

Question Bank For PG Course

Mathematics

Paper-7B

INTEGRAL EQUATIONS AND GENERALISED FUNCTIONS : PGMT-VIIB

Question 1

Find the solution of the homogeneous
Fredholm integral equation $u(x) = -$
 $\int_0^1 u(t) dt$

Question 2

Let φ satisfy

$$\varphi(x) = f(x) + \int_0^x \sin(x-t)\varphi(t) dt.$$

Then find φ .

Question 3

Consider the integral equation $y(x) =$
 $x^3 + \int_0^x \sin(x-t)y(t) dt$, $0 \leq x \leq \pi$.
Then, find the value of $y(1)$

Question 4

What is the value of λ for which $u(x) =$
 $1 + \lambda x$ is a solution of $x =$
 $\int_0^x e^{x-t} u(t) dt$?

Question 5

Find the resolvent kernel $R(x, t, \lambda)$ for
the Volterra integral equation $\varphi(x) =$
 $x + \lambda \int_a^x \varphi(s) ds$,

Question 6

For the integral equation $y(x) =$
 $1 + x^3 + \int_0^x K(x, t)y(t) dt$, with kernel
 $K(x, t) = 2^{x-t}$, find the iterated kernel
 $K_3(x, t)$

Question 7

Find the characteristic number
homogeneous integral equation $\varphi(x) -$
 $\lambda \int_0^1 (3x-2)t\varphi(t) dt = 0$

Question 8

Let λ_1, λ_2 be the characteristic numbers for the homogeneous integral equation $\varphi(x) - \lambda \int_0^1 (xt + 2x^2)\varphi(t)dt = 0$. Then find λ_1, λ_2 .

Question 9

Which of the following is correct?

1. $u(x) = \int_0^1 e^{xt}u(t)dt$ is a Volterra integral equation
2. $u(x) = \int_0^x xt u(t)dt$ is a Fredholm integral equation
3. $u(x) = \lambda \int_a^b K(x, t)u(t)dt$ is a Fredholm integral equation
4. $u(x) = \lambda \int_0^1 xt u(t)dt$ is a singular integral equation

Question 10

The homogeneous Fredholm integral equation $u(x) =$

$\lambda \int_a^b K(x, t)u(t)dt$ has only trivial solution. Then what could be said about the solution/solutions of the non-homogeneous equation $u(x) = f(x) + \lambda \int_a^b K(x, t)u(t)dt$?

Question 11

Which of the following kernels are degenerate?

1. $K(x, t) = \sin(xt)$
2. $K(x, t) = e^{x+t}$
3. $K(x, t) = e^{xt}$
4. $K(x, t) = \cos(xt)$

Question 12

Find the Volterra integral equation equivalent to the I.V.P. $\frac{dy}{dx} - 2xy = e^{x^2}$, $y(0) = 1$.

Question 13

Find the kernel of the Fredholm Integral equation equivalent to the B.V.P,

$y''(x) + 9y(x) = \cos(x)$,
 $y(0) = y(1) = 0$, is

Question 14

Find the eigenvalues of $y(x) = \lambda \int_0^{2\pi} \sin(x+t) y(t) dt$

Question 15

For the homogeneous Fredholm equation $\varphi(x) = \lambda \int_0^1 e^{x+t} \varphi(t) dt$, a nontrivial solution exists, when λ has the value

Question 16

What is the Volterra integral equation equivalent to the I.V.P, $\varphi''(x) = x\varphi(x)$, $\varphi(0) = 1$, $\varphi'(0) = 0$?

Question 17

Find the Neumann series solution of the integral equation $\phi(x) = 1 + \lambda \int_0^1 xt\phi(t) dt$.

Question 18

Find the Neumann series solution of the integral equation $\phi(x) = 1 + \lambda \int_0^1 e^{x-t} \phi(t) dt$.

Question 19

Find the solution of the integral equation $\phi(x) = 1 + \int_0^x (x-t)\phi(t) dt$, $0 \leq x \leq 1$.

Question 20

Find the solution of the integral equation $\phi(x) = 1 + \int_0^x 2t\phi(t) dt$, $0 \leq x \leq 1$.

Question 21

For the integral equation
 $y(x) = f(x) +$
 $\lambda \int_0^1 K(x, t)y(t)dt$, with
kernel $K(x, t) = xe^t$, what
is the value of iterated
kernel?

Question 22

What is the characteristic
number of the
homogeneous integral
equation $\varphi(x) -$
 $\lambda \int_0^1 (3 - \frac{3x}{2})t\varphi(t)dt = 0$?

Question 23

Let λ_1, λ_2 be the characteristic numbers
for the homogeneous integral equation
 $\varphi(x) - \lambda \int_0^1 (xt + 2x^2)\varphi(t)dt = 0$.
What are the value of λ_1, λ_2 ?

Question 24

Which of the following is
correct?

Question 25

Find the condition that the
integral equation $\varphi(x) =$
 $f(x) +$
 $\lambda \int_0^1 t \min(x, t) \phi(t)dt$,
 $0 \leq x \leq 1$ has unique
solution.

Question 26

Which of the following
kernels are degenerate?

Question 27

Using the resolvent kernel, find the solution of the Fredholm integral equation

$$\varphi(x) + \int_0^1 e^{x-t} \varphi(t) dt = e^x.$$

Question 28

Find the kernel of the Fredholm Integral equation equivalent to the B.V.P,

$$y''(x) + \lambda y(x) = 0, \\ y(0) = y(1) = 0.$$

Question 29

Find the eigen values

$$\text{of } \phi(x) = \lambda \int_0^\pi \cos(x+t) \phi(t) dt.$$

Question 30

For the Fredholm

equation $\varphi(x) =$

$$\lambda \int_{-1}^1 x e^t \varphi(t) dt + x,$$

no solution exists, what is the value of λ ?