Question Bank For PG Course

Mathematics

Paper-1A

ABSTRACT ALGEBRA : PGMT-IA

Question 1

What are the generators of the cyclic group Z of integers?

Question 2

For a prime number p, the ring of integers modulo p i.e., Z_p is

Question 3

How many disjoint cycles do yield the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$ as their product?

Question 4

If Z be the set of integers, then number of elements of the quotient group $Z/3Z$ is

Question 5

Let G be a group and $a \in G$, $b \in G$. Then the derived group of G is the subgroup of G generated by elements of some form .Find that form ..

Question 6

Let R^* be the multiplicative group of non- zero real numbers. Find the kernel of the group homomorphism

$f: R^* \to R^*$ given by f

$(x) = |x|, \quad \forall x \in R^*$.

Question 7

What is the characteristic of an integral domain?

Question 8

Let M be a maximal ideal of a non-zero commutative ring R with unity. Then what is R/M ?

Question 9

. Let R be the ring of 2×2 matrices of the form $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$, where $a \in F, \quad b \in F, \quad F$ being a field. Then what may be said about the ideal $M = \{\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} : \quad b \in F\}$?

Question 10

. Let the field K be a finite extension of degree n over a finite field F with m elements. Then how many elements does K have?

Question 11

What may be said about the multiplicative group of all non-zero elements of a finite field?

Question 12

Let G_1 be a finite extension field over a field F and G_2 be a finite extension field over the field F . Then find $[G_2 : F]$.

Question 13

What is the degree of extension of the splitting field $x^3 - 2 \in Q[x]$ over Q, where Q is the field of rational numbers?

Question 14

Which of the following field is a prime field:

(a) field of rational numbers Q,

(b) field of real numbers R;

(c) field of complex numbers C;

(d) none of these.

Question 15

. Let G be a finite extension field over a field F of degree m. Then what is the dimension of the vector space G over the field F ?

Question 16

Find the number of generators of the cyclic group $\mathbb{Z}_n$ with addition modulo n.

Question 17

How many cyclic subgroups does the group $(S,.)$ have where $S = \{1, i, -1, -i\}$?

Question 18

Find the number of disjoint cycles,of length at least two, whose composition yields the permutation $\begin{pmatrix} 1 & 2 & 34 & 5 & 6 \\ 2 & 4 & 31 & 6 & 5 \end{pmatrix}$.

Question 19

Let $G = (\mathbb{Z}_6, +)$ and $H = \{\bar{0}, \bar{3}\}$. Then find $[G:H]$.

Question 20

Let $G = (\mathbb{Z}, +)$ and $H = (2\mathbb{Z}, +)$. Then find $[G:H]$.

Question 21

Let G be a group and $a \in G$ with $o(a) = r$. If m be a positive integer then find $o(a^m)$.

Question 22

Let $\mathbb{R}[x]$ be the additive group of all polynomials with real coefficients and $f'(x)$ be the derivative of $f(x)$, where $f(x) \in \mathbb{R}[x]$. Find the kernel of the homomorphism ϕ: $\mathbb{R}[x] \to \mathbb{R}[x]$ given by $\phi(f) = f'$.

Question 23

Let a group G has only one element a of order $n > 1$. Find the value(s) of n.

Question 24

How many ideals does a field have?

Question 25

Let $\mathbb{Z}[x]$ be the ring of all polynomials with integer coefficients. If $S_1 \& S_2$ be the subsets (of $\mathbb{Z}[x]$) consisting of polynomials with odd and even constant terms respectively. Then which ones of $S_1 \& S_2$ are ideals of $\mathbb{Z}[x]$?

Question 26

What are the prime ideals of the ring of integers $(\mathbb{Z}, +, .)$?

Question 27

What is the characteristic of the ring $\mathbb{Z}$ of integers?

Question 28

What is the characteristic of the ring $\mathbb{Z}_5$ of integers modulo 5?

Question 29

What are the maximal ideals of the ring of integers $(\mathbb{Z}, +, .)$?

Question 30

Find the number of elements of the quotient ring $\mathbb{Z}_3[x]/(x^2 + 1)$.