QP Code: 22/PT/13/VB

POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-5B : ELEMENTS OF CONTINUUM MECHANICS & SPECIAL THEORY OF RELATIVITY

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Prove that

   $\in_{iks} \in_{mks} = 2\delta_{im}$

   b) By taking differential of the Lorentz transformation, show that the quantity 'ds' transforms to 'ds'' where

   $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ and

   $ds'^2 = c^2dt'^2 - dx'^2 - dy'^2 - dz'^2$.

   c) Show that $\frac{\partial u_i}{\partial x_j}$ is a tensor of second order and $\frac{\partial u_i}{\partial x_i}$ is a scalar.

   d) Two particles are moving towards each other, with each speed 0·9c with respect to the laboratory. What is their relative speed ?

   e) Show that the principal directions of strain at each point of a linearly isotropic elastic body are coincident with the principal directions of stress.

   f) Show that for the velocity field given by $v_1 = ax_3 - bx_2$, $v_2 = bx_1 - cx_3$, $v_3 = cx_2 - ax_1$, the motion is rotational.

g) Determine the Cauchy's stress quadratic at P for a state of stress

$$\left(T_{ij}\right) = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

where a, b and c all are of same sign.

2. a) Calculate the volumetric strain for small strain deformations. 5

   b) Find the equation of continuity in Lagrangian method. 5

3. a) Describe briefly time dilation. 5

   b) Show that if fluid motion is irrotational then the velocity potential must exist. 5

4. a) Find for viscous fluid the rate of circulation Γ round a closed circuit in a flow with velocity $\vec{v} = (v_1, v_2, v_3)$. 5

   b) State and prove the quotient law for tensors. 5

5. What is stream function ? Find stream function for a two-dimensional source given by the velocity potential $\varphi = -\frac{m}{2r}\log r$ where ' r ' is the distance from the source point. 10

6. a) Obtain the Euler's equations of motion for a perfect fluid in Eulerian method. 5

   b) For homogeneous incompressible fluid moving steadily under the action of gravity only, find the Bernoulli's equation along stream-line. 5

7. For a relativistic particle, find out the components of force perpendicular and parallel to the direction of motion. Show that in the relativistic region the acceleration vector is not parallel to the force vector. 10