Question Bank For PG Course

Mathematics

Paper-3A

ORDINARY DIFFERENTIAL EQUATIONS : PGMT-IIIA

Question 1

The differential equation $(y - px)x = y$ where $p = \frac{dy}{dx} \neq 0$ can be transformed into homogeneous equation by which equation?

Question 2

What is the necessary condition for the existence of a singular solution of the differential equation $(x, y, p) = 0$, $p = \frac{dy}{dx}$?

Question 3

By the substitution $x^2 = u$, $y^2 = v$, $P = \frac{dv}{du}$, the differential equation $(px - y)(x - py) = 2p$ where $p = \frac{dy}{dx}$ transforms to the clairaut's form. Write down the form.

Question 4

Find two linearly independent solutions for the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0$

Question 5

Find a fundamental matrix of the homogeneous linear vector differential equation $\frac{dx}{dt} = \begin{pmatrix} -2 & 3 \\ 3 & -2 \end{pmatrix} x$ where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

Question 6

In the homogeneous linear system $\frac{dx}{dt} = Ax$ where $A = \begin{pmatrix} 3 & 2 \\ -5 & 1 \end{pmatrix}$ and $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, find the characteristic roots of the matrix A.

Question 7

For interval $0 \leq x \leq \pi$, the system of differential equation $\frac{d^2y}{dx^2} + \lambda x = 0$ with boundary conditions $y(0) = 0$ and $y(\pi) = 0$find the eigenfunctions and the eigen values.

Question 8

Write down the condition for which a second oder Sturn-Liouville equation

$$\frac{d}{dx}\left\{p(x)\frac{dy}{dx}\right\} + \{q(x) + \lambda r(x)\}y = 0$$

is said to be regular in the interval $\leq x \leq b$.

Question 9

Find the Green's function for the equation $\frac{d^2u}{dx^2} = f(x)$ subject to the boundary condition $u(0) = u(1) = 0$.

Question 10

If $\delta(x)$ be the Dirac-delta function when $a < x < b$; find the value of

$$\int_a^b f(x)\ \delta(x - x_0)\, dx$$

Question 11

Find the critical point of the system $\frac{dx}{dt} = 5x - 6y + 2$, $\frac{dy}{dt} = 4x - 5y + 3$.

Question 12

Determine the nature of the critical point $(0, 0)$ of the autonomous system

$$\frac{dx}{dt} = -3x + 2y$$

$$\frac{dy}{dt} = -x - 4y$$

Question 13

In Bessel equation
$z^2\frac{d^2y}{dz^2}+z\frac{dy}{dz}+(z^2-\gamma^2)y=0$,Find the indicial equation corresponding to the singularity $z=0$.

Question 14

In Laguerre equation
$z\frac{d^2w}{dz^2}+(1-z)\frac{dw}{dz}+rw=0$, what is z=0 ?

Question 15

The Legendre polynomials $P_m(z)$ and $P_n(z)$ are orthogonal in the interval $-1\le z\le 1$, for positive integers m and n if $\int_{-1}^{1}P_m(z)P_n(z)dz=0$ for $m\neq n$. Now for $m=n$ find the value of
$\int_{-1}^{1}P_m(z)P_n(z)dz$.

Question 16

Find the general and singular solution of $4p2=9x$.

Question 17

Using the Picard's method of successive approximation, find the second approximation of the solution of the equation
$\frac{dy}{dx}=x+y^2,\ y(0)=0.$

Question 18

It is obvious that $y=x$ is a solution of $(x^2+1)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$. Find a linearly independent solution by reducing the order.

Question 19

Consider the three vector functions

$\vec{\emptyset}_1(t) = \begin{pmatrix} e^{2t} \\ 2e^{2t} \\ e^{2t} \end{pmatrix}$, $\vec{\emptyset}_2(t) = \begin{pmatrix} 0 \\ e^{3t} \\ e^{3t} \end{pmatrix}$

, $\vec{\emptyset}_3(t) = \begin{pmatrix} e^{t} \\ e^{t} \\ e^{t} \end{pmatrix}$

What is the value of the Wronskian $(\vec{\emptyset}_1, \vec{\emptyset}_2, \vec{\emptyset}_3)$?

Question 20

Find the general solution of the non homogeneous system

$\frac{dx}{dt} = Ax + F(t)$ where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

, $A = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}$, $F(t) = \begin{pmatrix} e^{5t} \\ 4 \end{pmatrix}$

Question 21

What is the value of

$\sum_{k=1}^{n} coskx - \frac{\sin\left\{(2n+1)\frac{x}{2}\right\}}{2\sin\left(\frac{x}{2}\right)}$?

Question 22

Find the eigen values and eigen functions of the Sturm-Liouville problem

$\frac{d^2u}{dx^2} + \lambda u = 0,\ u'(0) = 0,$

$u'(\pi) = 0,\ 0 \le x \le \pi$ and $\lambda > 0.$

Question 23

If $\delta(x)$ be the Dirac-delta function and a, b are negative and positive real numbers respectively then what is the value of

$\int_a^b f(x)\delta(x)dx$?

Question 24

What is the nature of the critical point (0,0) of the autonomous system

$\frac{dx}{dt} = x + 3y$

$\frac{dy}{dt} = 4x + 5y$?

Question 25

What is the equation of the phase paths of the system

$$\frac{dx}{dt} = 5x - 6y + 2$$

$$\frac{dy}{dt} = 4x - 5y + 3 \ ?$$

Question 26

Find the Hermite polynomials $H_n(z)$ for $n = 2$ and $n = 3$.

Question 27

If $\omega = \sum_{m=0}^{\infty} a_m z^{k+m}$, $a_0 \neq 0$ be a series of Laguerre's equation

$$z\frac{d^2\omega}{dz^2} + (1-z)\frac{d\omega}{dz} + \gamma\omega = 0$$

then what is the indicial equation?

Question 28

What is the Rodrigue's formula for Laguerre polynomial of degree *n* ?

Question 29

If the general solution of the Bessel's equation

$z^2\frac{d^2w}{dz^2} + z\frac{dw}{dz} + (z^2 - r^2)w =$ 0 be $w = AJ_r(z) + BJ_{-r}(z)$ where A, B are arbitrary constants then what is the value of $J_r(z)$?

Question 30

If $w = \sum_{r=0}^{\infty} a_r z^r$, $a_0 \neq 0$ be a series solution of the differential equations

$$(1-z^2)\frac{d^2w}{dz^2} - 2z\frac{dw}{dz} +$$

$n(n+1)w = 0$ then what is the relation between a_{r+2} and a_r for $r = 0, 1, 2, \ldots \ldots \ldots$?