Question Bank For PG Course

Mathematics

Paper-2B

COMPLEX ANALYSIS : PGMT-IIB

Question 1

Is the function $f(z) = |z|^4$ analytic at $z = 0$?

Question 2

Given that $f(z)$ is analytic. Then find the value of $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|f(z)|^2$.

Question 3

Given that C is the circle $|z - 1| = 3$.

Then find the value of $\oint_C \frac{e^z}{(z+1)^2} dz$.

Question 4

Find the value of

$\oint_C \frac{z}{(9-z^2)(z+i)} dz$ where C is the circle $|z| = 2$.

Question 5

Find the radius of convergence of the power series $\sum_n \frac{(-1)^n}{n}(z - 2i)^n$.

Question 6

Let $P(x)$be a polynomial of real variable x of degree $k \geq 1$. Consider the power series $f(z) = \sum_{n=0}^{\infty} P(n)z^n$where z is a complex variable. Then find the radius of convergence of $f(z)$.

Question 7

Discuss the singularity of the function $f(z) = \log(z^2 + z - 2)$.

Question 8

Given $f(z) = \frac{z^2-2}{z^3+3z+1}$.Discuss role of the point at infinity.

Question 9

Find the residue of the function $f(z) = \frac{e^{iz}}{z^2+1}$ at $z = -i$.

Question 10

Find the residue of the function $f(z) = z^3 sin\frac{1}{z-1}$ at its singular point.

Question 11

Discuss the effect of the transformation $f(z) = \frac{z-i}{z+i}$ maps on various half planes.

Question 12

Find the bilinear transformation which maps points $z = 0, -1, -i$ into $w = i, 0, 1$.

Question 13

Find the fixed points of the transformation $f(z) = \frac{2z+3}{z+4}$.

Question 14

When do the roots of the equation $z^5 + az + 1 = 0$ lie within $|z| = r$?

Question 15

If $f(z) = \frac{(z^2+1)^2}{(z^2+5z+6)^3}$ then find the value of $\int_{|z|=4} \frac{f'(z)}{f(z)} dz$

Question 16

At what point in the complex plane the function $f(z) = |z|^2$ is differentiable?

Question 17

Given that $f(z)$ is analytic. Then what is the value of $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|Re\, f(z)|^2$?

Question 18

Given that C is the circle $|z| = 3$. Then what is the value of $\oint_C \frac{e^{2z}}{(z+1)^4}\, dz$?

Question 19

Find the value of $\oint_C \frac{\log z}{(z-1)^3}\, dz$ where C is the circle $|z-1| = \frac{1}{2}$.

Question 20

What is the radius of convergence of the power series $\sum_n (5 + (-1)^n) z^n$?

Question 21

Consider the power series $f(z) = \sum_{n=0}^{\infty} n log\, n\, z^n$ where z is a complex variable. Then find the radius of convergence of $f(z)$.

Question 22

At what point or points the function $f(z) = \log(z^2 - 5z + 6)$ has singularity?

Question 23

Given $f(z) = \frac{z^2+2z}{z^4+3z+1}$. Find the nature of the point at infinity in $f(z)$.

Question 24

Find the fixed points of the transformation $f(z) = \frac{2z+5}{z+6}$.

Question 25

Find the bilinear transformation which maps points $z=i,-i,\infty$ into $w=i,-i,1$ respectively.

Question 26

Given that f is an entire function which maps real line into real line and imaginary line into imaginary line. Find the nature of f.

Question 27

In what condition the roots of the equation $z^5 + az + 1 = 0$ lie within $|z| = r$?

Question 28

If $f(z) = \frac{(z^2+1)^2}{(z^2-4z+3)^3}$ then what is the value of $\int_{|z|=2} \frac{f'(z)}{f(z)}\, dz$?

Question 29

Find the residue of the function $f(z) = \frac{z}{z^4-1}$ at $z = -i$.

Question 30

Find the residue of the function $f(z) = e^{-1/z}$ at $z = 0$.