

## Question Bank For PG Course

Mathematics

Paper-10A(i)

Special Paper: Pure Mathematics

ADVANCED DIFFERENTIAL GEOMETRY: PGMT-XA

### Question 1

Is the function  $f: R \rightarrow R$  defined by  
 $f(x) = x^3$ ,  $\forall x \in R$  a homeomorphism  
from  $R$  to  $R$ ?

### Question 2

What is the tangent vector to the curve  
 $\gamma(t) = (2 + t, t, t^2)$  in  $R^3$  at the point  
(1,1,1)?

### Question 3

If  $(x_1, \dots, x_n)$  is a local coordinate  
system in a neighbourhood  $U$  of  $p$  in the  
manifold  $M$ , then find the standard  
basis of the tangent space  $T_p M$ .

### Question 4

Find the integral curve  $\gamma(t)$  of the  
vector field  $\frac{\partial}{\partial x}$  in  $R^2$ .

### Question 5

For two smooth functions  $f: M \rightarrow$   
 $N$  and  $g: N \rightarrow K$ ,  
which of the followings is true?

1.  $(g \circ f)^* = f^* \circ g^*$
2.  $(g \circ f)^* = g^* \circ f^*$
3.  $(g \circ f)^* = f^* \circ g^* \circ f^*$

### Question 6

If  $(x_1, \dots, x_n)$  is a local coordinate  
system in a neighbourhood  $U$  of  $p$  in the  
manifold  $M$ , then find the standard  
basis of the dual tangent space  $T_p^* M$ .

### Question 7

Compute the following  
 $(2dx + dy) \wedge (dx - dy)$

### Question 8

What is the dimension of the following manifold?

$$\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$$

### Question 9

Which of the following condition is satisfied by linear connection  $\nabla$  on  $M$ ?

1.  $\nabla_{fX} Y = f \nabla_X Y$
2.  $\nabla_{fX} Y = \nabla_X Y$
3.  $\nabla_{fX} Y = f \nabla_Y X$

### Question 10

"Every Riemannian metric admits a unique Riemannian connection"- is this statement true or false?

### Question 11

Which of the following expression is/ are true for Riemannian curvature tensor?

1.  $R(X, Y)Z + R(Y, Z)X + R(Z, X)Y = 0$
2.  $R(X, Y)Z + R(Z, X)Y = 0$
3.  $R(X, Y)Z + R(Y, Z)X + R(Z, X)Y = 1$

### Question 12

What is the scalar curvature of the Euclidean Space?

### Question 13

Which of the following expression is/ are true for Riemannian curvature tensor field ?

1.  $R(X, Y, Z, W) = -R(Y, X, Z, W)$
2.  $R(X, Y, Z, W) = R(Y, X, Z, W)$
3.  $R(X, Y, Z, W) = R(Y, Z, Z, W)$
4.  $R(X, Y, Z, W) = -R(X, X, Z, W)$

### Question 14

If  $\omega$  is a 1-form, then which of the followings is true?

1.  $\omega \wedge \omega = 1$
2.  $\omega \wedge \omega = 0$
3.  $\omega \wedge \omega = -1$

**Question 15**

What is the dimension of the tangent space of the manifold  
 $M = \{(x, y) \in \mathbb{R}^2: y > 0\}$ ?

**Question 16**

Let  $S^1$  is the unit circle in the Euclidean plane  $\mathbb{R}^2$ . Does there exist a diffeomorphism between  $S^1$  and  $\mathbb{R}^2$ ?

**Question 17**

Let  $f$  be a constant function in  $M$  and  $X \in T_p M$  for  $p \in M$ . Find the value of  $X_p f$ .

**Question 18**

Let  $(x_1, \dots, x_n)$  be a local coordinate system in a neighbourhood  $U$  of  $p$  in the manifold  $M$ . Are the following vectors

$$\left(\frac{\partial}{\partial x_1}\right)_p, \dots, \left(\frac{\partial}{\partial x_{n-1}}\right)_p$$

linearly dependent or independent in the tangent space  $T_p M$ ?

**Question 19**

Let  $X, Y, Z$  be vector fields in  $M$  and  $f, g$  be smooth functions in  $M$ . Which of the following is/are true?

- (i)  $[fX, Y] = f[X, Y] - (Yf)X$
- (ii)  $[X, fX] = 0$
- (iii)  $[X, fX] = (Xf)X$

**Question 20**

For two smooth functions  $f: M \rightarrow N$  and  $g: N \rightarrow K$ , which of the following is true?

1.  $(g \circ f)_* = g_* \circ f_*$
2.  $(g \circ f)^* = g^* \circ f^*$
3.  $(g \circ f)^* = f^* \circ g^*$

### Question 21

Let  $X$  be any vector field on unit sphere  $S^2$ . Which of the followings is/are true?

- (i)  $X$  is complete in  $S^2$
- (ii)  $X$  is not complete in  $S^2$
- (iii)  $X$  is not complete in  $S^2$  but complete in  $S^2 - \{p\}$  for some point  $p \in S^2$

### Question 22

Let  $\{\omega_1, \dots, \omega_k\}$  be a set of 1-forms with  $\omega_1 \wedge \dots \wedge \omega_k \neq 0$ . Which of the following is/are true?

- (i)  $\{\omega_1, \dots, \omega_k\}$  is linearly dependent
- (ii)  $\{\omega_1, \dots, \omega_k\}$  is linearly independent
- (iii)  $\{\omega_1, \dots, \omega_l\}$  is linearly dependent for some  $l < k$ .

### Question 23

Let  $\omega$  and  $\lambda$  be  $k$ -form and  $l$ -form respectively and  $f$  be smooth function on  $M$ . Calculate

$$d^2(f\omega \wedge \lambda)$$

### Question 24

Which of the following is/are lie groups?

- (i)  $GL(n, R)$
- (ii)  $\{A \in GL(n, R) : \det(A) = 1\}$
- (iii)  $R^n$

### Question 25

When is a linear connection said to be symmetric?

### Question 26

What is the degree of the Riemannian curvature tensor field of 1st kind?

### Question 27

What is the curvature of 3-dimensional Einstein manifold?

### Question 28

What is the dimension of the tangent space  $T_I(GL(n, R))$ , where  $I$  is the identity matrix?

**Question 29**

If  $\omega$  and  $\mu$  are 1-forms, then which of the following is true?

1.  $\omega \wedge \mu = -\mu \wedge \omega$

2.  $(\omega \wedge \omega) \wedge \mu = 0$

3.  $\mu \wedge \mu = -1$

**Question 30**

Does the following set form a manifold?

$$M = \{(x, 0) : x \in \mathbb{R}\} \cup \{(0, y) : y \in \mathbb{R}\}$$