the Zeta functions, the classical theory of quachotic forms as Cauchy's Theorem. Though brilliant, many of his theorems on the theroy of prime numbers were wrong.

Reference :

Isaiah Lankham, Bruno Nachtergaele etc —"Some Common Mathematical Symbols and Abbreviations (with History), 2007. MAT 067, University of California, Davis.

● Boyer, C.B. "Fundamental steps in the Development of Numeration.

Unit-2 ❐ Objectives and Instruction Planning in Mathematics

Structure :

2.1 Aims and Objectives of Teaching Mathematics in Elementary and Secondary Schools.

2.1.1 Introduction

2.1.2 Aims of teaching Mathematics

2.1.3 Objectives of Teaching Mathematics

2.1.4 Objectives of Teaching Mathematics at the Elementary Stage

2.1.5 Objectives at the Secondary Stage

2.1 Aims and Objectives of Teaching Mathematics in Elementary and Secondary Schools.

2.1.1 Introduction

We know that education is meant to acquire 3 Rs reading, writing and Arithmetic.

The National policy in Education (1986 NPE) has mentioned that Mathematics should be taught to fulfill the aim that each learner is trained to think, on reason, analyze and articulate logically. As far as possible the discovery approach is to the followed. The skills of drawing, measuring and estimation have to be developed.

To remembers the knowledge of terms, concepts, principles, processes and symbols. To respect the contribution of great mathematicians. To develop the skills in handling devices like calculators Computers etc. To help the learner to utilize the knowledge of Mathematics and skills to solve problems of daily life. To be fit to understand the content of higher Mathematics.

The aims of teaching Mathematics are the goals, targets or universal purpose that may be fulfilled by the teaching of Mathematics. So aims are like ideals. Their attainment needs a stepwise as phasewise a long-term planning. Objectives are the short-term or immediate goals that may be attained within specified classroom activities.

2.1.2 Aims of teaching Mathematics

Besides helping the students to acquire mathematical knowledge, the teaching of Mathematics also help the students to attain the broad aims of Mathematics.

a. **Disciplinary aim :** The teaching of Mathematics will provide the learner to think logically and will develop the intellectual habits.

b. **Utilitarism aim** - The students will be given mathematical knowledge to solve the real life the real life problems also. Thus the study of Mathematics will be functional and purposeful.

c. **Social aim.**

The knowledge of Mathematics will serve both the individual in particular as well as the society in general.

d. **Moral aim**

The contental and the process solution will imbibe morality.

e. **Aesthetic aim**

The teaching of mathematics will create intervest and the learners will remain engaged even is their leisure time

f. **Cultural aim**

The teaching of Mathematics will also help the learner to understand the contribution of Mathematics in development culture.

g. **Vocational aim**

It will help the learner to be fit for any location of his choice.

h. **Inter disciplinary aim**

The learner will realise the application of Mathematics in different subjects.

2.1.3 Objectives of Teaching Mathematics

The objectives of teaching mathematics at the whole school stage may be classified as follows :

(a) Knowledge and understanding objectives

(b) Skill objectives

(c) Application objectives

(d) Attitude objectives

(e) Appreciation and interest objectives

These objectives are to be expressed in behavioural terms.

The instructional objectives are those objectives which the students is expected to active. These objectives are developed for classroomteaching.

By teaching knowledge and understanding of Mathematics to the learner it is expected that the pupil acquire the following.

A. KNOWLEDGE AND UNDER STANDING

(a) Language of Mathematics – symbols, formula, terms, definitions statements etc

(b) Different mathematical concepts like numbers, sets, measurements, directions etc

(c) Mathematical process and relationships

(d) Development of Mathematics and contribution of mathematicians

(e) The nature of Mathematics and its scope

(f) Interrelationship between different branches of Mathematics and with different subjects

B. SKILL OBJECTIVES

Skill means the ability to perform a given act with case and precision.

The teaching Mathematics helps the learners to develop the following skills

(1) He acquires and develops speed, neatness, accuracy and precision is mathematical calculations

(2) He develops skill in the use and understanding mathematical language

(3) He develops the ability to estimate, check and verify results

(4) He develops the ability to perform calculations orally and mentally

(5) The learners develops ability to think correctly, to draw conclusions, generalisations and interences.

(6) He develops skills to use mathematical tools and apparatuses skillfully.

(7) He develops skill is drawing, meaning, weighing and use of different mathematical tools.

(8) He develops necessary skill in drawing geometrical figures and mathematical concept maps.

C. APPLICATION OBJECTIVES

It is the ability to apply learned knowledge, concepts principles and skills in new and similar situations. It involbs the application of abstruct learning is reallife situations.

Learningoutcomes is this area require higher level of understanding of mathematical concepts. The students are enable to :

(1) Analyze, draw inferences, generalise from collected data.

(2) Use mathematical concepts is every day life

(3) Solve mathematical problems independent by

(4) Apply mathematics is his fortune vocational life.

(5) Make use of mathematics is learning of other subjects and equip the self for higher education.

D. ATTITUDE OBJECTIVES

The development of Mathematics attitude makes learner open minded, helps him to make critical observation and to develop curiosity and impartial thinking

The students will be able to

(1) To analyse the problem independently

(2) Develop the habit of systematic thinking and reasoning

(3) Develop huristic attitude

(4) Develop originality, independent thinking and creativity

(5) Develop mathematical perspective while observing real life physical phenomena.

E. APPRECIATION AND INTEREST OBJECTIVES

The students will be able to appreciate

1. The value of mathematics as the science of all sciences and art of all arts.
2. The recreational value of mathematics and the utility for leisure time activities.
3. Actively participate in mathematical debates, contests, quiz, quest.
4. The role of Mathematics in every day life.
5. The student will develop interest in mathematical learning and activities.

2.1.4. Objectives of Teaching Mathematics at the Elementary Stage.

The main objectives at the elementary stage are the following :

A. KNOWLEDGE AND UNDERSTANDING OBJECTIVES

1. Develop The knowledge and understanding of mathematical concepts of number, units of measurement, shape, size, direction, distance, fractions, equal, grouping and sub grouping etc.
2. Develops the knowledge, understanding of mathematical terms, symbols, digits, fractions, percentages etc
3. Develops the knowledge and understanding of mathematical facts like four fundamental operations, percentage, unitary method, mensuration etc
4. Develops the knowledge and understanding of mathematical relationshps, notations etc.

B. SKILL OBJECTIVES

The learner develops the following skills

1. Skill of four fundamental operations with speed and accuracy
2. Ability in counting, reading, writing numbers and simple calculations
3. Skill in the use of mathematical tables
4. Proficenary in makimg estimates of size, distance and weights.

C. APPLICATION OBJECTIVES

1. The learner is able to solve both oral and simple writtens mathematical problems independently.
2. Try to apply elementary mathematical knowledge is every day real life situation
3. Try to estimate the length, mass of visible objects.

D. ATTITUDE OBJECTIVES

1. To solve a problem, he tries to follow the systematic steps of solving the problem like reading the verbal problems carefully, analyse the facts, collect data and draws tentative inference.
2. Develop gradually the habit of logical thinking and reasoning
3. Develops the habits of neatness, honesty and regularity.
4. Gradually develops self-confidence for solving elementary mathematical problems.

E. **APPRECIATION AND INTEREST OBJECTIVES.**

1. Appreciates the knowledge of Mathematics is solving problems of daily life.
2. Appreceiates the recretional value of Mathematics and try to utilise his leisure time.
3. Develops interest in the learning of Mathematics.
4. Appreciates the contribution of mathematicians and get inspired.

2.1.5 Objectives at the Secondary Stage

A. KNOWLEDGE AND UNDERSTANDING OBJECTIVES

1. He understands relationship of concepts, formulas, axioms and properties in the field of mathematics.
2. Gudge sufficiency, superfluency, relevence, inconsistency is the given data.
3. Detects errors in definitions, concepts, principles, process etc.
4. Translate mathematical relationship into symbolic forms and vice-versa.
5. Interpret graphs, charts, tables etc.

B. SKILL OBJECTIVES

1. He develops skill in solving the same problem by various methods but with reasonable speed.
2. Make readings from the lable quickly and correctly.
3. Draw the geometrical figures nealy and systematically.
4. Check the calculation quickly and can use the instruments and appliances.

C. APPLICATION OBJECTIVES

1. He learns the application of Mathematics is daily life, vocational, occupational and recretional life.
2. Formulate hypothesis from observed data and then draw in ference.
3. Develop designs using mathematical relationship and principles
4. Solve new problems

D. **ATTITUDE OBJECTIVES**

1. Develops interest and positive attitude towards learning of Mathematics
2. Learner gains confidence in the learning of Mathematics
3. Observe mathematical relationship is the environment
4. Accepts mistakes and point out errors unhsitatingly.

E. APPRECIATION AND INTEREST OBJECTIVES

1. He enjoys solving mathematical problems of different types
2. Derive pleasure is observing, finding, interpreting mathematical relationship
3. Express joy and pride over great contribution of mathematicians
4. Regard great mathematicians and mathematics teacher with respect.

National curriculum Framework has put forword the objectives of teaching school Mathematics as follows Developing Childre's abilities for mathematisation is the main goal of mathematics education. The narrow aim of school mathemaics is to develop useful capabilities particularly those relating to numeracy-numbers, number operation, measurement, decimals and percentages. The higher aim is to develop the child's

resources to think and reason mathematically, to pursue assumptions to logical conclusions and to handle abstruction. It includes a way of doing things and the ability and the attitude to formulate and solve problems. Accordingly the vision for school Mathematics are :

- children learn to enjoy mathematics rather than fear it.
- children learn important mathematics.
- Mathematics is more than formulas and machanical procedures.
- Children see Mathematics as something to talk about, to communicate, to discuss among themselves, to work togethers on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relationship to see structure, to reason out things, to argue the truth or falsity of statements.
- Teachers engage every child in class with the conviction that everyone can learn mathematics.

2.2 Bloom's Taxonomy of Educational objectives and writing objectives in Behavioural terms

Structure :

2.2.1 Bloom Taxonomy

2.2.2 There six behavioural objectives are related to specific mental process or ability which can be identified by using related action verbs.

2.2.3 Behavioural objectives under Affective Domain Krathwohl and his associates developed this objectives. These objectives are concerned with feeling aspect i,e changes in interests, attitude and values and also development of approciations and adjustments.

2.2.4 Behavioural objectives under Psycho-motordomain.

2.2.5 Simpson's Taxonomy to Psychomotor Domain

2.2.6 Writing objectives in Behavioural terms

2.2.1 Bloom Taxonomy

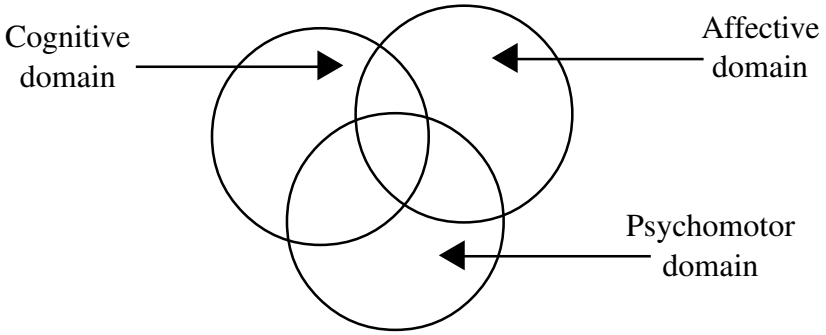
B.S. Bloom has included Taxonomy in educational objectives. The word Taxonomy has been derived from two Greek words : 'Taxis means configuration on arrangement and 'Nomas' means Law or rule. So Taxonomy means arrangement based on law.

Know that, Bloom and Maric (1964) defined ''A Taxonomy is a set of classification based on one on more principles.''

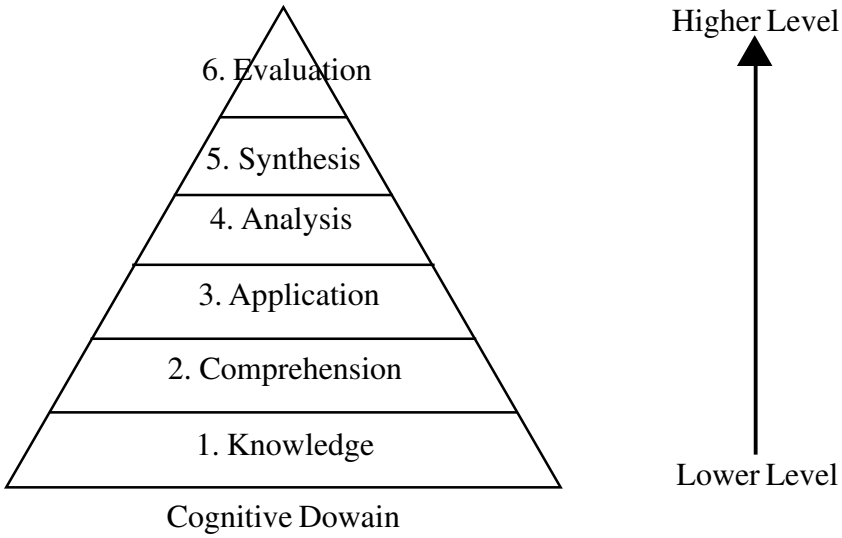
According to Bloom ''The Taxonomy places the behavioural aspect of the objective within a hierarchical frame work : each category is assumed to include behaviour more complex, abstract or internalised than the previous catagory. These categories are arranged along a continuum from simple to complex.''

In his book ''Taxonomy of Educational objectives'' (1956). Bloom has defined the educational objectives in terms of behavioural objectives. He mentioned three domains of Taxonomy namely i) Cognitive domain ii) Affective dowain and iii) Psychomotor domain. Bloom and other have mentioned different stages of congitive domain. ''The cognitive domain includes those objectives which deal with recall or recognition of

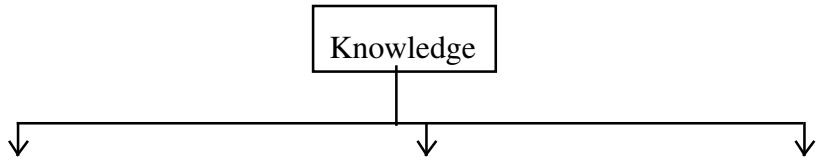
knowledge and the development of intelectual abilities and skills'' – B.S. Bloom & others 1956.



1. Knowledge is defined as the remembering of previously learned information. If stresses the Psychological process of remembering. Knowledge can be divided into three components.



Cognitive Dowain



A. Knowledge of specifics	B. knowledge of ways and means dealing with specifics.	C. Knowledge of the universl and abstraction in a field.
i) Knowledge of terminology Area, Triangle, Ratio Interest etc.	a) Knowledge of conventions (lengeth) breath, in a rectangle.	i) knowledge of principles and generalisations the effect of temperature on a volume of gas when pressure remains constant etc. the relation between area of a circle with that of its radious.
ii) Knowledge of specific facts.	b) Knowledge of trends and sequence. $-3 < -2 < -1 < 0 < 1 < 2$	ii) Knowledge of theories and structure
Area of triangle, Volume of a cube, $(a + b)^3 = ?$ etc.	c) Knowledge of classification and categories Area of circle, rectangle, triangle etc.	
	d) Knowledge of criteria Rational and irrational number, equation of inequation etc.	
	e) Knowledge of methodology To prove a geometrical theorem.	

2. Comprehension

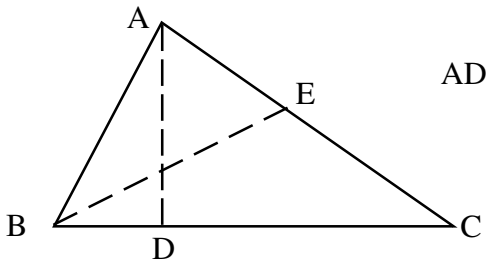
It is expressed as the ability to grasp the meaning of a concept. If represents the lowest level of understanding. This domain has three sub components

i) Translation

The abstract ideas can be translated into concrete ideas.

a) Translation from one level of abstration to another. The ability to convent verval statement into mathematical statement.

b) Translation from symbolic form to another form or vice versa.



$AD \perp BC$ $BF \perp BC$ etc.

c) Translation from one verbal from to another translate the therom from Bengali to English $5x + 3y = 15$ Give the graphical representations of the expression.

ii) Interpretation

It involves a re-ordering, rearrangement as explanation of concept for understanding. Interpret the statement, the correlation co efficient (r) is – 0.85.

Interpret the nature of a group of the expression

$Y = mx + c$

iii) Extrapolation of trends or tendencies beyond the given data for determining the implication or consequences on the original expression. $\frac{x}{a}+\frac{y}{b}=1$ Find the co-ordinates of points which lies on the state line as to justify whether a particular point lies on the state line.

3. Application: It involves the application of a concept to new or similar situations. The concept includes the following types :

i) General idea (Classification of numbers into rational and irrational, Types of triangles as the basis sides as angles.)

ii) The rules of procedure

iii) The Generalized method

Calculation of interest, or principal using the formula

iv) The technical principles

To draw the graphical representation of circle, Parabola, Hyperbola from giving sets of points.

v) The ideas

The reciprocals of rational numbers are less than the original numbers but it is not true for fractions.

vi) The theories/Laws

Based on a theorems, the teacher sets different hiders to test the application ability of students.

4. Analysis

It implies the breaking down of a mathematical concept into its constituent subconcept or parts so that relative hierarchy of ideas/concepts is made more clear. It has three components.

i) Analysis of elements

In a mathematical problem, There are some elements which do not directly related to the concept but to solve the problem relationship among the dependent variables is essential.Variables is essential.

For example incalculating the total interest of say Rs 5000/- for a period from January 15 to October 25 of 1999 at the rate of Rs 8/- per anum, the year 1999 is not related, but if the year be 2000 then it will affect the calculation.

ii) Analysis of relationship

In a mathematical problem, sometimes there may be two variables, each of which is related to other function independently. For solving the problems two relationship are to be taken into account for example, 12 women can finish a piece of work in 5 days, but 10 men can finish the same work in 4 days. Then in how a many days 7 women and 6 men can finish the work? Here rate of finishing a work is different for men and women. So independent relationship has man and women is to be calculated.

iii) Analysis of Organisational princeples. In teaching-learning of Mathematics, the domain of analysis of organisational principles includes the way the concept is to be presented, steps to be followed in sequence, scope of learners to take part in the discussion and the feedback system etc.

For example in proving a geometrical Theorem. The organisational principles will be what is going to proof must be known to learner.

- To test the necessary background knowledge for the theorem of the students. To proceed logically with active co-operation of others. etc

5. Synthesis

It refers to the ability to rejoin parts togethers to form a new whole. It gives emphasis to the formulation of new pattern. It is the fifth stage of cognitive domain. It has three components.

i) The production of unique communication :

- In this stage, the learn gains the ability to express his experience of learning to others
- Can construct new problems based on learnt concept.

ii) Production of a planned or proposed set of operations

- In this stage, the learner achieves confidence to set operations for understanding / solving similar mathematical problems.
- Can form unit plan for teaching a unit

iii) Derivation of a set of abstract relations

- In this stage of teaching-learning, mathematical concept are classified, abstract relations using symbols are established.
- Students gain the ability of generalisation, discovery approach and problem solving.

6) Evaluation.

It is concern with the ability to judge the value for a given purpose. Judgement are to be based on specific criteria like consistency, sequence and justification or internal determinents. It is of two types :

i) Judgement is terms of internal evidence. Here judgement is made considering the accuracy, justification, sequence of the concept presentation. In a class where a geometrical theorem is proved. Judgement will be made considering the internal evidences like whether the necessary back ground knowledge is tested, necessary construction is made, logical proof as well as verification is done, students, participation is adequate and expected outcome has been attained by the learner.

ii) Judgement in terms of external criteria. At the finishing stage, to what extent expected outcomes has been achieved, that will be evaluated summatively. The judgement value well be given considering the retaintion, recall value of the content.

2.2.2 There six behavioural objectives are related to specific mental process or ability which can be identified by using related action verbs.

Objectives	**Mental process / ability**	**Associated action Verbs**	**Examples**
1. Knowledge	1.1 Recall 1.2 Recognize	1.a Define 1.b State 1.c Recall 1.d Recognize 1.e Label 1.f Measure	1.a Define rational numbers 1.b State the parallel lines 1.c Recall the meaning of 5% per year 1.d Recognize the figure of triangle from the following 1.e Label the names of the geometrical figures 1.f Measure the length of the sides of the following triangle.
2. Comprehension	2.1 See relationship 2.2 Cite example 2.3 Discriminate 2.4 Classify 2.5 Verify 2.6 Generalise	2.1 Identify 2.2 Justify 2.3 Illustrate 2.4 Formulate 2.5 Judge 2.6 Classify	2.1 Identify the might angled triangle from the following figures 2.2 Justify alternative angles are equal 2.3 Illustrate the perpen-dicular distance is the shortest distance 2.4 Formulate the relation among 1, p, R. 2.5 Judge the relationship between external angle with that sum of intension angles. 2.6 Classify the triangles according to sides and angles. 3.1 Predict whether it will

3. Applications	3.1 Reason 3.2 Formulate 3.3 Establish 3.4 Inter 3.5 Predict	3.1 Predict 3.2 Assess 3.3 Explain 3.4 Show 3.5 Construct 3.6 Compute	be the angle of elevation or depresion when a man sees the top of a tower form top of a roof which is taller than the lower. 3.2 Assess the number of days from 2nd February to 16 October is 2004. 3.3 Explain the term 'Ratio' and 'Proportion' with examples 3.4 Show the relationship between the angles subtained at the circumtance and at the centre of a circle from a chord. 3.5 Construct a square equal in area of a given rectangle. 3.6 Compute the interest of a principal amounting Rs 6250/- for a period of $2\frac{1}{2}$ yrs at the rali of 8 .25% per anum.
4. Analysis	4.1 analysis	4.1 Analyse 4.2 Differentiate 4.3 Compre and contrust 4.4 Resolve	4. ~~Analyse the number of~~ terms of the expression $(ax + by)^7$ as $(ax + by)^5$ 4.2 Differentiate between Rombus and square 4.3 Compare and contrust between 'Frequency polygon and Histogram 4.4 Resolve into factors $(ax + b)^3$ 5.1 Combine the total

5. Synthesis	5.1 Synthesize	5.1 Combine 5.2 Argue 5.3 Generalize 5.4 Conclude	volumes of two cones formed by relating a night angle triangle along its hypotenuse. 5.2 Argue in favour of the statement that only in equilaliral triangle the angle bisectors and the lines joining the vertex and mid point of the opposite side meet in the same point. 5.3 Generalise the sum of interior angles of a polygon have n equal sides. 5.4 Conclude the measure of each angle of a right angled isosceles trangle.
6. Evaluation	6.1 Evaluate	6.1 Judge 6.2 Evaluate 6.3 Defend	6.1 Judge the value of which fraction is greater than 1. $\frac{3}{4}, \frac{7}{9}, \frac{26}{25}$,.9,0.99 6.2 Evalute the value of $X^3 - \frac{1}{X^3}$ When $X - \frac{1}{X} = 1$ 6.3 Defend the relation between different sets under Rational numbers NCWCZCQ with diagram where α = set of Rational numbers Z = Set of Integers W = Set of whole numbers N = Set of Natural number

2.2.3 Behavioural objectives under Affective Domain Krathwohl and his associates developed this objectives. These objectives are concerned with feeling aspect i,e changes in interests, attitude and values and also development of approciations and adjustments.

Affective taxonomy is divided into tive major classes arranged in an hierchical order characteristation by a value complex (attains the integration of his beliefs and attitude).

Organisation of a value systems (organisation of the value into a system, to determine the interrelationship and to establish the dominent value.

Valuing (acceptance of a value, preference for a value and commitment to certain value)

Responding (Response beyond attending, willingness to respond and satisfaction is response)

Receiving (it includes a wareness willingness to receive and selected affection.

Presentation of material in the classroom

Through teaching-learning of Mathematics, the behavioural objective under Affective domain will inculcate the value system among the learners. It will include to create interest in mathematics, to respect mathematicians to appreciate the recentional value of mathematics and appreciate the knowledge of mathematics in solving problems of daily life.

While teaching Mathematics, the teacher will try to remove the fear of Mathematics and engage every child with the conviction that every one can learn Mathematics.

2.2.4 Behavioural objectives under Psycho-motordomain.

The Psychmotor domain includes those behavioural objectives which deal with manual and motor skills.

In Mathematics teaching-leaving we include skill for development of precission and accuracy in calculation as well as for adopting alternative methods for solving a mathematical proble.

According to Dr R.H. Dove of the NCERT, the behavioural objectives under psychomotor domain are the following :

1) Imitation of an action, performance

The students of Mathematics class also at the initial stage imitale the action of his teachers in respect of calculation, drawing of mathematical figures of diagrams.

2) Manipulation of an act. Here the students will select the proper one among various movements.

For accuracy is calculation how to use calculators,Vedic mathematics etc.

3) Precission in reproducing agiven act.

This includes the motor activities in accurate calculation, exactness in performance by using calculators, computers etc.

4) Articulation among different acts.

It includes co-ordination, sequence and balance among acts. How to use instruments is drawing figures, skill of black board use, apporximation of the result and follow the requence.

5) Naturalisation : The motor activities in calculation, drawing figures, using mathematical instruments/aids will altain highest level of proficiency. It will become automatic and will be carried out unconsciously.

2.2.5 Simpson's Taxonomy to Psychomotor Domain

Simpson (1966-67) divided the domain in the following ways.

i) Perception - by utilising sense organs three types.

a) Sensory stimulation – Knowledge about cocrete things by sense organs.

b) Cue selection - for doing any mathematical operation a cue may be selected.

c) Translation - transfer of realactivities in to the virtual activities.

For examples a learner can visualise is how may seconds a water filled up tank having smaller incoming tap and larger outgoing tap will be emplyed. But for calculating such mathematical problem this real life knowledge will be translated into virtual activities.

ii) Set : For doing any mathematical activity or learning a mathematical concept, the learners need psysical, mental art emotional set of mind. There are three types of set.

a) Mental set – For transfer of knowledge from real life experience to mathematics class room activities a mental set is essential.

b) Physical set / Anatomical set - whether the learner is physicaly capable for perforing any mathematical activities is to be ascertained.

c) Emotional set : For performing the transfer of knowledge emotional set or disposition is essential.

All the three set may be treated as mind set for the transfer.

iii) Guided response : By initation of the activities of the teacher or by trial and error method the learn will be acquainted with the guided response in the mathematics classroom.

iv) Mechanism - The motor skill will be spontaneous by repetation or by habit formation.

For example use of calculator, computer will be accurate after prolong practice with a limited technical know how.

v) Complex over/response

At this stage the learner will acquire the motoractivity for performing accurate, rapid and speedy movement pattern. The performance will be spontaneous, no hagitation and speed will be higher. Indoing calculation, manipulation, drawing diagrams and solving problems is alternative way, the learner will be efficient.

vi) Adaptation : In new situation the transfer of motor activity will occur is a modified form.

vii) Origination : Creative ability to meet specific situation will also be transfered by motor activity. The skill of drawing geometrical diagram will also help the learner to draw creative picture.

2.2.6 Writing objectives in Behavioural terms

2.2.6.1 Introduction

The objectives of teaching mathematics at a particular stage will definitely give guidance to the mathematics teacher for planning of his/her classroom activities. It will help in planning what to teach, how to teach, how to teach and when to teach. Objectives have to be formulated for every lesson before teaching. Hence a teacher must know the procedure of writing the objective is terms of behavioural objectives so that the expected outcomes of teaching may be evluated while writing such behavioural objectives, the teacher must keep in mind the following points :

a) The nature of the subject matter as topic to be taught

b) The need and interest of his pupil and

c) The availability of resources :

d) It must follow the outline of objectives of teaching Math

2.2.6.2 Definition of Behavioural as Instructional objectives

These are the statements which will express the expected behavioural changes of the learners after the lesson. The characteristics are

i) The Broad objectives should be broken down into specific ones

ii) It must be observable

iii) It must be measureable

2.2.6.3 Guidelines for writing behavioural / Instructional objectives in Mathematics

i) Every statement of objectives should not be to large and general.

ii) They must fulfill the specific purpose of learning for that particular unit

iii) Objectives must be written and the outcomes will be achievable, and observable

iv) A teacher must keep is mind the entry behaviour (necessary background knowledge) of the learner.

v) It must mention the teaching points of the lesson, the expected behavioural changes of pupils, learning experiences to be organised etc.

vi) The statement of an objective should start with an the action verb.

vii) The elements of writing specific behavioural objective are a) performer i,e teacher and taught.

b) Task (which topic or subtopic to be taught like 1st lesson of Height and distance, Volume of cone etc)

c) Condition - (What previous knowledge will be required, how subtopics to be arranged etc)

d) Actions related objectives to be discussed.

e) Criteria for judgement of the teaching outcome,
specification of objectives

In setting of behavioural objectives, all cognitive three domains of honourable of objective cognitive affective and psychomotor domains are not utilised in details.

1. Knowledge objectives

a) The student will be able recognize different figures, symbols etc

b) The students will be able to recall different definitions, laws, mathematical tablets etc.

2. Understanding Objectives

The student will be able

i) to see relationship between like prohit and loss, interest, principal, rate of interest, time etc.

ii) to classify rectangles etc.

iii) to classify triangles according to sides, angles etc.

iv) to discriminated between rational and irrational number etc.

v) to verify the result obtained from the problem.

vi) to generalize the law like $(a+b)^3 = ?$

$(a + b)^n = ?$

3. Application objectives :

i) The learners will be able to formulate hypothesis like sum of intenterior angles of a polygon having n sides.

ii) The learner will be able to reason out why reciprocal of natural number is less than reciprocal of a fraction.

iii) The learner will be able to infer about the statement angle opposite to greater side is greater them the angle oppositst to smaller side of a triangle.

iv) The learner will be able to predict about say value of size, sin 45, sin 60 and sin 90 etc.

4. Skill (from psychomotor domain)

i) The learner will be able to handle the instruments of Geometry Box for drawing figure accurately.

ii) The learner will be able to calculate more correctly and precisely

iii) The learner will be in a position to solve similar problems in alternative ways.

5. Interest, attitude and appreciation (from affective domain)

i) The student will create positive attitude, interest towards solving such problems

ii) Mathematics phobia will be reduced.

iii) The learner will appreciate the problem solving nature of Mathematics etc.

2.3 ❐ Lesson Planning – Importance and Basic steps. Planning Lesson of Arithmetic, Algebra and Geometry

Structure :

2.3.1 Introduction

2.3.2 Importance of Lesson plan and characteristics

2.3.3 Basic steps in lesson planning

2.3.4 Planning Lesson Arithmetic

2.3.5 Planning Lesson an Algebra

2.3.1 Introduction

Mathematics must be taught by a suitable planning Teaching mathematics without preparation and proper planning is like a load without a ladder.

To make the classteaching more effective prior planning is essential. A planning may be of the following types.

1. Long range / term planning : The total content is to be divided into few Units
2. Topic as Unit planning : Each topic is to divided into few submit is to be completed in a class
3. Lesson planning : It is a written note prepared by the teacher containing the subunit to be taught, method to be used, objective in behavioural term, teaching activities, questions to evaluate and home work (assignment)

According to N.L. Bossing ‘‘Lesson plan is the title given to a statement of the achievements to be realised and the specific means by which these to be attained as a result of the activities engaged in day-by-day under the guidance of the teacher’’.

It is a detailed planning of a lesson.

It is a basic unit of planning for teaching.

It is the teacher's mental and emotional presumption of classroom experience. It is a plan of action which reflects the philosophy of the teachers, his knowledge of the content to be presented and the ability to use proper method of teaching.

Characteristics of Lesson plan.

- Lesson plan should be written.
- It is must give a clear picture of the activities to be performed by the teachers and the learner. It should contain clearly stated behavioural objectives. It should be related to previous knowledge. Home work / assignment must be related to the next days activities in the introduction stage. It should provide. It must cater the individual differences. Age Level and mental level of the students should be kept in.

2.3.2 Importance of Lesson plan and characteristics

i) For preparing a lessonplan, teacher has to study the content indetails which helps the teacher is the classroom.

ii) Prior to teaching, the teacher gets enough time to thinkover the content, method to be used.

iii) Lesson plan helps the teacher to know the different objectives of teaching a particular subtopic.

iv) It helps the teacher to know the necessary background knowledge of student prior to teaching.

v) Teacher gets the opportunity to think over the content and method to be used as well as to select the teaching aids necessary during the teaching.

vi) It helps the teacher to be acquainted with the expected out of the students.

vii) It helps the pupils to understand the process of teaching expected outcome and

viii) It ensures a proper connection of the new knowledge with the previous knowledge acquired in the class.

ix) It helps to maintain steady process with in schedule time.

2.3.3 Basic steps in lesson planning

Psychologist Herbart initially mentioned the following steps of 'Lesson Planning'. These steps are 1) clearness 2) Association 3) Systematisation and 4) Method

Later the associates of Herbart modified the steps and recommended the following six steps :

(a) Preparation or motivation or Introduction (b) Presentation

(c) Comparison (d) Generalisation and

(e) Application (f) Recapitulation

Mathematics is a continuous and sequential content based subject. To maintain the continuity, the lesson plain follows the concept of module preparation. The objectives are stated in terms of behavioural objectives.

So, the format of lesson plan will follow the following steps :

Format of Lesson plan	**School**
General Information	Unit
Class	Sub Units
Teacher	Name of the Teacher
No of students	....
Date	Today's Lesson

Small description of the content (Major concepts)

preparation / Introduction

Necessary background knowledge for the sub topic.

To test the background knowledge, following question will be asked

Instructional

Objectives

Cognitive domain

Knowledge

Understanding

Application

Affective domain

To develop interest, attitude and appreciation of the students

Psychomotor—

Skill

Methodology, Presentation by lecture method followed by question and answer method.

Teaching aid

	Teachers activity	Student's activity
Content Module-1 ↓ Module-2 ↓ Module-3 etc.		

Evaluation

By asking questions

Questions will be asked relevent to each module and then few probing questions will be asked.

Homework

Question of different types like short answer type. (fillup the blanks, match the pair, multiple choice type) and long answer type (may be from Mathematic Text book) may be given.

Unit Test in worksheet F.M = 10

A unit Test (F.M. = 10) may be given to students for immediate feed back.

2.3.4 Planning Lesson on Arithmetic

Introduction

Arithmeitc is the science of nember and the art and craft of computation. It deals with a system of counting. It is very essential in daily life. So the teaching of Arithmetic has to fulfield the following objectives.

i) To inculcate the ability of approximation and counting

ii) To develop the power of socialisation through number experiences.

Befor teaching Arithmetic in the class, the teachers and student must be aquainted with the aims of Teaching Arithmetics. These are :

1) To help the learner to understand verbal statement, to analyse them and to express it in mathematical statement.

2) To guide the student for simple computations using four basic operation.
3) To use arithmetic as simple tool in business and daily life problem.
4) To prepare for higher mathematics.

Lesson Part - I

A. **Subject : Arithmetic**	**Sub Units**
Class VI Unit - Addition of Fractions have different neumenators Time - 40 minutes Name of the teacher - MVX	i) Nature of fraction ii) Pure, impure and mixed fractions iii) To express one form to other form iv) To compare the fractions into smaller and greater form v) Addition of fractions

B. **Objectives**

i) Knowledge : a) The student will be able to given the definition of different forms of fractions

b) They will be able to identity which fraction is higher and which is lower in value.

ii) **Understanding :**

i) The students will be able to understand different types of fractions

ii) They will be able to distinguish between pure and impure and compound fractions

iii) They will be able to add different type of fractions

iv) Skill

a) The students will be able to add different fractions correctly and with speed

b) They will be able to draw diagrams showing different types of fraction

iv) Application

a) The students will be able to add fraction from a varbal statement

b) They will be able to convert fractions into decimals

v) Interest, Attitude and appreciation.

a) The students will be able to get interest from addition of fractions is their daily life.

b) They will be motivated to workout such problems

c) They will appreciate the existance of such fractional concept in nature and in daily life.

C. Necessary background knowledge

To test the background knowledge the teacher may ask following type of Question using charts, diagram paper cutting etc.

i) Using a quare paper whose part is coloured the ....teachers will ask how many part of the paper is coloured?

ii) If another part is coloured, then what portion of the paper is coloured?

iii) Express different types fraction using the number 7 and 9.

iv) Convert the following paise into rupees

a) 48 paise b) 125 paisa c) 60 paises etc

D. Teaching aids

a) Chalk, duster, Blackboard

b) Low cost and no cost teaching aids like paper cutting, Mathematics tool box.

E. Announcement of to-days lesson.

Today we shall discuss the diferent types of fraction and their addition

F. Method

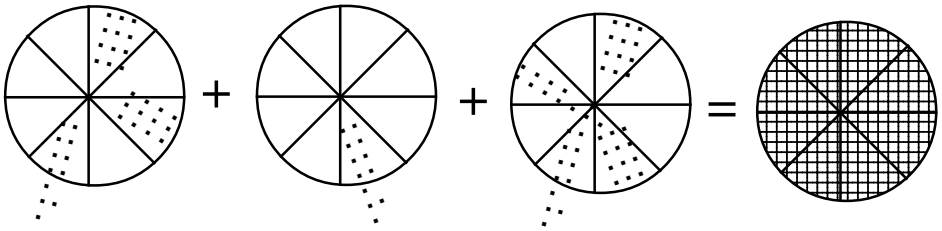
Presentation : Discussion followed by question answers method

	Content	**Teacher's activity**	**Method** **Student's activity**
P R E S E N T A T	Module I Fromation of fraction Module 2 Process of addition i) Find the L.C.M of dinominatior ii) Convert all fractions having	Write in fraction form 7 th by 8th part, 10 th by 16th part 15th part by 60th part L.C.M. of 8, 16, 60 $= \frac{210+150+60}{240}$	 240

I O N	as dinomination iii) Add the fractions Module 3 Problem 2 $\frac{2}{3}+\frac{4}{5}+\frac{6}{7}=?$ Module 4 Verbal problem.	$=\frac{^{4}320}{_{3}240}$ $=\frac{4}{3}=$ What is the answer?	How the operation can be verified using origimi/ Paper cuttings.

Evaluation

Add i)

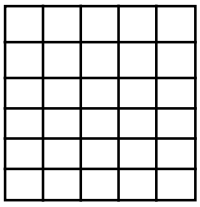


$\frac{\text{Shaded}}{\text{clean portion}} \Rightarrow \frac{3}{8}$ + + = 1

$\frac{5}{14}+1=?$

ii)

Probing Questions

1. 

   Shade 5/30 Part, = $\frac{5}{30}$

   1 Part by 30 th part =

   7/30 =

   Add all Count the shaded small squares

   '' '' unshaded '' ''

2. A man gives Part this capital to his elder son, $\frac{2}{9}$th part to his servant, $\frac{1}{2}$ to his wife. How much capital the mankeeps for himself ?

Home Work

1. Add ·3 + ·4 + ·04 =

2.

3. In Prpoer fraction N > D; N < D; N = D
4. In mix fraction N > D; N < D; N = D

Work sheet
F.M = 10

5×2=10

a) Add $\frac{5}{7}\text{cm} + \frac{9}{14}\text{cm} + \frac{5}{21}\text{cm}$

b) When N > D The Fraction is ———

c) When N = D The nature Fraction is ———

d) When N < D The Fraction is ———

e) Convert paise into Rs. and them add. 17 paise + 23 Paise + 20 paise = Rs.———

2.3.5 Planning Lesson on Algebra

Introduction : The word 'Algebra' has originated from the 'Arabic' word 'Al-Jabr' al-muquabulah' where 'al' means 'the', 'Jabar' refers to the operation of transferring a quantity from one side of an equation to another and 'muqabulah' refers the process of subtracting similar quantities from both sides of an equation.

Objecives of teaching Algebra.

- To formulate the problems into equations and then solving we get the result.
- To generalisation of arithmetic problems and also to use in other branches of mathematics trade and industries.
- To inculcate the power of analysis and to develop the confidence among students by verifying the results to simple way.
- To help the students to express a new symbolic relationship of abstract ideas.

PEDAGOGY OF TEACHING MATHEMATICS

LESSON - 2

Subject - Algebra
Class - IX
Unit : Formation of Euration
School -
Time 40 minutes
Name of the teacher Mry

Sub - Units

1) Meaning and order of Equation
2) Formation of simple equation
3) Solution of a equation in algebra
4) Solution of a arithmetic problem and problem of others discipline

2. **B. Objectives (Instructional)**

(i) knowledge

(a) The students will be able to name different components of an algebraic equation.

(b) They will be able to know the different sides i.e L.H.S and R.H.S side of the equation.

(ii) Understanding

(a) The students will be able to understand the relationship among different variables

(b) They will be able to form mathematical statement from the verbal statement

(iii) Application

(a) The students will be able to solve the algebraic equation formed by themselves

(b) They will be able solve the mathematical equation formed by themselves covering daily life problems.

(iv) Skill

(a) The students will be able ot solve the equation more accurately.

(b) They will be able to form two equations for finding two unknown quantity.

(v) Attitude

(a) The students will be able to create a positive attitude towards the formation of an algebraic equation has solving problems of asithemetc and even of geometry.

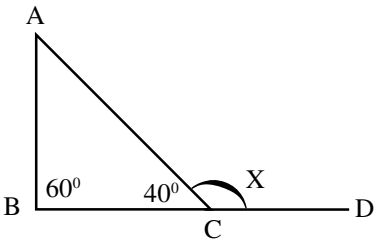
(b) Mathematics probia

Love for Mathematics will be generated and this will dispel the mathematics probia among the students.

3. Testing previous knowledge (related to the topic by means of asking questions, drawing figures– ankering)

   (i) What will the algebraic equation of the following verbal statements 8 years more than The age of the father is three times that of his elder son and four times that of his daughters.

   (ii)



   What is the value of ∠ACD if ^{0} and ^{0}

   (iii) For finding the values of two unknown how many equations are to be formed.

4. Introduction

   (Presenting the overview of the content)

In different branches of Mathematics like arithematic, algebra and geometry etc mathematical problems are solved by using specific procedure. But by using algebraic equation those problems can be solved easily.

Mainsteps of this algebrac procedure are

(i) To transform the verbal statement into algebraic equation/s.

(ii) To solve the equations

(iii) To verify the equations as relationship with the help of the find out solution.

(Major concepts (Modulwise)
Presentation of the content

To day we shall discuss the solution of arithematic problem by formation of equation.

5. Teaching method

   Problem solving / discussion followed by question-answer

6. Teaching acts : Graph papers, Mathematics tool box, Mathematics concept mops. etc

7. Presentation of the content

(Transaction of lesson)

Teachingpoints	Teacher's activity	Students' activity
• Arithmetical statement to algebraic equation prof I. The costof 5 tables and 3 chairs is 13000/- Again 4 tables 4 chairs cost 12000/- Form the equations • Prob 2 In a co-operative firm Rs 40,000/- was invested for the productions of rice and wheat in the last year. Meeting all expenditure a profit of 15% on rice and 20% on Wheat (1) Were obtained - In the (2) Present year the rate of expenditure is same but the total profit is 20% and Rs 1200 more profit is obtained. Find the expenditure in each of the cultivation. The teacher will write the relevant portion of the question-answer on the black board	(i) What is the algebraic equation? Let cort if each table be Rs T and Cort of each check is Rs C ii) Try to solve the equations 5T + 3c = 13000 20T + 12c = 52000- (III) 20 T + 20 c = 60000 (IV) Eq (IV) – Eq (V) What wegef iii) What is the cost of each chairs? using graph paper the teacher will show the point of interaction of two equations 2.1 If the expenditure for the cultivation of rice be Rs X, then what is the expenditure for wheat 2.2 What is the % of profit is rice? 2.3 What is the amount of profit is rice 2.4 What is% of profit in wheat?	5T + 3C = 13000 4T + 4C = 12000 20T + 20C = 60,000/- 20T + 12C = 52,000/ 8C = 8000 or, = 1000/- 2.1 Rs (4000–X) 2.2 15% 2.3 Rs. 2.4 20% 2.5 Rs. $(40,000-X)\times\frac{120}{100}$ Students activities $\frac{115X}{100}+(40,000-X)\frac{120}{100}$ 2.8 Rs. $40,000\times\frac{120}{100}$ 2.9 Rs. 1200/-

$C=\frac{8000}{\frac{115}{100}}$

	2.5 What is the amount of profit in wheat? Teacher's activity 2.6 What is the total profit on both rice and Wheat? 2.7 The teacher will use the graph paper to show the students the amount of profit is wheat. 2.8 What is the total profit is the production is the present year on the whole? 2.9 What is the excess profit is this year? 2.10 Companing the amount of profit of both the years, establish the relation. 2.11 Establish the algebraic equation relating to profits The teachers will solve the equations through question-answer method on the black board. 2.12 What is the value of X=? 2.13 What the amount of expenditure for the production of rice 2.14 What is the amount of expenditure for the cultivation of wheat?	2.10 The difference of profit will be Rs 1200/- 2.11 Rs. $\left\{40{,}000 \times \frac{120}{100}\right\}$ $\times \frac{120}{100}\Big\} = 1200$ 2.12 X = 24,000 2.13. Rs 24,000/- 2.14 Rs (40,000 – 24,000) = Rs 16,000/-

8. Generalisation (Consolidation of contents) The Teacher will generalise the Mathematical concepts of formation of equation on and the steps of solution

9. Application (is other branches of mathematics and is relevant or real life situation)

The teacher will give examples showing the solution of geometry, arithmetic problems.

i) If the length of a rectangular field be increased by 2 meter and breath be increased by 3 meters, the area will increase by 75 sq. meter. But if the length be less by 2 meter and breath be increased by 3 meter, the area will increase by 15 sq. meter. Find the length and breath of the field. This type of mensuration can also be solved by this method also.

10. Recapitulation (Evaluation of how much of the content has been grasped. To ask few profing type questions.)

a) What are the steps to be followed for the solution of a quachatic equation of two unknown.

b) Find the equation of the following statement.

If three times of a number is subtracted from the square of the numbers, the value will be 18.

c) A starts from a place P to go to a place M; at the same time B starts from M for P. After meeting they arrived at their destinations is 2 and 3 hours respectively. Show that the ratio of this speed is $\sqrt{3}:\sqrt{2}$

10. Home assignment –

The Home assignment will comprise different type of questions like objective type, short answer type and long answertypes questions. To fulfill the objectives belonging to cognitive as well as affective domain some activities may be included.

Q1. State four consquitive numbers

Q 2. Make a diagram to show that $\frac{1}{2}+\frac{1}{3}-\frac{1}{4}=\frac{7}{12}$

Q 3. Some problems of the Text book

Q 4. Divide 48 into two parts such that of one part be multiplied by 8 and the others by 5, the sum of the products shall be 180.

Lesson - 3

Class IX	**Sub unit**
Subject - Geometry School Time - 40 minutes	Relation between extension angle of atriangle with opposite angles.

2. Objectives (Instructional)

1) Knowledge

a) The students will be able to define extension and opposite angles of a triangle

b) They will be able to establish relation between extension and opposite intension angles

ii) Understanding

a) The students will be able to understand the logical proof of the theorem

b) They will be enable to proceed sequentially for establishing the relations

iii) Application.

a) They will be able to apply the theorem in real life situation

iv) Skill

a) The student will aquire the skill for drawing the geometrical figures with instruments.

b) They will be able to draw necessary figures with speed and accupacy.

v) Attitude :

a) The students will be able to create a positive attitude towards the development of the power of reasoning and analytical thinking.

b) To introduce students to the significance of regour, intelligent generalisation and critical evaluation.

3. Testing previous knowledge (related to the topic)

The necessary knowledge for the proposition is

i) The sum of the angles of a triangle is equal to two right angles

ii) A straight angle is equal to two right angles

iii) When a straight line stands on another st. line, the sum of adjacent angles so formed is equal to two right angles. For this the following questions will be asked showing the diagram.

a) What is the sum of the angles of a triangle?

b) Draw the figure of a straight angle?

c) What is the sum of supplementary angles?

d) If a straight line stands on another straight, what is the sum of adjacent angles so formed?

4. Introduction

The teacher will draw different types of triangle and will produce one of the side.He may ask the students that today we shall findout the relation between the exterior angle with that of the sum of oppositive angles of a triangle.

5. Teaching method

Deductive method followed by question answers method.

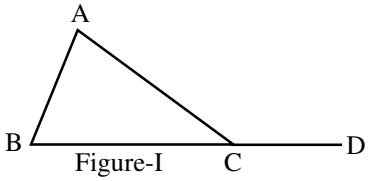
6. Teaching aid.

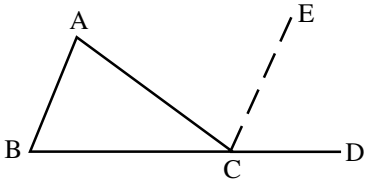
∠ACD∠B

a) Math Tool Box b) Model showing the relationship c) Geometry Box d) Usual classroom equipment

7. Presentation

(Transaction of lesson)

Teaching Pts	**Teacher's activity Board works**	**Students activity / Evaluation**
1) Conclusion through veritication 2) Theoretical proof 3) Alternative Proof Application	The teacher will draw  Figure-I ΔABC and the side BC is produced up to D A 1.1 What is the measure of the angle ?	The students will measure the The students will measure A side of a triangle is produced so as to form an exterior angle

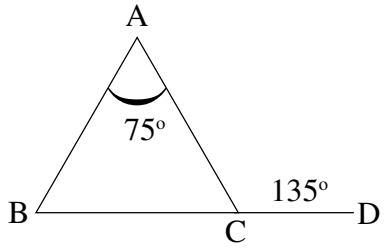
	1.2 What is the measure of and ? 1.3 What is the total measure of 1.4 What relationship is obtained? 2.1 What is given is the Fig-I 2.2 What exterior angle is produced 2.3 What is to be proved? 2.4 What is the sum of three angles of a triangle? 2.5 Find out another group of angles is this diagram whose sum is equal to 2 pt. angles 2.6 What is the relation between two equ (i) and (ii) 2.7 If we subtract the angle from both sides what will happen? 2.8 What is ference can we draw? 3.1 The teacher will draw the following Fig-II  	2.1 Intriangle Bc is produced to D. 2.2 exterior angle is formed. 2.3 You prove ACD = CAB + ABC The exterior angle=sum of the two opposite angles. 2.4 ABC + BCA + CAB = 2 rt. angles.....(i) 2.5 rt. angles as AC stands an (ii) the line BD 2.6 ACD + ACB = ABC + ACB + CAB = 2rt. angles 2.7 ACD = ABC + CAB 2.8 The exterior angle = Sum of the two opposite angles.

	Where CE ‖ BA. 3.2 The teacher will proceed refering the corresponding angles.	

8. Generalisation (Consolidation of the contents) The teacher will generalise the theorem to the students with examples

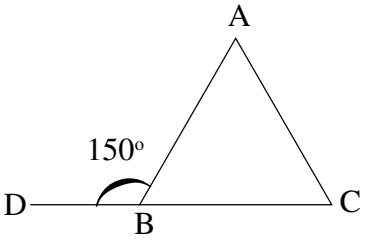
9. **Application**

(i)



In the triangle angle ^{0} What is the value of

(ii)

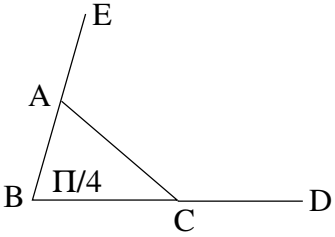


is an social triangle where Find the value of

10. Recapitulation.

The teachers may ask the following profing questions to the students.

i)



Gives $\angle ABC = \frac{\Pi}{4}0$

$\angle ACB = \frac{\Pi}{6}0$

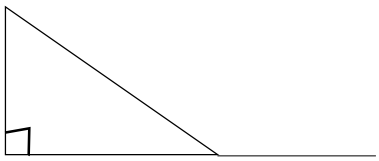
Find the value a) ∠CAE b)

ii) State the theorem and mention the steps of ‘‘given’’ and ‘‘to prove’ is this case.

11. Homework

i) In an equilational triangle what will be the value exterior angle?

ii) In an isosocial right angled triangle what will be the value of exterior angle opposite to the right angle triangle?



iii) Prove the theorem is alternative way taking the corresponding value of the paralled.

2.4 ❐ Unit Planning - Format of a Unit plan

Structure :

2.4.1 Introduction

2.4.2 Characteristics of a Unit

2.4.3 Unit Plan

2.5 Pedagogical Analysis

2.5.1 Meaning :

2.5.2 Need of pedalogical analysis

2.5.3 Types of pedagogy

2.5.4 Procedue for conducting Pedagogical analysis of Mathematics

2.5.4.1 Distribution of Content into Unit and sub Units

2.5.4.2 Writing the Instructional objects for any subunit for the preparation Lesson :

2.5.4.3 Test of Entry Level behaviour Necessary background knowledge for the topic

2.5.4.4 Selection of relevent strategy for teaching learning process and appropriate teaching aids.

2.4.1 Introduction

Prof. H.C. Morrison of Chicago University is the founder of unit approach of teaching. According to him 'Unit is a comprehensive and significant aspect of the environment of an organised Science and Art''

According to Wesley, ''Unit is an organised body of information and experience designed to effect significant outcome of the learner" The Dictionary of Education states a Unit as ''an organisation of Varried activities, experiences and types, learning around a central problem, purpose, developed co-operatively by a group pupils under teacher leadership, involve planning, execution of plans and evaluation of results'' Thus, Unit is comprehensive instructional plan specifying the 'What', 'how' and 'when' of teaching and organised body of information.

2.4.2 Characteristics of a Unit

1. A unit of a curriculum is a purposeful learning activity.
2. It has significant programme.
3. It is designed to effect significant out come of the learner.
4. It also provides opportunities for creative experience.
5. A unit is functional.
6. A unit enables learners to adjust to a life situation more effectively.
7. Every Unit is a contract or obligation to study.

2.4.3 Unit Plan

(a) Definition

A unit plan is the mapping process that begins with long-term plan which provides a sense of direction and organisation to achieve significant academic gains with is a particular time period.

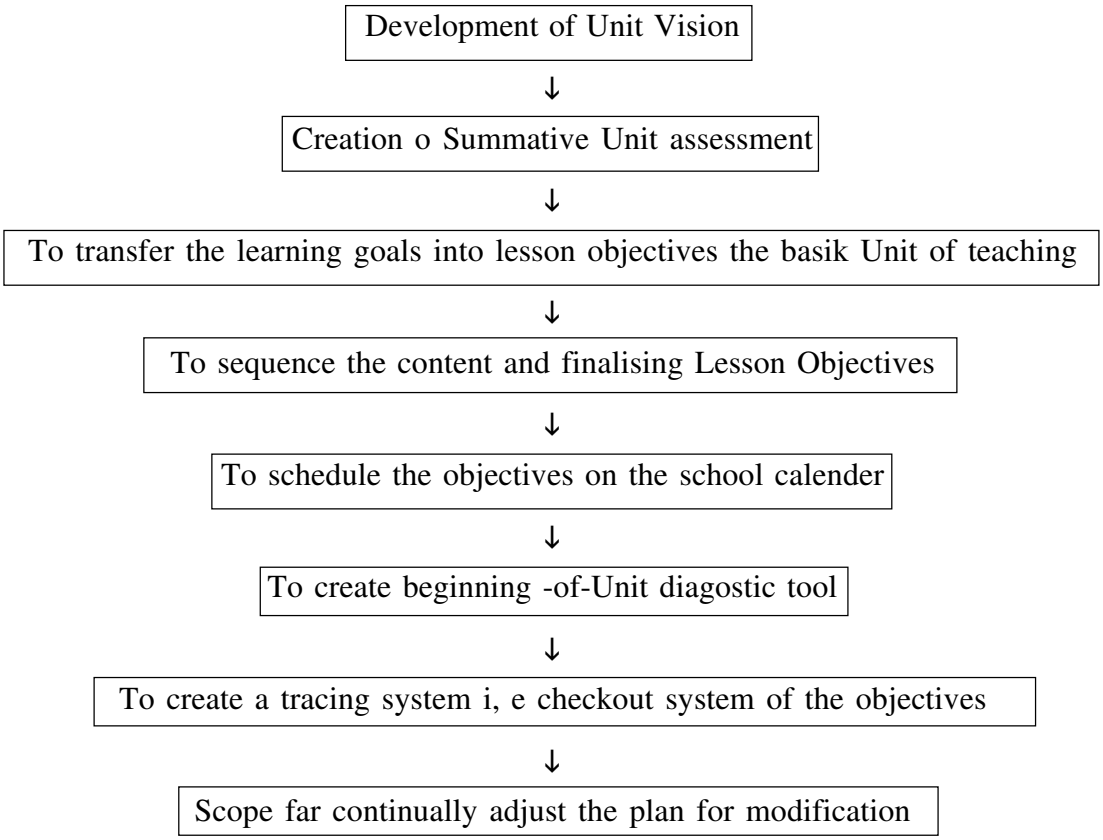
Before the excution of lesson, The teachers plans for the period. So unit planning begins with identifying the particular content to be taught to fulfill the goals for learning out comes. Goals relates the rational for teaching the particular content. It will cover the intended objectives, activities to be performed, time estimation, material needed, assessment procedure and alternatives for students having different ability levels or interests. So it learning tragectory.

(b) Characteristics of Unit plan

- It is a brood ‘road map’.
- It designs a structure covering key content, skills, ways of assessment, what students need to know, class time and its productive use.
- It can be used with revision

(c) Format of Unit Plan.

For creating Unit plans we have to follow the eight interdependent steps namely



So the teacher will plan out several Units over the course of a term or even the entire year.

(d) Standard Format of Unit Plan

1. Subject area :
   Grade level-
2. Unit / Title / Name
   Period : From – to –
3. Unit goals –

Lory term goals. What are the goals for this unit of instructioin? How does this Unit fit with year long goals?

4. Specific ways to relate goals to curriculum
5. Characteristics of students

To list the needs for the students is the cognitive social, emotional and physical areas—

a) Student Goals

i) will be able to define ii) will be able to identify iii) will be able to describe iv) will be able to draw/calculate etc.

b) Student objectives.

i) Will be able to pass a Unit test.

ii) Will be able to create.

iii) Will be able to understand / apply.

6. Introductory procedures.

   How the teacher will introduce the Unit and goals to the students.

7. Materials/Media to be used daywise.
8. Assessment and evaluation.

Selection of methods to identify students learning levels and needs when teaching will occur during the Unit.

9. Assessment instrument to measure the outcomes.

(e) Benefits of Unit planning–

i) Unit planning will help the teachers to take decesion about what to teach and how to teach it.

ii) A unit plan keeps teachers on pace to fulfill unit and untimately long term goals.

iii) To help the teachers to reflect on what they want to accomplish in each unit.

iv) How the classtime to be utilised as productive as possible.

v) Unit plan provides an opportunity to stimulate students interest through rearrangement of convent that is relevant to students.

vi) Will help the teacher to realise What type of revision and adaptation are necessary in the content.

vii) Unit plans help the teachers to organise individual lesson into a coherent structure i,e linking each lesson plan to the next.

viii) Will help the teacher to keep on tract for the formative and summative evaluation.

ix) Will help the teachers to realise how much materials can be realistically fit into a unit.

x) A unit plan after execution may help teacher to realise how much time should be allocated for a unit. In practice, a given concept takes more or less time to teach than anticipaled.

2.5 Pedagogical Analysis

2.5.1 Meaning :

The word 'pedagogy' has derived from the Green word 'paidagogia' means a slave who accompanied a Greek child to school.

'Paidos' means 'Child'

'ago' means 'to lead'

So it means 'to lead the child'

Pedagogy is the discipline that deals with theory and practice of education. It concerns the study and practice of how best to teach. Pedagogy comprises what teachers do in the classroom but also refers their ideas, knowledge and attitude in relation to the learner. the teaching and learnin process and the curriculum. Pedagogy is the art of teaching (creative and intuisive part), The science of teaching (research, decesion making and theoretical understanding) and the craft of teachin (To produce skill and practice far habit formation.

2.5.2 Need of pedalogical analysis

- Pedagogical analysis not only gives stress on the analysis of content but also how the content can be best presented to the students.
- It does not give stress on rote learning but on (i) meaningful learning (ii) discovery learning and (iii) problem solving learning.
- Pedagogical analysis not only help the teacher to teach the content systematically but alsohelp the learner to modify theirs experihences and to apply the new knowledge is real life situation.
- It helps the learner for sequential development of individual mental process such as recognising, recalling, analysing, reflecting, applying, creating, assimilating prior knowledge with new knowledge.
- Pedagogy develops metacognition in teachers i.e the ability to learn how to learn.
- Pedagogy is a basis for auditing teacher's practice.

2.5.3 Types of pedagogy

Depending on the are of application, it can be classified into four types

a) General pedagogy
b) Andragogy - the art and science of helping adult
c) Inclusive pedagogy - an alternative approach that has the potential to reduce educational inequalities by enhancing learning opportunities has everyone.
d) E-Pedagogy

2.5.4 Procedue for conducting Pedagogical analysis of Mathematics

Pedagogical analysis of Mathematics the process by which the content of Mathematics for a particular class is analysed by Educational psychology, follow the stategy of teaching in the class room and create an educational environment for transaction of the content.

2.5.4.1 Distribution of Content into Unit and sub Units

The whole syllabus of Mathematics for a particular class has been divided into different branches of Mathematics like Arithmetic, Algebra, Geometry, Mensmation, Trigonometry etc.

Concepts/Units	Concepts/Sub-Units	Experted Learning Outcomes
Factorisation	Meaning of factorization, finding the factors for the given algebraic expression, factorization of the expressions of the form $a^2 + 2ab + b^2$, $a^2 + 2ab + b^2$ and $a^2 - b^2$ using identities	Learns the meaning of factorisation, finds the factors for the given algebraic expression, performs the factorization of the expressions of the form $a^2 + 2ab+b^2$, $a^2-2ab+b^2$ and a^2-b^2 using identities.
Factorisation of $a^3 + b^3$ using identities	Finding the product of expressions of the form (x+a) (x+b) (x+c) using the identily. Finding the co-efficients of x^2 and x using the identity (x+a) (x+b) (x+c) Finding the factors of the expressions of the forms $a^3 \pm b^3$ using identies.	Finds the product of expressions of the form (x+a) (x+b) (x+c) using the identity Finds the coefficients of x^2 and x using the identity (x+a) (x+b) (x+c) and factors of the expressions of the forms $a^3 \pm b^3$ using identities
HCF and LCM	Meaning of HCF and LCM of algebraic expressions Finding the HCF and LCM of binomial and trinomials	Learns the meaning of HCF and LCM Finds the HCF and LCM of binomials and trinomials

Divisionof polynomials	Division of a monomial by monomial, division of polynomial by a monomial division of a polynomial by a binomial and application of division of polynomials	Learns the division of a monomial by a monomial, division of a polynomial by a monomial, division of a polynomial by a bimormial.
Simultaneous Linear Equations	Framing simultaneous liner equations solving simultaneous linear equations by elimination method and graphically, verification of results of simultaneous equations.	Frames simultaneous linear equation, solves simultaneous linear equations by elimination method and verifies results of simultaneous equations.
Algebraic structure	Verification of closure property w.r.t. a given operation. Definition of binary operation and algebraic structure.	verifies closure property w.r.t. a given operation. Defines binary operation and algebraic structure.

Concepts / Units	**Content / Sub-Units**	**Experted Learning Qutcomes**
Polygons	Meaning of polygons, identification of polygons, difference between regular and irregular polygons, inscribing regular pentagon, hexagon and octagon	Learns the meaning of polygons, identifies given polygons. Differentiates between regular and irregular polygons and inscribes regular pentagon, hexogon and octagon.
Quadrilaterals	Basic concepts of quadrilaterals, identification of elements of a given quardrilateral, properties of quardrilaterals,	Learns the basic concepts of quadrilaterals, identifies elements of a given quadrilateral, states the properties of quadrilaterals, constructs quadrilaterals to given measurements and calculates the area of a

	construction of quardrilaterals to given measurements, calculation of area of a quardrilateral using the formula.	quadrilateral using the formula. Improves skill of constructing more gemetrical figures given certain data/ conditions.
Different types of quadrilaterals	Identification of different types of quardrilaterals, properties ofparallelogram, rhombus and trapezium, construction of parallelogram, calculation of area of a parallelogram, construction of rhombus, finding the area of rhombus, construction of a trapezium and finding the area of traperzium	Identifies different type of quadrilaterals, states the properties of paralleglogram. rhombus and trapezium, constructs paralleglogram, rhombus and traperzium and calculates the area of these quadrilaterals using the formula.
Theorem on parallelogram	Proving of the different properties of the parallelogram logically Meaning of corollary, corollaries of the theorems, problems and riders based on the theorem	Proves the different properties of the paralleogram logically, Learns the meaning of corollary, states the corollaries of the theorems, solves the problems andlrider based on the theorem
Areas of Parallelogram	theorem on areas of parallelogram; corollaries of the theorem, problems and riders based on the theorem	States and proves the midpoint theorem, states the coverse of mid-point theorem, solves the problems and riders based on midpoint theorm.
Mid-point theorem	Statement of mid-point theorem, proof of mid-point theorem, converse of mid-point theorem, problems and riders abased on midpoint theorem.	States the definition of circle, radius, circumference, diameter, chord, arc, segmenj, etc. Identifies the chord property of the circle, central angle and inscribed angle.

Circles	Definition of circle, radius, circumference, diameter, chord, arc segment etc. Identifcation of chord property of the circle central angle and inscribed angle	Define cyclic quadrilateral, states the properties of cyclic quadrilateral logically, constructs cyclic quadrilaterals, and solves problems and riders based on the theorem on cyclic quadrilateral.
Cyclic quadrilateral	Definition of cyclic quadrilateral, properties of cycil quadrilateral, theorem on cyclic quadrilateral logically, construction of cyclic quadrilaterals, problems and riders based on the theorem on cyclic quadrilateral	States the definition of prism and pyramid, states the properties of prism and pyramid, differentiates between prism and pyramid, calculates the surface areas of prism and pyramid and volume of prisms and pyramids.
Surface areas and volumes of solids	Definition of prism and pyramid properties of prism and pyramid, differentation between prism and pyramid, calculation of surface areas of prisms and pyramids, calculation of the volume of prisms and pyramids	

In the Unit planning of each branch of Mathematics, the content is analysed interms of concepts/Units, Content /Subunits and corresponding Learning out comes. For example the content of 9th stand and mathematics, the Algebra can be analysed is terms of concepts / Unit like Factorisation, Factorisation of $a^3 + b^3$ using identities, HCF and LCM, Division of polynomials, Simultaneous Linear equations, Algebraic structure similarly, the content of Geometry can be divided into concept/Units like polygons, Quadrilaterals, Different types of Quedrilaterals, Theorems on parallelogram, Area of parallelogram, Midpoint theorem, circles, cyclic quadrilateral, Though in the present curriculum of Mathematic under West-Bengal Board of Secondary ..... There are no such grouping of Arithmatics, Algebra, Geometry etc.

For Pedagogical analyris of a particular unit, the unit is to be divided inot corresponding subunits in such a way that a sub unit is to be completed is one period by forming a lesson plan. So the number of periods to be required for the completion of the unit/is to be mantioned is the pedagogical analysis.

2.5.4.2 Writing the Instructional objects for any subunit for the preparation Lesson :

Instructional objectives are the objectives expressed interms of behavioural objects for class room teachering Mathematics to realise through activities Mathematics to realise through activities and working out the problems in the class room. These instructional objectives like knowledge understanding, skill, application, appreciation etc will follow the Bloom's Taxonomy. In Mathematics, the instructional objective will include under the following dimension knowledge means acquiring knowledge of symbols, terms, difinitions, relationships, principles, formulae, aims and properties. The specifications on actionverbs are recall and recognition.

Understanding means to develop an understanding for symbals, terms, concepts, principles etc. The specification or action verbs pertaining to this domain are illustrate, detect errors, translation mathematical relationships to symbolic forms vice versa, express given statement in different forms, compare, identify, classfy, substitute, analyse, judging the sufficiency of data, presentation of proofs, verify results, select proper formula in finding out solution of a problem etc.

Developing skills include to be able to do oral or written calculation, drawing neat figure, to select proper table, quick and accurate checking from the table etc.

Application : Developing ability to apply mathematical relationship to different situations. The specification as action verb to be used are suggest, modify, establish new relationships, inference, formulates hypothesis, find new application, solve new problems.

Application : Developing appreciation of mathematical relationship, achievements, develop self reliance through the habit of verification, develop the power of analysis, ability to work accuality sees relationship between mathematics and environment, makes reasonable estimation etc.

2.5.4.3 Test of Entry Level behaviour Necessary background knowledge for the topic

To test the background knowledge teacher will select few questions. Teacher will up date the students with necessary knowledge. He will also test the 'concept mapping' of the students for the topic.

2.5.4.4 Selection of relevent strategy for teaching learning process and appropriate teaching aids.

There are many methods of teaching Mathematics. They are :

a) Dogmatic Psychological Mehtod

b) Inductive and Deductive methods

c) Analytical and Synthetic methods

d) Neuristic mehtod

e) Laboratory method

f) Lecture method

g) Project method

h) Play way method

The content categories to be taught in Mathematics are the following and the appropriate method of teaching.

Content area	Method of Teaching
a) Concepts	i) Inductive reasonin
b) Generalization	ii) Verifying specific examples
c) Mathemacical probles	iii) Discovery activities
d) Geometric Constructions	iv) Developing skill of using instruments
e) Proof	v) Following logical sequence

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b) Generalization	ii) Verifying specific examples
c) Mathematical problems	iii) Discovery activities
d) Geometric Constructions	iv) Developing skill of using instruments
e) Proof	v) Following logical sequence

2.5.4.5 Objectives in pegagogical analysis

The knowledge dimension following revised Bloom's Taxonomy-2001 in given below.

2.5.4.5.1 Factual Knowledge

The Core knowledge which is essential for solving related problems. This includes :

a) knowledge of terminology (mathematical symbols, mathematical Vocabulary etc)

b) knowledge of specific and details of elements)

Like Types of triangles, Number system

2.5.4.5.2 Conceptual knowledge

Formula showing relationship among variables $I = \frac{PTR}{100}$; Interest = $\frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$

Relation of angles oppsite to sides.

The components are

a) Knowledge of classification and categories.

- Different types of teaching aids used in mathematics class
- Types of interest
- Types of quadrilaterals
- Types of reasoning (deductive and inductive etc.)

b) Knowledge of principles and generalization partinent points and their generalization viz. Rules of variation, logarithem, congruance of triangles etc.

c) Knowledge of theory, model and structure Viz Effect of increase or decrease in neumenators as denominatop, mathematical modelling, structure of three dimension bodies etc.

2.5.4.5.3 Procedural knowledge

How to proced for a proof, for drawing a graph of a line...equation. This includes

a) Knowledge of subject specific kills and algorithms

Examples

i) Procedure for solving quachatic equations

ii) To determine the volume of a sphere as prism

iii) To find out the ratio of the Components of a mixture etc

b) Knowledge of subject specific techniques and methods.

Examples

i) To find out the value of unknown number in arithmetic and in algebraic problems

ii) To solve a problem using synthetic, method

iii) To solve a problem using deductive and inductive method etc

c) Knowledge of criteria for determining when to use appropriate procedure.

Examples

i) Statistical analysis for ariving at a conclusion

ii) To verify the laws of Bialogy or others subject thought Mathematics

2.5.4.5.3 Metacognitive knowledge

Deep rooted cognitive knowledge to understand the inner meaning of a content and / or express the self creativity is known as Metacognitive knowledge.

It includes three types of knowledge objective namely : i) Strategic knowledge ii)Knowledge about cognitive tasks including appropriate contextual and conditional knowledge and iii) Self knowledge.

i) Strategic knowledge helps the learner a) to retain the information / Content b) to understand the inner meaning of the content c) to develop divergent thinking on the learnt content etc.

The strategic knowledge can be expressed in three ways a) through Rehearsal b) through elaboration and c) through organizational ways.

In the Rehearsal procedure repeation with earnestness is essential. This is not applicable in highes studies/education.

ii) In the elaboration way, the learners tries to go into the deep of the content by abstracting the content or thinking is details. This helps the learner to graph the content is details. In Mathematics, This is very helpful for creating a deep understanding.

iii) Through organisational procedure the learner tries to a) create concept map of the Content....b) tries to improve the cognitive structure (schemata) by incorporating the new content with his/her old concept. c) tries to rearrange the content pattern. He realises 'Product or multiplication is the shortest process of addition as division the shortest process of subdivissions. Verbal statement of mathematics can be organised as algebraic equation etc.

ii) Knowledge about cognitive tasks including appropriate contextual and conditional knowledge This dimension of objective helps the learner to select the appropriate steps for solving as understanding the problem. For example, the students put 'x' in place of what is to be find out? sometimes, we select a statement keeping a parameter fixed. When temperature increases, the volume of a gas is also increases keeping the presure fixed or unchanged.

In solving a mathematical problem, the student must reached, what steps to be followed? Why this is appropriate? What is the limitation of this process? etc

iii) Self knowledge :

To acquire Metacognitive knowledge is the highest order is dimension. In the dimention the learner will be able to reallise his own strength and weakness of knowledg. This will increase his self confidence at the sametime he will be eager to overcome the weaknesses. This self knowledge will initiate his creativity and the power of selection of might person, might direction and to think over right person for his/her guidance. To enhance this qualities, the students must invite relevent person.

2.5.4.6 Objectives in the cognitive process Dimention of Modified Bloom's Taxonomy.

There are six domains in the cognitive process dimension

2.5.4.6.1 Remember

It means to recall the exact facts from long term memory. It includes :

a) Recognizing

From the following geometrical figures recognise the circle.

$\sqrt{\ }$ is the symbol for...

$\sqrt[3]{\ }$ is the symbol far...

Which figure is an example for collinear

Fig 1

Fig 2

b) Recalling

i) What is prime number?

ii) In the given matmix, which are the row and column elements

$$\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

2.5.4.6.2 Understanding

The students will be able to understand the concept of inequation, as any mathematical term with the charls! model etc. It includes the following

a) Classifying

Mathematical concepts can be classified by the learner exampls. Triangles can be classified with respect to sides as angles.

Math the named the polygons and sum of the measures of interior angles.

Quadilateral 720^0

Hexagon 360^0

Octagon 360^0

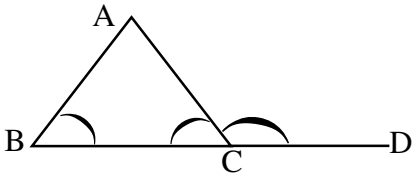
b) Interpreting

- Students will be able to interpret verbal statement into algebraric equation
- A geometrical statement can be interpreted by a geometrical diagram.
- Students will be able to draw the concept map releating to a mathematical problem.

c) Examplefying

Students will be able to give examples of different mathematical concepts.

- From the figure, identify the relation between the angles DCA, ABC, BAC



Observe the pattern and fill in the blanks

$1^2 = 1$

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234$ - - - -

$11111^2 = 12345$ - - - -

d) Summarizing

If $(a + b)^2 = a^2 + 2ab + b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Then $(a + b)^4 = 21$

What will be the roots of a quadratic equation

$ax^2 + bx + c = 0$

$x = -\dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

e) Inferring

Understanding the mathematical relation, a student will be able to inter the result

Observe the pattern in the following and fill in the blanks

$2^2 = 4 = 1 + 2 + 1$

$3^2 = 9 = 1 + 2 + 3 + 2 + 1$

$4^2 = 16 = 1 + 2 + 3 + 4 +$ - - -

$5^2 =$ —— = - - - - - - -

$6^2 =$ —— = - - - - - - -

Write the two numbers in the space provided between which the square roots of the following numbers lie.

$\boxed{3} \angle \sqrt{11} \angle \boxed{4}$

$\square \angle \sqrt{53} \angle \square$

$\square \angle \sqrt{18} \angle \square$

f) Comparing

- Students will be able to use Venn diagram for comparing real and imaginary numbers.
- Compare the reciprocals of realnumber and that of a fruction.
- Compare the areas of different circles with that of their is squere of radius.

$$\frac{\Pi 4^2}{4^2} = \frac{\Pi 5^2}{5^2} = \frac{\Pi 6^2}{6^2} =$$

g) Explaining

Students will be able to explain the cause and effect of any mathematical operation.

Explain with examples why the square of a real number increases but this square of a fraction as decimal numbers decreases Than the original number

$5^2 > 5$; but $(0.5)^2 < 0.5$; $\left(\frac{1}{5}\right)^2 < \frac{1}{5}$

$6^2 > 6$; but $(0.6)^2 < 0.6$; $\left(\frac{1}{6}\right)^2 < \frac{1}{6}$

2.5.4.6.3 Analysing

Students will be able to break a mathematical ideas into its components and to analyse the parts with the whole concepts. This objective will include.

a) Differentialing

- What is the different between ratio and proportion?
- What is the difference between Histogram and frequency polygon?

b) Organising

Students will be able to organise different mathematical activities for realising the inner concepts.

- How a mathematical tool box can be used to clear the idea of speed and time?

- By organising a 'Mathematical quest competition' the teacher can enhance the reasoning power of students.

c) Attributing

Students will be able to be motivated from the contribution of great mathematicians, or history of mathematics.

- During Vedic period (3000 B.C to 1000 B.C), at one place of samhita we see the relationship $39^2 = 36^2 + 25^2$.

  Can you mention this is a particular case of which geometrical theorem? An : Pythagoras theorem.

  This will motivate the students to be acquainted with ancient Indian Mathematics.

- A comprehension test in Mathematics may be given to the students.

2.5.4.6.4 Evaluating

Students will be able to evaluate the performance for giving judgement value. This can be done by the following ways.

a) Checking

After completing or solving a mathematical problem, The students must check different steps or realise by reasoning the correctness of the solutions.

- What is the value of $(999)^2$?

The student will check whether the result is below $(1000)^2 = 1000000$ and above $(900)^2 = 810000$.

b) Critiquing

Students will be able to criticise the pros and cons as suitability of a process for solving a complex mathematical problem.

Example : a) For solving an arithematical problem whether algebraic equation formation is more suitable that can be justified.

b) That there is a wide difference between astronomy and astrology that can be established by the learner.

2.5.4.6.5 Creating

This objective is the highest order in cognitive dimension of Bloom's Taxonomy. This is the manifestation of mathematical creativity. Here the learner will be able to reconcile the small concepts into a general or new mathematical concepts. The graphical representation of linear equation includes the concepts of straight line, nature of straight line, the locus of a point, the co-ordinates of different points which lie on the st-raight line etc. It comprises of the following sub themes.

a) Planning

- For enhancing the creativity the students will be able to submit a project proposal say for using multimedia is learning Mathematics or depecting concept maps for different mathematical concepts.
- To fulfill the self inquiritiveness student may submit a proposal for showing the contribution of Sir Asutosh Mukherjee on Mathematics.

b) Producing

The students will be able to produce new or novel mehtod / thought provocating activities etc.

- Alternative ways of solving a mathematical problem.
- Will be able to produce self learning materials for slow or advance learner.

c) Generating

- Students will be able to generate alternative hypothesis for judgement of objectives.
- Students will be able to generate the self evaluating mechanism etc.

2.5.4.6 Achievement Test

a) The next step of pedagogical analysis is the construction of Achievement Test. For time saving combined Taxonomic Table is to the prepared is which both the necessary objectives for a particular Unit along with number of questions together with total marks will be defected.

Cognitive dimension Knowled dimension	Remember	Understand	Apply	Analyze Total marks for Analyze	Create	Total	% of marks
Fractual Knowledg (F.K.)	No. of questions (total marks)					Total Marks for PK	
Conceptual Knowledge (C K)						Total Marks for CK	
Procedural Knowledge (PK)						Total Marks for PK	
Metacognitived know MK			No. of question (total marks)			Total Mark for MK	
Total	Total Marks for remember	Total Marks has Understand	Total marks for Apply		Total marks for create	Grand total	
% of marks							100%

All the objectives may not be suitable for questions is a topic. Find out the number of questions suitable for each type of objectives and decide on marks for each question.

Now in your combined Taxonomic Table, in each cell, fit in the number of questions and put the total marks for those questions is brackets.

The questions you will give must correspond to the objectives you have written. An objective may also account for more than one question. write the total marks for each categogy of cognitive dimension and knowledge dimention is the extra column and row :

Similarly percentages of mark is each column and row are to be noted

b) Write items (questions or items) to match the information is each cell of the Taxonomic Table.

Beside each questions write the objective number from which it arises. Also write the answer and marks for the question.

c) Make the final version of the Test so that it can be presentable to the students as a complete question paper.

The items should be suitabley arranged eg, easy to difficult, same type of questions put togethers etc.

Instruction is to be given at the top of the such question popers mentioning the class, with time, total marks etc.

d) Probing questions with answers

2-3 questions are to be set with answer as brainstorming questions.

2.5.4.7 Pedagogical Analysis following Revised Bloom's Taxonomy.

Subject Mathematics (Geometry) Class X

Unit Theorems related to circle and angles.

Unit analysis

	Subjects	**Period**
1.	The Pedagogical analysis on Sub Unit I is made. The angle subtended by ass are at the centre is double the angle subtended by it at any point on the remaining part of the circle	1
2.	The angle in a semi-circle is a right angle and its converse	1
3.	Angles in the same segment of a circle are equal	1
4.	The sum of the either pair of the opposite angles of a cyclic queadri lateral is 180^0	1
5.	If a pair of opposite angles of a queadrilateral is supplementary, then the quadrilageral is cyclic.	1
6.	A remedial test on the achievement of above five theorems.	1
Previous knowledge :		6

The following are the necessary background knowledge far the therem

i) Students can define arch, circumference, centre etc

ii) They can pointout the angle subtendead by an are at the centre and on the circumstence

iii) They know the measure of an external angle is equal to sum of two opposite interrior angles.

iv) They are able to draw geometrical figures with the help of geometrical instruments.

To Test the necessary background knowledge the teacher may ask the following questions even by drawing figures.

a) Draw a circle, name an arch, pointout the angle subtended by the each at the centre.

b) Draw any angle subtended by the arch at any point on the remaining part of the circle.

c) What is the relation between extension angle and interior opposite angles of a triangle.

Objectives :

After the completion of this subunit, the students will be able to acquire the following objective both is cognitive domains as well as is the know ledge domain.

1. Remembering

1.1 (Fractual) The students will be able to define arch and angle subtended at the centre.

1.2 (Conceptual) That an arch can subtend many angles on the circumference of a circle.

1.3 (Procedural) Which instruments are essential for chawing circle and arch?

2. Understanding

2.1 (Fractual) The students will understand that there is relation between two angles subtended by an arch.

2.2 (Conceptual) Students will be able to state the theorem.

3. Applying

3.1 (Fractual) Students will be able to verify the theorem with the help of models

3.2 (Conceptual)

Students will be in a position to apply the theorem in solving relevent riders.

3.3 (Procedural) Students will be able to prove the theorem by drawing different figures

3.4 (Metacognition) Students will be able to apply the theorem is solving critical relevant problems.

4. Analyzing

4.1 (Fractual)

Students will be able to analyse the relationship of this theorem with others theorems of subtopics.

4.2 (Conceptual)

The expected out come of the theorem will help the students to form the mathematical concepts relating to subtended angles at the centre and on the circumsference.

4.3 (Procedural) The students will be able to analyse the necessary and sufficients conditions for proofing the theorem.

5. Creating

5.1 (Fractual) Students will be able to draw different figures by changing the arch. They will be able to prepare model on the theorem.

5.2 (Conceptual) They will be able to create different problems by changing the arch and the case when the arch will be semi circle.

5.3 (Procedural)
They will be able to proof the theorem by drawing different figures (like Pig 1, 2 & 3) etc.

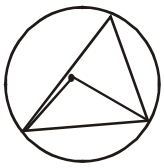


Fig 1

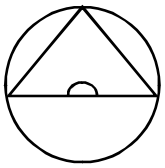


Fig 2

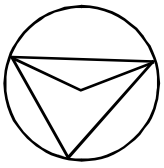


Fig 3

5.4 (Metacognition)

Students will be able to frame different probing questions to express their mathematical creativity.

Affective domain :

a. Interest

1. Students will show their interest in proofing such geometrical theorem.

2. They will be interested to develop their logical argument.

b. Attitude.

1. Students will generate possitive attitude towords Geometry
2. They will develop the logical and sequential argument is establishing a geometrical theorem.

c. Appreciation.

1. They will appreciate the necessary and sufficient argument infavour of proofing a theorem.
2. They will appraise the contribution of great mathematicians

d. Habit

Students will form the habit is establishing a fact by logical way.

Psycho motor Domain :

Students will be able to use geometrical instruments in drawing accurate figures with speed.

Teaching Strategy :

Module related to math concept	**Teaching Method**	**Teaching aid**
1. Verbal statement to mathematical statement of the theorem	Question-answer method	Geometrical instrument C
2. Drawing figure and necessary constructions	Analytic method	o A B
3. Verification with model	Experimentation	D Showing model.
4. Logical Proof	Problem solving Relation between $\angle AOD = OAC +$ $OCA = 2 \quad OCA$ $BOD = 2 \quad OCB$ $\therefore \quad AOB = 2 \quad ACB$	Colour Chalk, Black board, model showing the relationship

Probing questions with answer

1. How will you prove angle subtended by a semicircle is the greatest angle.

   Ans. $\angle MOP = 2 \quad MNP$

   $2 \quad MNP = 180^0$

   $\therefore \quad MNP = 90^0 = I$ right angle.

   N

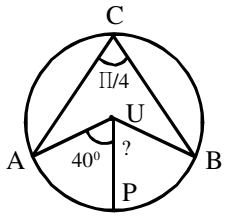
   M O P

2. O is the centre. The angle subtended by the are AOB at the circumference is . The angle $\angle AOP$ is 40^0, what is the measure of POB ?

   Ans. : $ACB = \quad = 45^0$

   $\angle AOB = 2 \times 45^0 = 90^0$

   $AOP = 40^0 \ \therefore \quad BOP = 90^0 - 40^0 = 50^0$



Combined Taxonomic Table

Cognitive dimension / Knowledge dimension	Remember	Understanding	Apply	Analyze	Create	Total	% of Mark]
Fractual Knowledge (F K)	(Q 1)^3 (Q 2)^2						20%
Conceptual (CK)		(Q 4)^2	(Q 6)^2	(Q 7)^2			24%
Procedual (PK)		(Q 9)^3	(Q 3)^2	(Q 5)^3			32%
Metacognitive Knowledge (MK)				(Q 8)^3	(Q 10)^3		24%
Total	Total marks for Remember 5	Total marks for understand 5	Total marks for apply 4	Total marks for Analyzes	Total marks for create 3	25	
% of mark	20%	20%	10%	32%	12%		100%

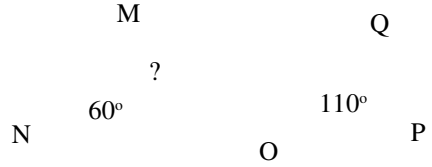
Number within the bracket indicates number of question and number outside the bracket indicates value of the question.

Question Paper

Subject : Mathematics
Subject : Relation between the angles subtended by an are at the centre and at circumference.

Class X,
F.M = 25,
Time - I hour

1. **Fill up the gaps :**

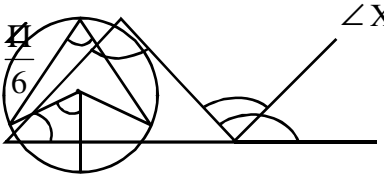
(i) 

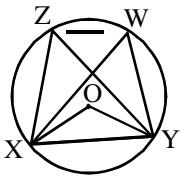
NMO = _____° 1×3=3

(ii) MOQ = ? When NM 11 OQ.

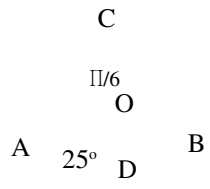
(iii) Largest arc of a circle is _____

2. Define arc of a circle. 2
3. Draw a circle and mention angles subtended by an are at the centre and at the circumsference of the cirel 2
4. In the adjacent figure find the ratio of two angles ∠XZP and XWY.

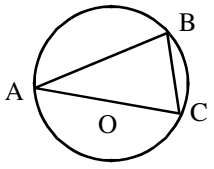




5. Prove that the angle subtended by an are at the centre is double by the angle subtended by the are at the circumference. 3
6. In the adjacent figure ACB = O, ∠AOD = 25°, Final BOD. 2



7. In the adjacent figure AC is the diameter. Prove That ABC : AOC = 1 : 2 2
8. Prove that the angle subtended by an are which is not a semicircle at the circumference is an acquile angle. 3
9. O is the Centre of the circumcircle of the triangle ABC. If BOC is 110°, find the valuable BAC. 3



10. Prepare a teaching aid to verify the theorem. 3

Unit 3 ❑ Strategies for Learning and Teaching Mathematics

Structure

3.1 Concept Formation and Concept Attainment

3.1.1 Introduction

3.1.2 Meaning of Concept

3.1.3 Characteristics of Concept

3.1.4 Steps of teaching concepts

3.1.5 Concepts Attainment Model for Learning and Teaching of concept.

3.1.5.1 Strategy of concept Attainment Model. Bruners as mentioned four strategies of the Model namely

3.1.5.2 Types of concept attainment model.

3.1.5.3 The Reception model of concept attainment.

3.1 Concept Formation and Concept Attainment

3.1.1 Introduction

The Pioneer of 'concept attainment' is Georm. S. Bruner and his associates Jackue line Goodnow and George Austin. The concept is developed from their research 'A study of Thinking' (1967) the basic considerations of concept Attainment model are :

i) Man posses the ability of discrimination and classification of things, persons in groups.

ii) For adjustment with the environment, a person encounter varried situations and experiences. It is impossible for a person to restore such huge experiences. So he tries to assimilate such experiences and try to adjust with the environment with these assimilated experiences. In general, the assimilation of such experiences is called concept.

iii) A concept has three elements a) example b) attributes and c) attribute values. For example, triangle is concept then each triangle is an example. Here rectangle and pentagon are negative and equilateral triangle is positive.

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1st Proof SPW 111 08.07.2016

Triangles have all the sides are equal is the attribute and all angles are also equal be the attribute values.

iv) The categorizing activity has two components (a) The act of category formation (concept formation) and (b) The act of concept attainment. The concept formation is the first step for concept attainment.

3.1.2 Meaning of Concept

According to Bruner ''A concept is a class as grouping response, an act of categorization, involves rendering different Things equivalent.''

Archer (1969) mentioned ''A concept is simply the level of a set of things that has something is common'' A concept is different from a fact, a principle and even a generatization. Halse states 'A concept is a set of feature connected by some rules. A concept of quadrilatiral is a geometrical figure bounded by four straight lines selvens (1993) has cleared the idea is the following ways ''A concept consists of an individual's organised information about one or more things, objects, ideas, or relations that enable the individual to discriminate a particular thing or class of things and as classes of things.

Thus concept is a class or category of all the members of which share a particular combination of attributes or critical properties not shared by another class.

3.1.3 Characteristics of Concept

Knowledge of common feature

i) It is the knowledge of common feature of person, object, animal or circumstances of similar characteritics

Characteristics observational experiences

ii) It is observational experiences. Greater the extend of observtional area greater will be the concept area. In framing the concept of triangle the barner must pay attention minutily to the length of different sides of a triangle.

iii) Individual differences

In concept formation individual differences play active role. Different persons form mathematical concepts in their own ways to some extend.

iv) Experience of object

At the initial stage a child forms the concept based on the experiences of real

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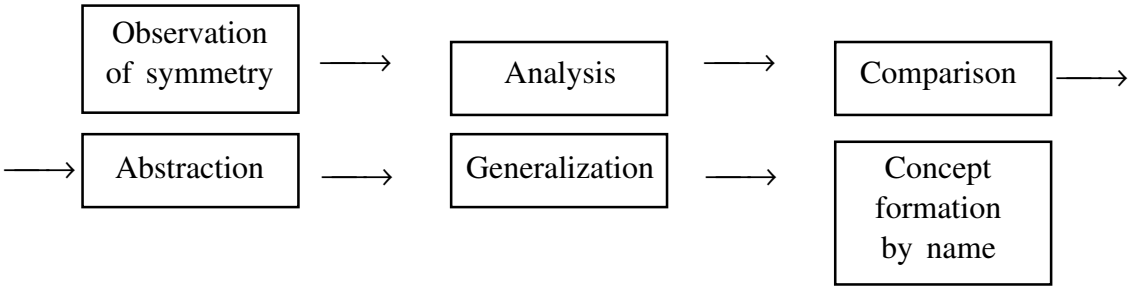
object. At the later stage due to development, he forms the concept is absence of real objects. At the begining, a child forms the concept of circle as triangle by observing circular as trianglular objects but later stage by imagination only.

v) Complex process : Mathematical

Concept formation is a complex process. If follows the following route.



vi) Concept formation may be of three types

a) Conjunctive (Combination of different traits, Geometrical figures included may type of figures)

b) Disjunctive (A concept may be subdivided into many sub concepts. Fractions may be subdivided is to proper and improper or deciwal etc)

c) Relational concept (When there is/are relation among concepts, (Rectangle, Trapezium, Rhombus, Square

vii) Conceptual Hierarchies.

Each concept is not found in isolation, but rather is relation to other concepts. Simth describes this process of forming new concepts as one of breaking down old categories into smaller and more specialized one. One way of describing the relationship of concepts formed is in terms of super ordinate, co-ordinate and sub-ordinate concepts. These terms refers not only to the scope or inclusiveness of a concept but also to its relationship to other concepts. For example, quadrilateral is more inclusive and also subsumes the concept of parallelogram which in term subsumes the concept of rectangle. In this respect rectangle is described as a sub-ordinate concept of quadrilateral. Related concepts such as parallelograms and trapezium form the co-ordinate concepts. In the conceptual hierarchy of quadrilaterals, parallelogram forms the super-ordinate concept to square.

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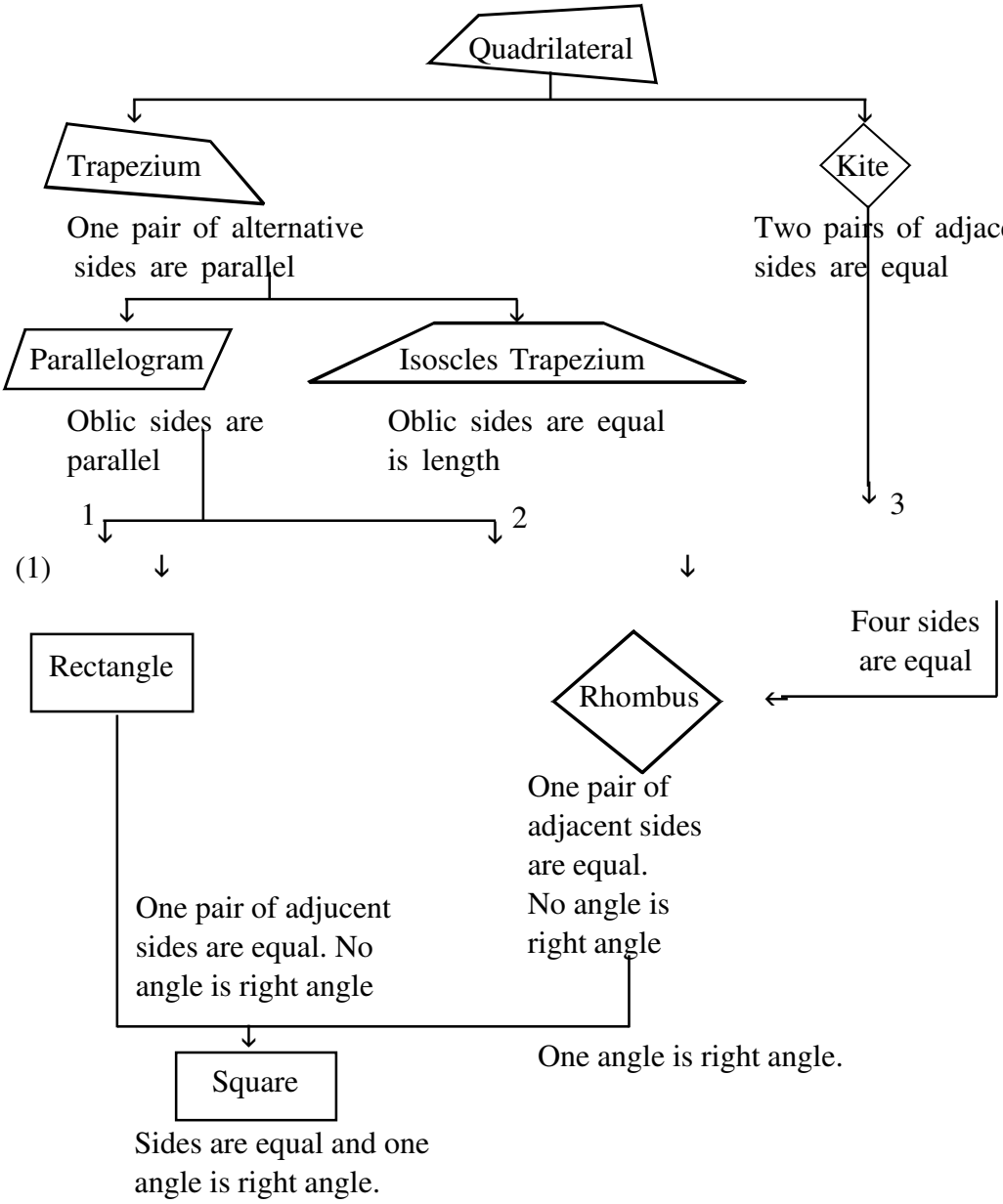
Conceptual Hierarchy of Quadrilaterals



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VIII) Concept Name

Every concept have a name. The concept name is the word used to symbolize the given concept. A concept is an idea or obstraction that exists is people's mind, while the concept name is the word that we arbitrarily use to designate the concept. For example, parallelogram is a concept name designated to a quadrilateral is which both pairs of opposite sides are equal. There are two ways which the concept name is attached to a concept are

a) The child learns the concept first and later learns to attach the name to the concept

b) The child learns the symbol and then learns the concept

IX) **Concept definition**

The deffinition of a concept means to describe typically is verbal statement, the meaning of the concept. If focuses the summarization of important aspects of the experiences which ignoring others. It describes the boundaries of a concept. It helps to determine set inclusion and set exclusion. For example, when we define a quadrilateral as four sided closed geometrical figure, it means that all four sides closed geometrical figures like square, trapezium, parallelogram, rectangle etc can be included is the set of quadrilaterals. But others geometrical figures which are not closed and which do not have four sides can not be included is the set of quadrilaterals.

X) **Process of observation.**

Through the process of observation, the essential characteristics i,e the attributes are identified. Identifying the attributes of a concept is essential becauses without the knowledge of the attributes of a concept, the process of understanding the concept is difficult. For example, the characteristics or attributes of the concept square are four sides which are equal is length and angles are right angle each. These Characteristics are important or essential set where as characteristics such us size colour or special orientation are not important is the set of square.

XI) **Example in a concept :**

Examples of a concept are those members of a class which are positive instances of the concept. Examples can appear is word form, is pictorial form or is real life form.

XII) **Concept formation**

Concept formation takes place when members of a category are groued togethers

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and similarities are acceplied which ignoring the differences among the members of the categories. Considering the similarities a rule is formed and conceptualization takes place.

For example, $-\frac{7}{3}, -\frac{5}{3}, -\frac{1}{2}, 0, \frac{1}{3}, \frac{7}{4}$ etc are the examples of a concept rational members. Rational numbers are defined as numbers expressed is the form of p/q, where p and q are integers and p and q do not have common factors and $q \neq o$.

3.1.4 Steps of teaching concepts

Please add page 592 to 598 (Content cum Methodology of Teaching Math B.Ed MC - 06/07 (09) NSOV.

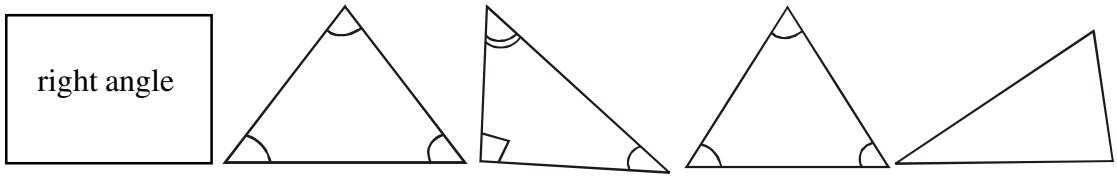
3.1.5 Concepts Attainment Model for Learning and Teaching of concept.

The concept learning includes both the phases i,e concept formation and concept attainment.

3.1.5.1 Strategy of concept Attainment Model. Bruners as mentioned four strategies of the Model namely

i) Simultaneous Scanning Strategy

In this strategy for a particular concept formation, the learners will be given a branch of cards is which different attributes and attribute values are mentioned. The learner will select the cards having positive attributes as well as positive attribute values and will reject which are negative attributes. Concept of Isosoul right angle triangle.



Card No 1 Card No. 2 Card No. 3 Card No 4 Card No 5

For right angle triangle he will select card No 3 and Card No. 4 In card No. 3, one angle is right angle but the others two angles are unequal. So considering the attribute value he will reject So considering the attribute value he will reject card No 3 and will select card No. 4.

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But he will reject card No. 1 as it is not triangle. He will reject Card No. 2 as it is equilatiral triangle and also reject card No 5 as it is not right angle triangle.

ii) **Successive scanning strategy**

Here the learner only select the cards having. Positive attributes successively. In this straligy the learner must know the necessary and sufficient positive attributes of the concept. Though he mentally rejects the negative attributes oriented cards

iii) **Conservative Focusing Strategy :**

Here every card will contain only one attribute of positive or negative nature. Considering the positive instance he will reject all the negative all the negative instances or non examples. By showing the non example cards he will reject those with classification.

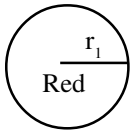

Fig 1

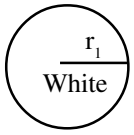

Fig 2

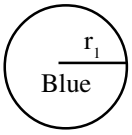

Fig 3

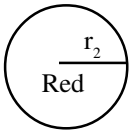

Fig 4 $r_2 > r_1$

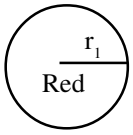

Fig 5

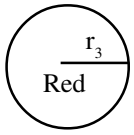

Fig 6 $r_3 > r_1$

Student is asked to pick up red circulrs dise having less diameter. He will reject circular dise No 1 to 5 by showing reasons and will select dise No 6 as positive instance.

iv) Focus Gambling - Here is each card all the attributes (positive example and negative attribute/non examples) are mentions and in only one and only the positive attributes are mentioned.

Students will be asked to pick up the right and as quickly as possible. Here like gambing chance factors will be effective. If adequate time is given, students will be able to born the right concept of circular dise.

3.1.5.2 Types of concept attainment model.

On the basis of research work done by Bruners and his associates several models of teaching have been developed. The concept attainment model has three variations.

i) The reception model

ii) The Selection model

iii) The model for unorganized material.

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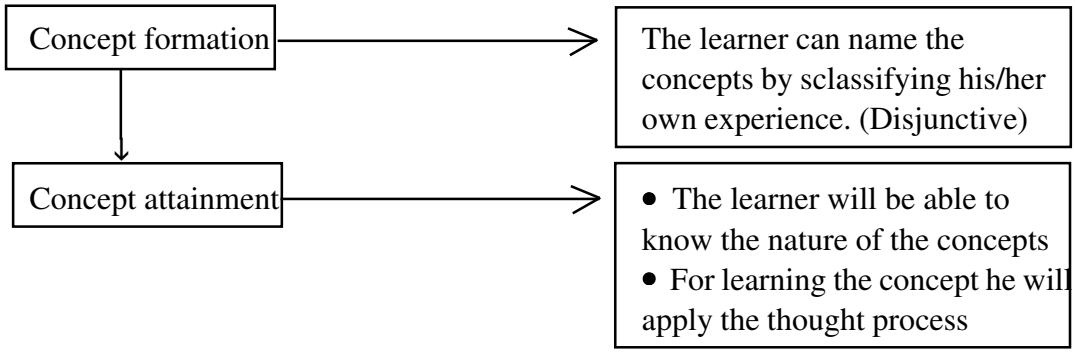
3.1.5.3 The Reception model of concept attainment.

A. Focus : Here weightage is given on the development of logical powers. Training is given how conclusion can be drawn by inductive reasoning through use of examples. The focus of the model is to form conception through the stages a) observation b) analysis c) comparison d) abstraction e) generalization and finally naming. There fore, in addition to help the students is the attainment of a particular concept, it enables them to become aware of the process of conceptualizing.



B. Syntax.

The sequence of thes phase and activities are given in the table.

Phase one.	Presentation of data and identification	**Activities** i) Presenting examples with 'yes' or 'no' lables by the teacher is a pre-ar ranged orders ii) Ask the students to compairs at tributes in positive and negative examples iii) Students will frame hypothesis and compare with teacher's hypothesis

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		iv) Naming the concept by students v) Will state the rule as definition of the concept according to needed attributes.
Phase Two	Testing for attainment of inner concept	a) Students will frame hypothesis for identifying the similaristics and dissimi larities of the facts. b) Considering each hypothesis, at tributes will be judge. c) By analysis and synthesis of the attributes they will find out the inner concepts
Phase 3	General Statement about the concept	d) Will be able to generate own examples. i) Teacher will give more information but will not mention the concept like before. ii) Teacher will ask to identify which information attribute is related to which concept iii) If the students role is statisfactory he will ask to give more examples iv) Teacher will give general statement about the concept
Phase 4	You aware about the process of concept formation and thinking strategy	a) Students will be asked to analyse and discuss the process by which They have attained the concept. b) Students will discuss the role of hypothesns and attributes. c) Students will evaluate the process concept attainment. d) They will be encouraged to follow the process for obtaining experiences by this way.

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C. Principle of reactions :

The important principles of reaction are the following

i) The teacher will be supportive to the student's hypothetical in nature.

ii) He will help the students to distinguish between two hypothesis and to evaluate the thinking process by themselves.

iii) He has to maintain record about the focus attention on specific feature of examples.

iv) He is to encourage the students to enalysis the merits of various strategies.

D. Social System

In this model, in most part of the teaching, the teacher has to exercise control over the social system. The teacher has to collect information as well as examples has a fixed up conception. He has to divide the experiences into positive and negative attributes for the concept. He will communicate the students that the solution of the problem of identifying concepts lies not within the teacher but in the data (examples). The teacher gradually relax the controll and encourage the students to work independently and collaborately. The functions of teacher will include a) to record the activities b) to give clue for collecting attributes c) and to give extra information is the form of examples. The social system functions on the mutual interaction between teacher and taught. For concept attainment, the teacher gives the possitive and negative examples or attributes to the students.

The students explain the examples and justify his hypothesis on the basis of positive attributes The interactions are noted on the black board as on the Tag board.

The concept is deduced and rule or definition is framed.

E. Support systems

In this model students will not discover a new concept but will deduce the concept on the basis of examples/experiences/attributes put forward by the teachers.

In the first stage, keeping is mind the appropriateconcept, the teacher will give different information or examples to the students. This will include both positive and negative attributes which are essential. By discriminating the positive i,e relevant attributes, the concept is formed. By giving more examples by the students, the teachers from the definition of the concept so is the concept attainment model, the support system is the relevant information is the form of examples.

F. Application Concept

Ioyce and will are of the opinion that the concept attainment model is an excellent evaluation tool when teachers want to determine whether important ideas introduced

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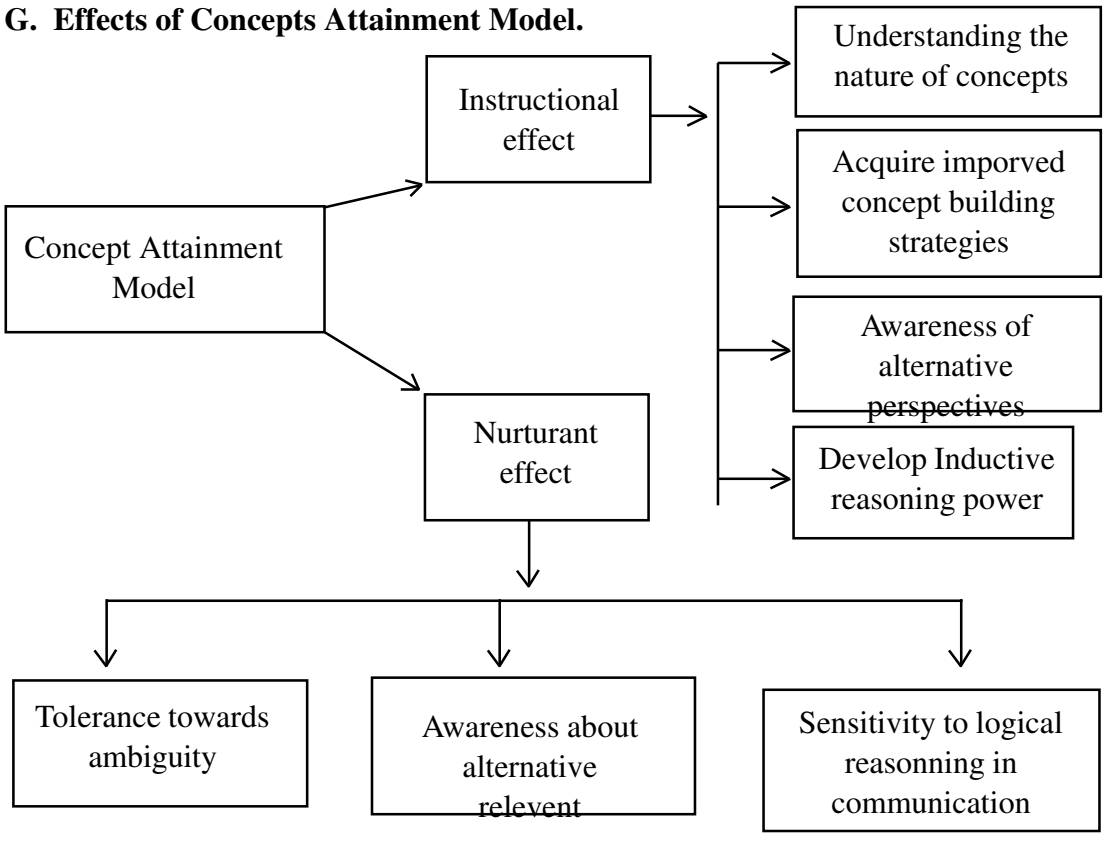
earlier have been mastered. It quickly reveals the depth of students understanding and reinforces their previous knowledge'.

Mathematics contains a good number of concepts inter related to each other. The model includes both concept formation and concept attainment. In most cases concept is formedby inducting reasoning and follow the process of generalisation and discrimination.

1) So the model is effective is teaching mathematical concepts, grammer and language.
2) For the introduction of this model in class-room teaching, the students may be divided in to small groups so that each of them can participate in the discussion.
3) Before teaching following the model, the teacher may explain the different phases examples (both positive and negative) for a concept.
4) For each group, there will be one leaders who will controll the group discussion, but the teacher will supervise all the group activities.

G. Effects of Concepts Attainment Model.



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3.1.5.4 Adventages/Merits of concept Attainment Model

i) Normal environment of class teaching-learning process is maintained.

ii) Reasoning powerspecially inducting reasoning power of students is developed

iii) In this model the power of observation and imagination of the students are increased

iv) In this model the students can apply his/her knowledge is real life is tuation.

v) Students get the opportunity to justify their own hypothesis by judging the experiences and drawing conclusions.

3.1.5.5 Disadvantages

The model demands the participation of all students and teacher in the teaching learning process. But a good number of students generally remain pasive. So the introvert and under achiever students are not benefited is the model of teaching.

Limitation of concept attainment model

i) To follow the model both the teacher and students are to take enough responsibility

ii) There are individual differences among the learners. So the introvert type students do not participate is this model of teaching

iii) All the contents of the syllabus can not be taught using this model.

But the model has revolusionised the face to face teaching process.

3.2 Learning By Exposition : Advance Organizer's Model

3.2.1 Introduction

There are two terms 'Models of teaching' and 'teaching models' used is teaching learning process. 'Teaching models' are just instructional design where as 'models of teaching' consists of guidelins for designing educational activities and environment. It also specifies ways of teaching and learning for fulfilling the intended goal of teaching. So the function of models of teaching is depected below.

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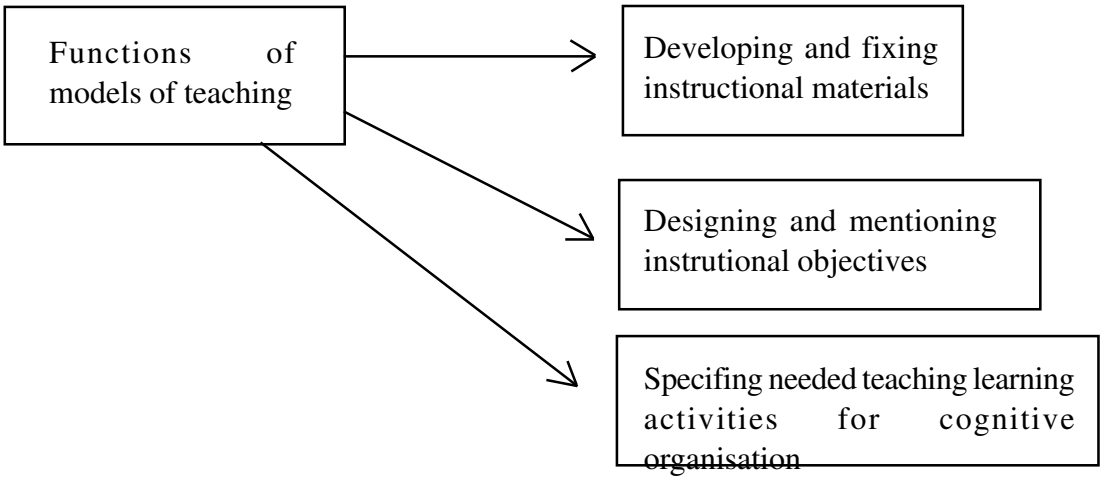

It has been accepted that Mathematics should be taught in such a way that each learner is trained to think, reason, analyse and articulate logically. As per as possible the discovery approach followed by teaching through designing and planning suitable aids and models.

The fundamentals essential for learning mathematics and solving of problems of daily life should be borne is mind while teaching one should aim to learn the concepts at the mastery level. Activity oriented programme should be used in teaching Mathematics. The most essential thing is that joy and achievement should prevail while learning Mathematics.

3.2.2 Advanced Organizer's Model of Ausubel–David Ausubel designed this model to increase the efficiency of information processing capacities of children.

The ideas of this model have emerged in his book 'Theory of Meaningful Learning' According to him the types of learning material related with the content can be learned by the learner quite effectively if presented through. Visual graphics, charts and picture, film, audio tapes and transparencies.

He is of opinion that for meaningful verbal learning, the teacher must know the following three issues (i) How knowledge (content of curriculum) (i) How knowledge (content of curriculum) is organised - (ii) How the mind works during the process of presenting new information (learning) and c) How the above two can be presented to the students (instruction)

According to A usubel new ideas can be learned or retained only to the extent that those ideas can be related to already available cognitive structure of the learner. That acts as necessary linkage or anchors.

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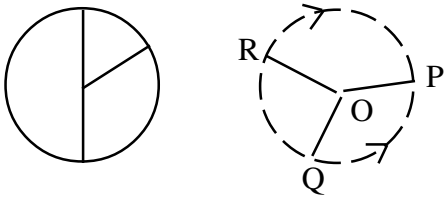
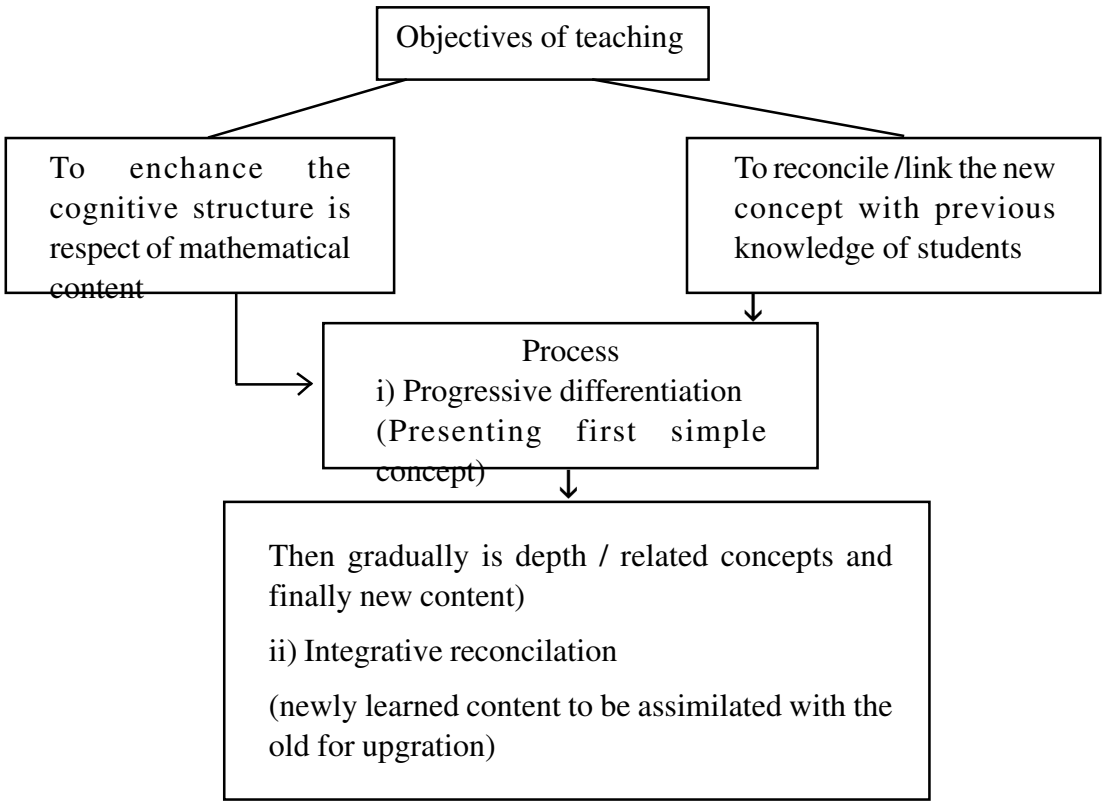
Advance organisers, as Ausubel maintains, are the primary means of strengthening learners's cognitive structure. The advance organiser may be of two types i) expository (which is helpful in providing the basic concepts at the highest leaved of abstraction and essential for understanding new nontent) and ii) Comparative (designed to discriminate between the old and new concepts to present confusion.

For teaching the equation of a circle, expository advance organiser may defferent shaped circular dise and as comparative advance organiser a diagram showing geometrical figure of circle and the locus of points which are equidistance from a fixed point, called its centre.



3.2.3 Before presenting the Advance organisor Model of teaching we must keep in mind the following



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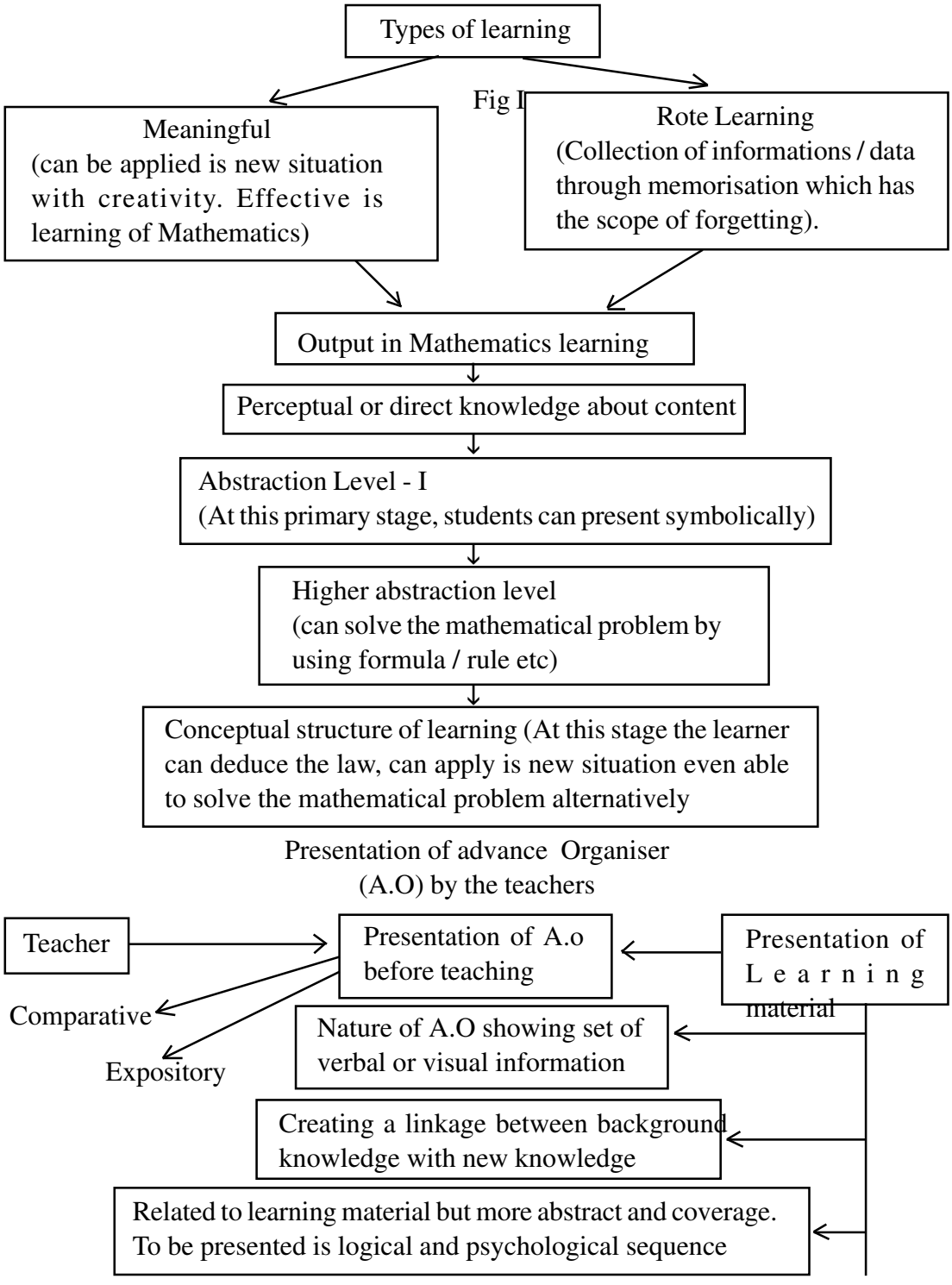

Fig I

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3.2.4 Fundamental elements of Advance organiser model

3.2.4.1 Focus

The model focuses

1. Assisting the teacher for improving the method of presentations for this must know how contacts are organised
2. Organising large amount of content information teacher must realize meaningfully. For this what is the cognitive structure of students in respect of new content
3. To help the learner to strengthen their existing cognitive structure. For this relevant questions are to be asked and necessary information to be provided.

3.2.4.2 Syntex

Activities are to be performed sequentially is the following three phases.

Phase : Presentation of the advance organisers

i) the aim of the lesson is to be classified. By stating the aims and objectives of the lesson, the students will be motivated for acquisition of the presented material

ii) Presentation of the advancne organiser. The teacher will present the advance organiser (expository or comparative) to the students which contain ideas more distinct and inclusive than the learning content for understanding and grasping the information.

ii) Prompting awareness learner's knowledge. The teacher will realise learner's existing cognitive structure interms of their previous knowledge and experience through questioning related to advance organiser.

Phase II presentation of learning task. (Exceutive

i) The teacher will present the content stage) modulewise through appropriate method like discussion, question answer method as problem solving method with necessary teaching aids

ii) Presentation will be at per with existing cognitive structure as well as the information supplied to them through advance organiser.

iii) As mentioned by goyce and will (2003) "The organisation of the learning material needs to be made explicit to the students so that they have an overall sense of direction and can see the logical order of the material and how the organization relates to the advance organiser.

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Phase III Strengthering Cognitive organisation (Follow-up stage

Joyce and will states ‘‘The purpose of phase three is to anchor the new learning mathematics is the student existing cognitive structure, that is to strengthen the students’ cognitive organization’’.

The following activities are performed is this phase

i) Integrative reconciliation

Teacher will relate process of solving solving a math problem taking asimilar problem.

ii) He may ask the students the definition of term, process of solution. He may repeat the concept once again.

Teacher may ask the student the relevance of Advance organizer as the teaching aids with the present topic.

ii) Active reception learning

The teacher may ask students to give additional examples related to the discussion For promoting active reception, the teacher will allow the student to ask question for their reception of facts, while teaching conqruence of a triangle, the teacher may ask ‘Why the triangles having three angles equal are not congruent? He may ask the students to make a teaching aid for understanding the concept.

iii) Eliciting critical approach to content knowledge or the learning material

The teacher may ask the students

a) How the problem as geometrical theorem can be proved is alternative way?

b) Without discussing the concept of probability. The teacher may ask the students to toss a coin 10 times on 20 times and recond the result is respect of head’ on ‘tail’.

c) Contradiction or exception of a rule may be asked to explain like $5 > 3 > 2 > 1$ but $\frac{1}{5} < \frac{1}{3} < \frac{1}{2} < 1$

iv) Classification of confusion or fomce of errors .

In this stage the teacher will apply all the techniques, strategies or methods to clarify the misconception, source of error etc. He may apply error correction test or show some practical demonestration to eradicati mis understanding.

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a) $7 \times 7 = 7^2 > 7$

$5 \times 5 = 5^2 > 5$

but $o.7 \times 0.7 = .49 < 0.7$

or $0.5 \times .05 = .25 < 0.5$

b) $\sqrt{25} = \pm 5$ or $\sqrt{49} = \pm 7$

but $\sqrt{-25} \neq \pm 5$ or $\sqrt{-49} \neq \pm 7$

3.2.4.3 Social System

The social system of the advance organiser model is structured and teacher centered. He relates the advance organiser to the learning material. This acts as an ancher between existing knowledge of the learner to the new knowledge. The social system starts with interaction with the students. Students may ask questions, may improve the advance organiser for their understanding and assimilation of new knowledge. So to devise the advance organiser is a difficult activity which will be appropriate to the students as well as to new content. Moreover, deliberation, discussion and relating the A.O with new learning materials require the efficiencies of the teachers.

3.2.4.4 Support system :

To make the model more effective and meaningful is teaching Mathematics the following support system is essential.

1. A well thought and structured learning material
2. Preparation of a well thought, relavent and appropriate advance organiser is the form of story telling, model, software, activity based performance etc
3. Integrating the advance organiser with learning material and presenting before the students keeping in mind their level of understanding
4. The appropriate competencies, teaching skill.

Professional efficiency and concern for student's preparedness will help the teacher to make the model more effective.

3.2.4.5 Learning output / Effect of Ashubel's Advance organiser Model

The effect is expressed in the following diagram.

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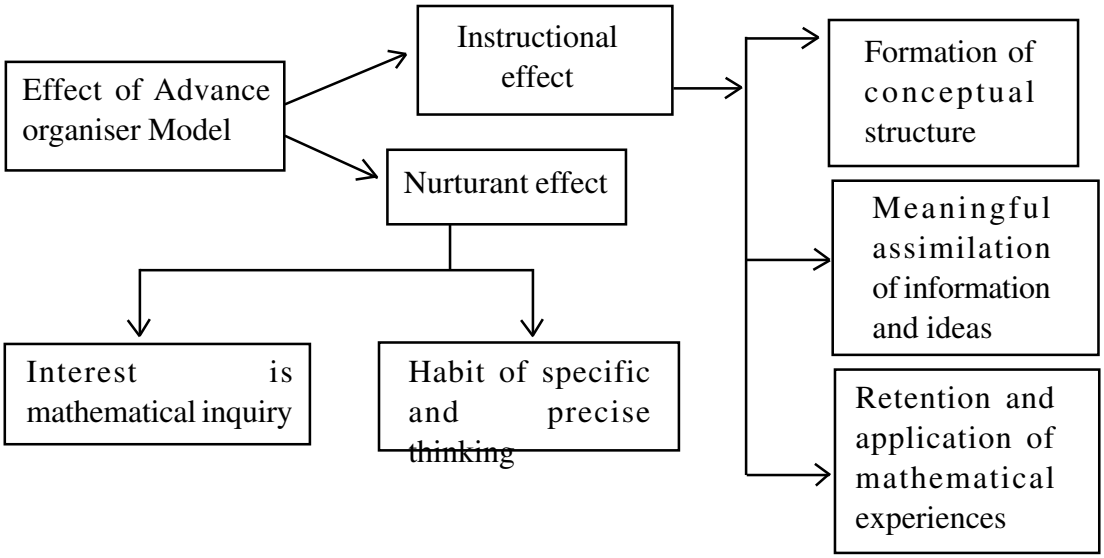

3.2.4.6 Advantages

The wide application area of the model as suggested by Joyce and will (2003) can be stated is teaching Mathematics.

i) This model can be used almost all franches of mathematics systematically in normal classroom situation having mixed ability group.

ii) It can help the teachers to understand the existing necessary background knowledge before presenting the advance organiser

iii) It follows the mechanism of direct instruction and reorgaganise the cognitive structure in though advance organiser.

iv) Though the Advance organiser at the initial stage act as deductive way on subsequent discussion it is subject to inductive concept attainment stage and evaluate the student's acquisition of new knowledge

v) It has both instructional and nurturant effects is different domains.

vi) It helps the student to develop critical thinking and interest is mathematical inquiry.

3.2.4.6 Disadvantages

i) Sometimes it is difficult to prepare relevent advance organiser of mathematical tpoics.

ii) It is different from ordinary teaching aids

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iii) Prior to apply the model, the mathematics curruculum is to be reorganised and teacher must know the interdisciplenary nature of Mathemaics

3.3 Analytic-synthetic, Problem-solving and Project.

3.3.1 Analytic-Synthetic

Introduction

Most Mathematics orginates from the ideas and concepts associated with physical form, shape and size of objects. Those concepts are present as a systematic abstractructure in logico-deductive form Analysis and synthesis are methods which use reasoning and systematic arguments arts to find out relationship Meaning.

The word 'analytic' means to take a part as to seperate the things that are together or 'breaking up' of the problem so that it gets connected with already known. Analysis is a a process of breaking a things into its smaller parts. It proceeds from unknown to known and conclusion to hypotheus. Thorndike says that all the highest intellectual activities of the mind are analysis. 'To analyse' means to loosen or seperate things that are together.

Procedure :

Ex 1. If $\frac{a}{b}=\frac{c}{d}$, Proved that $\frac{ac-2b^2}{b}=\frac{b^2-2bd}{d}$

The analysis will start from the unknown part of the given statement

$\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$ is to be proved $\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$ will be true $acd - 2b^2d = bc^2 - 2b^2d$ (Cross multiplication)

What is the next possibility of further simplification?

$\therefore$ $acd = bc^2$ ('c' can be cancelled on both sides as common)

$\therefore$ ad = b c will be ture

Dividing by b d on both sides we get $\frac{a}{b}=\frac{c}{d}$ $\therefore$ $\frac{a}{b}=\frac{c}{d}$ which is known and true.

$\therefore$ By going back through the chain of argument.

We can say that $\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$ is also true

Example 2. Prove that the sum of the three angles of a triangle is two right angles.

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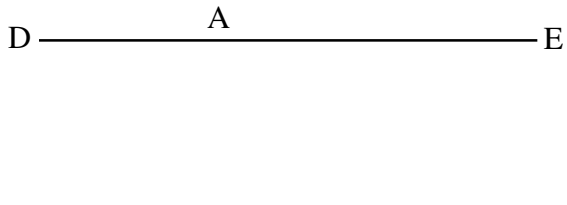
In analysis we start from the conclusion and break it up into simpler arguments for establishing connections with the relationships taken is the hypothesis. For this, we have to find out the missing logical connections and formulate a pattern for the proof.



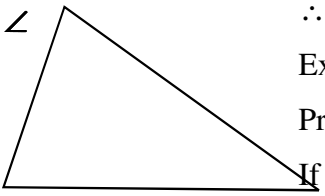
Assuming that the angle sum is $180^{0.}$ a straight angle, then the angle sum of the triangle equals the angle sum on one side of a straight line such as D E.

Again, if D E passes through the vertex A, which is parallel to the base or opposite side. From the properties of the parallel lines, it can be said that the correcponding alternate angles are equal in pair. A line D E is drawn paralled to B C through A.

$\therefore \angle ABC =$ DAB; ACB = EAC

But DAC + BAC + CAE = 180^{o} (alternate angles)

$\therefore$ BAC + ABC + BCA = 180^{o} (straight angles)



Example 3. If $a + b + c = 0$, Then prove that $a^3 + b^3 + c^3 = 3abc$

Proof $a^3 + b^3 + c^3 = 3abc$

If $a^3 + b^3 + c^3 - 3abc = 0$

$(a+b)^3 - 3ab\,(a + b) + c^3 - 3abc = 0$

or $(a +b)^3 + c^3 - 3ab\,(a+b+c) = 0$

$(a + b)^3 + c^3 = 0$ ($\therefore a + b + c = 0$)

$\therefore (a + b)^3 = - c^3$ or, $a + b = - c$

or, $a+b+c = 0$ which is ture.

Advantages of the Analytic method : -

1. It is a logical mehtod and there is no doubts in teaching a content
2. It motivates the learners to discover and improves the level of understanding.
3. If does not depend on cramming. Each step in its procedure has its reason and justification.

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4. In this method students are always guided by the questions like "How to simplify the two sides of an equation?" How to simplify the two sides of an equation? How to prove the equality of two sides? "What are the possible ways of resolving a statement into simpler elements" etc
5. The method is applicable to all types of learners and content of Mathematics.

Disadvantages / Drawbacks

1. It is lengthy method
2. It is difficult to acquire efficiency and speed
3. The method may not be applicable to all topics equally well.
4. This method is not suitable to the begininers as during the process many doubts may arise in the minds of the learners which can not be explained property.

Synthetic Method : Meaning and Procedure Meaning.

In synthesis, the small constituents orparts are combined so as to give something new. Here one proceeds from known to unknown. It proceeds with the data available or known and connects the same with the conclusions. In the process we start with hypothesis to conclusion. In practice, synthesis is the complement of analysis.

Procedure :

Ex-1 If $\frac{a}{b}=\frac{c}{d}$, Proved that $\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$

Synthetic proof :

$\frac{a}{b}=\frac{c}{d}$ (It is known, and hence the standing point)

Subtracting $\frac{2b}{c}$ on both sides. (But the question why? Why and how should the child remember to subtract $\frac{2b}{c}$ and not any other quantity?)

or $\frac{a}{b}-\frac{2b}{c}=\frac{c}{d}-\frac{2b}{c}$

or $\frac{ac-2b^2}{bc}=\frac{c^2-2bd}{cd}$ or $\frac{ac-2b^2}{b}=\frac{bd^2-c^2}{d}$

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Cancelling $\frac{1}{c}$ on both sides)

Hence the identity is proved.

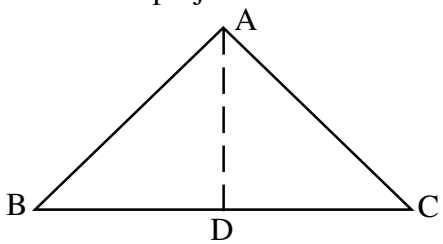
Ex 2. In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing the acute angle minus twice the rectangle containing by one of these sides and the projection upon it.

Proof.

Given ΔABC, acute ∠d at B. BD is the projection of AB on BC.

To prove $Ac^2 = AB^2 + Bc^2$

$-2BC.\ BD$



In ΔACD is a right Angle triangle ADC is not angle $Ac^2 = AD^2 + CD^2$ (But why we have taken this as first step is not clear)

Now $AC^2 = AD^2 + CD^2$ is expanded

We have $Ac^2 = AB^2 - BD^2 + (BC - BD)^2$

$\therefore$ CD = BC – BD

$= AB^2 - BD^2 + BC^2 + BD^2 - 2BC.BD$

$= AB^2 + BC^2 - 2\ BC.\ BD$

(No justification is given for each and every = step)

Ex 3. If a + b + c = 0 Prove that $a^3 + b^3 + c^3 = 3abc$

a + b + c = 0 (It is known) $\therefore$ a + b = – c

$\therefore (a + b)^3 = (-c)^3$ (Cubing both sides)

$\therefore a^3 + b^3 + 3ab\ (a+b) = -c^3$

as $a^3 + b^3 - 3abc = 0$ ($\therefore$ a + b = – c)

$\therefore a^3 + b^3 + c^3 = 3abc$ (proved)

Merits of synthetic method

1) It is a short and elegant method

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2) It is a short and elegant method

3) It is logical and psychological method because it starts from known to unknown

4) It glorifies memory

5) It is applicable to most of the topics

6) It suits both the teacher and students

7) It follows the same process as mentioned is the text books

Demerits of Synthetic Method :

1) It leaves many doubts is the mind of the learner and can not be explained property

2) It does not provide full understanding

3) There is little scope of discovery and thinking in the process

4) Memory work and home work are heavy

5) It is not suitable for learner's full understanding

6) It is not suitable for all students and all the topics of Mathematics

Conclusion :

Synthesis is the complement of of analysis and is teaching of Mathematics, the two methods, the two methds should always go together. Analysis leads to synthesis and synthesis makes the process of teaching learning more clear and complete. Analysis helps in understanding and synthesis helps is retaining knowledge. Analysis forms the beginning and synthesis advances the follow-up work.

3.3.1 A comparative study of Analytic and Synthetic Method.

Analytic Method	Synthetic Method
1) Analysis means breaking up into similar elements	1) Synthesis means building up seperate elements as combination of seperate elements to get something new.
2) It proceeds from inknown to the known facts	
3) It starts from the conclusion and goes to the hypothesis	2) It proceeds from known to the unknown facts

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Analytic Method	Synthetic Method
4) It is a general method	3) It starts with the hypothesis and ends with the conclusion
5) It is a method of discovery and requires thinking	4) It is a special method
6) It is a proceess of thinking (exploration)	5) It is a process of presentation of the previous by discovered facts.
7) It is a lengthy method which involves trial and error and time consuming	6) It is a product of thought
8) It is a method for the thinkers and discoverer	7) It is concise, elegant, straight forward and does not follow trial and error.
9) If question arises then it answers states factorily	8) It is a method for the crammers. No scope for thinking and discovering
10) There are close contacts between the teacher and taught	9) It does not starts by the doubts and question evolved in the mind of learner.
11) The students can recall and recounstruct easily any step if forgotten	10) There are little scope for such intimate
12) It is psychological	11) It is not easy to recall or reconstruct any forgotten step.
13) In this method we apply inductive reasoning	12) It is logical
14) It is formational	13) We apply deductive reasoning
15) Each step is explained with 'Why' and 'how'.	14) It is informational
16) It is based on heuristic lines	15) 'Why' and 'how' are not explained clearly
17) It develops originality	16) There is no heuristic approach is it
18) It is the forerunner of synthesis	17) If develops memory
19) It builds up a scientific attitude and creativity among students	18) It is the follower of analysis.
	19) Little scopes one there.

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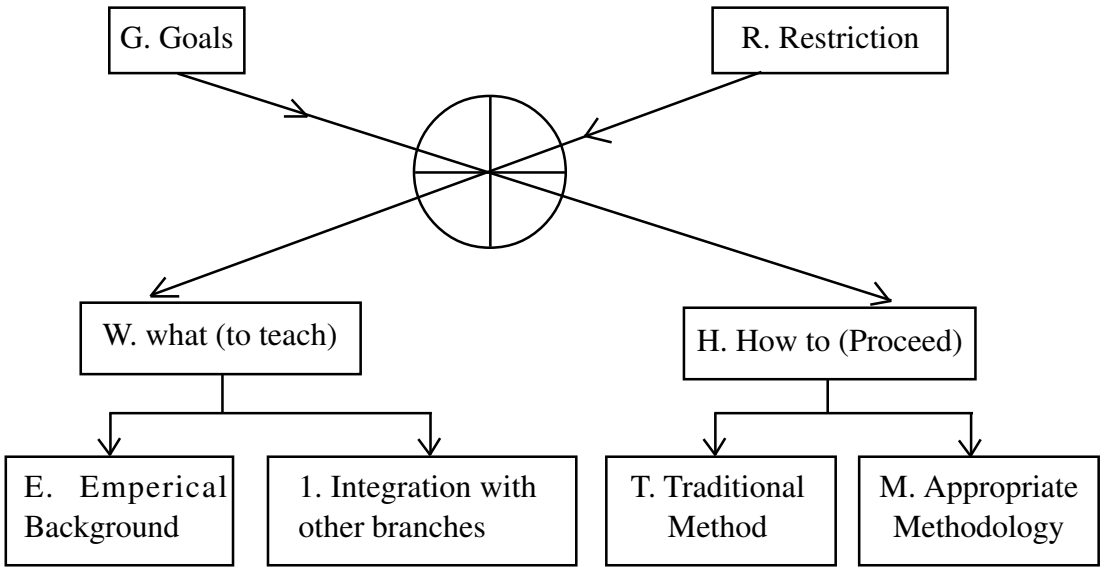
3.3.2 Problem-Solving

3.2.2.1 Introduction

Teaching any subject is general and teaching Mathematics is particular, we must consider the following diagram.



Teaching of Mathematics is generally done by traditional discussion method using chalk and talk. Some teachers follow rigid and sterotyped content and methods. It is felt that problem-solving is mathematics may be helpful to both the teachers and taught at the secondary level.

3.2.2.2 Menaning and Definition of problem solving

Problem-solving is an individual or a small group activity, most efficient when done co-operatively with the scope of discussion. It is a method of thinking mathematically, analysing and of learning how to find out the answer of a mathematical problem using known ideas.

The productive work involved is the evaluation of the situation and the strategy worked out to reach one's set goals is collecively known as problems solving.

Wood worth and Marquis (1948) : Problem. solving occuurs is novel ordiffirent situation in which a situation is not obtainable by havitual methods of applying concepts and priciples derived frome past experience is very similar situations.

Skinner (1968) is of the opinion problem solving is a process of overcoming difficulties that appear to interfere with the attainment of a goal. It is a procedure of

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making adjustment inspite of interferences. Lester (1975) states ''A problem is a situation in which an individual or group is called upon to perform a task for which there is no ready accessible algorithm which determines completely the method of solution :' So problem solving typically involves performing sets of actions to arrive at a solution to some particular task.

Laster has defined problem solving with a cognitive view point and clearly stresses the mental process rather than any overt behaviour of the problem solver.

The cognitive mathematical behaviour can be classified into three broad categories.

First, the memorisation of facts, definitions, rules and procedures. At this level the child is assumed to reproduce what has been taught.

What has been taught

The second level of mathematical cognitive behaviour is the mental activity of generalising or transfering learning from are context to another. The mental activity of recoginzing and restructuring will for form relationship which will help is finding a solution in 3rd level.

The third level is terned as 'open search' is the crucial stage is problem solving process. So problem solving behaviour may be said to be a deliberate and purposeful act on the part of an individual to realise the set goals or objectives by inventing some novel methods or ..ptematically following some planned step for removal of the interferences and obstacles in the path of the realization of then goals when usual methods like trial and error, habit formation and conditioning tail.

2. Natural and Characteristics of problem-solving behaviour

i) Problem-solving behaviour arises when there is serious interference or obstacles are percived to solve the purposeful goal.

ii) One has to utilise some well-organised steps for the removal of the difficulties and obstacles.

iii) It involves quite deliberate and serious efforts on the part of the problem solves.

iv) It helps an individual to reach his goals and also contributes to the process and development of the society.

The psychological view point of problem solving :–

Gagne (1966) has presented a model where production of a solution depends on the learner already knowing 'subordinal' rules, searching his memory to find relevant

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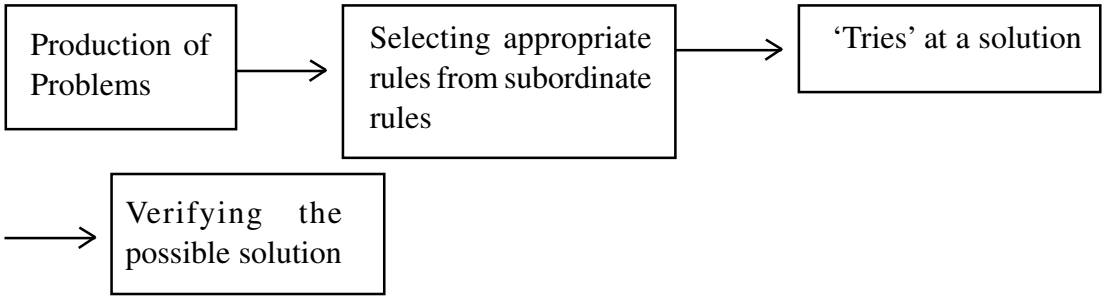
rules, selecting the appropriate rules form among the rellevant remembered rules, combining the rules to form 'tries' at a solution and finally verifying the posible solution.



In mathematic education **George Polya** (1957), in his famous work 'How to solve It', outlined a four-stage model for problem 'solving'.

i) understanding the problem ii) Devising a plan iii) Carrying out the plan iv) Looking back

Research findings as sub sequent years show that students benefited from all poiya's strategies except 'looking back' which was not realy used by the students.

In 1962, 1965 Polya pubblished a much, more detailed two-volume work 'Mathematical Discovery : On understanding, Learning and Teaching problem solving (Vol I & Vol Vol II)

After a careful review of several models, Lester proposed six distinct stages, not necessarily sequential.

i) Problem awareness ii) Problem comprehension iii) Goal analysis iv) Plan development etc.

Merits of Problem - solving Method :

The problem Method aims at presenting the knowledge to be learnt is the form of a problem. It begins with a problematic situation and consists of continuous, meaningful, well-integrated activity. The problems are set to the students is a natural way. Math is a subject of problems Efficiency and ability is solving problems is a guarantee for success is learning this subject.

i) The method stimulates thinking, reasoning and critical judgement is the students.

ii) It develops qualities of initiative and self-dependence is the students

iii) It is a method of learning by self effort

iv) It is a stimulating method. It acts as a great motivating forces.

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v) It develops desirable study habits in the students. It engaged the students is the analysis of the problem, reflectiong thinking, systematic data gathering, verification and critical study

vi) It is a method of experience-based learning.

vii) There is possibility of close contact between the teacher and taught

viii) The students get valuable social experiences like patience, co-operation, self-condifidence et.

Limitation a) It is difficult to roganise the contents according to the requirements of this method.

b) It is time consuming and slow. (c) All the topics and subject areas cannot be covered by this method (d) Teacher's burden becomes heavier (e) Mental activity dominates and there will be neglect of physical and practical experiences.

He main objection has been that the Lester's model does not provide specific information about the diagnosis or the development of specific abilities necessary for solving mathematical problems. With this rationate, kulm and Bussmann have formulated a model called the 'Phase-Ability' model for watching specific abilities corresponding to specific problem solving process.

STEPS IN EFFECTIVE PROBLEM-SOLVING BEHAVIOUR

In general the following steps may be followed is the task of problem solving.

i) Problem-awareness – (Semsing the problem)

He must be faced with some obstacle in the path of the realization of his goals consequently he must be consious of the difficulty of problem.

ii) Problem-Understanding – (Interpreting, defining and delimiting the problem)

All the difficulties and obstancles in the path of the goal or solution must he properly naned and identified.

iii) Collection of the relevant information – (Gathering data is a systemating manner)

He is required to collect all the relevant information about the problem by all possible means. He may consult experienced persons, read the available literature, re call his own expereience etc

iv) Formation of hypothesis or hunch for possible solution–

(Organising and evalusing the data)

He may start some cognitive activities to think out the various solutions to the problem.

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v) Selection of the correct solution – (Formulating tentative solutions)

a) Identify the conclusion that completely satisfies all the demands of the problem. b) Find out whether the so....is consistent

c) Make a deliberate search for negative aspects

vi) Verification of the concluded solution as hypothesis :–

The solution must be further verified far the solution if similar problems and then to be accepted for future solving.

5. FACTORS AFFECTING PROBLEM - SOLVING :-

There are foru interacting categories of factors (variables)

a) Task Variable (The nature of the problem)

b) The Subject Variables (The child readiness)

c) The process variables (the behaviour of the child)

d) The instructional Variables (to make the child a good problem solves)

Problem-Solving Guide

UNDERSTANDING THE PROBLEM

- Read the problem
- Decide what you are trying to find
- Find the important data

SOLVING THE PROBLEM

- Look for a pattern
- Guess and check
- se logical reasoning
- works back words
- Draw picture
- Make an organized list
- Use object or acto out
- Simplify the problem

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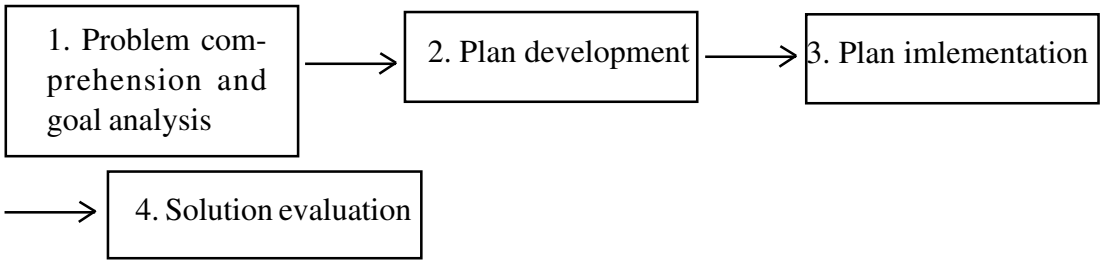
ANSWERING THE PROBLEM AND EVALUATING THE ANSWER

- Be sure you used all the important information
- Check your work
- Decide whether the answer makes sense
- Write the answer is a complete sentence

Problem Solving Stages



Guidelines for teachers in helping students solve proms :-

Students may loose interest if they do not understand the questions. So the maxims will be :

1. Make sure students understand the problems.

   For this

   a) Students a should understand the meaning of the terms of the mathematical problem.

   b) Students must take into consideration all the relevent information. If the student think that trapezium is isoceles, then their idea will lead to rhombus.

   c) They should be able to mentions what the problem is seeking to solve.

   d) Stdents should be able to state the problems is their own words.

2. To help students to gather relevant though material (mathematical concepts) for creating the plan.

   a) To assist the students is gathering information is order to analyse the given condition of the problem.

   b) To help the students to obtain is formation by analysing an analogous mathematical problem.

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c) To help the students to analyse a problem form a different point of view if is not solved by a particular approach.

$x > y > z > p$ but

3. To provide students an appropriate atmosphere for solving a problem.
4. To encourage the students to verify solutions obtained by inductive process and search for alternative.

   To find the sum of n natural numbers by induction and using the formula of A P series :

5. Help the students to general mathematical problems from real life situation.
6. To use the mathematical puzzles, quiz as interesting activities.

Project

Project method is based on John Dewey's philosophy of pragmatism. According to Dr kilpatrick, ‘‘A project is a unit of whole hearted purposeful activity, carried an preferably, in its natural setting. Stevenson defined it as ‘‘A problematic act carried to the completetion is its natural setting’’. Balland described ‘‘A project is a bit ob real life that has been imported into the school’’ project is a modefied form of ‘‘concecentration of studie’s the main feature of this studies is that some subject is considered as the core or centre of all other school subjects. The principles of correction has been given a practical shape through this method.

Project method is based on the principles of

(i) Learning by doing (iii) Learning by living and (iii) Association, activity and co operative learning. It is based on the fact that the different branches of knowledge are not seperable, though they are studied seperately has convenience.

The project may be classified as

i) Individual project which is to be cried out by the individual and

ii) Social projects or group project which are carried out by a group of pupil.

Step of the project Method :

follow page NV. 306 to 310 (To be added)

Content cum Methodology of Teaching

Mulh. B. Ed MC 06/07 (09)

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Initiation of a Project —

Project propsal

1. Title of the project / Name of the topic
2. Elaboration of the Content
   - Focus of the problem
   - Content of the problem
   - Purpose to be coverd
     (Area of coverage)
3. Objectives

   i)

   ii)

   iii) etc.
4. Equipment / Tools/ Resource required
   a) Questionnaire
   b) Information from different sources (Website, Report, documents)
   c) Syllabus curriculum
   d) Statical package
   e) Compules f) Calculator etc

5) **Stratigies**
   a) Hypothesis b) Population c) Sample d) procedure for conducting project / Execution e) Collection of data f) Analysis of data g) Findings covering objectives h) conclusion i) Submitting report.
6) Reflection and Feedback on the project Based on Expected out.
7) Limitation

   Obstacles faced

   Strategies to be adopted to overcome
8) Conclusion / Epilog.

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3.4 Techniques of Teaching Mathematics oral work, written work, Drill work, Brainstorming and computer Assisted Instruction (CAI)

3.4.1 Introduction

In teaching Mathemaitcs, teacher may adopt a particular method or a combination of methods to make the teaching effective effective and worthwhile. The claity mathematics lesson in tends to follow a standard pattern. Such lesson plan is prepared keeping is view the previously taught lessons. Hences, adquate practice or drill of previously learnt mathematical skills are important task. Similerly, for fulfilling the expected out come or gainin mastery of new kills some techniques are used for teaching of Mathematic. Some of them are oral work, Written work, Drill work etc. They are discussed below.

3.4.2 Oral Work

It is the work which is done orally without the help of written work and record. It is the mental work, where in a problem is solved orally or mentally. In mathematical learning much of mathematical work has to be coupleted mentally and many tables have to be learnt by heart : In teaching elementary Mathe it is very essential. Oral work helps each child work at the optimum rate which gives maximum accuracy.

Function of Oral Work –

1. At the introductory stage of teaching, oral questions are asked students to test the necessary back ground knowledge for today's lesson. Oral questions are also asked at the developmental stage module wise and at the recapitutatory stage.
2. It has an appeal for the eye and ear which is liked by students.
3. Some-time are saved by oral work.
4. Oral questions help the teacher to judge the level of understanding of the students is calssroom situation.
5. It arises interest of publish.
6. It is a good mental exercise because it develops atertness, readiness of mind, quick hearing, quick thinking and quick responding.

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7. A mathematical idea can be effectively illustrated through a sufficient number of oral examples on questions without much loss of time.
8. It is an effective means of maintaining class discipline
9. It encourages healthy competition among the students.
10. Oral work provides a rapid drill designed to habituate a fundamental process
11. It helps is completing morework is any given period.
12. Spontanity is grasping the data and organization of thought is a limited time, are important aspect of oral question-answer
13. Any individual difficulties can be identified and effectively removed by oral work
14. A teacher can throughout remain active is the class with the help of oral question answer.

Good planning and adequate preparation are necessary for constructing oral work for the students.

3.4.3 Written Work

We know the principle ‘‘Reading makes a full man, conversation a ready man and writing an exact mas Oral work is not enough to understand and measure the higher order of learning is Mathematics when a teacher requires to check work done by each child or to give children practice is independent work, written work becomes a necessary. Hence oral work is to be supplemented by written work. In Mathematics, too much written work is needed. Written work should be considered as an extention of oral work. They are complimentary to each other. The teacher is Mathematics class may follow the sequence

1. Oral fundamentals matter
2. Written fundamentals matter
3. Oral problems presentation
4. Written problem presentation

Both will work is combination

Importance of Written Work :

i) Throughout written work accuracy is computation, legibility of figures and symbols develop

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ii) It facilities deep understanding of different mathematical concepts and rules

iii) It improves speed consistency with accuracy, proper algorithm and neatness of work

iv) It fosters thinking and reasoning power

v) It motivates the learners to take active participation

vi) If helps the learner to maintain proper logical and sequentgial arrangement of steps is the mathematical solution

vii) If fosters desirable attitude towards Mathematics

viii) Written works also keep a collective record for assessing student's propgress over a period.

ix) If helps the student for self correction and identification of errors committed by him.

x) It helps to develop good study habit for improving achievement is Mathematics.

3.4.4 Drill work

Drill is one of the most essential methods of learning Mathematics Drill is the process of repetition to make automatic certain process or activities. Drill work is the most efficient means of fixing the impression is mind. One can not expect to achieve speed and accuracy is solving mathematical problems without. Teacher teaches mathematics concepts, rules as application of those. After this he has to evaluate whethers the knowledge given to students has been fixed is their minds and apply those is similar situation. For this drill work and followup action have to be carried out through drill work. Drill work are of three types

The first type of lessons for obtaining mastery of basic subject matter like multiplication fables, addition combinations, percentages, factorization, fraction to decimal, construction is geometry etc. Those subject matters are to be learnt at mastery level with respect to speed and accuracy for future learning.

The second category includes topics as mathematical concepts for the mastery of procedures. In this type of skill the students will be mastered is translate verbal problems is to symbolic form, systematic arrangement of steps, apply correct algorithms, to serutinise and check each step for finding error, sort out data, to label correctly the geometrical diagram, practice short cuts, back calculation etc.

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The most important i, e third type of drill consists of lesson which develop the power of thinking, reasoning, generalisation and interest, positive attitude of learner etc. Example of such skills are quizze, puzzle, math, talk etc.

Teachers must be careful is developing few functional or meaningful drill in mathematics classes. These are prior understanding of content knowledge and its appropriate application, the necessary and sufficient condition for mathematical proof etc.

Considerations to be kept is mind for making Drill work more effective.

1. Drill should follow learning as well as understanding of basic principles. It must not must rote memorization without understanding.
2. It should be individualised and follow the principles of reward and punishment
3. Drill should be varied and systematic Mare routine procedures make the learning monotonous and uninteresting
4. If must be sufficient is quantity. For better results the drill work may be divided into parts of appropriate interval.
5. Drill periods should not be planned merely to keep the students 'busy' at work. It must be based upon thought provoking situation.
6. Drill may provide students the diagnostic information and self checking
7. Drill should not be given in the form of punishment
8. Students should be given proper environment for individual and group drill work
9. Mistakes is drill work must be carefully checked and evaluated at an early time.

3.4.5 Brainstorming.

3.4.5.1 Introduction

A.F. Osborn (1963) popularized this strategy through his writing 'Applied Imagination'. It is indicates storming of the brain to generate a number of ideas as quickly as possible without passing any judgement

3.4.5.2 Definition

This is a strategy for the development of higher cognitive abilities like reflective thinking, creative imagination and problem solving capabilities. This stragegy is used with a group of students to explore a good number of ideas for solution of a problem.

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3.4.5.3 Procedure for using brainstorming as a teaching strategy

i) At first a small group of students (10-15 students) of a particular class is formed. They will be asked to sit is a group and will be given a focus topic say.

"How will you findout the hight of a tower without climbing it".

2) The teacher will then asks the students to think about the solution of the problem and give their ideas one by one or to list out the solution is a paper. They may be instructed as follows :

i) The problem is placed before you, think about the possible solution on solutions as you may think suitable.

ii) This is not an examination. Don't care for the criticism. Write down the possible solutions without any heritation even if they seem to you quite new or unusual.

iii) Students are also free to alter or modify their ideas and solutions given by them earlier in the session.

iv) Student members are also free to alter or modify their ideas after discussion with others.

3) In this way, students will be encouraged and inspired for submitting as many as ideas or solution procedures as possible. The group members and the teacher as leader are supposed to collect the different solutions so that :

a) All the solutions as ideas are to be encouraged and there will be no criticism during the brainstorming session.

b) Ideas are to be listed without any judgement or passing remaks.

c) Members are encouraged to supplement this ideas with others.

d) All the alternatives or solutions are to be reconded properly on the blackboard for free discussion.

4) At the end of the brains storming session, all the solutions and ideas collected from the group under the guidence of the group leader i, e the teacher will be dis usal for the approval of the exparts. Thus a variety of the solution or ideas are evolved.

Advantages of the brainstroming strategy.

1) Students become active and discovers of the solution of the problem or new ideas an concepts.

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2) Teachers act as guide.

3) The strategy helps the students to develop higher order cognitive abilities like think, analyse and synthaesize independently.

4) It helps the students to develop their creativity, originality, potentialities and problem solving ability.

5) As it is a group activity, there are scopes for exchange of views, cooperative spirit and development of reasoning power.

6) The student acqures a real understanding and clear notion of the subject as well as mastery of what he has discovered.

Disadvantages and limitations :

Brainforming strategy has the following dis advantages and limitations.

i) It is a time consuming process and the syllabus may not be completed within the period of time

ii) The group members may not be homogeneous with respect to cognitive level required for the discussion of the content.

iii) The output of the brainstorming session may not be as per with teacher's expectations on expected outcomes.

iv) At the concluding session, the result may not be the actual solution of the problem

v) This stragegy can not be applied in large class having 50-60 students

vi) All the members of the group may not be equally interested for find the solution

vii) All the topics of Mathematics may not be covered by this strategy.

3.4.8 Computer Assisted Instruction

Introduction :

With the introduction of New Education Policy is 1986, initiatives have been taken to use computer in the teaching-learning activities. In Mathematics, the instructional work so carried out with the help of computer is generally known as computer - assisted Instruction (CAI)

Definition

Computer-assisted instruction is a method of instruction is which there will be a purposeful interaction between a learner and the teaching material as software of the

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computer. It helps the individual learner to achieve the expected instructional objectives designed by the teacher with student's own pace and abilities at his command.

Characteristics :

i) It is an interaction between a student and a computer controlled display materials.

ii) The individual student observes the displayed material and responds to it

iii) The instructional material as software is prepard by the teacher keeping in view the multidimentional need and capbilities of the learners.

iv) It is a auto-individualised instructions which provides instruction to a large number of learners at a time.

v) It provides the opportunity for automatic recoding of the learners performance.

vi) It provides a wide variety of methods and approaches for imparting instruction

vii) The computer-assisted instruction helps the individual learner to achieve the objectives with his own pace and abilities

viii) This type of instruction involves three types of technologies namely, hardware, software and courseware.

Fields of Instruction is Mathemaics teaching through computer assisted instruction.

For providing self individualized instruction to a learner, computer assisted instruction is Mathematics helps in the following fields

1) Discrimination of information related to Mathematics content.

The mainpurpose of this type of C A 1 is to provide essential information as the context for example, a student wants to know the symmetrick Matrix. The definition like symmetric matrix is a square matix is which corresponding eliments above and before the principle diagonal are equal should be mentioned.

As Symmetrik matrix is a square mattriz is which the transpose of that is that it self is to be mentioned.

Example $A = \begin{Bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 7 \end{Bmatrix}$, $B = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$, $C = \begin{bmatrix} 4 & 5 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 9 \end{bmatrix}$

Similarly, Super-ordinate and sub ordinate concepts are to be given in the software.

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2) Drill and practice programme.

CAI provides different gypes of drill and practice programme covering specific topics.

Example : To draw the conceptual Hierarchy of quadrilaterals like parallelogram, Trapezium, Rhombus, Square etc.

3) Simulation type instruction

Such type of instructional activities, carefully prepared programme are given to students. They practice it and are trained.

4) Problem solving type

Here, the students are provided with programme that will allow them tothink about the ways and means of solving the problem systematicaly like drawing of groups with two equators.

5) Tutorial type instruction

The tutorial programme are prepared, where the students can play effectively through interaction and dialogue. The programme also provides remedial instruction.

6) Practical work related instruction CAI can provide help is supplementing practical work like drawing geometrical figures, calculations, checking the result, consultation with tables etc.

Limitation

1) It is expensive and uneconomical.
2) It is machines oriented is can nevers match the human beings. No simpathy expathy or human touch are available.
2) It is machine oriented it can never match the human beings. No simpathy empathy or human touch are available.
3) It is basically a learners - controlled instructionm, There is little scope to check the learner causing wasting-controalled instruction, there is little scope to check the learner causing wasting of time.
4) Chances of machine failure are there causing a set back is the system.

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Unit 3 ❑ Strategies for Learning and Teaching Mathematics

Structure

3.1 Concept Formation and Concept Attainment

3.1.1 Introduction

3.1.2 Meaning of Concept

3.1.3 Characteristics of Concept

3.1.4 Steps of teaching concepts

3.1.5 Concepts Attainment Model for Learning and Teaching of concept.

3.1.5.1 Strategy of concept Attainment Model. Bruners as mentioned four strategies of the Model namely

3.1.5.2 Types of concept attainment model.

3.1.5.3 The Reception model of concept attainment.

3.1 Concept Formation and Concept Attainment

3.1.1 Introduction

The Pioneer of 'concept attainment' is Georm. S. Bruner and his associates Jackue line Goodnow and George Austin. The concept is developed from their research 'A study of Thinking' (1967) the basic considerations of concept Attainment model are :

i) Man posses the ability of discrimination and classification of things, persons in groups.

ii) For adjustment with the environment, a person encounter varried situations and experiences. It is impossible for a person to restore such huge experiences. So he tries to assimilate such experiences and try to adjust with the environment with these assimilated experiences. In general, the assimilation of such experiences is called concept.

iii) A concept has three elements a) example b) attributes and c) attribute values. For example, triangle is concept then each triangle is an example. Here rectangle and pentagon are negative and equilateral triangle is positive.

Triangles have all the sides are equal is the attribute and all angles are also equal be the attribute values.

iv) The categorizing activity has two components (a) The act of category formation (concept formation) and (b) The act of concept attainment. The concept formation is the first step for concept attainment.

3.1.2 Meaning of Concept

According to Bruner ''A concept is a class as grouping response, an act of categorization, involves rendering different Things equivalent.''

Archer (1969) mentioned ''A concept is simply the level of a set of things that has something is common'' A concept is different from a fact, a principle and even a generatization. Halse states 'A concept is a set of feature connected by some rules. A concept of quadrilatiral is a geometrical figure bounded by four straight lines selvens (1993) has cleared the idea is the following ways ''A concept consists of an individual's organised information about one or more things, objects, ideas, or relations that enable the individual to discriminate a particular thing or class of things and as classes of things.

Thus concept is a class or category of all the members of which share a particular combination of attributes or critical properties not shared by another class.

3.1.3 Characteristics of Concept

Knowledge of common feature

i) It is the knowledge of common feature of person, object, animal or circumstances of similar characteritics

Characteristics observational experiences

ii) It is observational experiences. Greater the extend of observtional area greater will be the concept area. In framing the concept of triangle the barner must pay attention minutily to the length of different sides of a triangle.

iii) Individual differences

In concept formation individual differences play active role. Different persons form mathematical concepts in their own ways to some extend.

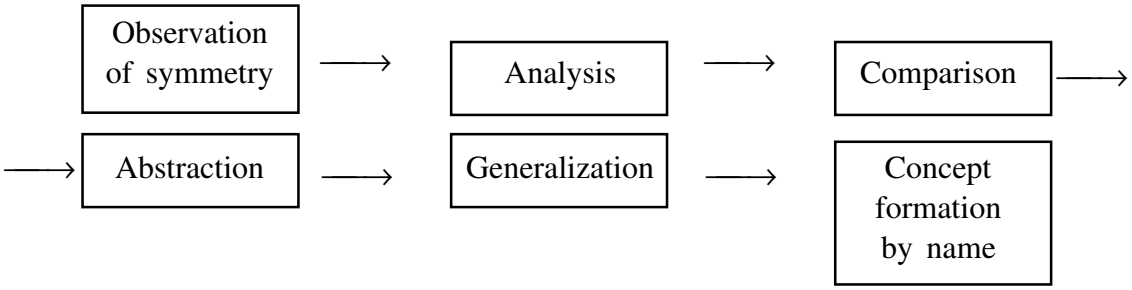
iv) Experience of object

At the initial stage a child forms the concept based on the experiences of real

object. At the later stage due to development, he forms the concept is absence of real objects. At the begining, a child forms the concept of circle as triangle by observing circular as trianglular objects but later stage by imagination only.

v) Complex process : Mathematical

Concept formation is a complex process. If follows the following route.



vi) Concept formation may be of three types

a) Conjunctive (Combination of different traits, Geometrical figures included may type of figures)

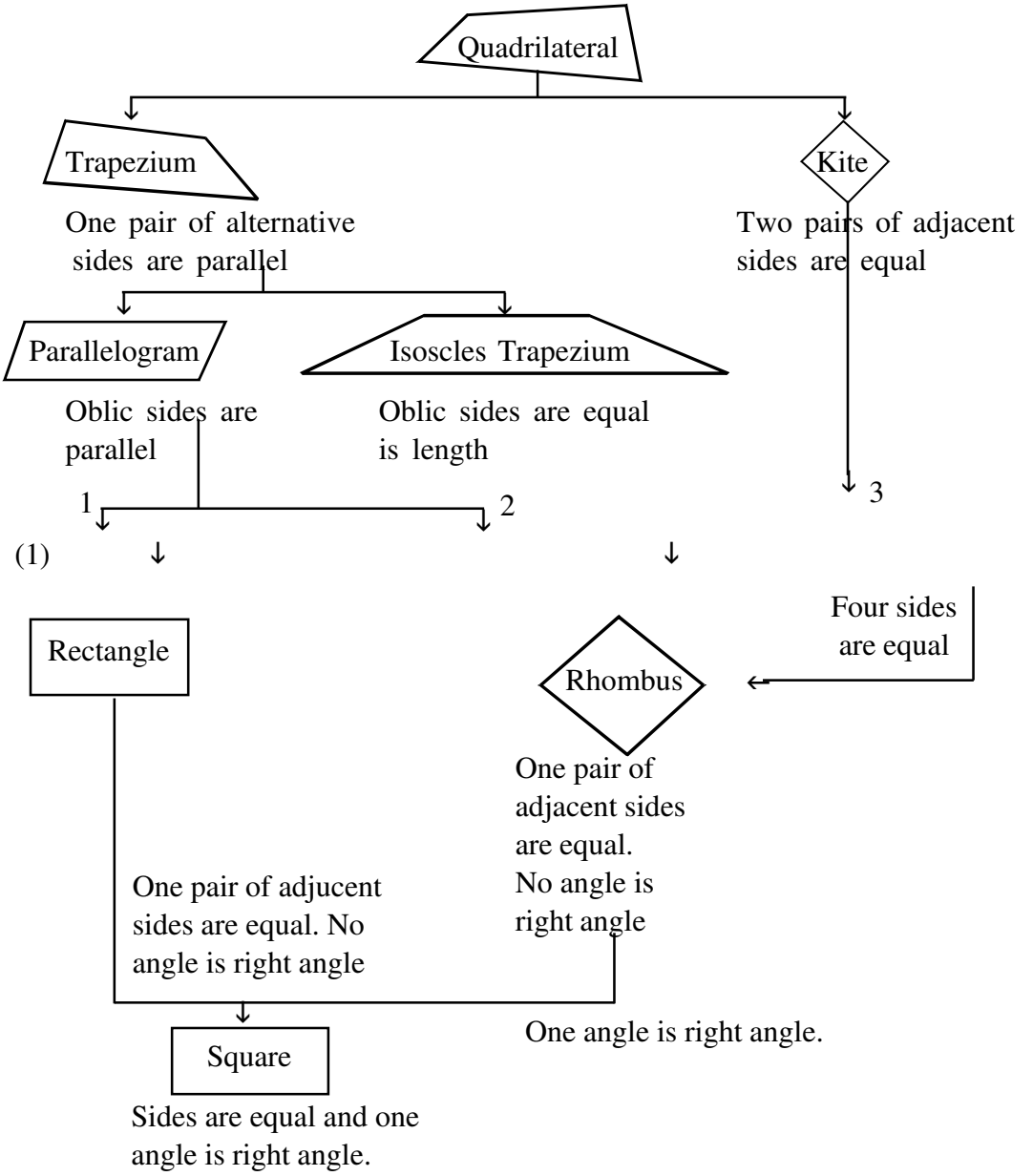
b) Disjunctive (A concept may be subdivided into many sub concepts. Fractions may be subdivided is to proper and improper or deciwal etc)

c) Relational concept (When there is/are relation among concepts, (Rectangle, Trapezium, Rhombus, Square

vii) Conceptual Hierarchies.

Each concept is not found in isolation, but rather is relation to other concepts. Simth describes this process of forming new concepts as one of breaking down old categories into smaller and more specialized one. One way of describing the relationship of concepts formed is in terms of super ordinate, co-ordinate and sub-ordinate concepts. These terms refers not only to the scope or inclusiveness of a concept but also to its relationship to other concepts. For example, quadrilateral is more inclusive and also subsumes the concept of parallelogram which in term subsumes the concept of rectangle. In this respect rectangle is described as a sub-ordinate concept of quadrilateral. Related concepts such as parallelograms and trapezium form the co-ordinate concepts. In the conceptual hierarchy of quadrilaterals, parallelogram forms the super-ordinate concept to square.

Conceptual Hierarchy of Quadrilaterals



VIII) Concept Name

Every concept have a name. The concept name is the word used to symbolize the given concept. A concept is an idea or obstraction that exists is people's mind, while the concept name is the word that we arbitrarily use to designate the concept. For example, parallelogram is a concept name designated to a quadrilateral is which both pairs of opposite sides are equal. There are two ways which the concept name is attached to a concept are

a) The child learns the concept first and later learns to attach the name to the concept

b) The child learns the symbol and then learns the concept

IX) **Concept definition**

The deffinition of a concept means to describe typically is verbal statement, the meaning of the concept. If focuses the summarization of important aspects of the experiences which ignoring others. It describes the boundaries of a concept. It helps to determine set inclusion and set exclusion. For example, when we define a quadrilateral as four sided closed geometrical figure, it means that all four sides closed geometrical figures like square, trapezium, parallelogram, rectangle etc can be included is the set of quadrilaterals. But others geometrical figures which are not closed and which do not have four sides can not be included is the set of quadrilaterals.

X) **Process of observation.**

Through the process of observation, the essential characteristics i,e the attributes are identified. Identifying the attributes of a concept is essential becauses without the knowledge of the attributes of a concept, the process of understanding the concept is difficult. For example, the characteristics or attributes of the concept square are four sides which are equal is length and angles are right angle each. These Characteristics are important or essential set where as characteristics such us size colour or special orientation are not important is the set of square.

XI) **Example in a concept :**

Examples of a concept are those members of a class which are positive instances of the concept. Examples can appear is word form, is pictorial form or is real life form.

XII) **Concept formation**

Concept formation takes place when members of a category are groued togethers

and similarities are acceplied which ignoring the differences among the members of the categories. Considering the similarities a rule is formed and conceptualization takes place.

For example, $-\frac{7}{3}, -\frac{5}{3}, -\frac{1}{2}, 0, \frac{1}{3}, \frac{7}{4}$ etc are the examples of a concept rational members. Rational numbers are defined as numbers expressed is the form of p/q, where p and q are integers and p and q do not have common factors and $q \neq o$.

3.1.4 Steps of teaching concepts

Please add page 592 to 598 (Content cum Methodology of Teaching Math B.Ed MC - 06/07 (09) NSOV.

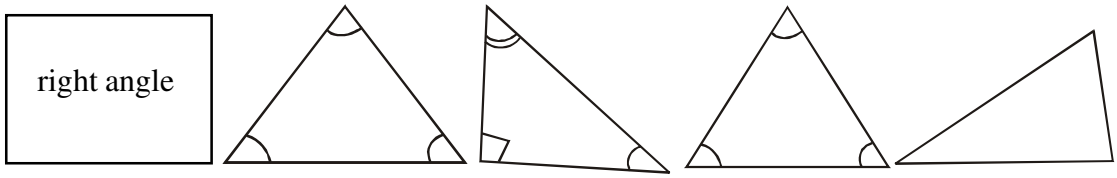
3.1.5 Concepts Attainment Model for Learning and Teaching of concept.

The concept learning includes both the phases i,e concept formation and concept attainment.

3.1.5.1 Strategy of concept Attainment Model. Bruners as mentioned four strategies of the Model namely

i) Simultaneous Scanning Strategy

In this strategy for a particular concept formation, the learners will be given a branch of cards is which different attributes and attribute values are mentioned. The learner will select the cards having positive attributes as well as positive attribute values and will reject which are negative attributes. Concept of Isosoul right angle triangle.



Card No 1 Card No. 2 Card No. 3 Card No 4 Card No 5

For right angle triangle he will select card No 3 and Card No. 4 In card No. 3, one angle is right angle but the others two angles are unequal. So considering the attribute value he will reject So considering the attribute value he will reject card No 3 and will select card No. 4.

But he will reject card No. 1 as it is not triangle. He will reject Card No. 2 as it is equilatiral triangle and also reject card No 5 as it is not right angle triangle.

ii) **Successive scanning strategy**

Here the learner only select the cards having. Positive attributes successively. In this straligy the learner must know the necessary and sufficient positive attributes of the concept. Though he mentally rejects the negative attributes oriented cards

iii) **Conservative Focusing Strategy :**

Here every card will contain only one attribute of positive or negative nature. Considering the positive instance he will reject all the negative all the negative instances or non examples. By showing the non example cards he will reject those with classification.

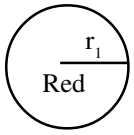


Fig 1

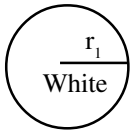


Fig 2

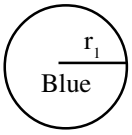


Fig 3

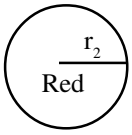


Fig 4 $r_2 > r_1$

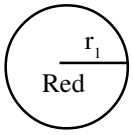


Fig 5

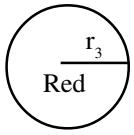


Fig 6 $r_3 > r_1$

Student is asked to pick up red circulrs dise having less diameter. He will reject circular dise No 1 to 5 by showing reasons and will select dise No 6 as positive instance.

iv) Focus Gambling - Here is each card all the attributes (positive example and negative attribute/non examples) are mentions and in only one and only the positive attributes are mentioned.

Students will be asked to pick up the right and as quickly as possible. Here like gambing chance factors will be effective. If adequate time is given, students will be able to born the right concept of circular dise.

3.1.5.2 Types of concept attainment model.

On the basis of research work done by Bruners and his associates several models of teaching have been developed. The concept attainment model has three variations.

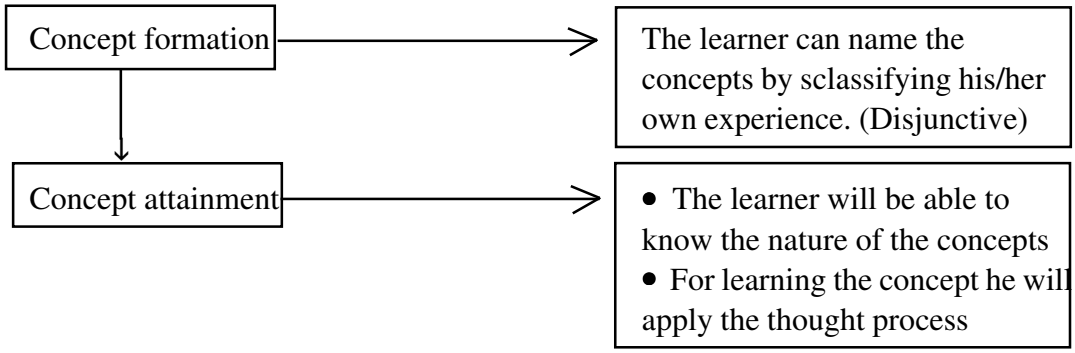
i) The reception model

ii) The Selection model

iii) The model for unorganized material.

3.1.5.3 The Reception model of concept attainment.

A. Focus : Here weightage is given on the development of logical powers. Training is given how conclusion can be drawn by inductive reasoning through use of examples. The focus of the model is to form conception through the stages a) observation b) analysis c) comparison d) abstraction e) generalization and finally naming. There fore, in addition to help the students is the attainment of a particular concept, it enables them to become aware of the process of conceptualizing.



B. Syntax.

The sequence of thes phase and activities are given in the table.

Phase one.	Presentation of data and identification	**Activities** i) Presenting examples with 'yes' or 'no' lables by the teacher is a pre-ar ranged orders ii) Ask the students to compairs at tributes in positive and negative examples iii) Students will frame hypothesis and compare with teacher's hypothesis

		iv) Naming the concept by students v) Will state the rule as definition of the concept according to needed attributes.
Phase Two	Testing for attainment of inner concept	a) Students will frame hypothesis for identifying the similaristics and dissimi larities of the facts. b) Considering each hypothesis, at tributes will be judge. c) By analysis and synthesis of the attributes they will find out the inner concepts
Phase 3	General Statement about the concept	d) Will be able to generate own examples. i) Teacher will give more information but will not mention the concept like before. ii) Teacher will ask to identify which information attribute is related to which concept iii) If the students role is statisfactory he will ask to give more examples iv) Teacher will give general statement about the concept
Phase 4	You aware about the process of concept formation and thinking strategy	a) Students will be asked to analyse and discuss the process by which They have attained the concept. b) Students will discuss the role of hypothesns and attributes. c) Students will evaluate the process concept attainment. d) They will be encouraged to follow the process for obtaining experiences by this way.

C. Principle of reactions :

The important principles of reaction are the following

i) The teacher will be supportive to the student's hypothetical in nature.

ii) He will help the students to distinguish between two hypothesis and to evaluate the thinking process by themselves.

iii) He has to maintain record about the focus attention on specific feature of examples.

iv) He is to encourage the students to enalysis the merits of various strategies.

D. Social System

In this model, in most part of the teaching, the teacher has to exercise control over the social system. The teacher has to collect information as well as examples has a fixed up conception. He has to divide the experiences into positive and negative attributes for the concept. He will communicate the students that the solution of the problem of identifying concepts lies not within the teacher but in the data (examples). The teacher gradually relax the controll and encourage the students to work independently and collaborately. The functions of teacher will include a) to record the activities b) to give clue for collecting attributes c) and to give extra information is the form of examples. The social system functions on the mutual interaction between teacher and taught. For concept attainment, the teacher gives the possitive and negative examples or attributes to the students.

The students explain the examples and justify his hypothesis on the basis of positive attributes The interactions are noted on the black board as on the Tag board.

The concept is deduced and rule or definition is framed.

E. Support systems

In this model students will not discover a new concept but will deduce the concept on the basis of examples/experiences/attributes put forward by the teachers.

In the first stage, keeping is mind the appropriateconcept, the teacher will give different information or examples to the students. This will include both positive and negative attributes which are essential. By discriminating the positive i,e relevant attributes, the concept is formed. By giving more examples by the students, the teachers from the definition of the concept so is the concept attainment model, the support system is the relevant information is the form of examples.

F. Application Concept

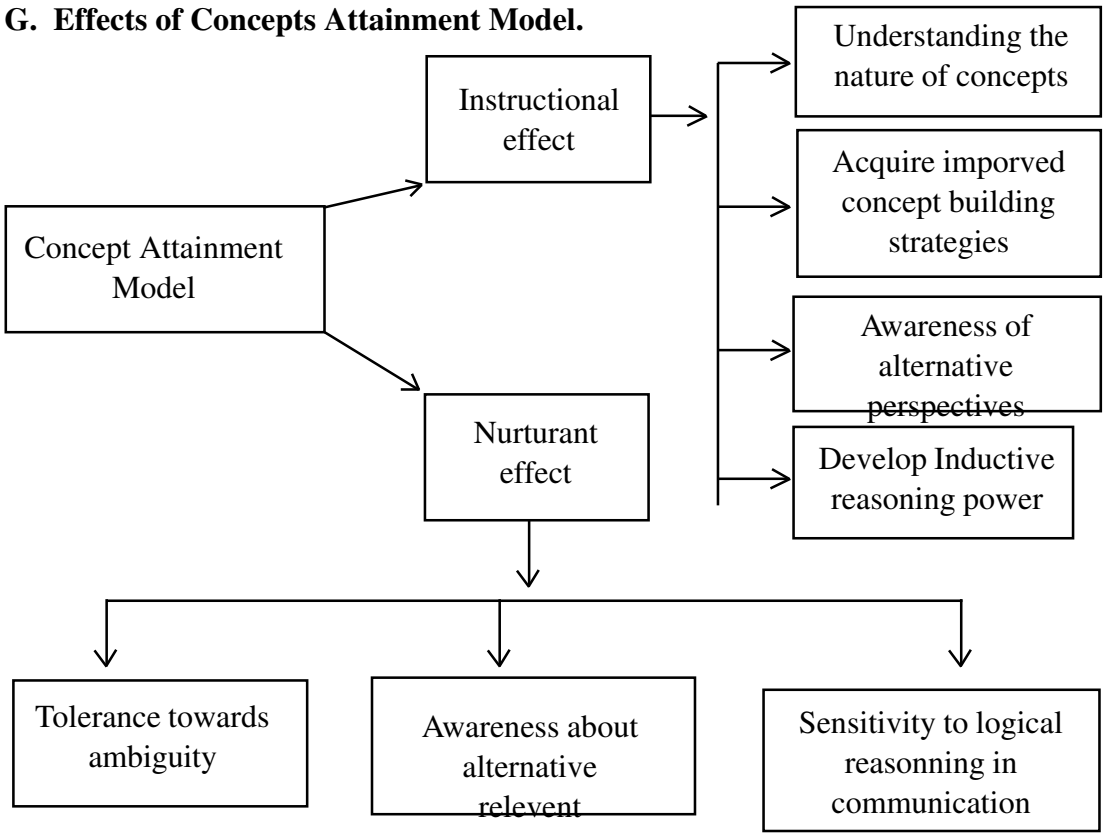
Ioyce and will are of the opinion that the concept attainment model is an excellent evaluation tool when teachers want to determine whether important ideas introduced

earlier have been mastered. It quickly reveals the depth of students understanding and reinforces their previous knowledge'.

Mathematics contains a good number of concepts inter related to each other. The model includes both concept formation and concept attainment. In most cases concept is formedby inducting reasoning and follow the process of generalisation and discrimination.

1) So the model is effective is teaching mathematical concepts, grammer and language.
2) For the introduction of this model in class-room teaching, the students may be divided in to small groups so that each of them can participate in the discussion.
3) Before teaching following the model, the teacher may explain the different phases examples (both positive and negative) for a concept.
4) For each group, there will be one leaders who will controll the group discussion, but the teacher will supervise all the group activities.

G. Effects of Concepts Attainment Model.



3.1.5.4 Adventages/Merits of concept Attainment Model

i) Normal environment of class teaching-learning process is maintained.

ii) Reasoning powerspecially inducting reasoning power of students is developed

iii) In this model the power of observation and imagination of the students are increased

iv) In this model the students can apply his/her knowledge is real life is tuation.

v) Students get the opportunity to justify their own hypothesis by judging the experiences and drawing conclusions.

3.1.5.5 Disadvantages

The model demands the participation of all students and teacher in the teaching learning process. But a good number of students generally remain pasive. So the introvert and under achiever students are not benefited is the model of teaching.

Limitation of concept attainment model

i) To follow the model both the teacher and students are to take enough responsibility

ii) There are individual differences among the learners. So the introvert type students do not participate is this model of teaching

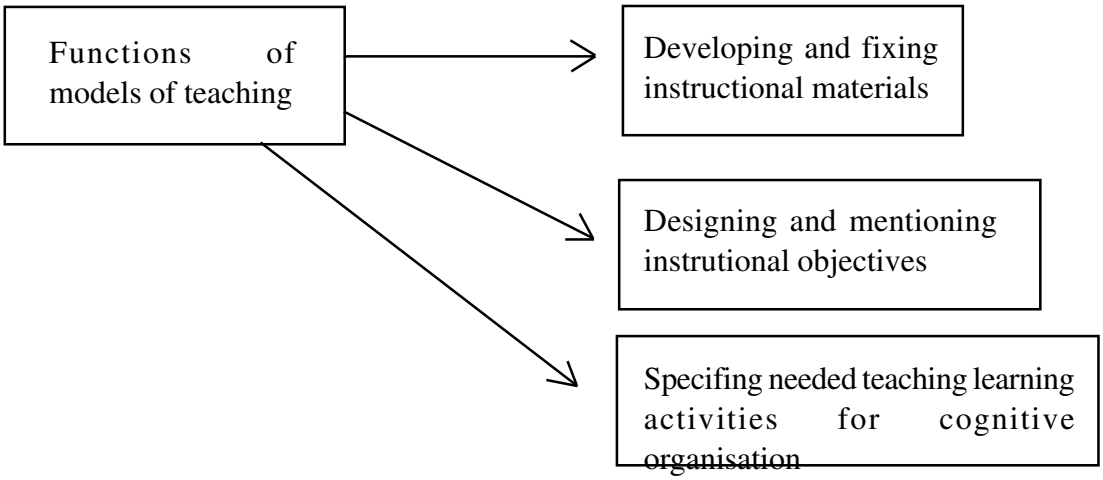
iii) All the contents of the syllabus can not be taught using this model.

But the model has revolusionised the face to face teaching process.

3.2 Learning By Exposition : Advance Organizer's Model

3.2.1 Introduction

There are two terms 'Models of teaching' and 'teaching models' used is teaching learning process. 'Teaching models' are just instructional design where as 'models of teaching' consists of guidelins for designing educational activities and environment. It also specifies ways of teaching and learning for fulfilling the intended goal of teaching. So the function of models of teaching is depected below.



It has been accepted that Mathematics should be taught in such a way that each learner is trained to think, reason, analyse and articulate logically. As per as possible the discovery approach followed by teaching through designing and planning suitable aids and models.

The fundamentals essential for learning mathematics and solving of problems of daily life should be borne is mind while teaching one should aim to learn the concepts at the mastery level. Activity oriented programme should be used in teaching Mathematics. The most essential thing is that joy and achievement should prevail while learning Mathematics.

3.2.2 Advanced Organizer's Model of Ausubel–David Ausubel designed this model to increase the efficiency of information processing capacities of children.

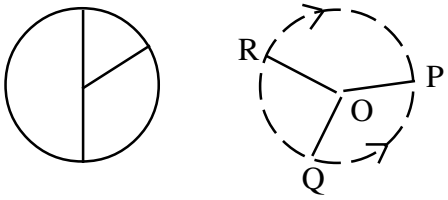
The ideas of this model have emerged in his book 'Theory of Meaningful Learning' According to him the types of learning material related with the content can be learned by the learner quite effectively if presented through. Visual graphics, charts and picture, film, audio tapes and transparencies.

He is of opinion that for meaningful verbal learning, the teacher must know the following three issues (i) How knowledge (content of curriculum) (i) How knowledge (content of curriculum) is organised - (ii) How the mind works during the process of presenting new information (learning) and c) How the above two can be presented to the students (instruction)

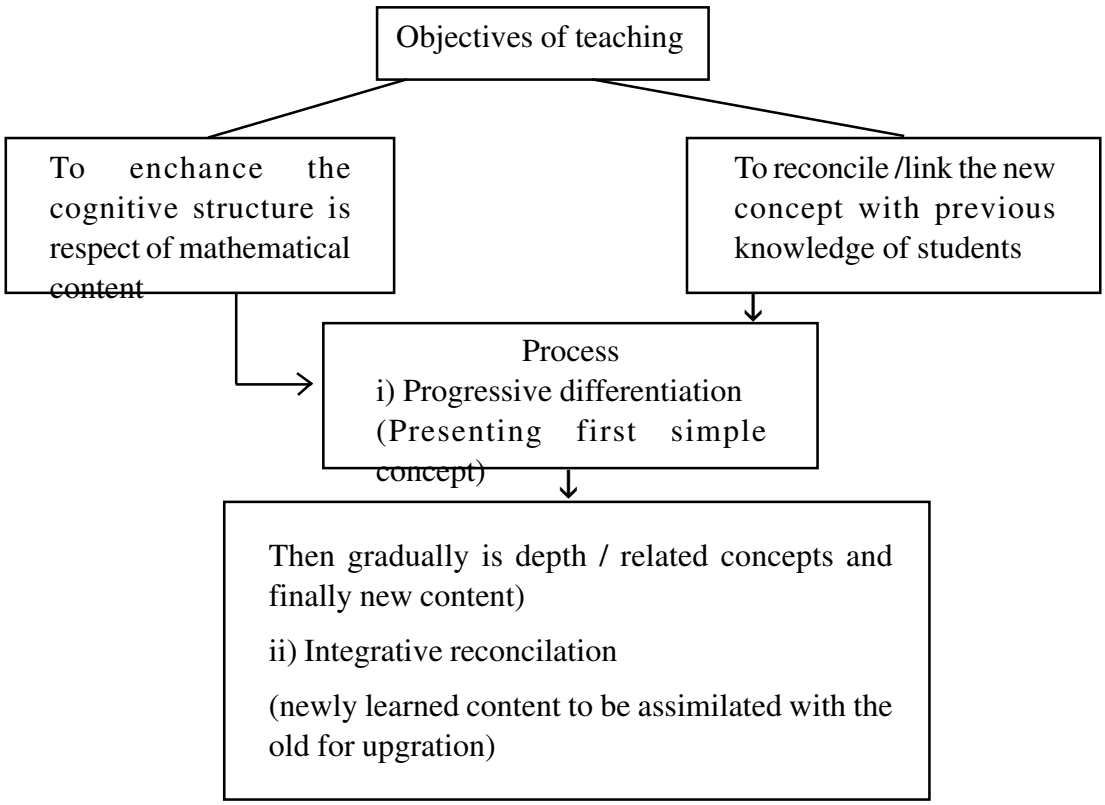
According to A usubel new ideas can be learned or retained only to the extent that those ideas can be related to already available cognitive structure of the learner. That acts as necessary linkage or anchors.

Advance organisers, as Ausubel maintains, are the primary means of strengthening learners's cognitive structure. The advance organiser may be of two types i) expository (which is helpful in providing the basic concepts at the highest leaved of abstraction and essential for understanding new nontent) and ii) Comparative (designed to discriminate between the old and new concepts to present confusion.

For teaching the equation of a circle, expository advance organiser may defferent shaped circular dise and as comparative advance organiser a diagram showing geometrical figure of circle and the locus of points which are equidistance from a fixed point, called its centre.



3.2.3 Before presenting the Advance organisor Model of teaching we must keep in mind the following



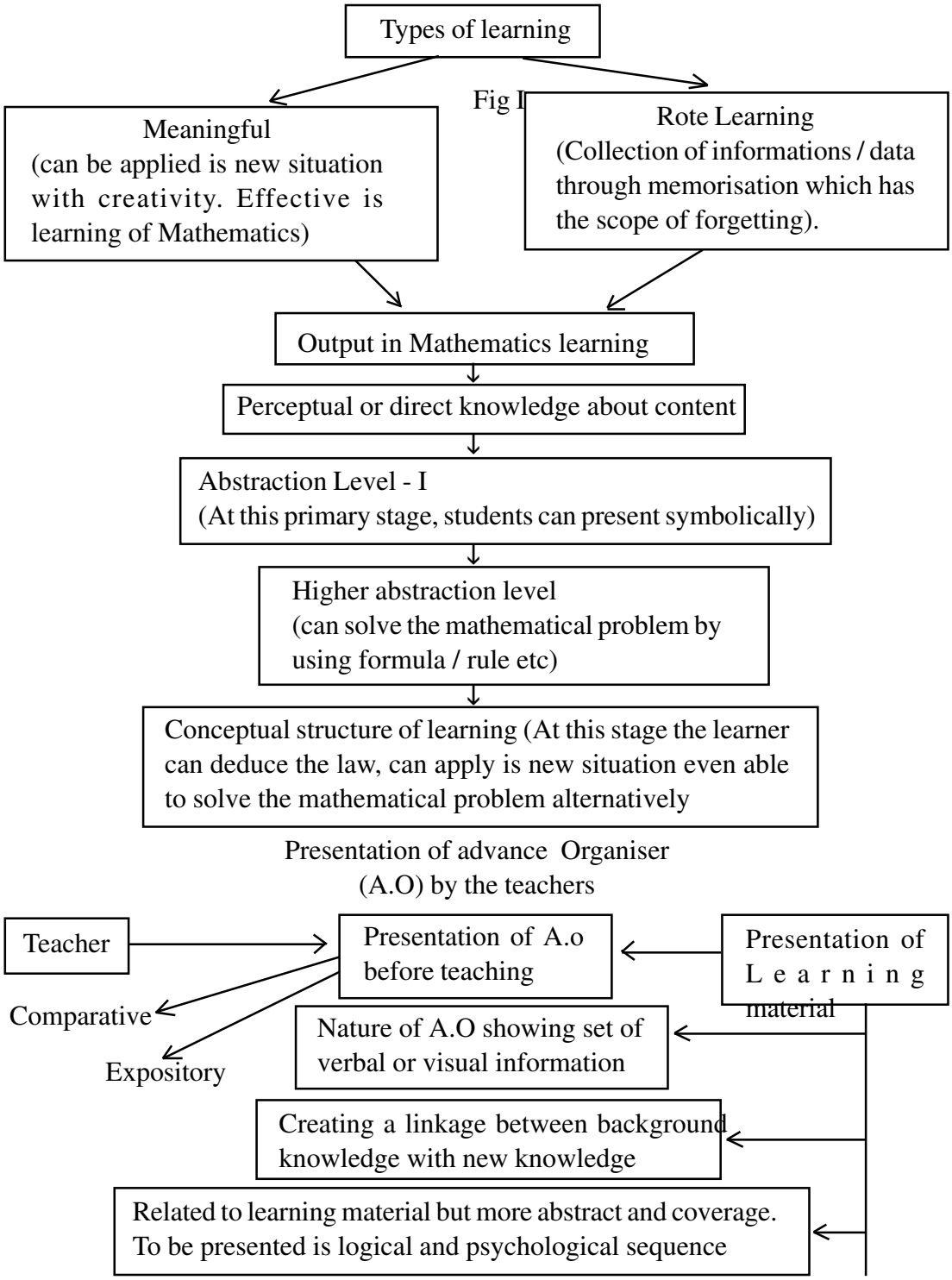

Types of learning
Meaningful
(can be applied is new situation with creativity. Effective is learning of Mathematics)
Rote Learning
(Collection of informations / data through memorisation which has the scope of forgetting).
Output in Mathematics learning
Perceptual or direct knowledge about content
Abstraction Level - I
(At this primary stage, students can present symbolically)
Higher abstraction level
(can solve the mathematical problem by using formula / rule etc)
Conceptual structure of learning (At this stage the learner can deduce the law, can apply is new situation even able to solve the mathematical problem alternatively
Presentation of advance Organiser
(A.O) by the teachers
Teacher
Presentation of A.o before teaching
Presentation of Learning material
Comparative
Expository
Nature of A.O showing set of verbal or visual information
Creating a linkage between background knowledge with new knowledge
Related to learning material but more abstract and coverage.
To be presented is logical and psychological sequence

Fig I

3.2.4 Fundamental elements of Advance organiser model

3.2.4.1 Focus

The model focuses

1. Assisting the teacher for improving the method of presentations for this must know how contacts are organised
2. Organising large amount of content information teacher must realize meaningfully. For this what is the cognitive structure of students in respect of new content
3. To help the learner to strengthen their existing cognitive structure. For this relevant questions are to be asked and necessary information to be provided.

3.2.4.2 Syntex

Activities are to be performed sequentially is the following three phases.

Phase : Presentation of the advance organisers

i) the aim of the lesson is to be classified. By stating the aims and objectives of the lesson, the students will be motivated for acquisition of the presented material

ii) Presentation of the advancne organiser. The teacher will present the advance organiser (expository or comparative) to the students which contain ideas more distinct and inclusive than the learning content for understanding and grasping the information.

ii) Prompting awareness learner's knowledge. The teacher will realise learner's existing cognitive structure interms of their previous knowledge and experience through questioning related to advance organiser.

Phase II presentation of learning task. (Exceutive

i) The teacher will present the content stage) modulewise through appropriate method like discussion, question answer method as problem solving method with necessary teaching aids

ii) Presentation will be at per with existing cognitive structure as well as the information supplied to them through advance organiser.

iii) As mentioned by goyce and will (2003) "The organisation of the learning material needs to be made explicit to the students so that they have an overall sense of direction and can see the logical order of the material and how the organization relates to the advance organiser.

Phase III Strengthering Cognitive organisation (Follow-up stage

Joyce and will states ‘‘The purpose of phase three is to anchor the new learning mathematics is the student existing cognitive structure, that is to strengthen the students’ cognitive organization’’.

The following activities are performed is this phase

i) Integrative reconciliation

Teacher will relate process of solving solving a math problem taking asimilar problem.

ii) He may ask the students the definition of term, process of solution. He may repeat the concept once again.

Teacher may ask the student the relevance of Advance organizer as the teaching aids with the present topic.

ii) Active reception learning

The teacher may ask students to give additional examples related to the discussion For promoting active reception, the teacher will allow the student to ask question for their reception of facts, while teaching conqruence of a triangle, the teacher may ask ‘Why the triangles having three angles equal are not congruent? He may ask the students to make a teaching aid for understanding the concept.

iii) Eliciting critical approach to content knowledge or the learning material

The teacher may ask the students

a) How the problem as geometrical theorem can be proved is alternative way?

b) Without discussing the concept of probability. The teacher may ask the students to toss a coin 10 times on 20 times and recond the result is respect of head’ on ‘tail’.

c) Contradiction or exception of a rule may be asked to explain like $5 > 3 > 2 > 1$ but $\frac{1}{5} < \frac{1}{3} < \frac{1}{2} < 1$

iv) Classification of confusion or fomce of errors .

In this stage the teacher will apply all the techniques, strategies or methods to clarify the misconception, source of error etc. He may apply error correction test or show some practical demonestration to eradicati mis understanding.

a) $7 \times 7 = 7^2 > 7$

$5 \times 5 = 5^2 > 5$

but $o.7 \times 0.7 = .49 < 0.7$

or $0.5 \times .05 = .25 < 0.5$

b) $\sqrt{25} = \pm 5$ or $\sqrt{49} = \pm 7$

but $\sqrt{-25} \neq \pm 5$ or $\sqrt{-49} \neq \pm 7$

3.2.4.3 Social System

The social system of the advance organiser model is structured and teacher centered. He relates the advance organiser to the learning material. This acts as an ancher between existing knowledge of the learner to the new knowledge. The social system starts with interaction with the students. Students may ask questions, may improve the advance organiser for their understanding and assimilation of new knowledge. So to devise the advance organiser is a difficult activity which will be appropriate to the students as well as to new content. Moreover, deliberation, discussion and relating the A.O with new learning materials require the efficiencies of the teachers.

3.2.4.4 Support system :

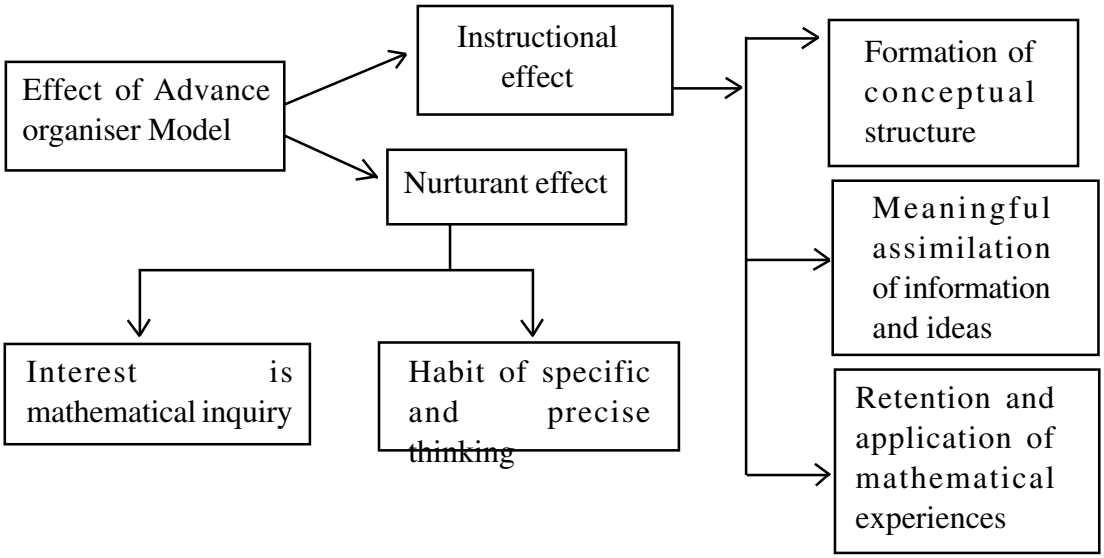
To make the model more effective and meaningful is teaching Mathematics the following support system is essential.

1. A well thought and structured learning material
2. Preparation of a well thought, relavent and appropriate advance organiser is the form of story telling, model, software, activity based performance etc
3. Integrating the advance organiser with learning material and presenting before the students keeping in mind their level of understanding
4. The appropriate competencies, teaching skill.

Professional efficiency and concern for student's preparedness will help the teacher to make the model more effective.

3.2.4.5 Learning output / Effect of Ashubel's Advance organiser Model

The effect is expressed in the following diagram.



3.2.4.6 Advantages

The wide application area of the model as suggested by Joyce and will (2003) can be stated is teaching Mathematics.

i) This model can be used almost all franches of mathematics systematically in normal classroom situation having mixed ability group.

ii) It can help the teachers to understand the existing necessary background knowledge before presenting the advance organiser

iii) It follows the mechanism of direct instruction and reorgaganise the cognitive structure in though advance organiser.

iv) Though the Advance organiser at the initial stage act as deductive way on subsequent discussion it is subject to inductive concept attainment stage and evaluate the student's acquisition of new knowledge

v) It has both instructional and nurturant effects is different domains.

vi) It helps the student to develop critical thinking and interest is mathematical inquiry.

3.2.4.6 Disadvantages

i) Sometimes it is difficult to prepare relevent advance organiser of mathematical tpoics.

ii) It is different from ordinary teaching aids

iii) Prior to apply the model, the mathematics curruculum is to be reorganised and teacher must know the interdisciplenary nature of Mathemaics

3.3 Analytic-synthetic, Problem-solving and Project.

3.3.1 Analytic-Synthetic

Introduction

Most Mathematics orginates from the ideas and concepts associated with physical form, shape and size of objects. Those concepts are present as a systematic abstractructure in logico-deductive form Analysis and synthesis are methods which use reasoning and systematic arguments arts to find out relationship Meaning.

The word 'analytic' means to take a part as to seperate the things that are together or 'breaking up' of the problem so that it gets connected with already known. Analysis is a a process of breaking a things into its smaller parts. It proceeds from unknown to known and conclusion to hypotheus. Thorndike says that all the highest intellectual activities of the mind are analysis. 'To analyse' means to loosen or seperate things that are together.

Procedure :

Ex 1. If $\frac{a}{b}=\frac{c}{d}$, Proved that $\frac{ac-2b^2}{b}=\frac{b^2-2bd}{d}$

The analysis will start from the unknown part of the given statement

$\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$ is to be proved $\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$ will be true $acd-2b^2d=bc^2-2b^2d$ (Cross multiplication)

What is the next possibility of further simplification?

$\therefore$ $acd = bc^2$ ('c' can be cancelled on both sides as common)

$\therefore$ ad = b c will be ture

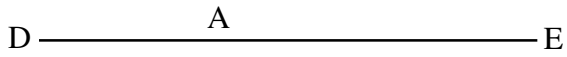
Dividing by b d on both sides we get $\frac{a}{b}=\frac{c}{d}$ $\therefore$ $\frac{a}{b}=\frac{c}{d}$ which is known and true.

$\therefore$ By going back through the chain of argument.

We can say that $\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$ is also true

Example 2. Prove that the sum of the three angles of a triangle is two right angles.

In analysis we start from the conclusion and break it up into simpler arguments for establishing connections with the relationships taken is the hypothesis. For this, we have to find out the missing logical connections and formulate a pattern for the proof.



B C

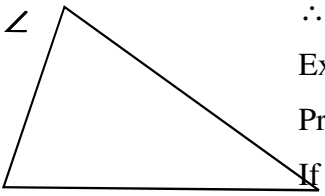
Assuming that the angle sum is $180^{0,}$ a straight angle, then the angle sum of the triangle equals the angle sum on one side of a straight line such as D E.

Again, if D E passes through the vertex A, which is parallel to the base or opposite side. From the properties of the parallel lines, it can be said that the correcponding alternate angles are equal in pair. A line D E is drawn paralled to B C through A.

$\therefore \angle ABC =$ DAB; ACB = EAC

But DAC + BAC + CAE = 180^{o} (alternate angles)

$\therefore$ BAC + ABC + BCA = 180^{o} (straight angles)



Example 3. If a + b + c = 0, Then prove that $a^3 + b^3 + c^3 = 3abc$

Proof $a^3 + b^3 + c^3 = 3abc$

If $a^3 + b^3 + c^3 - 3abc = 0$

$(a+b)^3 - 3ab\ (a + b) + c^3 - 3abc = 0$

or $(a +b)^3 + c^3 - 3ab\ (a+b+c) = 0$

$(a + b)^3 + c^3 = 0$ ($\therefore a + b + c = 0$)

$\therefore (a + b)^3 = - c^3$ or, a + b = – c

or, a+b+c = 0 which is ture.

Advantages of the Analytic method : -

1. It is a logical mehtod and there is no doubts in teaching a content
2. It motivates the learners to discover and improves the level of understanding.
3. If does not depend on cramming. Each step in its procedure has its reason and justification.

4. In this method students are always guided by the questions like "How to simplify the two sides of an equation?" How to simplify the two sides of an equation? How to prove the equality of two sides? "What are the possible ways of resolving a statement into simpler elements" etc
5. The method is applicable to all types of learners and content of Mathematics.

Disadvantages / Drawbacks

1. It is lengthy method
2. It is difficult to acquire efficiency and speed
3. The method may not be applicable to all topics equally well.
4. This method is not suitable to the begininers as during the process many doubts may arise in the minds of the learners which can not be explained property.

Synthetic Method : Meaning and Procedure Meaning.

In synthesis, the small constituents orparts are combined so as to give something new. Here one proceeds from known to unknown. It proceeds with the data available or known and connects the same with the conclusions. In the process we start with hypothesis to conclusion. In practice, synthesis is the complement of analysis.

Procedure :

Ex-1 If $\frac{a}{b}=\frac{c}{d}$, Proved that $\frac{ac-2b^2}{b}=\frac{c^2-2bd}{d}$

Synthetic proof :

$\frac{a}{b}=\frac{c}{d}$ (It is known, and hence the standing point)

Subtracting $\frac{2b}{c}$ on both sides. (But the question why? Why and how should the child remember to subtract $\frac{2b}{c}$ and not any other quantity?)

or $\frac{a}{b}-\frac{2b}{c}=\frac{c}{d}-\frac{2b}{c}$

or $\frac{ac-2b^2}{bc}=\frac{c^2-2bd}{cd}$ or $\frac{ac-2b^2}{b}=\frac{bd^2-c^2}{d}$

Cancelling $\frac{1}{c}$ on both sides)

Hence the identity is proved.

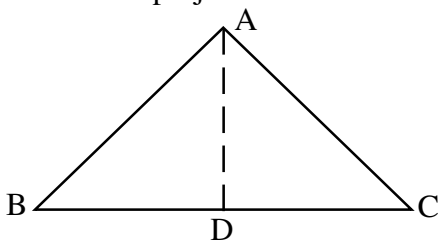
Ex 2. In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing the acute angle minus twice the rectangle containing by one of these sides and the projection upon it.

Proof.

Given ΔABC, acute ∠d at B. BD is the projection of AB on BC.

To prove $Ac^2 = AB^2 + Bc^2$

$-2BC.\ BD$



In ΔACD is a right Angle triangle ADC is not angle $Ac^2 = AD^2 + CD^2$ (But why we have taken this as first step is not clear)

Now $AC^2 = AD^2 + CD^2$ is expanded

We have $Ac^2 = AB^2 - BD^2 + (BC - BD)^2$

$\therefore$ CD = BC – BD

$= AB^2 - BD^2 + BC^2 + BD^2 - 2BC.BD$

$= AB^2 + BC^2 - 2\ BC.\ BD$

(No justification is given for each and every = step)

Ex 3. If a + b + c = 0 Prove that $a^3 + b^3 + c^3 = 3abc$

a + b + c = 0 (It is known) $\therefore$ a + b = – c

$\therefore (a + b)^3 = (-c)^3$ (Cubing both sides)

$\therefore a^3 + b^3 + 3ab\ (a+b) = -c^3$

as $a^3 + b^3 - 3abc = 0$ ($\therefore$ a + b = – c)

$\therefore a^3 + b^3 + c^3 = 3abc$ (proved)

Merits of synthetic method

1) It is a short and elegant method

2) It is a short and elegant method

3) It is logical and psychological method because it starts from known to unknown

4) It glorifies memory

5) It is applicable to most of the topics

6) It suits both the teacher and students

7) It follows the same process as mentioned is the text books

Demerits of Synthetic Method :

1) It leaves many doubts is the mind of the learner and can not be explained property

2) It does not provide full understanding

3) There is little scope of discovery and thinking in the process

4) Memory work and home work are heavy

5) It is not suitable for learner's full understanding

6) It is not suitable for all students and all the topics of Mathematics

Conclusion :

Synthesis is the complement of of analysis and is teaching of Mathematics, the two methods, the two methds should always go together. Analysis leads to synthesis and synthesis makes the process of teaching learning more clear and complete. Analysis helps in understanding and synthesis helps is retaining knowledge. Analysis forms the beginning and synthesis advances the follow-up work.

3.3.1 A comparative study of Analytic and Synthetic Method.

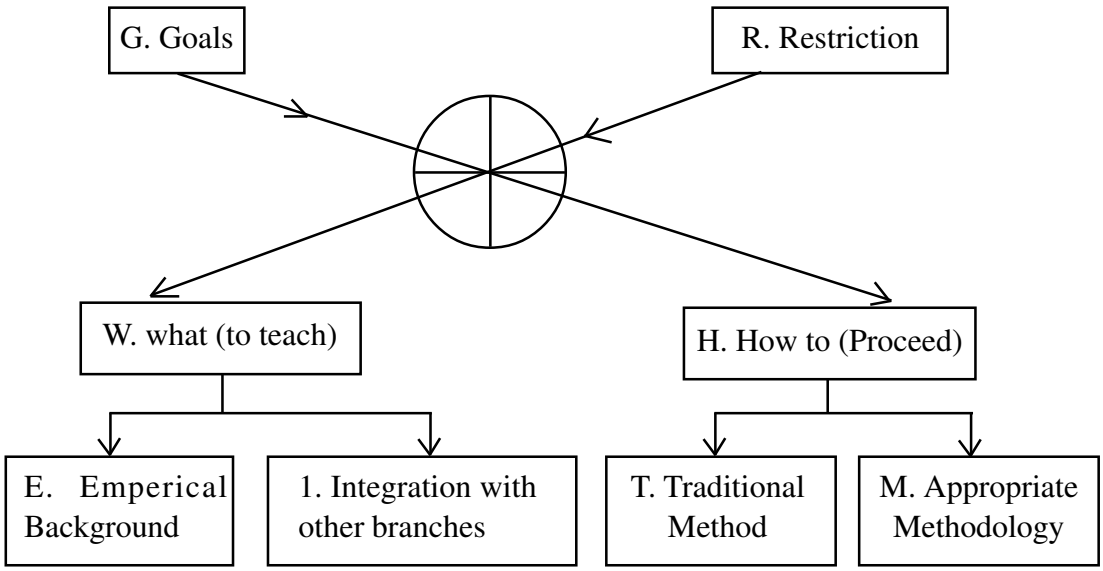
Analytic Method	Synthetic Method
1) Analysis means breaking up into similar elements	1) Synthesis means building up seperate elements as combination of seperate elements to get something new.
2) It proceeds from inknown to the known facts	
3) It starts from the conclusion and goes to the hypothesis	2) It proceeds from known to the unknown facts

Analytic Method	Synthetic Method
4) It is a general method	3) It starts with the hypothesis and ends with the conclusion
5) It is a method of discovery and requires thinking	4) It is a special method
6) It is a proceess of thinking (exploration)	5) It is a process of presentation of the previous by discovered facts.
7) It is a lengthy method which involves trial and error and time consuming	6) It is a product of thought
8) It is a method for the thinkers and discoverer	7) It is concise, elegant, straight forward and does not follow trial and error.
9) If question arises then it answers states factorily	8) It is a method for the crammers. No scope for thinking and discovering
10) There are close contacts between the teacher and taught	9) It does not starts by the doubts and question evolved in the mind of learner.
11) The students can recall and recounstruct easily any step if forgotten	10) There are little scope for such intimate
12) It is psychological	11) It is not easy to recall or reconstruct any forgotten step.
13) In this method we apply inductive reasoning	12) It is logical
14) It is formational	13) We apply deductive reasoning
15) Each step is explained with 'Why' and 'how'.	14) It is informational
16) It is based on heuristic lines	15) 'Why' and 'how' are not explained clearly
17) It develops originality	16) There is no heuristic approach is it
18) It is the forerunner of synthesis	17) If develops memory
19) It builds up a scientific attitude and creativity among students	18) It is the follower of analysis.
	19) Little scopes one there.

3.3.2 Problem-Solving

3.2.2.1 Introduction

Teaching any subject is general and teaching Mathematics is particular, we must consider the following diagram.



Teaching of Mathematics is generally done by traditional discussion method using chalk and talk. Some teachers follow rigid and sterotyped content and methods. It is felt that problem-solving is mathematics may be helpful to both the teachers and taught at the secondary level.

3.2.2.2 Menaning and Definition of problem solving

Problem-solving is an individual or a small group activity, most efficient when done co-operatively with the scope of discussion. It is a method of thinking mathematically, analysing and of learning how to find out the answer of a mathematical problem using known ideas.

The productive work involved is the evaluation of the situation and the strategy worked out to reach one's set goals is collecively known as problems solving.

Wood worth and Marquis (1948) : Problem. solving occuurs is novel ordiffirent situation in which a situation is not obtainable by havitual methods of applying concepts and priciples derived frome past experience is very similar situations.

Skinner (1968) is of the opinion problem solving is a process of overcoming difficulties that appear to interfere with the attainment of a goal. It is a procedure of

making adjustment inspite of interferences. Lester (1975) states ''A problem is a situation in which an individual or group is called upon to perform a task for which there is no ready accessible algorithm which determines completely the method of solution :' So problem solving typically involves performing sets of actions to arrive at a solution to some particular task.

Laster has defined problem solving with a cognitive view point and clearly stresses the mental process rather than any overt behaviour of the problem solver.

The cognitive mathematical behaviour can be classified into three broad categories.

First, the memorisation of facts, definitions, rules and procedures. At this level the child is assumed to reproduce what has been taught.

What has been taught

The second level of mathematical cognitive behaviour is the mental activity of generalising or transfering learning from are context to another. The mental activity of recoginzing and restructuring will for form relationship which will help is finding a solution in 3rd level.

The third level is terned as 'open search' is the crucial stage is problem solving process. So problem solving behaviour may be said to be a deliberate and purposeful act on the part of an individual to realise the set goals or objectives by inventing some novel methods or ..ptematically following some planned step for removal of the interferences and obstacles in the path of the realization of then goals when usual methods like trial and error, habit formation and conditioning tail.

2. Natural and Characteristics of problem-solving behaviour

i) Problem-solving behaviour arises when there is serious interference or obstacles are percived to solve the purposeful goal.

ii) One has to utilise some well-organised steps for the removal of the difficulties and obstacles.

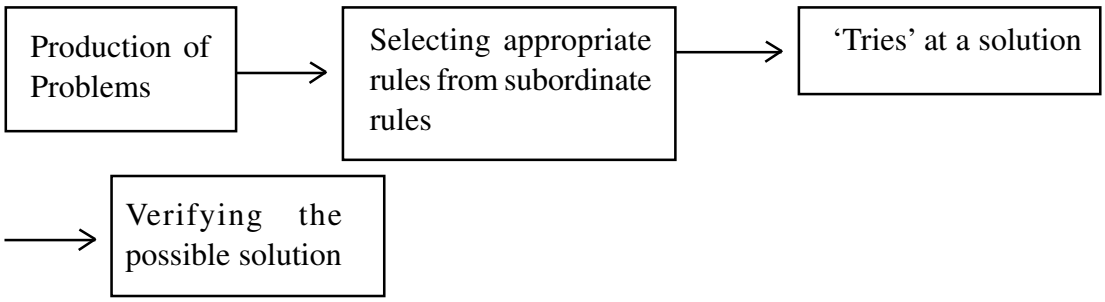
iii) It involves quite deliberate and serious efforts on the part of the problem solves.

iv) It helps an individual to reach his goals and also contributes to the process and development of the society.

The psychological view point of problem solving :–

Gagne (1966) has presented a model where production of a solution depends on the learner already knowing 'subordinal' rules, searching his memory to find relevant

rules, selecting the appropriate rules form among the rellevant remembered rules, combining the rules to form 'tries' at a solution and finally verifying the posible solution.



In mathematic education **George Polya** (1957), in his famous work 'How to solve It', outlined a four-stage model for problem 'solving'.

i) understanding the problem ii) Devising a plan iii) Carrying out the plan iv) Looking back

Research findings as sub sequent years show that students benefited from all poiya's strategies except 'looking back' which was not realy used by the students.

In 1962, 1965 Polya pubblished a much, more detailed two-volume work 'Mathematical Discovery : On understanding, Learning and Teaching problem solving (Vol I & Vol Vol II)

After a careful review of several models, Lester proposed six distinct stages, not necessarily sequential.

i) Problem awareness ii) Problem comprehension iii) Goal analysis iv) Plan development etc.

Merits of Problem - solving Method :

The problem Method aims at presenting the knowledge to be learnt is the form of a problem. It begins with a problematic situation and consists of continuous, meaningful, well-integrated activity. The problems are set to the students is a natural way. Math is a subject of problems Efficiency and ability is solving problems is a guarantee for success is learning this subject.

i) The method stimulates thinking, reasoning and critical judgement is the students.

ii) It develops qualities of initiative and self-dependence is the students

iii) It is a method of learning by self effort

iv) It is a stimulating method. It acts as a great motivating forces.

v) It develops desirable study habits in the students. It engaged the students is the analysis of the problem, reflectiong thinking, systematic data gathering, verification and critical study

vi) It is a method of experience-based learning.

vii) There is possibility of close contact between the teacher and taught

viii) The students get valuable social experiences like patience, co-operation, self-condifidence et.

Limitation a) It is difficult to roganise the contents according to the requirements of this method.

b) It is time consuming and slow. (c) All the topics and subject areas cannot be covered by this method (d) Teacher's burden becomes heavier (e) Mental activity dominates and there will be neglect of physical and practical experiences.

He main objection has been that the Lester's model does not provide specific information about the diagnosis or the development of specific abilities necessary for solving mathematical problems. With this rationate, kulm and Bussmann have formulated a model called the 'Phase-Ability' model for watching specific abilities corresponding to specific problem solving process.

STEPS IN EFFECTIVE PROBLEM-SOLVING BEHAVIOUR

In general the following steps may be followed is the task of problem solving.

i) Problem-awareness – (Semsing the problem)

He must be faced with some obstacle in the path of the realization of his goals consequently he must be consious of the difficulty of problem.

ii) Problem-Understanding – (Interpreting, defining and delimiting the problem)

All the difficulties and obstancles in the path of the goal or solution must he properly naned and identified.

iii) Collection of the relevant information – (Gathering data is a systemating manner)

He is required to collect all the relevant information about the problem by all possible means. He may consult experienced persons, read the available literature, re call his own expereience etc

iv) Formation of hypothesis or hunch for possible solution–

(Organising and evalusing the data)

He may start some cognitive activities to think out the various solutions to the problem.

v) Selection of the correct solution – (Formulating tentative solutions)

a) Identify the conclusion that completely satisfies all the demands of the problem. b) Find out whether the so....is consistent

c) Make a deliberate search for negative aspects

vi) Verification of the concluded solution as hypothesis :–

The solution must be further verified far the solution if similar problems and then to be accepted for future solving.

5. FACTORS AFFECTING PROBLEM - SOLVING :-

There are foru interacting categories of factors (variables)

a) Task Variable (The nature of the problem)

b) The Subject Variables (The child readiness)

c) The process variables (the behaviour of the child)

d) The instructional Variables (to make the child a good problem solves)

Problem-Solving Guide

UNDERSTANDING THE PROBLEM

- Read the problem
- Decide what you are trying to find
- Find the important data

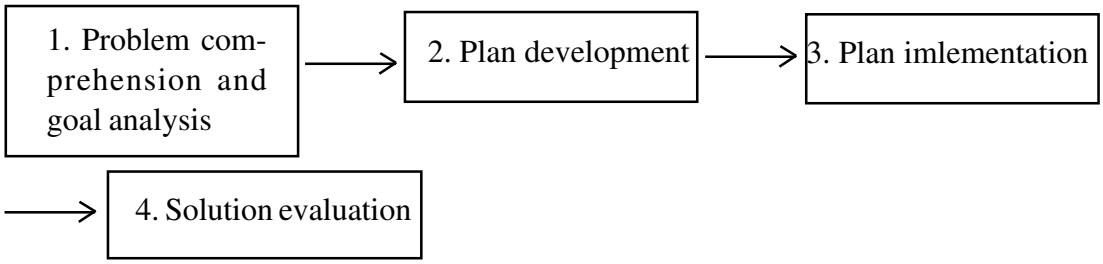
SOLVING THE PROBLEM

- Look for a pattern
- Guess and check
- se logical reasoning
- works back words
- Draw picture
- Make an organized list
- Use object or acto out
- Simplify the problem

ANSWERING THE PROBLEM AND EVALUATING THE ANSWER

- Be sure you used all the important information
- Check your work
- Decide whether the answer makes sense
- Write the answer is a complete sentence

Problem Solving Stages



Guidelines for teachers in helping students solve proms :-

Students may loose interest if they do not understand the questions. So the maxims will be :

1. Make sure students understand the problems.

   For this

   a) Students a should understand the meaning of the terms of the mathematical problem.

   b) Students must take into consideration all the relevent information. If the student think that trapezium is isoceles, then their idea will lead to rhombus.

   c) They should be able to mentions what the problem is seeking to solve.

   d) Stdents should be able to state the problems is their own words.

2. To help students to gather relevant though material (mathematical concepts) for creating the plan.

   a) To assist the students is gathering information is order to analyse the given condition of the problem.

   b) To help the students to obtain is formation by analysing an analogous mathematical problem.

c) To help the students to analyse a problem form a different point of view if is not solved by a particular approach.

$x > y > z > p$ but

3. To provide students an appropriate atmosphere for solving a problem.
4. To encourage the students to verify solutions obtained by inductive process and search for alternative.

   To find the sum of n natural numbers by induction and using the formula of A P series :

5. Help the students to general mathematical problems from real life situation.
6. To use the mathematical puzzles, quiz as interesting activities.

Project

Project method is based on John Dewey's philosophy of pragmatism. According to Dr kilpatrick, ‘‘A project is a unit of whole hearted purposeful activity, carried an preferably, in its natural setting. Stevenson defined it as ‘‘A problematic act carried to the completetion is its natural setting’’. Balland described ‘‘A project is a bit ob real life that has been imported into the school’’ project is a modefied form of ‘‘concecentration of studie’s the main feature of this studies is that some subject is considered as the core or centre of all other school subjects. The principles of correction has been given a practical shape through this method.

Project method is based on the principles of

(i) Learning by doing (iii) Learning by living and (iii) Association, activity and co operative learning. It is based on the fact that the different branches of knowledge are not seperable, though they are studied seperately has convenience.

The project may be classified as

i) Individual project which is to be cried out by the individual and

ii) Social projects or group project which are carried out by a group of pupil.

Step of the project Method :

follow page NV. 306 to 310 (To be added)

Initiation of a Project —

Project propsal

1. Title of the project / Name of the topic
2. Elaboration of the Content
   - Focus of the problem
   - Content of the problem
   - Purpose to be coverd
     (Area of coverage)
3. Objectives

   i)

   ii)

   iii) etc.
4. Equipment / Tools/ Resource required
   a) Questionnaire
   b) Information from different sources (Website, Report, documents)
   c) Syllabus curriculum
   d) Statical package
   e) Compules f) Calculator etc

5) **Stratigies**
   a) Hypothesis b) Population c) Sample d) procedure for conducting project / Execution e) Collection of data f) Analysis of data g) Findings covering objectives h) conclusion i) Submitting report.
6) Reflection and Feedback on the project Based on Expected out.
7) Limitation

   Obstacles faced

   Strategies to be adopted to overcome
8) Conclusion / Epilog.

3.4 Techniques of Teaching Mathematics oral work, written work, Drill work, Brainstorming and computer Assisted Instruction (CAI)

3.4.1 Introduction

In teaching Mathemaitcs, teacher may adopt a particular method or a combination of methods to make the teaching effective effective and worthwhile. The claity mathematics lesson in tends to follow a standard pattern. Such lesson plan is prepared keeping is view the previously taught lessons. Hences, adquate practice or drill of previously learnt mathematical skills are important task. Similerly, for fulfilling the expected out come or gainin mastery of new kills some techniques are used for teaching of Mathematic. Some of them are oral work, Written work, Drill work etc. They are discussed below.

3.4.2 Oral Work

It is the work which is done orally without the help of written work and record. It is the mental work, where in a problem is solved orally or mentally. In mathematical learning much of mathematical work has to be coupleted mentally and many tables have to be learnt by heart : In teaching elementary Mathe it is very essential. Oral work helps each child work at the optimum rate which gives maximum accuracy.

Function of Oral Work –

1. At the introductory stage of teaching, oral questions are asked students to test the necessary back ground knowledge for today's lesson. Oral questions are also asked at the developmental stage module wise and at the recapitutatory stage.
2. It has an appeal for the eye and ear which is liked by students.
3. Some-time are saved by oral work.
4. Oral questions help the teacher to judge the level of understanding of the students is calssroom situation.
5. It arises interest of publish.
6. It is a good mental exercise because it develops atertness, readiness of mind, quick hearing, quick thinking and quick responding.

7. A mathematical idea can be effectively illustrated through a sufficient number of oral examples on questions without much loss of time.
8. It is an effective means of maintaining class discipline
9. It encourages healthy competition among the students.
10. Oral work provides a rapid drill designed to habituate a fundamental process
11. It helps is completing morework is any given period.
12. Spontanity is grasping the data and organization of thought is a limited time, are important aspect of oral question-answer
13. Any individual difficulties can be identified and effectively removed by oral work
14. A teacher can throughout remain active is the class with the help of oral question answer.

Good planning and adequate preparation are necessary for constructing oral work for the students.

3.4.3 Written Work

We know the principle ‘‘Reading makes a full man, conversation a ready man and writing an exact mas Oral work is not enough to understand and measure the higher order of learning is Mathematics when a teacher requires to check work done by each child or to give children practice is independent work, written work becomes a necessary. Hence oral work is to be supplemented by written work. In Mathematics, too much written work is needed. Written work should be considered as an extention of oral work. They are complimentary to each other. The teacher is Mathematics class may follow the sequence

1. Oral fundamentals matter
2. Written fundamentals matter
3. Oral problems presentation
4. Written problem presentation

Both will work is combination

Importance of Written Work :

i) Throughout written work accuracy is computation, legibility of figures and symbols develop

ii) It facilities deep understanding of different mathematical concepts and rules

iii) It improves speed consistency with accuracy, proper algorithm and neatness of work

iv) It fosters thinking and reasoning power

v) It motivates the learners to take active participation

vi) If helps the learner to maintain proper logical and sequentgial arrangement of steps is the mathematical solution

vii) If fosters desirable attitude towards Mathematics

viii) Written works also keep a collective record for assessing student's propgress over a period.

ix) If helps the student for self correction and identification of errors committed by him.

x) It helps to develop good study habit for improving achievement is Mathematics.

3.4.4 Drill work

Drill is one of the most essential methods of learning Mathematics Drill is the process of repetition to make automatic certain process or activities. Drill work is the most efficient means of fixing the impression is mind. One can not expect to achieve speed and accuracy is solving mathematical problems without. Teacher teaches mathematics concepts, rules as application of those. After this he has to evaluate whethers the knowledge given to students has been fixed is their minds and apply those is similar situation. For this drill work and followup action have to be carried out through drill work. Drill work are of three types

The first type of lessons for obtaining mastery of basic subject matter like multiplication fables, addition combinations, percentages, factorization, fraction to decimal, construction is geometry etc. Those subject matters are to be learnt at mastery level with respect to speed and accuracy for future learning.

The second category includes topics as mathematical concepts for the mastery of procedures. In this type of skill the students will be mastered is translate verbal problems is to symbolic form, systematic arrangement of steps, apply correct algorithms, to serutinise and check each step for finding error, sort out data, to label correctly the geometrical diagram, practice short cuts, back calculation etc.

The most important i, e third type of drill consists of lesson which develop the power of thinking, reasoning, generalisation and interest, positive attitude of learner etc. Example of such skills are quizze, puzzle, math, talk etc.

Teachers must be careful is developing few functional or meaningful drill in mathematics classes. These are prior understanding of content knowledge and its appropriate application, the necessary and sufficient condition for mathematical proof etc.

Considerations to be kept is mind for making Drill work more effective.

1. Drill should follow learning as well as understanding of basic principles. It must not must rote memorization without understanding.
2. It should be individualised and follow the principles of reward and punishment
3. Drill should be varied and systematic Mare routine procedures make the learning monotonous and uninteresting
4. If must be sufficient is quantity. For better results the drill work may be divided into parts of appropriate interval.
5. Drill periods should not be planned merely to keep the students 'busy' at work. It must be based upon thought provoking situation.
6. Drill may provide students the diagnostic information and self checking
7. Drill should not be given in the form of punishment
8. Students should be given proper environment for individual and group drill work
9. Mistakes is drill work must be carefully checked and evaluated at an early time.

3.4.5 Brainstorming.

3.4.5.1 Introduction

A.F. Osborn (1963) popularized this strategy through his writing 'Applied Imagination'. It is indicates storming of the brain to generate a number of ideas as quickly as possible without passing any judgement

3.4.5.2 Definition

This is a strategy for the development of higher cognitive abilities like reflective thinking, creative imagination and problem solving capabilities. This stragegy is used with a group of students to explore a good number of ideas for solution of a problem.

3.4.5.3 Procedure for using brainstorming as a teaching strategy

i) At first a small group of students (10-15 students) of a particular class is formed. They will be asked to sit is a group and will be given a focus topic say.

"How will you findout the hight of a tower without climbing it".

2) The teacher will then asks the students to think about the solution of the problem and give their ideas one by one or to list out the solution is a paper. They may be instructed as follows :

   i) The problem is placed before you, think about the possible solution on solutions as you may think suitable.

   ii) This is not an examination. Don't care for the criticism. Write down the possible solutions without any heritation even if they seem to you quite new or unusual.

   iii) Students are also free to alter or modify their ideas and solutions given by them earlier in the session.

   iv) Student members are also free to alter or modify their ideas after discussion with others.

3) In this way, students will be encouraged and inspired for submitting as many as ideas or solution procedures as possible. The group members and the teacher as leader are supposed to collect the different solutions so that :

   a) All the solutions as ideas are to be encouraged and there will be no criticism during the brainstorming session.

   b) Ideas are to be listed without any judgement or passing remaks.

   c) Members are encouraged to supplement this ideas with others.

   d) All the alternatives or solutions are to be reconded properly on the blackboard for free discussion.

4) At the end of the brains storming session, all the solutions and ideas collected from the group under the guidence of the group leader i, e the teacher will be dis usal for the approval of the exparts. Thus a variety of the solution or ideas are evolved.

Advantages of the brainstroming strategy.

1) Students become active and discovers of the solution of the problem or new ideas an concepts.

2) Teachers act as guide.

3) The strategy helps the students to develop higher order cognitive abilities like think, analyse and synthaesize independently.

4) It helps the students to develop their creativity, originality, potentialities and problem solving ability.

5) As it is a group activity, there are scopes for exchange of views, cooperative spirit and development of reasoning power.

6) The student acqures a real understanding and clear notion of the subject as well as mastery of what he has discovered.

Disadvantages and limitations :

Brainforming strategy has the following dis advantages and limitations.

i) It is a time consuming process and the syllabus may not be completed within the period of time

ii) The group members may not be homogeneous with respect to cognitive level required for the discussion of the content.

iii) The output of the brainstorming session may not be as per with teacher's expectations on expected outcomes.

iv) At the concluding session, the result may not be the actual solution of the problem

v) This stragegy can not be applied in large class having 50-60 students

vi) All the members of the group may not be equally interested for find the solution

vii) All the topics of Mathematics may not be covered by this strategy.

3.4.8 Computer Assisted Instruction

Introduction :

With the introduction of New Education Policy is 1986, initiatives have been taken to use computer in the teaching-learning activities. In Mathematics, the instructional work so carried out with the help of computer is generally known as computer - assisted Instruction (CAI)

Definition

Computer-assisted instruction is a method of instruction is which there will be a purposeful interaction between a learner and the teaching material as software of the

computer. It helps the individual learner to achieve the expected instructional objectives designed by the teacher with student's own pace and abilities at his command.

Characteristics :

i) It is an interaction between a student and a computer controlled display materials.

ii) The individual student observes the displayed material and responds to it

iii) The instructional material as software is prepard by the teacher keeping in view the multidimentional need and capbilities of the learners.

iv) It is a auto-individualised instructions which provides instruction to a large number of learners at a time.

v) It provides the opportunity for automatic recoding of the learners performance.

vi) It provides a wide variety of methods and approaches for imparting instruction

vii) The computer-assisted instruction helps the individual learner to achieve the objectives with his own pace and abilities

viii) This type of instruction involves three types of technologies namely, hardware, software and courseware.

Fields of Instruction is Mathemaics teaching through computer assisted instruction.

For providing self individualized instruction to a learner, computer assisted instruction is Mathematics helps in the following fields

1) Discrimination of information related to Mathematics content.

The mainpurpose of this type of C A 1 is to provide essential information as the context for example, a student wants to know the symmetrick Matrix. The definition like symmetric matrix is a square matix is which corresponding eliments above and before the principle diagonal are equal should be mentioned.

As Symmetrik matrix is a square mattriz is which the transpose of that is that it self is to be mentioned.

$$\text{Example } A = \begin{Bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 7 \end{Bmatrix}, B = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, C = \begin{bmatrix} 4 & 5 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 9 \end{bmatrix}$$

Similarly, Super-ordinate and sub ordinate concepts are to be given in the software.

2) Drill and practice programme.

CAI provides different gypes of drill and practice programme covering specific topics.

Example : To draw the conceptual Hierarchy of quadrilaterals like parallelogram, Trapezium, Rhombus, Square etc.

3) Simulation type instruction

Such type of instructional activities, carefully prepared programme are given to students. They practice it and are trained.

4) Problem solving type

Here, the students are provided with programme that will allow them tothink about the ways and means of solving the problem systematicaly like drawing of groups with two equators.

5) Tutorial type instruction

The tutorial programme are prepared, where the students can play effectively through interaction and dialogue. The programme also provides remedial instruction.

6) Practical work related instruction CAI can provide help is supplementing practical work like drawing geometrical figures, calculations, checking the result, consultation with tables etc.

Limitation

1) It is expensive and uneconomical.
2) It is machines oriented is can nevers match the human beings. No simpathy expathy or human touch are available.
2) It is machine oriented it can never match the human beings. No simpathy empathy or human touch are available.
3) It is basically a learners - controlled instructionm, There is little scope to check the learner causing wasting-controalled instruction, there is little scope to check the learner causing wasting of time.
4) Chances of machine failure are there causing a set back is the system.

3.5 Creating Different Situations of Learning Engagement

3.5.1 Group Learning

Students grouped together, who are emotionally intellectually engaged is solving a mathematical problem is called group lurning, such students grouped together working on usually one project. It is an assembly in which each lanner learns autonomously and through the ways of learning of others.

In teaching of Mathematics, often teaching one or two Units of contents, the teacher prepare some problems covering the mathematical concepts of those Units and asses the students to solve the problems in groups. For example, after discussing the topic 'Height and Distance'. The teacher may form two groups comprising of atleast 20 students in each groups have similar emotional and intellectual lurls.

Topics of discussion :

1) Device a plan for finding the height of a towers without climbing on the top of a tower.

2) Solve the following problems related to 'Height and distance' and give the suitable diagrams.

a) Find out the height of a tower situated at a distance of 3 meters whose angle of elivition when viewed from point is 60^{o} and 30^{o} when viewed from the top of a house (height 10 mts)

b) The second group of students will be assigned to frame a question paper consisting of items as per table of specifications or the test blue print.

Unit Trigonometry Class IX

Behaviour/ Concept	Recall	Identify	Change	Find	Verify	Total
Magnitude of am angle	4	4	—	—		8
Unit measure of an angle	2	—	4	—		6
Trigonometric ratio	4	—	2	—	4	10

Variations in trigonometric ratio for $0 \leq \theta \leq 90^{o}$	2	2	—	—	—	4
Trigonometric ratio of specific angles	2	2	—	4	4	12
Angles of elevation and depression	2	—	—	2	—	4
Simple cases of heights and distances	—	—	—	4	2	6
	16	8	6	10	10	50

Features of group learning—

1. Members of learning groups include adult (teacher) as well as students of the class.
2. Documentations of learning outcomes must be maintained to evaluate the progress.
3. Members are to be engaged in cognitive, affective and psychomotor dimentions of learning.
4. Learning will be extended beyond the larning of individuals to create a collective body of knowledge.
5. The make up of the group (i,e size, competencies, interest, motivation etc.) is an important consideration.
6. Group learning focuses on how to learn in groups and understanding of others.

Advantages of Group learning in Mathematics teaching :

(a) Group learning encourages peer dependence, especially by students with learning difficulties.

(b) Working in a group in Mathematics teaching and learning provides learners the opportunities to articulate ideas and understanding.

(c) It helps to negotiate with others to create new understanding as reach consensus.

(d) It proves the scope to uncover assumptions and misconceptions.

(e) It enables the students to discover deeper concepts of Mathematics in the context and improves thinking power.

(f) Group learning engages students with higher level content i,e thought provoking, difficult to understand on the multiple interpretations of a simple or concept.

3.5.2 Individual Learning :

3.5.2.1 Meaning

Individual learning is a strategy of instruction in which content, materials and pace of learning are based upon the abilites and interest of each individual learner. This method presumes that needs and capabilities of individual students are different with respect to mathematics learning and thus be differently addressed.

3.5.2.2 Perspective of individual learning

1. Each student learn differently.
2. All students are talented in Mathematics in different ways.
3. This process will meet the unique mathematical needs of the child.
4. This process will meet the unique educational needs of the child in Mathematics.
5. Careful progress are to be carried out in the process.

Thus Individualised instruction strategy refers to those classroom practices of teaching which recognise the uniqueness of each student learner and for this provide adequeate guidance and other support services suited to them.

3.5.2.3 Characteristics of Individualised Learner :

Mathematics develops the ability to each and every student :

(a) How to analyse a situation

(b) To make proper estimates and approximations

(c) To devise and use formulae, rules of procedure and methods of making comparisons.

(d) To represent designs and spatial relations by drawing.

(e) To be accurate and to be systematic in our work habits.

In this perspective the characteristics as purposes of individual learning are the following :

(i) Individual learning provides opportunities to learn at this own pace, in their own way and to be successful.

(ii) To prove a learning environment that will maximise the potential for student success in Mathematics.

(iii) In this strategy teachers do not stick to the same pattern of teaching rather adopt new ways so that students get multiple options.

(iv) The process enables the teacher to explain a lesson or demonstrate to small group of students at a time.

(v) In individual learning, importance is given to a child as individual not as group or class.

(vi) The method intends to maximise each student's growth and success.

(vii) In this process each student has to note down what he/she usually understands which helps the long term retention of mathematical concepts, laws or proceduse.

(viii) Individual learning is concept focused and principle driven.

3.5.2.4 Principles of Individual Learning

In executing the individual learning strategy, the following principles are adopted :

(a) To make the students clear about the key concept and necessary knowledge for gaining strong understading.

(b) Assesment should occur before, during and following the instructional strategy.

(c) Emphasis and stress will be given more on critical and creative thinking of the students while preparing a lesson plan.

(d) Engaging all learners is essential.

(e) Guiding factors will be (Tomlinson 2001)

(i) Contents (include concepts, generalization of principles, mathematical law, rule and skill)

(ii) Process (Varying learning activities to provide appropriate methods for students to explore the concepts)

(iii) Products : Below grade students show reduced performance.

Above grade students exhibit more advance thinking, understanding and application.

3.5.2.5 Steps of Individual learning

Eight steps are followed to support individual learning.

1. Read the curriculum

   Teachers to be entrusted for the individual learning strategy must be conversent of the goals, subject mattern (content) and evaluation criteria of the whole course.

2. Define the core skills of the course :

   Expectation to be taught. For this the specific content, facts, concepts and skill are to be identified.

3. Mix and match a wide range of learning methods: Any single method can not be the best way for us to learn. Various methods are to be adopted for complete learning.

4. Revise the skills.

   Repeations is necessary for obtaining mastery on a skill.

5. Allow the students to set this own learning goals.

6. Teach, coach, encourage experiments and evaluate progress at every phase.

7. Give continuous feed back through monitoring and evaluation.

8. Give move support and give more challenging tasks individually.

3.5.2.6 Advantages of Individual Learning

(i) Individual Learning is student centric.

Focus is given on the academic needs and learning abilities of every individual student.

(ii) Raises the standard of learning in a big way.

(iii) It meets the needs and interests of diverse learns.

(iv) Individual learning provides the opportunity for students to learn at this own pace.

(v) The strategy recognises students, varying background knowledge readines and preferences in learning.

(vi) The method maximises each student's growth and success.

(vii) Individual learning helps students in provides the opportunity for students to learn at this own pace.

(v) The strategy recognises students, varying background knowledge, readiness and preterences in learning.

(vi) The method maximises each student's growth and success.

(vii) Individual learning helps students is providing opportunities for the uniqueness of each child interms of his learning style, potentials, talent as well as learning deficiencies.

3.5.2.7 Criticism against individual learning (Disadvantages)

- In the individual learning method, students will get insufficient interactions with peers.
- The students feel lonelines and the strategy may create boring.
- It neglects the norms and values of all round development.
- The method requires self discipline which may not be available in mathematics learning.
- The strategy of individual learning focuses on self-interest and personal success and ignores the success and/or failues of others.
- Class size and teaching load are two biggest constraints in the method.
- There is a time constraint in the process.
- Teachers preparedness is difficult for the process.

- In case of drill and practice oriented content, this method is time consuming and not economic.

3.5.3 Small group learning

3.5.3.1 Introduction

Like the group learning, small group learning is an educational approach. Here the class is to be divided into few small groups consisting of 5-7 students in each small group. After teaching a unit of Mathematics in a class, the strategy of small group teaching is to be applied for more altertion to individual learner.

3.5.3.2 Procedure :

The small group work has to be carefully planned and requires a facilitator to enome group progress group function needs to assessed and evaluated. A structured activity is to be to the members of small group to work together. They are individually accountable for their work and the work of the group as a whole. Teachers become learners at times and learners sometime teach.

The meta-analysis demonstrate that various forms of small group learning are effective in promoting greater academic achievement in Mathematics. A fovourable attitude towords mathematics learning will be developed. In this process every member will respect others. Members must draw upon their past experience and knowledge. Group members are to be invested in their own learning. Diversity is to be celebrated and all contributions by members to be given due value.

Advantages

1. Small group learning allows students to develop problem solving ability, interpersonal, presentational and communication skills.
2. It offers tolerance and positive interaction facilities among participent learners.
3. In small group learning students become capable of owenership of new knowledge and skills.
4. They get the opportunity to solve real world problems.
5. Positive attitude towords the content and motivation to learn are also developed.

6. Each member gets the opportunity to contribute is small group learning. They learn to deal with confilict. Each of the group members gets the opportunities for personal feed back about this ideas.

Disadvantages—

1. Some claim, small group work is an avoidance of teaching. It allows the teacher to escape his/her responsibility.
2. In this learning students' abilities are not accurately reflected. Both high and low achiever get same reward.

Suggestion : Teacher will resolve small groups conficts as soon as they arise. They must help the students to reflect on their progress on a regular basis.

Gigsaw Cooperative Learning :

Introduction : Gigsaw puzzle is a mysterious problem that can only be solved by connecting several pieces of information.

1. **Introduction :**

   It is a research based Co-operative learning technique invented and developed in the early 1970s by Elliot Aronson and his students at The Univ. of Texas and The Univ. of California. It is used with great success. In teaching Mathematics, this technique promotes better learning, improves student motivation thus removes mathematics anxiety and increases enjoyment of the learning experience.

2. **Principles :**

- Like Gigsaw puzzle, each part of a lesson as well as each student's participation is essential for the completion and complete understanding of the learnt topic.
- As each student's participation in the class room teaching is essential so students' absentecism is reduced.
- To shift the emphases froms a quite competative environment of Mathematics class to a more Co-operative one.

3. **Steps in Gigsaw techniques of teaching :**

   The are 10 easy stpeps in the technique.

Step one

The students of the class is to be divided into 5 or 6 person Jigsaw groups. The groups should be diverse in terms of gender, race and mathematical ability.

Step two :

One student from each group is to be appointed as the groups leader. Initially, this groups leader should be matured and advanced level student in Mathematics.

Step three :

(i) The day's lesson is to be divided into 5-6 segments. For example in teaching the area and circumference of a circle, the segments may be i) examples of circular figures and objects.

(ii) Definition of circle, circumference, area, radius, Diameter etc.

(iii) To deduce the formula of the area of a circle with geometrical concepts.

(iv) To deduce the formula of the circumference of a circle with geometrical concepts.

(v) Practical Verification of the area and circumference with aids.

(vi) To solve the problems related to the formulee (Simple to Complex).

Step Four

Each student is to be assigned with one segment. Care is to be taken so that students have direct access only to their own segment.

Step Five

Students will be given time to go through or thinkover their segment and become familiar with the content part. Thus is no need to memorise the segment.

Step Six

Temporary ''expert groups' are to be formed having are student from each gigsaw group as leader. Other students will be assigned to the respective segment.

Students of these expert groups will be given time to discuss the main points of their segment and to rehearse the presentation that they will give to their gigsaw group.

Step Seven

All the students are to bring back into their Gigsaw groups.

Step Eight

Each student will be asked to present his or her segment to the group. Other students of the group will be encourage to ask questions for clarification.

Step Nine

The teacher/instructor will float from group to group for observing the process. If any group is having trouble (for example, a member is dominating, disrupting or not presenting up to the mark), then the teacher will take appropriate intervention. It will be best for the group leader to handle the task. The leaders can be trained by whispering an Instruction on how to intervenl or controll by the teachers untid it is solved.

Step Ten

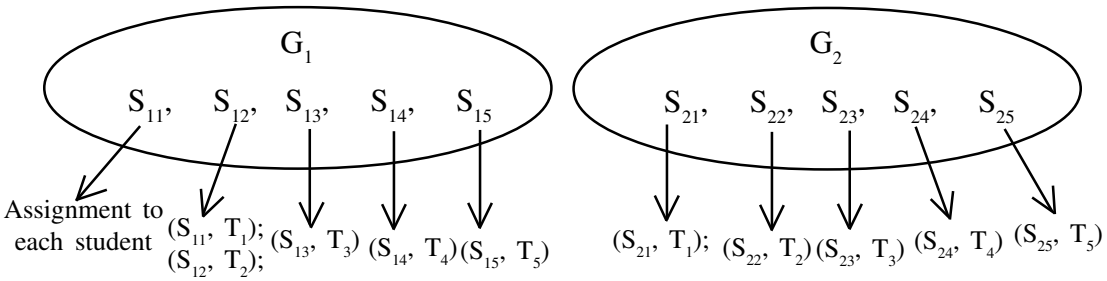
At the end of the session, a quiz sesson on the topic will be organised. The students will particepate and will come to realise quickly that though the session is enjoyable but it is not just a fun but realy essential.

4. Flow chart of the process

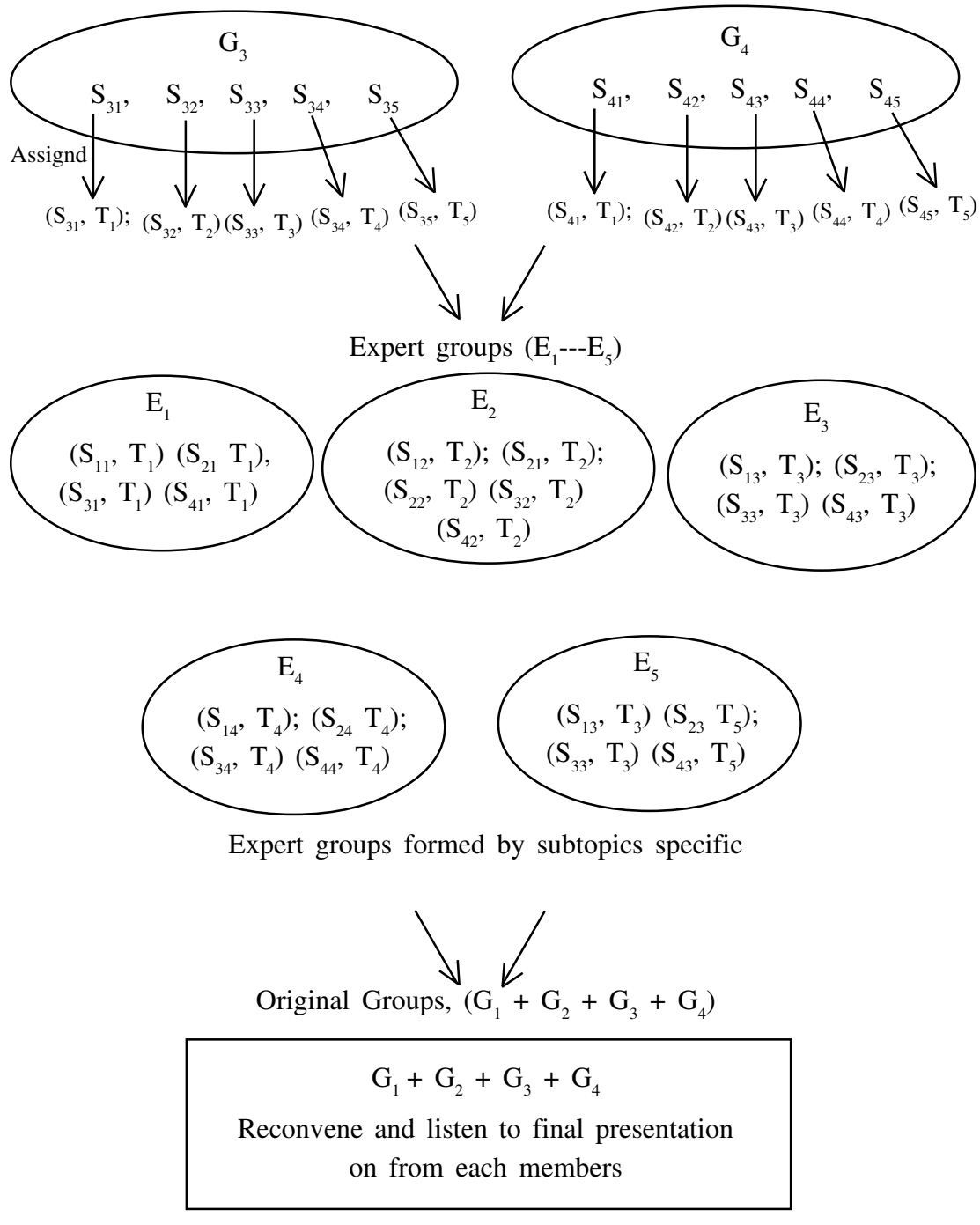
Today's Lesson (Topic T) is divided into few sub topics (T_1, T_2, T_3, T_4 T_5) based on sub concepts of Mathematics.

Students of the class (S) is divided into few groups having 5-6 students in each groups. These groups are called gigsaw groups (G_1, G_2, G_3, G_4 etc.) Expert Group (E) will be five (E_1, E_2, E_3, E_4, E_5) for each sub concepts.

Gigsaw Groups (G_1....G_4)



Similarly for gigsaw groups



5. **Advantages**

1. The techniques works even if only used for a period per day.
2. It can be applied in teaching different subjects along with Mathematics.
3. The teachers feel comfortable by woming with it
4. It is very simple but effective.
5. It is an efficient way for students to become engaged in this learning.
6. Students learn a lot of material quickly.
7. Gigsaw method maximizes intereaction and establishes an atmosphere of Co-operation and respect for others students.
8. This method maximises accountability for students learning.
9. It is verified that students performed significantly better, their interpersonal skill and communication skills are developed.
10. This method helps the students to minimise listing time but expands their thinking and understanding.

6. **Limitation**

(i) It is very difficult to control when a dominant student will talk too much or to control the whole group.

(ii) The weak or Underachiever may not be able to create a good report for their group.

(iii) The talent group on the other hand may be bored working with slower students.

(iv) Teacher may not be satisfied with the expected out comes from the students.

7. **Conclusion :**

The theoretical aspect of gigsaw method has been included in B.Ed curriculum. But to find out the efficacy of the method in teaching Mathematics, the allthough has undertaken a project under centre for pedagogical studies in Mathematics (CPSM).

মানুষের জ্ঞান ও ভাবকে বইয়ের মধ্যে সঞ্চিত করিবার যে একটা প্রচুর সুবিধা আছে, সে কথা কেহই অস্বীকার করিতে পারে না। কিন্তু সেই সুবিধার দ্বারা মনের স্বাভাবিক শক্তিকে একেবারে আচ্ছন্ন করিয়া ফেলিলে বুদ্ধিকে বাবু করিয়া তোলা হয়।

— রবীন্দ্রনাথ ঠাকুর

ভারতের একটা mission আছে, একটা গৌরবময় ভবিষ্যৎ আছে, সেই ভবিষ্যৎ ভারতের উত্তরাধিকারী আমরাই। নূতন ভারতের মুক্তির ইতিহাস আমরাই রচনা করছি এবং করব। এই বিশ্বাস আছে বলেই আমরা সব দুঃখ কষ্ট সহ্য করতে পারি, অন্ধকারময় বর্তমানকে অগ্রাহ্য করতে পারি, বাস্তবের নিষ্ঠুর সত্যগুলি আদর্শের কঠিন আঘাতে ধূলিসাৎ করতে পারি।

— সুভাষচন্দ্র বসু

Any system of education which ignores Indian conditions, requirements, history and sociology is too unscientific to commend itself to any rational support.

— Subhas Chandra Bose

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